Computational Fluid Dynamics Modeling of Cathodic Arc Jet in Stationary Atmospheric Pressure Gas

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In this paper a computational fluid dynamic (CFD) model of the cathodic arc jet is developed. The solver is based on an incompressible solver modified with an external body force representing the ion-neutral momentum transfer in the region of plasma-gas interaction. The flow properties – pressure and velocity are qualitatively analyzed in different periods of simulation time. The CFD jet model of the CAJ predicts the observed experimental phenomena – *i.e.* forming of an initial plasma-gas boundary and the steady conditions in the CAJ region after $t = 100\ \mu s$. The compatibility of the results with the Taylor-Sedov blast model pressure front position and velocity is validated. This simplified empirical model can be used to model the CAJ even without modeling the plasma explicitly, greatly reducing the computation requirements.

I. Introduction

Active manipulation of flows is an important area of aeronautical research [1]. One prominent example is the use of flow manipulation to reduce drag by delaying and/or eliminating separation zones [2]. The literature provides several examples of flow-control devices including: synthetic jets, bleed/suction and plasma actuators. In contrary to other actuators, plasma actuators can operate on the flow by three different mechanisms [3]: momentum, shock and chemical effects. Momentum effects induce near surface flow velocities. Shock effects induce very high local gas pressure and temperature gradients in the background air. Chemical effects introduce new species, such as ions, electrons, and excited particles into the flow field. The most studied plasma actuator configuration is the dielectric barrier discharge (DBD) which is capable of inducing $\sim 10\ m/s$ flows [3–6], too low to be effective at high subsonic regimes.

In a recent study [7], cathodic arcs operating at atmospheric pressure environment were shown to produce fast jets of gas. This so called cathodic arc jet (CAJ) induces a local flow field velocities of $\geq 100\ m/s$ (see Fig. 1). The jet direction was also shown to be controlled by the application of an external magnetic field. A physical model of the CAJ was developed in a recent paper [8], relating the cathode and background gas properties to the observed CAJ. Further results suggest the possibility of using the CAJ as a flow-control device for external subsonic flows [9].

In this paper a computational fluid dynamic (CFD) model is developed. A comparison with available measurement data of the spatial distribution of flow parameters and a Taylor-Sedov blast wave model [10, 11] is made. The goal is to examine if the main mechanism of CAJ formation is ion-gas momentum transfer collisions.

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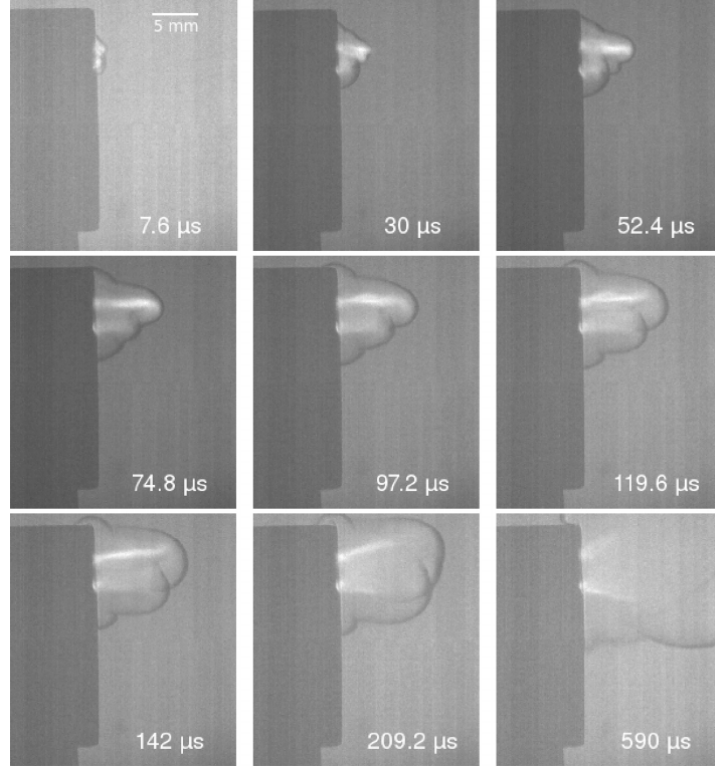[†]Assistant Professor. E-mail: kronhaus@technion.ac.il

Figure 1: Schlieren images of the cathodic arc jet in air. The luminous plasma shows white. The time is counted from the moment of ignition. Reproduced from [7].

