DIFFERENTIATION TUTORIAL :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \qquad \frac{dy}{dx}$$

$$2ax + 2h1y + 2hxy' + 2yby' + 2g + 2fy' + 0 = 0$$

$$y'(2hx + 2f + 2yb) + 2ax + 2hy + 2g = 0$$

$$y' = -\frac{(2ax + 2hy + 2g)}{2hx + 2by + 2f}.$$

$$= -\left(\frac{ax + hy + g}{hx + by + f}\right)$$

Comprehension Pg. 159

$$-te^s = 1 - s$$
$$1 - te^s = 2 - s$$

(1) $s = 1 + t \cdot e^s$ $\qquad \dfrac{d^2 s}{dt^2} = ?$ $\qquad \dfrac{ds}{dt} = s'$ $\qquad \dfrac{d^2s}{dt^2} = s''$

$$1 \cdot s' = 0 + 1 \cdot e^s + t \cdot e^s \cdot s'$$

$$s' = e^s + (te^s)s' \longrightarrow s' = \frac{e^s}{1 - te^s}. \qquad t = \frac{s-1}{e^s}$$

$$s'' = e^s \cdot s' + (e^s + te^s s')s' + (te^s)s''$$

$$s''(1 - te^s) = s'(2e^s + te^s s')$$

$$s''(2-s) = \frac{e^s}{(2-s)}\left(2e^s + \frac{s-1}{\cancel{e^s}}\cancel{e^s} \cdot \frac{e^s}{2-s}\right)$$

$$s''(2-s) = \frac{e^s}{2-s}\left(2e^s + e^s\left(\frac{s-1}{2-s}\right)\right) \Rightarrow s'' = \frac{e^{2s}(3-s)}{(2-s)^3}$$

② $x^2+y^2=r^2$ $\qquad \dfrac{d^3y}{dx^3} = ?$ $\quad \dfrac{dy}{dx}$ $\quad y'$ $\quad y''$ $\quad y'''$

$\qquad\qquad\qquad d^2y/dx^2$

$\qquad y^{(4)} \quad y^{(5)} \quad y^{(6)}$

$$2x + 2y \cdot y' = 0 \quad \text{— ①} \rightarrow y' = -x/y$$

$$2 + 2y' \cdot y' + 2y \cdot y'' = 0 \quad \text{— ②} \rightarrow y'' = -\frac{(1+y'^2)}{y}$$

$$= -\frac{\left(1+\frac{x^2}{y^2}\right)}{y}$$

$$= -\frac{(x^2+y^2)}{y^3}$$

$$= -\frac{r^2}{y^3}$$

$$0 + \cancel{4}\, y' \cdot y'' + 2y' \cdot y'' + 2y \cdot y''' = 0 \quad \text{— ③}$$

$$3\, \cancel{6}\, y'y'' + \cancel{2}\, y \cdot y''' = 0$$

$$3\left(-\frac{x}{y}\right)\left(-\frac{r^2}{y^3}\right) + y \cdot y''' = 0$$

$$y''' = -\frac{3xr^2}{y^5} \qquad \text{D}$$

③ $$\frac{dy}{dx} = -\frac{(\cancel{\text{BQ}}\ ax + hy + g)}{hx + by + f}$$

$$d\ \frac{u}{v} = \frac{vu' - uv'}{v^2}$$

$$-\frac{d^2y}{dx^2} = \frac{(hx+by+f)(a+hy') - (ax+hy+g)(h+by')}{(hx+by+f)^2 = d^2}$$

$$-\frac{d^2y}{dx^2} = \frac{(hx+by+f)\left(a - h\frac{(ax+hy+g)}{(hx+by+f)}\right) - (ax+hy+g)\left(h - \frac{b(ax+hy+g)}{hx+by+f}\right)}{d^2}$$

$$\frac{(hx+by+f)\left\{\frac{a\cancel{hx}+aby+af-\cancel{ahx}-h^2y-gh}{\cancel{hx+by+f}}\right\}-(ax+hy+g)\left\{\frac{h^2x+\cancel{bhy}+hf-abx-\cancel{bhy}-bg}{\cancel{hx+by+f}}\right\}}{d^3}$$

$$\frac{\begin{aligned}&x^2\{a^2b-ah^2\}+y^2\{ab^2-bh^2\}\\&+xy\{abh-ah^3-h^3+abh\}\\&+x\{-h^2g+\cancel{afh}+abg-gh^2-\cancel{ahf}+abg\}\\&+y\{-\cancel{bgh}+abf-fh^2+\cancel{bgh}+fab-fh^2\}\\&-(+2fgh-af^2-bg^2)\end{aligned}}{d^3}$$

