3

CURRENT ELECTRICITY

• Current Electricity: - The study of electric charges in motion is called current electricity.

Electric current

The flow of electric charges through a conductor constitutes electric current.

Quantitatively,

Electric current across an area held perpendicular to the direction of flow of charge is defined as the amount of charge flowing across that area per unit time.

For a steady flow of charge $\boxed{I = \frac{q}{t}}$

Current are not always steady and hence more generally we define the current as follows.

Let ΔQ be the net charge flowing across a cross-section of a conductor during the time interval Δt and $t + \Delta t$. Then the current at a time 't' across the cross-section of the conductor is defined as

$$I(t) = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}.$$

S.I unit of current :

Current $I = \frac{q}{t}$

$= \frac{\text{Coulomb (C)}}{\text{Second (S)}} =$ Ampere (A)

Definition of 1 Ampere.

If one coulomb of charge crosses an area in one second, then the current through that area is one Ampere.

$$1\,\text{ampere} = \frac{1\ \text{Coulomb}}{1\ \text{second}}$$

Conventional Current and Electronic current.

The direction of motion of positive charges is taken as the direction of conventional current. Electrons being negatively charged so the direction of electric current or electronic current is opposite to that of the conventional current.

Q. Is electric current a scalar or a vector quantity? Give reason.

Electric current has both magnitude and direction but it is consider as a scalar quantity because it does not obey the laws of vector addition.

Ohm's law.

The law states that the current flowing through a conductor is directly proportional to the potential difference applied across its ends provided the temperature and other physical conditions remains unchanged.

Thus Potential difference $\propto$ current.

$$V \propto I$$

or $\boxed{V = RI}$

The proportionality constant R is called the resistance of the conductor.

Resistance:- It is the property by virtue of which a conductor opposes the flow of charges through it. It is equal to the ratio of the potential difference applied across the conductor to the current flowing through it.

$$\therefore R = \frac{V}{I}.$$

S.I unit of Resistance is Ohm (Ω).

i.e. S.I unit of resistance = $\frac{\text{Volt}}{\text{Ampere}}$ = Ohm (Ω)

Definition of 1 Ohm.

The resistance of a conductor is 1Ω if a current of 1 ampere flows through it on applying a potential difference of 1 volt across its ends.

$$1\,\Omega = \frac{1\,V}{1\,A}$$

Factors affecting the resistance.

1. The resistance R of a conductor is directly proportional to the length 'l'
   i.e. $R \propto l$.

2. The resistance R of a conductor is inversly propor

tional to its area of cross-section.

ie $R \propto \frac{1}{A}$.

3. Nature of the material.

Combining the above factors

$$R \propto \frac{l}{A}$$

$$\boxed{R = \frac{\rho l}{A}}$$

where ρ is the constant of proportionality, called resistivity or specific resistance of the material of the conductor.

Resistivity or Specific resistance.

We know that $R = \frac{\rho l}{A}$.

if $l = 1m$ and $A = 1m^2$.

$\therefore R = \rho$.

Thus the resistivity or specific resistance of a material may be defined as the resistance of a conductor, having unit length and unit area of crossection.

S.I unit of resistivity

$$R = \frac{\rho l}{A}$$

$$\therefore \rho = \frac{RA}{l}$$

S.I unit of $\rho = \frac{\Omega m^2}{m}$

$= \Omega m$.

Current Density:

Current per unit area is called current density. It is denoted by j

$$\therefore \boxed{j = \frac{I}{A}}.$$

S.I unit of current density $= A/m^2$.

Conductance

It is defined as the reciprocal of the resistance

ie Conductance $G = \frac{1}{\text{Resistance}}$

$= \frac{1}{R}$

The S.I unit of conductance is $\frac{1}{\Omega} = \Omega^{-1}$

or, mho (Ω^{-1}) or siemens (S)

Conductivity.

The reciprocal of the resistivity of a material is called its conductivity.

ie Conductivity $\sigma = \frac{1}{\text{resistivity}}$

$= \frac{1}{\rho}$

The S.I unit of conductivity is $\frac{1}{\Omega m} = \Omega^{-1}m^{-1}$

$=$ mho m^{-1}

$= S m^{-1}$

Ohm's law in vector form.

If E is the magnitude of electric field in a conductor of length 'l', then the potential difference across its ends is

$$V = El \quad \text{—(1)}$$

Also from Ohm's law

$$V = IR$$

$$El = \frac{I\rho l}{A} \quad [\text{from (1)}]$$

$$\therefore E = \frac{I}{A}\rho$$

$$\therefore E = j\rho \quad \text{where } j = I/A \text{ current density.}$$

As the direction of current density $\vec{j}$ is same as that of electric field $\vec{E}$,

$$\therefore \vec{E} = \rho\vec{j}$$

or $$\vec{j} = \frac{\vec{E}}{\rho}$$

$$\boxed{\vec{j} = \sigma\vec{E}} \quad \text{where } \sigma = \frac{1}{\rho} \text{ called conductivity.}$$

The above equation is the vector form of Ohm's law. It is equivalent to the scalar form $V = RI$.

Classification of solids on the basis of resistivity.

On the basis of their resistivity values, solids can be classified in to three categories.

1. **Conductors:-** Metals have low resistivities in the range $10^{-8}\ \Omega m$ to $10^{-6}\ \Omega m$. These are known as good conductors of electricity.

2. **Insulators:-** These are the substances which have large resistivities, more than $10^{4}\ \Omega m$. Insulators like glass and rubber have resistivities in the range 10^{14} to $10^{16}\ \Omega m$.

3. **Semiconductors:-** These are the substances having resistivities between those of conductors and insulators i.e. between $10^{-6}\ \Omega m$ to $10^{4}\ \Omega m$. Germanium and silicon are typical semiconductors.

Drift velocity and Relaxation time.

In the absence of any electric field, the free electrons of a metal are in a state of continuous random motion. The average random velocity of free electrons is zero.

i.e. $$\vec{u} = \frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \cdots + \vec{u}_n}{N} = 0$$

or $$\frac{1}{N}\sum_{i=1}^{n}\vec{u}_i = 0$$

Thus there is no net flow of charge in any direction.

In the presence of an external electric field $\vec{E}$ each electron experience a force $F = q\vec{E}$
$= -e\vec{E}$ in the opposite direction of $\vec{E}$, and undergoes an acceleration $\vec{a}$ is given by

$$\vec{a} = \frac{-e\vec{E}}{m} \qquad \left\{ \begin{array}{l} F = ma \\ a = F/m \end{array} \right\}$$

where 'm' is the mass of an electron.
As the electrons accelerate, they frequently collide with the other electrons of the metal. Between two successive collisions, an electron gains a velocity component in a direction opposite to $\vec{E}$. However, the gain in velocity lasts for a short time and is lost in the next collision. After each collision each electron makes a fresh start.

The average time interval between successive collisions of an electron is called relaxation time. It is given by.

$$\tau = \frac{t_1 + t_2 + \dots + t_N}{N}$$

During the relaxation time 'τ', an electron gains an average velocity given by.

$v = u + at$.

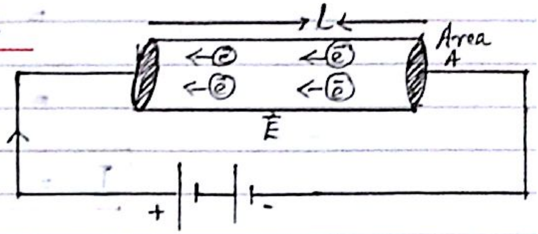
$$\vec{v_d} = \vec{u} + \vec{a}\tau$$
$$= 0 + \vec{a}\tau$$
$$= \vec{a}\tau$$
$$\boxed{\vec{v_d} = -\frac{e\vec{E}}{m}\tau}$$

The parameter $\vec{v_d}$ is called drift velocity of electrons.
Drift velocity of electrons is defined as the average velocity gained by the free electrons of a conductor in the opposite direction of the externally applied electric field.

Relation between electric current and drift velocity.

Consider a conductor of length 'l' and area of cross-section A applied to a potential difference 'V'.
The electric field 'E' set up inside the conductor is given by

$$E = \frac{V}{l}$$



Under the influence of field $\vec{E}$ the free electrons begin to drift in the opposite direction of $\vec{E}$.

Number of electrons per unit volume $= n$.
(Electron density)
Charge of an electron $= e$.

Electron density 'n' = $\frac{\text{Total number of electrons}}{\text{Volume of the conductor}}$

$$'n' = \frac{N}{Al}$$ Volume = Area × length

$$N = nAl$$

Total charge 'q' $= N \times$ charge of an electron
$= \cancel{N \times e}$
$= N \times e$
$= nAl \times e = nAle$.

All the electrons which enter the conductor at the right end will pass through the conductor at the left end in time.

$$t = \frac{\text{distance}}{\text{velocity}}.$$

$$= \frac{l}{v_d}.$$

$\therefore$ Current $I = \frac{q}{t}$.

$$= \frac{nAle}{l/v_d}.$$

$$= \frac{nAle \times v_d}{l}$$

$$\boxed{I = nAv_d e.}$$

This equation relates the current I with the drift velocity v_d.

Current density $j = \frac{I}{A}$

$$= \frac{nAv_d e}{A}$$

$$\boxed{j = nv_d e.}$$

Deduction of Ohm's law.

Consider a conductor of length 'l' and area of cross-section A containing 'n' free electrons per unit volume. Let a potential difference 'V' be applied between its ends.

Electric field set up inside the conductor. $E = \frac{V}{l}$ — ①

If 'm' is the mass of an electron and 'τ' is the relaxation time then drift velocity

$$v_d = \frac{eE}{m}\tau. \quad \text{— ②}$$

But $E = \frac{V}{l}$

$\therefore$ the above eqn becomes. $v_d = \frac{e}{m}\frac{V}{l}\cdot\tau$ — ③

Current $I = nAv_d e$.

$$= nA\left(\frac{e}{m}\frac{V}{l}\tau\right)e \quad \text{from equ (3)}$$

$$I = \frac{ne^2\tau A}{ml}\cdot V$$

$$\boxed{\frac{V}{I} = \frac{ml}{ne^2\tau A}}$$ At a fixed temperature the quantities 'm', 'l', 'n', 'e' τ and A all have constant values for a given conductor.