II.   Physical Model

The schlieren time sequence images shown in Fig. 1 allow us to postulate the following process for the formation of the CAJ:

1. Plasma formation in a bounded region

2. CAJ plume gas-dynamic expansion towards the background gas

In the first event a small yet energized volume of plasma particles is generated. A pressure boundary is formed with a radius size determined by the cathode parameters, discharge current and the background gas pressure. Subsequently, [12], the second event begins with a gas-dynamic expansion of the initial energized volume. This expansion is followed by the formation of a jet caused by a momentum and energy transfer of the plasma particles to the gas. It is seen that after the initial boundary expansion certain *steady-state* conditions are met in the jet plume region.

The evolution of the initial pressure wave due to the formation of plasma can be modeled using a Taylor-Sedov blast wave model [10, 11]:

$$r^5 t^{-2} \;\;=\;\; \text{constant} \tag{1}$$

The pressure wave initially starts with a sonic velocity of $\approx$ 500 m/s at the plasma boundary [8], and quickly decays to $\approx$ 50 m/s [7,9] at $\approx$ 10 mm as visualized in Fig. 1. Following the pressure wave expansion, the CAJ is formed at the plasma-gas boundary.

Close to the plasma-gas boundary, the CAJ is assumed to follow the initial pressure wave expansion velocity. The steady-state results, summarized in Table 1 and the physical model developed in [8] suggest that the CAJ is formed mainly due to momentum transfer from ions to the gas.

The force exerted on the plasma particles can be written as (similar to the derivation in [12]):

$$F_i = \frac{M_i V_i f}{Ze} I_d \tag{2}$$

Where M_i is the ion mass, V_i is the ion velocity, f is the ion current fraction, Z is the charge state, I_d is the current magnitude and e is the electron charge. Assuming an equilibrium between the plasma and gas induced forces, Eq. (2) can be used to find the area A for which the equilibrium is achieved:

$$F_g = F_i \tag{3}$$
$$pA = \frac{M_i V_i f}{Ze} I_d \tag{4}$$
$$A = \frac{M_i V_i f}{Ze} \frac{1}{p} I_d \tag{5}$$

Where F_g and F_i are the background gas pressure force and the ion momentum transfer force, respectively. Assuming hemi-spherical distribution of the gas pressure force on the plasma, i.e $A = \pi r^2$, the following expression is obtained for the radius, r:

$$r = \mathcal{C}\sqrt{I_d} \tag{6}$$

Where $\mathcal{C} = \sqrt{\frac{1}{\pi} \frac{M_i V_i f}{Ze} \frac{1}{p}}$ is a constant defined by the cathode and experimental parameters. This allows us to relate the boundary radius r to the discharge current, I_d, thus resulting in a simplified relation between the two, for a given set of cathode parameters. Results for the given parameters in the experiments [7, 8] are shown in Table 1. From Table 1 it is expected that the plasma-gas boundary is less than 0.5 mm.

In steady state conditions at the plasma-gas boundary, with constant current, I_d, we can express the exerted force on the gas with respect to velocity as:

$$F_i = F_g \tag{7}$$
$$= \dot{m} V_g \tag{8}$$

Substituting the mass flux $\dot{m}$ for $\rho V_g A_o$ – where A_o represents a circular cross section through which the jet flows and ρ is the fluid density, yields:

$$F_g = \rho V_g^2 A_o \tag{9}$$

As the boundary radius is defined by the equivalence between the pressure force and the ion force (Eq. (3)), Eq. (9) thus yields:

Table 1: Calculated CAJ boundary radius

Parameter	Case 1	Case 2
I_d [A]	230	40
p [Pa]	101×10^3	101×10^3
T [K]	850	850
γ	1.365	1.365
M	0.856	0.856
M_i [kg]	1.055×10^{-25}	1.055×10^{-25}
V_i [m/s]	12.5×10^3	12.5×10^3
f	0.08	0.08
Z	1.8	1.8
M	0.856	0.856
r [mm]	0.51	0.21

$$pA \;=\; \rho V_g^2 A_o \tag{10}$$

The CAJ initial temperature can be estimated by substituting the areas A and A_o with the initial hemispherical:

$$p \cdot \pi r^2 \;=\; \rho V_g^2 \cdot \pi r^2 \tag{11}$$

$$\frac{p}{\rho} \;=\; V_g^2 \tag{12}$$

Using the state equation for an ideal gas, $p = \rho RT$, we get:

$$T \;=\; \frac{V_g^2}{R} \tag{13}$$

Where T is the temperature and R is the specific gas constant. Eq. (13) relates the gas temperature to the jet velocity. Substituting $R = 287 \ J \cdot kg^{-1} K^{-1}$ and the expansion velocity measured in [8], $V_g = 500 \ m/s$ results in a temperature values of around 850 K. From Eq. (13) we can express the Mach number at the boundary, as:

$$M \;=\; \frac{V_g}{a} \tag{14}$$

$$\approx \frac{\sqrt{RT}}{\sqrt{\gamma RT}} \tag{15}$$

$$= \sqrt{\frac{1}{\gamma}} \tag{16}$$

Where a is the speed of sound. For most cases, the heat capacity ratio $\gamma \geq 1$. Values of Mach and Temperature are summarized in Table 1 for dry air. The Mach number is therefore smaller than 1.

