ONE DIMENSIONAL MOTION

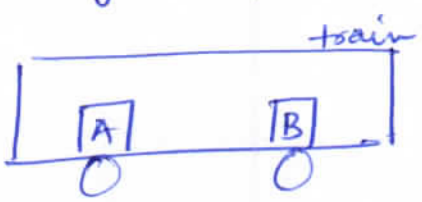
MECHANICS → Object in Rest / Motion

- STATICS
  - Cases when object is at rest.
  - 
- KINEMATICS
  - Rest
  - Motion
  - Relationship between displacement velocity acceleration but not the cause of motion.
- DYNAMICS
  - Here we will study the cause of motion.

KINEMATICS

- Object is at rest
- Object is in motion

A body is said to at rest if it is ~~changing~~ not changing its position with respect to the surrounding



A is in rest w.r.t B

A & B are in motion w.r.t C

TYPES OF MOTION

FAN IS IN MOTION (CIRCULAR MOTION)

When a point mass is rotating about a fixed point.

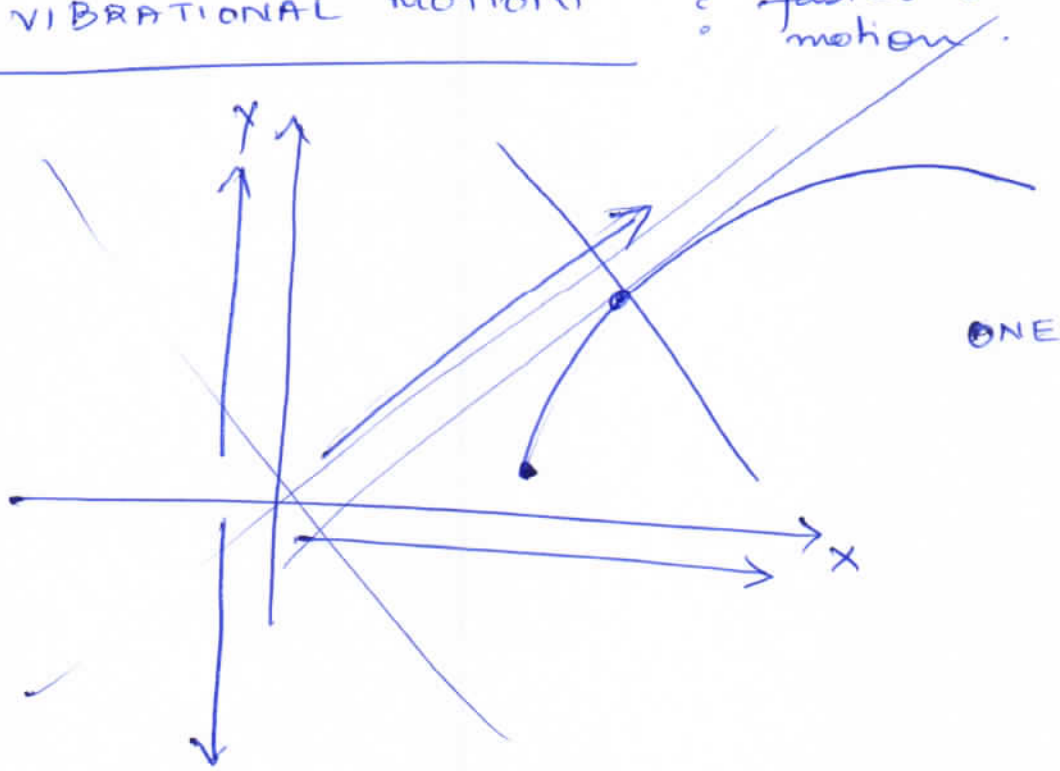
RECTILINEAR MOTION : If a ~~body~~ point object is moving in a straight line.

Point mass — If the object size is very less as compared to the distances being measured in the given problem.

TRANSLATIONAL MOTION : If a body (set of point masses) is moving in a straight line

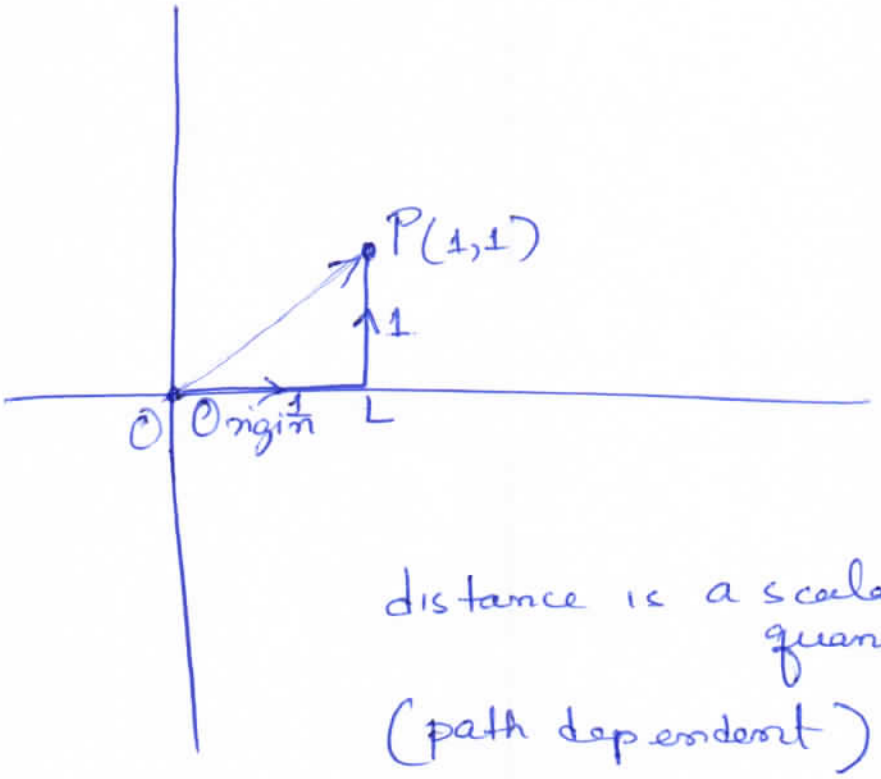
OSCILLATORY MOTION : To & fro motion.

VIBRATIONAL MOTION : faster version of oscillatory motion.



ONE DIMENSIONAL MOTION

CONCEPT OF ORIGIN & POSITION



Distance OP is 2 units.

Displacement $\vec{OP}$

distance is a scalar quantity

(path dependent)

$OL + LP = 1 + 1$

$= 2\ m$.

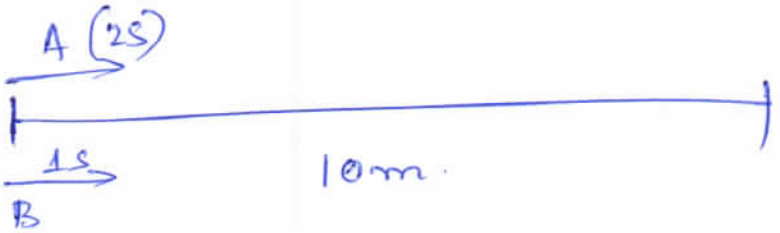
displacement is vector.

(path independent)

$|\vec{OP}| = \sqrt{2}$

$\vec{OP} = i + j$

Speed (Characteristic of a body in motion) **Velocity**



Displacement / (Time)

$$\text{Speed (A)} = \frac{10}{2} = 5\ m/s$$

$$\text{Speed (B)} = \frac{10}{1} = 10\ m/s.$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time taken}}$$

Uniform Speed : Object covering equal distance in equal interval of time.