$\therefore$ $\frac{V}{I}$ = a constant called the resistance R.

This proves Ohm's law for a conductor and hence $\boxed{R = \frac{ml}{ne^2\tau A}}$ is the resistance of the conductor.

The resistance R of a conductor of length 'l', area of crossection A and resistivity ρ is given by.

$$R = \rho\frac{l}{A}$$

But $R = \frac{ml}{ne^2\tau A}$

where τ is the relaxation time. Comparing the above two equations we get. $\frac{\rho l}{A} = \frac{m l}{n e^2 \tau A}$

$$\boxed{\rho = \frac{m}{n e^2 \tau}}$$

Obviously 'ρ' is independent of the dimensions of the conductor but depends on its two parameters.

1. Number of free electrons per unit volume or electron density of the conductor.

2. The relaxation time 'τ', the average time between two successive collisions of an electron.

Mobility:

The mobility of a charge carrier is the drift velocity per unit electric field.

$$\mu = \frac{v_d}{E}$$

$$= \frac{\frac{eE\tau}{m}}{E} \quad \text{where } v_d = \frac{eE\tau}{m}$$

$$\therefore \boxed{\mu = \frac{e\tau}{m}}$$

where τ is the relaxation time. Mobility is always positive

S.I unit of mobility $= \frac{m/s}{V/m} = \frac{m}{s} \times \frac{m}{V}$

$= m^2 s^{-1} V^{-1}$

Practical unit of mobility $= cm^2 s^{-1} V^{-1}$

CBSE. Qu. What do you understand by the resistivity of a conductor? Discuss its temperature dependence for a (i) metallic conductor
(ii) Semiconductor
(iii) ionic conductor
(iv) electrolyte.

Ans: Resistivity of a material is the resistance of a conductor of that material, having unit length and unit area of cross-section.

It is given by $\rho = \frac{m}{n e^2 \tau}$.

(i) In metallic conductor ie in metals the number of free electrons is fixed. As the temperature increase the amplitude of vibration of the atom increases the collision of electrons with these atoms become more frequent. The relaxation time τ decreases. Hence the resistivity of a metallic conductor increases with the increase of temperature.

(ii) In case of semiconductors, the relaxation time τ does not change with temperature but the number density of free electrons increases exponentially with the increase in temperature. Consequently the conductivity increases or resistivity decreases exponentially with the increase in temperature.

(iii) Ionic conductors:- An ionic conductor has both positive and negative ions as the charge carriers. As the temperature increases, the electrostatic attraction between cations and anions decreases, the ions are more free to move and so the conductivity increases or resistivity decreases.

(iv) Electrolytes: As the temperature increases the inter ionic attractions decrease and also the viscous forces decrease the ions move more freely. Hence conductivity increases or the resistivity decreases as the temperature of an electrolytic solution increases.

Temperature coefficient of resistivity.

The temperature coefficient of resistivity may be defined as the increase in resistivity per unit resistivity per degree rise in temperature

ie $\boxed{\alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)}}$

ie $\alpha \rho_0 (T - T_0) = \rho - \rho_0$

or $\rho = \rho_0 + \alpha \rho_0 (T - T_0)$ — (1)

$\rho = \rho_0 [1 + \alpha (T - T_0)]$ this eqn represents the resistivity ρ at any temperature T and ρ_0 is the resistivity at a lower temperature T_0.

As $R = \rho . \frac{l}{A}$

ie $R \propto \rho$.

Hence eqn (1) can be written in terms of resistances as:

$$R_t = R_0 [1 + \alpha t]$$

where R_t = the resistance at $t°C$

R_0 = the resistance at $0°C$ and

t = the rise in temperature = $(T - T_0)$

S.I unit and dimension of 'α'.

We know that

$$\alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)}$$

S.I unit of $\alpha = \frac{\Omega m}{\Omega m . °C} = °C^{-1}$.

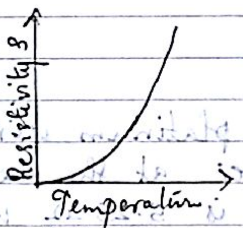
and dimension = $(\text{Temperature})^{-1} = K^{-1}$

Distinguish between metals semiconductors and alloys on the basis of their 'α' values.

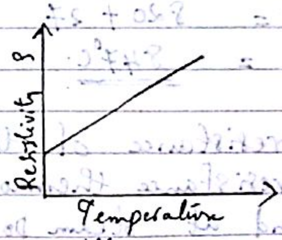
For metals α is positive, ie resistance of metals increases with the increase in temperature

For semiconductors and insulators 'α' is negative, the resistance decreases with the increase in temperature

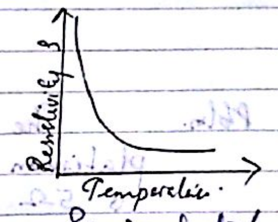
For alloys like constantan, nichrome and maganin the temperature coefficient of resistance α is very small, so they are used for making standard resistors.



Metals.



Alloys.



Semiconductors/Insulators.

Pblm: An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature is found to be 75.3 Ω. When the toaster is connected to a 230 V supply the current settles after a few seconds to a steady value of 2.68 A. What is the steady temperature of the nichrome element? α of nichrome = $1.70 \times 10^{-4}\ °C^{-1}$

$R_0 = 75.3\ \Omega$

$R_t = \frac{230}{2.68} = 85.8\ \Omega$

$\alpha = 1.70 \times 10^{-4}\ °C^{-1}$

$R_t = R_0(1+\alpha t)$

$R_t = R_0 + R_0 \alpha t$

$R_t - R_0 = R_0 \alpha t$

$t = \frac{R_t - R_0}{R_0 \alpha} = \frac{85.8 - 75.3}{75.3 \times 1.70 \times 10^{-4}} = 820°C$

$T - T_0 = 820°C$

$T = 820 + T_0$

$= 820 + 27$

$= 847°C$

Pblm. The resistance of the platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.23 Ω. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is 5.795 Ω. Calculate the temperature of the bath.

Solu: $R_0 = 5\ \Omega$ $R_{100} = 5.23\ \Omega$

$R_t = 5.795\ \Omega$.

As $R_t = R_0(1+\alpha t)$

$R_t - R_0 = R_0 \alpha t$ —①.

and $R_{100} - R_0 = R_0 \alpha \times 100$ —②.

$\frac{①}{②} \Rightarrow \frac{R_t - R_0}{R_{100} - R_0} = \frac{R_0 \alpha t}{R_0 \alpha \times 100}$

$\frac{R_t - R_0}{R_{100} - R_0} = \frac{t}{100}$

$\therefore t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$

$= \frac{5.795 - 5}{5.23 - 5} \times 100$

$= \frac{0.795}{0.23} \times 100$

$= 345.65°C$.

Qu: Why alloys like constantan or manganin are used for making standard resistor?

Ans: This is because of the following reasons.

(i) These alloys have high value of resistivity

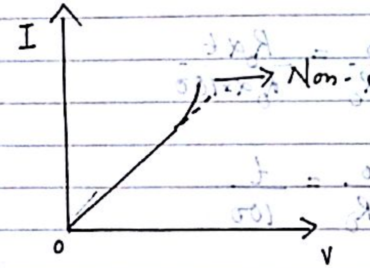
(ii) They have very small temperature coefficient

(ii) So their resistance does not change appreciably

even for several degrees rise of temperature.
(iii) They are least affected by atmospheric conditions like air, moisture etc.

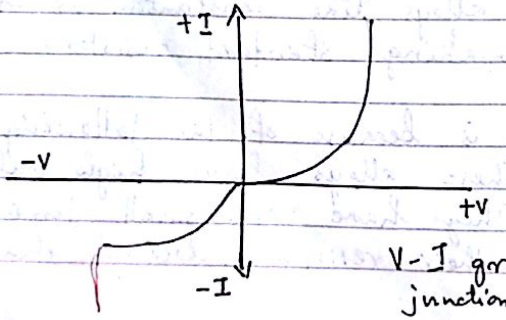
Limitations of Ohm's law.

(i) When a large current flows through a metallic conductor, it gets heated up and its resistance increases. The V-I graph becomes non-linear ie the conductor becomes non-ohmic at higher currents.



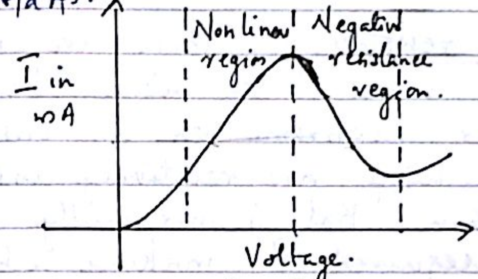
V-I graph for a metallic conductor

(ii) The relation between V and I depends on the sign of V. In other words, if I is the current for a certain V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction.
eg:- V-I graph for a junction diode.



V-I graph for a junction diode.

(iii) The relation between V and I is not unique ie there is more than one value of V for the same current. A material exhibiting such behaviour is GaAs.



V-I graph for GaAs.

Resistivity of Various Materials.

On the basis of their resistivity values, solids can be classified into following three categories.

1. Conductors:- Metals have low resistivities in the range of $10^{-8}\,\Omega m$ to $10^{-6}\,\Omega m$. These are known as good conductors of electricity.

2. Insulators:- These are the substances which have large resistivities more than $10^{4}\,\Omega m$. Insulators like glass and rubber have resistivities in the range $10^{14}\,\Omega m$ to $10^{16}\,\Omega m$.

3. Semiconductors:- These are the substances having resistivities between those of conductors and insulators ie between $10^{-6}\,\Omega m$ to $10^{4}\,\Omega m$. Germanium and silicon are typical semiconductors.

