CS555B1 Data Analysis and Visualization

Lecture 9

One-Way Analysis of Variance

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One-Way Analysis of Variance

- Analysis of variance is a general term which involves breaking down the overall **variability** in a particular continuous outcome into pieces.

- It involves comparing the variability after accounting for a characteristic versus the remaining variability not explained by the characteristic and just inherent to the outcome.

- In **one-way analysis** of variance (**ANOVA**), we study groups that are defined based on the **value of one factor**.

- The goal of a one-way ANOVA is to **compare means across groups**.

- Within the ANOVA framework, we seek to make comparisons **across several groups** while **considering all of the data together**

One-Way Analysis of Variance

- To compare data across multiple groups, we will test the null hypothesis that the underlying population means are all equal versus the alternative that at least two of the underlying population means differ.

- If we have k groups and we denote μ_i as the true population mean for group i, then the hypotheses for the one-way ANOVA can be written as follows:

$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$ (All underlying population means are equal)

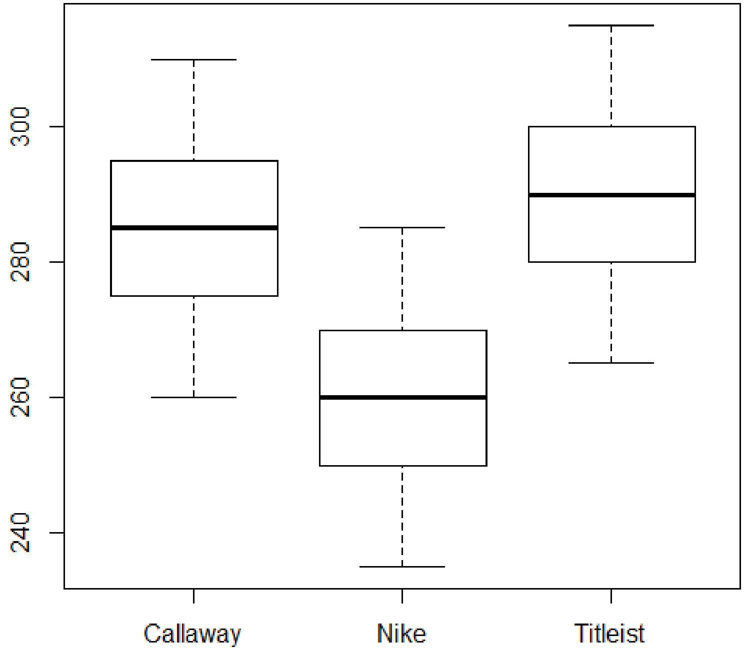
$H_1 : \mu_i \neq \mu_j$ for some i and j
(At least two of the k underlying population means are different or not all of the underlying population means are the same/equal)

One-Way Analysis of Variance - Example

Commercials aired on TV entice potential buyers to consider purchasing specific brand of **golf balls** by claiming increased **driving distances**. In order to see which brand of golf balls is best (as measured by distance traveled), an experiment is set up where a mechanical driver **hits 5 balls** of each of 3 brands. The distance in yards achieved after each strike is measured.

Table A. Distance by golf ball brand

Observation	Brand		
	Callaway	Nike	Titleist
1	275	235	265
2	310	285	300
3	285	270	280
4	260	250	315
5	295	260	290
Mean	285	260	290
Standard Deviation	19.0	19.0	19.0



