Chapter 3
Basic Positioning Techniques

If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

John von Neumann

Indoor positioning is the task of inferring the location of a mobile device inside a building. Often, indoor positioning is identified with indoor navigation as many people seem to believe that positioning is the only missing prerequisite to provide guidance services comparable to a GPS-based navigation system outside buildings.

Still, a lot of questions remain open once indoor positioning becomes widely available. This is mainly due to the fact that the semantic structure of a building is much more complex than a road network. Without additional ideas and algorithms, an indoor positioning system would have to provide accuracy below 1 m of expected error. This accuracy is not even available outside buildings.

Another aspect of indoor positioning is the fact that making indoor positioning available inside one building induces the additional challenge of seamless positioning: When should a mobile device switch over to the indoor system? How can the positioning system for the outside space deliver bootstrap information to the indoor positioning system when a user is walking into a building? And what happens, when the user leaves the building?

But before we start to discuss the vast number of possibilities to infer a mobile device inside buildings, let us look at a simple question, which should always be asked before heading for indoor positioning technology:

> Why should we introduce navigation inside a building?

Most buildings are either used by a small group of users with varying tasks (e.g., home, bureau, etc.) or by a large group of users with a similar task (e.g., airport, hotel, etc.). In both cases, classical signage or some smart digital signage will be enough to guide the majority of users. Hence, indoor positioning is for different application scenarios and should, hence, not be discussed isolated from a concrete application. This is due to the fact that the most important trade-off before choosing an indoor positioning system is between cost and accuracy. Moreover, the needed accuracy is dictated by the application.

For example, a proximity marketing solution could be provided to users using a positioning technology. However, the application is not at all interested in the

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position in the sense of location coordinates but much more into the fact of being near to some point. It can be a good idea in such cases to distribute digital beacons at these places, for example, based on Wi-Fi, Bluetooth, or Radio-Frequency Identification (RFID) technology.

A lot of serious interest in positioning technology stems from security applications, quality control in manufacturing, and safety applications in high-risk areas. In all of these cases, the location of a mobile device is used as an additional feature inside an existing process reducing errors there. For example, quality control is usually a complex process based on education, management, self-control, and motivation. In these applications, the decision between different indoor navigation systems is guided by the need to find out enough information to truly aid the process and, which is often more difficult to achieve, by the invented errors based on wrong location estimates. The critical effect of seldom wrong location measurements is best illustrated in the area of ambient assisted living (AAL). Assume a location system is tracking an elderly person inside his own room and wrongly detects that the person does not move anymore. Then, an ambulance might be sent to the elderly person in error leading to high cost.

Altogether, the central question before deploying any indoor positioning system is the following:

> Does the system provide a sufficient advantage in the average case to accept the disadvantages the system will introduce?

An indoor positioning system does not only introduce disadvantages due to wrong location estimates: It is very important to incorporate privacy discussions into the design step of any indoor positioning infrastructure. Unfortunately, for many applications, the advantage of indoor positioning does not directly aid the one whose privacy might be violated. Therefore, a lot of deployments face the problem of communicating the need and getting the systems disadvantages accepted by the users.

The rest of this chapter is organized as follows: Sect. 3.1 introduces the basic algorithms of location determination. These algorithms are applicable inside and outside buildings in the same manner. Section 3.2 explains properties which can be used to compare indoor positioning techniques with each other. Section 3.3 describes a short selection of real-world indoor positioning systems and techniques.

3.1 Methods for Location Determination

As suggested by the previous exposition, there exists a large variety of positioning techniques and positioning systems. However, there is only a limited number of algorithms and methods to infer location information from measurements. Therefore, we will organize this section along these algorithms.

A central problem of inferring location is that this inference is usually based on a set of measurements of physical sizes. And these measurements usually contain a considerable amount of noise or even systematic errors of measurement. For the algorithm of circular lateration, for example, one assumes that the mobile device is located on circles around known locations for which the radius has been measured as the distance between the mobile device and a respective reference location. Due to noise, these circles will merely never intersect in a single point. To successfully deal with this type of problem, the next Sect. 3.1.1 introduces the method of least square estimation. This method enables us to calculate the most probable solution to an overdetermined and possibly inconsistent system of linear equations. This method of least squares is the central tool to enable a wide range of geometric location determination algorithms.

3.1.1 Method of Least Squares

Observing the world by means of measuring physical sizes is generally subject to different classes of errors. First of all, the device used to measure a physical size can introduce errors. Moreover, measurements are often stored and communicated in digital form introducing limited precision. Due to these errors, a reliable observation of the world can often only be achieved by repeated measurements. These repeated measurements will contradict each other, and these contradictions need to be resolved.

Let us assume that there is a linear relationship between the values to be observed and the values actually measured. Then, a sufficient number of measurements leads to an overdetermined linear equation

$$Ax = b,$$

where the vector b contains the actual measurements, the vector x contains the value to be determined, and A expresses the theoretic or expected relationship between both. For this situation, Carl Friedrich Gauss and Adrien Marie Legendre found a method in 1795 merely at the same time and apparently independent from each other to find the most probable value of x.

To remove some subtleties from the discussion, we will assume from now on that the matrix A has maximal rank. In other words, the column vectors of A are linear independent. If this is not the case, redundant columns can be removed without introducing problems. In fact, this is usually performed automatically by computer libraries implementing least squares.

As a perfect solution x with $Ax = b$ does not exist due to measurement errors, the objective is simplified and formulated as follows: find the value x, which minimizes the norm of the error function

$$||r(x)|| = ||b - Ax||.$$

A perfect solution corresponds to a vanishing error function and a norm of zero in this expression. When using the Euclidean norm, we can simplify the exposition by squaring both sides. The norm is the square root of the scalar product. Hence, a minimal norm corresponds to a minimal scalar product. As the norm is defined to be the square root of the scalar product, we can calculate

$$||r(x)||^2 = (b - Ax)^T(b - Ax) = x^T A^T Ax - 2x^t A^t b + b^T b \to \min .$$

We want to find the spot x, where this expression is minimal, and we can just use differential calculus therefore: a necessary condition for a function to reach the minimum is that the first derivative vanishes. The first derivative of the above formula can easily be derived and requiring this first derivate to vanish results in the following equation:

$$2A^t Ax - 2A^t b = 0.$$

This equation is called normal equation of the overdetermined system of linear equations $Ax = b$ and is usually given in the equivalent form:

$$A^t Ax = A^t b. \tag{3.1}$$

It is easy to see that $A^t A$ is a positive semi-definite, symmetric matrix. As a result, the normal equation can be easily solved for x, as this equation is not overdetermined anymore. One can show that this x actually minimizes the error function norm. The most important result justifying the use of this approach, especially the choice of error function, is known as the Gauss–Markov theorem. This proves that the given algorithm provides a best, linear, unbiased estimate (BLUE) of the value x. The preconditions of this theorem, namely, that the errors have zero expectation and equal variance, are usually fulfilled when using similar physical sizes.

