GRAVITATION

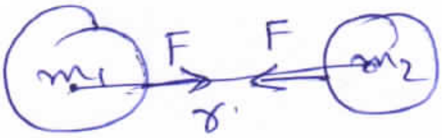
phenomenon due to which object having masses attract each other

The force with which they attract is called the force of gravitation.

'Newton's law of Gravitation'

1) Force of gravitation is directly proportional to the product of their masses

$$F \propto m_1 m_2$$



2) Force of gravitation is inversely proportional to the square of distance between the bodies

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{m_1 m_2}{r^2}$$ (acts along the line joining their CM's)

$$\boxed{F = G \frac{m_1 m_2}{r^2}}$$

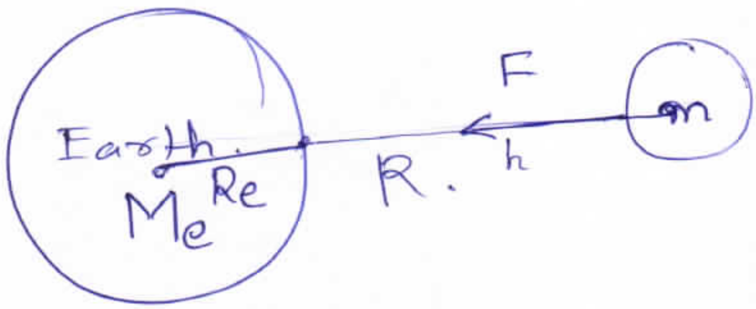
Gravitational Const. (Universal Const)

$$G = 6.67 \times 10^{-11} \ N\,m^2\,kg^{-2}$$

Force of gravity

Gravitational pull (force) by the Earth.

$R_e \approx 6400\,km$



$$F = \frac{G M_e m}{(R_e + h)^2}$$

i) If $h << R_e$ (very close to surface of earth)

$$F = \frac{G M_e m}{R_e^2}$$ (as $R_e + h \approx R_e$ when $h << R_e$)

$$F = m a_m$$

$$a_m = \frac{\frac{G M_e \cancel{m}}{R_e^2}}{\cancel{m}} = \frac{G M_e}{R_e^2} = g = 9.81\,m/s^2$$

g is $9.81\,m/s^2$ $= \frac{G M_e}{R_e^2}$ near the surface of the earth.

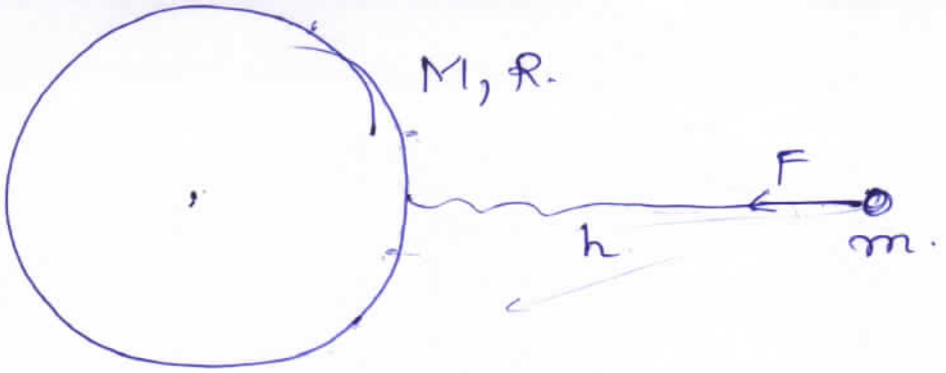
$h << R_e$

$$g = \frac{G M_e}{R_e^2}$$

ii) When 'h' is comparable but not negligible.

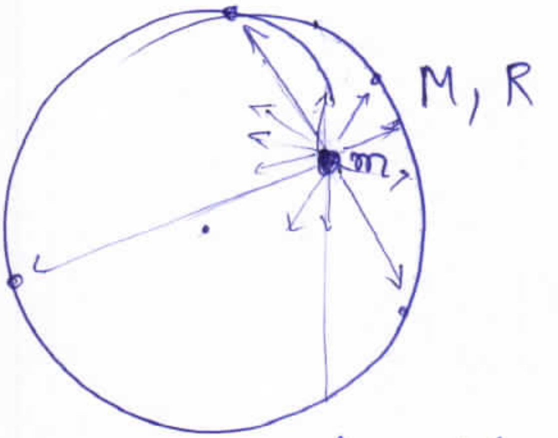
$$F_h = \frac{G M_e m}{(R_e+h)^2} = m g_h$$

$h \to$ ~~5%~~ 8% of R_e	Other Cases. $h >$ 8% R_e.
$g_h = \frac{G M_e}{(R_e+h)^2}$	$g_h = \frac{G M_e}{(R_e+h)^2}$
$g_h = \frac{G M_e}{R_e^2\left(1+\frac{h}{R_e}\right)^2}$	$g_h = \frac{G M_e}{R_e^2\left(1+\frac{h}{R_e}\right)^2}$
$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$	$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$
$g_h = g\left(1+\frac{h}{R_e}\right)^{-2}$ $\simeq g\left(1-\frac{2h}{R_e}\right)$	accurate & will give actual acceleration due to gravity at height h.
approximate. (will give close results to actual if $h <$ 7-8% R_e	



$$F = \frac{GMm}{(R+h)^2}$$

spherical shell.

