

December 3, 2021

Task 4

Consider the cover

$$C = \{i \mid y_i^* = 1\}.$$

This is indeed a cover, since the \mathbf{y}^* solution must satisfy

$$\sum_{i \in C} w_i = \sum_{i \in C} w_i + \sum_{i \notin C} w_i(0) = \sum_{i=1}^n w_i y_i^* > c.$$

Now if $\sum_{i=1}^n (1 - x_i^*) y_i^* < 1$, we can make the following calculation

$$\begin{aligned} & \sum_{i=1}^n (1 - x_i^*) y_i^* < 1 \\ \implies & \sum_{i \in C} (1 - x_i^*) < 1 \\ \implies & |C| - \sum_{i \in C} x_i^* < 1 \\ \implies & - \sum_{i \in C} x_i^* < 1 - |C| \\ \implies & \sum_{i \in C} x_i^* > |C| - 1 \end{aligned}$$

Therefore \mathbf{x}^* violates the inequality corresponding to C . So to construct a cover, one must simply choose the variables i where $y_i^* = 1$.