For details regarding efficient implementations and numerical stability of results, the interested reader should consult introductory books on numerical mathematics such as [7, 14, 18].

3.1.2 Lateration

Lateration is the process of estimating the location of a mobile device given distance measurements to a set of points with known location. Figure 3.1 depicts this situation. The distance measurements are erroneous and limit the locus of the mobile device to the area between the dashed lines. Hence, a good approximation to a system of equation needs to be found in which the locus of the mobile device is bound to circles with radius given by measurements around the known locations.

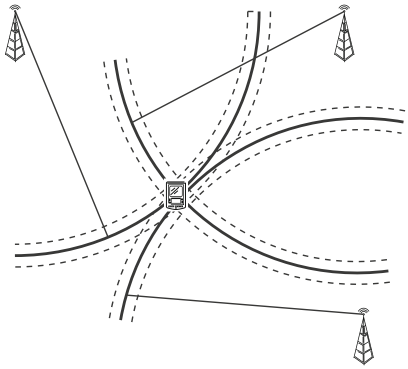

Fig. 3.1 Lateration: three erroneous distance estimates to base stations of known location result in an estimate of the location of the mobile device

Put formally, the location $p = (x, y)$ of the mobile device must fulfill all equations k_i describing the circles around the base station locations given as $p_i = (x_i, y_i)$ with radii d_i given by measurements:

$$d_i = k_i(x, y) = \sqrt{(x_i - x)^2 + (y_i - y)^2}\; i = 1 \ldots k. \tag{3.2}$$

This is a nonlinear problem, and hence, the least squares algorithm cannot be applied directly. To overcome this problem, it is customary to use an iterative approach based on having an initial coarse estimate of the location. Then the Taylor approximation around this current estimate can be used to linearize the system of Equations (3.2). This linearization is only valid around the previous location, and the resulting linear system of equations can be solved using the techniques of least squares described in the previous section. This linearization is based on the Taylor expansion given by

$$f(x) = \sum_{i=1\ldots n} \frac{f^{(i)}(x_0)}{i!}(x - x_0)^i + R_{n+1}(x, x_0).$$

The term R collects the remaining error of using a finite sum. To linearize a system of equations using Taylor expansion, one can set $n = 1$, and for the case of lateration, one needs the partial derivatives in both directions to construct the Taylor sum as a vector expression. These partial derivatives of Eq. (3.2) are given as follows:

$$\frac{\partial}{\partial x} k_i(x, y) = -\frac{x_i - x}{\sqrt{(x_i - x)^2 + (y_i - y)^2}}$$

$$\frac{\partial}{\partial y} k_i(x, y) = -\frac{y_i - y}{\sqrt{(x_i - x)^2 + (y_i - y)^2}}.$$

Let now $(\tilde{x}, \tilde{y})$ denote the current estimate of location. Then using the measurements d_i, one is left with the following system of linear equations:

$$d_i = k_i(\tilde{x}, \tilde{y}) + \frac{\partial}{\partial x} k_i(\tilde{x}, \tilde{y})(x - \tilde{x}) + \frac{\partial}{\partial y} k_i(\tilde{x}, \tilde{y})(y - \tilde{y}) \; i = 1 \ldots k. \tag{3.3}$$

Introducing the notation $\hat{x} = (x - \tilde{x})$ as well as $\hat{y} = (y - \tilde{y})$, this results in an overdetermined system of linear equations for which the method of least squares explained in Sect. 3.1.1 can be applied directly. Applying this technique results in a vector expressing the correction of the current location estimate $(\hat{x}, \hat{y})$. This can be added to the last estimate of location, and the process can be iterated. The following section provides a detailed example of this approach.

Example 3.1 The following table provides the coordinates of the location of four base stations as well as erroneous distance measurements of a mobile entity located at coordinates $(2, 2)$.

Component	Coordinate	Measurement (d_j)
Base station 1	$(x_1, y_1) = (0/0)$	2.92
Base station 2	$(x_2, y_2) = (10/0)$	8.14
Base station 3	$(x_3, y_3) = (15/10)$	15.46
Base station 4	$(x_4, y_4) = (0/12)$	9.89
Mobile device	$(x_e, y_e) = (2/2)$	–
Initial location estimate	$(\tilde{x}, \tilde{y}) = (20, 20)$	–

As a first step, one calculates the following factors:

$$\alpha_i = \sqrt{(x_i - \tilde{x})^2 + (y_i - \tilde{y})^2} \text{ for } i = 1 \ldots 4$$

coming up in the expressions of the partial derivatives in (3.3). From a geometric perspective, these factors provide the distance of the current location estimate to the different base stations. An optimal estimate for the complete problem is reached when $\alpha_i \approx d_i$.

Using the data from the given table, the first step calculates

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} \sqrt{800} \\ \sqrt{500} \\ \sqrt{125} \\ \sqrt{464} \end{pmatrix} \approx \begin{pmatrix} 28.284 \\ 22.361 \\ 11.180 \\ 21.541 \end{pmatrix}.$$

Using these factors, one can build up the needed linear system as described in Eq. (3.3) consisting of the matrix A given by the rows A_i as follows:

$$A_i = \left(-\frac{(x_i - \tilde{x})}{\alpha_i}, -\frac{(y_i - \tilde{y})}{\alpha_i} \right).$$

For the given example, this results in the following matrix:

$$A = \begin{pmatrix} 0.70711 & 0.70711 \\ 0.44721 & 0.89443 \\ 0.44721 & 0.89443 \\ 0.92848 & 0.37139 \end{pmatrix}.$$

The right-hand side of the linear equation is given by assembling a vector b row by row as

$$b_i = d_i - \alpha_i .$$

For the given example, this results in the following vector:

$$b = \begin{pmatrix} -25.364 \\ -14.221 \\ 4.28 \\ -11.65 \end{pmatrix}.$$

Solving this system of linear equations using least squares approach, the correction vector $(\hat{x}, \hat{y})$ inside the system

$$A \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = b$$

has the following concrete value:

$$\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} -18.62235 \\ -0.23377 \end{pmatrix}.$$

The next location estimate is accordingly given by

$$\begin{pmatrix} \tilde{x}_2 \\ \tilde{y}_2 \end{pmatrix} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} + \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} 1.3777 \\ 19.7662 \end{pmatrix}.$$

Iterating this approach, one gets results as given in the following table:

Step number	Location estimate	Correction vector	Norm of the residuum
1	(20.00, 20.00)	(−18.62, −0.23)	25.456
2	(1.38, 19.77)	(0.38, −8.82)	17.777
3	(1.75, 10.95)	(−0.38, −8.88)	8.954
4	(1.37, 2.07)	(0.57, 0.09)	0.631
5	(1.94, 2.16)	(−0.01, −0.01)	0.168
6	(1.94, 2.15)	(0.00, 0.00)	0.163
7	(1.94, 2.15)	(−0.00, −0.00)	0.163

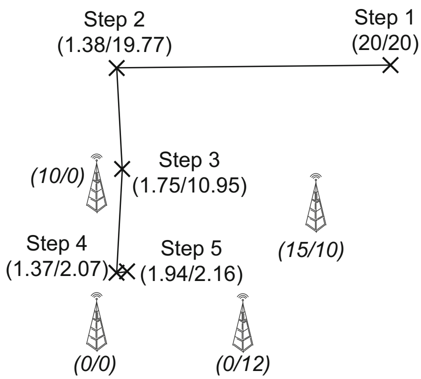

Fig. 3.2 The example and the iterative process of lateration

This table shows that the result approaches the true location $(2, 2)$, though it will never reach it due to the inherent errors in the system. The residuum is the distance of the estimate to the true location. From the seventh step, the result does not change inside the printed precision, and hence, the iteration can be stopped. The overall result is then given by the last estimate $(1.94, 2.15)$.

Figure 3.2 depicts the base stations, the starting position, and the iteration steps.

3.1.3 Hyperbolic Lateration

Hyperbolic lateration is a variant of lateration in which the measurement input does not consist of distance estimates to known locations but estimates of distance difference. Assuming that some infrastructure is tightly synchronized and produces events at the same time, which can be received at different times by a mobile station, this is a quite common variant known as time difference of arrival. The most important advantage of this measurement of time differences is that the mobile device does not need to be time synchronized with the sender of a signal. When for two base stations the difference Δd of the distances between the mobile device and both base stations is known, then the mobile entity resides on the hyperbel defined by this distance difference as depicted in Fig. 3.3.

In order to calculate the position, one uses the same approach as used for circular lateration in the previous section. As a first step, the following system of equations is established limiting the location of the mobile device onto the hyperbels defined from the measurements:

$$\begin{aligned}\Delta d_{ij} &= k_i(x, y) - k_j(x, y)\\ &= \sqrt{(x_i - x)^2 + (y_i - y)^2} - \sqrt{(x_j - x)^2 + (y_j - y)^2}\; i, j = 1 \dots k, i < j.\end{aligned} \tag{3.4}$$

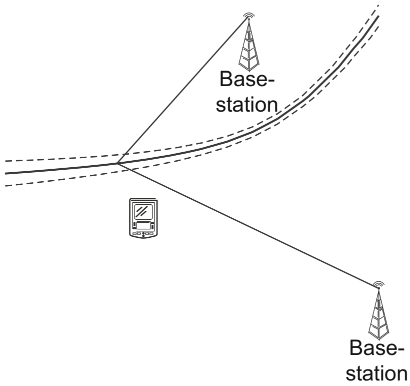

Fig. 3.3 Example for hyperbolic lateration: the distance difference between the drawn lines is known fixing the locus of the mobile device onto a hyperbel

It is customary to only include the distance measurements with respect to one fixed station, say, $i = 1$. This nonlinear, overdetermined system of equations in erroneous measurements is then solved along the same lines as in the previous paragraph. As a first step, the Taylor expansion is calculated and then used to linearize the problem at some estimated location in order to iteratively update these estimations until the process converges.

From a system implementation perspective, the main difference between hyperbolic lateration and lateration is given by which components have to be time synchronized. In lateration using signal propagation delay, each pair of sender and the receiver have to be synchronized. For hyperbolic lateration, the complete set of anchors need to be synchronized, while the mobile device does not need a reference to that time.

3.1.4 Angulation

Angulation is another very common class of positioning approaches in which measured angles between known base stations and mobile devices are used to infer the location of the mobile device. For angulation, there are two general perspectives regarding angles: either the angle between fixed points and mobile devices is measured at those fixed locations or the mobile device measures angles with respect to the incoming signals of base stations. We will discuss only the more common case that the distributed infrastructure measures angles in this section. The other case is, however, very similar. Figure 3.4 depicts an instance of this case.

Ignoring complications such as multipath effects, measurements of angles of incoming radio signals at base stations limit possible locations of the mobile device onto a ray starting at the base station. This can, again, be expressed as a nonlinear system of equations as follows:

$$\alpha_i = \arctan \frac{y_i - y}{x_i - x} \; i = 1 \ldots k. \tag{3.5}$$

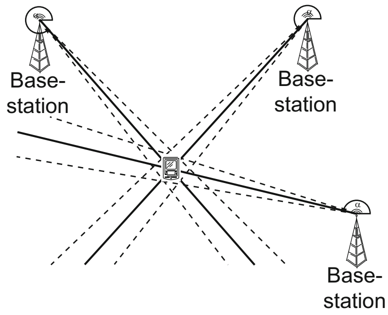

Fig. 3.4 Example of angulation: three erroneous estimates of angles provide a position of a mobile device

This system of equations can again be linearized by a first-order Taylor expansion leading to an iterative algorithm which refines an initial location estimate by approximating the nonlinear system of equations around these estimates and solving the linearized system there in order to update the location estimate.

3.1.5 Proximity Detection

Proximity detection is a class of location determination algorithms which are based purely on the proximity of the mobile device to previously known locations. The visibility of a Wi-Fi network, for example, results in proximity to the access point as the Wi-Fi signal is limited to a region around the access point. Consequently, proximity detection does not provide location in form of coordinates but rather in form of sets of possible locations. Proximity to a given Wi-Fi access point limits location of the user to a large and complex region. Therefore, proximity to several different locations can be used to intersect these sets and find smaller regions of possible residence of the mobile device. A common simplification is given by assuming that the range of a wireless infrastructure would be well represented by a circle of given radius r. Then proximity results in being located inside this circle, and for several circles, one can limit the possible location to the intersection of the different circles

$$p \in \bigcap_i \{x \text{ mit } ||x - p_i|| < r\}.$$

This simplified situation is depicted in Fig. 3.5.

In extreme cases, proximity detection can be used with very many possible objects and very small radii r such that the location of a device can be tracked down to centimeter accuracy. However, this implies that a large infrastructure is deployed to distinguish between these small areas of location.

