BANGALORE SAHODAYA SCHOOLS COMPLEX
PRE-BOARD EXAMINATION 2016-17
SUBJECT:MATHEMATICS

SET-1

CLASS: XII **MARKS:100**
DATE: 13.1.2017 **TIME:3 HOURS**

General Instructions

(i)**All** questions are compulsory.
(ii)This question papercontains**29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each

Section-A

Questions 1 to 4 carry 1 mark each.

1. Check whether the relation R in the set of natural numbers, defined by R = {(x, y), x, y, ϵ Z, $x \le y^2$} is transitive. Justify.
2. If A is a square matrix of order 3 such that $|\text{adj } A| = 225$, find $|A^T|$ and $|AA^T|$.
3. In what ratio does vector $3\vec{a} + 2\vec{b}$ divides the line segment joining the points $2\vec{a} + 3\vec{b}$ and $\vec{a} + 4\vec{b}$
4. Let $f: R \to R$ be the function defined by f(x) = $\frac{1}{2-\cos x}$, $x \in R$. Then find the range of f.

Section-B

Questions 5 to 12 carry 2 marks each.

5. Evaluate $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$
6. The probability that at least one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P(\bar{A}) + P(\bar{B})$.
7. Write the order and degree of the differential equation $y = px + \sqrt{x^2p^2 + b^2}$ where p = $\frac{dy}{dx}$
8. Find all the points of discontinuity of the function f(x) = $[x^2]$ on [1,2), where [ ] denotes the greatest integer function.
9. The two vectors $3\hat{i} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of ΔABC. Find the length of the median through A.

10. If $0 < x < 1$ and if $\tan^{-1}(1 - x)$, $\tan^{-1}x$ and $\tan^{-1}(1 + x)$ are in arithmetic progression, prove that $x^3+x^2 =1$.

11. Write the value of x + y + z if $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

12. Using differentials, find the approximate value of $\sqrt[3]{(7.968)^4}$

Section-C

Questions 13 to 23 carry 4 marks each.

13. Find the equation of a curve passing through the point (1, 1) if the perpendicular distance of the origin from the normal at any point P(x, y) of the curve is equal to the distance of P from the x – axis

OR

Find a particular solution of the differential equation $(x - y)(dx+dy) = dx - dy$, given that $y = -1$, when $x = 0$

14. Evaluate $\int_0^7 f(x)dx$ if $f(x) = \begin{cases} x - |1 - 2x|, & x < 2 \\ 3x - 2, & \text{if } x > 5 \\ -5, & \text{otherwise} \end{cases}$

OR

Show that $\int_0^{\pi} \frac{x \sin x}{1+\sin x} = \frac{\pi}{2}(\pi - 2)$

15. Find the values of a and b so that the function $f(x) = \begin{cases} x^2 + 3x + a, & \text{for} \quad x \leq 1 \\ bx + 2, & \text{for} \quad x > 1 \end{cases}$ is differentiable for $x \in R$

16. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for, $-1 < x < y < 1$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

17. A factory produces three types of toys T1, T2 and T3 using three types of plastic materials P1, P2, and P3. They are transported to different depots for sales. Find the production of all the three types of toys using Matrices. The plastic requirement for each type of toy and the total availability of plastic of all the three types is summarized in the following table.

Plastic	T1	T2	T3	Total available Plastic
P1	1	1	1	6
P2	2	5	5	27
P3	2	5	11	45

18. Find the equation to the normal to the curve $x^2 = 4y$ passing through the point (1, 2).

19. A farmer wants to divide his circular land into four parts as three segments and one triangle such that the rice is grown in triangle field and vegetables in the rest. Prove that areas of all segments are equal such that minimum area is used to grow vegetables.

OR

A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}\frac{1}{2}$. Water is poured into it at a constant rate of 5 cubic meters per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m.

20. Find the equation of the plane which contains the line of intersection of planes, $\bar{r}\cdot(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) - 4 = 0$ and $\bar{r}\cdot(-2\hat{\imath} + \hat{\jmath} + \hat{k}) = 5$ and whose intercept on x-axis is equal to that on y-axis

21. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of points A, B, C, prove that $(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$ is a vector perpendicular to the plane of triangle ABC.

22. If A, B and C throw a coin and a dice alternately and wins the game till one of them gets head on coin and odd prime on dice . Find the respective probabilities of winning if A starts first.

23. Two groups are competing for the positions on the board of directors of a company. The probabilities that the first and second group win are 0.4 and 0.6 respectively. Further, if the first group wins the probability of introducing a new product is 0.8, and the corresponding probability is 0.6 if the second group wins. Find the probability that the new product is introduced. Also, find the probability that the second group introduces the product.

Section-D

Questions 24 to 29 carry 6 marks each.

24. Let *f and g* : $R \rightarrow R$ be a function defined as $f(x) = |x| + x$ and $g(x) = |x| - x$ for all $x \in R$. Then find *fog and gof*

OR

Functions $f, g : R \rightarrow R$ are defined, respectively, by $f(x) = x^2 - 3x - 4$, $g(x) = 2x - 3$, prove that fog is not invertible modify the domain and co-domain of *fog* to make invertible and hence find its inverse.

25. A farmer mixes two brands P and Q of cattle feed. Brand P costing ₹ 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B, and 2 units of element C. Brand Q costing ₹ 200 per bag, contains 1.5 units of nutritional element A, 11.25 units of element B and 3 units of element C. The minimum requirement of nutrients A, B and C are 18units, 45units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag? What value is depicted here?

26. Using Integrals, find the area bounded by $|x-y| \leq 2, \quad y \leq \sqrt{x}, \ and\ y \geq 0.$

27. Find the shortest distance between the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and the 'line passes through the image of (1,6,3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$and perpendicular to the plane 2x−y+3z=2'.

OR

Show that the lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{y-b-c}{\beta+\gamma}$ are coplanar.

28. If $\frac{d}{dx} f(x) = \frac{1}{(x^2-1)\sqrt{x^2+1}}$ then find f(x)

OR

Evaluate $\int \frac{cosx-cos2x}{1-cosx} dx$

29. Show that:

$$\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$