# N-Body Simulations

A Thesis
Presented to
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# Acknowledgements

I want to thank a few people.

# Preface

This is an example of a thesis setup to use the reed thesis document class.

# List of Abbreviations

You can always change the way your abbreviations are formatted. Play around with it yourself, use tables, or come to CUS if you'd like to change the way it looks. You can also completely remove this chapter if you have no need for a list of abbreviations. Here is an example of what this could look like:

| | |
|---|---|
| **ABC** | American Broadcasting Company |
| **CBS** | Columbia Broadcasting System |
| **CDC** | Center for Disease Control |
| **CIA** | Central Intelligence Agency |
| **CLBR** | Center for Life Beyond Reed |
| **CUS** | Computer User Services |
| **FBI** | Federal Bureau of Investigation |
| **NBC** | National Broadcasting Corporation |

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# Abstract

The preface pretty much says it all.

# Dedication

You can have a dedication here if you wish.

# Introduction

The Milky Way is an unextraordinary spiral galaxy. Its age, structure, and size are all similar to other nearby galaxies, and its dynamics match those seen in other spiral galaxies. Even

## Disks

The Milky Way has two main disks (seen as the infamous spiral arms.) The *thin disk* is the more visible of the two, composed mainly of main-sequence stars and clouds of gas and dust. Its vertical density scale height—the distance over which its density decreases by a certain factor—is about 350 pc—very thin compared to its radius of about 25 kpc. This thinness comes from its young age, since the stars that compose it are less likely to have had their orbits perturbed—especially in the chaotic period about 9 Gyr ago. The thin disk accounts for about 97% of the galaxy's (normal) mass and holds nearly all galactic dynamism and stellar formation.

The other disk, referred to as the *dark disk*, is far older and less dynamic. Composed of stars formed 10 Gyr to 12 Gyr ago, it is very faint and hard to detect—all the bright stars burned out long ago, and the only ones left are low-magnitude red dwarfs and K-class stars. These stars' orbits also tend to be less regular, since they've had time to be perturbed and pushed into new orbits, especially during the chaotic initial organization of the Milky Way 10 Gyr ago—resulting in a scale height of about 1 kpc. Because its stars are so steady-burning and the complete lack of gas and dust, the thick disk is very stable and exhibits none of the dynamism seen in the thin disk. It accounts for only about 3% of the regular matter in the galaxy, with a total mass of about $1 \times 10^{10}\ M_\odot$.

## Metallicity and Age

The easiest way to determine which disk a star is in is to look at its metallicity (the amount of metal in the star.) This can be measured by looking at the strength of various emission spectra, since elements like iron have very distinct emission lines. These heavy elements are only formed when large stars reach the end of their life, and so their concentration has steadily increased over time as more large stars form and die. Old stars dating back to the formation of the galaxy (like those of the thick disk) tend to have very "clean" emission spectra, with very little other than hydrogen and helium, while younger stars have strong magnesium and iron lines.

Metallicity isn't a perfect way to measure the age of a star. Metal concentrations vary widely across both space and time, and two stars forming at the same time may have very different metallicities. However, when looking at a large and statistically representative sample of stars, a high metallicity indicates a younger age (Carroll & Ostlie, 2006, pp. 885).

## The Stellar Halo

The disks extend to about 25 kpc from the galactic center, and contain practically all the mass within that radius. Past that point, however, stars are distributed far more chaotically. The thin plane disappears, and stellar orbits become more spherically distributed. The composition of individual stars in the halo is similar to those in the thick disk—old, dim, and unchanging. However, the halo is also home to many globular clusters, groups of tens or hundreds of thousands of stars that act like small galaxies in their own right. These clusters continue to create new stars, and most of the light coming from the halo is from globular clusters.

Measurements of stellar velocities have long predicted that the mass of the halo dominates the mass of the galaxy—about 95% of galactic mass must be in the halo for the observed velocity curves to hold. Since the halo was known not to be made up of gas and dust (otherwise its extinctive properties would be easy to measure), astronomers long though that the halo contained vast numbers of dense, dark bodies, such as lone planets, dim stars, black holes, and neutron stars—referred to as MACHOs, or Massive Astrophysical Compact Halo Object. Gravitational lensing observations disproved this theory, however, when they capped the mass percentage of MACHOs at about 15% of the mass of the galaxy. The remainder of the mass was some strange material, spherically distributed throughout the universe, that interacts with nothing but gravity.

# Dark Matter

As that strange material was further studied (as much as it is possible to study something so elusive), more of its properties were discovered. This *dark matter* seems to be made up of WIMPS (Weakly Interacting Massive Particles), which only interact via gravity and the weak force. Being collisionless (since it does not interact with the electromagnetic force), it does not coalesce and form clouds and stars like regular matter, which is how it has maintained its spherical distribution for so long.

This dark matter has a density of

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{a}\right)\left(1 + \frac{r}{a}\right)^2} \tag{1}$$

where $\rho_0$ is the maximum density and $a$ is proportional to the size of the galaxy. This function is referred to as the NFW profile (named after its creators, Navarro, Frank, and White), and is highly accurate for all observed spiral galaxies. For the Milky Way, the simplified equation

$$\rho(r) = \frac{\rho_0}{1 + \left(\frac{r}{1}\right)^2} \tag{2}$$

also returns satisfactory results. However, both these equations appear to suffer from a major problem. If we use them to calculate the total mass of the dark matter in a galaxy, integrating from 0 to $\infty$, it appears that a galaxy has an infinite mass, as follows:

$$\int_0^\infty \rho(r) 4\pi r^2 \, \mathrm{d}r = \infty \tag{3}$$

Since we know this is not true, there must be a cutoff point where the law no longer holds. As it turns out, in local groups like our own, the dark matter halos are so large that they border one another—providing a natural cutoff point and a solution to out problem (Sparke & Gallagher, 2000, pp. 196).

