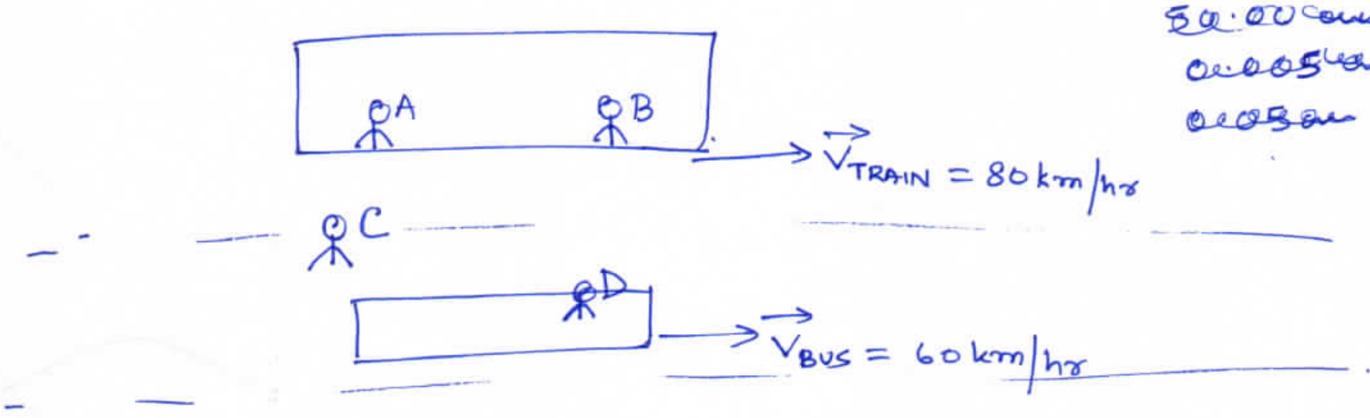
RELATIVE VELOCITY



Relative velocity of B w.r.t A $= \vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$$\vec{V}_{DC} = \vec{V}_D - \vec{V}_C$$
$$= +60 - 0 = +60 \text{ km/hr}$$

$$\vec{V}_{AD} = \vec{V}_A - \vec{V}_D = +80 - (+60) = +20 \text{ km/hr}$$

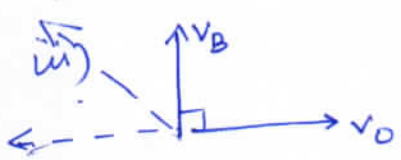
$$\vec{V}_{BO} = \vec{V}_B - \vec{V}_O$$

i) 

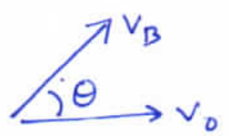
$\vec{V}_{BO} = \vec{V}_B - \vec{V}_O = V_B\hat{i} - V_O\hat{i} = (V_B - V_O)\hat{i}$

ii) 

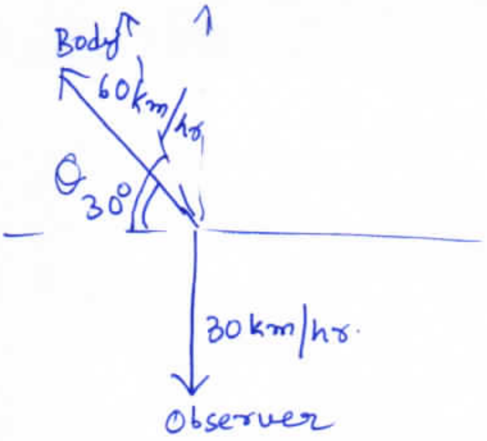
$\vec{V}_{BO} = \vec{V}_B - \vec{V}_O = V_B\hat{i} - (-V_O\hat{i}) = (V_B + V_O)\hat{i}$

iii) 

$\vec{V}_{BO} = \vec{V}_B - \vec{V}_O = V_B\hat{j} - V_O\hat{i}$

iv) 

$\vec{V}_{BO} = \vec{V}_B - \vec{V}_O = (V_B\cos\theta\,\hat{i} + V_B\sin\theta\,\hat{j}) - V_O\hat{i}$
$= (V_B\cos\theta - V_O)\hat{i} + V_B\sin\theta\,\hat{j}$



$$\vec{V}_{BO} = ?$$

$$= \vec{V}_B - \vec{V}_O$$

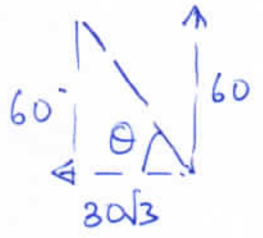
$$= \left(-60\cos 30^\circ \hat{i} + 60\sin 30 \hat{j}\right) - \left(-30\hat{j}\right)$$

