Entropic Information Functionals Combined with Neural Networks Explainability to Detect Events in Time Series

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Introduction

Entropic, Information, and Complexity (EIC) measures have shown the capability to describe non-stationary signals, but they have not been sufficiently investigated as feature extraction methods for deep neural network models. In this work, we aim to:

- Use an exhaustive set of EIC measures to evaluate their effectiveness as feature extraction methods for deep learning algorithms.
- Compare EIC-based features with other features extraction methods to assess the efficacy of this approach.
- Explore which EIC functionals are most important for classification using different explainability solutions.

All of these methods are tested with a binary classification algorithm to detect epileptic seizures in time series using the CHB-MIT Scalp EEG Database [Shoeb & Guttag, 2019].

EIC Feature extraction

Given a discrete-time signal composed of N equispaced samples,

$$\{x_i\}_{i=1}^{N} = \{x(t_i) = x_i \in \mathbb{R},\ i = 1, \dots, N\} \tag{1}$$

1. Apply overlapping window division, defined through a sliding temporal window as:

$$W_j(\delta, \Delta) = \{x_i,\ i = 1 + j\delta - \Delta, \dots, j\delta\}\,. \tag{2}$$

2. Compute a PDF via KDE non-parametric inference

$$\hat{p}(x) = \mathcal{K}_h[W_j(\delta, \Delta)] = \frac{1}{h\Delta}\sum_{i=1+j\delta-\Delta}^{1+j\delta} K\left(\frac{x - x_i}{h}\right), \tag{3}$$

3. Apply the entropic, information and complexity functionals

$$a_j = \mathbb{H}\left[\hat{p}_j(x)\right], \tag{4}$$

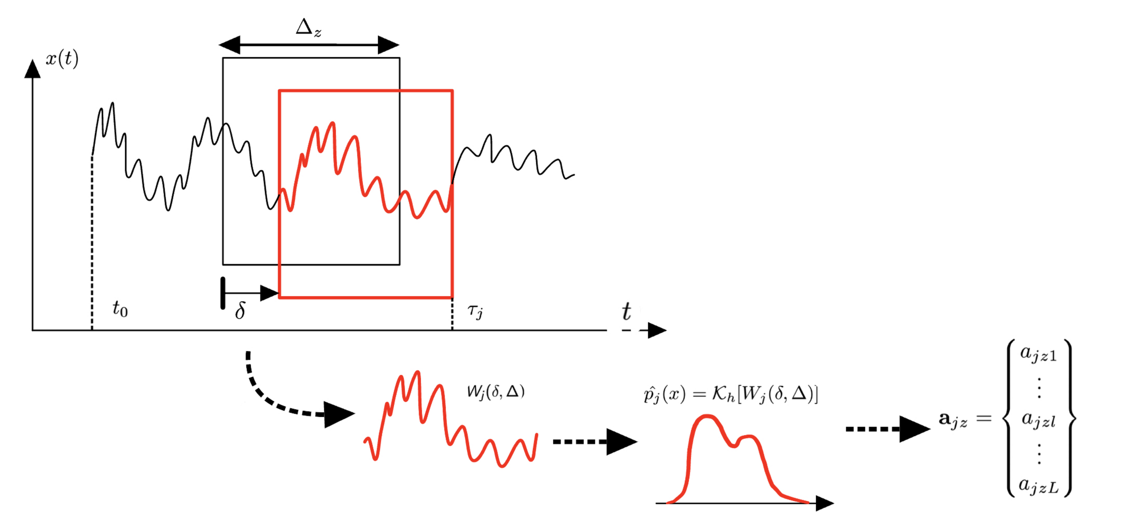

Figure: This figure illustrates the methodology employed.

Repeat the methodology considering the following:

- C channels, in case of multichannel time series.
- L desired continuous entropic, information and complexity measures.

The simultaneous usage of various measures reveals unique characteristics of the underlying time signal dynamics [Guignard et al., 2020; Rosso et al., 2006].

EICs Considered

The following list of EICs were considered, where $p(x)$ represents a PDF ($\int_\Omega p(x) = 1$ and $p(x) \geq 0\ \ \forall x \in \Omega \subseteq \mathbb{R}$).

