SEQUENCES & SERIES TUTORIAL

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Pg 31

Comp 1

$$\sum_{x=1}^{n} f(a+x) = \underline{2^p(2^k-1)} \qquad f(1)=2$$

$$f(a+1) + f(a+2) + f(a+3) + \cdots f(a+n)$$

$$f(a)\{2 + 2^2 + 2^3 + \cdots\cdots 2^n\}$$

$$= f(a) \times 2\frac{(2^n-1)}{2-1} = f(a)\cdot 2(2^n-1)$$

$$f(x+y) = f(x)\cdot f(y) \qquad = 2^a \cdot 2(2^n-1)$$

$$f(a+x) = f(a)\cdot f(x) \qquad = \underline{2^{a+1}(2^n-1)}$$

$$f(a+1) = f(a)f(1) \qquad f(1+1) = f(1)f(1)$$

$$= 2f(a) \qquad = 2^2$$

$$f(a+2) = f(a)f(2) \qquad f(2+1) = f(2)f(1)$$

$$= 2^2 f(a) \qquad = 2^2\cdot 2$$

$$= 2^3$$

$$f(a+3) = f(a)f(3)$$

$$= 2^3 f(a)$$

$$f(a+n) = 2^n f(a)$$

① $k = n$.

② $p = a+1$ $\qquad a+1=3$, $\underline{a=2}$

③ $2^{a+1}(2^n-1) = 120 = 2^3(2^4-1)$

(20) $a>0$, $b>0$, $c>0$

p.t

i) $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$

a, b, c

$$\frac{a+b+c}{3} \geq (abc)^{1/3}$$

$$(a+b+c) \geq 3(abc)^{1/3}. \quad \text{—— (1)}$$

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$.

$$\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3} \geq \left(\frac{1}{a}\frac{1}{b}\frac{1}{c}\right)^{1/3}$$

$$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 3\left(\frac{1}{abc}\right)^{1/3}. \quad \text{—— (2)}$$

Multiply (1) & (2)

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9 \times 1$$

$$\frac{b+c}{a} + \frac{a+c}{b} + \frac{b+a}{c} \geq 6$$

$$\left(\frac{b}{a} + \frac{a}{b}\right) + \left(\frac{c}{a} + \frac{a}{c}\right) + \left(\frac{c}{b} + \frac{b}{c}\right)$$

$$\left\{\begin{aligned}
\frac{\frac{b}{a} + \frac{a}{b}}{2} &\geq \left(\frac{a}{b} \times \frac{b}{a}\right)^{1/2} \\
\frac{\frac{c}{a} + \frac{a}{c}}{2} &\geq \left(\frac{a}{c} \times \frac{c}{a}\right)^{1/2} \\
\frac{\frac{c}{b} + \frac{b}{c}}{2} &\geq \left(\frac{c}{b} \times \frac{b}{c}\right)^{1/2}
\end{aligned}\right.$$

$$\frac{b+c}{a} + \frac{a+c}{b} + \frac{b+a}{c} \geq 6$$

ii) $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$

$\geq \frac{1}{2}$ $\geq \frac{1}{2}$ $\geq \frac{1}{2}$

$\frac{a}{b+c} \geq \frac{1}{2}$.

$a > b \geq c$

$2a \geq b+c$

$a \geq \frac{b+c}{2}$.

$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} + 1 + 1 + 1 \geq \frac{3}{2} + 3$

$\frac{a+b+c}{b+c} + \frac{a+b+c}{a+c} + \frac{a+b+c}{a+b} \geq \frac{9}{2}$

$(a+b+c)\left[\frac{1}{b+c} + \frac{1}{a+c} + \frac{1}{a+b}\right] \geq \frac{9}{2}$

$(a+b+c)\left[\frac{(a+b+c)^2 + (ab+bc+ca)}{(a+b)(b+c)(c+a)}\right] \geq \frac{9}{2}$.