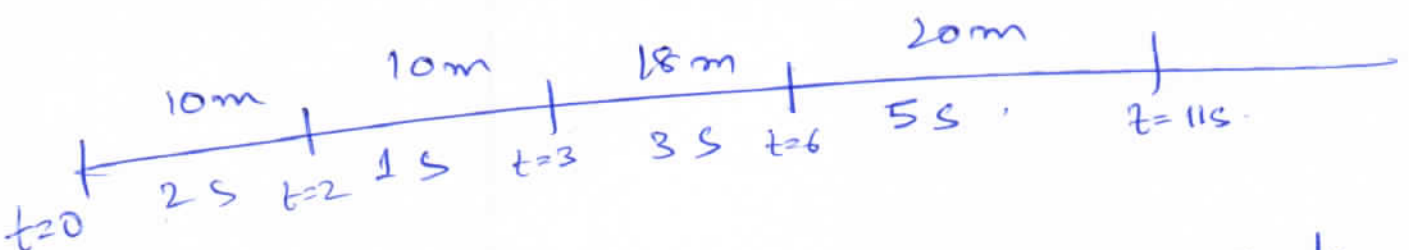
↓ Speed is not changing

Variable Speed

↓

Speed is changing.

Average Speed = $\frac{\text{Total Distance}}{\text{Total time}}$.



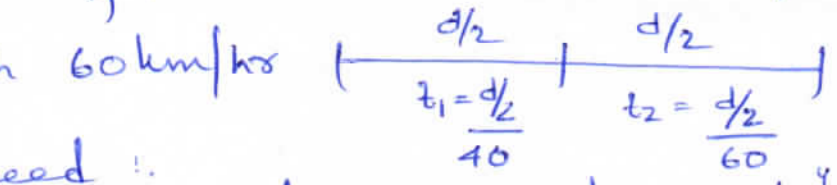
$= \frac{10+10+18+20}{2+1+3+5} = \frac{58}{11} = 5.27 \text{ m/s}$.

Instantaneous Speed : Speed at any instant of time.

velocity = $\frac{\text{Displacement}}{\text{Time}}$

Speed / Velocity is used interchangingly in one dimensional motion.

Q1 i) Car travels half the distance at 40 km/hr & half with 60 km/hr

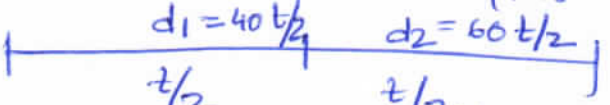


Find avg speed :

$$\text{Avg speed} = \frac{d}{\frac{d}{80} + \frac{d}{120}} = \frac{120 \times 80 d}{200 d} = 48 \text{ km/hr}$$

(ii) Car travels half the time at 40 km/hr & the other half the time at 60 km/hr

Find avg speed

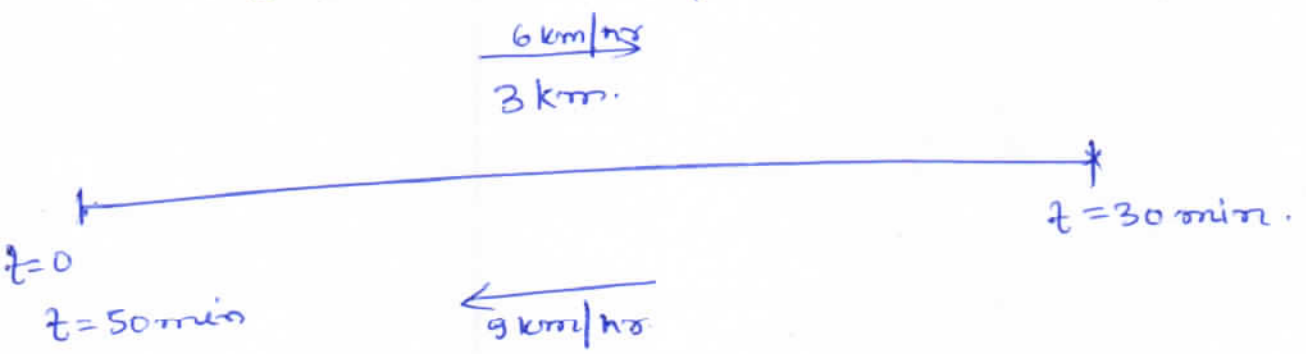


$$\text{Avg speed} = \frac{20t + 30t}{t/2 + t/2} = 50 \text{ km/hr}$$

Q2) A man walks on a straight road for 3 km with speed 6 km/hr. Returns immediately with speed of 9 km/hr.

What is magnitude of avg. velocity & avg speed over the following time intervals

i) 0 to 30 min ii) 0 to 50 min iii) 0 to 40 min



i) 0 - 30 min

$$\text{Avg Speed} = \frac{3}{1/2} = 6 \text{ km/hr}$$

$$\text{Avg Velocity} = \frac{3}{1/2} = 6 \text{ km/hr}$$

ii) 0 - 50 min

$$\text{Avg speed} = \frac{6}{5/6} = \frac{36}{5} = 7.2 \text{ km/hr}$$

$$\text{Avg Velocity} = \frac{0}{5/6} = 0 \text{ km/hr}$$

iii) 0 – 40 min.

0 – 30 min → 3 km (→)

30 – 40 min → $\frac{1}{6} \times 9 = 1.5$ km (←)

$$\text{Avg Speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{4.5}{2/3} = 6.75 \text{ km/hr}$$

$$\text{Avg Velocity} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{1.5}{2/3} = 2.25 \text{ km/hr}.$$

ACCELERATED MOTION

↓

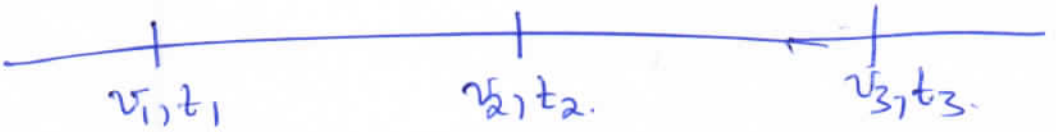
VELOCITY IS CHANGING.

VELOCITY IS INCREASING → ACCELERATED MOTION.

VELOCITY IS DECREASING → RETARDATION

$$\text{ACCELERATION} = \frac{\text{Change in velocity}}{\text{time}} = \frac{v_2 - v_1}{t_2 - t_1}$$

Unit of acceleration $= \frac{m/s}{s} = m/s^2$



UNIFORMLY ACCELERATED MOTION

↓

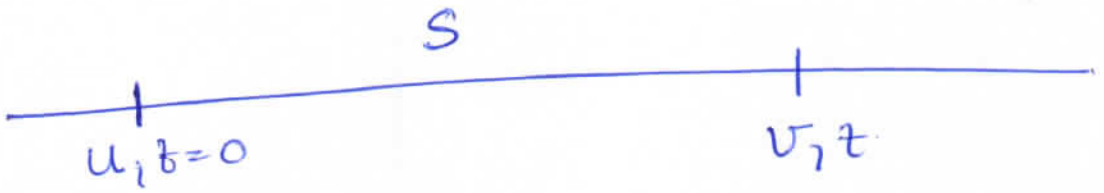
Acceleration is const.

velocity is changing at uniform rate.