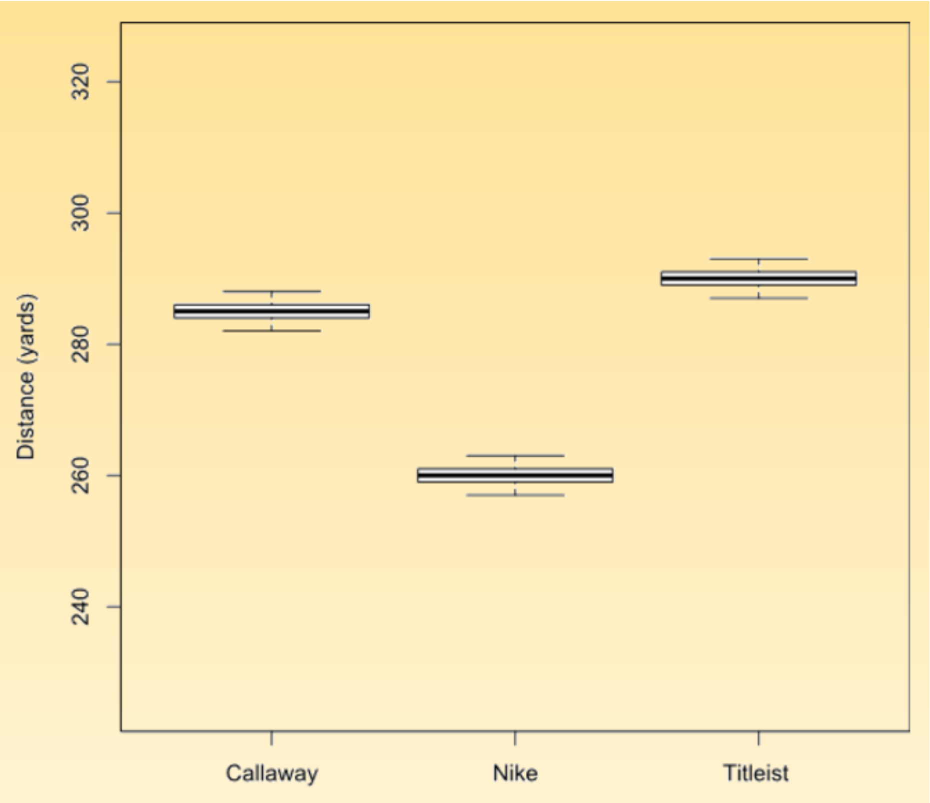
One-Way Analysis of Variance

In both cases, the sample means for each of the brands is the same. The difference between the two versions of the examples are **the amount of variability in the outcome.**

When the variability is small relative to the differences between means, **we become increasingly likely to declare that groups differ from each other.**

Table B. Distance by golf ball brand

Observation	Brand		
	Callaway	Nike	Titleist
1	288	257	291
2	286	263	293
3	284	261	287
4	282	260	289
5	285	259	290
Mean	285	260	290
Standard Deviation	2.2	2.2	2.2

