

ARIMA processes

PRACTICAL TIME SERIES ANALYSIS

THISTLETON AND SADIGOV

Objectives

- ▶ Describe autoregressive, integrated, moving average models
- ▶ Rewrite autoregressive, integrated, moving average models using backshift and difference operators

ARMA processes

Remember ARMA(p, q) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + Z_t + \beta_1 Z_{t-1} + \cdots + \beta_q Z_{t-q}$$

can be written as

$$\phi(B)X_t = \beta(B)Z_t$$

where

$$\beta(B) = \beta_0 + \beta_1 B + \cdots + \beta_q B^q$$

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$$

- ▶ z – complex variable
- ▶ Roots of the polynomials $\beta(z)$ and $\phi(z)$ lie outside of the unit circle
- ▶ ARMA(p, q) process will be stationary and invertible

Non-stationary data

- ▶ Real life datasets are non stationary
- ▶ They might have a systematic change in trend
- ▶ We need to remove trend
- ▶ Difference operator $\nabla = 1 - B$

Difference operator

Remember

$$\nabla X_t = X_t - X_{t-1} = (1 - B)X_t$$

So, the random walk model

$$X_t = X_{t-1} + Z_t$$

can be written

$$\nabla X_t = Z_t$$

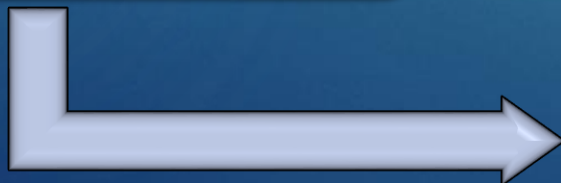
ARIMA(p, d, q) process

A process X_t is Autoregressive INTEGRATED Moving Average of order (p, d, q) if

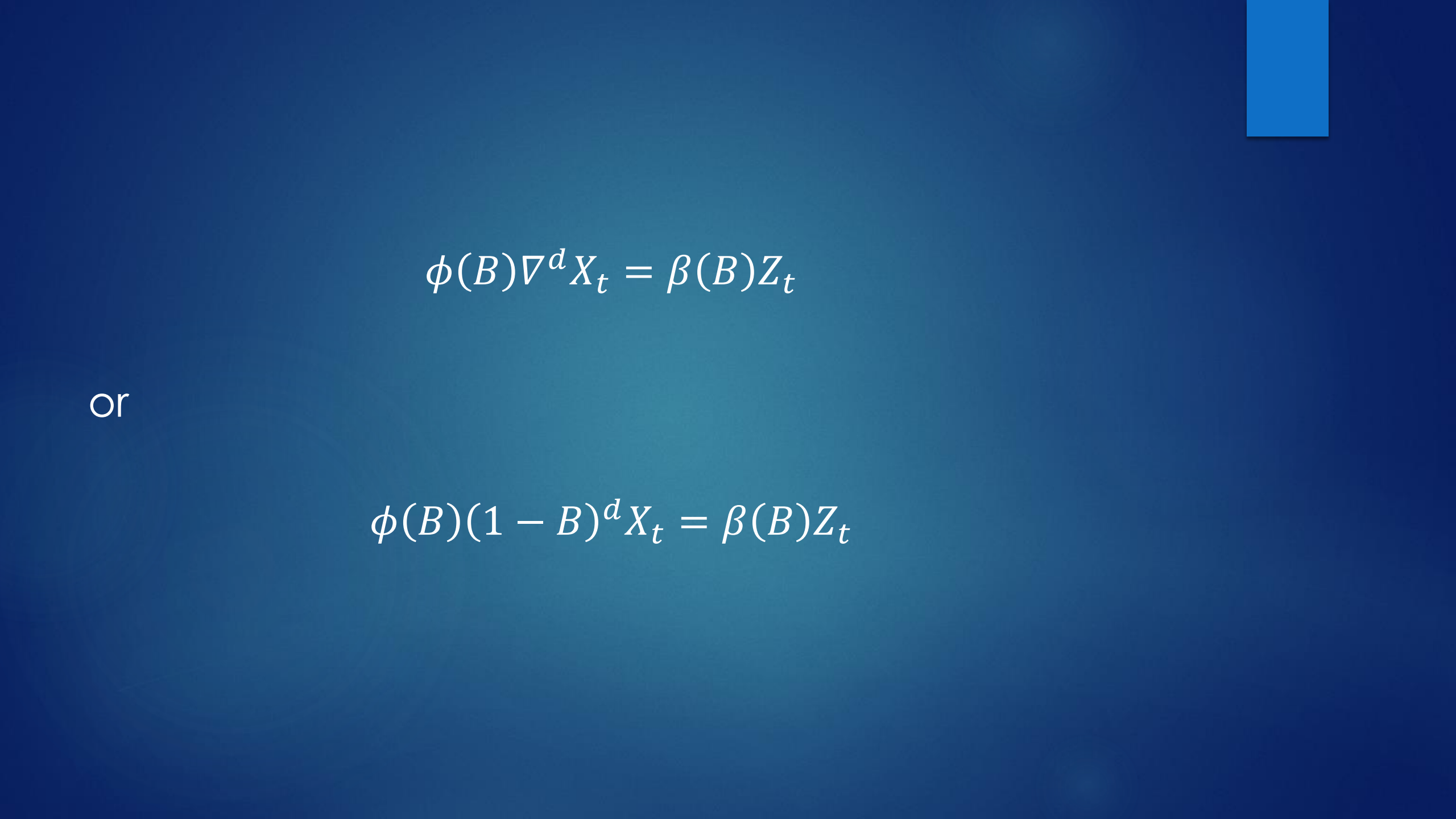
$$Y_t := \nabla^d X_t = (1 - B)^d X_t$$

is ARMA(p, q).