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = v_2 - v_1$$

$$\Delta t = t_2 - t_1$$



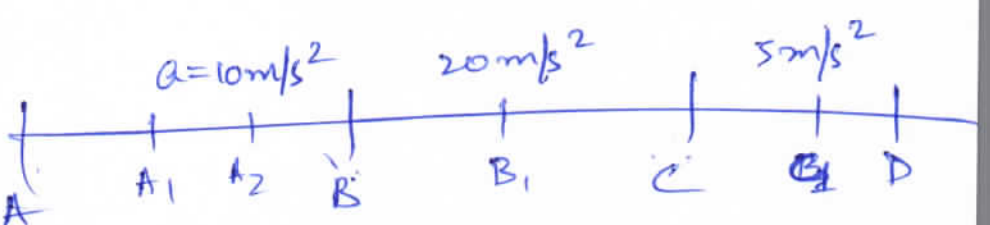
$$a = \frac{v-u}{t-0} = \frac{v-u}{t}$$

final velocity

$$v = u + at \leftarrow \text{time}$$

(1)

initial velocity — acceleration



A A_1 ✓

A B ✓

A B_1 ⟶ AB + BB_1

✗ ✓ ✓

$$s = \left(\frac{v+u}{2}\right)t$$

$$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \frac{v^2-u^2}{2a} \Rightarrow \boxed{v^2-u^2 = 2as} \quad (2)$$

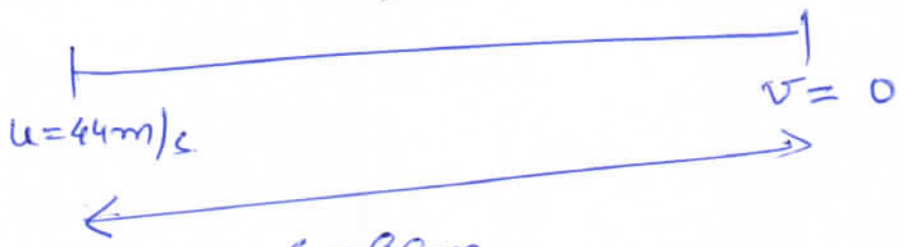
$$s = \left(\frac{u+at+u}{2}\right)t$$

$$\boxed{s = ut + \frac{1}{2}at^2} \quad (3)$$

Q1 A motorist at speed 44 m/s spots a child on the road 100m ahead He applies its break & stops within 1m of the child. Calculate time required to stop.

Q2 A train moving at 72km/hr starts retarding. It takes 200m to make its velocity 36 km/hr. How much distance it will go further before coming to rest.

Ans1



$v = u + at$
$v^2 - u^2 = 2as$
$s = ut + \frac{1}{2}at^2$

$v^2 - u^2 = 2as$.

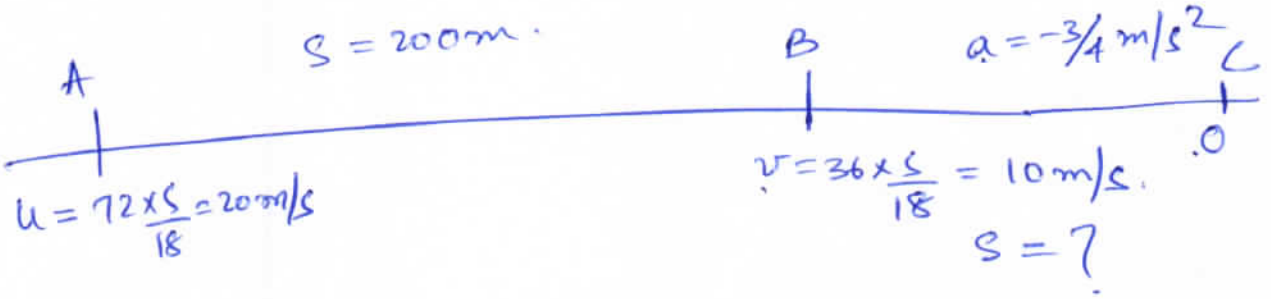
$0^2 - 44^2 = 2a(99)$

$a = -\frac{44^2}{2 \times 99}$

$v = u + at$

$0 = 44 - \frac{44^2}{2 \times 99} t \Rightarrow t = \frac{44 \times 2 \times 99}{44^2} = 4.5\,s$

Ans2



AB

$10^2 - 20^2 = 2a(200)$

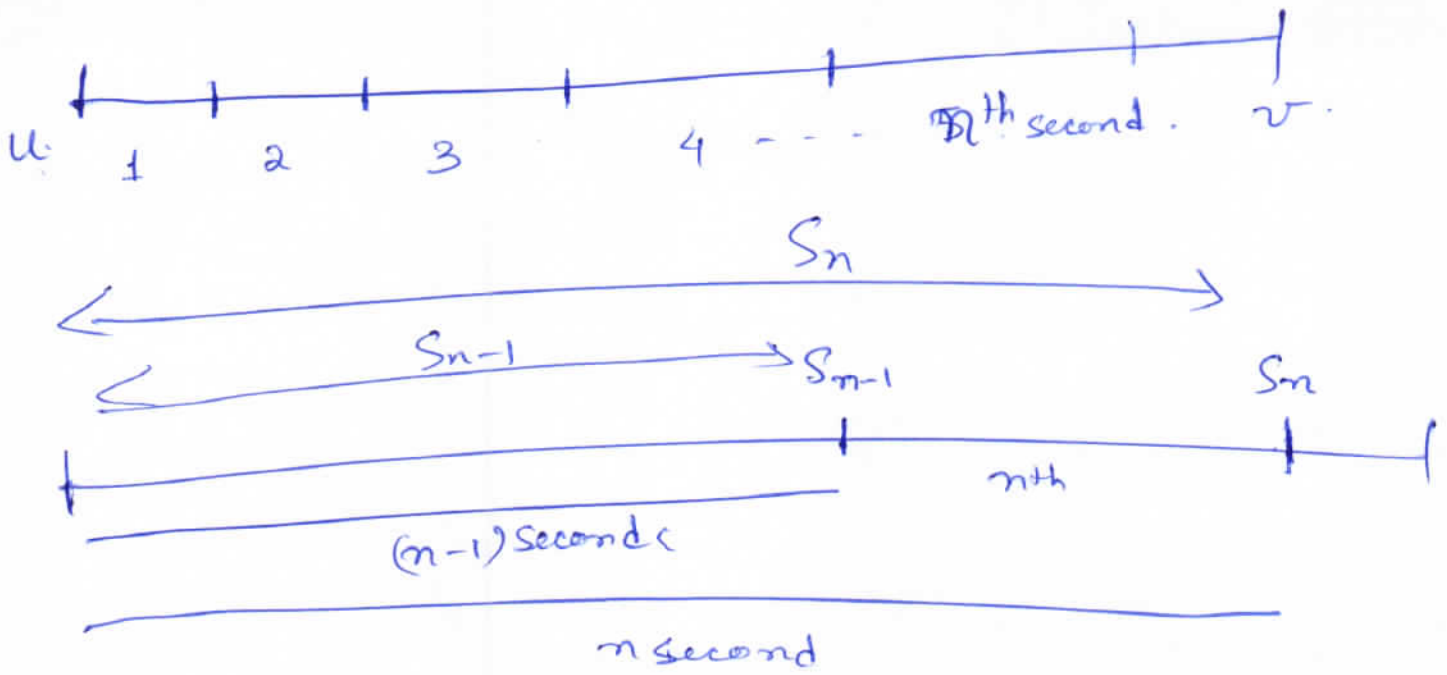
$a = -\frac{300}{400} = -\frac{3}{4}\ m/s^2$

BC

$v^2 - u^2 = 2as$.

$0^2 - 10^2 = 2\left(\frac{-3}{4}\right)s$

$s = \frac{200}{3}\ m.$



In n^{th} second $= S_n - S_{n-1}$

$$= \left[u(n) + \frac{1}{2}a(n)^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= u + \frac{1}{2}a(2n-1)$$

distance travelled in n^{th} second.

