COMP 2019 Workbook Exercises Week 4 – Constraint Satisfaction Search Answers

(a)

As no two queens may be placed in the same column, and the number of queens equals the number of columns, we may assume that there is exactly one queen in each column. Therefore, we can fix the column for each queen, and only need to determine the row in which each queen can be placed. In the following, we will assume that queen 1 is placed in column 1, etc. (We could phrase a similar model with the rows fixed and the variables representing the columns.)

One decision variable per queen: $Q1, Q2, Q3, Q4$.
The variables' values indicate the row in which the queen is placed.

The domain (=set of possible values) for each variable is {1,2,3,4}.

The constraints are as follows:

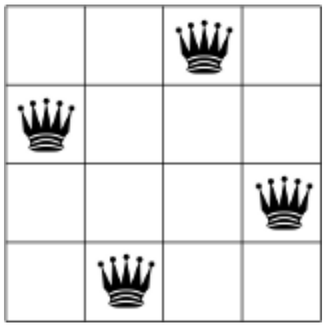
(i) No two queens can be placed in the same column:
No constraint is necessary, as this restriction is already implicit in the model we have created.

(ii) No two queens can be placed in the same row:
$Q_i \neq Q_k$ for each *i,k* in {1,2,3,4} with $i \neq k$
or, alternatively, the
all_different(Q1,Q2,Q3,Q4) constraint can be asserted.

(iii) No two queens can be placed in the same diagonal:
$|Q_i - Q_k| \neq |i - k|$ for each i, k in $\{1,2,3,4\}$ with $i \neq k$
Here, $|x|$ denotes the absolute value of expression *x*.

(b)

1. Perform AC3. No values can be eliminated from any domain.
2. As all variables have equal domain, pick the first variable in alphabetic order and assign the smallest possible value (Q1=1). Then, perform AC3. We obtain the following domains after eliminating inconsistent values from the remaining variables' domains:
   Q2 in {4}, Q3 in {2}, Q4 in {}. Note that AC3 is stronger than Forward Checking, which would derive only Q2 in {3,4}, Q3 in {2,4}, Q4 in {2,3}.
3. Since Q4 has no remaining legal values, we must backtrack. Undo the last assignment to Q1 and continue with the next alternative assignment.
4. Assign Q1=2. AC3 propagation yields: Q2 in {4}, Q3 in {1}, Q4 in {3}.
5. All variables have domain sizes of 1, hence we proceed with the next variable in alphabetic order and assign it the only remaining possible value: Q2=4. AC3 cannot eliminate any values from Q3 and Q4.
6. Assign Q3=1 and perform AC3 (no changes).
7. Assign Q4=3. A solution has been found.



If we continued searching, we would find the second solution. (As the problem is symmetric, the 2nd solution can be obtained by flipping the board horizontally, vertically, or via the main diagonal.)