T.P

$$(a+b+c)^3 \geq \frac{9}{2}(a+b)(b+c)(c+a)$$
$$+ (a+b+c)(ab+bc+ca)$$

let $(a+b)$, $(b+c)$, $(c+a)$ be 3 number.

$$\frac{(a+b)+(b+c)+(c+a)}{3} \geq \{(a+b)(b+c)(c+a)\}^{\frac{1}{3}}$$

$$\frac{2}{3}(a+b+c) \geq \{(a+b)(b+c)(c+a)\}^{1/3}$$

$$\frac{8}{27}(a+b+c)^3 \geq (a+b)(b+c)(c+a)$$

$$(a+b+c)^3 \geq \frac{27}{8}(a+b)(b+c)(c+a)$$

$\cancel{a+b+}$ $(a+b+c)(ab+bc+ca)$

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$a+2b+3c=1$ $\quad a>0, b>0, c>0$

a^3b^2c

$$\frac{a+b+b+c+c+c}{6} \geq (ab^2c^3)^{1/6}$$

$\cancel{\frac{1}{6} \geq a}$

$$a + 2b + 3c = 1$$

$$\frac{\frac{a}{3} + \frac{a}{3} + \frac{a}{3} + b + b + 3c}{6} \geq \left(\frac{a^3}{27} b^2 3c\right)^{1/6}$$

$$\left(\frac{a^3b^2c}{9}\right)^{1/6} \leq \frac{1}{6}$$

$$a^3b^2c \leq \frac{9}{6^6}$$

(22) $a + b + c = 1$ $\quad ab^2c^3$

$$\frac{a + \frac{b}{2} + \frac{b}{2} + \frac{c}{3} + \frac{c}{3} + \frac{c}{3}}{6} \geq \left(a \frac{b^2}{4} \frac{c^3}{27}\right)^{1/6}$$

$$\frac{ab^2c^3}{4 \times 27} \leq \frac{1}{6^6}$$

$$ab^2c^3 \leq \frac{\cancel{4} \times \cancel{27}^{\cancel{9}\,3}}{\cancel{6} \times \cancel{6} \times 6^4} = \frac{\cancel{3}}{2\,\cancel{6} \times 6^3} = \frac{1}{432}$$

(25)

$$S = 1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + \cdots n(n+1)(n+2)$$

$$-4S = 1\cdot2\cdot3\cdot(0-4) + 2\cdot3\cdot4(1-5) + \cdots n(n+1)(n+2)\{(n-1)-(n+3)\}$$

$$-4S = 0\cdot1\cdot2\cdot3 - \cancel{1\cdot2\cdot3\cdot4} + \cancel{1\cdot2\cdot3\cdot4} - \cancel{2\cdot3\cdot4\cdot5} + \cdots \cancel{(n-1)n(n+1)(n+2)} - n(n+1)(n+2)(n+3)$$

$$-4S = 0 - n(n+1)(n+2)(n+3)$$

$$S = \frac{n(n+1)(n+2)(n+3)}{4}$$

$$T_n = n(n+1)(n+2)$$

$$T_n = n^3 + 3n^2 + 2n$$

$$S_n = \sum_1 T_n = \sum_1 n^3 + 3\sum_1 n^2 + 2\sum_1 n$$

$$= \left\{\frac{n(n+1)}{2}\right\}^2 + 3\left\{\frac{n(n+1)(2n+1)}{6}\right\} + \frac{2n(n+1)}{2}$$

$$\downarrow$$

$$\frac{n(n+1)(n+2)(n+3)}{4}$$

ii) $T_n = (2n-1)(2n+1)^2$

$= (2n-1)(4n^2+4n+1)$

$T_n = 8n^3+4n^2-2n-1$

$S = \sum_1 T_n = 8\sum_1 n^3 + 4\sum_1 n^2 - 2\sum_1 n - n$

$= \cancel{8}^{2} \dfrac{n^2(n+1)^2}{\cancel{4}\,1} + \cancel{4}^{2} \dfrac{n(n+1)(2n+1)}{\cancel{6}\,3} - \cancel{2} \dfrac{n(n+1)}{\cancel{2}}$

