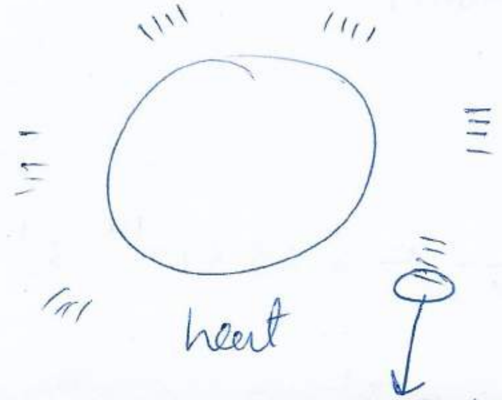
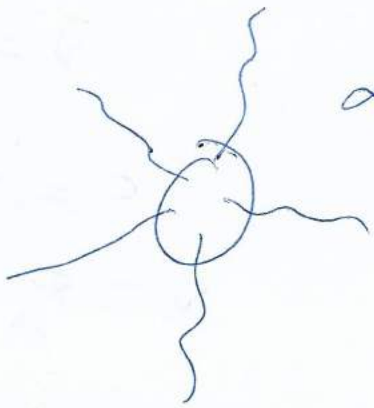
Atomic structure

Quantum theory of radiation

 or particle



heat



α

a photon (a quantum)

• Plural → quanta (many photons)

$E = h\nu \rightarrow$ frequency of radiation

h → Planck's constant

$\nu = \frac{c}{\lambda} \Rightarrow E = \frac{hc}{\lambda}$

λ → wavelength

$E = h\nu,\ 2h\nu,\ 3h\nu\ ------\ nh\nu$

→ Quantisation of Energy

$h = 6.626 \times 10^{-34}\ J.s = 6.626 \times 10^{-27}\ erg.s$

SI — CGS

Example 2. *Calculate the frequency and wave number of radiation with wavelength 480 nm.*

Solution: Given,

$$\lambda = 480\text{ nm} = 480\times10^{-9}\text{ m} \quad [\because 1\text{ nm} = 10^{-9}\text{ m}]$$

$$c = 3\times10^{8}\text{ m/sec}$$

$$\text{Frequency, } \nu = \frac{c}{\lambda} = \frac{3\times10^{8}\text{ ms}^{-1}}{480\times10^{-9}\text{ m}} = 6.25\times10^{14}\text{ s}^{-1}$$

$$= 6.25\times10^{14}\text{ Hz}$$

Example 3. *Calculate the energy associated with photon of light having a wavelength 6000 Å. [$h = 6.624\times10^{-27}$ erg-sec.]*

Solution: We know that, $E = h\nu = h\cdot\frac{c}{\lambda}$

$h = 6.624\times10^{-27}$ erg-sec; $c = 3\times10^{10}$ cm/sec;

$\lambda = 6000\text{ Å} = 6000\times10^{-8}$ cm

$$\text{So, } E = \frac{(6.624\times10^{-27})\times(3\times10^{10})}{6\times10^{-5}} = 3.312\times10^{-12}\text{ erg.}$$

Example 4. *Which has a higher energy, a photon of violet light with wavelength 4000 Å or a photon of red light with wavelength 7000 Å? [$h = 6.62\times10^{-34}$ Js]*

Solution: We know that, $E = h\nu = h\cdot\frac{c}{\lambda}$

Given, $h = 6.62\times10^{-34}$ Js, $c = 3\times10^{8}$ ms^{-1}

For a photon of violet light,

$$\lambda = 4000\text{ Å} = 4000\times10^{-10}\text{ m}$$

$$E = 6.62\times10^{-34}\times\frac{3\times10^{8}}{4\times10^{-7}} = 4.96\times10^{-19}\text{ J}$$

For a photon of red light,

$$\lambda = 7000\text{ Å} = 7000\times10^{-10}\text{ m}$$

$$E = 6.62\times10^{-34}\times\frac{3\times10^{8}}{7000\times10^{-10}} = 2.83\times10^{-19}\text{ J}$$

Hence, photon of violet light has higher energy than the photon of red light.

Example 5. *What is the ratio between the energies of two radiations one with a wavelength of 6000 Å and other with 2000 Å?*

Solution: $\lambda_1 = 6000\text{ Å}$ and $\lambda_2 = 2000\text{ Å}$

$$E_1 = h\cdot\frac{c}{\lambda_1} \text{ and } E_2 = h\cdot\frac{c}{\lambda_2}$$

$$\text{Ratio, } \frac{E_1}{E_2} = \frac{h\cdot c}{\lambda_1}\times\frac{\lambda_2}{h\cdot c} = \frac{\lambda_2}{\lambda_1} = \frac{2000}{6000} = \frac{1}{3}$$

or $E_2 = 3E_1$

Example 6. *Calculate the wavelength, wave number and frequency of photon having an energy equal to three electron volt. ($h = 6.62\times10^{-27}$ erg-sec.)*

Solution: We know that,

$$E = h\cdot\nu$$

$$\nu = \frac{E}{h} \quad (1\text{ eV} = 1.602\times10^{-12}\text{ erg})$$

$$= \frac{3\times(1.602\times10^{-12})}{6.62\times10^{-27}}$$

$$= 7.26\times10^{14}\text{ s}^{-1}$$

$$= 7.26\times10^{14}\text{ Hz}$$

$$\lambda = \frac{c}{\nu} = \frac{3\times10^{10}}{7.26\times10^{14}} = 4.132\times10^{-5}\text{ cm}$$

$$\bar{\nu} = \frac{1}{\lambda} = \frac{1}{4.132\times10^{-5}} = 2.42\times10^{4}\text{ cm}^{-1}$$

Example 7. *Calculate the energy in kilocalorie per mol of the photons of an electromagnetic radiation of wavelength 7600 Å.*

