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**Tutorial Class:** TUT05, Tuesday 2-4pm

**Question 1**
Why do we define stress and strain? **[4%]**

We introduce the concepts of stress and strain to exclude the specimen size effect on the mechanical properties of the material.

For example, consider two metal rods made from the same material, each with an equal length, but one with a larger diameter (one rod is thicker). Upon application of an identical lateral force, it is intuitive that the rod with the larger diameter will deform less. Similarly, for two rods identical except for their length, upon application of the same tensile axial force, it is intuitive that the longer rod will deform more. Both of these examples contribute to the *specimen size effect*. As material engineers are interested in generalising results made for specific **materials** rather than for specific specimens with certain lengths and diameters, we define the units of stress and strain.

Stress is defined by

$$\sigma = \frac{F}{A_0} \qquad \left(\mathrm{Nm}^{-2} \text{ or } \mathrm{Pa}\right) \tag{1}$$

where $F$ is the instantaneous load applied perpendicular to the original cross-section and $A_0$ is the original cross-sectional area. Notice that by dividing through by the original cross-sectional area, we account for variation in differing thicknesses and sizes of specimen (also known as the *dimensional effect*).

Strain is defined by

$$\varepsilon = \frac{\Delta l}{l_0} \qquad \text{(dimensionless)} \tag{2}$$

where $\Delta l$ is the deformation elongation and $l_0$ is the original length.

**Question 2**
Fatigue accounts for  90% of mechanical engineering failures. Please list 3 measures that may be taken to increase the resistance to fatigue of metallic materials. Please also briefly explain how each of these measures improves fatigue performance. **[6%]**

To understand how to increase a metallic material's resistance to fatigue, one must recognise that:

(i) Cracks associated with fatigue failure almost always nucleate on the surface. This is because the maximum stress within a structure under load often occurs at its surface.

(ii) At the surface, cracks form at stress concentration/amplification sites. These are microscopic flaws (e.g. surface scratches, sharp fillets, keyways, threads, dents etc) which have the effect of 'raising' the applied stress.

Knowing this, it is evident that the *condition* and *configuration* of the surface of the metal has a large impact on its fatigue life. This leads to the two main categories of measures one can take to increase a metallic material's resistance to fatigue: surface treatments and design criteria.

### Q2.1 Surface Treatments

The most basic surface treatment one can perform is polishing. When a metal is being worked on, it is unavoidable that small scratches or grooves make their way onto the surface of the piece. As per (ii), it is these small flaws which are able to act as 'stress raisers' and ultimately as origin points of cracks. Indeed, simply polishing the metal down after work on it is completed improves the surface finish, which ultimately enhances fatigue life.

A more complex treatment is known as *case hardening*. In simple terms, case hardening is the process whereby a carbon- or nitrogen-rich outer surface — known as the case — is introduced onto the surface of the original material. This process involves 'carburising' or 'nitriding': essentially exposing the material (often steel) to gaseous carbon or nitrogen molecules at a very high temperature. The free carbon/nitrides penetrate the steel surface and, after cooling, form a very hard outer layer on the metal approximately 1 mm deep. It is this hard layer (which is much harder than the inner core) which resists 'flaws' and thus, per (i) and (ii), prevents crack formation and increases resistance to fatigue.

Another method to prevent fatigue failure is to pre-emptively introduce residual compressive stresses into the outer layer of the metal's surface. Any induced tensile stress (which acts to form a crack) will then be partially negated by this compression, thus reducing the chance of crack formation, and ultimately increases fatigue life. This is often done through the process of *shot peening*, in which small particles are projected at high velocities into the surface of the metal, introducing these desired compressive stresses.

### Q2.2 Design Criteria

The formula which dictates stress concentration (assuming the crack is approximately an elliptical hole perpendicular to the original tensile stress) is

$$\sigma_m = 2\sigma_0 \left(\frac{a}{\rho_t}\right)^{\frac{1}{2}} \tag{3}$$

where $\sigma_m$ is the maximum stress at the point of concentration, $\sigma_0$ is the original applied tensile stress, $a$ is the half the length of the crack and $\rho_t$ is the radius of curvature of the crack.

Looking at the stress concentration factor $K_t = \frac{\sigma_m}{\sigma_0} = 2\left(\frac{a}{\rho_t}\right)^{\frac{1}{2}}$, it is evident that to decrease $K_t$ it is necessary to increase $\rho_t$, the radius of curvature. This motivates the use of 'fillets' during manufacture, as compared to sharp corners (see Figure 1). Indeed, sharp edges are extremely dangerous for stress concentration and therefore crack formation, and so should be avoided whenever possible during manufacture.