Common Commercial resistors.

The commercial resistors are of two types

1. **Wire bound resistors :-** There are made by winding the wires of an alloy like manganin constantan or nichrome on an insulating base. These alloys are relatively insensitive to temperature. But inconveniently large length is required for making a high resistance

2. **Carbon resistors :-** They are made from mixture of carbon black, clay and resin binder which are pressed and then ~~mounted~~ moulded into cylindrical rods by heating. The rods are enclosed in a ceramic or plastic jacket.

The carbon resistors are widely used in electronic circuits of radio receivers amplifiers etc.

They have the following advantages.

* They can be made with resistance values ranging from few ohms to several million ohms.
* They are quite cheap and compact
* They are good enough for many purposes.

Colour code for resistors.

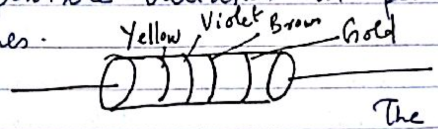
A colour code is used to indicate the resistance value of a carbon resistor and its percentage accuracy.

Resistor colour codes.

Colour	Number	Multiplier	Tolerance (%)
Black	0	$10^0 = 1$	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold			5
Silver			10
No Colour			20.

The resistors have a set of co-axial coloured rings. The first two bands from the end indicate the first two significant figures of the resistance in Ohms. The third band indicates the decimal multiplier. The last band stands for tolerance or possible variation in percentage about the indicated values.

eg.



The resistance value is

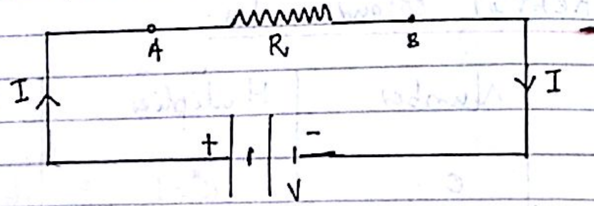
$R = (47 \times 10\,\Omega) \pm 5\%$.

$= 470\,\Omega \pm 5\%$

Electrical Energy and Power.

Electrical Energy.

Consider a conductor AB of resistance R, shown in figure in which a current I is flowing from A to B.



A source of emf maintains a potential difference "V" between its ends A and B and sends a steady current I

Clearly $V_{(A)} > V_{(B)}$

and the potential difference across AB is

$$V = V_{(A)} - V_{(B)} > 0. \quad \text{—— ①}$$

The amount of charge that flows from A to B in time 'Δt' is

$$\Delta Q = I\Delta t \quad \text{—— ②}$$

Potential energy of the charge at $A = \Delta Q\, V_{(A)}$

Potential energy of the charge at $B = \Delta Q\, V_{(B)}$

Change in its potential energy

$$\begin{aligned}\Delta U_{pot} &= \text{Final P.E} - \text{Initial P.E} \\ &= \Delta Q\, V_{(B)} - \Delta Q\, V_{(A)} \\ &= \Delta Q\left[V_{(B)} - V_{(A)}\right] \\ &= \Delta Q \times -V \\ &= -\Delta Q V \\ &= -IV\Delta t < 0 \quad \text{from ②}\end{aligned}$$

If the charges move through the conductor without suffering collision, their K.E would change

By conservation of energy, the change in K.E must be

$$\Delta K = -\Delta U_{pot}$$

that is, $\Delta K = IV\Delta t > 0$

Thus in case, charges were moving freely through the conductor under the action of the electric field, their K.E would increase as they move. But on the average, the electrons move with a steady drift velocity. This is because of the collisions of electrons with ions and atoms during the course of their motion. The K.E gained by the electrons is shared with the metal ions. These ions vibrate more vigorously and the conductor gets heated up. The amount of energy dissipated as heat in conductor in time Δt is

$$\boxed{\Delta W = IV\Delta t}$$

According to Ohm's law $V = IR$

$$\therefore \boxed{\Delta W = \text{Heat} = I^2R\Delta t}$$

The above equation is known as Joule's law of heating.

According to this law, the heat produced in a resistor is

1. directly proportional to the square of current
2. directly proportional to the resistance R
3. directly proportional to the time 't' for which the current flows through the resistor.

Electric Power.

The energy dissipated per unit time is the power

$$\therefore P = \frac{\Delta W}{\Delta t}$$

$$= \frac{IV\Delta t}{\Delta t}$$

$$\therefore \boxed{P = IV} \quad \text{——— (1)}$$

Using Ohm's law $V = IR$, we get

$$P = I^2R \quad \text{——— (2)}$$

$$P = \frac{V^2}{R} \quad \text{——— (3)}$$

as the power loss (Ohmic loss) in a conductor of resistance R carrying a current I

High Voltage power transmission.

Electric power is transmitted from power stations to homes and factories through transmission cables. These cables have resistance. Power is wasted in them as heat If we wants to minimise the power loss in the transmission cables connecting the power stations to homes and factories.

Let us consider a device R to which a power P is to be delivered via transmission cables having a resistance R_c to be dissipated by it finally. If V is the voltage across R and I current through it. then $P = VI$ ——— (1)

The power wasted in transmission cables is

$$P_c = I^2R_c$$

$$= \frac{P^2R_c}{V^2} \quad \text{from equ (1).}$$

Thus power wasted in the transmission cables is inversly proportional to the square of voltage. Hence to minimise the power loss, electric power is transmitted to distant places at high voltages and low currents. These voltages are stepped down by transformers before supplying to homes and factories.

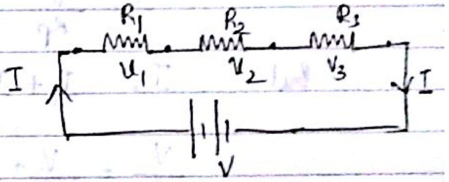
Combination of Resistors.

1. Resistance in Series.

Consider three resistances R_1, R_2 and R_3 connected in series. When a potential difference V is applied across the combination, the same current I flows through each resistance.

By Ohm's law, the potential drops across the three resistances are

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3.$$



If R_s is the equivalent resistance of the series combination, then we must have

$$V = IR_s.$$

But $V = V_1 + V_2 + V_3$.

$$IR_s = IR_1 + IR_2 + IR_3$$

$$\therefore \boxed{R_s = R_1 + R_2 + R_3.}$$

Thus when a number of resistances are connected in series their equivalent resistance is equal to the sum of the individual resistances.

2. Resistances in parallel.

Consider three resistances R_1, R_2 and R_3 connected in parallel between points A & B. Let V be the potential difference applied across the combination.

Let I_1, I_2 and I_3 be the currents through the resistances R_1, R_2 and R_3 respectively. Then the current $I = I_1 + I_2 + I_3$.

Since all the resistances have been connected between the same two points A and B

∴ potential drop V is same across each of them.

By Ohm's law, the currents through the individual resistances will be.

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

If R_p is the equivalent resistance of the parallel combination then we must have

$$I = \frac{V}{R_p}$$

But $I = I_1 + I_2 + I_3$

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or

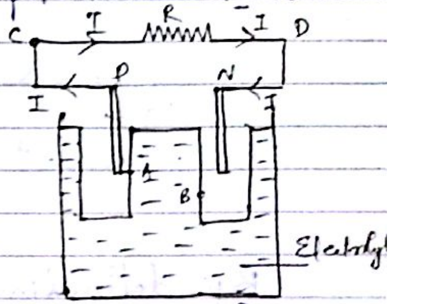
$$\boxed{\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Thus when a number of resistances are connected in parallel, the reciprocal of the equivalent resistance of the parallel combination is equal to the sum of the reciprocals of the individual resistances.

Cells, emf and Internal Resistance.

A simple device to maintain a steady current in an electric circuit is the electrolytic cell. A cell has two electrodes called (P) positive and (N) negative. They are immersed in an electrolytic solution. Dipped in the solution, the electrodes exchange charges with the electrolyte.

The positive electrode has a potential V_+ between itself and the electrolyte solution. Similarly negative electrode develops a negative potential $-V_-$ relative to the electrolyte.



When there is no current the electrolyte has the same potential throughout so the potential difference between P and N is

$$V_+ - (-V_-) = V_+ + V_-$$

This difference is called the electromotive force (emf) of the cell and is denoted by ε

Thus $\varepsilon = V_+ + V_- > 0$

Internal resistance (r): The resistance offered by the electrolyte of a cell to the flow of current between its electrodes is called internal resistance of the cell. The internal resistance of a cell depends on the following factors.

1. Nature of the electrolyte.
2. Concentration of the electrolyte (directly)
3. distance between the two electrodes (directly)
4. Area of the electrodes (inversely)
5. Temperature of the electrolyte (inversely)

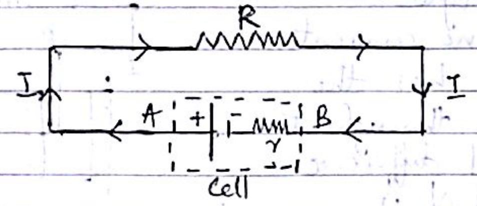
Terminal Potential difference (V).

The potential drop across the terminals of a cell when a current is being drawn form it is called its terminal potential difference.

Relation between r, $\mathcal{E}$ and V.

IMP

Consider a cell of emf $\mathcal{E}$ and internal resistance 'r' connected to an external resistance R.



By definition of emf.