Solution: $\lambda = 7600\text{ Å} = 7600\times10^{-8}$ cm

$c = 3\times10^{10}$ cm s^{-1}

$$\text{Frequency, } \nu = \frac{c}{\lambda} = \frac{3\times10^{10}}{7600\times10^{-8}} = 3.947\times10^{14}\text{ s}^{-1}$$

$$\text{Energy of one photon} = h\nu = 6.62\times10^{-27}\times3.947\times10^{14}$$

$$= 2.61\times10^{-12}\text{ erg}$$

$$\text{Energy of one mole of photons} = 2.61\times10^{-12}\times6.02\times10^{23}$$

$$= 15.71\times10^{11}\text{ erg}$$

Energy of one mole of photons in kilocalorie

$$= \frac{15.71\times10^{11}}{4.185\times10^{10}} \quad [1\text{ kcal} = 4.185\times10^{10}\text{ erg}]$$

$$= 37.538\text{ kcal per mol}$$

Example 8. *Electromagnetic radiation of wavelength 242 nm is just sufficient to ionise the sodium atom. Calculate the ionisation energy in kJ mol^{-1}. $h = 6.6256\times10^{-34}$ Js.*

Solution: $\lambda = 242\text{ nm} = 242\times10^{-9}$ m

$c = 3\times10^{8}$ ms^{-1}

$$E = h\nu = h\cdot\frac{c}{\lambda} = 6.6256\times10^{-34}\times\frac{3\times10^{8}}{242\times10^{-9}}$$

$$= 0.082\times10^{-17}\text{ J} = 0.082\times10^{-20}\text{ kJ}$$

$$\text{Energy per mole for ionisation} = 0.082\times10^{-20}\times6.02\times10^{23}$$

$$= 493.6\text{ kJ mol}^{-1}$$

Example 9. *How many photons of light having a wavelength 4000 Å are necessary to provide 1.00 J of energy?*

Solution: Energy of one photon

$$= h\nu = h\cdot\frac{c}{\lambda}$$

$$= \frac{(6.62\times10^{-34})(3.0\times10^{8})}{4000\times10^{-10}}$$

$$= 4.965\times10^{-19}\text{ J}$$

$$\text{Number of photons} = \frac{1.00}{4.965\times10^{-19}} = 2.01\times10^{18}$$

$$\boxed{1\ \text{erg} = 10^{-7}\ \text{J}} \rightarrow \text{Energy}$$

CGS SI

<u>Ex. 2</u>

$$E = h\nu = \frac{hc}{\lambda} = hc\bar{\nu}$$

$$\bar{\nu} = \frac{1}{\lambda} \text{ (wave number)}$$

$$c = \nu\lambda$$

$$\nu = c/\lambda = \frac{3\times10^{8}\ \text{m/s}}{480\times10^{-9}\ \text{m}} = 6.25\times10^{14}\ \text{s}^{-1}$$

$$\bar{\nu} = \frac{1}{\lambda} = \frac{1}{480}\ \text{nm}^{-1}$$

<u>CGS</u>

<u>Ex 3</u>

$$E = \frac{hc}{\lambda} = \frac{6.624\times10^{-27}\times3\times10^{10}}{6000\times10^{-8}}\ \text{erg}$$

$$= 3.312\times10^{-12}\ \text{erg}$$

<u>Ex 6.</u>

$$\boxed{1\ \text{eV} = 1.6\times10^{-19}\ \text{J}}$$

$$1\ \text{erg} = 10^{-7}\ \text{J}$$

$$1\ \text{J} = 10^{7}\ \text{erg}$$

$$E = h\nu$$

$\boxed{\text{CGS}}$

$$3\times1.6\times10^{-19}\ \text{J}\times10^{7} = 6.62\times10^{-27}\times\nu$$

$$\nu = 7.26\times10^{14}\ \text{Hz}$$

$$E = \frac{hc}{\lambda}$$

$$(\text{cm}) \quad \lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-27} \times 3 \times 10^{10}}{3 \times 1.6 \times 10^{-19} \times 10^{7}} \text{ cm.}$$

$$= 4.132 \times 10^{-5} \text{ cm}$$

$$\bar{\nu} = \frac{1}{\lambda} = 2.42 \times 10^{4} \text{ cm}^{-1}$$

Ex 7 $\boxed{1 \text{ cal} = 4.184 \text{ J}}$

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J.s} \times 3 \times 10^{8} \text{ m/s}}{7600 \times 10^{-10}} \text{ J}$$

$$E\,(\text{kcal/mol}) = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8} \times N_A}{7600 \times 10^{-10} \times \underbrace{4.184}_{\text{cal}} \times \underbrace{10^{3}}_{\text{kcal}}}$$

$$= 37.538 \text{ kcal/mol}$$

Ex 8 $$E = \frac{6.6256 \times 10^{-34} \times 3 \times 10^{8} \times N_A}{242 \times 10^{-9} \times 10^{3}}$$

Ex 9 $$\underset{\text{photons}}{n} \times h\nu = 1 \text{ J} = n\frac{hc}{\lambda}$$

$$n \times \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{4000 \times 10^{-10}} = 1 \Rightarrow n = 2.01 \times 10^{18}$$

Example 10. *Find the number of quanta of radiations of frequency 4.67×10^{13} s^{-1}, that must be absorbed in order to melt 5 g of ice. The energy required to melt 1 g of ice is 333 J.*

Solution: Energy required to melt 5 g of ice

$$= 5 \times 333 = 1665 \text{ J}$$

Energy associated with one quantum

$$= h\nu = (6.62 \times 10^{-34}) \times (4.67 \times 10^{13})$$

$$= 30.91 \times 10^{-21} \text{ J}$$

Number of quanta required to melt 5 g of ice

$$= \frac{1665}{30.91 \times 10^{-21}} = 53.8 \times 10^{21} = 5.38 \times 10^{22}$$