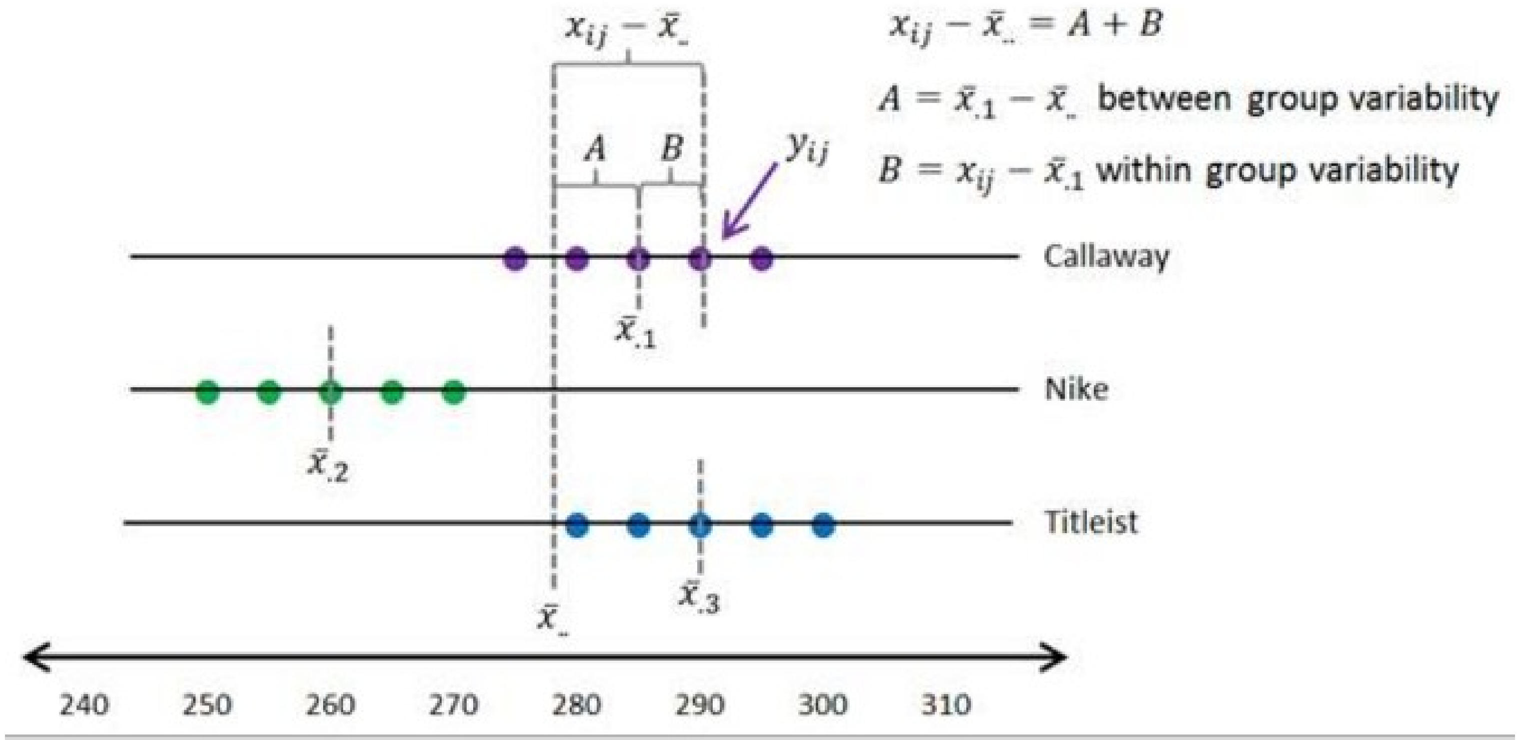


One-Way Analysis of Variance – an example

the deviation of any given point, x_{ij}, from the overall mean, $\bar{x}_{..}$, can be represented as:

$$x_{ij}-\bar{x}_{..}=x_{ij}-\bar{x}_{.j}+\bar{x}_{.j}-\bar{x}_{..}=(x_{ij}-\bar{x}_{.j})+(\bar{x}_{.j}-\bar{x}_{..})$$

The deviation between an individual point and the overall mean is comprised of two parts:
(1) the deviation between the individual point and the respective group mean
(2) the deviation between the group mean and the overall mean.



One-Way Analysis of Variance

- If the **variability between groups** is **small relative to** the variability in the measurements **within groups**, we are less inclined to conclude that there is a ***difference between them***.

- On the other hand, if the **variability between groups** is **large in comparison to** the variability **within each individual group**, it is easier to see and conclude that ***there is a difference***.

- **In ANOVA, we use the F-statistic** for this test where the F-statistic is calculated as

$$\begin{aligned} F &= \frac{s_b^2}{s_w^2} \\ &= \frac{\text{between group variance}}{\text{within group variance}} \\ &= \frac{\text{mean square between}}{\text{mean square within}} \end{aligned}$$

One-Way Analysis of Variance

- The numerator of the F-statistic is an estimate of the between group variation. It measures the variation among the k sample means.
- For this estimate of the variance, we want to measure how far the group means are from the overall mean (across all groups).

$$\begin{aligned} s_b^2 &= \text{mean square between} \\ &= \frac{\text{SSB}}{k-1} \\ &= \frac{\text{sum of squares between}}{\text{number of groups} - 1} \\ &= \frac{\sum_{j=1}^{k} n_{.j}(\bar{x}_{.j} - \bar{x}_{..})^2}{k-1} \end{aligned}$$

where $n_{.j}$ is the number of observations in group j , $\bar{x}_{.j}$ is the sample mean for group j, $\bar{x}_{..}$ is the overall mean (using all observations across all groups), and k is the number of groups.

