LIMITS TUTORIAL.

Pg 97, 98, 99 — 1, 3, 6, 9, 11, 17, 18, 21, 22, 25, 30

Pg 100 – 103 — 35, 38, 43, 44

7, 8, 12, 13, 14

Pg 105 — Comp ~~1~~ 2

Pg 109 — 1, 2, 4

Pg 97, 98, 99

(1) $$\lim_{n\to\infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$$

$$\lim_{n\to\infty} \frac{n(4n^2+4n+1)}{n^3+3n^2-n+2n^2+6n-2}$$

$$\lim_{n\to\infty} \frac{4n^3+4n^2+n}{n^3+5n^2+5n-2}$$

$$\lim_{n\to\infty} \frac{4+\frac{4}{n}+\frac{1}{n^2}}{1+\frac{5}{n}+\frac{5}{n^2}-\frac{2}{n^3}} = \frac{4}{1} = 4$$

(3) $$\lim_{x\to 0} \left[\frac{1}{x} - \frac{\ln(1+x)}{x^2}\right]$$

$$\lim_{x\to 0} \left[\frac{x-\ln(1+x)}{x^2}\right] \qquad \frac{0}{0}$$

L.H.

$$\lim_{x\to 0} \left[\frac{1-\frac{1}{1+x}\times 1}{2x}\right] \qquad \frac{0}{0} \qquad x^n \quad nx^{n-1}$$

L.H.

$$\lim_{x\to 0} \left[\frac{0+(1+x)^{-2}\times 1}{2}\right] = \frac{1}{2} \qquad \frac{1}{1+x} = (1+x)^{-1}$$

(6) $\lim_{x\to 5} \dfrac{x^k - 5^k}{x-5} = 500$

$$k(5)^{k-1} = 500.$$

$$\underline{k = 4}.$$

(9)
$$\lim_{x\to\infty} \frac{2x^3 - 5x^2 + 14}{3x^3 + 12x - 15} = \lim_{x\to\infty} \frac{2 - \frac{5}{x} + \frac{14}{x^3}}{3 + \frac{12}{x^2} - \frac{15}{x^3}}$$

$$= \frac{2}{3}$$

(11) $\lim_{x\to a} \dfrac{(x+2)^{5/3} - (a+2)^{5/3}}{x-a.} =$

$$\lim_{x+2 \to a+2.} \frac{(x+2)^{5/3} - (a+2)^{5/3}}{(x+2) - (a+2)} = \tfrac{5}{3}(a+2)^{5/3 - 1}$$

$$= \frac{5}{3}(a+2)^{2/3}$$

D

(17) $\lim_{x\to\infty} \dfrac{\sqrt{x}}{\sqrt{x + \sqrt{x + \sqrt{x}}}} = \lim_{x\to\infty} \dfrac{x^{1/2}/x^{1/2}}{\sqrt{\frac{x}{x} + \sqrt{\frac{x}{x^2} + \sqrt{\frac{x}{x^4}}}}}$

$$= \frac{1}{\sqrt{1 + \sqrt{0 + \sqrt{0}}}} = 1$$

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}} = \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}}.$$

$$= \sqrt{\frac{x}{x}+\sqrt{\frac{x+\sqrt{x}}{x^2}}}$$

$$= \sqrt{\frac{x}{x}+\sqrt{\frac{x}{x^2}+\sqrt{\frac{x}{x^4}}}}$$

(18) $$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3} = \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}-\sin x}{x^3}$$

$$\lim_{x\to 0}\frac{\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots\right)-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}\cdots\right)}{x^3}$$

$$\lim_{x\to 0}\frac{x^3\left(\frac{1}{3}+\frac{1}{3!}\right)+x^5\left(\frac{2}{15}-\frac{1}{5!}\right)+\cdots}{x^3}$$