Example 11. *Calculate the wavelength of the spectral line, when the electron in the hydrogen atom undergoes a transition from the energy level 4 to energy level 2.*

Solution: According to Rydberg equation,

$$\frac{1}{\lambda} = R\left(\frac{1}{x^2} - \frac{1}{y^2}\right)$$

$R = 109678 \text{ cm}^{-1}$; $x = 2$; $y = 4$

$$\frac{1}{\lambda} = 109678\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$= 109678 \times \frac{3}{16}$$

On solving, $\lambda = 486 \text{ nm}$

Example 12. *A bulb emits light of wavelength $\lambda = 4500$ Å. The bulb is rated as 150 watt and 8% of the energy is emitted as light. How many photons are emitted by the bulb per second?*

Solution: Energy emitted per second by the bulb

$$= 150 \times \frac{8}{100} \text{ J}$$

Energy of 1 photon $= \dfrac{hc}{\lambda} = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^8}{4500 \times 10^{-10}}$

$$= 4.42 \times 10^{-19} \text{ joule}$$

Let n photons be evolved per second.

$$\therefore \quad n \times 4.42 \times 10^{-19} = 150 \times \frac{8}{100}$$

$$n = 27.2 \times 10^{18}$$

Example 13. *A near ultraviolet photon of 300 nm is absorbed by a gas and then remitted as two photons. One photon is red with wavelength of 760 nm. What would be the wave number of the second photon?*

Solution:

Energy absorbed = Sum of energy of two quanta

$$\frac{hc}{300 \times 10^{-9}} = \frac{hc}{760 \times 10^{-9}} + \frac{hc}{\lambda \times 10^{-9}}$$

On solving, we get,

$$\bar{\nu} \text{ (wave number)} = \frac{1}{\lambda} = 2.02 \times 10^{-3} \text{ m}^{-1}$$

Example 14. *Calculate the wavelength of the radiation which would cause the photodissociation of chlorine molecule if the Cl—Cl bond energy is 243 kJ mol $^{-1}$.*

Solution: Energy required to break one Cl—Cl bond

$$= \frac{\text{Bond energy per mole}}{\text{Avogadro's number}}$$

$$= \frac{243}{6.023 \times 10^{23}} \text{ kJ} = \frac{243 \times 10^3}{6.023 \times 10^{23}} \text{ J}$$

Let the wavelength of the photon to cause rupture of one Cl—Cl bond be λ.

We know that, $\lambda = \dfrac{hc}{E} = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 6.023 \times 10^{23}}{243 \times 10^3}$

$$= 4.90 \times 10^{-7} \text{ m} = 490 \text{ nm}$$

Example 15. *How many moles of photon would contain sufficient energy to raise the temperature of 225 g of water 21°C to 96°C? Specific heat of water is 4.18 J g^{-1} K^{-1} and frequency of light radiation used is 2.45×10^9 s^{-1}.*

Solution: Energy associated with one mole of photons

$$= N_0 \times h \times \nu$$

$$= 6.02 \times 10^{23} \times 6.626 \times 10^{-34} \times 2.45 \times 10^9$$

$$= 97.727 \times 10^{-2} \text{ J mol}^{-1}$$

Energy required to raise the temperature of 225 g of water by 75°C $= m \times s \times t = 225 \times 4.18 \times 75 = 70537.5$ J

Hence, number of moles of photons required

$$= \frac{mst}{N_0 h\nu} = \frac{70537.5}{97.727 \times 10^{-2}} = 7.22 \times 10^4 \text{ mol}$$

Example 16. *During photosynthesis, chlorophyll absorbs light of wavelength 440 nm and emits light of wavelength 670 nm. What is the energy available for photosynthesis from the absorption-emission of a mole of photons?*

Solution: $\Delta E = \left[\dfrac{Nhc}{\lambda}\right]_{\text{absorbed}} - \left[\dfrac{Nhc}{\lambda}\right]_{\text{evolved}}$

$$= Nhc\left[\frac{1}{\lambda_{\text{absorbed}}} - \frac{1}{\lambda_{\text{evolved}}}\right]$$

$$= 6.023 \times 10^{23} \times 6.626 \times 10^{-34} \times$$

$$3 \times 10^8 \left[\frac{1}{440 \times 10^{-9}} - \frac{1}{670 \times 10^{-9}}\right]$$

$$= 0.1197[2.272 \times 10^6 - 1.492 \times 10^6]$$

$$= 0.0933 \times 10^6 \text{ J/mol} = 93.3 \text{ kJ/mol}$$

Example 17. *Photochromic sunglasses, which darken when exposed to light, contain a small amount of colourless AgCl (s) embedded in the glass. When irradiated with light, metallic silver atoms are produced and the glass darkens.*

$$AgCl(s) \longrightarrow Ag(s) + Cl$$

Escape of chlorine atoms is prevented by the rigid structure of the glass and the reaction therefore, reverses as soon as the light is removed. If 310 kJ / mol of energy is required to make the reaction proceed, what wavelength of light is necessary?