III.   Numerical Model

The CFD numerical approach is derived from the assumption that the CAJ is predominantly influenced by gas dynamics effects after being accelerated by momentum transfer collisions between ions and gas inside the plasma-gas boundary region, and therefore can be simulated by the application of an acceleration field which causes the gas to reach the velocities measured in [7]. This corresponds to adding a body force acceleration to the Navier-Stokes momentum equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} \;\; = \;\; -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 \boldsymbol{u} + \boldsymbol{F_b} \qquad (17)$$

Where $\boldsymbol{u}$ is the flow field velocity and $\boldsymbol{F_b}$ represents the acceleration field.

The numerical solution is carried out using a modified `OpenFOAM` [13] open source toolbox solver, based on the `icoFoam` solver. The `icoFoam` is a pressure based solver for transient incompressible flows which was modified to include an external force field acting on the flow. An incompressible solver was used due to its simplicity and fast calculation time. It is therefore important to note that compressibility effects are not taken into account. The use of an incompressible solver can be justified by the fact that M < 1 at boundary, as described in Section II.

The numerical domain consists of 200 × 300 grid points representing a 15 mm × 30 mm dimensioned physical domain. Grid dimensions were obtained by conducting grid size sensitivity runs, analyzing the CAJ profile. The grid density is adjusted at the wall region to yield a maximal cell height of 0.1 mm.

We simulate the plasma-gas interaction (ions momentum transfer collisions) by assuming a constant acceleration field just upstream the CAJ expansion plume region. The field properties are calculated to correspond with the plasma-gas boundary given in Eq. (6). For the observed parameters represent the experiment [7] of Fig. 1, the field was placed at $y < 0.5$ mm at a width of $r = 0.5$ mm, centered around $x = 0$ mm. These values correspond to a discharge current of 230 A (Case 1 in Table 1). Following the measurement data in [8], the acceleration field is adjusted to yield a velocity of 500 m/s. It is assumed that the discharge current and gas pressure affect only the initial plasma-gas boundary location. The background gas is stationary, with $p_a = 101 \cdot 10^3 \; Pa$ and $T_a = 300$ K.

IV.   Results & Discussion

The simulation was executed using adaptive time steps with a Courant number $\mathcal{C} \leq 0.1$, for a total simulation time of $t = 500 \; \mu s$. The solution region of interest includes the CAJ gas dynamic expansion region ($y > 1$mm). The velocity and pressure field spatial distributions, obtained at $t = 500 \; \mu s$, are shown in Fig. 2a and Fig. 2b, respectively. The acceleration field results in the formation of a time evolving axisymmetric jet that reaches a quasi-steady-state condition at $t = 500 \; \mu s$. The CFD pressure front position and velocity results are shown in Fig. 3a and Fig. 3b, respectively. Good correspondence is obtained between the Taylor-Sedov blast model and the pressure/velocity front velocities obtained in the simulation. The measurement results taken from [7], show good agreement with the simulation results, as well.

The pressure front velocity and the CAJ velocity profile evolution are shown in Fig. 4. We observe that the velocity profile developed in the CAJ follows the same trend as

the pressure's front velocity. Moreover, the CAJ steady state velocity is higher in the entire region. The two velocities converge as the CAJ reaches > 18 mm.

The CAJ axial velocity gradient with respect to axial distance from the cathode is shown in Fig. 5, for several times. For each instantaneous moment, we can observe a sharp decrease of the gradient magnitude at a certain distance from the cathode spot. We note that the gradient values converge to a distance of about 6 mm (dotted vertical line). It is interesting to note the correspondence of this location with the termination of the bright spot in the CAJ observed in Fig. 1.
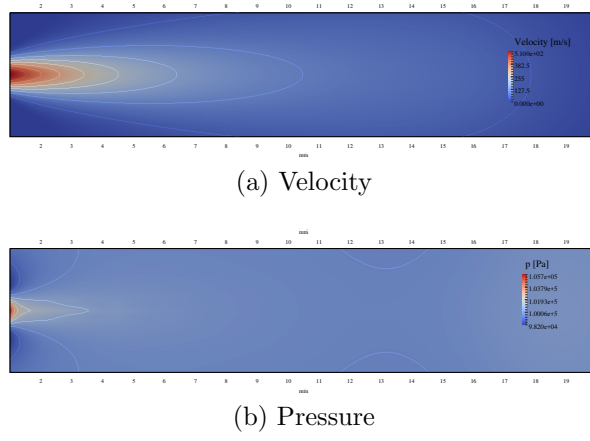

