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Tutorial - 4

Q1 $T(n) = 3T(n/2) + n^2$

$T_n = aT(n/b) + f(n)$

$a = 3, \quad b = 2$ ($a >= 1$ and $b > 1$)

(Condition satisfied)

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^2$

case 3 satisfied : $f(n) > n^c$.

$T(n) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a = 4, \quad b = 2$ ($a >= 1, b > 1$) (satisfied)

$c = \log_b a = \log_2 4 = 2 \log_2 2 = 2$

$n^c = n^2$

case 2 satisfied.

$T(n) = \Theta(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

$T(n) = aT(n/b) + f(n)$

$a = 1, \quad b = 2$ $a >= 1, b > 1$

(satisfied).

$c = \log_b a = \log_2 1 = \log_2 2^0 = 0 \log_2 2$
$= 0$

$n^c = n^0 = 1$

$f(n) = 2^n$

Case 3 satisfied $f(n) >> n^c$

$T(n) = \Theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$T(n) = a\,T(n/b) + f(n)$

$a = 2^n$ $\quad a >= 1$ $\quad 2^0 = 1$

$b = 2$ $\quad b > 1$ (satisfied)

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

Case 2 satisfied because $f(n) >= n^c$

$T(n) = \Theta(n^n \log n)$

5. $T(n) = 16\,T(n/4) + n$

$T(n) = a\,T(n/b) + f(n)$

$a = 16$ $\quad (a >= 1)$

$b = 4$ $\quad (b > 1)$

$c = \log_b a = \log_4 16 = \log_4 4^2 = 2$

$n^c = n^2$

Case 1 satisfied $f(n) < n^c$

$T(n) = \Theta(n^2)$

6. $T(n) = 2\,T(n/2) + n \log n$

$T(n) = a\,T(n/b) + f(n)$

$a = 2, \; b = 2$ $\quad a >= 1, \; b > 1$

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

case 3 : $f(n) > n^c$

$T(n) = \Theta(n \log n)$

7. $T(n) = 2\,T(n/2) + \frac{n}{\log n}$

$T(n) = a\,T(n/b) + f(n)$

$a = 2, \; b = 2$ $\quad (a >= 1, \; b > 1)$

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

case 1 : $f(n) < n^c$

$T(n) = \Theta(n)$

8. $T(n) = 2\,T(n/4) + n^{0.51}$

$T(n) = a\,T(n/b) + f(n^c)$

$a = 2$ $\quad b = 4$ $\quad a >= 1, \; b > 1$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

case 2 = $f(n) = n^c$

$T(n) = \theta(n^{0.5} \log n)$

9. $T(n) = 0.5T(n/2) + 1/n$

$T(n) = aT(n/b) + f(n)$

$a = 0.5 \quad b = 2$ $(a > \not< 1)$ (condition not satisfied)

(Master's Theorem can't be applied)

10. $T(n) = 16T(n/4) + n!$

$T(n) = aT(n/b) + f(n)$

$a = 16 \quad b = 4$

$c = \log_b a = \log_4 16 = \log_4 4^2 = 2$

$n^c = n^2$

$n = 5 \; ; \; n^2 = 25$

$n! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$f(n) > n^c$ case (3)

$T(n) = \theta(n!)$

11. $T(n) = 4T(n/2) + \log n)$

$T(n) = aT(n/b) + f(n)$

$a = 4 \quad b = 2$

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2$

$n^c = n^2$

$f(n) < n^c$ = case 1.

$T(n) = \theta(n^2)$

12. $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

$a = \sqrt{n} \quad b = 2$

$a \not\geq 1, b > 1$

$c = \log_2 \sqrt{n}$

$n^c = n^{\log_2 \sqrt{n}}$

13. $T(n) = 3T(n/2) + n$

$a = 3, b = 2 \quad a >= 1, b > 1$

$c = \log_b a = \log_2 3 = 1.58$

$n^{1.58} = n^c$

$f(n) < n^c$ > case 1

$T(n) = \theta(n^{1.58})$

14. $T(n) = 4T(n/2) + cn$

$a = 4 \; b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2 \qquad f(n) = 2n$

Case 1 : $f(n) < n^c$

$T(n) = \theta(n^2 \log n)$

(15) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3 \quad b = 3 \qquad a >= 1, b > 1$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

case 1 $\quad f(n) < n$

$T(n) = \Theta(n)$

(16) $T(n) = 3T(n/4) + n\log n$

$a = 3, \quad b = 4, \quad a >= 1, \quad b > 1$

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

For $n = 5 \qquad (5)^{0.79} = 3.56$

$5 \times \log 5 = 3.49$

For $n = 20 = (20)^{0.79} = 10.66$

$= 20 \log 20 = 26.020$

$f(n) > n^c$ = case 3

$T(n) = \Theta(n \log n)$

(17) $T(n) = 3T(n/2) + \frac{n}{2}$

$a = 3 \quad b = 2$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58}$

$f(n) < n^c$, case 1.

$T(n) = \Theta(n^{1.58})$

(18) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3 \qquad (a >= 1, b > 1)$

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$f(n) > n^c \qquad$ (case 3)

$T(n) = \Theta(n^2 \log n)$

(19) $T(n) = 4T(n/2) + \frac{n}{\log n}$

$a = 4, \quad b = 2 \qquad a >= 1, b > 1$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

For $n = 10 = n^2 = 100$

$\frac{n}{\log n} = \frac{100}{\log 10} = 10$

$f(n) < n^c$ (case 1) $\quad \therefore \boxed{T(n) = \Theta(n^2)}$

20 $T(n) = 64T(n/8) - n^2 \log n$

Masters Method cannot be applied, because it cannot do negative work

21 $T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

$f(n) > n^c$ case 3.

$T(n) = \Theta(n^2)$

22 $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2 \qquad a >= 1, b > 1$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$ (case 3)

$T(n) = \Theta(n(2 - \cos n))$