$$\vec{V}_{BO} = \left(-30\sqrt{3}\,\hat{i} + 60\hat{j}\right) \text{ km/hr.}$$

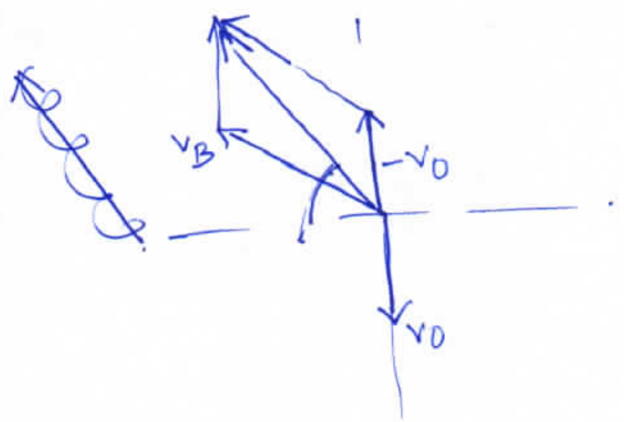
$$|\vec{V}_{BO}| = \sqrt{30^2 \times 3 + 60^2}$$

$$= 30\sqrt{3+4}$$

$$= 30\sqrt{7} \text{ km/hr}$$



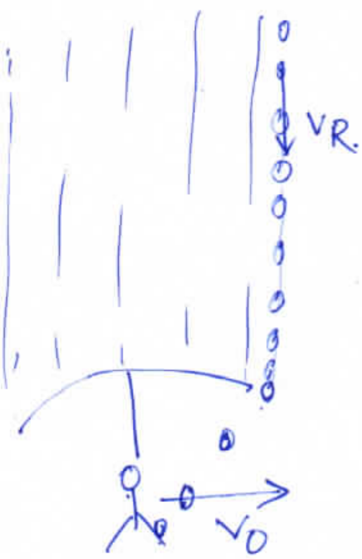
$$\tan\theta = \frac{60}{30\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

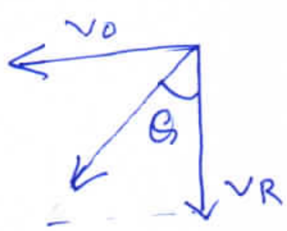


APPLICATION OF RELATIVE VELOCITY

1) RAIN PROBLEM

$$\vec{V}_{RO} = \vec{V_R} - \vec{V_0}$$
$$= -V_R\hat{j} - 0$$
$$= -V_R\hat{j}$$





$$\tan\theta = \frac{V_O}{V_R}$$

$$\theta = \tan^{-1}\frac{V_O}{V_R}$$

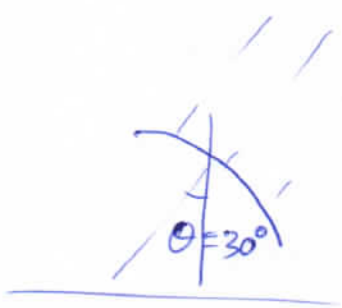
$$\vec{V}_{RO} = \vec{V_R} - \vec{V_O}$$
$$= -V_R\hat{j} - V_O\hat{i}$$
$$=$$

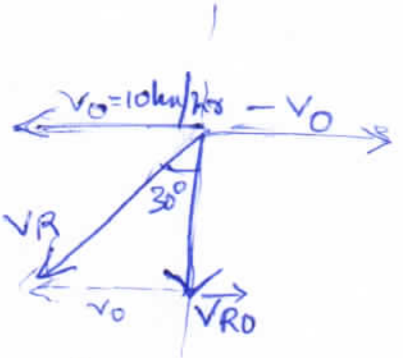
Q1 When a boy is standing he holds his umbrella at 30° with vertical. Now the boy starts running at 10 km/hr and the rain appears to be coming down vertically. Find actual velocity of rain (magnitude)

Q2 If a man is moving at 5 m/s forward & rain appears to be coming down vertically at 10 m/s

Find actual direction of rain with vertical.