Figure 1: Filleting

**Question 3**

From one of the previous Materials 1 lectures, we learnt that Hooke's law can be described as $\sigma = E\varepsilon$, in which $\sigma$ is the stress applied to a specimen, $E$ is the Young's modulus of the specimen, and $\varepsilon$ is the resulting strain. In some other sources, Hooke's law is expressed as $F = k\Delta x$, where $F$ is the force applied to the specimen, $k$ is a constant, and $x$ is the displacement caused by the force. Please discuss the similarities and differences between these two types of description. **[10%]**

$$F = k\Delta x \tag{4}$$

$$\sigma = E\varepsilon \tag{5}$$

Both equations 4 and 5 are variations of Hooke's law. Both understand and describe the phenomenon that upon application of a force $F$ or $\sigma$, an object will experience some deformation/elongation $\Delta x$ or $\varepsilon$. However, equation 5 is a more general form of equation 4 in that it allows calculation using any specimen's length and cross-sectional area, parameters which would have otherwise been accounted for in Hooke's constant of proportionality $k$. Consider the following derivation:

$$\sigma = E\varepsilon \tag{6}$$

$$\frac{F}{A_0} = E \cdot \frac{\Delta l}{l_0} \tag{7}$$

$$\implies F = \frac{EA_0}{l_0} \cdot \Delta l \tag{8}$$

which is essentially of the form:

$$F = k\Delta x \quad \text{with} \quad k \propto \frac{A_0}{l_0} \tag{9}$$

This shows that the constant of proportionality $k$ in equation 4 subsumes these factors of original cross-sectional area and original length, making the original Hooke's law a much more specific equation as it is only applicable to specific specimen sizes.

Young postulated that it was not simply the length of extension that the imposed force affected, but actually the length of extension *in comparison to the unextended material*. In doing so, equation 5 excludes the dimensional effect of the specimen (see question 1) and thus **Young's constant of proportionality $E$ is constant for each material**[1]. In simple terms, while the value of $k$ varies from object to object (based on length, width, thickness etc), the value of $E$ remains constant for every material.

[1] at a constant temperature. Varying temperatures will have an effect on the intermolecular bonding of the material and thus changes the Young's modulus. This is discussed further in question 7.

**Question 4**

A long cylindrical rod with a diameter of 10 mm is cut into two cylindrical rods. The length of one rod (Rod 1) is a half of the length of the other one (Rod 2). Tensile stress of 600 MPa is applied to both rods. Rod 1 experiences only elastic deformation with elongation of 1 mm and strain of 0.5%. What are the elongation and strain values of Rod 2? **[10%]**

Note that as both rods have originated from the same material, they each have identical moduli of elasticity.

$$E_1 = E_2 \tag{10}$$

$$\implies \frac{\sigma_1}{\varepsilon_1} = \frac{\sigma_2}{\varepsilon_2} \tag{11}$$

As both rod 1 and rod 2 have identical forces applied to them and have identical cross-sectional areas, the applied stresses must too be identical. That is,

$$\sigma_1 = \sigma_2 \tag{12}$$

Hence, from equations 11 and 12 we see:

$$\varepsilon_1 = \varepsilon_2 \tag{13}$$

Now let us examine $\varepsilon_1$. The question states that $\varepsilon_1 = 0.5\%$ and $\Delta l_1 = 1\text{mm}$. Thus,

$$\varepsilon_1 = \frac{\Delta l_1}{l_{0_1}} \tag{14}$$

$$0.5\% = \frac{1 \times 10^{-3}}{l_{0_1}} \tag{15}$$

$$\implies l_{0_1} = 0.2\text{m} \tag{16}$$

We are given that $l_{0_1} = \frac{1}{2} l_{0_2} \implies l_{0_2} = 2 l_{0_1}$. Returning to equation 13,

$$\varepsilon_1 = \frac{\Delta l_2}{l_{0_2}} \tag{17}$$

$$\implies \Delta l_2 = \varepsilon_1 \cdot 2 l_{0_1} \tag{18}$$

From equation 16 we know $l_{0_1} = 0.2\text{m}$. Thence,

$$\Delta l_2 = 0.005 \cdot 2(0.2) \tag{19}$$

$$\implies \Delta l_2 = 0.002\text{m} \tag{20}$$

$\therefore$ The elongation value for Rod 2 is 2mm and the strain value for Rod 2 is 0.5%.