One-Way Analysis of Variance

- The denominator of the F-statistic is an estimate of the within group variation. It measures the variation among individual measurements in the same group.
- For this estimate of the variance, we want to measure how far each individual data point is from the corresponding group's mean and take a weighted average.

$$
\begin{aligned}
s_w^2 &= \text{mean square within} \\
&= \frac{\text{SSW}}{n-k} \\
&= \frac{\text{sum of squares within}}{\text{number of observations} - \text{number of groups}} \\
&= \frac{\sum\sum(x_{ij} - \bar{x}_{.j})^2}{n-k} \\
&= \frac{\sum_{j=1}^{k}(n_{.j} - 1){s_j}^2}{n-k}
\end{aligned}
$$

where n is the number of observations across all groups, x_{ij} is the ith observation in the group j, $\bar{x}_{.j}$ is the sample mean for group j, $n_{.j}$ is the number of observations in group j, and k is the number of groups.

One-Way Analysis of Variance – an example

The overall mean, $\bar{x}_{..}$, is 287.33.

Observation	Brand		
	Callaway (1)	**Nike (2)**	**Titleist (3)**
1	280	260	280
2	275	255	290
3	290	270	295
4	295	265	300
5	285	250	285
Mean	285	260	290
Standard Deviation	7.9	7.9	7.9
Variance	62.5	62.5	62.5

$$
\begin{aligned}
\text{mean square between} &= s_b^2 \\
&= \frac{\text{sum of squares between}}{\text{number of groups} - 1} \\
&= \frac{\text{SSB}}{k-1} \\
&= \frac{\sum_{j=1}^{k} n_{.j}(\bar{x}_{.j} - \bar{x}_{..})^2}{k-1} \\
&= \frac{5 \cdot (285 - 287.33)^2 + 5 \cdot (260 - 287.33)^2 + 5 \cdot (290 - 287.33)^2}{3-1} \\
&= \frac{222.4445 + 1679.9445 + 680.9445}{2} \\
&= \frac{2583.3335}{2} \\
&= 1291.6668
\end{aligned}
$$

One-Way Analysis of Variance – an example

$$
\begin{aligned}
\text{mean square within} &= s_w^2 \\
&= \frac{\text{sum of squares within}}{\text{number of observations} - \text{number of groups}} \\
&= \frac{\text{SSW}}{n-k} \\
&= \frac{\sum\sum(x_{ij} - \bar{x}_{.j})^2}{n-k} \\
&= \frac{(280-285)^2 + (275-285)^2 + \cdots + (285-285)^2 + \cdots + (280-290)^2 + (290-290)^2 + \cdots + (285-290)^2}{15-3} \\
&= \frac{750}{12} \\
&= 62.5
\end{aligned}
$$

One-Way Analysis of Variance – an example

Equivalently

$$
\begin{aligned}
\text{mean square within} &= s_w^2 \\
&= \frac{\text{sum of squares within}}{\text{number of observations} - \text{number of groups}} \\
&= \frac{\text{SSW}}{n-k} \\
&= \frac{\sum_{j=1}^{k} (n_{\cdot j} - 1) s_j{}^2}{n-k} \\
&= \frac{(5-1)\cdot 62.5 + (5-1)\cdot 62.5 + (5-1)\cdot 62.5}{15-3} \\
&= \frac{4\cdot 62.5 + 4\cdot 62.5 + 4\cdot 62.5}{12} \\
&= \frac{750}{12} \\
&= 62.5
\end{aligned}
$$

One-Way Analysis of Variance – Characteristics

- The deviation between the individual point and the respective group mean **is representative** of the **within group variability**. The deviation between the group mean and the overall mean **is representative** of the **between group variability**.

- If the between group variability is large and the within group variability is small, then the **null hypothesis is often rejected** (in favor of the alternative hypothesis that the underlying means across group are not all equal).

- Large values of the F-statistic indicate that the variation between groups is larger than the variation within each individual group.

- To know how large is large enough to reject the null hypothesis, we use the **F-distribution** with **$k-1$ and $n-k$ degrees of freedom**.

Inference

- In ANOVA, the F-test derived from the ANOVA table is sometimes **referred to as the global test**.
- The global **F-test only has the ability to conclude that there are group differences** (if the null hypothesis is rejected), but **does not allow one to know which groups are different** without taking further steps to investigate where the differences lie.