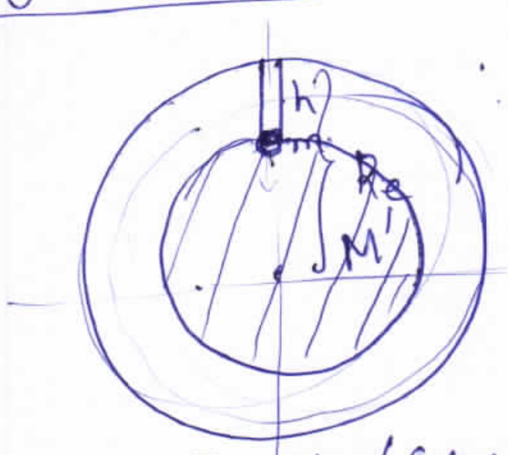


spherical shell.

Resultant of all forces on a mass inside a spherical shell is zero due to the shell.

$$\underline{F_{net} = 0}$$

→ gravity a height h inside the earth from the surface



Earth. (Solid sphere)

$$F = \underbrace{\frac{GM'm}{(R_e-h)^2}}_{\text{solid part}} + \underbrace{0+0+0}_{\text{due to Spherical shells}}$$

$$F = mg_{-h}$$

$$M' = \frac{M_e}{R_e^3}(R_e-h)^3$$

$$\frac{GM'\cancel{m}}{(R_e-h)^2} = \cancel{m}g_{-h}$$

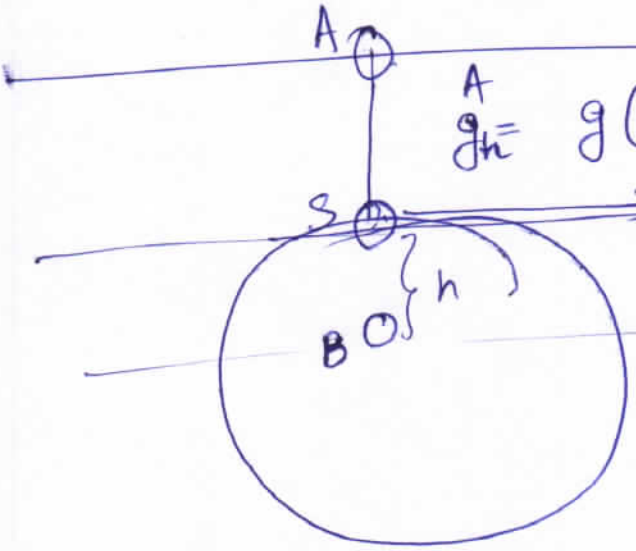
$$\frac{GM_e}{(R_e-h)^2}\left(\frac{R_e-h}{R_e}\right)^3 = g_{-h}.$$

$$\frac{GM_e}{R_e^3}(R_e-h) = g_{-h}$$

$$\frac{GM_e}{R_e^2}\left(\frac{R_e-h}{R_e}\right) = g_{-h}$$

$$\boxed{g\left(1-\frac{h}{R_e}\right) = g_{-h}}$$

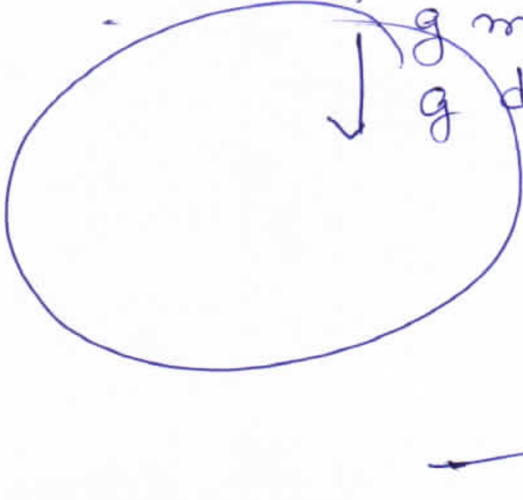
actual result (NOT APROXIMATE FORMULA)



A $g_h = g\left(1-\frac{2h}{R_e}\right)$ or $g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$