**Question 5**

For the engineering stress-strain curve shown in Figure 2, the values of the engineering strain at point $M$ (just before necking) and point $F$ (the fracture point) are 35% and 53%, respectively. Can we calculate the true strain values at these two points using formula $\varepsilon_T = \ln(1+\varepsilon)$? If yes, provide the true strain values. If no, provide the reason. **[10%]**

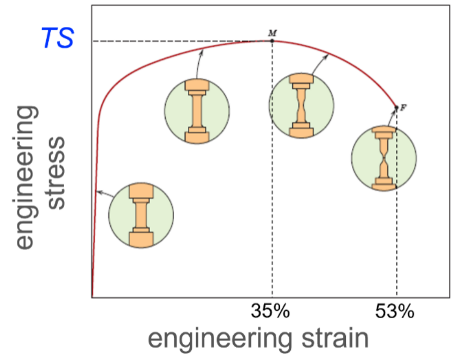


Figure 2: Engineering stress-strain curve for calculating true strain

To understand when we can calculate the true strain value at a certain point simply by looking at the engineering strain value, we must first understand where the formula for true strain originates from.

Unlike engineering strain, which only considers the change in length of the entire specimen during tensile testing, true strain considers each instantaneous change in length:

$$\varepsilon_T = \sum_{i=1}^{n} \frac{l_{i+1} - l_i}{l_i} \tag{21}$$

$$= \sum_{i=1}^{n} \frac{\Delta l_i}{l_i} \tag{22}$$

Considering the limit as $\Delta l_i \to 0$, the equation can be simplified through integration:

$$\varepsilon_T = \lim_{\Delta l_i \to 0} \sum_{i=1}^{n} \frac{\Delta l_i}{l_i} \tag{23}$$

$$= \int_{l_0}^{l_i} \frac{dl}{l} \tag{24}$$

$$= \ln\left(\frac{l_i}{l_0}\right) \tag{25}$$

$$= \ln\left(\frac{l_i - l_0}{l_0} + 1\right) \tag{26}$$

$$= \ln(1+\varepsilon) \tag{27}$$

Now, note that throughout this derivation we have relied on determining the value of $\Delta l_i$. While deformation is uniform, we can easily find this value. However, after necking, deformation is no longer uniform. Therefore, $\Delta l_i$ at each point has its own value. Different parts have different true strain values. Hence, any calculation using the formula becomes meaningless. Thus, for Figure 2, we can calculate the true strain value at point $M$ but not point $F$.

True strain at point M:

$$\begin{aligned} \varepsilon_{T_M} &= \ln(1+\varepsilon) && (28)\\ &= \ln(1.35) && (29)\\ \implies \varepsilon_{T_M} &\approx 0.300 && (30) \end{aligned}$$

**Question 6**

Please explain what mechanical properties of materials that we can acquire from a tensile engineering stress–strain curve (such as the one shown in Figure 3) and how. Please also provide the definition of these mechanical properties. **[20%]**

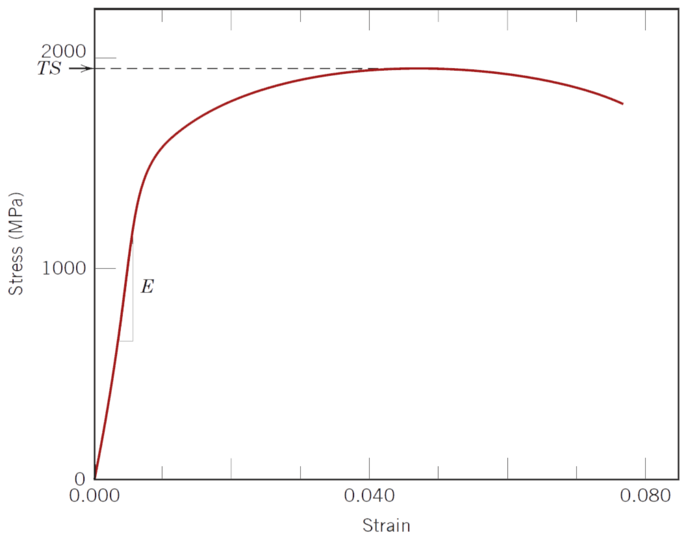


Figure 3: Engineering stress-strain curve for determining mechanical properties

### Q6.1 Young's modulus

The degree to which a structure *strains* depends on the magnitude of the imposed *stress*. In other words, stress and strain are proportional to one another. We denote the proportionality constant as $E$, known as the modulus of elasticity or Young's modulus.