Entropy/Uncertainty Measures

$$\mathbb{H}[p] = -\int_\Omega p(x) \ln p(x)\, dx, \tag{5}$$

$$\mathbb{H}_q[p] = \frac{1}{1-q}\left(1 - \int_\Omega p(x)^q\, dx\right) \qquad q \in \mathbb{R}, \tag{6}$$

$$\mathbb{H}_\alpha[p] = \frac{1}{1-\alpha} \ln\left(\int_\Omega p(x)^\alpha\, dx\right) \qquad \alpha \in \mathbb{R}, \tag{7}$$

$$\mathbb{V}[p] = \mathbb{E}[x^2] - \mathbb{E}[x]^2. \tag{8}$$

Information Measures

$$\mathbb{I}[p] = \int_\Omega \frac{\left(\frac{d}{dx}p(x)\right)^2}{p(x)}\, dx, \tag{9}$$

$$\mathbb{D}[p] = \int_\Omega (p(x))^2\, dx. \tag{10}$$

Complexity Measures

$$\mathbb{C}_{CR}[p] = \mathbb{I}[p] \times \mathbb{V}[p], \tag{11}$$

$$\mathbb{C}_{LMC}[p] = e^{\mathbb{H}[p]} \times \mathbb{D}[p], \tag{12}$$

$$\mathbb{C}_{FS}[p] = \mathbb{I}[p] \times \frac{1}{2\pi e} e^{2\mathbb{H}[p]}. \tag{13}$$

Other Feature Extraction Solutions

In addition to EIC, the same model will be compared with two other methods:

- **Raw:** Windowed time series.
- **FFT:** Fourier Transform with amplitude and phase values.

Additional information:

- Non-overlapping 2-second time windows.
- Patient-specific models (seizure occurrences are consistent within a patient).
- Training conducted solely on records containing seizure events (140 records).
- Leave-one-record-out strategy used to evaluate generalization performance.
- Extensive hyperparameter grid search over the model that uses *chb01_03* record as test.
- The bandwidth h of the KDE is selected with Silverman's rule.

Results

Average results were obtained over 140 models, utilizing the three types of feature extraction solutions: EIC Measures (**EIM**), raw discrete-time signals (**raw**) and Fourier transform (**FFT**).

	accuracy	recall	specificity	gmean	0-Recall
EIM	92.1%	63.1%	92.6%	71.3%	5
FFT	78.3%	53.0%	78.8%	56.7%	12
raw	88.8%	29.4%	90.2%	37.0%	28

The results using EIM features improve for all performance measures!!!

Feature Importance Analysis

- The brain is a complex system, and epileptic seizures exhibit significant inter-patient variability.
- The following figures illustrate the feature importance for two distinct patients using Kernel SHapley Additive exPlanations (KernelSHAP) [Lundberg & Lee, 21017].
- Note how importance can be differently distributed across channels and EIC features. After aggregating the importance across channels differences emerge, but a degree of coherence prevails.

