Homework 1

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CS 230 : Discrete Computational Structures

Fall Semester, 2024
HOMEWORK ASSIGNMENT #1
Due Date: Friday, September 6

Suggested Reading: Rosen Sections 1.1 - 1.3; LLM Sections 1.1, 3.1 - 3.4

These are the problems that you need to hand in for grading. Always explain your answers and show your reasoning.

1. **[6 Pts]** Translate the following English sentences into logic. First, define your basic propositions and use logical operations to connect them.
   (a) You will pass the class only if you attend class regularly and do your homework.
   (b) Being a good speaker is sufficient for being elected as president and for being a successful teacher.
   (c) Being a good programmer and having good communication skills are necessary to get a good job.

2. **[6 Pts]** State the converse, inverse and contrapositive of the statement "if it is Sunday, you go to a movie."

3. **[6 Pts]** Determine whether $((p \to q) \land (q \to r)) \leftrightarrow (p \to r)$ is a tautology using truth tables.

4. **[6 Pts]** Prove that $(p \to (q \to r))$ and $(q \to (p \to r))$ are logically equivalent by deduction using a series of logical equivalences studied in class (truth tables will not be allowed).

5. **[10 Pts]** Use logical reasoning to solve the following puzzle:
   A detective has interviewed four witnesses to a crime. From the stories

of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook. The cook and the gardener are either both telling the truth or they are both lying. Either the gardener or the handyman is telling the truth, but not both. If the handyman is lying then so is the cook. Determine which witnesses are lying and which are telling the truth. Define four basic propositions and use these to describe the four statements above. Then use deductive reasoning, not truth tables, to derive the truth of each of those basic propositions.
Hint: Start by assuming that the cook is telling the truth. This leads to a contradiction, so you conclude that the cook is lying. Now, you can go on to conclude the facts about the other three witnesses.

6. **[6 Pts]** Prove that $\{\leftrightarrow, \vee, \text{FALSE}\}$ is functionally complete, i.e.,any propositional formula is equivalent to one whose only connectives are $\leftrightarrow$ and $\vee$, along with the constant FALSE. Prove using a series of logical equivalences. You may assume any logical equivalences we studied in class and the fact that any formula is equivalent to some formula in DNF. *Note: If you make the statement that a particular set of operators is functionally complete, and use this in your proof, then you need to justify your statement.*

7. **[10 Pts]** Consider the proposition $(p \vee q) \rightarrow (\neg q \wedge r)$.
   (a) Construct a truth table for the proposition above.
   (b) We showed in class that any compound proposition is logically equivalent to some proposition in DNF. Use the procedure to construct a DNF proposition that is equivalent to the proposition above, starting with the truth table (show your work).
   (c) A proposition is said to be in CNF (conjunctive normal form) if it is a conjunction (and) of one or more clauses, where each clause is a disjunction (or) of basic propositions or their negations. For example, $(p \vee \neg q) \wedge (\neg p \vee \neg q \vee r) \wedge (q \vee r)$ is in CNF. Any compound proposition is also logically equivalent to some proposition in CNF. Come up with a procedure to construct a CNF proposition that is equivalent to the proposition above, starting with the truth table (show your work).

For more practice, you are encouraged to work on the problems given at the end of Rosen, Sections 1.1 - 1.3.
Question 1:

a.)
p: You will pass the class
q: You attend class regularly
r: You do your homework

$$p \rightarrow (q \wedge r)$$

b.)
t: Being a good speaker
u: Being elected president
v: Being a successful teacher

$$t \rightarrow (u \wedge v)$$

p	q	r	$(p \to q)$	$(q \to r)$	$(p \to r)$	$(p \to q) \wedge (q \to r)$	$((p \to q) \wedge (q \to r)) \leftrightarrow (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	F
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Table 1: Question 3: Not a tautology

c.)
x: Being a good programmer
y: Having good communication skills
z: Get a job

$$z \to (x \wedge y)$$

Question 2:

p: It is Sunday
q: You go to a movie

$$Converse(q \to p)$$

$$Inverse : (\neg p \to \neg q$$

$$Contrapositive : (\neg q \to \neg p)$$

Question 3: TABLE 1

Question 4:

Breaking up the logic of

$$(p \to (q \to r))$$

we have that

$$(p \to (q \to r)) = \neg p \vee (\neg q \vee r)$$

And breaking up the logic of

$$(q \to (p \to r)) = \neg q \vee (\neg p \vee r)$$

Since the logic or is commutative

$$\neg p \vee \neg q \vee r = \neg q \vee \neg p \vee r$$

we can say that the 2 propositions are logically equivalent using disjunction.

Question 5:

b: Butler is telling the truth
c: Cook is telling the truth
g: Gardener is telling the truth
h: Handyman is telling the truth
Propositions as logic:
$(Statement 1) b \rightarrow c$
$(Statement 2) c \leftrightarrow g$
$(Statement 3) g \oplus h$
$(Statement 4) \neg h \rightarrow \neg c$
Starting and assuming that the cook is telling the truth so (2) cook and the gardener are telling the truth, but (3) is only one telling the truth, so the handyman is lying since only it is and XOR. But (4) completely contradicts our statement of the cook telling the truth so the cook is lying.

Reevaluating (2) we now assume the cook is lying, meaning the gardener is lying also, so if(3) is an XOR we have the handyman telling the truth, and with (4) there would not be a contradiction to the cook, and (1) is the butler is lying if the cook is lying.
CONCLUSION:
Butler is lying
Cook is lying
Gardener is lying
Handyman is telling the truth

Question 6:

First I am proving that NOT, AND, and OR can be constructed using the connectives.
Constructing NOT:
1.) $p \leftrightarrow FALSE$ this behaves like the proposition $\neg p$

$$\neg p = p \leftrightarrow FALSE$$

Constructing AND:
1.) Using DeMorgan's Law we have that $p \wedge q = \neg(\neg p \vee \neg q)$
2.) $\neg p \vee \neg q = (p \leftrightarrow FALSE) \vee (q \leftrightarrow FALSE)$
3.) $p \wedge q = ((p \leftrightarrow FALSE) \vee (q \leftrightarrow FALSE)) \leftrightarrow FALSE$
Constructing OR: 1.) It has already been given to us as a proposition in the set
CONCLUSION: $\leftrightarrow, \vee, FALSE$ **is functionally complete based on the proven series of logical equivalences above.**

Question 7:

a.)TABLE 2
b.) There are 3 total propostions that can be constructed for a DNF.
CONCLUSION:$(p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$
c.) Using the truth table we have that the CNF is:
CONCLUSION: $(p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r)$

p	q	r	$\neg q$	$(p \vee q)$	$(\neg q \wedge r)$	$(p \vee q) \rightarrow (\neg q \wedge r)$
T	T	T	F	T	F	F
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	T	F	F	T	F	F
F	F	T	T	F	T	T
F	F	F	T	F	F	T

Table 2: Truth Table for Question 7 Part a