$$\lim_{x\to 0}\frac{\cancel{x^3}\left(\frac{1}{2}\right)}{\cancel{x^3}}+\frac{x^{\cancel{5}\,2}}{\cancel{x^3}}\left(\frac{2}{15}-\frac{1}{5!}\right)+\frac{x^{\cancel{7}\,4}}{\cancel{x^3}}\left(\right.$$

$$=\frac{1}{2}.$$

$$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}(1-\cos x)}{x^3}$$

$$= \lim_{x\to 0}\left(\frac{\tan x}{x}\right)\left(\frac{1-\cos x}{x^2}\right)$$

$$= \lim_{x\to 0}\left(\frac{\tan x}{x}\right)\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$$

$$= 1 \times \frac{1}{2} = \frac{1}{2}$$

(21) $$\lim_{x\to 0} \frac{\cos^{1/2} x - \cos^{1/3} x}{\sin^2 x}$$

$\cos^{1/6} x = t$

$x \to 0$

$\cos^{1/6} x \to 1$

$t \to 1$

$$\lim_{t\to 1} \frac{t^3 - t^2}{1 - t^{12}}$$

$$\lim_{t\to 1} \frac{t^2(t-1)}{-(t^{12}-1)}$$

$1 - \cos^2 x$

$$\lim_{t\to 1} \frac{t^2}{-\frac{(t^{12}-1)}{t-1}} = \frac{\lim_{t\to 1} t^2}{\lim_{t\to 1} -\frac{(t^{12}-1^{12})}{t-1}} = \frac{1}{-12(1)^{12-1}}$$

$$= -\frac{1}{12}$$

(22) $$\lim_{x\to \pi/4} \frac{\sqrt{2} - \cos x - \sin x}{(4x-\pi)^2}$$

L.H.

$$\lim_{x\to \pi/4} \frac{0 + \sin x - \cos x}{2(4x-\pi)^1 \times 4}$$

L.H

$$\lim_{x\to \pi/4} \frac{\cos x + \sin x}{8 \times 4} = \frac{\sqrt{2}}{32}$$

$$\lim_{x\to \pi/4} \frac{\sqrt{2} - \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)}{(4x-\pi)^2}$$

$$\lim_{x\to \frac{\pi}{4}} \frac{\sqrt{2}\left(1 - \cos\left(x - \frac{\pi}{4}\right)\right)}{16\left(x - \frac{\pi}{4}\right)^2}$$

$$\frac{\sqrt{2}}{16} \times \frac{1}{2} = \frac{\sqrt{2}}{32}$$

$$\sin x + \cos x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$$

$$= \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$$

---

(25) $$\lim_{x\to 3} \frac{(x^3+27)\ln(x-2)}{(x^2-9)}$$

$$\lim_{x\to 3}\left(\frac{x^3+27}{(x+3)}\right)\left(\frac{\ln(1+x-3)}{(x-3)}\right)$$

$$\left(\frac{3^3+27}{3+3}\right)\times 1 \quad = \quad 9$$

(30) $$\lim_{x\to -1} \frac{\sqrt{\pi} - \sqrt{\cos^{-1}x}}{\sqrt{x+1}}$$

$\cos^{-1}x = t.$

$x = \cos t.$

$x \to -1$

$t \to \pi.$

$$\lim_{t\to\pi} \frac{\sqrt{\pi} - \sqrt{t}}{\sqrt{1+\cos t}}$$

$$\lim_{t\to\pi} \frac{\cancel{-}\frac{1}{\cancel{2}\sqrt{t}}}{\frac{\cancel{-}\sin t}{\cancel{2}\sqrt{1+\cos t}}} \Rightarrow \lim_{t\to\pi} \frac{\sqrt{1+\cos t}}{\sqrt{t}\sin t}$$

$t^{1/2}$

$$\frac{1}{2}t^{1/2-1} = \frac{1}{2}t^{-1/2}$$

$$= \frac{1}{2\sqrt{t}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\lim_{x\to -1} \frac{\sqrt{\pi} - \sqrt{\cos^{-1}x}}{\sqrt{x+1}}$$