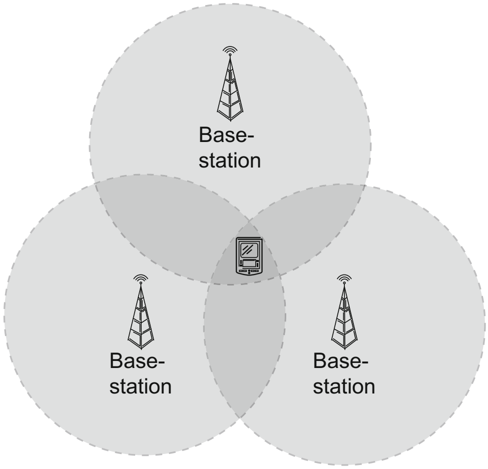

Fig. 3.5 Example of proximity detection

3.1.6 Inertial Navigation

Inertial navigation systems are based on estimating the location of the mobile device using only measurements made inside the inertial system of the mobile device. Therefore, the location, speed, and orientation at the starting time are known, and measurements are used to update this complete movement state. Inertial navigation is usually based on measuring acceleration and rotation. Sometimes, odometrical measurements including steps as well as steering angles can also be used.

The most important advantage of inertial navigation lies in the fact that the mobile device can operate completely autonomous. It does not depend on any infrastructure. The most important drawback of inertial navigation is that the location of a device cannot be observed directly from within the inertial frame of the mobile device. Hence, measurement errors in sensor data will accumulate over time rendering inertial navigation systems useless after a specific amount of time.

Typical inertial navigation systems are based on using an inertial measurement unit (IMU) containing six elementary sensors measuring acceleration in three pairwise orthogonal directions and three gyroscopes each measuring rotation around one axis. Altogether, these IMUs can be used to update the location and pose of the mobile device.

Inertial navigation systems in general use the directly measured acceleration vector $\ddot{p}$ and calculate the current velocity using the following integral:

$$\dot{p}(t) = \int_{t_0}^{t} \ddot{p}(\tau) d\tau + \dot{p}_0.$$

This amounts to knowing the velocity vector at time t. In the same way, one can again integrate the velocity vector to obtain the location function assigning location to the time variable t as follows:

$$p(t) = \int_{t_0}^{t} \dot{p}(\tau)d\tau + p_0.$$

In order to use such an approach, one needs two information about the initial state of the mobile device:

- p_0: the location vector describing the location of the mobile device at time $t = t_0$
- $\dot{p}_0$: the initial velocity vector describing the velocity of the mobile entity at time $t = t_0$

For all practical purposes, it is pretty difficult to use inertial navigation systems due to the fact that the doubled integration is also applied to the error vectors inherently contained in the measurements of acceleration. These can quickly accumulate to a wrong movement state and over time to arbitrary large errors in position. If, for example, the mobile device is accelerated and then again stopped, the errors will seldom cancel and the estimated speed will be nonzero; hence, the estimated location drifts away from the actual location. This is even more problematic, when the earth acceleration has to be removed from the accelerometer readings based on other erroneous measurements.

Due to these problems, inertial navigation is often used only together with another positioning system able to recalibrate the movement state of the mobile device. One prominent example of this is given by inertial navigation using foot-mounted IMUs. In these cases, it is possible to reset the complete movement state to zero during the time where the foot remains in contact with the floor. These zero velocity updates (ZUPT) can lead to very good overall system performance as the errors can only accumulate during the short time frame in which a foot moves freely in air.

3.1.7 Fingerprinting

The previously described approaches are all based on observing a known physical relation between some measurable size and location. In contrast to that, a lot of approaches exist, which do not rely on any such relation but are rather based on reproducibility of patterns of measurable variables. In this way, the more complex an environment and the behavior of underlying physics becomes the more difficult can the physical laws be used to infer the location. However, these complexities make data locally unique and distinguishable leading to a new technique of location determination known as fingerprinting.

The set of measurements at a specific location is similar to the set of measurements taken at the same location at another time or with another device, but not

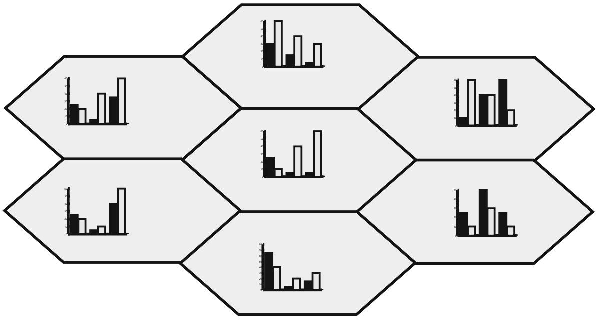

Fig. 3.6 Example of using scene analysis techniques

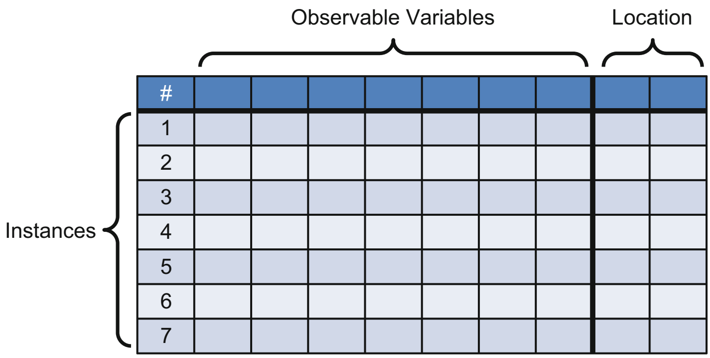

Fig. 3.7 Data mining problems represented by a table

similar to measurements taken at another place. Figure 3.6 illustrates this concept using a cell subdivision of an area and assigning a histogram describing each location cell. This histogram could be given by signal strength of GSM base stations or similar location-dependent measurements.

In general, fingerprinting is a class of algorithms, which are often formulated as classification or regression problems in the form of a data mining problem. These are typically based on two phases: a training phase, in which relations between measurements, coordinates, and labels are collected and stored in a training dataset. Such a dataset is typically a table of measured variables (or variables directly calculated from atomic measurements) and location. Figure 3.7 depicts this situation. The rows of the table are called instances and combine a set of data at a given situation. In the case of localization, this consists of all measurements made at one fixed location called observable variables in the figure, as well as a description of location given by a label or coordinates. When several parameters cannot be observed in one instance, the cells can be marked as unknown. Some data mining schemes can perfectly deal with missing values in both the observable variables and the label or location.

A data mining algorithm is now given such a table including the rows describing location. Using this data, the data mining algorithm takes an instance containing only observable variables as input and tries to provide a reasonable estimate of location.

The algorithms for this type of inference problems can be roughly grouped into the following three classes depending on the question, which information is actually available at the time of building a model out of the training data:

- *Supervised Learning*: For each instance of the training dataset, the location is given.
- *Unsupervised Learning*: Instead of searching for a model predicting location out of observable variables, unsupervised learning tries to find relations between attributes of instances.
- *Semi-supervised Learning*: Both approaches are combined—a prediction model is built from the training data containing location, and unsupervised learning is being used to enhance this model further.