Ans 1



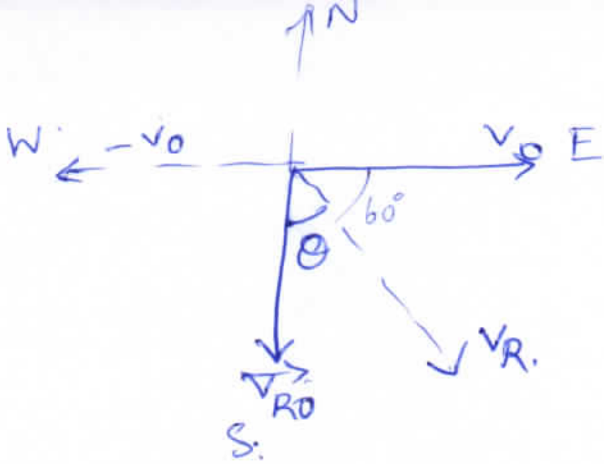


$$\vec{V}_{RO} = \vec{V}_R - \vec{V}_O$$

$$\frac{v_O}{v_R} = \sin 30°$$

$$v_R = \frac{v_O}{\sin 30°} = \frac{10}{\sin 30°} = \frac{10}{1/2} = 20 \text{ km/hr}$$

Ans2

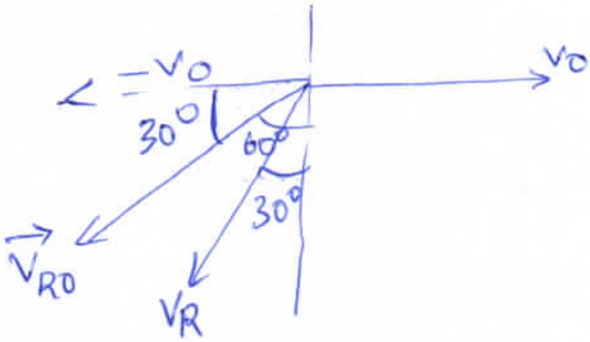


$$V_R \sin\theta = v_o$$

$$\sin\theta = \frac{v_o}{V_R} = \frac{5}{10} = \frac{1}{2}$$

$$\theta = 30^\circ \text{ east of south.}$$

Q3 When body is running at 10 km/hr forward he observes the rain to fall at at angle 60° west of south. If rain originally was falling at 30° with the vertical. Find actual velocity of rain.



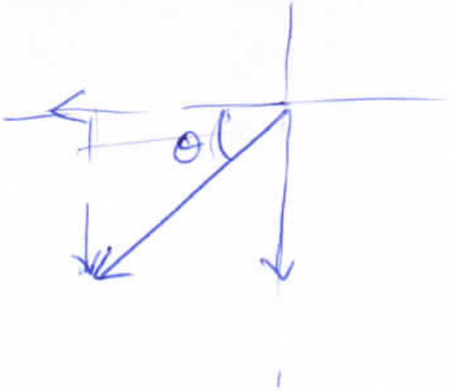
$$\vec{V}_{RO} = \vec{V}_R - \vec{V}_O$$

$$= \left(-V_R \cos 30\, \hat{j} + V_R \sin 30\, \hat{i}\right)$$

$$- 10\ \hat{i}$$

$$\vec{V}_{RO} = \left(\frac{V_R}{2} - 10\right)\hat{i} - \frac{V_R\sqrt{3}}{2}\hat{j}$$

$$\vec{V}_{RO} = -V_{RO}\sin 30\, \hat{j} - V_{RO}\cos 30\, \hat{i}$$



$xi + yj$

$$\tan 30^\circ = \frac{|y|}{|x|}$$

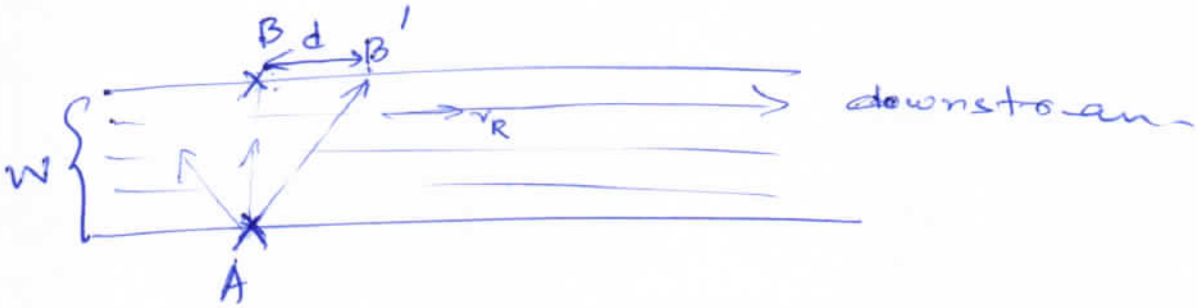
$$\tan 30^\circ = \frac{\frac{V_R\sqrt{3}}{2}}{\frac{V_R}{2} + 10}$$

$$\frac{1}{\sqrt{3}} = \frac{\frac{V_R\sqrt{3}}{\cancel{2}}}{\frac{V_R+20}{\cancel{2}}}$$

$V_R + 20 = 3V_R$

$V_R = 10\,km/hr.$

② RIVER PROBLEM



V_M is velocity generated by the man.