L.H.

$$\lim_{x\to -1} \frac{0 - \frac{1}{2\sqrt{\cos^{-1}x}} \times \left(-\frac{1}{\sqrt{1-x^2}}\right)}{\frac{1}{2\sqrt{x+1}} \times 1}$$

$$\lim_{x\to -1} \frac{\cancel{\sqrt{x+1}}}{\sqrt{\cos^{-1}x}\,\sqrt{1-x}\,\cancel{\sqrt{1+x}}} = \frac{1}{\sqrt{\pi}\sqrt{2}} = \frac{1}{\sqrt{2\pi}}.$$

Pg 100-103.

(35) $$\lim_{x\to 2} \frac{\sin(e^{x-2}-1)}{\ln(x-1)}$$

$$\lim_{x\to 2} \frac{\sin(e^{x-2}-1)}{(e^{x-2}-1)} \times \frac{\frac{(e^{x-2}-1)}{(x-2)}}{\frac{\ln(x-1)}{(x-2)}}$$

$x-1$
$= 1+x-2$

$$= \frac{1 \times 1}{1} = 1$$

(38) $$\lim_{x\to \pi/2} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x}.$$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$$

$$\lim_{x\to \pi/2} \frac{\left(a^{(\cot x - \cos x)} - 1\right)}{(\cot x - \cos x)} a^{\cos x}$$

$\ln a \times 1. = \log_e a$

(43) $\lim_{x\to 0}\left(\frac{1}{2}\left(2^{\sin x}+3^{\tan x}\right)\right)^{\operatorname{cosec} x}$.

$$= e^{\lim_{x\to 0} g(x)\{f(x)-1\}} \qquad \lim_{x\to 0} g(x)\{f(x)-1\} \to \alpha. \qquad = e^{\alpha}.$$

$$\alpha = \lim_{x\to 0} \operatorname{cosec} x\left\{\frac{1}{2}\left(2^{\sin x}+3^{\tan x}\right)-1\right\}$$

$$= \lim_{x\to 0}\left\{\frac{2^{\sin x}+3^{\tan x}-2}{2\sin x}\right\}$$

$$= \lim_{x\to 0}\frac{1}{2}\left\{\left(\frac{2^{\sin x}-1}{\sin x}\right)+\left(\frac{3^{\tan x}-1}{\left(\frac{\sin x}{\cos x}\right)\cos x}\right)\right\}$$

$$= \frac{1}{2}\left\{\ln 2 + \ln 3\right\}$$

$$\alpha = \frac{1}{2}\ln 6 = \ln\sqrt{6}$$

$$e^{\alpha} = e^{\ln\sqrt{6}} = \sqrt{6}$$

$e^{\ln x} = y.$

$\ln x \cancel{\ln e} = \ln y$

$x = y.$

44

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$$

$$\cancel{\lim_{x\to}\{f(x)\}^{g(x)}}$$

$$\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^{\lim_{x\to 0}\frac{\sin x}{x-\sin x}}$$

$$\lim_{x\to 0}\left(\frac{\sin x}{x-\sin x}\right) = \alpha$$

$$1 \qquad\qquad 1^{\alpha}$$

$$\alpha = \lim_{x\to 0}\frac{\sin x}{x-\sin x}$$

$$= \lim_{x\to 0}\frac{\cos x}{1-\cos x}$$

$$\to \infty$$

$$e^{\alpha}$$

$$\alpha = \lim_{x\to 0}\frac{\sin x}{x-\sin x}\left(\frac{\sin x}{x}-1\right)$$

$$\alpha = \lim_{x\to 0}\frac{\sin x}{x-\sin x}\cdot\left(\frac{\sin x - x}{x}\right) = -1$$