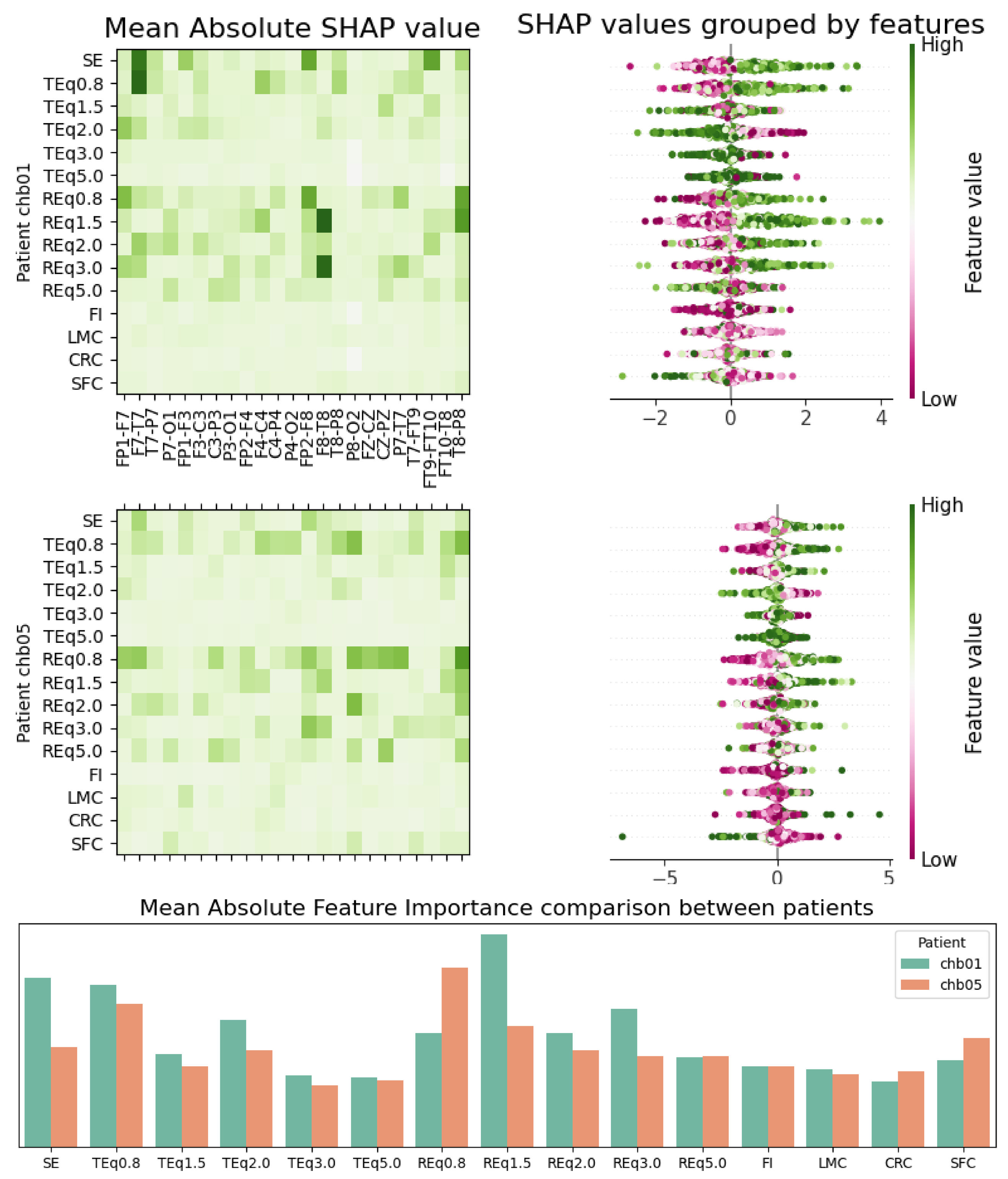

Figure: Feature Importance comparison between patients chb01 and chb05.

- To further understand the role of EIC measures in epileptic seizure detection, we aggregated the importance of all patients to obtain a global value.
- We repeated the process using Permutation Feature Importance (PI) [Breiman, 2001], KernelSHAP and GradientSHAP [Lundberg & Lee, 2017].
- The following graph shows the features most relevant to the problem on average and indicates a general agreement across different methods.

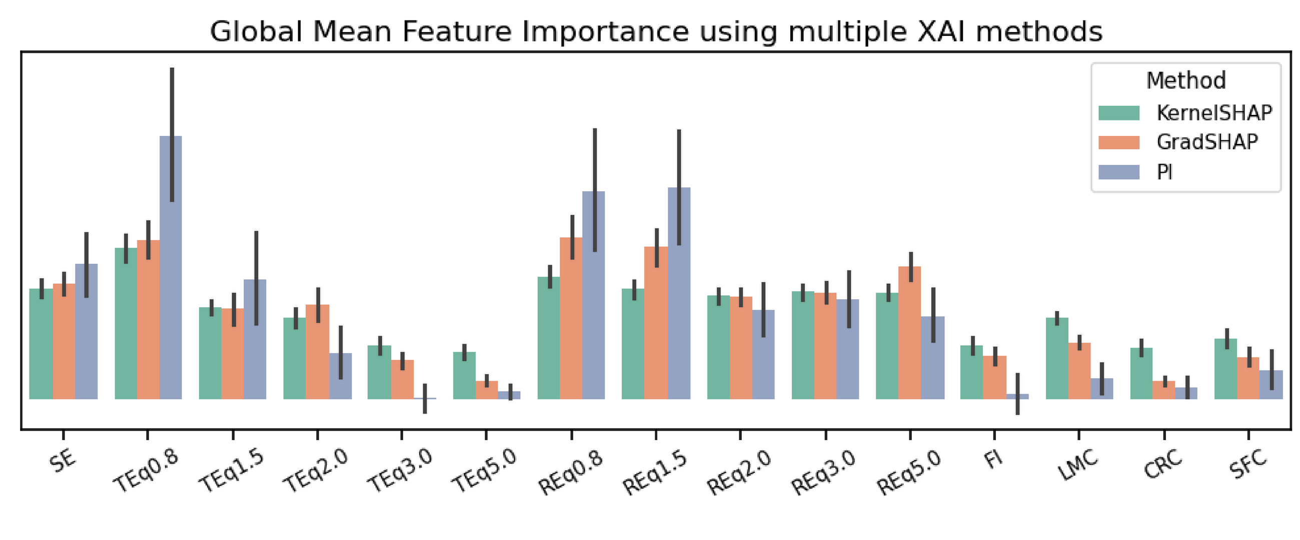

Figure: EIC Measures Global Importance. The values are scaled across methods to ease the comparison.

Conclusions

- EIC-based models shown to be suitable to train deep learning models, confirming previous results of the literature about the capability of those feature extraction solutions.
- EIC-based models outperform other feature extraction solutions.
- There are EIC metrics that are, on average, most relevant to the epileptic seizure detection problem.

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Grant "PID2023-153035NB-I00" funded by:
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Cofinanciado por la Unión Europea
AGENCIA ESTATAL DE INVESTIGACIÓN