$\mathcal{E}$ = Work done in carrying a unit charge from A to B against external resistance R + Work done in carrying a unit charge from B to A against internal resistance r

$$\mathcal{E} = V + V' \quad \text{—(1)}$$

$$\mathcal{E} = IR + Ir$$

$$= I(R+r)$$

∴ Current in the circuit is

$$I = \frac{\mathcal{E}}{R+r}$$

Terminal P.d of the cell is given by

$$V = IR$$

$$= \frac{\mathcal{E}}{R+r} \cdot R = \frac{\mathcal{E}R}{R+r} \quad \text{—(2)}$$

From (1)

Terminal P.d

$$V = \mathcal{E} - V'$$

$$\boxed{V = \mathcal{E} - Ir}$$

$$Ir = \mathcal{E} - V$$

$$r = \frac{\mathcal{E} - V}{I}$$

$$= \frac{\mathcal{E} - V}{V/R}$$

$$= \left(\frac{\mathcal{E} - V}{V}\right)R = \left(\frac{\mathcal{E}}{V} - 1\right)R.$$

∴ The internal resistance

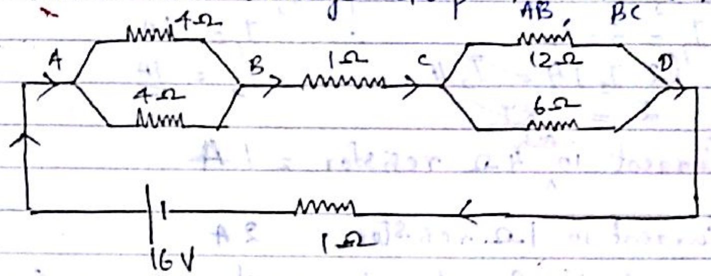
$$\boxed{r = \left(\frac{\mathcal{E}}{V} - 1\right)R.}$$

Pblm. A network of resistors is connected to a 16 V battery with internal resistance 1Ω as shown in figure.

(a) Compute the equivalent resistance of the network

(b) Obtain the current in each resistor.

(c) Obtain the voltage drop V_{AB}, V_{BC} and V_{CD}.



(a). 4Ω and 4Ω are in parallel.

$$\frac{1}{R'} = \frac{1}{4} + \frac{1}{4} = \frac{4+4}{4\times4} = \frac{8}{16} = \frac{1}{2}$$

$R' = 2\,\Omega$

$12\,\Omega$ and $6\,\Omega$ are in parallel.

$$\frac{1}{R'''} = \frac{1}{12} + \frac{1}{6}$$

$$= \frac{1+2}{12} = \frac{3}{12}$$

$$\therefore R''' = \frac{12}{3} = 4\,\Omega$$

$\therefore$ Equivalent resistance $R = R' + R'' + R'''$

$$= 2 + 1 + 4 = \underline{\underline{7\,\Omega}}$$

(b) Total current $I = \dfrac{\varepsilon}{R+r}$

$$= \frac{16}{(7+1)}$$

$$= \frac{16}{8} = \underline{\underline{2A}}$$

Consider the resistors between A and B. voltage same.

$I_1 \times 4 = I_2 \times 4$.

$I_1 = I_2$

$\cancel{I_1 + I_2 = 2}$

$\cancel{I_1 = 2 - I_2}$

$\cancel{(2 - I_2)4 = I_2 4}$

$\cancel{2 = 8I_2}$

$I_1 + I_2 = 2$

$\therefore I_1 = 1A$

$I_2 = \underline{1A}$

$\therefore$ Current in each $4\,\Omega$ resistor $= 1A$

Current in $1\,\Omega$ resistor $= 2A$

~~Current in~~ Consider the resistor between C and D.

$I_3 \times 12 = I_4 \times 6$.

$2I_3 = I_4$

$I_3 + I_4 = 2$

$2I_3 = 2 - I_3$.

$3I_3 = 2$

$I_3 = \frac{2}{3} A$

$\therefore I_4 = \frac{4}{3} A$

Current in $12\,\Omega = 2/3\,A$

Current in $6\,\Omega = 4/3\,A$

(c) $V_{AB} = 4 \times 1 = 4V$ (because both are $4\,\Omega$ resistors)

$V_{BC} = 1 \times 2 = 2V$

$V_{CD} = 12 \times \frac{2}{3} = \underline{\underline{8V}}$

$6 \times \frac{4}{3}$

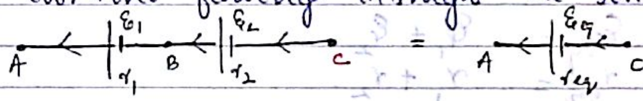
$= 2 \times 4 = \underline{8V}$

$\frac{2}{6} \times \frac{4}{3}$

Cells in Series and in Parallel.

Cells in Series.

Suppose two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in series between points A and C. Let I be the current flowing through the series combination.



Let V(A), V(B), V(C) be the potentials at points A, B and C shown in figure.

The potential difference between the positive and negative terminals of the first cell.

$V_{AB} = V(A) - V(B)$

$\quad\;\; = \mathcal{E}_1 - I r_1$

lly

$V_{BC} = V(B) - V(C)$

$\quad\;\; = \mathcal{E}_2 - I r_2$

Hence P.d between the terminals A and C of the combination is

$V_{AC} = V(A) - V(C)$

$\quad\;\; = [V(A) - V(B) + V(B) - V(C)]$

$\quad\;\; = \mathcal{E}_1 - I r_1 + \mathcal{E}_2 - I r_2$

$\quad\;\; = (\mathcal{E}_1 + \mathcal{E}_2) - I(r_1 + r_2)$ —— (1)

If we wish to replace the combination by a single cell between A and C of emf $\mathcal{E}_{eq}$ and internal resistance r_{eq}, then

$V_{AC} = \mathcal{E}_{eq} - I r_{eq}$ —— (2)

Comparing the equ's (1) and (2) we get

$\mathcal{E}_{eq} = \mathcal{E}_1 + \mathcal{E}_2$

and $r_{eq} = r_1 + r_2$.

For a series combination of n cells

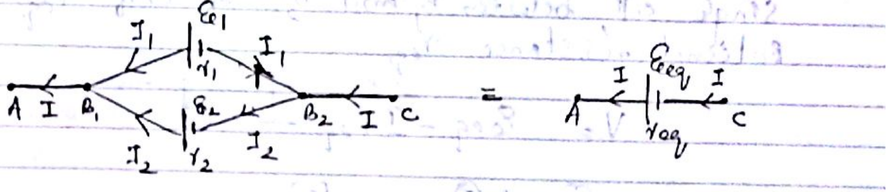
$$\boxed{\begin{aligned} \mathcal{E}_{eq} &= \mathcal{E}_1 + \mathcal{E}_2 + \dots + \mathcal{E}_n \\ r_{eq} &= r_1 + r_2 + \dots + r_n \end{aligned}}$$

i.e. (1) the equivalent emf of a series combination of n cells is the sum of their individual emf's.
(2) The equivalent internal resistance of a series combination of n cells is the sum of their internal resistances.

Cells in Parallel.

Suppose two cells of emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 are connected in parallel between two points. Suppose the currents I_1 and I_2 from the positive terminals of the two cells flow towards the junction B and current I flows out.

Thus $I = I_1 + I_2$ —— (1)



Let $V(B_1)$ and $V(B_2)$ be the potentials at B_1 and B_2 then P.d across its terminals of the first cell.

$V = V(B_1) - V(B_2)$

$\quad = \mathcal{E}_1 - I_1 r_1 \qquad \therefore I_1 = \dfrac{\mathcal{E}_1 - V}{r_1}$ —— (2)

P.d across its terminals of the second cell

$V = V(B_1) - V(B_2)$

$\quad = \mathcal{E}_2 - I_2 r_2 \qquad \therefore I_2 = \dfrac{\mathcal{E}_2 - V}{r_2}$ —— (3)

$\therefore$ equ (1) becomes

$$I = \frac{\mathcal{E}_1 - V}{r_1} + \frac{\mathcal{E}_2 - V}{r_2}$$

$$I = \left(\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}\right) - V\left[\frac{1}{r_1}+\frac{1}{r_2}\right] \quad \text{—— (4)}$$

If we want to replace the combination by a single cell between B_1 and B_2 of emf ε_{equ} and internal resistance r_{equ}.

$$V = \varepsilon_{equ} - I r_{equ}$$

$$I r_{equ} = \varepsilon_{equ} - V$$

$$I = \frac{\varepsilon_{equ} - V}{r_{equ}}$$

$$I = \frac{\varepsilon_{equ}}{r_{equ}} - \frac{V}{r_{equ}}$$

$$= \frac{\varepsilon_{equ}}{r_{equ}} - V\left[\frac{1}{r_{equ}}\right] \quad \text{—— (5)}$$

Comparing equ. (4) and (5)

We can express the above results in a simpler way

$$\boxed{\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}$$

and $$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

For a parallel combination of n cells, we have

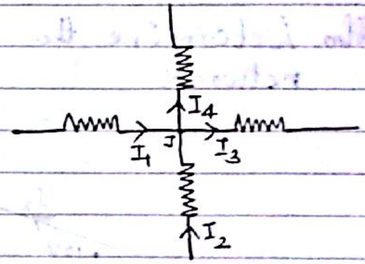
$$\boxed{\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \cdots + \frac{\varepsilon_n}{r_n}}$$

and $$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots + \frac{1}{r_n}}$$

Kirchhoff's Rules.

1. Junction rule :- In an electric circuit, the algebraic sum of currents at any junction is zero. Or the sum of currents entering a junction is equal to the sum of currents leaving that junction

Applying junction rule to the junction J of fig we get.



$$\Sigma I = 0$$

$$I_1 + I_2 - I_3 - I_4 = 0$$

$$I_1 + I_2 = I_3 + I_4$$

Current entering the junction	=	Current leaving the junction

2. Loop rule :- (Mesh rule) - Around any closed loop of a network the algebraic sum of changes in potential must be zero. Or the algebraic sum of the emf's in any loop of a circuit is equal to the sum of the products of

currents and resistances in it.

