Task 5

So the strict knapsack problem is represented as

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} \lambda_i y_i \\ & \sum_{i=1}^{n} \omega_i y_i < C \\ & y_i \in \{0,1\} \quad \forall i \in \{1,\ldots,n\} \end{aligned}$$

With profits λ, weights ω and capacity C.

The connection between the above and the cover inequality model is easy to see. So let $\lambda_i = -(1 - x_i^*)$, $\omega_i = -w_i$ and $C = -c$. Then the above model can be written as

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} -(1 - x_i^*) y_i \\ & \sum_{i=1}^{n} -w_i y_i < -c_i \\ & y_i \in \{0,1\} \quad \forall i \in \{1,\ldots,n\} \end{aligned}$$

Using basic mathematical properties, we can see that we recover the cover inequality model.

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} (1 - x_i^*) y_i \\ & \sum_{i=1}^{n} w_i y_i > c_i \\ & y_i \in \{0,1\} \quad \forall i \in \{1,\ldots,n\} \end{aligned}$$

Now a constraint with a strict inequality like $\mathbf{a}^T\mathbf{x} < b$ can be made into one with a weak inequality with the following procedures.

1. Pick some $\varepsilon > 0$ that is small enough such that any number i in the computation is equivalent to $i \pm \varepsilon$.
2. Let $\mathbf{a}^T\mathbf{x} \leq b - \varepsilon$.
3. We must have that $\mathbf{a}^T\mathbf{x} \leq b - \varepsilon$ is equivalent to $\mathbf{a}^T\mathbf{x} < b$, since any TODO MAKE THIS MAKE SENSE.