DMO Project

Antoine Carnec, Leon Karreman,
Richard Orlando Gil Vergara, Marilyn Medrano,
Juan Piccio, Ricardo Sanchez

Task 1

Task 2

Consider the classical knapsack problem program with the following parameters:

$$\begin{aligned} n &= 2 \\ c &= 9 \\ \mathbf{w} &= \begin{bmatrix} 3 & 10 \end{bmatrix}^T \\ \mathbf{p} &= \begin{bmatrix} 2 & 20 \end{bmatrix}^T \end{aligned}$$

. The solution to this model, which we verify with gurobi is

$$\mathbf{x}^* = [0, 0.9].$$

Obviously $C = \{2\}$ is a cover (since $w_2 > c$), and the corresponding cover inequality is $x_2 \leq 0$; which x_2^* violates.

Task 3

Yes this is a valid integer programming. The objective function (5) and constraint (6) are both linear functions, since x_i^* is fixed, it is a constant not a variable.

What is the motivation of this linear program?

Task 4

Consider the cover

$$C = \{i \mid y_i^* = 1\}.$$

This is indeed a cover, since the $\mathbf{y}^*$ solution must satisfy

$$\sum_{i \in C} w_i = \sum_{i \in C} w_i + \sum_{i \notin C} w_i(0) = \sum_{i=1}^{n} w_i y_i^* > c.$$

Now if $\sum_{i=1}^{n}(1-x_i^*)y_i^* < 1$, we can make the following calculation

$$\begin{aligned}
\sum_{i=1}^{n}(1-x_i^*)y_i^* &< 1 \\
\implies \sum_{i\in C}(1-x_i^*) &< 1 \\
\implies |C| - \sum_{i\in C} x_i^* &< 1 \\
\implies -\sum_{i\in C} x_i^* &< 1 - |C| \\
\implies \sum_{i\in C} x_i^* &> |C| - 1
\end{aligned}$$

Therefore $\mathbf{x}^*$ violates the inequality corresponding to C. So to construct a cover that violates the solution $\mathbf{x}^*$, one must simply choose the variables i where $y_i^* = 1$.

Task 5