$ab\,(ax^2+by^2+2hxy+2gx+2fy+c)-abc$

$-h^2(ax^2+by^2+2hxy+2gx+2fy+c)+ch^2$

$-2fgh+af^2+bg^2$

$$\frac{d^2y}{dx^2}=+\frac{abc+2fgh-af^2-bg^2-ch^2}{d^3}=\frac{\Delta}{d^3}$$

(B)

Pg 163-167

① $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \frac{x^n}{n!}$

$$\frac{dy}{dx} = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \cdots \frac{nx^{n-1}}{n!}$$

$$\frac{dy}{dx} = 1 + x + \frac{x^2}{2!} + \cdots \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!} - \frac{x^n}{n!}$$

$$= y - \frac{x^n}{n!}$$

$\log_b a$
$= \frac{\log a}{\log b}$

④ $f(x) = \log_x(\ln x)$

$$= \frac{\log(\ln x)}{\log x} = \frac{\ln(\ln x)}{\ln x} = \frac{u}{v}$$

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \frac{(\ln x) \times \frac{1}{\ln x} \times \frac{1}{x} - \ln(\ln x) \times \frac{1}{x}}{(\ln x)^2}$$

$$f'(e) = \frac{1 \times \frac{1}{1} \times \frac{1}{e} - 0 \times \frac{1}{e}}{1^2} = \frac{1}{e}$$

(10) $\cos(x+y) = y\sin x \qquad \frac{dy}{dx}.$

$$-\sin(x+y)\{1+y'\} = y\cos x + \sin x\, y'$$

$$y'\{\sin x + \sin(x+y)\} = -\sin(x+y) - y\cos x$$

$$y' = \frac{-\{\sin(x+y) + y\cos x\}}{\sin(x+y) + \sin x} \qquad \text{(A)}$$

(12) $y = x^x \qquad \frac{dy}{dx} \qquad \frac{d}{dx} f(x)^{g(x)} = f(x)^{g(x)} \ln f(x) \times g'(x) + g(x)\{f(x)\}^{g(x)-1} \times f'(x)$

$$\frac{dy}{dx} = x^x \ln x + x(x)^{x-1} \times 1.$$

$$= x^x \ln x + \cancel{x} \cdot \frac{x^x}{\cancel{x}} \Rightarrow x^x(1+\ln x)$$

$$\Rightarrow x^x(\ln e + \ln x)$$

$$\Rightarrow x^x(\ln ex)$$

(16) $y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$ (A)

$$y \times \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} + \sqrt{x^2+1} \cdot y' = \frac{\frac{x}{\sqrt{x^2+1}} - 1}{\sqrt{x^2+1} - x}$$

$$\frac{xy + (x^2+1)y'}{\sqrt{x^2+1}} \qquad = -\frac{\cancel{(x - \sqrt{x^2+1})}}{\sqrt{x^2+1}\,\cancel{(\sqrt{x^2+1} - x)}} = -\frac{1}{\sqrt{x^2+1}}$$

(18) $f(x) = a_1x^2 + a_2x + a_3$ | $a_3 - a_2 = a_2 - a_1$

$f'(x) = 2a_1x + a_2$

$f'(a_1) = 2a_1^2 + a_2$ $\quad f'(a_2) = 2a_1a_2 + a_2$

$f'(a_3) = 2a_1a_3 + a_2$

$2a_1^2 + a_2 \quad , \quad 2a_1a_2 + a_2 \quad , \quad 2a_1a_3 + a_2$

$2a_1a_2 + \cancel{a_2} - 2a_1^2 - \cancel{a_2}$

$2a_1(a_2 - a_1)$

$2a_1a_3 + \cancel{a_2} - 2a_1\cancel{a_2} - \cancel{a_2}$

$2a_1(a_3 - a_2)$

$2a_1(a_2 - a_1)$

A.P.