S g

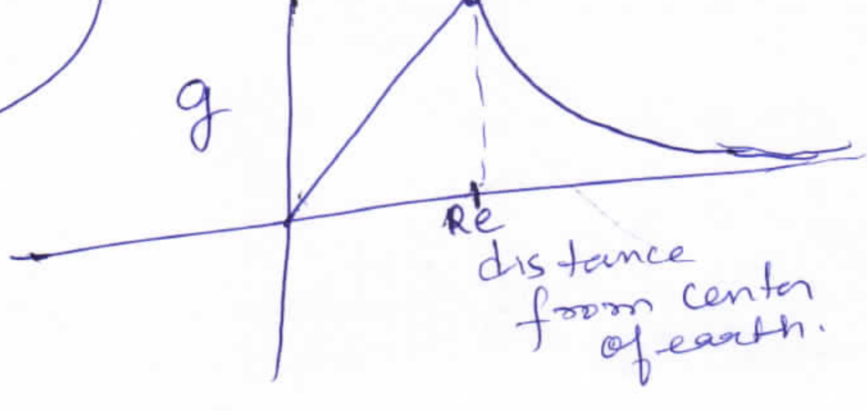
B $g_{-h} = g\left(1-\frac{h}{R_e}\right)$



↑ g decreases

g max at surface

↓ g decreases as we go below



Q) At what height h, the value of g becomes $g/4$

Q) What is the value of g at a height $h = R_e/4$ above surface of earth.

$$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} = g/4$$

$$\frac{1}{\left(1+\frac{h}{R_e}\right)^{\cancel{2}}} = \frac{1}{\pm 2^{\cancel{2}}}$$

$$1+\frac{h}{R_e} = \pm 2$$

$$1+\frac{h}{R_e} = +2 \qquad\qquad 1+\frac{h}{R_e} = -2$$

$$\underline{h = R_e} \longrightarrow \underline{h = -3R_e}$$

$$g_{-h} = g\left(1-\frac{h}{R_e}\right) = \frac{g}{4}$$

$$1-\frac{h}{R_e} = \frac{1}{4}$$

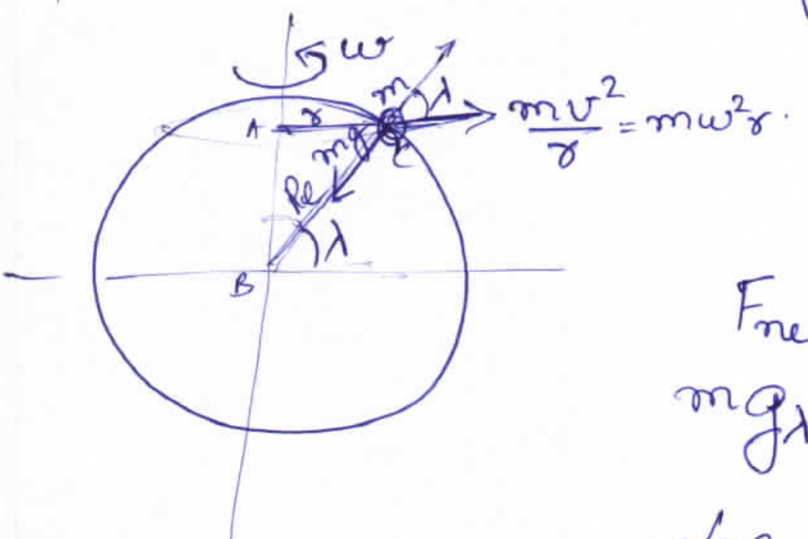
$$\frac{3}{4} = \frac{h}{R_e} \Rightarrow h = \frac{3R_e}{4}$$

Q)

$$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} = \frac{g}{\left(1+\frac{\frac{R_e}{4}}{R_e}\right)^2} = \frac{g}{\left(1+\frac{1}{4}\right)^2} = \frac{16g}{25}$$

Variation in g.

i) due to Rotation of Earth.



$$\omega = \frac{2\pi}{24 \times 60 \times 60} \text{ rad/s.}$$

$$F_{net} = mg - m\omega^2 r \cos\lambda.$$

$$mg_\lambda = mg - m\omega^2 r \cos\lambda.$$

$$\cancel{m}g_\lambda = \cancel{m}g - \cancel{m}\omega^2 (R_e \cos\lambda)\cos\lambda$$

$$\boxed{g_\lambda = g - \omega^2 R_e \cos^2\lambda}$$

In ΔABC

$$\frac{r}{R_e} = \sin(90-\lambda)$$

$$r = R_e \cos\lambda.$$

If $\lambda = 90^\circ$ poles.

g is maximum.

$\lambda = 0^\circ$ equator

g is minimum.

ii) due to shape of earth.

$R_p < R_{eq}$

$\Rightarrow g_p > g_{eq}$