# Coordinate Systems

To identify and keep track of objects in the sky, we need to create a coordinate system. A number of natural possibilities spring to mind. The three most useful are detailed below.

## Equatorial Coordinates

An equatorial coordinate has two components—a *right ascension*, and a *declination*. To find this coordinate, find the location on the Earth's surface where the object of interest is directly overhead, in the very middle of the sky. The right ascension is the angle between the nearest point on the equator and the vernal equinox (which is defined to be the point where the equator crosses the ecliptic). The declination is then the angle between the current point and that nearest equatorial point. This coordinate system is very ancient, dating back millennia. Figure 1 shows the process of finding such a coordinate.

## Galactic Coordinates

This coordinate system is similar to the equatorial system, but the declination is measured from the galactic plane instead of the equatorial plane. The right ascension is then defined to be the angle between the projection of the body of interest on the galactic plane and the vector between the Sun and galactic center. Figure 2 shows this process in more detail. Although harder to calculate from the surface of the earth, this system is more natural when studying bodies traveling close to the Sun. The standard notation and conversions between the two systems are given below:

| Measurement | Equatorial Notation | Galactic Notation |
|---|---|---|
| Right Ascension | $\delta$ | $b$ |
| Declination | $\alpha$ | $\ell$ |

$$\sin b = \sin \delta_{NGP} \sin \delta + \cos \delta_{NGP} \cos \delta \cos(\alpha - \alpha_{NGP}) \tag{4}$$

$$\sin \delta = \sin \delta_{NGP} \sin b + \cos \delta_{NGP} \cos b \cos \ell_{NCP} - \ell \tag{5}$$

Where $\delta_{NGP} = 27°7'41.7''$ and $\ell_{NCP} = 123°55'55.2''$, as determined by the tilt of the earth and its orientation relative to the galactic plane. These equations can also be inverted to find $\ell$ and $\alpha$ (Carroll & Ostlie, 2006, pp. 900).

## Cylindrical Coordinates

The two coordinate systems discussed earlier are well-suited for positional observations from the Earth at a given point in time, but perform poorly over long timeframes. As the sun travels around its orbit, the coordinates of an object change even if it has not moved at all—not an ideal behavior from a reference system. The cylindrical coordinate system, with a reference point at the center of the universe solves these concerns. Unlike the others, it is a three-component coordinate system: $R$ is the radial distance along the plane, increasing outwards; $\theta$ is the angular position, and increases in the direction of rotation; and $z$ is the height above (or below) the plane, increasing towards the north, as shown in figure 3. These coordinates also produce a natural (and commonly used) velocity coordinate system, as described below (Carroll & Ostlie, 2006):

$$\Pi \equiv \frac{dR}{dt} \qquad \Theta \equiv R\frac{d\theta}{dt} \qquad Z \equiv \frac{dz}{dt} \tag{6}$$

Note that, because the galaxy rotates clockwise when viewed from the north pole, this is a left-handed coordinate system rather than a more-standard right-hand system. Fortunately, we do not need to take any cross-products, so this does not cause any problems.

## Local Standard of Rest

Now that we have a definition of the cylindrical velocity coordinates, we can deine the Local Standard of Rest (LSR), an important concept in astrophysics. The LSR at a given moment is defined to be the velocity of a body in the sun's position, in a perfectly circular and on-plane orbit—which in practice means the $\Theta$-component of the sun's velocity, with $\Pi$ and $Z$ set to zero.

The velocity of a nearby star relative to the LSR is known as its *peculiar velocity*, and approximates the velocity of that star relative to the sun. Its coordinates are typically designated $(u, v, w)$, where

$$u = \Pi - \Pi_{LSR} \tag{7}$$

$$v = \Theta - \Theta_{LSR} \tag{8}$$

$$w = Z - Z_{LSR} \tag{9}$$

The average peculiar velocity for stars in the solar neighborhood is approximately zero, since the universe is mostly axisymmetric. However, individual peculiar velocities vary widely, with young main-sequence stars like the sun having low velocities and old, metal-poor red dwarfs having higher velocities. As discussed earlier, this is due to the additional orbital perturbations experienced by old stars, especially during the chaotic period of formation 9 Gyr ago.

Image coming soon?

Figure 1: This figures shows the process of finding an equatorial coordinate.

Image coming soon?

Figure 2: This figures shows the process of finding a galactic coordinate.

Image coming soon?

Figure 3: This figures shows the process of finding a cylindrical coordinate.

# Conclusion

Here's a conclusion, demonstrating the use of all that manual incrementing and table of contents adding that has to happen if you use the starred form of the chapter command. The deal is, the chapter command in LaTeX does a lot of things: it increments the chapter counter, it resets the section counter to zero, it puts the name of the chapter into the table of contents and the running headers, and probably some other stuff.

So, if you remove all that stuff because you don't like it to say "Chapter 4: Conclusion", then you have to manually add all the things LaTeX would normally do for you. Maybe someday we'll write a new chapter macro that doesn't add "Chapter X" to the beginning of every chapter title.

## 4.1 More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't be* indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

# Appendix A

# The First Appendix

# Appendix B

# The Second Appendix, for Fun

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