$$Y_t \sim \text{ARMA}(p, q)$$



$$X_t \sim \text{ARIMA}(p, d, q)$$


$$\phi(B)\nabla^d X_t = \beta(B)Z_t$$

or

$$\phi(B)(1 - B)^d X_t = \beta(B)Z_t$$

d – order of differencing

- ▶ $d = 1$ or $d = 2$
- ▶ Over differencing may introduce dependence
- ▶ ACF might also suggest differencing is needed
- ▶ $\phi(z)(1 - z)^d$ has unit root with multiplicity of d
- ▶ ACF will decay very slowly

Modeling

- ▶ Trend suggests differencing
- ▶ Variation in variance suggests transformation
- ▶ Common transformation: log, then differencing
- ▶ It is also known as log-return
- ▶ ACF suggests order of moving average process (q)
- ▶ PACF suggests order of autoregressive process (p)
- ▶ Akaike Information Criterion (AIC)
- ▶ Sum of squared errors (SSE)
- ▶ Ljung-Box Q-statistics (Next lecture)
- ▶ Estimation!

What We've Learned

- ▶ Describe autoregressive, integrated, moving average models
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Ljung-Box Q-statistic

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Objectives

- ▶ Define Ljung-Box Q-statistic
- ▶ Learn the decision rule to test the null hypothesis that several autocorrelation coefficients are zero
- ▶ Test the null hypothesis that several autocorrelation coefficients are zero using R

Portmanteau statistic

Box and Pierce (1970) proposed Portmanteau statistic

$$Q^*(m) = T \sum_{l=1}^m r_l^2$$

as a test statistic for the null hypothesis

$$H_0: \rho_1 = \rho_2 = \cdots = \rho_m = 0$$

against the alternative hypothesis

$$H_a: \rho_i \neq 0$$

for some $i \in \{1, 2, \dots, m\}$.

Under i.i.d condition of $\{r_t\}$,

$$Q^*(m) \sim \chi^2(df = m)$$

asymptotically.

Ljung and Box (1978) modified statistic to increase the power of the test in finite samples

$$Q(m) = T(T + 2) \sum_{l=1}^m \frac{r_l^2}{T - l}$$

Decision rule

We reject the null hypothesis if $Q(m)$ is large enough i.e.,

$$Q(m) > \chi^2_{\alpha}$$

where χ^2_{α} is $100(1 - \alpha)$ -th quantile of Chi-Squared distribution with m degrees of freedom.

Most packages will actually calculate p -value. We will reject the null hypothesis if the p -value is sufficiently small, i.e.

$$p < \alpha$$

where α is the significance level.

Choice of m and R routine

Usually we take

$$m \approx \ln(T)$$

R routine

Box.test(data, lag = log(T))

What We've Learned

- ▶ Define Ljung-Box Q-statistic
- ▶ Learn the decision rule to test the null hypothesis that several autocorrelation coefficients are zero
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ARIMA fitting: Daily female births in California in 1959

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Objectives

- ▶ Fit an ARIMA model to a real world data set
- ▶ Judge various fitting tools such as ACF, PACF and AIC
- ▶ Examine Ljung-Box test for testing autocorrelation in a time series

Modeling

- ▶ Trend suggests differencing
- ▶ Variation in variance suggests transformation
- ▶ Common transformation: log, then differencing
- ▶ It is also known as log-return
- ▶ ACF suggests order of moving average process (q)
- ▶ PACF suggests order of autoregressive process (p)
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- ▶ Sum of squared errors (SSE)
- ▶ Ljung-Box Q-statistics
- ▶ Estimation!

Daily female births in CA, 1959

- ▶ Time Series Data Library (TSDL)
- ▶ Created by Rob Hyndman, Professor of Statistics at Monash University, Australia.
- ▶ Link: <https://datamarket.com/data/list/?q=provider%3Atsdl>
- ▶ Category: Demography
- ▶ Name: [Daily total female births in California, 1959](#)
- ▶ 01 January 1959 – 31 December 1959
- ▶ Daily time series

Obtaining the data

- ▶ Click on the link: <https://datamarket.com/data/set/235k/daily-total-female-births-in-california-1959#!ds=235k&display=line>
- ▶ Export as CSV file
- ▶ Open the file, clean up the bottom row.
- ▶ Put the file into your working directory and read it to R
- ▶ OR read it directly from its path to R

ARIMA(0,1,2)

Then we have,

$$(1 - B)X_t = 0.015_{0.015} + Z_t - 0.8511_{0.0496} Z_{t-1} - 0.1113_{0.0502} Z_{t-2}$$

where moving average coefficients are significant in the level of 0.05, and indices are standard errors.

Thus, the fitted model is

$$X_t = X_{t-1} + 0.015 + Z_t - 0.8511 Z_{t-1} - 0.1113 Z_{t-2}$$

where

$$Z_t \sim \text{Normal}(0, 49.08)$$

What We've Learned

- ▶ How to fit an ARIMA model to a real world data set using various fitting tools such as ACF, PACF and AIC
- ▶ Examine Ljung-Box test for testing correlation in a time series