**Student ID:** 510415022
**Tutorial Time:** 10:00 to 11:00 (10-11am), Monday
**Location:** F07.03.355.Carslaw Building Carslaw Seminar Room 355
**Lecturers:** Haotian Wu and James Parkinson

1. Find the general solution (in explicit form) of the differential equation

$$\frac{dy}{dx} = 2xe^{-y}.$$

**Solution:**

The DE is separable with $f(x) = 2x$ and $g(y) = e^{-y}$.

$$\begin{aligned}
\frac{dy}{dx} &= 2xe^{-y} \\
\int e^y \ dy &= \int 2x \ dx \\
\implies e^y &= x^2 + C \\
\implies y &= \ln(x^2 + C)
\end{aligned}$$

2. Consider the differential equation

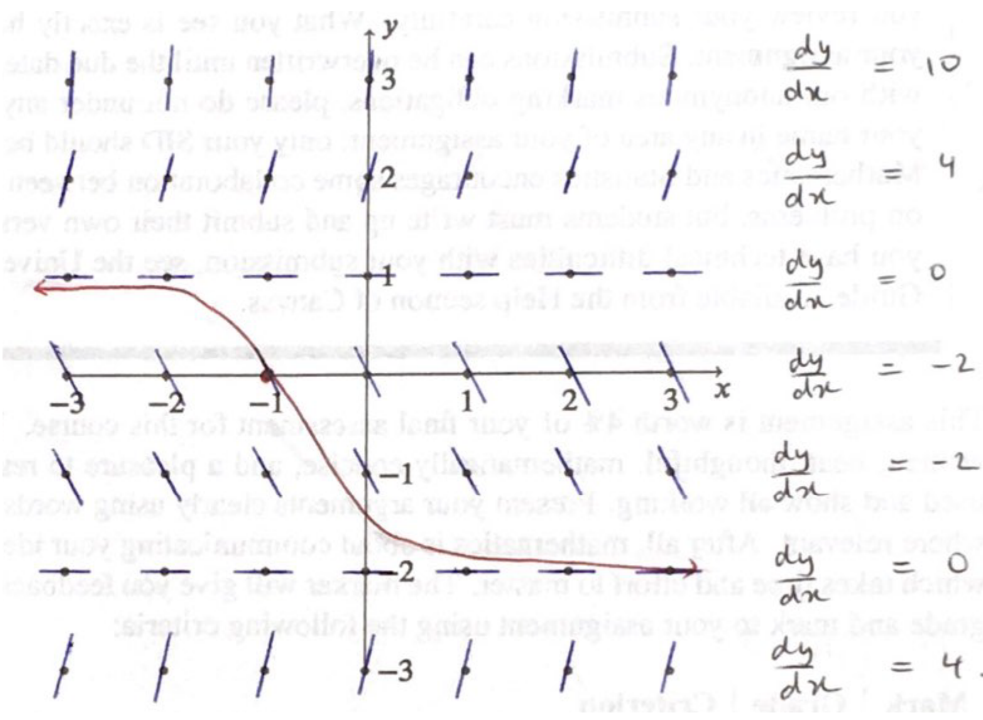
$$\frac{dy}{dx} = (y-1)(y+2).$$

   (a) Copy the grid below into your solutions, and use it to sketch the direction field of the differential equation. Your sketch needs to be hand-drawn (either on paper, or on a tablet), and you should draw the appropriate slope at each of the 49 points of the grid.

   (b) On your sketch in part (a), draw the solution curve passing through the point (-1, 0).

**Solution:**
Note that the DE is autonomous, so all slope lines along a single y-coordinate have equal gradient. Also,

$$\frac{dy}{dx} = 0 \iff y = 1 \quad \text{or} \quad y = -2.$$



3. A cylindrical tank of height 1 metre is full of water at time $t = 0$ seconds. A tap is turned on at the bottom of the tank, and water begins to drain from the tank. You are given that the differential equation governing the height $h(t)$ of water at time $t$ in the tank is given by

$$\frac{dh}{dt} = -\frac{\sqrt{h}}{30}.$$

How long does it take for the tank to be completely empty?

**Solution:**

The DE is separable with $f(t) = -\dfrac{1}{30}$ and $g(h) = \sqrt{h}$. Also note the initial condition $h(0) = 1$.

$$\begin{aligned}
\frac{dh}{dt} &= -\frac{\sqrt{h}}{30}, \quad h(0) = 1 \\
\int h^{-\frac{1}{2}}\ dh &= \int -\frac{1}{30}\ dt \\
\implies 2h^{\frac{1}{2}} &= -\frac{1}{30}t + C \\
\implies h^{\frac{1}{2}} &= -\frac{1}{60}t + D \qquad (D = C/2)
\end{aligned}$$

From the initial conditions, when $t = 0$, $h = 1 \implies D = 1$. So our IVP becomes

$$h^{\frac{1}{2}} = -\frac{1}{60}t + 1$$

For the tank to be completely empty, $h = 0$.

$$\begin{aligned}
0 &= -\frac{1}{60}t + 1 \\
\implies t &= 60
\end{aligned}$$

Therefore it takes 60 seconds or 1 minute for the tank to be completely empty.