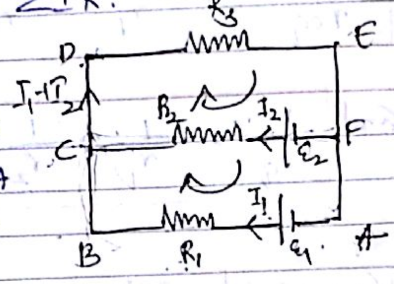
Mathematically

$$\Sigma \Delta V = 0$$

or $$\Sigma \mathcal{E} = \Sigma IR.$$

<u>Illustration</u>:

Consider the circuit



Applying Kirchhoff's loop rule to the mesh ABCFA

$$\mathcal{E}_1 - \mathcal{E}_2 = I_1R_1 - I_2R_2.$$

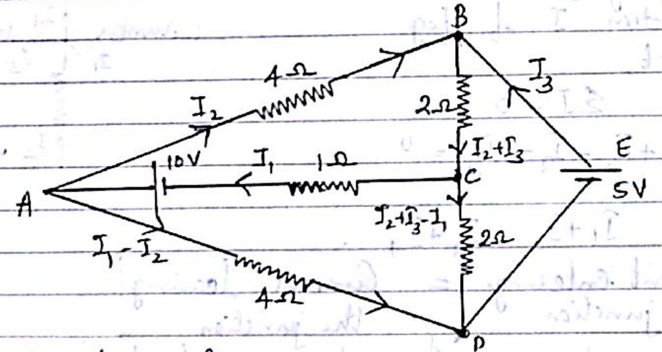
IIIly for the mesh CDEFC

$$\mathcal{E}_2 = I_2R_2 + (I_1 + I_2)R_3.$$

IIIly for the mesh ABDEA.

$$\mathcal{E}_1 = I_1R_1 + (I_1 + I_2)R_3.$$

Pblm. Determine the current in each Branch of the network.



For the loop ADCA

$$4(I_1 - I_2) - 2(I_2 + I_3 - I_1) + I_1 = 10$$

$$\Rightarrow \quad 7I_1 - 6I_2 - 2I_3 = 10 \quad \text{---} \quad (1)$$

For the mesh ABCA

$$4I_2 + 2(I_2 + I_3) + I_1 = 10$$

$$\Rightarrow \quad I_1 + 6I_2 + 2I_3 = 10 \quad \text{---} \quad (2)$$

For the mesh BCDEB

$$2(I_2 + I_3) + 2(I_2 + I_3 - I_1) = 5$$

$$\Rightarrow \quad 2I_1 - 4I_2 - 4I_3 = 5 \quad \text{---} \quad (3)$$

On solving $I_1 = 2.5\,A$, $I_2 = \frac{5}{8}A$ and $I_3 = \frac{15}{8}A$

The currents in the various branches

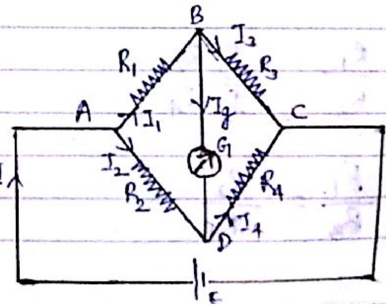
$AB = \frac{5}{8}A$ $\quad$ $CA = \frac{5}{2}A$ $\quad$ $DEB = \frac{15}{8}A$

$AD = \frac{15}{8}A$ $\quad$ $CD = 0\,A$ $\quad$ $BC = \frac{5}{2}A$.

Wheatstone Bridge

It is an arrangement of four resistances used to determine one of these resistances quickly and accurately in terms of the remaining three resistances.

A Wheatstone bridge consists of four resistances R_1, R_2, R_3 and R_4; connected to form the arms of a quadrilateral ABCD. The battery of emf $\mathcal{E}$ is connected between points A and C and a sensitive galvanometer between B and D as shown in figure.



The resistances are so adjusted that no current flows through the galvanometer. The bridge is then said to be balanced. In the balanced condition

$$\boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

Proof.

In accordance with Kirchhoff's first law, the currents through various branches are shown. Applying Kirchhoff's II law to the loop ABDA, we get

$$I_1R_1 + I_gG - I_2R_2 = 0$$

For the loop BCDB

$$I_3R_3 - I_4R_4 - I_gG = 0$$

In the balanced condition of the bridge

$$I_g = 0, \quad I_1 = I_3 \quad I_2 = I_4$$

the above eqn becomes.

$$I_1R_1 - I_2R_2 = 0$$

$$\therefore I_1R_1 = I_2R_2$$

$$\therefore I_1R_1 = I_2R_2 \quad \text{——} \; ①$$

$$I_3R_3 - I_4R_4 = 0$$

$$I_3R_3 = I_4R_4$$

$$I_1R_3 = I_2R_4 \quad \text{——} \; ②$$

$$\therefore \frac{①}{②} \quad \frac{I_1R_1}{I_1R_3} = \frac{I_2R_2}{I_2R_4}$$

$$\therefore \boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}}$$

This proves the condition for the balanced Wheatstone bridge.

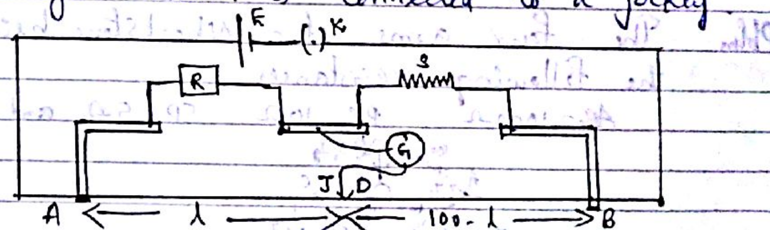
Meter Bridge.

It is the simplest practical application of the Wheatstone bridge that is used to measure an unknown resistance.

Principle: Its working is based on the principle of Wheatstone bridge. When the bridge is balanced

$$\boxed{\frac{R_1}{R_3} = \frac{R_2}{R_4}}$$

Construction:- It consists of usually 1m long manganin wire of uniform cross-section stretched along a metre scale fixed over a wooden board with its two ends clamped between two thick metallic strips bent at right angles. The metallic strip has two gaps across which resistors can be connected. The end points where the wire is clamped are connected to a cell through a key. One end of the galvanometer is connected to the metallic strip midway between the two gaps. The other end of the galvanometer is connected to a jockey.



Working:- After taking out a suitable resistance R from the resistance box the jockey is moved along the wire, till there is no deflection in the galvanometer. This is the balanced condition of the Wheatstone bridge.

Let D be the balance point on the wire distance 'l' cm from the end A. Then the portion AD of the wire has a resistance $R_{cm}\, l$, where R_{cm} is the resistance of the wire per unit cm. The portion DB of the wire similarly has a resistance $R_{cm}(100-l)$. Then according to Wheatstone's principle

$$\frac{R}{S} = \frac{R_{cm}\, l}{R_{cm}(100-l)}$$

$$\frac{R}{S} = \frac{l}{100-l}$$

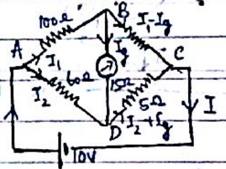
$$\cancel{\frac{R}{S} = \left(\frac{l}{100-l}\right)}$$

or the unknown resistance $\boxed{S = \frac{R(100-l)}{l}}$

Knowing l and R unknown resistance S can be determined.

Pblm: The four arms of a Wheatstone bridge have the following resistances

AB = 100 Ω BC = 10 Ω CD = 5 Ω and DA = 60 Ω



A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a p.d. of 10 V is maintained across AC.

Soln. Considering the mesh BADB, we have

$$100 I_1 + 15 I_g - 60 I_2 = 0$$
$$20 I_1 + 3 I_g - 12 I_2 = 0 \quad \text{—— ①}$$

For the mesh BCDB, we have

$$10(I_1 - I_g) - 15 I_g - 5(I_2 + I_g) = 0$$
$$10 I_1 - 30 I_g - 5 I_2 = 0$$
$$2 I_1 - 6 I_g - I_2 = 0 \quad \text{—— ②}$$

For the mesh ADCA

$$60 I_2 + 5(I_2 + I_g) = 10$$
$$65 I_2 + 5 I_g = 10$$
$$13 I_2 + I_g = 2 \quad \text{—— ③}$$

Xing eqn ② by 10

$$20 I_1 - 60 I_g - 10 I_2 = 0 \quad \text{—— ④}$$
$$20 I_1 + 3 I_g - 12 I_2 = 0 \quad \text{—— ①}$$

① - ④ ⇒ $63 I_g - 2 I_2 = 0$

$$2 I_2 = 63 I_g$$

$$I_2 = \frac{63}{2} I_g$$

Sub: I_2 in eqn ③

$$13 \times \frac{63}{2} I_g + I_g = 2$$

$$I_g \left[\frac{819}{2} + 1\right] = 2$$

$$I_g \left[\frac{821}{2}\right] = 2$$

$$I_g = \frac{4}{821}\ \text{A}$$

$$= 0.00487\ \text{A}$$
$$= 4.87 \times 10^{-3}\ \text{A}$$
$$= 4.87\ \text{mA}$$

13 ×
63
39
78
819

Potentiometer

A potentiometer is a device used to measure an unknown emf or potential difference accurately.

Principle:- The basic principle of a potentiometer is that when a constant current flows through a wire of uniform cross-sectional area and composition the potential drop across any length of the wire is directly proportional to that length.

By Ohm's law $V = IR$

$= I\frac{\rho l}{A}$ —— (1)

For a wire of uniform cross-section and uniform composition, resistivity ρ and area of cross-section A are constant.

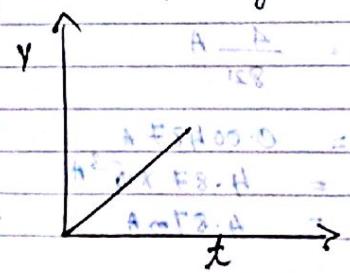
∴ When a steady current I flows through the wire

$\frac{I\rho}{A} =$ a Constant.

∴ equ (1) becomes

$\boxed{V \propto l}$.

This is the principle of a potentiometer. A graph drawn between V and l will be a straight line passing through the origin.



Potential gradient

The potential drop per unit length of the potentiometer wire is known as potential gradient. It is given by

$\boxed{K = \frac{V}{l}}$

S.I unit of potential gradient $= Vm^{-1}$

Practical unit of potential gradient $= Vcm^{-1}$.