$$e^{-1}$$

(7) $h(x) = \lim_{n\to\infty} \dfrac{x^{2n} f(x) + g(x)}{1+x^{2n}}$

$$= \lim_{n\to\infty} \dfrac{\dfrac{x^{2n} f(x)}{x^{2n}} + \dfrac{g(x)}{x^{2n}}}{\dfrac{1}{x^{2n}} + \dfrac{x^{2n}}{x^{2n}}} \quad |x| > 1$$

$$= \lim_{n\to\infty} \dfrac{f(x) + \dfrac{g(x)}{x^{2n}}}{1 + \dfrac{1}{x^{2n}}} = f(x)$$

$$h(x) = \lim_{n\to\infty} \dfrac{x^{2n} f(x) + g(x)}{1+x^{2n}} \quad |x| < 1$$

$$= \dfrac{0 \times f(x) + g(x)}{1+0}$$

$$= g(x) \quad |x| < 1$$

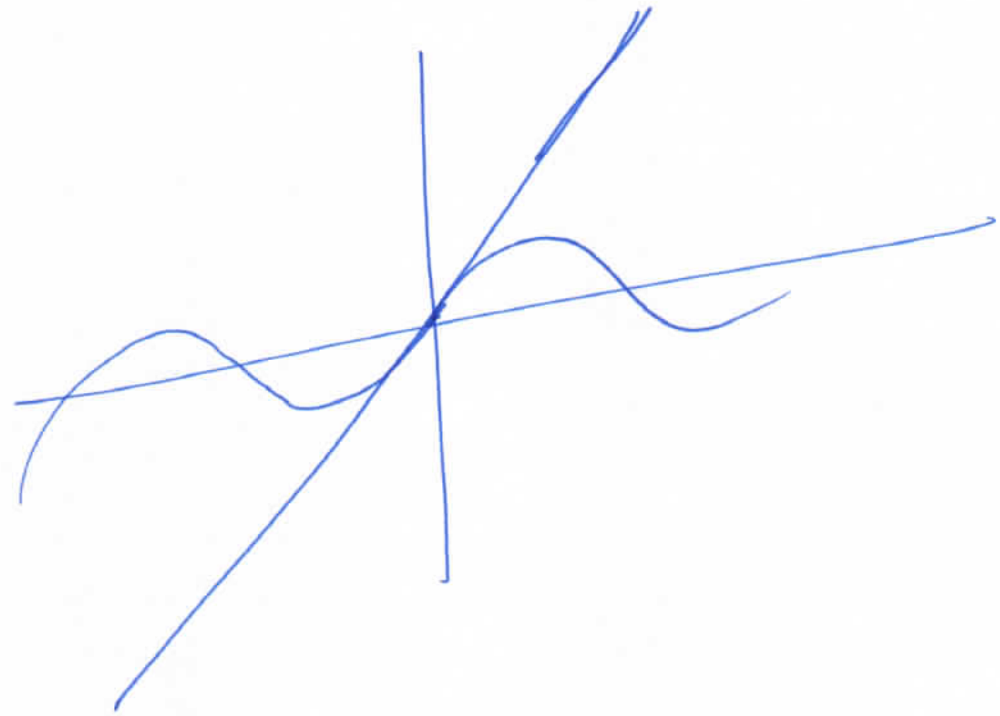
A B

$x = -1$

$x = 1$

$$h(x) = \dfrac{1 \times f(x) + g(x)}{1+1} = \dfrac{f(x)+g(x)}{2} \quad ABC$$

8) $\lim_{x\to 0^+}\left[m\,\frac{\sin x}{x}\right]$



$$\lim_{x\to 0^+}\left[m\,\frac{x-\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots}{x}\right]$$

$$\lim_{x\to 0^+}\left[m\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right)\right]$$

$[m_-]$

$m-1 \quad m>0$

$m. \quad m\leq 0$

AB

(12) $\lim_{x\to\infty}\left(1+\frac{\lambda}{x}+\frac{\mu}{x^2}\right)^{2x} = e^2$

$e^{\alpha} = e^2 \Rightarrow \alpha = 2$

$\alpha = \lim_{x\to\infty} 2x\left(1+\frac{\lambda}{x}+\frac{\mu}{x^2}-1\right)$

$= \lim_{x\to\infty} 2x\left(\frac{\lambda}{x}+\frac{\mu}{x^2}\right)$

$= \lim_{x\to\infty}\left(2\lambda+\frac{2\mu}{x}\right) = 2$

$\lambda = 1.$

$\mu =$ any real constant.