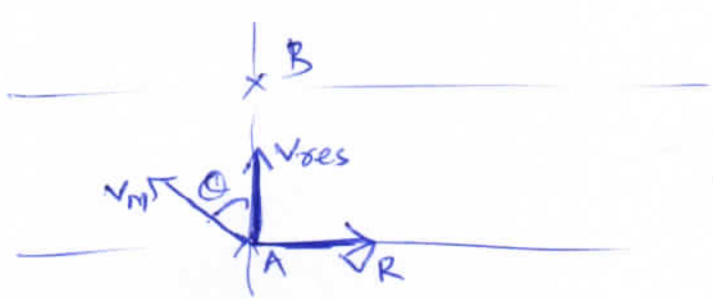
$$\frac{W}{V_M} = t$$

$$d = V_R t$$

$$d = V_R \times \frac{W}{V_M}$$

$$d = W\left(\frac{V_R}{V_M}\right)$$

Shortest distance.



$$V_M \sin\theta = V_R$$

$$\sin\theta = \frac{V_R}{V_M}$$

$$\theta = \sin^{-1}\frac{V_R}{V_M}$$

$$\cancel{t = \frac{W}{V_{res}} = \frac{W}{}}$$

$$\vec{V}_{res} = \vec{V}_M + \vec{V}_R$$

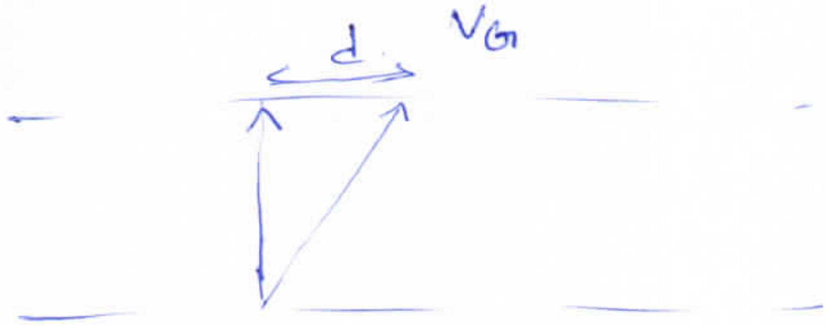
$$= V_M \cos\theta \,\hat{j} - V_M \sin\theta \,\hat{i} + V_R \,\hat{i}$$

$$V_{res}\,\hat{j} = V_M \cos\theta \,\hat{j} + \underline{(V_R - V_M \sin\theta)}\,\hat{i}$$

$$V_{res} = V_M \cos\theta .$$

$$t = \frac{W}{V_M \cos\theta}$$

<u>Shortest time path.</u>



$$t_R = \frac{W}{V_M}.$$

$$t_G = \frac{d}{V_G}.$$

$$t = t_R + t_G.$$

$$= \frac{W}{V_M} + W\left(\frac{V_R}{V_M}\right)\frac{1}{V_G}.$$

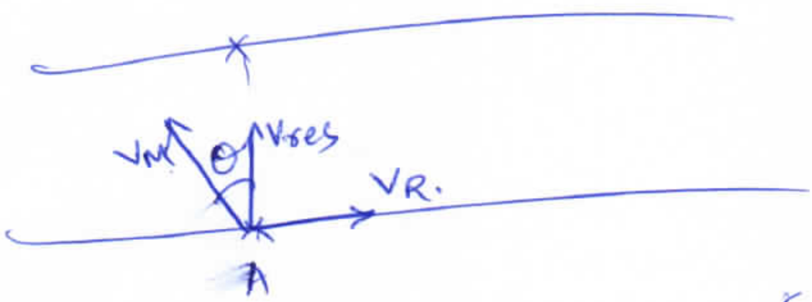
$$= \frac{W}{V_M}\left(1 + \frac{V_R}{V_G}\right)$$

Q1 Man wants to reach ~~other~~ opposite end of the river ($V_R = 3$ km/hr)

At what angle should he start swimming so that he reaches the opposite end (width of river = 1 km)

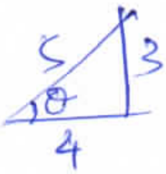
$V_M = 5$ km/hr

Also find time taken for him to reach opposite end.



$V_M \sin\theta = V_R$.

$\sin\theta = \frac{V_R}{V_M} = \frac{3}{5}$



$\sin\theta = \frac{3}{5}$

$t = \frac{W}{V_{res}} = \frac{1}{V_M \cos\theta} = \frac{1}{5 \times 4/5} = \frac{1}{4}$ hr

$= 0.25$ hr

$= 15$ min.