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CS 230 : Discrete Computational Structures

Fall Semester, 2024
HOMEWORK ASSIGNMENT #2
Due Date: Friday, Sept 13

Suggested Reading: Lecture Notes Logic 4 - 7; Rosen Sections 1.4 - 1.6; LLM Chapter 3

For the problems below, explain your answers and show your reasoning. Even if you get a problem wrong, you are more likely to get partial credit if you show your work.

1. [**10 Pts**] For the following problems, let $R(x)$, $C(x)$ and $D(x, y)$ be the statements "x has read the notes", "x is in your class" and "x has discussed problems with y", respectively. *Assume that $D(x, x)$ evaluates to false.*

   Let the domain be all people at ISU. Translate from English to logic.

   (a) Someone in your class has read the notes but has not discussed problems with anyone in your class.

   (b) There are two students in your class who have, between them, discussed problems with everyone in the class.

2. [**6 Pts**] Define propositions (eg., let f be 'I fail the test') and prove the following using the appropriate rules of inference:

   (1) If I fail the test, then I am sad and angry.
   (2) If I am sad, then I will not party tonight.
   (3) I party tonight.

   Prove that I did not fail the test.

3. [**12 Pts**] Define predicates and prove the following using the appropriate rules of inference:

   (a) [**6 Pts**] John, a student in class, plays tennis. Everyone who plays tennis is watching the US Open. Therefore, someone in class is watching the US Open.
   Universe: all ISU students

   (b) [**6 Pts**] All bears are good swimmers. If you can catch fish, you will not go hungry. If you can't catch fish, you are not a good swimmer. Therefore, no bears go hungry.
   Universe: all animals

4. [**6 Pts**] Prove, by giving a counterexample, that the propositions $\exists x(P(x) \wedge Q(x))$ and $\exists x P(x) \wedge \exists x Q(x)$ are not logically equivalent. In other words, show that there exist predicates $P(x)$ and $Q(x)$ for which the statements are not logically equivalent. *Hint*: Let your domain be the set of integers. Define $P(x)$ to be 'x is odd' and $Q(x)$ to be 'x is even'.

5. **[16 Pts]** Some of the kids at Moore Memorial Park in Ames are lined up for ice cream. There are at least two kids in line. Translate the assertions below into predicate formulas using $\exists$ and $\forall$ and logical operators $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$. You may use the equality predicate and the predicate $ahead(x, y)$, meaning "x is somewhere ahead of y in line". You can use the predicates you define first to define subsequent predicates. Assume that the universe is the set of all kids at a park.

   Give a short explanation of your construction.

   (a) **[4 Pts]** $behind(x, y)$ meaning 'x is somewhere behind y in line'

   (b) **[4 Pts]** $inline(x)$ meaning 'x is in line for ice cream'
   Hint: x is ahead or behind someone else, since there are at least two.

   (c) **[4 Pts]** $first(x)$ meaning 'x is first in line'

   (d) **[4 Pts]** $second(x)$ meaning 'x is second in line'

   (e) **Extra Credit [4 Pts]** $right\text{-}behind(x, y)$ meaning 'x is right behind y in line'

For more practice, work on the problems from Rosen Sections 1.4 - 1.6 and LLM Chapter 3.

1.)

Stating the propositions:
R(x) = x has read the notes
C(x) = x is in your class
D(x,y) = x has discussed the problems with y
a.)
$\exists x(R(x) \wedge C(x) \wedge \forall(y)(C(y) \rightarrow \neg D(x, y)))$
b.)
$\exists x \exists y(C(x) \wedge C(y) \wedge \forall z(C(z) \rightarrow D(x, z) \vee D(y, z))))$
The variable z represents everyone in the class

2.)