$$\sigma = E\varepsilon \tag{5}$$

$E$ has the units Pascals (Pa) but is usually reported in gigapascals (GPa). The reason it is known as the modulus of elasticity is that stress and strain are linearly proportional only during elastic deformation. Any stress value which surpasses the limit of elastic deformation (known as the yield strength; more on this later) will result in a deviation from linearity and the beginning of a curve. Therefore, on an engineering stress-strain plot, we can easily calculate Young's modulus by examining the linear section of the $\sigma$-$\varepsilon$ function and using the relationship $E = \frac{\sigma}{\varepsilon}$. For example, from Figure 3, we can approximate a point on the linear portion of the graph to be $(0.006, 1000 \times 10^6)$. Then, $E = \frac{1000\times10^6}{0.006} \approx 167\text{GPa}$.

Hence, from an engineering stress-strain curve, one can determine the modulus of elasticity of a material.

### Q6.2 Plastic and elastic strain

Though it may seem trivial, from an engineering $\sigma$-$\varepsilon$ curve, one can determine the amount of plastic ($\varepsilon_P$) and elastic ($\varepsilon_E$) strain a material experiences upon the application of some stress $\sigma$.

Observing Figure 4, notice that at any point after the linear elastic deformation part of the curve, *both elastic and plastic deformation continue to occur.* Hence, at any point $P$, to accurately determine $\varepsilon_P$ and $\varepsilon_E$ we must construct a line (known as the 'unloading curve') from $P$ to the $x$-axis parallel to Young's modulus (which is represented by the linear part of the curve). This is essentially accounting for the elastic deformation which occurred after plastic deformation began (the yield strength). It is then easy to graphically determine the elastic and plastic strains of the material.

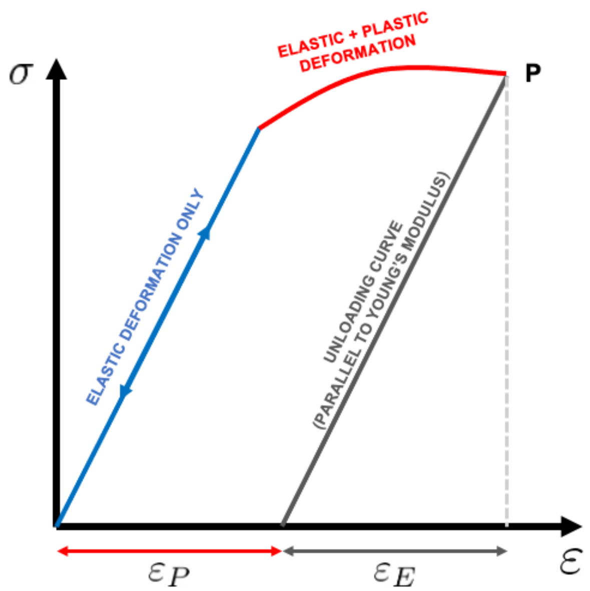


Figure 4: Determination of $\varepsilon_P$ and $\varepsilon_E$

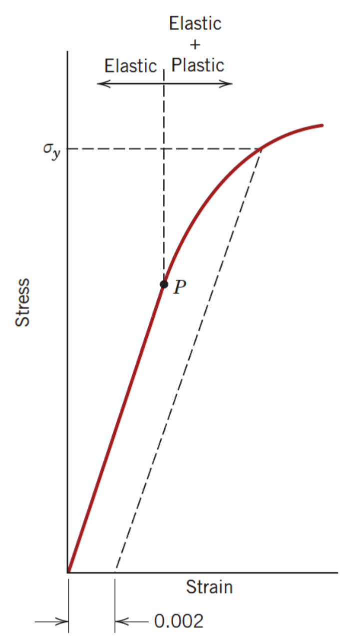


Figure 5: Determination of the yield strength $\sigma_y$

### Q6.3 Yield strength

*Yielding* describes the phenomenon where a material slowly starts to deform permanently i.e. undergo plastic deformation. The *yield strength* is therefore the stress level at which yielding first occurs, and is denoted by $\sigma_y$ with units of stress (Pascals). Similar to how the elastic modulus $E$ was a measure of a material's resistance to *elastic* deformation, the yield strength $\sigma_y$ is a measure of a material's resistance to *plastic* deformation.

The point of yielding is theoretically the point at which the stress-strain curve deviates from linearity (also known as the *proportional limit*, denoted by $P$) however it is incredibly difficult to measure this point accurately off of a graph. Thus, for consistency, engineers have defined the yield strength as the stress value at the intersection of the stress-strain curve and a new line parallel to Young's modulus at some specified strain offset — usually 0.002 (see Figure 5).