Mohan in gravity

$$a = -9.8\ m/s^2$$

$$|a| = 9.8\ m/s^2 = g$$

Q.1 Calculate distance travelled in 6th second if bike starts from rest and moves with constant acceleration of $20\ m/s^2$

Q2 An open elevator is ascending with constant velocity of 10m/s. A ball thrown vertically up by a boy when he is at height 10 m from ground. The velocity of projection is $v = 30$ m/s w.r.t to elevator. Find

a) Max height attained by the ball above ground.

b) time taken by ball to meet elevator again

c) time taken by the ball to reach the ground after crossing the elevator. (take $g = 10 m/s^2$)

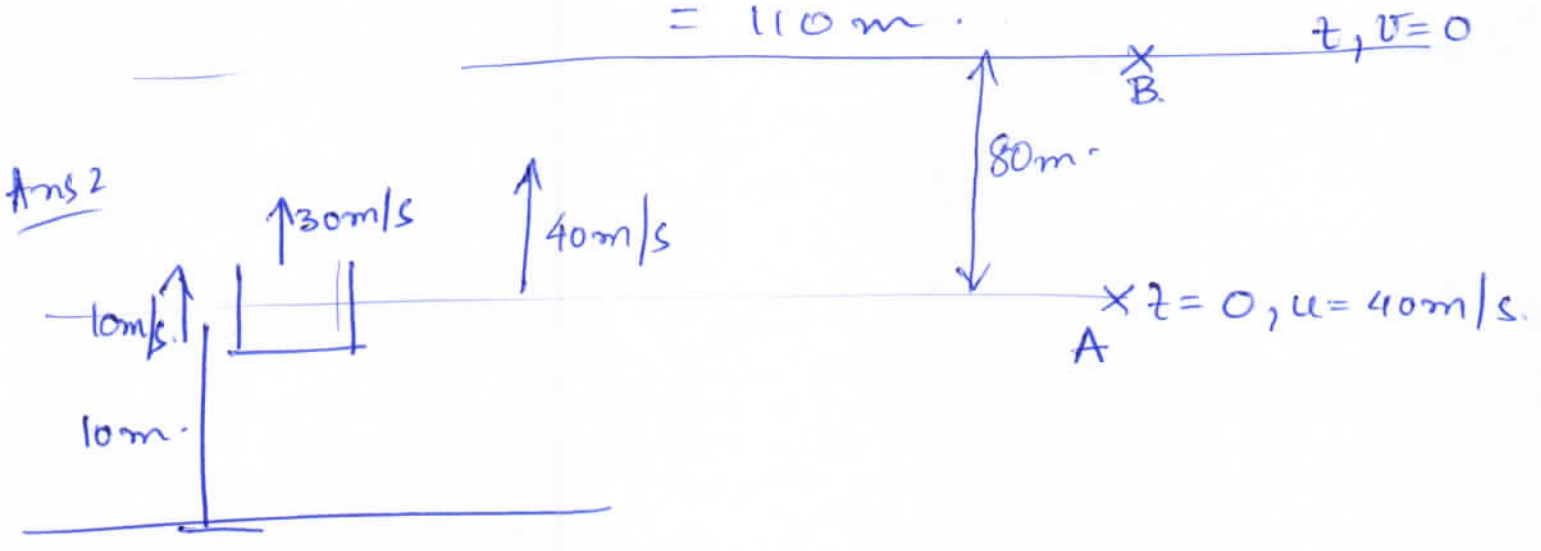
Ans 1

$$n^{th} \text{ second} = u + \frac{1}{2}a(2n-1)$$

$$= 0 + \frac{1}{2} \times 20 (2 \times 6 - 1)$$

$$= 110 \text{ m}.$$

Ans 2



A → B

$v = u + at$

$0 = 40 + (-10)t$

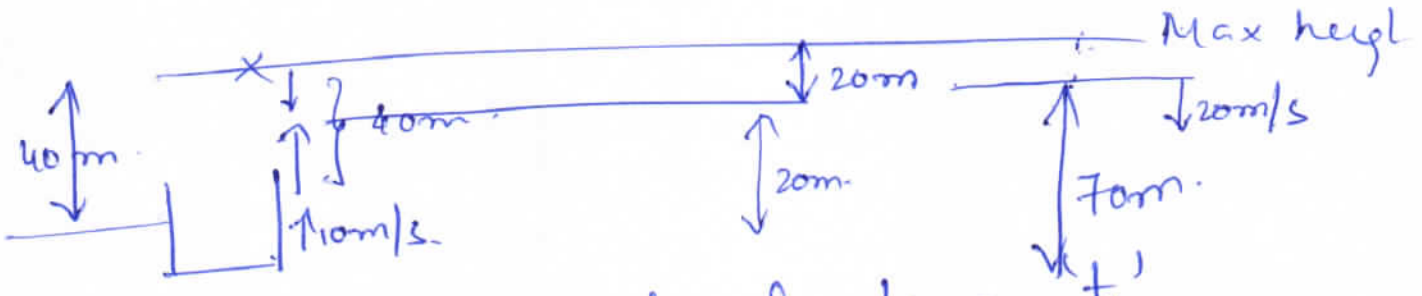
$t = 4s$

A → B

$v^2 - u^2 = 2as$

$0^2 - 40^2 = 2(-10)s \implies s = 80m$

a) Max height $= 10 + 80 = 90m$



Let them meet at time

$s = ut + \frac{1}{2}at^2$

Elevator moves up $= 10t$ m

$s = 0t + \frac{1}{2}(10)t^2$

$= 5t^2$

40m

Ball moves down $= 5t^2$ m.

$5t^2 + 10t = 40$

$t^2 + 2t - 8 = 0$

$t^2 + 4t - 2t - 8 = 0$

$t(t+4) - 2(t+4) = 0$

$(t-2)(t+4) = 0$

$t = 2s$

b) $4s + 2s = \underline{6s}$

c) $u = 20\ m/s$.

$s = 70\ m$

$a = +g = +10\ m/s$.

$t = ?$

$s = 20t + \frac{1}{2} \times 10 t^2$.

$70 = 20t + 5t^2$

$t^2 + 4t - 14 = 0$

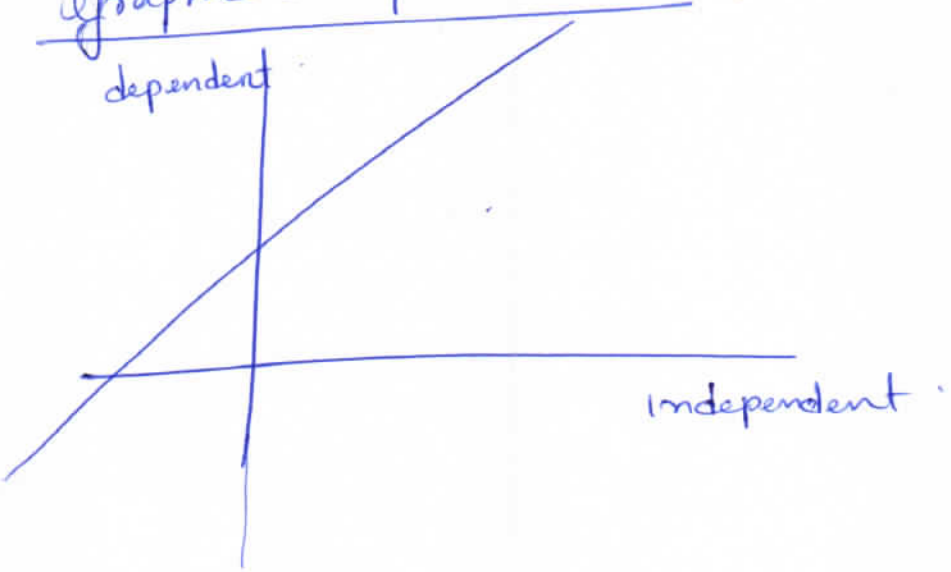
$$t = \frac{-4 + \sqrt{16 + 56}}{2}.$$

$$= \frac{-4 + 6\sqrt{2}}{2}.$$

$$= (-2 + 3\sqrt{2})\ s$$

velocity, acceleration, time, displacement.
(v) (a) (t) (s)