Applications of a Potentiometer.

1. Comparison of emf's of two primary cells.

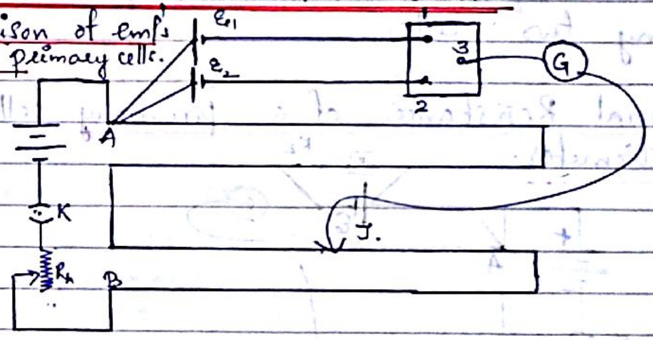


Figure shows the circuit diagram for comparing the emfs of two cells. A constant current is maintained in the potentiometer wire AB by means of a battery. Let ε_1 and ε_2 be the emfs of the two primary cells which are to be compared.

The points marked 1, 2, 3 form a two way key. Consider first position of the key where 1 & 3 are connected so that the galvanometer is connected to ε_1. The jockey is moved along the wire AB till the galvanometer shows no deflection. The balancing length is l_1 from A.

∴ $\varepsilon_1 \propto l_1$ —— (1)

Similarly
By inserting the plug between 2 and 3 so that the galvanometer is connected to $\mathcal{E}_2$, the balancing length l_2 is obtained for cell $\mathcal{E}_2$.

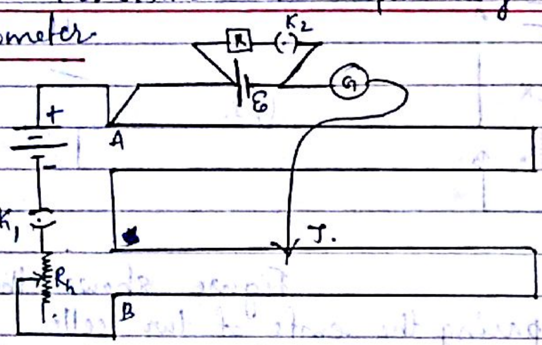
Then $\mathcal{E}_2 \propto l_2$ — ②

Hence $\dfrac{\mathcal{E}_1}{\mathcal{E}_2} = \dfrac{K l_1}{K l_2}$

or $\boxed{\dfrac{\mathcal{E}_1}{\mathcal{E}_2} = \dfrac{l_1}{l_2}}$

From this eq^n we have to compare the emf's of any two cells.

2. Internal Resistance of a primary cell by a potentiometer



The positive terminal of the cell of emf $\mathcal{E}$ whose internal resistance 'r' is to be measured is connected to the end A of the potentiometer wire and its negative terminal to the galvanometer G through a jockey J.

A resistance box R is connected across the cell through a key K_2.

When the key K_1 is closed a constant current flows through the potentiometer wire. With key K_2 kept open move the jockey along AB till it balances the emf $\mathcal{E}$ of the cell. Let l_1 be the balancing length of the wire

then $\mathcal{E} \propto l_1$ — ①

Introduce a resistance R and close key K_2. Find the balance point for the terminal potential difference V of the cell. If l_2 is the balancing length then $V \propto l_2$ — ②

$\therefore \dfrac{\mathcal{E}}{V} = \dfrac{l_1}{l_2}$

But $\mathcal{E} = I(r+R)$ where 'r' is the internal resistance of the cell.

and $V = IR$

$\therefore$ the above eq^n becomes

$\dfrac{I(r+R)}{IR} = \dfrac{l_1}{l_2}$

$\dfrac{r+R}{R} = \dfrac{l_1}{l_2}$

$\dfrac{r}{R} + 1 = \dfrac{l_1}{l_2}$

$\dfrac{r}{R} = \dfrac{l_1}{l_2} - 1$

or $\boxed{r = R\left[\dfrac{l_1}{l_2} - 1\right]}$

from this eq^n we can find the internal resistance of a given cell.

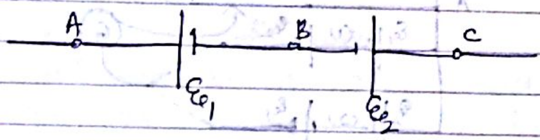
Q. Why is potentiometer preferred over a voltmeter for measuring the emf of a cell?

Ans: Potentiometer is a null method device. At null point it does not draw any current from the cell and thus there is no potential drop due to the internal resistance of the cell. It measures the p.d in an open circuit which is equal to the actual emf of the cell.
On the other hand, a voltmeter draws a small current from the cell for its operation. So it measures the terminal P.d in a closed circuit which is less than the emf of a cell. That is why a potentiometer is preferred over a voltmeter for measuring the emf of a cell.

Sensitiveness of a Potentiometer.
A potentiometer is sensitive if it is capable of measuring very small potential differences. The sensitivity of a potentiometer depends on the potential gradient along its wire. Smaller the potential gradient greater will be the sensitivity of the potentiometer.

Que. Why do we prefer a potentiometer with a longer bridge wire?
Ans. A potentiometer with a longer bridge wire has a small potential gradient $\left(K = \frac{V}{l}\right)$. Consequently it is more sensitive and hence preferred.
Q: How can the sensitivity of potentiometer be increased?
Ans. Sensitivity can be increased by
(i) Reducing the potential gradient
(ii) increasing the length of the wire
(iii) Reducing the current in the main circuit

Additional Numericals [Previous CBSE questions]
Q. When a potentiometer is connected between A & B the balancing length of the potentiometer wire is 300cm. On connecting the same potentiometer between A and C the balancing length is 100cm. Calculate the ratio of ε_1 and ε_2.



Solu: $\varepsilon \propto l$.
In Case I b/w A and B
$\varepsilon_1 \propto 300$

In Case II b/w A and C
$\varepsilon_1 - \varepsilon_2 \propto 100$

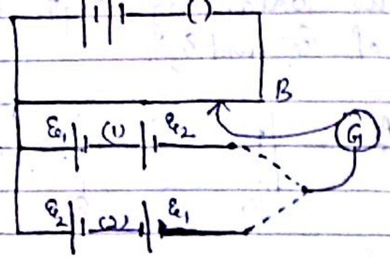
Hence $\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1} = \frac{100}{300}$

$$1 - \frac{\varepsilon_2}{\varepsilon_1} = \frac{1}{3}$$

$$\frac{\varepsilon_2}{\varepsilon_1} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\therefore \frac{\varepsilon_1}{\varepsilon_2} = \frac{3}{2} = 3:2$$

Q. A circuit using a potentiometer and battery of negligible internal resistance is set up as shown to develop a constant potential gradient along the wire AB. Two cells of emfs ε_1 and ε_2 are connected in series as shown in combination (1) and (2). The balance points are obtained respectively at 400 cm and 240 cm from the point A. Find (i) $\frac{E_1}{E_2}$ (ii) balancing length for the cell E_1 only



(1)

Solu: For the combination (1)

$$E_1 + E_2 \propto 400$$

For the combination (2)

$$E_2 - E_1 \propto 240$$

$$\frac{E_1+E_2}{E_2-E_1} = \frac{400}{240} = \frac{5}{3}$$

$$3(E_1+E_2) = 5[E_2 - E_1]$$

$$3E_1 + 3E_2 = 5E_2 - 5E_1$$

$$3E_1 + 5E_1 = 5E_2 - 3E_2$$

$$8E_1 = 2E_2$$

$$\frac{E_1}{E_2} = \frac{2}{8} = \frac{1}{4} = 1:4$$

(ii) $E_1 + E_2 \propto 400$

$$E_1 \propto l_1$$

$$\frac{E_1+E_2}{E_1} = \frac{400}{l_1} \quad \text{or} \quad \cancel{(E_1+E_2)\, l_1 = 400 E_1}$$

$$1 + \frac{E_2}{E_1} = \frac{400}{l_1}$$

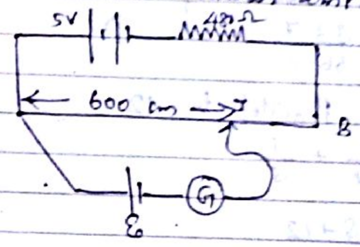
$$1 + 4 = \frac{400}{l_1}$$

$$5 = \frac{400}{l_1}$$

$$l_1 = \frac{400}{5} = 80 \text{ cm.}$$

Q. A 10m long wire of uniform cross-section of 20Ω resistance is used as a potentiometer wire. This wire is connected in series with a battery of 5V, along with an external resistance of 480Ω. If an unknown emf E is balanced at 600 cm of this wire. Calculate (i) the potential gradient of the potentiometer

(ii) the value of unknown emf E



Solu. Current $I = \frac{V}{R_{AB}+R} = \frac{5}{20+480} = \frac{5}{500} = 0.01\text{ A}$

PD across the wire $V = I R_{AB}$

$= 0.01 \times 20$

$= 0.2\text{ V}$

Length of the wire $= 10\text{m}$

Potential gradient $k = \frac{V}{l} = \frac{0.2}{10} = 0.02\text{ V/m}$

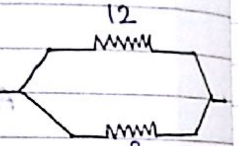
Unknown emf $\quad \varepsilon_0 \propto l$

$$\varepsilon_0 = kl$$
$$= 0.02 \times 600 \times 10^{-2}$$
$$= 2\times10^{-2} \times 600 \times 10^{-2}$$
$$= 1200 \times 10^{-4}$$
$$= \underline{0.12\ V}$$

Q. In a metre Bridge, the null point is found at a distance of 33.7 cm from A. If now resistance of 12 Ω is connected in parallel with S, the null point occurs at 51.9 cm. Determine the values of R and S.