In general, one must take care that the data mining model generated from a training set does not describe the training dataset too closely. Because then, the model will have difficulties to deal with unknown instances. Hence, the success rate on the training set will be rather high, while the model has problems in generalizing the extracted knowledge to unknown situations. This problem is very common and called *overfitting*. Overfitting becomes easily possible with small training datasets in which the randomness of noise influences cannot be observed and the actual noise sizes are used during classification.

For the case of classification, that is, assigning a label to an unlabeled instance using a training set of labeled instances, a simple and well-known method is given by *decision trees*. A decision tree is usually defined to be a binary tree in which every inner node including the root node contains a test comparing observable variables. For an unknown instance, these tests can be performed, and depending on the result, the left or ride subtree can be expanded until one reaches a leaf node. The leaf nodes then contain the label to be predicted. From a geometrical point of view, the set of training instances consists of points in a high-dimensional space, one dimension for each attribute. A decision tree is then usually constructed in a way such that each level splits the set of training instances into two similar parts. A very classical definition of how to split is the amount of information one gains with a split. The well-known ID3 algorithm splits on single attributes using a threshold. That is the comparisons inside the inner nodes are of the type $a < t_a$ for an attribute a and a threshold t_a. Instances with attribute value smaller than the threshold are descending the left subtree and instances with attribute value higher than the threshold descend the right tree. The split is constructed using the information gain that is a measure about how informative a split was. It is given as the difference between the entropy (see Sect. 2.1.2) of the current training set and the entropy of the set after splitting it using the current attribute. Let therefore A denote some attribute; S denote the current set of training instances; T be the splitting of S using A, that is, $S = \cup_{t \in T} t$; and $p(t)$ the relative count of elements of $t \in T$ with respect to T. Then let H

denote the entropy. The information gain of an attribute together with a splitting T using this attribute is given by

$$\textbf{InfoGain}(\text{attribute}) = H(S) - \sum_{t \in T} p(t)H(t).$$

The ID3 algorithm now uses the attribute, which provides the highest information gain.

Another fundamentally different method of learning from examples is based on the famous Bayes theorem:

$$P(I|M) = \frac{P(M|I)P(I)}{P(M)}. \tag{3.6}$$

This equation about probabilities means that the probability $P(I|M)$ that an event I happened when M has been measured can be calculated from the right-hand factors. From a practical point of view, this means that we can predict probabilities of events from measurements, when we are able to fill in probabilities to the right-hand side. The right-hand side contains the probability $P(M|I)$ that a measurement M has been made in cases where the event I actually happened and can easily be calculated from a training set. Note that a training set contains measurements associated to events. This has to be multiplied by the overall probability of event I happening. This again can be easily estimated from relative fractions calculated over a training set. This product is then to be divided by $P(M)$, a probability to get measurement result M in general.

Let, for example, I describe the location given by location labels and let M describe a measurement of signal strength of some Wi-Fi access points. Then $P(I|M)$ can be calculated for each location I and gives the probability of being at location I. The right-hand values can be estimated from the training dataset as explained before. Note that the denominator $P(M)$ of Eq. (3.6) can be safely ignored as it does not vary for a single measurement. It suffices to calculate the nominator of Eq. 3.6 for every location I and normalize the resulting values such that they sum up to one.

The *naive Bayes* algorithm makes an additional assumption. It assumes that the attributes are pairwise statistically independent. Let the attributes be named $a_1, \ldots, a_n$. Then, under the assumption of independence, the right-hand factor $P(M|I)$ can be further simplified to

$$P(M|I) = \prod_{i=1\ldots n} P(a_i, I),$$

which makes the calculation of relative counts possible on a per attribute basis in the dataset. This algorithm is called naïve as the assumption of independence is often not justified in practice. However, this algorithm still provides good results in many cases, and in other cases, one can try to automatically reduce redundancy of attributes beforehand such that the input attributes to the naïve Bayes algorithm are more or less independent.

When data mining approaches are used to infer the location of a mobile device inside buildings, they are usually called *fingerprinting* algorithms. The seminal work RADAR [1] can be seen as one of the first working indoor positioning systems based on Wi-Fi signal strength uses a weighted k next neighbor (kNN) approach to estimate the location from incoming signal strength. Therefore, for an instance containing signal strength information, the k next neighbors with respect to signal strength vectors are searched within the training database. Their distance in signal space to the incoming signal strength vector is then taken to calculate the location result for the incoming signal strength vector as the weighted sum of the location of the k next neighbor data points from the training set.

3.2 Properties and Evaluation of Positioning Systems

The evaluation of positioning systems is often based on experiments in which a specific error measure of location is used in order to compare the performance of the positioning system. Before using such a measure, however, one should always divide the possible errors into two parts and evaluate them independently, if possible. The quality of a positioning system can be good or bad in two different dimensions as depicted in Fig. 3.8. The two different properties of a positioning system are called precision and accuracy. Though they have a similar meaning in everyday life, they denote quite different aspects for positioning systems. *Precision* measures the deviation of location estimates for the same location from each other, while *accuracy* measures the deviation from the truth. A very accurate system can be used for long-term location determination where the precision is not too relevant and errors cancel out over time. On the opposite, a precise but inaccurate system can be used to guide local decision or find proximity between two mobile devices, while it cannot be used to provide a reliable link to map information. In optimal cases, positioning systems are precise and accurate at the same time.

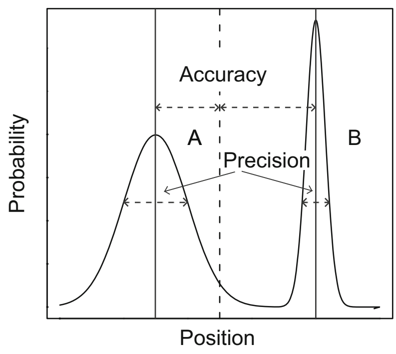

Fig. 3.8 Accuracy vs. precision for a one-dimensional positioning system

However, this is—in general—impossible. Another relevant property of positioning systems is the *spatial resolution* which is defined to be the minimal change of position of the mobile device that can be detected by the system. For proximity systems, this spatial resolution is rather large, while it can be quite small for systems based on lateration. Furthermore, the *temporal resolution* of a positioning system can be of interest: the minimum time for which a position must have changed before the position change can be detected. Again, if a proximity system is based on observing advertisements of a wireless infrastructure and these advertisements are only sent out once per minute, then it takes 1 min until the position can be updated. Furthermore, positioning algorithms can *lag* in time. The lag describes the time difference between an instance of having a specific position in reality and the instance of being informed by the positioning system about this location.