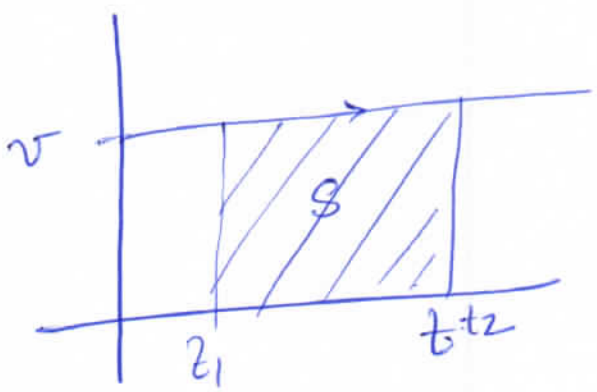
Graphical representation:

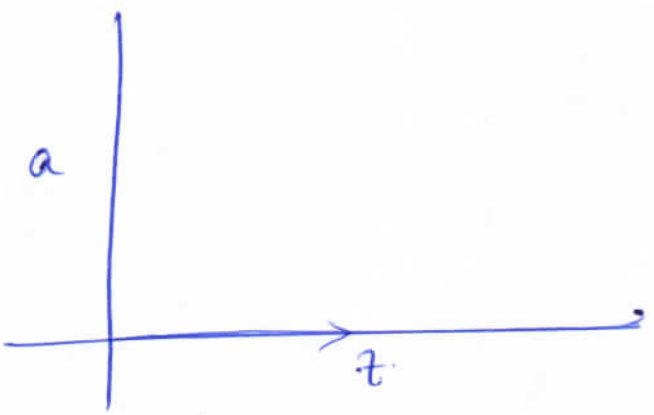


$f(x)$
$y = x + 4.$

UNIFORM VELOCITY

$V = \text{const.}$ $\quad a = 0$ $\quad s = vt$





s = area in v-t graph.

UNIFORM ACCELERATION

a = const. v t s.

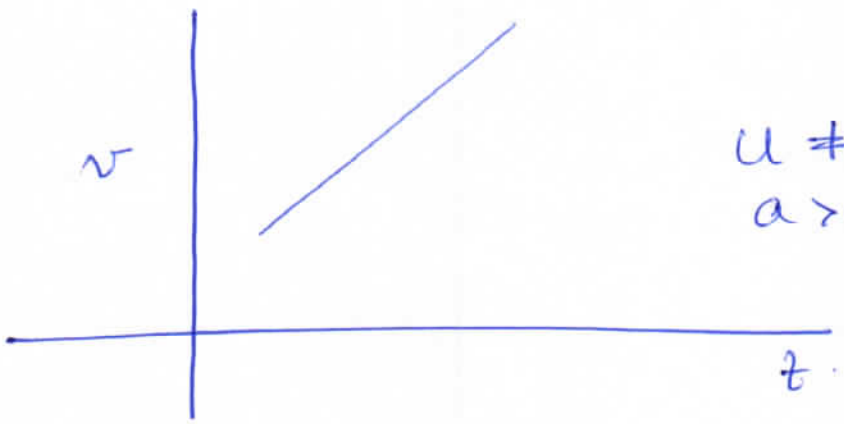
$v = u + at$

$s = ut + \frac{1}{2}at^2$

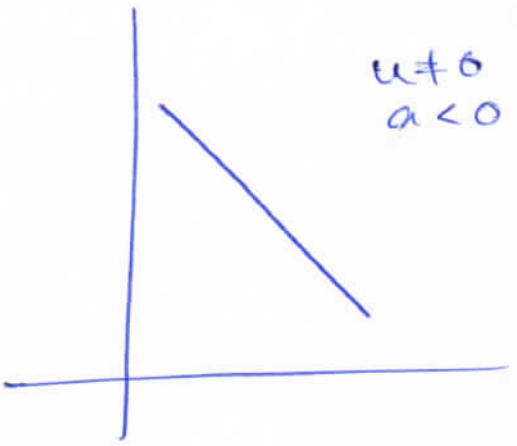
$v^2 - u^2 = 2as$

$v - u = at$

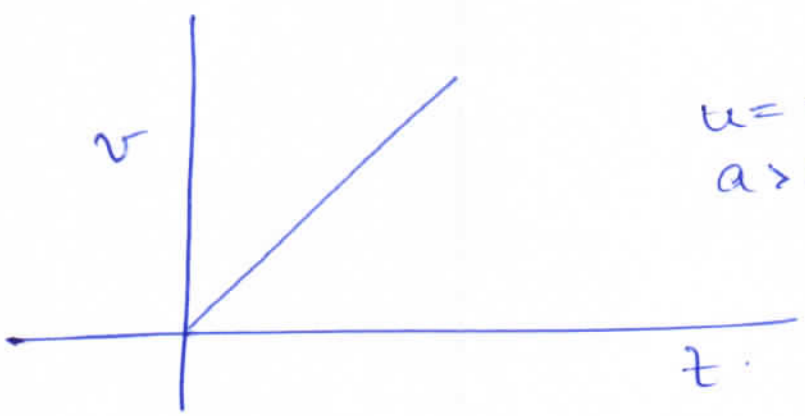
$\frac{v_2 - v_1}{t_2 - t_1} = \text{const} = a$.