	SS (Sum of Squares)	df (degrees of freedom)	MS (Mean Square)	F-statistic	p-value
Between	SSB	$\mathrm{SSB\ df} = k - 1$	$\mathrm{MSB} = \mathrm{SSB}/\mathrm{SSB\ df} = s_b^2$	$F = s_b^2 / s_w^2$	$P(F_{\mathrm{SSB\ df,\ SSW\ df},\ \alpha} > F$
Within	SSW	$\mathrm{SSW\ df} = n - k$	$\mathrm{MSW} = \mathrm{SSW}/\mathrm{SSW\ df} = s_w^2$		
Total	$\mathrm{Total\ SS} = \mathrm{SSB} + \mathrm{SSW}$				

F-test for ANOVA

The F-statistic is calculated as: $F = MSB/MSW$
which follows an F-distribution with $k-1$ and $n-k$ degrees of freedom under H_0.

The decision rule for a level α test is:
Reject $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$ if $F \geq F_{k-1,\, n-k,\, \alpha}$
Otherwise, do not reject H_0
(at least two of the k underlying population means are different)

where $F_{k-1,\, n-k,\, \alpha}$ is the value from the F-distribution table with $k-1$, $n-k$ degrees of freedom and associated with a right hand tail probability of α.

When $k=2$, the F-statistic above is the equivalent to the square of the t-statistic in the 2-sample test of means procedure under the assumption that the underlying standard deviations are the same.

Inference

- In ANOVA, the F-test derived from the ANOVA table is sometimes referred to as the global test.
- The global F-test only has the ability to conclude that there are group differences (if the null hypothesis is rejected), but does not allow one to know which groups are different without taking further steps to investigate where the differences lie.

	SS (Sum of Squares)	df (degrees of freedom)	MS (Mean Square)	F-statistic	p-value
Between	SSB	$\mathrm{SSB\ df} = k - 1$	$\mathrm{MSB} = \mathrm{SSB}/\mathrm{SSB\ df} = s_b^2$	$F = s_b^2 / s_w^2$	$P(F_{\mathrm{SSB\ df,\ SSW\ df},\ \alpha} > F$
Within	SSW	$\mathrm{Res\ df} = n - k$	$\mathrm{MSW} = \mathrm{SSW}/\mathrm{SSW\ df} = s_w^2$		
Total	$\mathrm{Total\ SS} = \mathrm{SSB} + \mathrm{SSW}$				

$$\mathrm{SSB} = \sum_{j=1}^{k} n_j (\bar{x}_{.j} - \bar{x}_{..})^2$$
$$\mathrm{SSW} = \sum\sum (x_{ij} - \bar{x}_{.j})^2 = \sum_{j=1}^{k} (n_j - 1) s_j{}^2$$
$$\mathrm{Total\ SS} = \sum\sum (x_{ij} - \bar{x}_{..})^2$$

SSB df = k-1 the degree of freedom of the sum of squares between.
SSW df = n-k the degree of freedom of the sum of squares within.
MSB=SSB/(k−1), the mean square between.
MSW=SSW/(n−k) the mean square within
F=MSB/MSW the F-statistic

p-value = the probability that the observed value of test statistic or a more extreme value could have been observed by chance.

ANOVA Table in R

Calculate using R commands:

```
> data <- read.csv("smoking_SBP.csv")
> is.factor(data$smoker)
> m<- aov(data$SBP~data$smoker, data=data)
> summary(m)
```

	SS (Sum of Squares)	df (degrees of freedom)	MS (Mean Square)	F
Between	$\text{SSB} = 2583.3335$	$\text{SSB df} = k - 1$ $= 3 - 1$ $= 2$	$\text{MSB} = \text{SSB}/\text{SSB df}$ $= 2583.3335/2$ $= 1291.6668$	$F = \text{MSB}/\text{MSW}$ $= 1291.6668/62.5$ $= 20.67$
Within	$\text{SSW} = 750$	$\text{Res df} = n - k$ $= 15 - 3$ $= 12$	$\text{MSW} = \text{SSW}/\text{SSW df}$ $= 750/12$ $= 62.5$	
Total	$\text{Total SS} = 2583.3335 + 750$ $= 3333.3335$			

Recap: factors in R

- Factors are variables in R which take on a limited number of different values; such variables are often referred to as categorical variables