Ex 10 $E = h\nu$

$$n \times 6.62 \times 10^{-34} \times 4.67 \times 10^{13} = 5 \times 333$$

$$n = 5.38 \times 10^{22}$$

Ex 12 $$150\ J/s \times \frac{8}{100} = n \times \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{4500 \times 10^{-10}}\ J$$

$$n = 27.2 \times 10^{18}$$

Ex 13 $$\frac{hc}{300} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{1}{300} = \frac{1}{760} + \frac{1}{\lambda_2}$$

$$\bar{\nu}_2 = \frac{1}{\lambda_2} = \frac{1}{300} - \frac{1}{760}$$

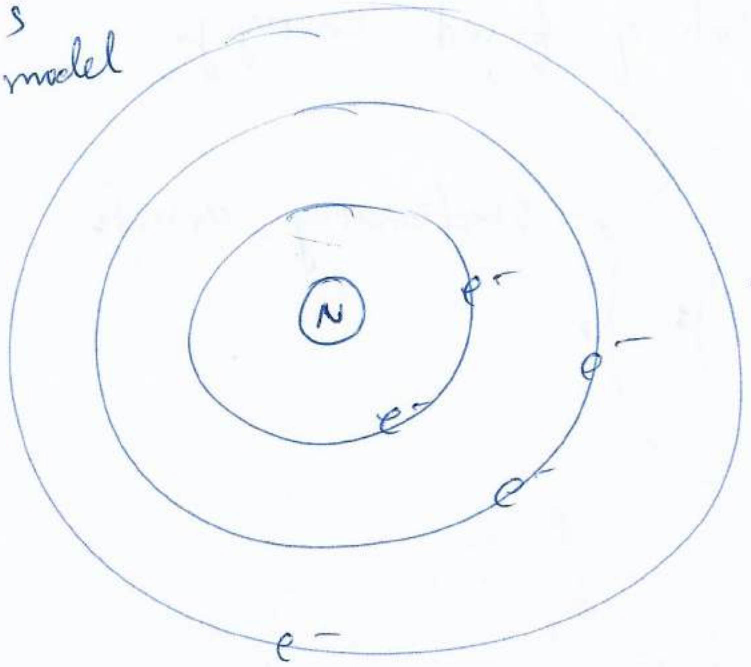
$$= \left(\frac{1}{300 \times 10^{-9}} - \frac{1}{760 \times 10^{-9}} \right)\ m^{-1}$$

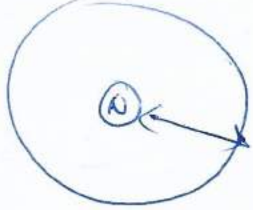
$$= 2.02 \times 10^{-3}\ m^{-1}$$

Bohr's Atomic Model

Postulates of Bohr's model

→





→ The e^- can revolve only in those orbits in which its angular momentum is an integral multiple of $h/2\pi$

momentum = mv

angular momentum = mvr

$r \rightarrow$ radius of circular orbit

$$\boxed{mvr = n\frac{h}{2\pi}}$$

$m \rightarrow$ mass of e^-
$v \rightarrow$ velocity of e^-

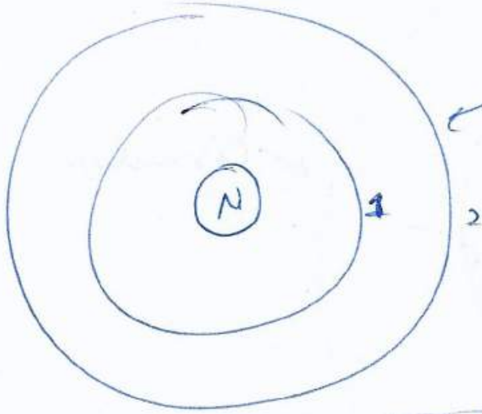
$n = 1, 2, 3, \text{-----}$ integral values
↳ orbit no.

$$mvr = \frac{h}{2\pi}, \frac{2h}{2\pi}, \frac{3h}{2\pi} \text{-----} n\frac{h}{2\pi}$$

