

Feature Crosses: Encoding Nonlinearity

Estimated Time: 7 minutes

In Figures 1 and 2, imagine the following:

- The blue dots represent sick trees.
- The orange dots represent healthy trees.



Figure 1. Is this a linear problem?

Can you draw a line that neatly separates the sick trees from the healthy trees? Sure. This is a linear problem. The line won't be perfect. A sick tree or two might be on the "healthy" side, but your line will be a good predictor.

Now look at the following figure:



Figure 2. Is this a linear problem?

Can you draw a single straight line that neatly separates the sick trees from the healthy trees? No, you can't. This is a nonlinear problem. Any line you draw will be a poor predictor of tree health.

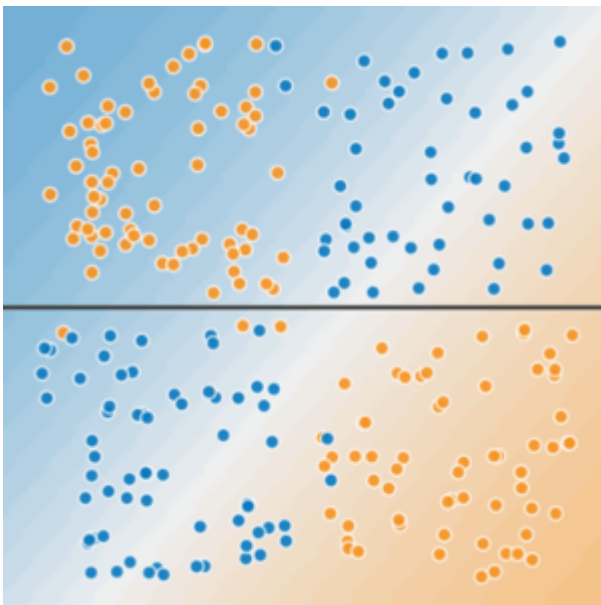


Figure 3. A single line can't separate the two classes.

To solve the nonlinear problem shown in Figure 2, create a feature cross. A **feature cross** is a synthetic feature that encodes nonlinearity in the feature space by multiplying two or more input features together. (The term *cross* comes from [cross product](https://wikipedia.org/wiki/Cross_product) (https://wikipedia.org/wiki/Cross_product).) Let's create a feature cross named x_3 by crossing x_1 and x_2 :

$$x_3 = x_1 x_2$$

We treat this newly minted x_3 feature cross just like any other feature. The linear formula becomes:

$$y = b + w_1 x_1 + w_2 x_2 + w_3 x_3$$

A linear algorithm can learn a weight for w_3 just as it would for w_1 and w_2 . In other words, although w_3 encodes nonlinear information, you don't need to change how the linear model trains to determine the value of w_3 .

Kinds of feature crosses

We can create many different kinds of feature crosses. For example:

- [A X B]: a feature cross formed by multiplying the values of two features.
- [A x B x C x D x E]: a feature cross formed by multiplying the values of five features.
- [A x A]: a feature cross formed by squaring a single feature.

Thanks to [stochastic gradient descent](https://developers.google.com/machine-learning/crash-course/reducing-loss/stochastic-gradient-descent?authuser=0)

(<https://developers.google.com/machine-learning/crash-course/reducing-loss/stochastic-gradient-descent?authuser=0>)

, linear models can be trained efficiently. Consequently, supplementing scaled linear models with feature crosses has traditionally been an efficient way to train on massive-scale data sets.

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[feature crosses](https://developers.google.com/machine-learning/glossary?authuser=0#feature_cross) (https://developers.google.com/machine-learning/glossary?authuser=0#feature_cross)

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