### Q6.4 Tensile strength and fracture strength

From Figure 3, we observe that after yielding, the curve increases to a maximum, before slightly decreasing until the fracture point. The stress at the maximum point represents the maximum stress that can be sustained by this material in tension, and therefore we call it the material's *tensile strength*. The tensile strength is important, as it is at this point that a small neck begins to form in the centre of the testing specimen in the process known as *necking*. This leads to the decrease of the external load applied to the specimen, and as such the engineering stress decreases until the fracture point[2]. The stress at the fracture point is known as the *fracture strength*.

From Figure 3, we can approximate the tensile strength to be $\approx$ 1900MPa and the fracture strength to be $\approx$ 1750MPa.

### Q6.5 Ductility

Ductility is a measure of the degree of **plastic** tensile deformation sustained by a material at fracture. Materials which exhibit a large plastic strain at the point of fracture are termed 'ductile' whereas those that do not are termed 'brittle.' See Figure 6 for an example of each.

The most common way to quantify ductility is through percentage elongation

$$\%EL = \frac{L_f - L_0}{L_0} \times 100\% \tag{31}$$

where $L_f$ is the value of the **plastic strain** strain at fracture. To determine this, the technique utilised in Figure 4 must be utilised, where we construct an unloading curve parallel to Young's modulus from the point of fracture to the $x$-axis. It is incredibly important not to use the whole tensile strain value, but rather only $\varepsilon_P$.

A less common method is to use percent reduction in area

$$\%RA = \frac{A_0 - A_f}{A_0} \times 100\% \tag{32}$$

however this method requires engineers to determine the cross sectional area at fracture, which is more difficult than a simple length measurement and increases uncertainties in calculations.

[2] Note that true stress will continue to increase as $\sigma_T$ is dependent on the instantaneous cross-sectional area.

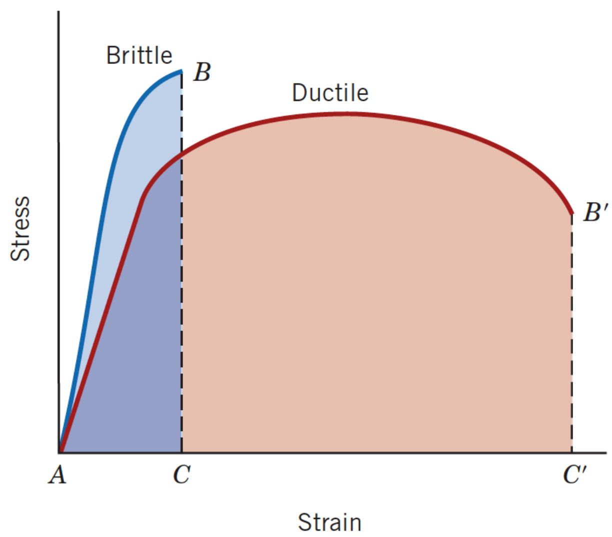


Figure 6: Comparison of a ductile and brittle material

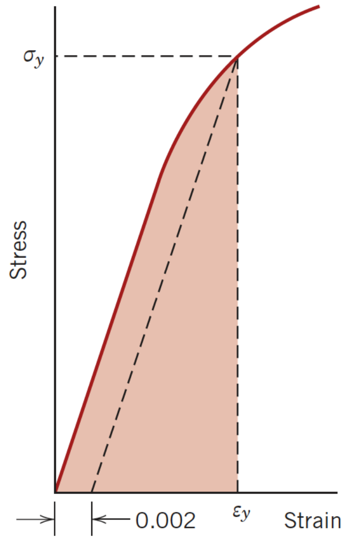


Figure 7: The approximation of resilience to a triangle

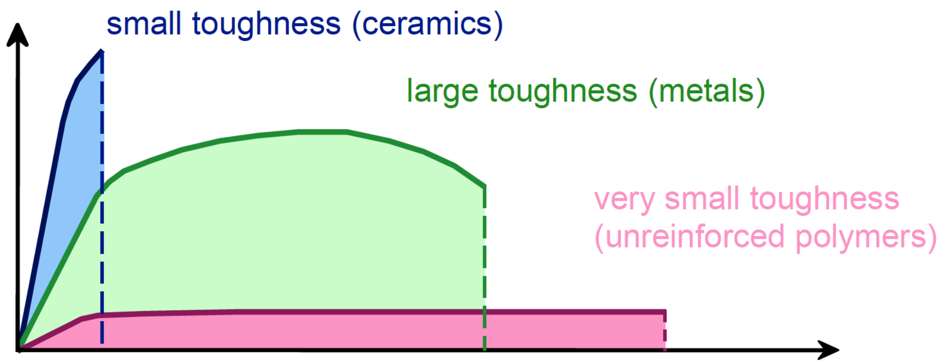


Figure 8: Toughness of various materials

### Q6.6 Toughness

The toughness of a material is the energy required to break a unit volume of the material. This energy can be approximated by the area underneath the material's stress-strain curve (see Figure 8):