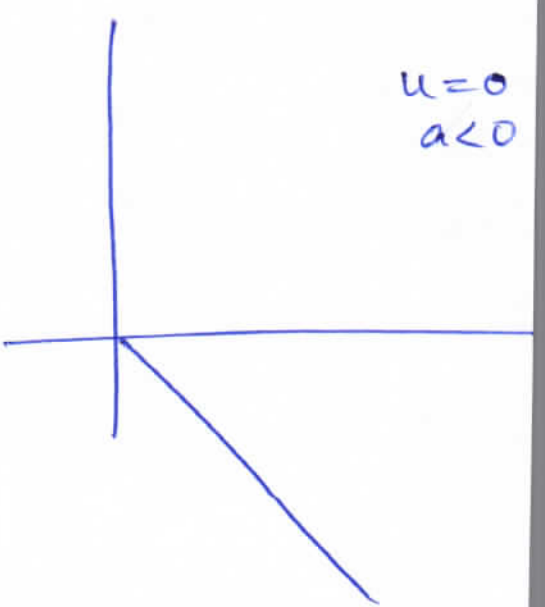
$u \neq 0$
$a > 0$



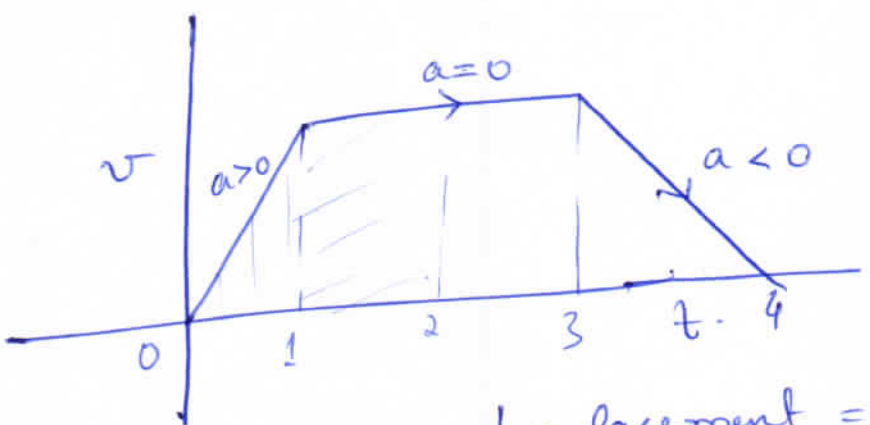
$u \neq 0$
$a < 0$



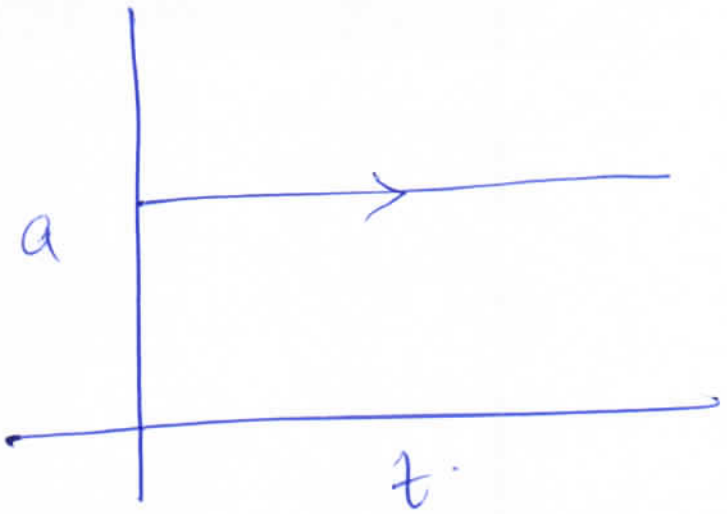
$u = 0$
$a > 0$

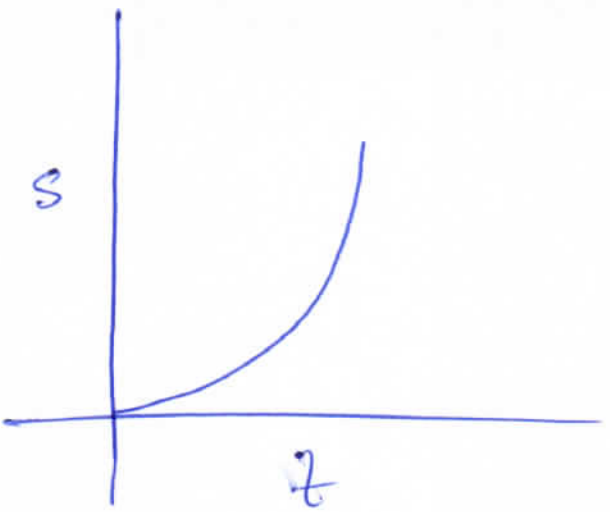


$u = 0$
$a < 0$



displacement = area under v-t graph.

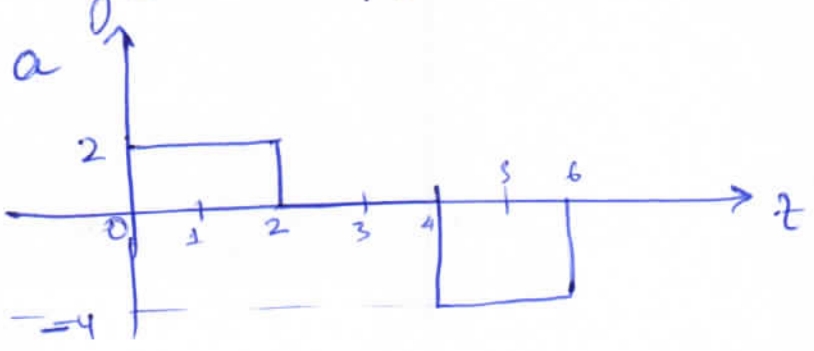




$$s = ut + \frac{1}{2}at^2$$

Q1 A ball is thrown vertically upward with initial velocity = 100 m/s. After how much time will it return? ($g = 10 m/s^2$)
Draw Velocity - time graph for the ball and find from the graph.
i) Maximum height attained by the ball
ii) height of ball after 15s.

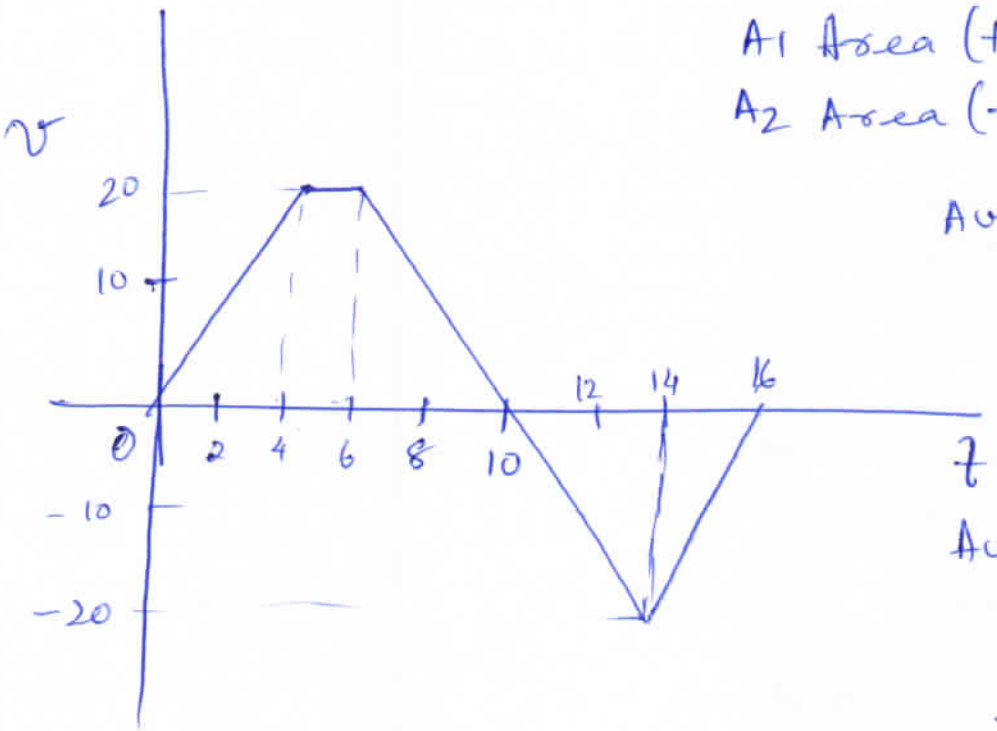
Q2 The acceleration versus time graph of particle moving along a straight line is given. Draw corresponding $v-t$ graph given $v = 0$ at $t = 0$



Q3 If v-t graph of a particle moving in a straight line is shown below.

Find a) Avg velocity

b) Avg Speed



A_1 Area (+ve) = 120 m.

A_2 Area (-ve) = 60 m.