$n(n+1)\left\{2n^2+2n+\frac{4n}{3}+\frac{2}{3}-1\right\} - n$

$\frac{n(n+1)}{3}\left\{6n^2+10n-1\right\} - n$

$\frac{n}{3}\left\{6n^3+6n^2+10n^2+10n-n-1-3\right\}$

$\frac{n}{3}\left\{6n^3+16n^2+9n-4\right\}$

iii) $1 + 9 + 24 + 46 + 75$

8 15 22 29

7 7 7

$T_n = a(n-1)(n-2) + b(n-1) + c$

$T_1 = c = 1$

$T_2 = b+c = 9 \Rightarrow b = 8$ $T_3 = 24 = 2a+2b+c$

$a = 7/2$

$$T_n = \frac{7}{2}(n-1)(n-2) + 8(n-1) + 1$$

iv) $2 + 5 + 14 + 41 + \cdots$

$3 \quad 9 \quad 27 \quad \cdots$

$$T_n = a + b \cdot 3^n$$

$T_1 = 2$ $\qquad$ $T_2 = 5$.

$a + b \cdot 3^1 = 2$ $\qquad$ $a + b \cdot 3^2 = 5$.

$a + 3b = 2$ $\qquad$ $a + 9b = 5$.

$a = \frac{1}{2}$. $\qquad$ $b = \frac{1}{2}$.

$$T_n = \frac{1}{2} + \frac{3^n}{2} = \frac{1+3^n}{2}$$

$$\Sigma_1 T_n = \Sigma_1 \frac{1}{2} + \frac{1}{2}\Sigma_1 3^n.$$

$$= \frac{n}{2} + \frac{1}{2} \times 3\frac{(3^n - 1)}{3-1}$$

$$= \frac{n}{2} + \frac{3}{4}(3^n - 1)$$

v) $1+\left(1+\frac{1}{2}+\frac{1}{2^2}\right)+\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}\right)$

$$T_n=\left\{1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\cdots\cdots\frac{1}{2^{2n-2}}\right\}$$

$$T_n=1\left(\frac{1-\left(\frac{1}{2}\right)^{2n-1}}{1-\frac{1}{2}}\right)$$

$$=2\cdot\left(1-\left(\frac{1}{2}\right)^{2n-1}\right)$$

$$T_n=2-\left(\frac{1}{2}\right)^{2n-2}$$

$$\sum_1 T_n=\sum_1 2-\sum_1\left(\frac{1}{2}\right)^{2n-2}$$

$$=2n-\frac{1}{4}\left\{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^4+\left(\frac{1}{2}\right)^6+\cdots\left(\frac{1}{2}\right)^{2n}\right\}$$

$$=2n-\frac{1}{4}\left\{\left(\frac{1}{2}\right)^2\frac{\left\{1-\left(\frac{1}{2}\right)^{2n}\right\}}{1-\left(\frac{1}{2}\right)^2}\right\}$$

$$=2n-\frac{1}{12}\left(1-\frac{1}{2^{2n}}\right)$$

10 i) $\left(\frac{2}{3}+\frac{2}{3^3}+\frac{2}{3^5}+\cdots\right)+\left(\frac{3}{3^2}+\frac{3}{3^4}+\frac{3}{3^6}+\cdots\right)$

$\frac{2}{3}\left(1+\frac{1}{3^2}+\frac{1}{3^4}+\cdots\right)+\frac{3}{3^2}\left(1+\frac{1}{3^2}+\frac{1}{3^4}+\cdots\right)$

$1\left(1+\frac{1}{3^2}+\frac{1}{3^4}+\cdots\right)$

$$\frac{1}{1-\frac{1}{3^2}}=\frac{1}{1-\frac{1}{9}}=\frac{9}{8}$$

11 $(666\cdots n \text{ digits})^2+(888\cdots n \text{ digits})$
$=(444\cdots 2n \text{ digits})$

$\cancel{66}$ $\frac{6666\cdots n\times 9}{6}$

$9999\cdots n$ di

$\frac{36}{81}(999\cdots n \text{ digs})^2+\frac{8}{9}(999\cdots n \text{ digits})$