- Factors are stored as a vector of integer values with a corresponding set of character values to use when the factor is displayed. The factor function is used to create a factor.

```
> data = c(1,2,2,3,1,2,3,3,1,2,3,3,1)
> fdata = factor(data)
> fdata
 [1] 1 2 2 3 1 2 3 3 1 2 3 3 1
Levels: 1 2 3
> is.factor(data)
[1] FALSE
> is.factor(fdata)
[1] TRUE
> rdata = factor(data,labels=c("I","II","III"))
> rdata
 [1] I   II  II  III I   II  III III I   II  III III I
Levels: I II III
```

F-test for ANOVA: an example

A random sample (n=19) of current light smokers, current heavy smokers, former smokers, and those who have never smoked was taken to determine if mean systolic blood pressure (SBP) differs across smoking status categories.

1. Set up the hypotheses and select the alpha level
$H_0: \mu_{heavy}=\mu_{light}=\mu_{former}=\mu_{never}$ (All underlying population means are equal)
$H_1: \mu_i \neq \mu_j$ for some i and j. (Not all of the underlying population means are equal)
$\alpha=0.05$

2. Select the appropriate test statistic
$F=\frac{MSB}{MSW}$ with k-1=3 and n−k=19-4=15 degrees of freedom

3. State the decision rule
F-distribution with 3, 15 degrees of freedom and associated with $\alpha=0.05$.
> qf(.95, df1=3, df2=15)
> $F_{3,15,0.05}=3.287$
Decision Rule: Reject H_0 if $F \geq 3.287$
Otherwise, do not reject H_0

F-test for ANOVA: an example (continued)

4. Compute the test statistic

$F = \frac{MSB}{MSW} = \frac{928.7}{43.2} = 21.49$

5. Conclusion

Reject H_0 since $21.49 \geq 3.287$. We have significant evidence at the $\alpha=0.05$ that there is a difference in SBP among current light smokers, current heavy smokers, former smokers, and those who have never smoked (here, $p < 0.001$ as calculated in R).

```
> data <- read.csv("smoking_SBP.csv")
> is.factor(data$group)
> m<- aov(data$SBP~data$group, data=data) #aov(data$response~data$group)
> summary(m)
```

```
            Df Sum Sq Mean Sq F value  Pr(>F)    
group        3 2786.2   928.7   21.49 1.1e-05 ***
Residuals   15  648.3    43.2                    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Evaluating group differences

If the global F-test is significant (that is, if the F-test testing rejects the null hypothesis in favor of the alternative, indicating that there are differences in group means), then it is of interest to further determine which of the population group means are different.

We perform testing on each pairwise comparison of interest. In order to test if $\mu_i = \mu_j$, we use a t statistic:

$$t = \frac{\bar{x}_i - \bar{x}_j}{\sqrt{s_p^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}}$$

where $\bar{x}_i \; and \; \bar{x}_j$ are the sample mean in groups i and j (respectively), s_p^2 is the estimate of the variance from the ANOVA model (and is equal to the mean square within, or s_w^2), and n_i and n_j are the number of observations in groups i and j (respectively), which follows a t-distribution with n−k degrees of freedom under H_0.

Evaluating group differences (continued)

The decision rule for a two-sided level α test is:

Reject $H_0: \mu_i = \mu_j$ if $|t| \geq t_{n-k,\alpha/2}$

Otherwise, do not reject $H_0: \mu_i = \mu_j$

where $t_{n-k,\alpha/2}$ is the value from the t-distribution table with n−k degrees of freedom and associated with a right hand tail probability of α/2.

We can also calculate the two-sided 100% × (1−α) confidence interval for the difference between means $(\mu_i - \mu_j)$ using the following formula:

$$(\bar{x}_i - \bar{x}_j) \pm t_{n-k,\alpha/2} \sqrt{s_p^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

We can say with 100% × (1−α) confidence that difference between the underlying means of groups i and j is between $(\bar{x}_i - \bar{x}_j) - t_{n-k,\alpha/2} \sqrt{s_p^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$ and

$(\bar{x}_i - \bar{x}_j) + t_{n-k,\alpha/2} \sqrt{s_p^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$

Example: t-test

The golf ball example.

1. Set up the hypotheses and select the alpha level

$H_0: \mu_{Titleist} = \mu_{Callaway}$

$H_1: \mu_{Titleist} \neq \mu_{Callaway}$

$\alpha=0.05$

2. Select the appropriate test statistic $t = \frac{\bar{x}_{Titleist} - \bar{x}_{Callaway}}{\sqrt{s_p^2\left(\frac{1}{n_{Titleist}} + \frac{1}{n_{Callaway}}\right)}}$

3. State the decision rule

Determine the appropriate value from the t-distribution table with $n-k=15-3=12$ degrees of freedom and associated with a right hand tail probability of $\alpha/2=0.05/2=0.025$

> qt(.975, df=12)
> $t_{12,0.025}=2.179$

Decision Rule: Reject H_0 if $|t| \geq 2.179$

Otherwise, do not reject H_0

Example: t-test (continued)

4. Compute the test statistic

$$t = \frac{\bar{x}_{Titleist} - \bar{x}_{Callaway}}{\sqrt{s_p^2\left(\frac{1}{n_{Titleist}} + \frac{1}{n_{Callaway}}\right)}} = \frac{290-285}{\sqrt{62.5\left(\frac{1}{5}+\frac{1}{5}\right)}} = 1$$

$$(\bar{x}_{Titleist} - \bar{x}_{Callaway}) \pm t_{n-k,\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_{Titleist}} + \frac{1}{n_{Callaway}}\right)} = (290\text{-}285) \pm 2.18\sqrt{62.5\left(\frac{1}{5} + \frac{1}{5}\right)}$$

=(-5.9, 15,9)

5. Conclusion

Do not reject H_0 since 1<2.179. We do not have significant evidence at the α=0.05 level that $\mu_{Titleist} \neq \mu_{Callaway}$. That is, we do not have evidence that the mean distances are different between Titleist golf balls and Callaway golf balls (p =0.34 as calculated using a software program). We are 95% confident that the true difference between the two is between 5.9 yards (favoring the Callaway balls) and 15.9 yards (favoring the Titleist balls).

Example: t-test R command