(20) $y = f\left(\frac{5x+1}{10x^2-3}\right)$ & $f'(x) = \cos x$

$$\frac{dy}{dx} = f'\left(\frac{5x+1}{10x^2-3}\right) \times \frac{(10x^2-3)5 - (5x+1)(20x)}{(10x^2-3)^2}$$

$$= \cos\left(\frac{5x+1}{10x^2-3}\right) \times \frac{d}{dx}\left(\frac{5x+1}{10x^2-3}\right)$$

(A)

(23)

$x = e^{y+e^{y+e^{y}\cdots\infty}}$

$x = e^{y+x}$

$1 = e^{(y+x)}\{y'+1\}$

$\downarrow$
x

$\cancel{y' = e^{-(y+x)}}$

$1 = x(y'+1)$

$y' = \frac{1}{x} - 1 = \frac{1-x}{x}$ (C)

(30) $y^2 = ax^2+bx+c$ —— ①

$2yy' = 2ax+b$ —— ②

$y^3\left(\frac{d^2y}{dx^2}\right)$

$\cancel{2}y'^2 + \cancel{2}yy'' = \cancel{2}a$ —— ③

$y'' = \frac{a-y'^2}{y}$

$= \frac{a-\left(\frac{2ax+b}{2y}\right)^2}{y}$

$= \frac{4ay^2-(2ax+b)^2}{4y^3}$

(A)

const

$y'' = \frac{4a(ax^2+bx+c)-4a^2x^2-b^2-4axb}{4y^3}$

$\Rightarrow y''y^3 = (\cancel{4a^2x^2}-\cancel{4a^2x^2}+4ac-b^2)/4 = \frac{4ac-b^2}{4}$

(32) $f(x) = \sin\left(\frac{\pi}{2}[x] - x^5\right)$ $\quad 1 < x < 2$

$= \sin\left(\frac{\pi}{2} - x^5\right)$

$= \cos x^5$

$f'(x) = -\sin x^5 \times (5x^4)$

$f'\left(\sqrt[5]{\frac{\pi}{2}}\right) = -\sin\frac{\pi}{2}. \times 5\left(\frac{\pi}{2}\right)^{4/5}.$

$= -5\left(\frac{\pi}{2}\right)^{4/5}$ $\quad$ (B)

(31) $xe^{xy} = y + \sin^2 x$ $\quad$ at $x=0$ $\quad \frac{dy}{dx} = ?$

$x\{e^{xy}(xy'+y)\} + e^{xy} \times 1$

$= y' + 2\sin x\cos x$

$y' = 1.$ $\quad$ (B)

(34) If $x^2+y^2 = t + \frac{1}{t}$ $\quad x^4+y^4 = t^2 + \frac{1}{t^2}$

$\frac{dy}{dx}.$ $\quad x^4+y^4+2x^2y^2 = t^2+\frac{1}{t^2}+2$

$\cancel{t^2+\frac{1}{t^2}} + 2x^2y^2 = \cancel{t^2+\frac{1}{t^2}} + 2$

$\cancel{2}x^2y^2 = \cancel{2}$ $\quad \rightarrow x^2y^2 = 1$

$xy = \pm 1.$

$xy' + y \cdot 1 = 0$

$y' = -\dfrac{y}{x}$ (A)

(40) If $y = \sin x$ $\quad \dfrac{d^2}{dy^2}(\cos^7 x)$

$f(x) = \cos^7 x.$ $\quad \dfrac{d\, f(x)}{d\, g(x)} = \dfrac{\frac{d\, f(x)}{dx}}{\frac{d\, g(x)}{dx}}.$

$f'(x) = \dfrac{7\cos^6 x\,(-\sin x)}{\cos x}$

$= \underline{7\cos^5 x}\,(\underline{-\sin x})$

$f''x = \dfrac{35\cos^{\cancel{4}\,3} x\,(-\sin x)(-\sin x) + 7\cos^{\cancel{5}\,4} x\,(-\cos x)}{\cancel{\cos x}}$

