1)　$y = x^4 \cos x + 3\sin x - \ln x$

$$\frac{dy}{dx} = 4x^3 \cos x + x^4 \cdot (-\sin x) + 3\cos x - \frac{1}{x}$$

$$\frac{dy}{dx} = 4x^3 \cos x - x^4 \sin x + 3\cos x - \frac{1}{x}.$$

2)　$f(x) = \sin(x^2 + 6x + 5)$

$$f'(x) = \cos(x^2 + 6x + 5) \cdot (x^2 + 6x + 5)'$$

$$= \cos(x^2 + 6x + 5) \cdot (2x + 6)$$

3)　$f(x) = \arctan x - x$

$$\therefore f'(x) = \frac{1}{1+x^2} - 1$$

$$= \frac{-x^2}{1+x^2} < 0$$

$\therefore f(x)$ 是严格递减.

5) $\int \left(\frac{1}{\sqrt{1-x^2}} - \frac{1}{1+x^2} \right) dx \qquad = \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{dx}{1+x^2}$

$= \text{arc} \sin x - \arctan x + c$

4)
$$\lim_{x \to 1} \frac{\ln x}{x-1}$$

$$= \lim_{x \to 1} \frac{(\ln x)'}{(x-1)'}$$

$$= \lim_{x \to 1} \frac{\frac{1}{x}}{1}$$

$$= 1.$$

6) $\int_{1}^{2} \left(x + \frac{1}{\sqrt{x}} \right) dx.$

$= \left| \frac{x^2}{2} + 2\sqrt{x} \right|_{1}^{2}$

$= 2 + 2\sqrt{2} - \left(\frac{1}{2} + 2 \right)$

$= 2 + 2 \cdot 2^{\frac{1}{2}} - \frac{1}{2} - 2$

$= 2^{\frac{3}{2}} - \frac{1}{2}.$

7. $\displaystyle\int \frac{1}{(3-2x)^2} \cdot dx$

$x \Rightarrow 3-2x$

$= -\frac{1}{2} \displaystyle\int \frac{d(3-2x)}{(3-2x)^2}$

$= -\frac{1}{2}\left(-\frac{1}{3-2x}\right) + c$

$= \cancel{\frac{1}{2}} \cdot \frac{1}{2} \cdot \frac{1}{3-2x} + c$

$= \frac{1}{2(3-2x)} + c \ .$

8) $\displaystyle\frac{d}{dx} \int_x^4 \frac{t^2}{\sqrt{1+t}} \cdot dt$

$= \displaystyle\frac{d}{dx}\left(-\int_4^x \frac{t^2}{\sqrt{1+t}} \cdot dt\right)$

$= -\displaystyle\frac{d}{dx}\int_4^x \frac{t^2}{\sqrt{1+t}} \cdot dt$

$= -\displaystyle\frac{x^2}{\sqrt{1+x}} \ .$

9. $\int \arctan x \cdot dx$

$= x \arctan x - \int x\, d\arctan x$

$= x \arctan x - \int \frac{x}{1+x^2} \cdot dx$

$= x \arctan x - \frac{1}{2} \int \frac{d(x^2+1)}{1+x^2}$

$= x \arctan x - \frac{1}{2} \ln(1+x^2) + c$

10. 1) $f(x) = (x-1)(x-2)(x-3)(x-4)$

$f(x) = 0$

$\therefore x = 1, 2, 3, 4$

$\therefore f(1) = f(2) = f(3) = f(4) = 0$

由罗尔定理可知:
$f'(x) = 0$ 在区间 $(1,2)$ 至少存在一个定根.

$\therefore f'(x) = 0$ 分别在区间 $(2,3)$, $(3,4)$ 都至少存在一个实根.

$\therefore f'(x) = 0$ 为三次方程, 其根至多三个.

$\therefore f'(x) = 0$ 至少有 3 个实根, 区间分别是 $(1,2)$, $(2,3)$, $(3,4)$.

10.

2) $\displaystyle\int \frac{dx}{a^2\sin^2 x + b^2\cos^2 x}$, $(a, b \neq 0)$.

$$= \frac{d\left(\frac{a}{b}\tan x\right)}{ab\left(1 + \left(\frac{a}{b}\right)^2 \tan^2 x\right)}$$

$= \dfrac{1}{ab}\arctan\left(\dfrac{a}{b}\tan x\right) + c$