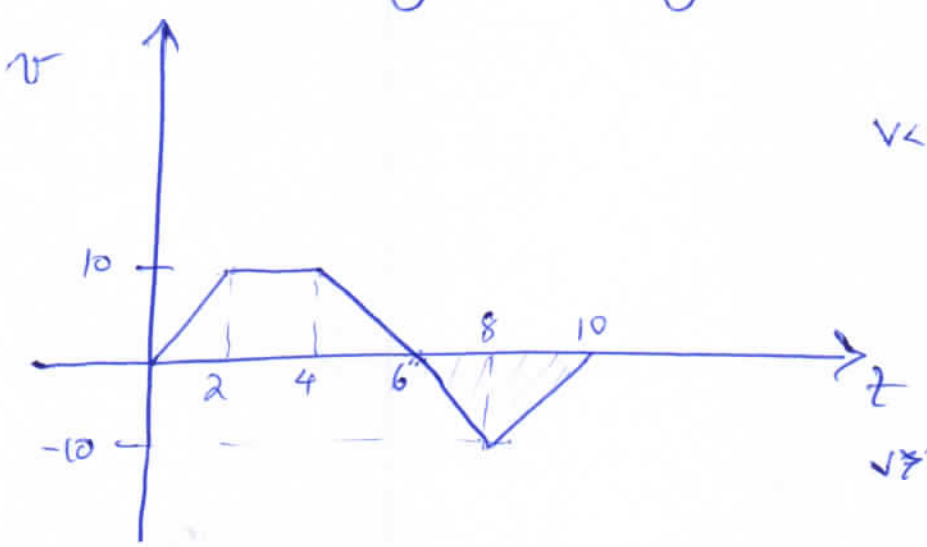
Avg Velocity $= \frac{120-60}{16}$

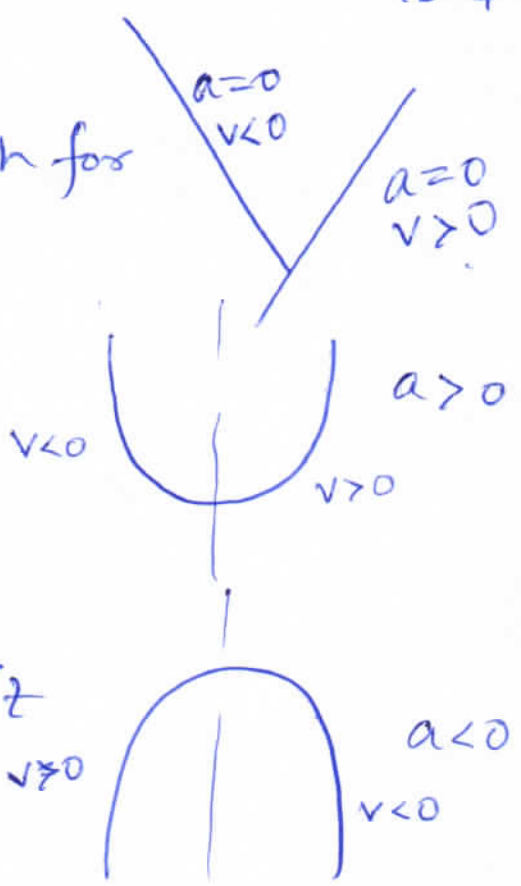
$= \frac{60}{16} = \frac{15}{4}$ m/s

Avg Speed $= \frac{120+60}{16}$

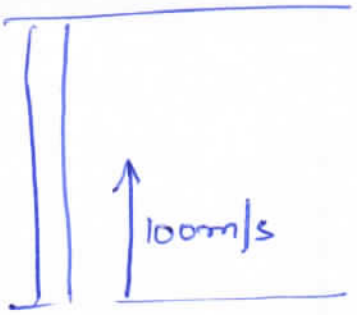
$= \frac{180}{16} = \frac{45}{4}$ m/s

Q4 Draw a-t & s-t graph for corresponding v-t graph.





Ans 1



let it return after t second.

↑ +ve.

$u = +100\ m/s$

$a = -10\ m/s^2$

$S = 0$

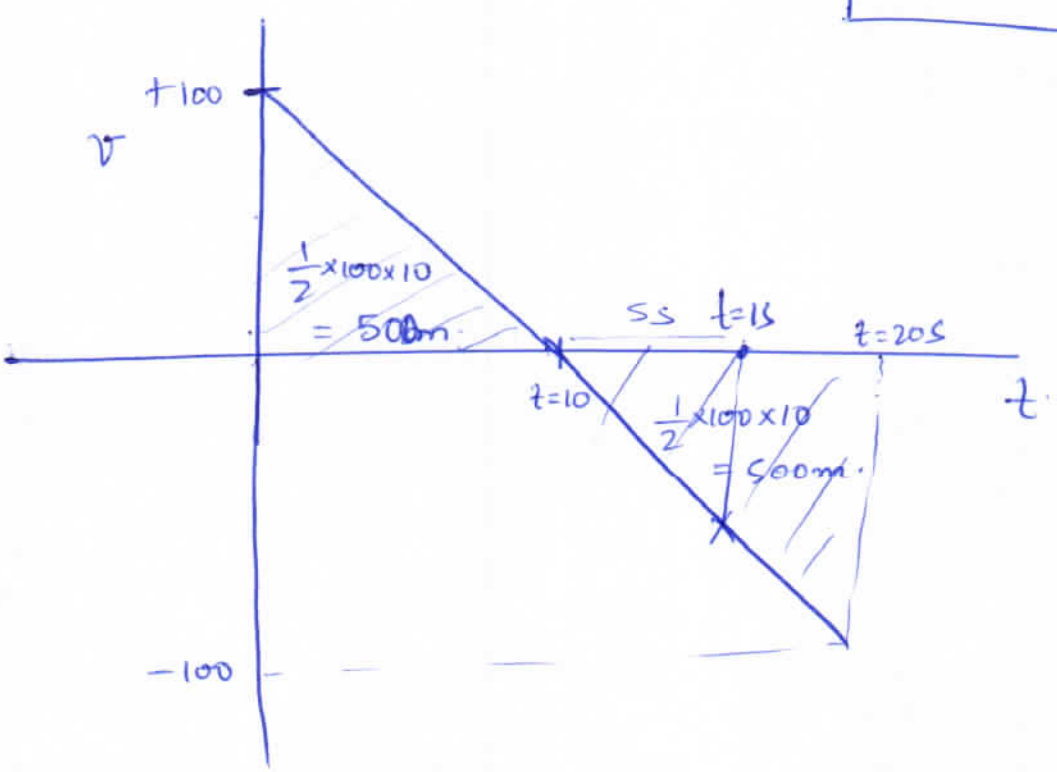
$$S = ut + \frac{1}{2}at^2$$

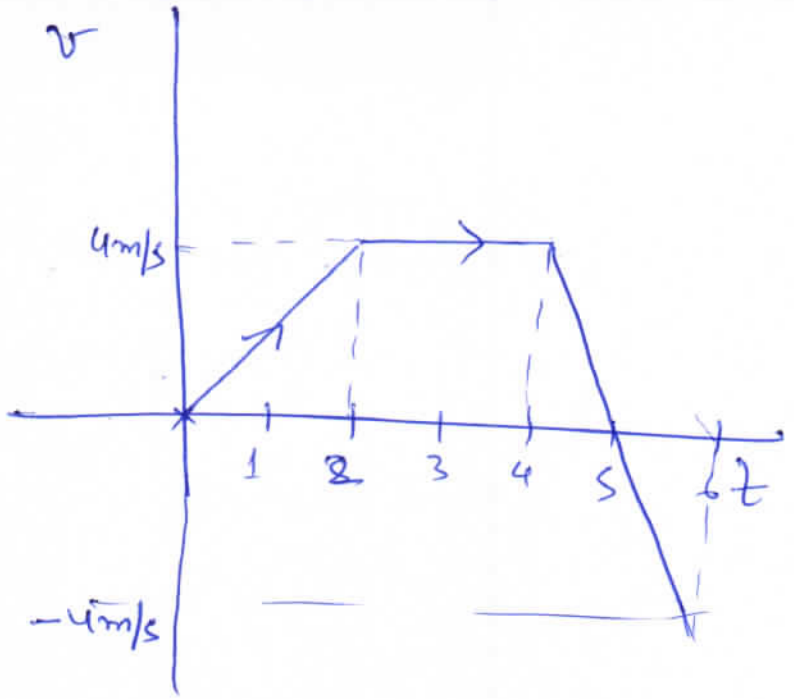
$$0 = 100t + \frac{1}{2}(-10)\cdot t^2$$

