

Homework

1) Simplify $\cos \frac{\pi}{13} \cdot \cos \frac{2\pi}{13} \cdot \cos \frac{3\pi}{13} \cdot \cos \frac{4\pi}{13} \cdot \cos \frac{5\pi}{13} \cdot \cos \frac{6\pi}{13}$

2) $\cos \theta - \sin \theta = a$
 $\sec \theta - \csc \theta = b$

Find $a^{2/3} b^{2/3} (a^{2/3} + b^{2/3})$

3) If $\tan x = \frac{m}{m+1}$ $\tan y = 2m+1$

$x+y = ?$

4) $\sin \theta + \csc \theta = 2$ $\sin^{10} \theta + \csc^{10} \theta = ?$

5) $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = ?$ options (A) 4 (B) 3 (C) 2 (D) 1

6) Find $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15}$

7) Find $\sin 12^\circ \sin 48^\circ \sin 54^\circ$

8) $\tan 100^\circ + \tan 125^\circ + \tan 100^\circ \times \tan 125^\circ =$

9) $\cos 5\theta = a \cos \theta + b \cos^3 \theta + c \cos^5 \theta + d$
find a, b, c, d .

10) $(1 + \tan 1^\circ)(1 + \tan 2^\circ)(1 + \tan 3^\circ) \dots (1 + \tan 45^\circ) = 2^{25-n}$
Find n

①

$$\frac{\cos \frac{\pi}{13} \cos \frac{2\pi}{13} \cos \frac{3\pi}{13} \cos \frac{4\pi}{13} \cos \frac{5\pi}{13} \cos \frac{6\pi}{13} \times 2^4 \sin \frac{\pi}{13}}{2^4 \sin \frac{\pi}{13}}$$

$$= \frac{2^3 \sin \frac{2\pi}{13} \cos \frac{2\pi}{13} \cos \frac{3\pi}{13} \cos \frac{4\pi}{13} \cos \frac{5\pi}{13} \cos \frac{6\pi}{13}}{2^4 \sin \frac{\pi}{13}}$$

$$= \frac{2^2 \sin \frac{4\pi}{13} \cos \frac{4\pi}{13} \cos \frac{3\pi}{13} \cos \frac{5\pi}{13} \cos \frac{6\pi}{13}}{2^4 \sin \frac{\pi}{13}}$$

$$= \frac{2 \sin \frac{8\pi}{13} \cos \frac{3\pi}{13} \cos \frac{6\pi}{13} \cos \frac{5\pi}{13}}{2^4 \sin \frac{\pi}{13}}$$

$$= \frac{2 \sin \frac{8\pi}{13} \cos \frac{3\pi}{13} \cos \frac{6\pi}{13} (-\cos(\pi - \frac{5\pi}{13}))}{2^4 \sin \frac{\pi}{13}}$$

$$= \frac{2 \sin \frac{8\pi}{13} \cos \frac{8\pi}{13} \cos \frac{3\pi}{13} \cos \frac{6\pi}{13}}{2^4 \sin \frac{\pi}{13}}$$

$$= \frac{-\sin \frac{16\pi}{13} \cos \frac{3\pi}{13} \cos \frac{6\pi}{13}}{2^4 \sin \frac{\pi}{13}}$$

$$= \frac{-\sin(\pi + \frac{3\pi}{13}) \cos \frac{3\pi}{13} \cos \frac{6\pi}{13} \times 2}{2^4 \sin \frac{\pi}{13} \times 2}$$

$$= \frac{\sin \frac{6\pi}{13} \cos \frac{6\pi}{13} \times 2}{2^6 \sin \frac{\pi}{13}} = \frac{\sin \frac{12\pi}{13} \times \sin(\pi - \frac{\pi}{13})}{2^6 \sin \frac{\pi}{13}} = \frac{\sin \frac{\pi}{13}}{2^6 \sin \frac{\pi}{13}} = \frac{1}{2^6}$$