→ Quantisation of angular momentum or angular momentum is quantised

→ Stationary orbits

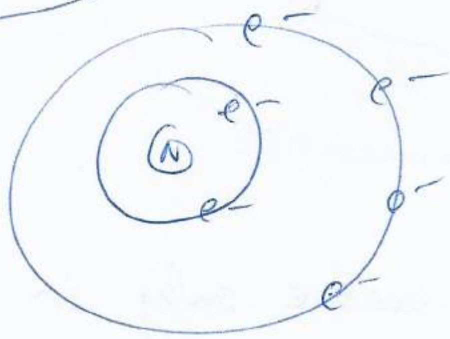
orbits of fixed energy



← Stationary orbits

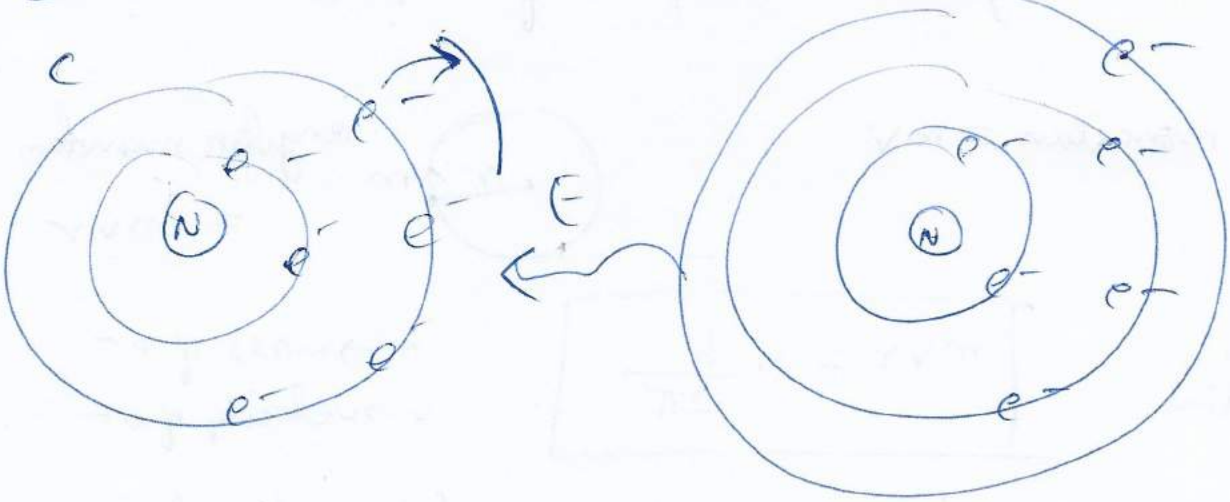
C − 6

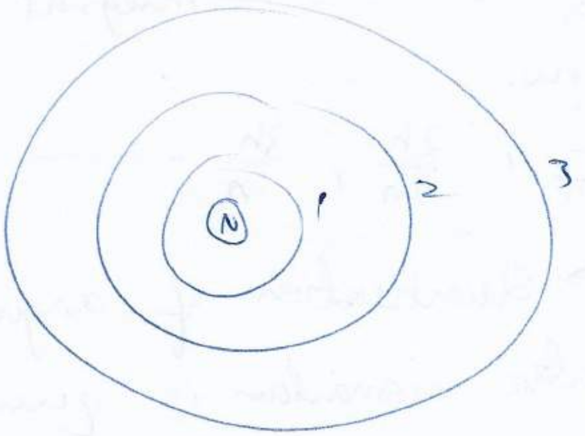
2,4



Ground state

Excited state





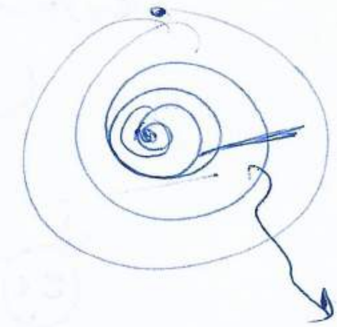
$E_1 < E_2 < E_3 \ldots$

$E_2 - E_1 > E_3 - E_2$

$> E_4 - E_3 \ldots$

→ The energy is absorbed or emitted only whe e^- moves from one stationary orbit into another stationary orbit

→ The e^- in stationary orbits do not radiate energy



<u>H-atoms</u> or <u>H like atoms</u>

<u>Radii of orbits</u>

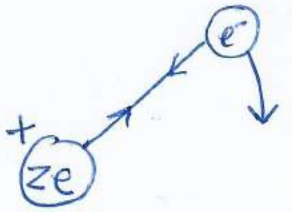
H	$1e^-$	1P
He^+	$1e^-$	2P
Li^{2+}	$1e^-$	3P
Be^{3+}	$1e^-$	4P

Species having one e^-

<u>H atom</u>

$Z \rightarrow$ atomic No.
= No. of protons

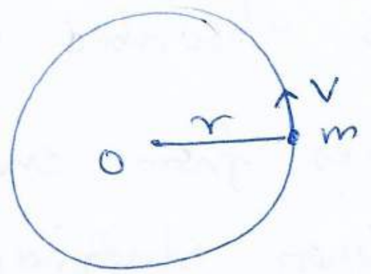
$\underline{Z}$



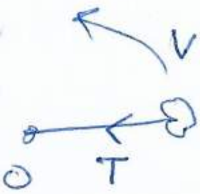
~~Each~~ each proton has a charge $= +1.602 \times 10^{-19}$ C $= e$

Electrostatic force $= \dfrac{k q_1 q_2}{r^2}$

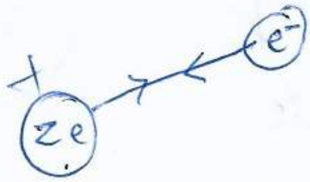
$k = 9 \times 10^9$ Nm^2/C^2



Circular motion



$$\text{Centripetal force } (F_c) = \frac{mv^2}{r}$$



$$F_c = \frac{k q_1 q_2}{r^2} = \frac{mv^2}{r}$$

$$q_1 = ze, \quad q_2 = -e$$

force is attractive

$$F_c = \frac{-kze^2}{r^2}$$

$$\frac{kze^2}{r^{\cancel{2}}} = \frac{mv^2}{\cancel{r}}$$

$$\Rightarrow r = \frac{kze^2}{mv^2}$$

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi m r}$$

$$\cancel{r} = \frac{kze^2 \times 4\pi^2 m^{\cancel{2}} r^{\cancel{2}}}{\cancel{m}\, n^2 h^2}$$

$$\Rightarrow r = \left(\frac{n^2}{Z}\right) \frac{h^2}{4\pi^2 m k e^2}$$

$$r = \frac{n^2}{Z}\left[\frac{(6.626 \times 10^{-34})^2}{4 \times (3.14)^2 \times (9.1 \times 10^{-31}) \times (9 \times 10^9) \times (1.602 \times 10^{-19})^2}\right]$$

$$\boxed{r_n = \frac{n^2}{Z} \times 0.529\ \text{Å}} \rightarrow$$ For H & H like atoms

H-atom $(Z=1)$

$r_1 = 0.529\ \text{Å}$

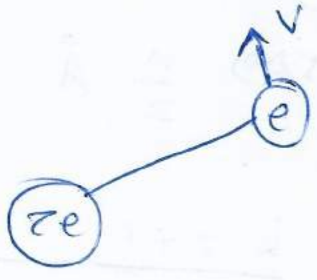
$r_2 = 4 \times 0.529\ \text{Å}$

He^+

$r_2 = \frac{2^{\cancel{2}}}{\cancel{2}} \times 0.529$

$= 2 \times 0.529$

Energy of an e^-



Energy $=$ Kinetic energy $+$ Potential energy
(E) (KE) (PE)