$$\text{Toughness} = \int_0^{\varepsilon_F} \sigma \ d\varepsilon \tag{33}$$

where $\varepsilon_F$ is the strain at fracture. Note that for materials with a very brittle fracture, the area under the curve and therefore the energy required mostly originates from elastic energy, whereas for materials associated with ductile fracture, the energy originates from both elastic and plastic energy.

### Q6.7 Resilience

Resilience is the ability of a material to store energy for elastic deformation. It is associated to a property known as the *modulus of resilience* denoted by $U_r$. The modulus of resilience is the maximum amount of energy per unit volume that a material can absorb and still return to its original position. Similar to how toughness was approximated, the modulus of resilience can be approximated by the area under the stress-strain curve until the point of yielding

$$U_r = \int_0^{\varepsilon_y} \sigma \ d\varepsilon \tag{34}$$

where $\varepsilon_y$ is the strain at yield point. Assuming a linear elastic region, this integral can be approximated by a triangle (see Figure 7):

$$U_r \approx \frac{1}{2}\sigma_y \varepsilon_y \tag{35}$$

From equation 5 we know that $\varepsilon = \dfrac{\sigma}{E}$ so

$$U_r = \frac{1}{2} \cdot \sigma_y \cdot \left(\frac{\sigma_y}{E}\right) \tag{36}$$

$$\implies U_r \approx \frac{\sigma_y^2}{2E} \tag{37}$$

From equation 37 it is clear that resilient materials are those with high yield strengths and low moduli of elasticity.

**Question 7**

Please download the following paper[a] from the University library website and read it carefully. Based on your reading and knowledge that you have learnt from Chapter 6, please provide detailed explanation of:

(a) if change in specimen dimensions affects the measured Young's modulus, yield strength, and tensile strength, and

(b) the reason for using specimens with standard dimensions for mechanical property testing.

**[30%]**

[a] Y. H. Zhao, Y. Z. Guo, Q. Wei, A. M. Dangelewicz, C. Xu, Y. T. Zhu, T. G. Langdon, Y. Z. Zhou, E. J. Lavernia, Influence of specimen dimensions on the tensile behavior of ultrafine-grained Cu, Scripta Materialia 59 (2008) 627-630 (DOI: 10.1016/j.scriptamat.2008.05.031)

### Q7.1 Young's modulus

In question 3) and 6) we described how by taking into consideration the *relative* change in length $\varepsilon$ and the force *per unit area* $\sigma$, the Young's modulus of a specific material becomes a constant (at a constant temperature). This constancy is also supported by an understanding of what the modulus of elasticity represents on an atomic level.

During elastic deformation of a particular material, interatomic bonds are stretched in a reversible process, and return to their initial state once the external load is reversed. $E$ is a measure of the resistance of this stretching and separation. Thus, on a force-separation chart such as the one shown in Figure 9, Young's modulus is proportional to the slope of the graph at the equilibrium spacing $r_0$ (the point at which the atoms feel no net force).

$$E \propto \left. \frac{dF}{dr} \right|_{r=r_0} \tag{38}$$

This slope, in turn, is dependent on the interatomic bonding strength of the material. Therefore, $E$ is **ultimately a measure of the material's interatomic bonding force.** This force has nothing to do with the dimensions of the material, thus validating the idea that $E$ is constant regardless of the specimen size.