A C D.

(13) $\lim_{x\to\infty}\left(\frac{|x|}{|x|+2}\right)^{x}$.

$\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^{x} = e^{\alpha}$ e^{-2}

$\alpha = \lim_{x\to\infty} x\left(\frac{x}{x+2}-1\right)$

$= \lim_{x\to\infty} x\left(\frac{-2}{x+2}\right) = -2$

Pg 105 Comp 2.

(4) $\lim_{x\to 0} \left(\frac{1}{x}\right)^{\tan x} = L$

$\lim_{x\to 0} \tan x \log\left(\frac{1}{x}\right) = \log L$

$\lim_{x\to 0} \frac{\log \frac{1}{x}}{\cot x} \xrightarrow{L.H} \lim_{x\to 0} \frac{\left(\frac{1}{1/x}\right)\left(\frac{-1}{x^2}\right)}{-\operatorname{cosec}^2 x}$

x^{-1}

$(-1)x^{-1-1}$

$-\frac{1}{x^2}$

$\lim_{x\to 0} \frac{1}{x} \times \sin^2 x$

$\lim_{x\to 0} \left(\frac{\sin x}{x}\right) \times \sin x = 0$

$\log L = 0 \Rightarrow L = 1.$

(5) $\lim_{x\to a} \frac{\cos x \log(x-a)}{\log(e^x - e^a)} = \lim_{x\to 0} \cos x \lim_{x\to a} \frac{\log(x-a)}{\log(e^x-e^a)}$

$\cos a \cdot \lim_{x\to a} \frac{\frac{1}{x-a}}{\frac{1}{e^x - e^a} \times e^x}$

$$\cos a \times \lim_{x\to a} \frac{e^x - e^a}{(x-a)e^x}$$

$$\cos a \times \lim_{x\to a} \frac{e^a}{e^x} \frac{(e^{x-a} - 1)}{(x-a)}$$

$\to 1$

$\cos a.$

(4)

Pg 109

(6) lim ... (1) $$\lim_{x\to\infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$$

$$\lim_{x\to\infty} \sqrt{\frac{\frac{x}{x} - \frac{\sin x}{x}}{\frac{x}{x} + \frac{\cos^2 x}{x}}}$$

$= 1$

(2) $G(x) = -\sqrt{25 - x^2}$

$$\lim_{x\to 1} \frac{-\sqrt{25-x^2} + \sqrt{25-1}}{x-1}$$

$$\lim_{x\to1}\left(\frac{\sqrt{24}-\sqrt{25-x^2}}{x-1}\right)\frac{\left(\sqrt{24}+\sqrt{25-x^2}\right)}{\left(\sqrt{24}+\sqrt{25-x^2}\right)}$$

$$\lim_{x\to1}\left(\frac{24-25+x^2}{x-1}\right)\left(\frac{1}{\sqrt{24}+\sqrt{25-x^2}}\right)$$

$$\lim_{x\to1}\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}\left(\sqrt{24}+\sqrt{25-x^2}\right)}$$

$$\frac{2}{\sqrt{24}+\sqrt{24}}=\frac{\cancel{2}}{\cancel{2}\sqrt{24}}=\frac{1}{\sqrt{24}}$$

$\Rightarrow$ D

4 $$\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x=e^{\alpha}$$

$$\alpha=\lim_{x\to\infty}x\left\{\frac{x-3}{x+2}-1\right\}$$

$$=\lim_{x\to\infty}x\left\{\frac{-5}{x+2}\right\}$$

$$=-5.$$

$$e^{-5}$$