$KE = \frac{1}{2}mv^2$, $\frac{(mv^2)}{r} = \frac{kZe^2}{r^{\cancel{2}}}$

$\Rightarrow mv^2 = \frac{kZe^2}{r}$

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\frac{kze^2}{r}$$

$$PE = \frac{q_1 \quad q_2}{r} = \frac{kq_1q_2}{r}$$

$$\underline{P.E} = \frac{k(ze)(-e)}{r}$$

$$P.E. = -\frac{kze^2}{r}$$

e^- is bound to the nucleus

$$E = K + P = \underbrace{\frac{1}{2}\frac{kze^2}{r}}_{K} - \underbrace{\frac{kze^2}{r}}_{PE}$$

$$= -\frac{1}{2}\frac{kze^2}{r}$$

$$\boxed{KE = -\frac{1}{2}PE = -E}$$

6

$$E = -\frac{1}{2}\frac{kze^2}{r}$$

$$r_n = 0.529\frac{n^2}{z}\ \text{Å}$$

$$E_n = -\frac{1}{2}\frac{kze^2}{0.529\frac{n^2}{z}\times10^{-10}} \quad J$$

$$= -\frac{1}{2}\frac{z^2}{n^2}\left(\frac{-9\times10^9\times(1.6\times10^{-19})^2}{0.529\times10^{-10}}\right)$$

$$\boxed{E_n = -\frac{z^2}{n^2}\times 2.18\times10^{-18}\ J/atom}$$

for H & H like atoms

$$\boxed{E_n = \frac{-Z^2}{n^2} \times 13.6 \text{ eV/atom}}$$

for H atom ($Z=1$)

$$E_n = -13.6/n^2 \text{ ev/atom}$$

e^- is bound to the nucleus

$$E_1 = -13.6 \text{ eV}$$

$$E_2 = -13.6/4 \text{ eV}$$

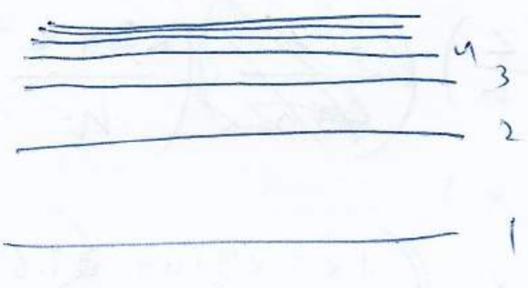
$$E_3 = -13.6/9 \text{ eV}$$

$$E_2 - E_1 = 13.6\left(1-\tfrac{1}{4}\right) = \tfrac{3}{4} \times 13.6 \text{ eV}$$

$$E_3 - E_2 = 13.6\left(\tfrac{1}{4}-\tfrac{1}{9}\right) = \tfrac{5}{36} \times 13.6 \text{ eV} \leftarrow$$

$$E_4 - E_3 = 13.6\left(\tfrac{1}{9}-\tfrac{1}{16}\right) = \tfrac{7}{144} \times 13.6 \text{ eV} \leftarrow$$

$$E_2 - E_1 > E_3 - E_2 > E_4 - E_3 > \ldots$$



Energy order

$n = \infty$

N

$$E_\infty = \frac{-13.6 \times Z}{(\infty)^2} \sim 0$$

$\rightarrow$ as $n \uparrow$ $E_n \downarrow$

$$\boxed{E_n = -13.6 \times \frac{Z^2}{n^2} \text{ eV/atom}}$$

also $n = 1, 2, 3 \ldots$

$\Rightarrow$ Energy is quantised

Velocity of e^-

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2} \quad -(i)$$

$k = 9 \times 10^9 \; Nm^2/C^2$
$\rightarrow$ SI

$$mvr = \frac{nh}{2\pi} \quad -(ii)$$

$$\boxed{\begin{array}{l}\text{In CGS} \\ k = 1\end{array}}$$

(i) $$\frac{mv^2}{\cancel{r}} = \frac{kZe^2}{r^{\cancel{2}}}$$

$$v(mvr) = kZe^2$$

$$v\left(\frac{nh}{2\pi}\right) = kZe^2$$

$$V_n = \left(\frac{Z}{n}\right) \cancel{\left(\frac{kZe^2}{2\pi h}\right)} \left(\frac{k 2\pi e^2}{h}\right)$$

in CGS $k = 1$

$$V_n = \frac{Z}{n} \times \left(\frac{1 \times 2 \times 3.14 \times (1.6 \times 10^{-19})^2}{6.62 \times 10^{-27} \text{ erg s}}\right)$$

$$\boxed{V_n = \frac{Z}{n} \times 2.188 \times 10^8 \text{ cm/s}}$$

Orbital frequency (ν_0)

No. of revolutions per second by an e^- in an orbit is called orbital frequency.

$$V \rightarrow v\ m/s$$

$$1\ rev = 2\pi r$$

$$2\pi r \longrightarrow 1\ rev$$

In 1s $\quad v\ m \longrightarrow \frac{v}{2\pi r}\ rev/s$

$$= \frac{\left(\frac{z}{n}\right)\left(\frac{2\pi e^2}{h}\right)}{2\pi r} \swarrow v$$

$$= \frac{ze^2}{nhr_n}$$

$$r_n = 0.529 \times \frac{n^2}{z}\ \mathring{A}$$

$$\nu_0 = \frac{ze^2}{nh \times 0.529 \times \frac{n^2}{z} \times 10^{-10}}\ s^{-1}$$

$$= \frac{z^2}{n^3}\left(\frac{e^2}{h \times 0.529 \times 10^{-10}}\right)$$

$$\boxed{\nu_0 = \frac{z^2}{n^3} \times 6.66 \times 10^{15}\ s^{-1}} \rightarrow \text{H \& H like atoms}$$

Time period

$$T = \frac{1}{\nu_0} \quad \text{or} \quad \nu_0 = \frac{1}{T}$$

$$\boxed{T = \frac{n^3}{Z^2} \times 1.5 \times 10^{-16}\ s}$$

time taken for one revolution.