In numerical experiments, one has to carefully select statistical values which can indicate the given properties. However, it is not easy to choose the right ones. It has become common practice in the indoor positioning domain to report the mean of the deviation from the truth as an indicator of accuracy and the standard deviation or variance as an indicator for precision. Using the mean of the positioning error is very illustrative as it is a good estimate of the expectation of error: a user of the system should expect this amount of error for the next measurement. In other domains, especially with more emphasis on the theoretical background, the root mean squared error is used. This is due to the fact that the quality can then be measured globally by comparing it to the Cramer–Rao bound. The Cramer–Rao bound states that the variance of an unbiased estimator is at least as high as the inverse of the Fisher information matrix. Furthermore, an unbiased estimator which achieves this bound minimizes the mean squared error among all unbiased methods and is hence a minimum-variance unbiased estimator. Therefore, all unbiased estimators which try to minimize variance that is increase precision can be compared to this theoretical bound.

Figure 3.8 depicts the estimation of a single location coordinate by two different positioning systems of different accuracy and precision. In this figure, the precision amounts to the spread of the location estimate. The system A is clearly less precise as compared to the system B. The accuracy is given by the distance between the truth depicted as a dashed line in the middle and the estimate of both systems. Clearly, the system B is less precise as compared to the system A.

Several non-numerical aspects are also important in describing the properties of positioning systems. One of the most important properties is *mobility*. Does the positioning system provide position estimates to a device on the move or does it provide location only to devices with limited mobility such as devices inside a specific area of coverage. Furthermore, the *scaling variables* can differ between different positioning systems: for systems with few mobile devices, the price of a high-quality inertial navigation system can be justified. Think of a robot exploring mars as an example. When, however, the number of mobile devices is very high as compared to the area in which they must be located, then it might be better to install some expensive positioning infrastructure and provide the mobile devices with cheap devices such as RFID tags or UWB beacons.

With respect to cost, scaling, and privacy properties of a positioning system, it is common practice to classify positioning systems into the following three classes depending on their working principle [12]:

- *Terminal-based positioning* in which the mobile device calculates position without depending on some infrastructure.
- *Terminal-assisted positioning* in which the positioning is distributed between infrastructure elements and the mobile device.
- *Terminal-free positioning* in which a mobile device is located while the mobile device is passive.

At first sight, the class of terminal-assisted positioning can be a bit confusing, and it might make sense to further subdivide this class. GPS, for example, is clearly a terminal-assisted positioning system. However, the GPS infrastructure does not learn any location information about the mobile devices. It rather provides data from which the device can locally calculate position anywhere on earth. In other terminal-assisted scenarios, the mobile device is used to measure some information, while the infrastructure actually calculates and hence knows the position of the mobile device.

3.3 Examples of Positioning Systems

Inside buildings it is often difficult to estimate the distance between two fixed points due to multipath effects. When trying to measure the distance between two points which are not in line of sight, one estimates the length of the transmission path, which can be quite different from the distance between the two points. Therefore, the lateration approaches often lead to imprecise results unless they are under in line-of-sight conditions. For the measurement of distances, there are two general types of approaches: one is based on signal timing. The distance is then defined by the propagation speed of the signal and the time between sending and receiving a signal. On the opposite, one can use signal strength measurements to estimate distance using a model for the signal strength.

For the timing approach, one considers three general classes depending on which time to measure, namely:

- *Time of Arrival:* The absolute point in time at which some signal (e.g., light, sound, radio) set out at some known place and time reaches the mobile device can be measured.
- *Time Difference of Arrival:* The time difference between two signals sent out from different places at the same time can be measured by the mobile device.
- *Roundtrip Time of Flight:* The time difference between sending out a signal and receiving a reflection of the same signal is measured. Note that reflections can be reflections of the signal in the physical sense or messages sent in response to the package such as ACK frames.

The distances estimated using a time-of-arrival approach together with the propagation speed of the signal can be together with the algorithm of lateration to estimate a location. The most important drawback of this type of positioning system is the fact that the infrastructure as well as the mobile device needs to be tightly time synchronized. If, for example, the clock error at the mobile device is 1 μs and the system uses radio communication for positioning, this time error introduces a length estimation error of

$$s = c_0 t = 300{,}000{,}000\,\mathrm{m/s} \cdot 0.000001\,\mathrm{s} = 300\,\mathrm{m}.$$

Using audio signals such as ultrasonics with the same type of approach leads to a much better localization estimation due to the slow propagation speed of sound of approximately 343 m/s:

$$s = c_0 t = 343\,\mathrm{m/s} \cdot 0.000001\,\mathrm{s} = 3.43\,\mathrm{mm}.$$

A very classical positioning system based on time of arrival is GPS, and the most challenging part of GPS is to let the satellites have a consistent clock. Therefore, each GPS satellites is equipped with an atomic clock, and the first phase of GPS localization is to get the mobile device tightly time synchronized to the satellites.

In many cases, it is infeasible to let the mobile device be synchronized with some infrastructure. One reason for that is given by the problem that time synchronization algorithms in computer networks are often based on timing of messages which change for mobile devices. For time-difference-of-arrival positioning, the mobile device does not need to be synchronized with any infrastructure as it uses its local clock only to measure time differences independent from a global time. This is especially useful in cases where the infrastructure is already time synchronized for other reasons. In cellular phone networks, time synchronization of base stations is often given and is relatively precise as the stations often use GPS for time synchronization. Then it is easy to let the infrastructure send out pilot signals at the same time, and a mobile device just measures time offsets between the reception of these messages.

Roundtrip time of flight-based positioning systems do not need any time synchronization which is their central strength. However, the drawback of this setting is that the timing information need not be correct when non-physical reflections are being used. The same influences and uncertainties occur as discussed together with the two-way message exchange time synchronization in Sect. 2.3.1.1.

The following section collects several typical indoor positioning systems for each of the previously explained approaches. The reader should also consult classical surveys such as [9, 10].

3.3.1 Pseudolites and High Sensitivity GNSS

The radio signals of different satellite-based positioning systems including GPS, GLONASS, and Galileo suffer very much from path loss and multipath effects inside buildings. The received signal strengths of GPS signals are often smaller than the sensitivity of typical GPS receivers. Moreover the assumption that the signal travels a direct line between the satellite and the position on earth is often wrong. The receiver often receives reflections and, thus, estimates wrong distances to the different satellites due to differences between the direct distance and the length of the propagation path of the signal.

In order to use satellite positioning systems inside buildings, these two problems need to be addressed. One idea is to produce highly sensitive signal receivers. If the sensitivity is high enough and the path loss is small enough, the line-of-sight signal could be received as it will be the first signal though not the strongest one. However, in large buildings with many of floors above a receiver, the line-of-sight signal cannot be received even by highly sensitive receivers. Therefore, this approach is applicable in halls with simple roofs. The most prevalent application domain is logistics where a coarse position with accuracy of about 10 m can easily suffice to locate a fork shift inside a hall.