```
> golf <- read.csv("golfball_C.csv")
> golf
> attach(golf)
> aggregate(dist, by=list(brand), summary)
> aggregate(dist, by=list(brand), var)

> pairwise.t.test(dist, brand, p.adj='none')

Pairwise comparisons using t tests with pooled SD

data:  dist and brand

         Callaway Nike
Nike     0.00031  -
Titleist 0.33705  6.2e-05

P value adjustment method: none
```

Type I and type II errors

Type I Error: rejecting the null hypothesis when it is true (false positive)

The probability of making a Type I Error is controlled by the significance level of the test. It is generally the error rate that we worry the most about.

The potential implications of this type of error explains why we **specifying small values of alpha** (the significance level).

The potential implications of this type of error necessitates the **need to report p-values in your summary** of results.

		Reality (unknown in practice)	
		H_0 is true	H_0 is false
Decision based on sample	Reject H_0	Type I Error	Correct Decision
	Fail to reject H_0	Correct Decision	Type II Error

Type I and type II errors

Type II Error: failing to reject the null hypothesis when it is not true (false negative)

The probably of making a Type II Error is controlled by the sample size. The probability decreases with increasing sample size.

The quantity 1 – **Type II Error is known as the power of a test** (the probability of rejecting the null hypothesis when the null hypothesis is not true).

When we fail to reject the null hypothesis, it is often difficult to know whether or not we didn’t have enough power or if the null hypothesis was indeed true. This is **why we don’t say we “accept” the null hypothesis** and we instead state our conclusion as **“failing to reject”** the null hypothesis or “we do not have sufficient evidence to reject the null hypothesis.”

Since the **probability of this type of error decreases as the sample size increases**, type II error rates of 10% or less are often selected for such settings. In most other cases, a type II error rate of 20% is generally considered reasonable for planning purposes.

Issues with multiple comparisons

If there are k groups, the number of possible pairwise comparisons is: **$k \cdot (k-1)/2$**

If each of the possible $k \cdot (k-1)/2$ pairwise comparisons are performed at a significance level of α, then the expected number of false positives increases (up to as many as $\alpha \cdot k \cdot (k-1)/2$). Thus the expected number of comparisons that are significant by chance alone increases.

To maintain strong control of the error rate across all pairwise comparisons (often referred to as **the experiment wise or family wise error rate**) at a selected level, the significance level of **each individual test needs to be adjusted.**

Procedures for controlling the family wise **type I error rate** at a pre-specified level are called multiple comparison procedures.

These procedures work by essentially **making it “harder” to find differences between groups**. In other words, **more evidence against the null hypothesis is needed to reject the null hypothesis** when these procedures are implemented.

Issues with multiple comparisons (Continued)

Bonferroni adjustment: one of the simplest and most commonly used multiple comparisons procedures

To control the family wise error rate at the α level, individual **tests are performed at the $\alpha^*=\alpha/c$ level of significance where c is the number of individual comparisons to be performed and α is the family wise error rate that you wish to maintain.**