The supplied paper provides support for this constancy. In figures 1a) and 1b) it is evident that all 4 curves, representing various thicknesses $T$ and lengths $L$, are identical up until the yield strength. The gradient of this curve represents $E$ and therefore the paper (Zhao, 2008) shows **no deviation of the measured Young's modulus for varying specimen dimensions.**

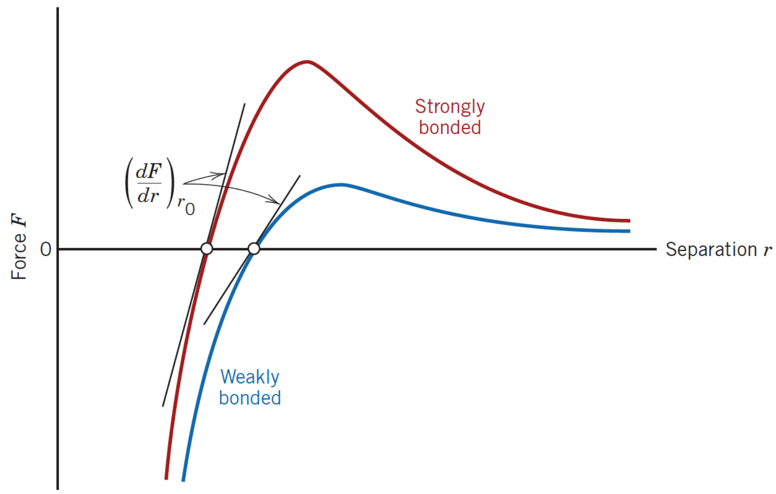


Figure 9: Force separation chart

### Q7.2 Yield strength

Yield strength, as described in question 6), is the stress level at which yielding first occurs. Similar to the elastic modulus, yield strength $\sigma_y$ is also a constant for a particular material. We understand why through a deeper understanding of the intermolecular interactions at this point.

Recall that the proportional limit $P$ is the exact point at which elastic deformation gives way to plastic deformation. From Q7.1, we understand that during elastic deformation, atomic bonds are being stretched only to a point where if the applied force is released, bonds can compress and return to their original state. But what occurs when bonds are stretched past this point (past the elastic deformation limit)? In metals, atoms are able to form a **dislocation.** Rows of atoms slip past one another, allowing the metal to be further stretched out in what is known as *dislocation motion*. This is a permanent mechanism and cannot be undone, which is why it results in permanent deformation. At the yield point, the force required to move a dislocation is less than the force required to stretch atomic bonds, so the dislocation motion is effected.

Once again, this yield point phenomena is inherently based on the interatomic bonding of the material, and has no correlation to the material dimension or thickness. Experimental evidence verifies this, as Zhao (2008) writes "…reducing $L$ and increasing $T$ increases the apparent strain hardening rate **without changing the yield strength of [the metal]** which is about 390 MPa."

### Q7.3 Tensile strength

Figure 1a) from the paper (see Figure 10 to the right) has some interesting discussion to offer regarding the effect of specimen thickness on a material's measured ultimate tensile strength. Observing the figure, which shows the engineering stress-strain curves for various samples with constant length but varying thicknesses, it is apparent that just after the yield strength (which was quoted to be "about 390 MPa") the 4 curves diverge. The blue curve, representing the thickest sample with $T = 1000\,\mu\text{m}$, seems to peak (i.e. $\frac{d\sigma}{d\varepsilon} = 0$) at the largest stress value, followed by the green, red, and finally the black curve. That is,

$$TS_{\text{blue}} > TS_{\text{green}} > TS_{\text{red}} > TS_{\text{black}} \quad (39)$$

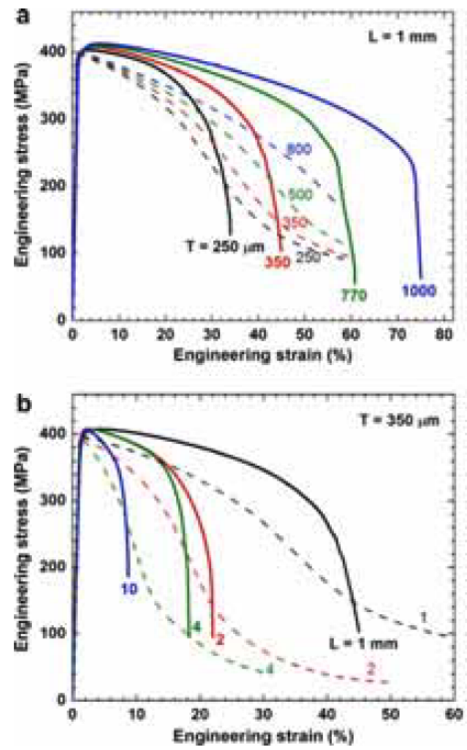


Figure 10: Figure 1a) and 1b) from Zhao's paper.