Another approach to enabling GNSS positioning inside buildings is by providing the signal with a fake infrastructure of pseudolites. A pseudolite emulates the signals a mobile device would receive outside from a satellite. Pseudolites provide quite accurate positioning inside a limited area, where the mobile device has free line of sight to the pseudolites. However, they are also very expensive due to the inherent sensitivity of GPS with respect to time delays. In complex multipath environments, pseudolites can only be used inside regions with line-of-sight conditions between the pseudolites and the mobile devices. Hence, for a complete coverage of a complex building, a lot of pseudolites are needed [3].

Another example of extending the coverage of GNSS systems inside buildings is given by the Locata system. This system is based on a time-synchronized network of base stations inside buildings sending signals similar to GPS but inside the license-free ISM band. These signals can then be used by special devices together with GPS signals to infer the position of a mobile device both outside and inside a building with very high accuracy [17].

Another approach of extending GNSS coverage into buildings is given by modeling all complexities of signal propagation. Therefore, very detailed three-dimensional models of buildings including building material and furniture can be used to calculate the expected dominant propagation path of the satellites. If these models are sufficiently correct, then high sensitivity GNSS receivers can be used without the assumption of direct line-of-sight connection to the satellites increasing accuracy [11, 21]. Though these approaches are promising as there is no new infrastructure as, for example, with the Locata system, the creation of sufficient models of buildings is very expensive, and positioning will still be limited to areas, where a high sensitivity GNSS receiver is able to detect the signals and the building model is sufficiently correct.

3.3.2 Light-Based Systems

Using special light-based techniques for indoor positioning is often motivated from the fact that most building material can reflect light. Moreover, these reflections are more deterministic as compared to other signals, and due to the high frequency of the radio waves, pulses can be quite short allowing for highly accurate line-of-sight ranging. The most prevalent systems use roundtrip time-of-flight approach together with physical reflection of modulated light waves. The modulation is only used to distinguish between scattered light and the reflection. These systems are often called *LiDAR systems (light detection and ranging)*. This name has been chosen for its similarity to RADAR. For RADAR systems, radio signal reflections are detected very similar to LiDAR but with higher range. A very typical LiDAR-based system is presented in the work [20] in which LiDAR depth information is used together with an inertial sensor system to derive position. As a natural extension, LiDAR systems can be used to generate maps of the surroundings using techniques of simultaneous localization and mapping (SLAM). A depth image similar to the depth matrix generated by LiDAR systems can be generated in a simpler fashion with limited range and accuracy. The Microsoft Kinect, for example, sends out a pattern of light. The reflections of this pattern are recollected using a camera. From the distortions of the light pattern, a distance is calculated for each pixel of the camera detecting the light pattern. Unfortunately, the range of such an approach is quite limited due to the resolution needed for the camera and the intensity of light needed to make distant patterns detectable to the camera.

A completely different approach to light-based positioning systems is given by light-based proximity detection. Therefore, a modulated light signal is sent out by a mobile device containing an identification of the mobile device. This signal is then captured by a sensor network, and the location of the sensors detecting the signal can be used to infer the location of the mobile device. One of the first systems of this type is the Active Badge system. In the Active Badge system, sensors are mounted to the ceiling detecting infrared signals from badges placed on the shoulder. Of course, this system can also be inverted by making the mobile device detect light signals sent out from a distributed infrastructure. However, for Active Badge, the badges do not need any capabilities except simple light modulation, whereas the other case is better suited for mobile devices with computational capabilities.

3.3.3 Camera-Based Systems

Camera-based systems aim to extract location and movement information out of the same information as a human being out of his visual sense. This approach is promising as it is known that human orientation is mainly based on visual information. However, we are not yet able to reach the same accuracy of orientation using camera systems.

In general, there are two possible deployments for a camera-based positioning system: either the camera is given to the mobile device and location of the mobile device is extracted from the point of view of the mobile device or the camera is mounted to the building and movement information is extracted from the location of a person or object inside the camera stream.

The former approach is often treated similar to the problem of scene analysis. The challenge is to assign location to a camera image. The localization part of the latter approach is often quite easy; however, the correct identification of mobile objects is challenging.

For a mobile camera system, some information is typically extracted from the camera pictures including landmarks, feature points, or geometric peculiarities. These are then compared to a database of these features referenced to location. In some systems, specific landmarks with high probability of reidentification are observed. Some approaches put synthetic landmarks such as barcodes into the environment, while others try to find natural, distinctive landmarks.

A third class of camera-based positioning systems consists of systems trying to extract the camera egomotion out of a sequence of images. Therefore, techniques such as optical flow extraction or SLAM can be used.

Some extraordinary camera-based positioning systems use the camera for example for monitoring the floor space comparably to an optical computer mouse [5, 15].

For the class of stationary camera systems, the extraction of location is relatively simple. An empty scene has to be recorded as a ground truth. Significant changes to this base scene are given by mobile objects; their depth can often be extracted from the X–Y coordinate inside the picture assuming that the object moves on a planar floor. The challenging problem here is the identification of a mobile object in order to assign a location not only to an observation but to an individual. Linked to this problem is also the reidentification of mobile objects across different cameras. A camera can only cover a fraction of a building, and hence, objects can enter the field of view of a camera and leave it again. The question is now how to reidentify objects leaving one camera's field of view and entering another camera's field of view. Typical areas of application are process observation applications where the actual reidentification of objects is not too important. In these cases, the camera system differentiates between moving objects inside the field of view and tries to detect noticeable objects moving differently from the majority of objects.

3.3.4 Radio-Based Systems

Today, most positioning systems inside buildings rely on radio technology. With radio technology, it is possible to reach extremely high accuracy. Moreover, radio technology has seen large-scale deployment resulting in relatively cheap radio hardware. Additionally, radio communication infrastructures are basically everywhere. GPS reaches the whole surface of the earth, Wi-Fi enables location

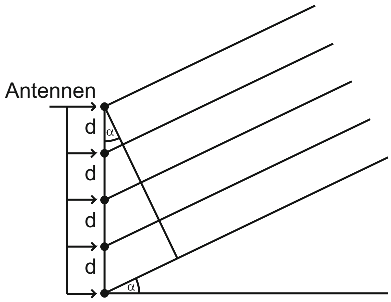

Fig. 3.9 Geometry of an antenna array

awareness without using GPS, and cell tower signals of cellular networks provide another wide-spread infrastructure of radio communication systems.

Positioning based on radio signals can be based on signal strength information as the signal energy decreases with distance. Other systems are based on accurate timing information. With ultra-wideband (UWB) signals, it is possible to transmit and detect very short pulses allowing very precise positioning due to the precise calculation of signal delays. Inside buildings, however, the length of the propagation path is not always a good indicator for the distance between the sender and the receiver. Thus, these systems are often limited to line-of-sight conditions, and a lot of infrastructure is needed to cover several rooms and floors.