$\frac{36}{81}(10^n-1)^2+\frac{8}{9}(10^n-1)$

$\frac{36}{81}(10^{2n}-2\cdot10^n+1)+\frac{72}{81}(10^n-1)$

$\frac{36\cdot10^{2n}+36-72}{81}=\frac{36\cdot10^{2n}-36}{81}=\frac{\cancel{36}^{4}}{\cancel{81}_{9}}(10^{2n}-$

(13) $S_1 = \dfrac{1}{1-\frac{1}{2}}$ $\qquad S_2 = \dfrac{2}{1-\frac{1}{3}}$ $\qquad S_3 = \dfrac{3}{1-\frac{1}{4}}$

$$S_k = \frac{k}{1-\frac{1}{k+1}} = \frac{\cancel{k}(k+1)}{\cancel{k}}$$

$$S = \sum_{n=1}^{k} S_k = \sum_{1} k+1 = \frac{n(n+1)}{2} + n$$

$$n\left(\frac{n+1}{2}+1\right)$$

$$= n\left(\frac{n+3}{2}\right)$$

$$= \frac{k(k+3)}{2}$$

(9) $T_r = \dfrac{1}{r(r+1)} = \dfrac{1}{r} - \dfrac{1}{r+1}$

$$S_n = \sum_1 T_r = T_1 + T_2 + T_3 + \cdots T_n$$

$$= \left(\frac{1}{1} - \cancel{\frac{1}{1+1}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{1+2}}\right) + \left(\cancel{1}\quad\right)$$

$$+ \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1}$$

$$= \frac{n}{n+1}$$

10) $S = \sum_1 \dfrac{1}{(2r-1)(2r+1)}$

$$2T_r = \frac{2}{(2r-1)(2r+1)}$$

$$2T_r = \frac{1}{(2r-1)} - \frac{1}{(2r+1)}$$

$$2\sum_1 T_r = \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \cdots \cancel{\left(\frac{1}{2n-1}\right.} - \left.\frac{1}{2n+1}\right)$$

$$2S = 1 - \frac{1}{2n+1}$$

$$S = \frac{n}{2n+1} = \frac{1}{2+\frac{1}{n}} \qquad S_\infty = \frac{1}{2}$$

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$$T_r = \frac{1}{r(r+1)(r+2)}$$

$$2T_r = \frac{(r+2) - r}{r(r+1)(r+2}$$

$$2T_r = \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$$

$$2S_n = \left(\frac{1}{1\cdot 2} - \cancel{\frac{1}{2\cdot 3}}\right) + \left(\cancel{\frac{1}{2\cdot 3}} - \cancel{\frac{1}{3\cdot 4}}\right) + \cancel{\cdots} \left(\cancel{\frac{1}{n(n+1)}} - \frac{1}{(n+1)(n+2)}\right)$$

$$2S_n = \frac{1}{1\cdot 2} - \frac{1}{(n+1)(n+2)}$$

$$S_n = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

$$S_\infty = \frac{1}{4}$$

(12) $\sum_{r=1}^{\infty} \left(\frac{r}{1+r^2+r^4} \right)$

$$T_r = \frac{r}{1+r^2+r^4} = \frac{r}{r^4+2r^2+1-r^2} = \frac{r}{(r^2+1)^2-r^2}$$

$$T_r = \frac{r}{(r^2+r+1)(r^2-r+1)}$$

$$2T_r = \frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}$$

$$2\sum_1 T_r = \left(\frac{1}{1} - \cancel{\frac{1}{3}} \right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{7}} \right) + \cdots + \left(\cancel{\frac{1}{n^2-n+1}} - \frac{1}{n^2+n+1} \right)$$

$$2S_n = 1 - \frac{1}{n^2+n+1}$$

$$2S_n = \frac{n^2+n}{n^2+n+1}$$

$$S_n = \frac{1}{2} \frac{(n^2+n)}{(n^2+n+1)}$$

$$S_\infty = \frac{1}{2} \left(\frac{1+\frac{1}{n}}{1+\frac{1}{n}+\frac{1}{n^2}} \right)$$

$$\underline{S_\infty = \frac{1}{2}}$$