(a) Velocity

(b) Pressure

Figure 2: (a) Velocity field, $t = 500$ μs; (b) Pressure field, $t = 500$ μs

(a) Position
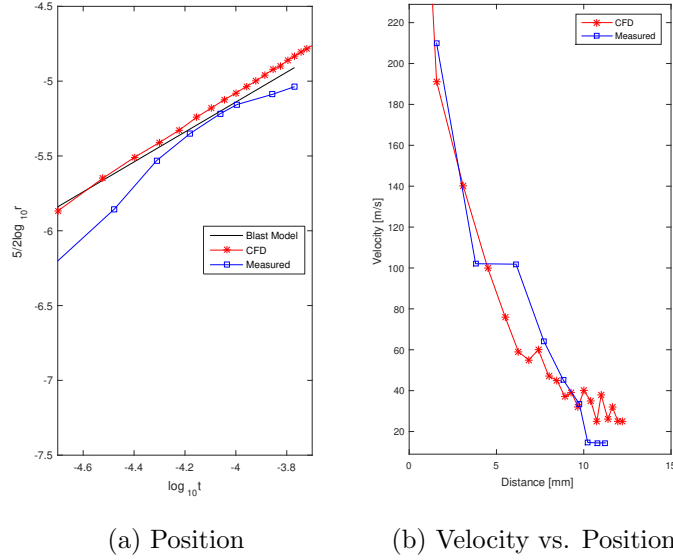
(b) Velocity vs. Position

Figure 3: (a) Time evolution of gas pressure front position (squares - experiment, stars - CFD) together with a linear fit of the blast wave model (black line); (b) the gas pressure front velocity versus position, measured in the direction of the plasma jet (squares) together with the computed CFD velocity of the front (stars). Experiment data reproduced from [7].
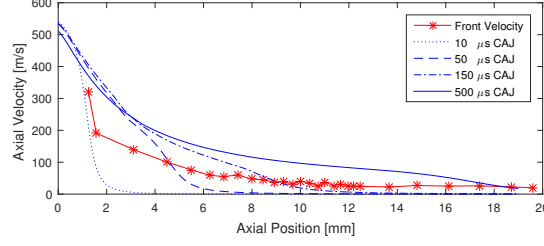
Figure 4: Simulation results of front expansion rate (red squares); Time evolution of the CAJ axial velocity versus axial position for sample times of 10 μs, 50 μs, 100 μs, 150 μs and Steady state conditions (blue line).
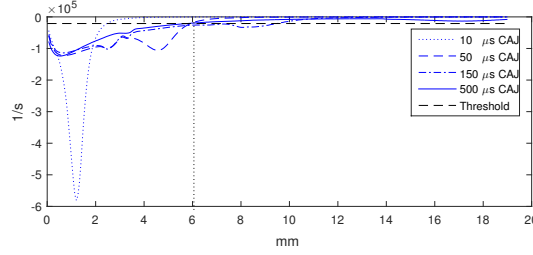

Figure 5: Simulation results of time evolution of the CAJ axial velocity gradient with respect to the axial distance from the cathode, for sample times of 10 μs, 50 μs, 100 μs, 150 μs and Steady state conditions (blue line); $-2 \cdot 10^4$ $1/s$ horizontal boundary line (black dash); the vertical line (black dotted) indicates the position where the gradient cross at different times.

V.   Conclusions

A CFD model of the cathodic arc jet was developed and its results were compared with measurement data, as well as with a Taylor-Sedov blast wave model. An `OpenFOAM` incompressible solver was modified to include a region of external force field, simulating the acceleration of the background gas due to the plasma ions momentum transfer. The flow properties – pressure and velocity were qualitatively analyzed in different periods of simulation time. The CFD results show a good correspondence with measurements data. The pressure front position, caused by the jet expansion with respect to time, show that the steady-state velocity is higher than the transient front velocity with $> 100 \ m/s$ at a distance of 10 mm. The CFD model predicts the experimentally observed CAJ phenomena – *i.e.* forming of an initial hemispherical expansion zone, the transient density gradient, front velocities and the steady conditions in the CAJ. Future studies will analyze the effects of a cross-flow on the CAJ based on the suggested implementation of the model.

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