In general, the family wise **error rate is calculated using the formula $1-(1-\alpha)^c$ where c is the number of individual comparisons** to be performed and α is the significance level of each individual test.

Tukey procedure (or the Studentized Range test or **Tukey's Test of Honest Significance Test**) is another common multiple comparisons procedure which is commonly used to control the family wise **type I error rate** for pairwise comparisons.

It tends to **have more statistical power (is less conservative) than the Bonferroni** method, especially when there are a large number of pairwise comparisons.

Issues with multiple comparisons: the golf example

In the golf ball example, the global F-test showed that there was a difference in mean distance between brands.

Then we can **perform three pairwise comparisons** (Titleist versus Callaway, Titleist versus Nike, and Callaway versus Nike). However, we did not account for the fact that we were doing all three comparisons and did each at the $\alpha=0.05$ level when really we had wanted to control the family wise type I error rate at $\alpha=0.05$ overall.

The **Bonferroni methodology** suggests that individual tests should be performed at the $\alpha^*=\alpha/c$ level of significance, $\alpha^*=\alpha/c=0.05/3\approx 0.0167$. The critical value that we should have used in each comparison should have been $t_{n-k,\, \alpha^*/2}=t_{12,0.00833}=2.78$ instead of $t_{12,0.025}=2.18$.

Example – SBP by smoking status

Check if grouping variable (smoking status) is a factor

```
> is.factor(data$group)
```

Numerical and graphical summaries (Module 1 and 2)

- **Calculate mean, SD of SBP by groups**
- **Box plots and histograms**

```
> aggregate(data$SBP, by=list(data$group), summary)
> aggregate(data$SBP, by=list(data$group), sd)
> boxplot(data$SBP~data$group, data=data, main="SBP by smoking status",
xlab="group", ylab="SBP", ylim=c(100, 160))
```

Perform one-way ANOVA and if necessary, calculate the associated pairwise comparisons

```
> m<- aov(data$SBP~data$group, data=data)
> summary(m)
> pairwise.t.test(data$SBP, $data$group, p.adj=“bonferroni”)
> TukeyHSD(m)
```

One-way ANOVA: R commands

Use the aov() fundtion

> m <- aov(data$response~$data$group) # $group must be a factor
> summary(m)

Or use lm() function with dummy variables

> m2<- lm(data$response~$data$dummy1+$data$dummy2...)
> summary(m2)

If model is significant, then use pairwise.t.test() to compare means

> pairwise.t.test(data$response, $data$group, p.adj=“method”)
Note: method = “holm", "hochberg", "hommel", "bonferroni", "BH", "BY", "fdr", "none")

> **TukeyHSD**(m, conf.level = 0.95)
Note: The input to TukeyHSD() needs to be generated using aov(), not lm().
conf.level: the family-wise confidence level to use; default is 0.95

The golf example R command

```
> pairwise.t.test(dist, brand, p.adj='none')
```

Pairwise comparisons using t tests with pooled SD

```
data:  dist and brand
         Callaway Nike
Nike     0.00031  -
Titleist 0.33705  6.2e-05
```

P value adjustment method: none

```
> pairwise.t.test(dist, brand, p.adj='bonferroni')
 Pairwise comparisons using t tests with pooled SD
data:  dist and brand
         Callaway Nike
Nike     0.00093  -
Titleist 1.00000  0.00019
```

P value adjustment method: bonferroni

```
> aov(dist~brand, data=golf)
> summary(g)
> TukeyHSD(g)
```