$$r_n = 0.529 \times \frac{n^2}{Z}\ \text{Å}$$

$$E_n = -13.6 \times \frac{Z^2}{n^2}\ \text{ev/atom}$$

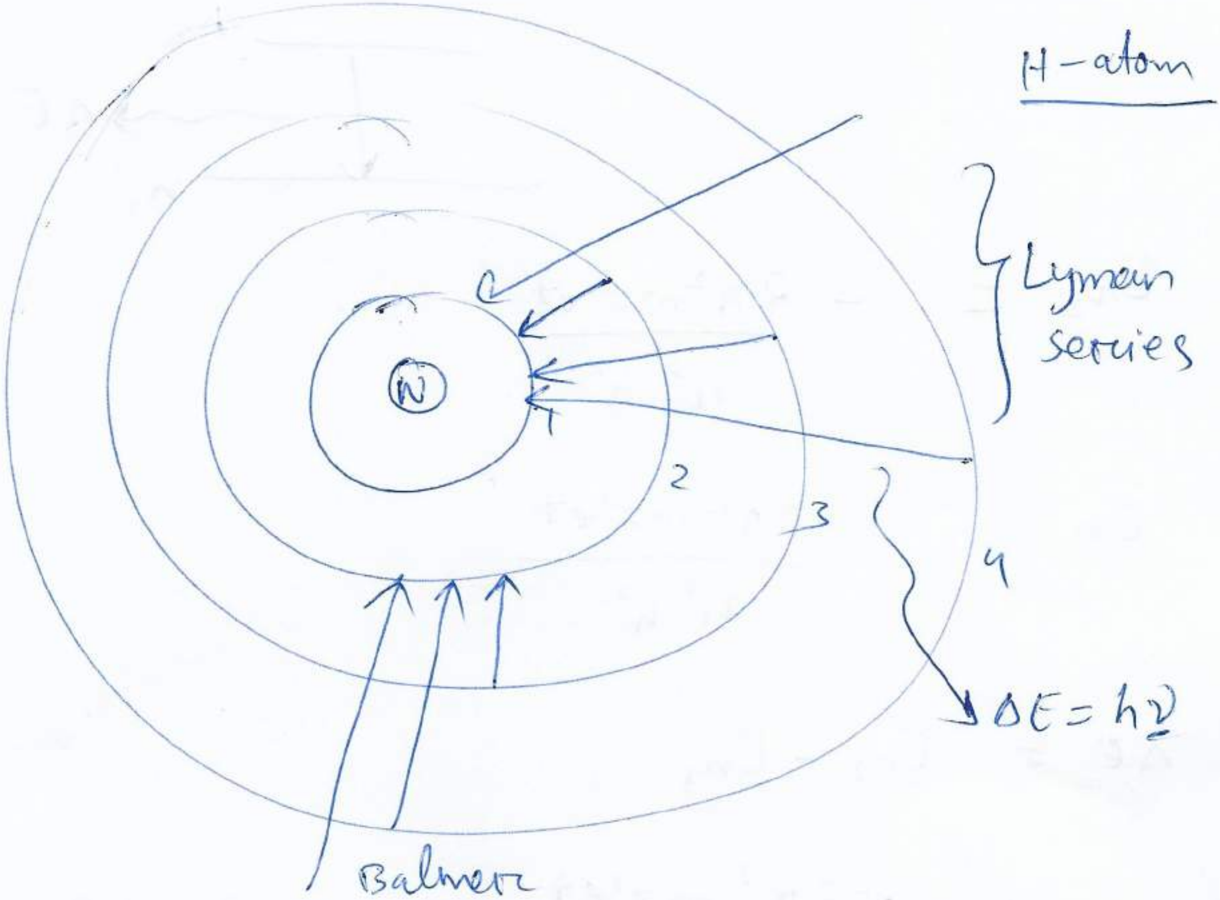
Remember

$$V_n = \frac{Z}{n} \times \underbrace{2.188 \times 10^8}_{V_1}\ \text{cm/s} = \frac{Z}{n} \times V_1$$

$$\nu_0 = \frac{Z^2}{n^3} \times \underbrace{6.66 \times 10^{15}}_{\nu_{01}}\ s^{-1} = \frac{Z^2}{n^3}\nu_{01}$$

$$T = \frac{n^3}{Z^2} \times \underbrace{1.5 \times 10^{-16}}_{T_1}\ s = \frac{n^3}{Z^2} T_1$$

Interpretation of Hydrogen spectrum



Series	Starting from n_2	to n_1
Lyman	$n = 2, 3, 4 \ldots \infty$	1
Balmer	$n = 3, 4, 5 \ldots \infty$	2
Paschen	$n = 4, 5, 6 \ldots \infty$	3
Brackett	$n = 5, 6, 7 \ldots \infty$	4
Pfund	$n = 6, 7, 8 \ldots \infty$	5

Silent

ΔE

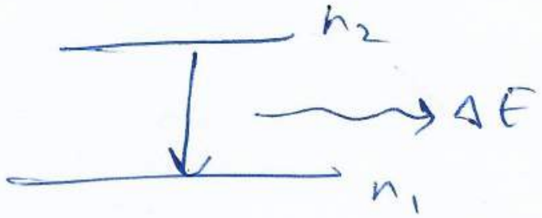
E_2

E_1

$$\Delta E = h\nu = \frac{hc}{\lambda}$$

→ line spectrum of H

Rydberg equation



$$E_{n_2} = -\frac{2\pi^2 m z^2 e^4}{n_2^2 h^2}$$

$$E_{n_1} = \frac{-2\pi^2 m z^2 e^4}{n_1^2 h^2}$$

$$\Delta E = E_{n_2} - E_{n_1}$$

$$= \frac{-2\pi^2 m z^2 e^4}{n_2^2 h^2} - -\frac{2\pi^2 m z^2 e^4}{n_1^2 h^2}$$

$$\frac{hc}{\lambda} = \frac{2\pi^2 m z^2 e^4}{h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda} = z^2 \underbrace{\left(\frac{2\pi^2 m e^4}{h^3 c}\right)}_{\text{Rydberg constant} = 109743\ cm^{-1}} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \bar{\nu} =$$

$$\boxed{\bar{\nu} = R z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \rightarrow \text{H \& H like atoms}$$

