Gravitation and Cosmology Notes

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Chapter 1

Special Relativity

The axioms of Special Relativity are:

1. The laws are invariant under change of inertial reference frames.

2. In inertial reference frames, there is an absolute speed of signal propogation $c = 1$.

1.1 Lorentz Transformations

Let x^μ, x'^μ be two coordinate systems and $c = 1$. For now we work in the classical vacuum and here experiments[1] point us to the fact that light travels at the absolute speed c. Suppose A, B are spacetime events representing emmission and absorption of light. By the second postulate, the distance between these two points is the same in both reference frames, and in particular, the quantity $(\Delta x^0)^2 - (\Delta x^1)^2 - (\Delta x^2)^2 - (\Delta x^3)^2 = 0 = (\Delta x'^0)^2 - (\Delta x'^1)^2 - (\Delta x'^2)^2 - (\Delta x'^3)^2$. This leads us to the definition of the **proper time**:

$$d\tau^2 := dt^2 - d\boldsymbol{x}^2 = -\eta_{\alpha\beta} dx^\alpha dx^\beta.$$

The notation $dx^\alpha dx^\beta$, in the mathematical sense, is a simple tensor $dx^\alpha \otimes dx^\beta$ and *not* the symmetric tensor. Using the Einstein summation convention and the fact that our metrics are always *symmetric*, this means that the contraction $\eta_{\alpha\beta} dx^\alpha dx^\beta = \eta_{00} dx^0 \otimes dx^0 + \eta_{10} dx^1 \otimes dx^0 + \eta_{01} dx^0 \otimes dx^1 + \cdots = \eta_{00} dx^0 \otimes dx^0 + \eta_{10}(dx^1 \otimes dx^0 + dx^0 \otimes dx^1) + \cdots$. Now we return back to proper time and see how far we can run with the concept.

Proposition 1. Let $\mathscr{L} \subseteq \mathrm{GL}_{\mathbb{R}}(3,1)$ denote the subgroup of linear operators that fix proper time: $d\tau'^2 = d\tau^2$. Then the set $\mathscr{L}$ can be described concretely:

$$\mathscr{L} = \{\Lambda \mid \Lambda^\alpha_\gamma \Lambda^\beta_\delta \eta_{\alpha\beta} = \eta_{\beta\delta}\}.$$

[1] Is there a better explanation?