A third class of radio positioning systems is based on angle estimation. An array of antennas can be used to determine the angle from which a radio signal has been sent out. Alternatively, the same antenna array can be used to transmit radio signals into a specific direction. If the sender and receiver are far enough from each other, it is admissible to model the radio waves propagation direction as parallel lines as depicted in Fig. 3.9. In this case, the time delay of the same signal (e.g., the phase difference) at those different antennas can be measured and results in an estimate of the angle. In the opposite case, the signal can be sent out into a specific direction by choosing appropriate delays for the different antennas. The superposition of waves is then maximally constructive into the direction given by the angle α. The situation depicted in Fig. 3.9 leads to the following equation relating the angle α to the time delay Δt:

$$\Delta t = \frac{d}{c_0} \sin \alpha.$$

Finally, some radio-based systems use the complexity of signal propagation inside buildings and try to collect reproducible features changing with location. A very classical example for this type of localization is RADAR [1]. RADAR collects Wi-Fi signal strength together with location in a training phase. The localization is then performed by assigning to incoming signal strength information the location of the most similar database entries. It has been shown in experiments that this information is actually characterizing location as long as there is not too much change to the building. In practice, systems based on fingerprinting are sensitive to changes in the surroundings and need regular recalibration. The main advantage of using Wi-Fi fingerprinting is, however, that an existing and ubiquitously available infrastructure can be used. Wi-Fi access points, in general, send out their identifications regularly, and the signal strength of these beacon packets can be measured by all Wi-Fi devices as this signal strength is also used to manage handover decisions for Wi-Fi networks with multiple access points. Furthermore, mobile devices regularly send out scan requests to the surrounding access points in order to quickly detect handover situations and to discover hidden access points. Therefore, the infrastructure of Wi-Fi access points is also able to collect signal strength information of mobile devices in many cases.

Another radio technology capable of positioning devices is given by *RFID* deployments. RFID systems are based on two components: RFID readers and RFID tags. The tags are small and cheap electronic components often only able to store and communicate an integer number used for identification of mobile objects. These tags can be either passive or active. Passive tags do not need a current supply and are powered by the RFID reader using induction. Active tags possess their own power supply and can be used more flexibly. The range of RFID systems is typically very limited making proximity-based positioning possible. Therefore, RFID readers are commonly placed in sensible locations such as doors and can be used to split the navigation space into zones. For each tag attached to some mobile object, the zone of residence is then known.

3.3.5 Inertial Navigation

Inertial navigation systems subsume all navigation systems based on measuring changes of the inertial system of the mobile device. As these changes reflect only relative measures, no absolute position can be calculated. Sensors for this type of navigation systems include accelerometer, gyroscopes, odometers, and magnetometers. In this area, only few and very specialized systems for the indoor area have been successfully demonstrated. This is due to the inherent inaccuracy of inertial navigation by errors accumulating over time. Therefore, the inertial navigation system needs external support. Sometimes, this can be provided from another positioning system; sometimes this is just given by points in time, where it is possible to reset the movement state to zero. These zero-update systems often use foot-mounted IMUs and reset movement to zero during the phase where the foot is fixed to the ground. It is also possible to integrate map information into the calculation of position using particle filters. Due to the quick change of sensor quality and the general interest in inertial navigation systems, a lot of work has been done in the last years including [4, 6, 8, 13, 19, 23, 24].

The area of integrated solutions consisting of some coordinate-based positioning system with possibly slow update rates aided with a high-update inertial navigation system between position fixes from the coordinate system is very promising and can lead to an overall system with high-update rate and limited error.

3.3.6 Audio-Based Systems

Audio-based systems use the propagation of audio waves in space in order to locate a mobile device. Especially with sound, several different physical effects can be used to infer the location of the mobile device. Simple audio-based systems provide ultrasonic signals not disturbing humans to identify locations and provide proximity-based awareness similar to the Active Badge system. Due to this similarity, one of the classical such systems is called Active Bat system [22]. This system is based on a sensor network of ultrasonic microphones and detects mobile devices sending out ultrasonic identifications up to centimeter accuracy. The propagation of sound inside buildings is very natural and often better than the propagation of light signals. A lot of building material reflects and scatters sound. In this way, semantic places such as rooms or hallways can easily be filled with the same sound identification, which is not spreading to adjacent rooms too much.

Due to the slow propagation speed of audio (approx. 343 m/s), it is possible to use several microphones to detect the angle out of which a specific audio signal has been received with great accuracy. This can even be used to localize the origin of a shoot in military applications [2].

Another approach to using audio information for indoor localization is given by recording the typical ambient noise of different rooms inside a building. Approaches of this type are quite similar to Wi-Fi fingerprinting in that they take an audio signal of some fixed duration and calculate a fingerprint out of this sound signal in a manner such that the fingerprint is characteristic for location: it changes across different locations while it is reproducible at a fixed location over time. This type of positioning system is, of course, only applicable in cases where a typical ambient noise exists and varies sufficiently between locations. The most promising application domain is thus given by industrial environments.

3.3.7 Pressure-Based Systems

An exceptional positioning system called Smartfloor is based on distributing pressure sensors across the floor measuring the presence of objects by their pressure induced against ground [16]. The main advantage of such a system is its unobtrusiveness. The localized objects do not need to interact in any non-natural way with the environment. Just as with camera infrastructures, the main problem with such an approach is with respect to identification and reidentification of objects inside crowded scenes. This approach is a very promising approach for applications of AAL as these overcrowded scenarios are less likely to occur and the risk for a patient is low when many other people are around. For these applications, the unobtrusiveness is the most important factor in order to allow a system to monitor the patient movement at any time.

Table 3.1 Most important characteristics of positioning approaches

Algorithm	Input sizes	Limitations
Lateration	Length, distance, time	Time synchronization, multipath
Hyperbolic lateration	Length differences, delays	Only infrastructure needs to be time synchronized
Angulation	Angles, phase differences	Multipath
Proximity detection	Visibility, physical proximity	Simple and reliable, often only coarse location
Inertial navigation	Acceleration, rotation, movement	Errors accumulate Independent from infrastructure
Fingerprinting	Feature vectors	Stable with respect to multipath and complexities, sensitive to (small) changes in the environment
SLAM	Inertial navigation, depth images	Often accurate, independent from infrastructure, computationally challenging

3.4 Summary

As described in this chapter, the available technologies and algorithms for detecting the position of a mobile device are manifold. The first part of this chapter introduced the main algorithms for determining position from measurements, while the second part introduced actual examples for organizing along the measurable variables. A very short summary of algorithms is given in Table 3.1 in which the most important advantages, limitations, or disadvantages of different schemes are mentioned.

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