Interestingly, this phenomenon is not observed at all in Figure 1b) of the paper, where the thickness of the sample is constant with varying lengths. What makes it more perplexing is that, to the best of our[3] knowledge, there is no evidence or reasoning behind why such an effect would be observed. All theory points to a constant ultimate tensile strength of a material at a constant temperature. Indeed, after taking into consideration how small the difference in tensile strengths are, as well as the fact that the simulated curves (shown in dashed lines) all have an equal tensile strength, this could very well be simply attributed to experimental error.

### Q7.4 Using specimens with standard dimensions

The results of the paper show that although Young's modulus, yield strength, and tensile strength are not affected by changes in specimen dimensions, characteristics such as total ductility, necking length, and failure mode are.

It is evident from examining Figures 1a) and 1b) from the study that the largest variability between the samples of varying thicknesses and lengths lies in the % strain value. As Zhao writes, "Both $T$ and $L$ tend to have significant effects on the post-necking elongation and

[3] an AMME1362 Materials 1 student.

total ductility . . . ” — indeed, the results show that the experimental ductility (related to the total necking length) of the material $\varepsilon_{\text{ef}}$ decreases from 75% to 34% simply by reducing the thickness of the sample by 3/4 of a mm, with similar results for changing $L$. This test is clearly incredibly sensitive to the thickness and length of the test specimen.

Furthermore, Figures 2a) and 3 highlight the effect of increasing the thickness of the specimen on the failure mode of the material. The thicker the specimen tested was, the more likely that the specimen would fail due to *normal* tensile failure rather than *shear* failure.

It is evident from reading Zhao's paper that during tensile tests, small changes in the specimen being tested can result in large changes to properties such as ductility and failure mode. This goes to show the need for using specimens with standard dimensions for mechanical property testing — no fair comparison of a characteristic such as ductility or failure mode can truly be made between one material and another unless they are of exactly the same size. This has various implications on the scientific process itself, affecting the repeatability and reproducibility of these tests, and also on the engineering process which often involves choosing a suitable material for the task at hand. As Zhao writes, “Our results and analyses suggest that the tensile specimen size effect should be considered when comparing mechanical properties, particularly tensile ductility, measured on non-standardised dog-bone specimens . . . **a standardised protocol should be adopted for use by the community at large.”**

**Question 8**

A material presents S–N fatigue behaviour shown in Figure 11 with the fatigue limit being 100 MPa. If a cylindrical bar made from the material which has a diameter of 20 mm is subjected to repeated tensile stress along its axis, calculate the maximum allowable tensile load to ensure no occurrence of fatigue failure. [**10%**]

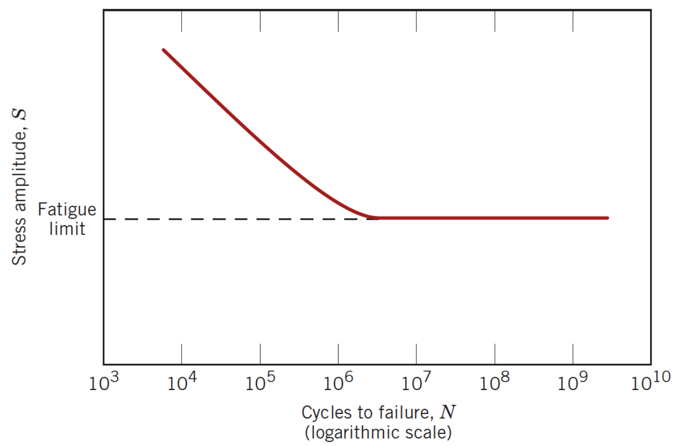


Figure 11: S-N curve

The cylindrical bar is under a repeated tensile stress cycle with $\sigma_{\text{min}} = 0$. Therefore, $\sigma_{\text{max}} = 2 \times \sigma_a = 2 \times S_{\text{fat}}$.

$$\sigma_{\text{max}} = \frac{F_{\text{max}}}{A_0} \tag{40}$$

$$\implies F_{\text{max}} = \sigma_{\text{max}} \cdot A_0 \tag{41}$$

$$= (2 \times S_{\text{fat}}) \cdot \frac{\pi d^2}{4} \tag{42}$$

$$= S_{\text{fat}} \cdot \frac{\pi d^2}{2} \tag{43}$$

$$F_{\text{max}} = (100 \times 10^6) \cdot \frac{\pi \cdot (20 \times 10^{-3})^2}{2} \tag{44}$$

$$\implies F_{\text{max}} = 62831.85\text{...kN} \approx 63000\text{kN} \tag{45}$$

The maximum allowable tensile load to ensure no occurrence of fatigue failure is approximately 63000kN.