$= +35\cos^3 x\sin^2 x - 7\cos^5 x.$

$= +35\cos^3 x\,(1-\cos^2 x) - 7\cos^5 x)$

$= +35\cos^3 x - 35\cos^5 x - 7\cos^5 x.$

$= 35\cos^3 x - 42\cos^5 x$ (B)

Pg 171

Comp 1

$f(x) = |3-x|$

$g(x) = f(f(x))$

$$f(x) = \begin{cases} 3-x & x<3 \\ x-3 & x \geq 3 \end{cases}$$

$$f(f(x)) = \begin{cases} 3-f(x) & f(x)<3 \\ f(x)-3 & f(x)\geq 3 \end{cases}$$

$3-x < 3$

$x > 0$

$0 < x < 3$

$3-(3-x)$

x

$g'(2) = 1$

$x-3 < 3$

$x < 6$

$3 \leq x < 6$

$3-(x-3)$

$6-x$

$3-x \geq 3$

$x \leq 0$

$(3-x)-3$

$-x$

$x-3 \geq 3$

$x \geq 6$

$(x-3)-3$

$x-6$

(2) $y = f\left(\frac{x-1}{x+1}\right)$ $\quad f'(x) = x^2$

$$y' = f'\left(\frac{x-1}{x+1}\right) \frac{d}{dx}\left(\frac{x-1}{x+1}\right)$$

$$y' = \left(\frac{x-1}{x+1}\right)^2 \times \frac{(x+1)(1) - (x-1)1}{(x+1)^2}$$

$$y' = \left(\frac{x-1}{x+1}\right)^2 \times \frac{2}{(x+1)^2}$$

(D)

$$y'(0) = \left(\frac{0-1}{0+1}\right)^2 \times \frac{2}{(0+1)^2} = 1 \times \frac{2}{1} = 2$$

(3) $f'(a) = \frac{1}{2}$ $\quad f(a) = b$ $\quad \underline{g(f(x))} = x$

$g'(b) = ?$

$$g'(f(x)) \cdot f'(x) = 1$$

$x = a \quad f(a) = b$

$$g'(b) \cdot f'(a) = 1$$

$$g'(b) \cdot \frac{1}{2} = 1$$

$$g'(b) = 2$$

(C)

Comp 3

7 $y = \dfrac{\ln x}{x}$

$$y' = \frac{x \times \frac{1}{x} - \ln x(1)}{x^2} = \frac{1 - \ln x}{x^2}$$

$$y'' = \frac{x^2\left(0 - \frac{1}{x}\right) - (1 - \ln x)2x}{x^4}$$

$$= \frac{-x - 2x + 2x\ln x}{x^4} = \frac{\cancel{x}(2\ln x - 3)}{\cancel{x^4}x^3}$$

$$y''(e) = \frac{2\ln e - 3}{e^3} = \frac{2-3}{e^3} = -\frac{1}{e^3}$$

8 $y = \dfrac{1}{3-4x}$ $\qquad y' = -\dfrac{1}{(3-4x)^2} \times -4 = \dfrac{4}{(3-4x)^2}$

$$y'' = \frac{4(-2) \times -4}{(3-4x)^3} = \frac{4^2}{(3-4x)^3}\,2$$

$$y^n = \frac{4^n}{(3-4x)^{n+1}}\,n!$$ A

(9)

$$y = \tan^{-1} x$$

$$y' = \frac{1}{1+x^2}$$

$$y'(0) = 1$$

$$y''(0) = 0$$

$$y'' = -\frac{1 \times 2x}{(1+x^2)^2} \Rightarrow y'' = -2x(y')^2$$

$$\cancel{y''' = \frac{-2}{(1+x^2)^4}}$$

$$\Rightarrow y''' = -2x\,2y' \cdot y'' - 2y'^2$$

$$y''' = -2y'\{2xy'' - y'\}.$$

$$= -2(1)\{0 - 1\}$$

$$= 2.$$

(D)