Ans: From the first balance point

$$\frac{R}{S} = \frac{33.7}{66.3} \quad \text{——— (1)}$$



S is connected parallel to 12

$$\frac{1}{S_{equ}} = \frac{1}{12} + \frac{1}{S}$$
$$= \frac{S+12}{12S}$$
$$\therefore S_{equ} = \frac{12S}{12+S}$$

and hence the new balance condition

$$\frac{51.9}{48.1} = \frac{R}{S_{equ}}$$

$$\frac{51.9}{48.1} = \frac{R(12+S)}{12S} \quad \text{——— (2)}$$

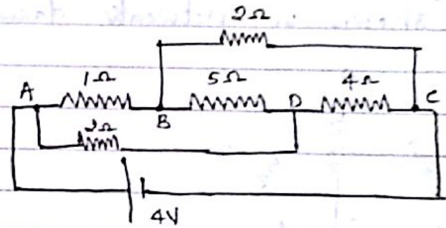
$$= \frac{51.9}{48.1} = \frac{12+S}{12} \times \frac{R}{S}$$

$$= \frac{12+S}{12} \times \frac{33.7}{66.3}$$

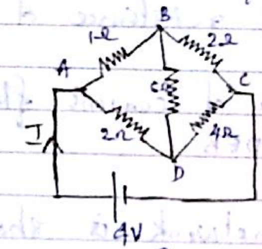
$$12+S = \frac{51.9 \times 12 \times 66.3}{48.1 \times 33.7} = \frac{41291.64}{1631.08}$$
$$= 25.31$$
$$\therefore S = 25.31 - 12$$
$$= \underline{13.31\ \Omega}$$

$$R = \frac{33.7}{66.3} \times S$$
$$= \frac{33.7}{66.3} \times 13.31$$
$$= \underline{6.77\ \Omega}$$

Q. Calculate the current drawn from the battery by the network of resistors shown in figure.



Soln: The given network is equivalent to the circuit shown



Now $\frac{1\Omega}{2\Omega} = \frac{2\Omega}{4\Omega} \quad \left(\frac{R_1}{R_2} = \frac{R_3}{R_4}\right)$

∴ the given circuit is balanced Wheatstone bridge

The resistance 5 Ω in arm BD is ineffective.

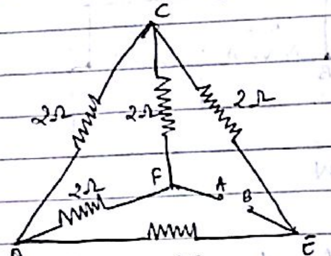
1Ω and 2Ω are in Series
$\therefore R_1 = 1+2 = 3\Omega$

2Ω and 4Ω are in Series.
$R_2 = 2+4 = 6\Omega$

R_1 and R_2 are in parallel.
$$\frac{1}{R} = \frac{1}{3}+\frac{1}{6} = \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$$
$\therefore R = 2\Omega$

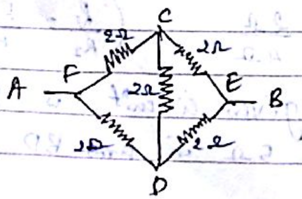
Current $I = \frac{V}{R} = \frac{4}{2} = \underline{\underline{2A}}$.

Q. A potential difference of 2V is applied between the points A and B shown in network drawn in figure



Calculate
(i) the equivalent resistance of the network between the points A and B
(ii) the magnitude of current flowing in the arms AFCEB and AFDEB.

Soln. The equivalent network is shown in



The Bridge is balanced because
$$\frac{2\Omega}{2\Omega} = \frac{2\Omega}{2\Omega}$$

Hence the resistance in arm CD is ineffective.

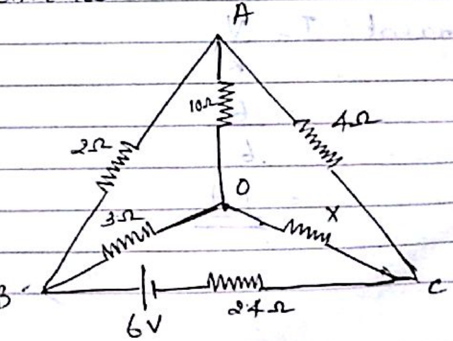
$R_1 = 2\Omega + 2\Omega = 4\Omega$
$R_2 = 2\Omega + 2\Omega = 4\Omega$

Total resistance $\frac{1}{R} = \frac{1}{R_1}+\frac{1}{R_2}$
$$= \frac{1}{4}+\frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$
$\therefore R = 2\Omega$

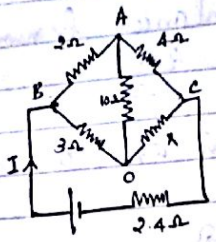
Total current $I = \frac{V}{R} = \frac{2V}{2\Omega} = 1A$

Current through arm AFCEB = Current through arm AFDEB
$$= \frac{1}{2}A$$
$$= \underline{\underline{0.5A}}$$

Q. Find the value of the unknown resistance X in the following circuit. If no current flows through the section AO. Also calculate the current drawn by the circuit from the battery of emf 6V and negligible internal resistance.



Soln:



If no current flows through the section AO

∴ the bridge is balanced $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$$\frac{2}{4} = \frac{3}{X}$$

or $X = \frac{4 \times 3}{2} = \underline{6\Omega}$

2Ω and 4Ω are in series

$R_1 = 2 + 4 = 6\Omega$

3Ω and X = 6Ω are in series

$R_2 = 3 + 6 = 9\Omega$

Total resistance of the arm

$$\therefore \frac{1}{R'} = \frac{1}{6} + \frac{1}{9}$$

$$= \frac{9+6}{9 \times 6}$$

or $R' = \frac{9 \times 6}{9+6} = \cancel{\frac{54}{15}} = \frac{18}{5} = \underline{3.6\Omega}$

Total resistance $R = 3.6 + 2.4$

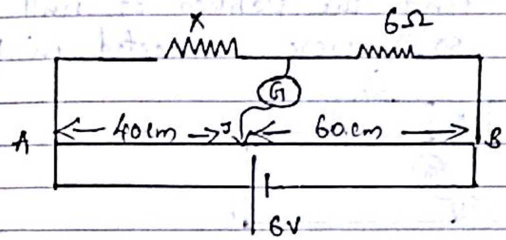
$= 6\Omega$

Current $I = \frac{V}{R}$

$= \frac{6}{6}$

$= \underline{\underline{1A}}$

Q. In the following circuit a metre bridge is shown in its balanced state. The metre bridge wire has a resistance of 1Ω/cm. Calculate the value of the unknown resistance X and the current draw from the battery of negligible external resistance.



Soln. $l = 40cm$. $R = X$

$100 - l = 60cm$. $S = 6$

$r = 1\Omega cm$.

In Balanced condition $\frac{R}{S} = \frac{l r}{(100-l) r}$

$$\frac{X}{6} = \frac{40 \times 1}{60 \times 1}$$

$$\frac{X}{6} = \frac{40}{60}$$

$$X = \frac{4\cancel{0} \times 6}{6\cancel{0}} = \underline{4\Omega}$$

4Ω and 6Ω are in series

∴ $R' = 4 + 6 = 10\Omega$

Resistance of the wire AB = 100Ω (100×1)

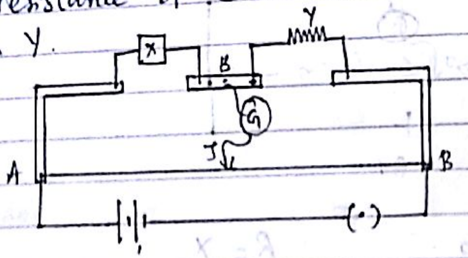
These R' and R_{AB} are $||^{el}$

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{100} = \frac{10+1}{100} = \frac{11}{100}$$

$$R = \frac{100}{11}\Omega$$

$$I = \frac{V}{R} = \frac{6}{\frac{100}{11}} = \frac{66}{100} = \underline{\underline{0.66A}}$$

Q. The given figure shows the experimental setup of a metre bridge. The null point is found to be 60cm away from the end A with X and Y in positions as shown. When the resistance of 15Ω is connected in series with Y, the null point is found to shift by 10cm towards the end A of the wire. Find the position of null point if resistance of 30Ω were connected in parallel with Y.



Soln. In I Case

$$\frac{X}{Y} = \frac{60}{40}$$

$$\frac{X}{Y} = \frac{3}{2} \quad \text{—①}$$

In II Case

$$\frac{X}{Y+15} = \frac{50}{50} = 1 \quad \text{—②}$$

$$\frac{①}{②} \quad \frac{X}{Y} \times \frac{Y+15}{X} = \frac{3}{2}$$

$$\frac{Y+15}{Y} = \frac{3}{2}$$

$$1 + \frac{15}{Y} = \frac{3}{2}$$

$$\frac{15}{Y} = \frac{3}{2} - 1 = \frac{3-2}{2} = \frac{1}{2}$$

$$\therefore Y = \underline{30\Omega}$$

$$\frac{X}{Y} = \frac{3}{2}$$

$$X = \frac{3 \times 30}{2} = \underline{45\Omega}$$

When a resistance of 30Ω is connected in parallel with Y

∴ the equivalent resistance

$$\frac{1}{Y'} = \frac{1}{30} + \frac{1}{Y}$$

$$= \frac{Y+30}{30Y}$$

$$Y' = \frac{30Y}{Y+30}$$

$$\frac{X}{\frac{30Y}{Y+30}} = \frac{l}{100-l}$$

$$\frac{45}{\frac{900}{60}} = \frac{l}{100-l}$$

$$\frac{45 \times 60}{900} = \frac{l}{100-l}$$

$$3(100-l) = l$$

$$300 - 3l = l$$

$$300 = 4l$$

$$l = \frac{300}{4} = \underline{75\,cm}$$

Q. In a discharge tube the no. of hydrogen ions (protons) drifting across a cross-section per second is 1×10^{18} while the number of electrons drifting in the opposite direction across another cross-section is 2.7×10^{18} per second. If the supply voltage is 230V, what is the effective resistance of the tube?