②

$$a^{2/3} b^{2/3} (a^{2/3} + b^{2/3})$$

$$\sec \theta - \sin \theta = a$$

$$\frac{1 - \sin^2 \theta}{\sin \theta} = a$$

$$\frac{\cos^2 \theta}{\sin \theta} = a$$

$$\sec \theta - \cos \theta = b$$

$$\frac{1 - \cos^2 \theta}{\cos \theta} = b$$

$$\frac{\sin^2 \theta}{\cos \theta} = b$$

$$\frac{\sin^2 \theta}{\cos \theta} = \frac{b}{a} = \tan^2 \theta$$

$$\tan \theta = \left(\frac{b}{a} \right)^{1/3}$$

$$\tan^2 \theta = \left(\frac{b}{a} \right)^{2/3}$$

$$\frac{\sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta} = ab$$

$$\sin \theta \cos \theta = ab$$

$$\sin 2\theta = 2ab$$

$$\frac{2 \tan \theta}{1 + \tan^2 \theta} = 2ab$$

$$1 + \tan^2 \theta = \frac{2 \tan \theta}{2ab}$$

$$1 + \tan^2 \theta = 1 + \frac{b^{2/3}}{a^{2/3}}$$

$$= (a^{2/3} + b^{2/3})$$

$$(a^{2/3} + b^{2/3}) = (a^{2/3}) (1 + \tan^2 \theta)$$

$$= (a^{2/3}) \left(\frac{\tan \theta}{ab} \right)$$

$$= a^{2/3} \left(\frac{b^{1/3}}{a^{1/3}} \right) \frac{1}{ab}$$

$$(a^{2/3} + b^{2/3}) = \frac{a^{-2/3} b^{-2/3}}{1}$$

$$\Rightarrow (a^{2/3} b^{2/3}) (a^{2/3} + b^{2/3}) = \underline{\underline{1}}$$

Ans 3

$$\tan x = \frac{m}{m+1}$$

$$\cot y = 2m+1$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x+y) = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)}$$

$$= \frac{2m^2 + m + m + 1}{2m^2 + 3m + 1 - m}$$

$$= \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$$

$$x+y = 45^\circ$$

Ans 4

$$\sin \theta + \operatorname{cosec} \theta = 2$$

$$\sin^{10} \theta + \operatorname{cosec}^{10} \theta = ? \leftarrow 4^p + 4^{10} = 2$$

$$\frac{1}{\sin \theta} + \sin \theta = 2$$

$$\sin^2 \theta + 1 = 2 \sin \theta$$

$$(\sin \theta - 1)^2 = 0$$

$$\sin \theta = 1 \Rightarrow \operatorname{cosec} \theta = 1$$

Ans 5

$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = ?$$

$$2 \left(\frac{1}{2 \sin 10^\circ} - \frac{\sqrt{3}}{2 \cos 10^\circ} \right)$$

$$2 \left(\frac{\sin 30^\circ}{\sin 10^\circ} - \frac{\cos 30^\circ}{\cos 10^\circ} \right)$$

$$2 \times 2 \left(\frac{\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ}{2 \cdot \sin 10^\circ \cos 10^\circ} \right)$$

$$4 \left(\frac{\sin(30^\circ - 10^\circ)}{\sin 20^\circ} \right)$$

$$4 \frac{\cancel{\sin 20^\circ}}{\cancel{\sin 20^\circ}} = 4$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

Ans 7

$$\frac{2 \sin 12^\circ \sin 48^\circ \sin 54^\circ}{2}$$

$$\left[\cos(48^\circ - 12^\circ) - \cos(48^\circ + 12^\circ) \right] \sin 54^\circ$$

$$\left(\cos 36^\circ - \frac{1}{2} \right) \sin 54^\circ$$

$$\frac{\sin^2 54^\circ - \frac{\sin 54^\circ}{2}}{2}$$

$$\frac{1}{2} \left(\frac{1 - \cos 108^\circ}{2} - \frac{\sin 54^\circ}{2} \right)$$

$$\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} (\cos 108^\circ + \sin 54^\circ) \right)$$

$$\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} (-\sin 18^\circ + \sin 54^\circ) \right)$$

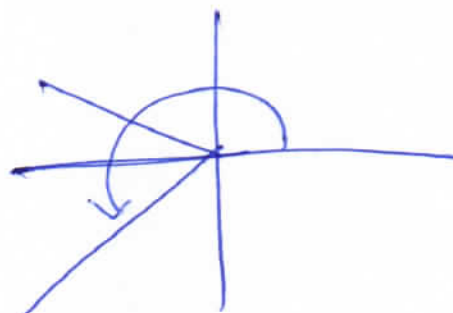
$$\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \left(-\frac{(\sqrt{5}-1)}{4} + \frac{(\sqrt{5}+1)}{4} \right) \right)$$

$$\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \left(\frac{1}{2} \right) \right)$$

$$\frac{1}{2} \left(\frac{1}{2} - \frac{1}{4} \right) = \frac{1}{2} \left(\frac{1}{4} \right) = \frac{1}{8}$$

Ans 8 $\tan 100^\circ + \tan 125^\circ + \tan 100^\circ \times \tan 125^\circ = ?$

$$\tan 225^\circ = 1$$



$$\tan (100^\circ + 125^\circ) = 1$$

$$\frac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ} = 1$$

$$\tan 100 + \tan 125 + \tan 100 \tan 125 = 1$$