$$5t^2 - 100t = 0$$

$$t(t-20) = 0$$

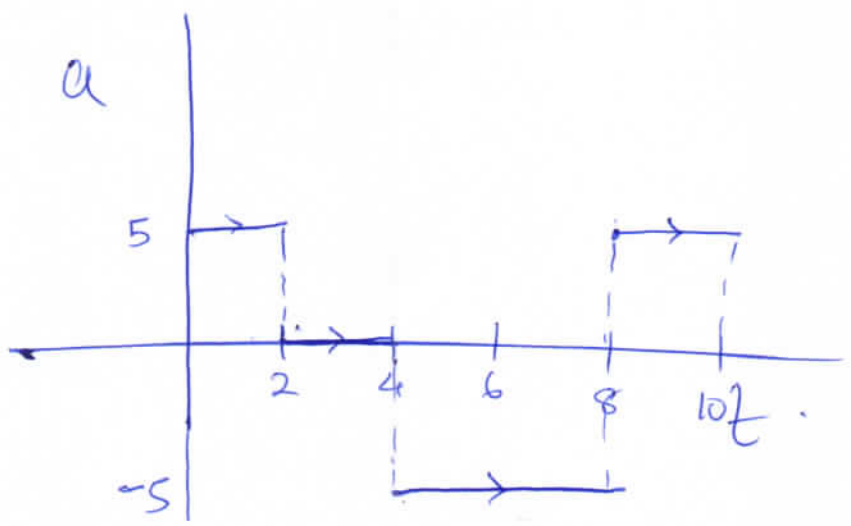
$t = 0s$ $\quad$ $\boxed{t = 20s}$.

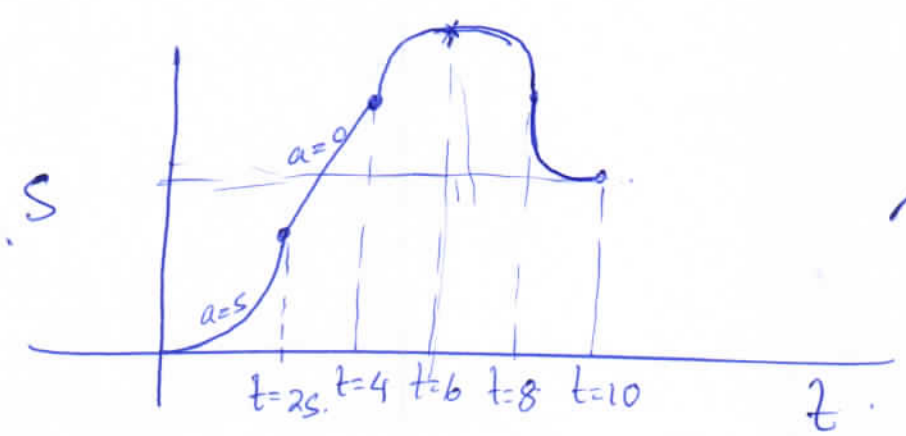




Ans4 a)

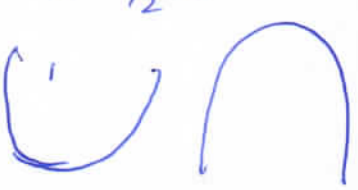
0 – 2 s	+5
2 – 4 s	0
4 – 8 s	−5
8 – 10 s	+5.





$s = ut + \frac{1}{2}at$

$s = \frac{1}{2}at^2$



NON-UNIFORMLY ACCELERATED MOTION

$a \neq 0$ $\quad v \neq 0$ $\quad v \neq \text{const}$.

$a \neq \text{const}$. $\quad$ or

$\Downarrow$

a = changing. $\quad v$ = changing.

$$a = \frac{\Delta v}{\Delta t}.$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad a = \frac{dv}{dt} \left\{ \begin{array}{l} \text{instantaneous} \\ \text{acceleration} \end{array} \right.$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad v = \frac{ds}{dt} \left\{ \begin{array}{l} \text{instantaneous} \\ \text{velocity.} \end{array} \right.$$

$$a = \frac{d\left(\frac{ds}{dt}\right)}{dt} = \frac{d^2 s}{dt^2} = \frac{dv}{dt}.$$

$$v = \frac{ds}{dt}.$$

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} \leftarrow v$$

$$\boxed{a = v\frac{dv}{ds}}$$

Q1) If velocity-time equation is $v = 10 + 2t + 3t^2$

Find a) a-t equation.

b) displacement of particle from mean position at time $t = 1$ sec

It is given displacement is 20 m at time $t = 0$ s.

Q2 If displacement-time equation of a particle moving along x-axis is given as $x = (20 + t^3 - 12t)$ m

a) Find position & velocity of particle at $t = 0$ sec

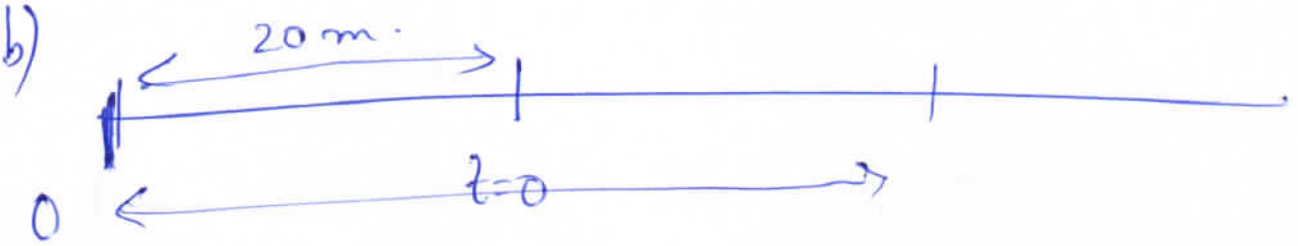
b) Is motion uniformly accelerated.

c) Find position of particle when $v = 0$ m/s.

Ans 4

a) $a = \frac{dv}{dt} = 0 + 2 + 6t = 6t + 2$

b)



$t = 1 \Rightarrow 20 + 1^3 = 21$

$= 20 - 11 = 12$

$v = \frac{ds}{dt}$

$s = \int v\,dt \uparrow \quad \downarrow ds/dt$

$v = \int a\,dt \uparrow \quad \downarrow \frac{dv}{dt}$

a.

$(10 + 2t + 3t^2) = \frac{ds}{dt}$.

$$\int_{s=20}^{s=s} 1\,ds = \int_{t=0}^{t=t} (10 + 2t + 3t^2)\,dt$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$s\Big]_{20}^{s} = 10t + \frac{2t^2}{2} + \frac{3t^3}{3}\Big]_0^1$$

$$s - 20 = [10(1) + (1)^2 + (1)^3] - [10(0) + (0)^2 + (0)^3]$$

$$s = 20 + 12 = 32\,m.$$

Ans2 $x = 20 + t^3 - 12t$

$$v = \frac{dx}{dt} = 0 + 3t^2 - 12 = 3t^2 - 12$$

$$a = \frac{dv}{dt} = 6t$$

a) $x(t=0) = 20 + (0)^3 - 12(0)$

$= \underline{20\ m}$

$v(t=0) = 3(0)^2 - 12 = \underline{-12\ m/s}$

b) $a = 6t \neq$ const $\Rightarrow$ non-uniformly accelerated.

c) $v = 3t^2 - 12$

$v = 0 \Rightarrow 3t^2 - 12 = 0$

$t = \sqrt{4} = 2$

$x(t=2) = 20 + 2^3 - 12(2)$

$= \underline{4\ m}$