Solu. $I = ne$

$= (n_e + n_p)e$

$= (2.7 \times 10^{18} + 1 \times 10^{18}) \times 1.6 \times 10^{-19}$

$= 3.7 \times 10^{18} \times 1.6 \times 10^{-19}$

$= 0.592$ A

$R = \frac{V}{I} = \frac{230}{0.592} = 388.5\ \Omega$

Que. A current of 2mA is passed through a colour coded carbon resistor with first, second and third rings of yellow, green and orange colours. What is the voltage drop across the resistor?

Sol. $R = 45 \times 10^{3}\ \Omega$

$I = 2$mA

$= 2 \times 10^{-3}$ A

$V = RI$

$= 45 \times 10^{3} \times 2 \times 10^{-3}$

$= 90$ V

Q. A wire of 10Ω resistance is stretched to thrice its original length. What will be its (i) new resistivity and (ii) new resistance

Ans. (i) Resistivity ρ remains unchanged because it is the property of the material.

(ii) $V = A'l' = Al$

$\frac{A'}{A} = \frac{l}{l'} = \frac{l}{3l} = \frac{1}{3}$

$\therefore A' = A/3$

$R' = \frac{\rho l'}{A'}$

$= \frac{\rho \times 3l}{A/3}$

$= 9 \times \frac{\rho l}{A}$

$= 9 \times 10 = 90\ \Omega$

Q. A wire has a resistance of 16Ω. It is melted and drawn into a wire of half its length. Calculate the resistance of the new wire. What is the percentage change in its resistance?

Solu. $V = A'l' = Al$

$\frac{A'}{A} = \frac{l}{l'} = \frac{l}{l/2} = 2$

$A' = 2A$

$R' = \frac{\rho l'}{A'} = \frac{\rho \times l/2}{2A} = \frac{1}{4} \frac{\rho l}{A}$

$= \frac{1}{4} \times 16 = 4\ \Omega$

change in resistance $= \frac{R - R'}{R} \times 100$

$= \frac{16 - 4}{16} \times 100$

$= 75\%$

Q. A Cu wire has a resistance of 10Ω and an area of cross-section $1\ mm^2$. A potential difference of 10V exists across the wire. Calculate the drift speed of electrons if the number of electrons per cubic metre in Cu is 8×10^{28} e's?

Solu $R = 10\,\Omega$

$A = 1\,mm^2$

$= 10^{-6}\,m^2$

$V = 10\,V$ $\quad n = 8\times10^{28}\,\bar{e}^{1}s/m^3$

$I = nAV_d e.$

$\frac{V}{R} = nAV_d e.$

$V_d = \frac{V}{nAeR} = \frac{10}{8\times10^{28}\times10^{-6}\times1.6\times10^{-19}\times10}$

$= \underline{\underline{0.078\times10^{-3}\,m/s}}$

Q. An aluminium wire of diameter 0.24 cm is connected in series to a cu wire of diameter 0.16 cm. The wires carry an electric current of 10 A. Find (i) current density in the Al wire (ii) drift velocity of $\bar{e}$'s in the copper wire. Given $n = 8.4\times10^{28}\,m^{-3}$

Solu $d = 0.24\,cm$

$r = 0.12\,cm = 0.12\times10^{-2}\,m$

Area $= \pi r^2$

$= 3.14\times(0.12\times10^{-2})^2$

$= 4.5\times10^{-6}\,m^2$

$I = 10\,A$

(i) $j = \frac{I}{A} = \frac{10}{4.5\times10^{-6}}$

$= \underline{\underline{2.2\times10^{6}\,A/m^2}}$

(ii) Area of Cu-wire $= \pi r^2$

$= \pi\times(0.08\times10^{-2})^2$

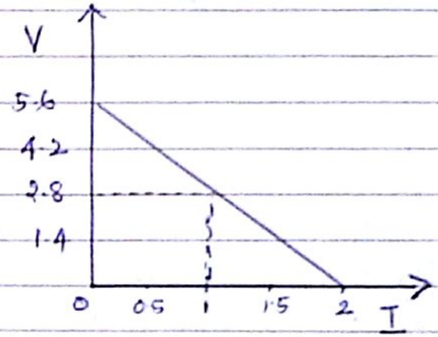
$= 2\times10^{-6}\,m^2$

$I = nAV_d e$

$V_d = \frac{I}{nAe} = \frac{10}{8.4\times10^{28}\times2\times10^{-6}\times1.6\times10^{-19}}$

$= \underline{\underline{3.7\times10^{-4}\,m/s}}.$

Q. 4 cells of identical emf $\mathcal{E}$, internal resistance r are connected in series to a variable resistor. The following graph shows the variation of terminal voltage of the combination with the current output:



(i) What is the emf of each cell used?

(ii) For what current from the cells, does maximum power dissipation occur in the circuit?

(iii) Calculate the internal resistance of each cell.

Solu: (i) When $I = 0$

total emf = terminal p.d

$4\mathcal{E} = 5.6$

$\mathcal{E} = \frac{5.6}{4} = \underline{1.4\,V}$

(iii) $r = \frac{\mathcal{E} - V}{I} =$ ~~$\frac{}{}$~~

If $I = 1A$ $\quad V = 2.8\,V$

ie for 4 cell $V = 2.8$

$\therefore 4V = 2.8$ $\quad V = \frac{2.8}{4} = \underline{0.7\,V}$

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$r = \frac{E - V}{I}$

$= \frac{1.4 - 0.7}{1} = \underline{0.7\,\Omega}$

(ii) The output power is maximum when external resistance = internal resistance

$R = 4r$

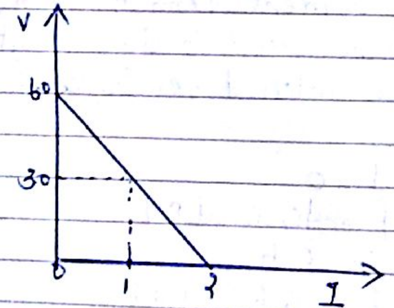
$I_{max} = \frac{E}{R + r} =$

Here $= I_{max} = \frac{4E}{4r + 4r}$

$= \frac{E}{2r} = \frac{1.4}{2 \times 0.7}$

$= \underline{1A}$

Q. The graph shows the variation of terminal potential difference V across a combination of three cells in series to a resistor versus the current I



(i) Calculate the emf of each cell.

(ii) For what current I will the power dissipation of the circuit be maximum?

Sol: (i) When $I = 0$

total emf = terminal p.d

$3E = 6$

$E = 6/3 = \underline{2V}$

(ii) The output power is maximum the external resistance = internal resistance

$R = 3r$

$\therefore I_{max} = \frac{E}{R + r}$ becomes.

$= \frac{3E}{3r + 3r}$

$= \frac{E}{2r}$

When $I = 1A$

$V = 3V$

ie for 3 cells $V = 3$ Volt

$\therefore 3V = 3$ Volt

$V = \frac{3}{3} = 1V$

$r = \frac{E - V}{I} = \frac{2 - 1}{1} = 1\,\Omega$

$\therefore I_{max} = \frac{2}{2 \times 1}$

$= \frac{2}{2} = \underline{1A}$

Q. Two wires of equal lengths, one of copper and the other of manganin have the same resistance. Which wire be thicker?

Solu. As $R = \frac{\rho l}{A}$

$A = \frac{\rho l}{R}$ —(1)

For both wires R and l are same

and $\rho_{cu} < \rho_{manganin}$.

from equ(1) $A_{cu} < A_{manganin}$

∴ manganin wire is thicker than copper wire.

Q: Two wires of equal cross-sectional area, one of copper and other of manganin have the same resistance. Which one will be longer?

Solu. $R = \frac{\rho l}{A}$

$l = \frac{RA}{\rho}$ —(1)

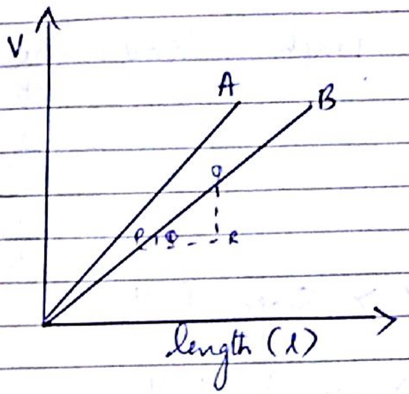
For both wires R and A are same

$\rho_{cu} < \rho_{mangani}$

∴ from equ(1) $l_{cu} > l_{manganin}$

∴ copper wire is longer than manganin.

Q: The variation of p.d with length in case of two potentiometers A and B is shown. Which of the two is preferred to find emf of a cell? Give reason for your answer.



Solu: Slope of the graph

$\tan\theta = \frac{QR}{PR}$

$= \frac{V}{l} = K$ (potential gradient)

Slope of line A > Slope of line B

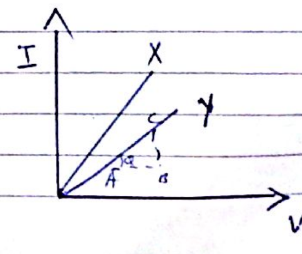
∴ K of A > K of B

As the potential gradient increases the sensitivity of potentiometer decreases.

∴ B has less potential gradient

∴ B is preferred over A.

Q. The voltage current variation of two metallic wires X and Y at constant temperature are shown in figure. Assuming that the wires have the same length and the same diameter, explain which of the two wires will have larger resistivity.



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Soln: Slope of the graph $\tan\theta = \frac{BC}{AB}$

$$= \frac{I}{V}$$

$$= \frac{1}{R}$$

Slope of X > Slope of Y

ie $\frac{1}{R}$ of X > $\frac{1}{R}$ of Y

or R of X < R of Y.

$\frac{\rho_x l}{A}$ of X < $\frac{\rho_y l}{A}$ of Y. (length and diameter same)

$\therefore \rho_x < \rho_y$

Thus wire Y has larger resistivity.