The propositions are:
f = I fail the test
s = I am sad
a = I am angry
p = I party tonight
Premise 1 = $f \rightarrow (s \wedge a)$
Premise 2 = $s \rightarrow \neg p$
Premise 3 = p

CONCLUSION: The steps and reason are: Premise 3 is p. By using MOdus Tollens if $s \rightarrow \neg p$, but p is true s must be false, Premise 1 states $f \rightarrow (s \wedge a)$, but we know s is false, so by Modus Tollens if we are proving that f is false $s \wedge a$

–	*Steps*	*Reason*
1	$s \to \neg p$	Premise 2
2	$\neg s$	Modus Tollens(1)(2)
3	$\neg f$	Modus Tollens(1)(2)

Table 1: Question 2

–	*Steps*	*Reason*
1	P(John) $\to$ W(John)	Universal Instantiation
2	W(John)	Modus Ponens(1)(2)
3	C(John) $\wedge$ W(John)	Conjunction

Table 2: Question 3 Part a

must be false.
3.)

a.)

C(x) = x student in class
P(x) = x plays tennis
W(x) = x is watching the US Open

Premise 1 = C(John) $\wedge$ P(John)
Premise 2 = $\forall x$ (P(x) $\to$ W(x))
We are trying to prove: $\exists x$ **(C(x)** $\wedge$ **W(x))**

CONCLUSION: In the end we can use Existential Generalization to say $\exists x$**(C(x)** $\wedge$ **W(x)) or There exist someone in the class and is watching the US Open.**

b.)

B(x) = x is a bear
S(x) = x is a good swimmer
C(x) = x can catch fish
H(x) = x is hungry
Premise 1 = $\forall$(B(x) $\to$ S(x))
Premise 2 = $\forall$(C(x) $\to \neg$ H(x))
Premise 3 = $\forall$($\neg$C(x) $\to \neg$ (S(x)
We are trying to prove $\forall$**(B(x)** $\to \neg$ **H(x))**

CONCLUSION: In the end were are able to prove that $\forall$**(B(x)** $\to \neg$ **H(x)) by using Universal Generalization**

–	*Steps*	*Reason*
1	B(x) $\rightarrow$ S(x)	Universal Instantiation
2	B(x) $\rightarrow \neg$ H(x)	Universal Instantiation
3	$\neg$ C(x) $\rightarrow \neg$ S(x)	Universal Instantiation
4	S(x) $\rightarrow$ C(x)	Contrapositive
5	B(x) $\rightarrow \neg$ H(x)	Hypothetical Syllogism (1)(4)

Table 3: Question 3 Part b

4.)

Proposition 1 states that $\exists x(P(x) \wedge Q(x))$ in English: There exist an x where both P(x) and Q(x) are true.
Proposition 2 states that $\exists x P(x) \wedge \exists x Q(x)$ in English: There exist some P(x) and there exist some Q(x)
P(x) = x is odd
Q(x) = x is even

In proposition 1 it says that there exists an integer that is odd and even, but that is FALSE because an integer cannot be odd and even at the same time.

In proposition 2 it states that there exists an integer that is odd and there exists an integer that is even, which is TRUE.

CONCLUSION: Since (1) is false and (2) is true we have proved that the 2 propositions are not logically equivalent.//

5.)

a.)
behind(x,y) $\leftrightarrow$ ahead(y,x)
CONCLUSION: x behind y is the inverse of y is head of x

b.)
inline(x) $\leftrightarrow$ $\exists$y(ahead(x,y) $\leftrightarrow$ ahead(y,x)
CONCLUSION: x is inline if there exists some y when x is either ahead or behind y.
c.)
first(x) $\leftrightarrow$ $\forall$x(ahead(x,y))
CONCLUSION: x is ahead for all y
d.)
second(x) $\leftrightarrow$ $\exists$y(first(y) $\wedge$ ahead(y,x) $\wedge\forall$z(ahead(z,x) $\vee$ ahead(y,z)))
CONCLUSION: There exist some y who is first and ahead of x, and x is always ahead of zm which is some other kid in line.

e.)

right behind(x,y) $\leftrightarrow \neg\exists$z(ahead(y,z) $\wedge$ ahead(z,x))

CONCLUSION: y is always ahead of x and behind y and there does not exist a kid,z, that is in between y and x in the line