First line of a series

first line

Lymann $2 \to 1$

2nd line = $3 \to 1$

⋮

last line = $\infty \to 1$
(series limit)

Balmer series first line $3 \to 2$

⋮

For H atom

series limit of Lyman series

$$\boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

n_2 ∞
↓
n_1 1

$$\bar{\nu} = \frac{1}{\lambda} = R \times 1^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$\bar{\nu} = R$$

Ionization energy

for H atom ($Z=1$)

$\Delta E = ? = IE$

$$E_n = -13.6 \times \frac{Z^2}{n^2}\ eV$$

$$E_\infty = 0$$

$$E_1 = -13.6\ eV$$

$$\Delta E = E_\infty - E_1 = 0-(-13.6)$$

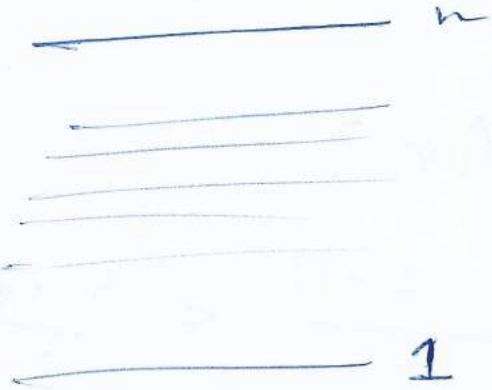
$$= \underline{13.6\ eV}\ (\text{for } 1^{st} \text{ orbit})$$

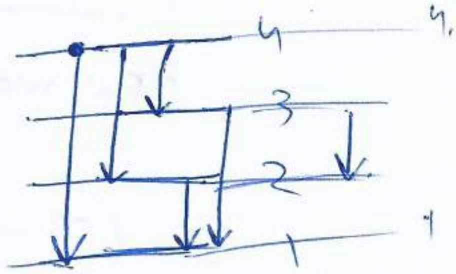
He^+ (1^{st} orbit)

$$E_1 = -13.6 \times \frac{2^2}{1^2}\ eV$$

$$IE = 13.6 \times 4\ eV$$

I.E. → the energy required to take an e^- from an orbit to ∞ (when atom is isolated & ~~ga~~ is in gaseous phase)





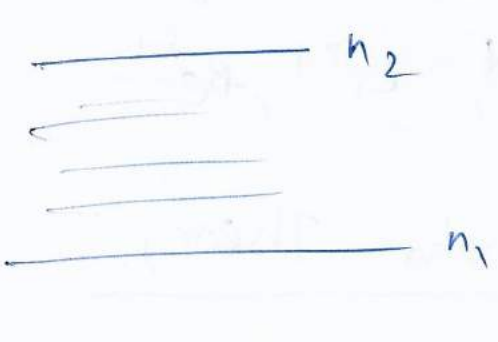
$$\underset{4-1}{3+2+1}$$

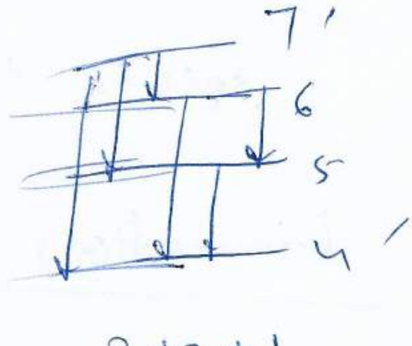
$(n-1) + ---- + 3+2+1$

$$1+2+3+ --- n = \frac{n(n+1)}{2}$$

$$\frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2}$$

total No. of lines produced when e^- comes from $n \rightarrow 1$





$$\underset{(7-4)}{3+2+1}$$

~~(n_2-n_1)~~

$(n_2-n_1) + --- + 3+2+1$

$$\frac{(n_2-n_1)(n_2-n_1+1)}{2}$$

total No. of lines $n_2 \rightarrow n_1$

$\underline{7 \rightarrow 2}$

How many line ?

$$\frac{(7-2)(7-2+1)}{2} = \frac{5 \times 6}{2}$$

$= 15$ <u>lines</u>

$6 \rightarrow 1$

$\frac{6 \times 5}{2} = \underline{15}$

Significance of Bohr Theory

→ Can explain H – spectrum

→ R (Rydberg constant) matches with experimental value

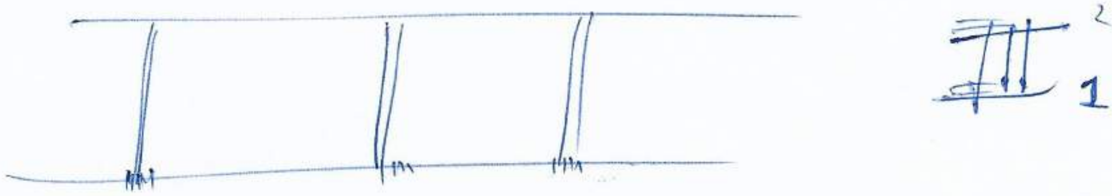
→ can explain spectrum of H like species e.g He^{+}, Li^{2+}, Be^{3+}

Limitations of Bohr Theory

→ does not explain spectra of multielectron atoms.

→ does not explain the fine spectrum of hydrogen.

high resolution spectroscope



→ Does not explain Zeeman effect

& Stark effect

↘ Electric field

~~mag~~ magnetic field

(Zeeman effect)

→ No justification why $mvr = \frac{nh}{2\pi}$

$$\frac{h}{2\pi} = \hbar \text{ (h bar)}$$

$$mvr = n\hbar$$