PROJECTILE MOTION TUTORIAL

Pg 100 — 4, 5, 7

Pg 101 — 9, 10, 12, 14

Pg 102 — 19, 20

Pg 103 — Comp 1

Pg 104 — Comp 2

Pg 106 — 40, 43

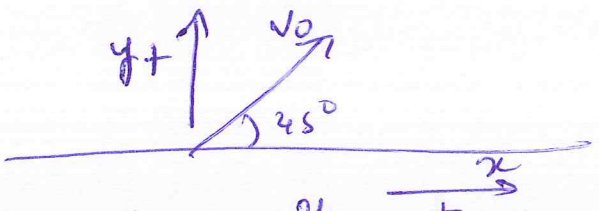
(4)

$h = Ax - Bx^2$

$A : B$

$v_0 = 20$ m/s

$\theta = 45^\circ$



x	y
$u_x = v_0 \cos 45 = \frac{20}{\sqrt{2}}$	$u_y = v_0 \sin 45 = \frac{20}{\sqrt{2}}$
$a_x = 0$	$a_y = -g$
$x = u_x t$	$h = u_y t + \frac{1}{2} a_y t^2$
$x = \frac{20}{\sqrt{2}} t$	$h = \frac{20}{\sqrt{2}} t - \frac{1}{2} g t^2$
$t = \frac{\sqrt{2}x}{20}$	

$$h = x - \frac{1}{2} g \left(\frac{\sqrt{2}x}{20}\right)^2$$

$$= x - \frac{g x^2}{400}$$

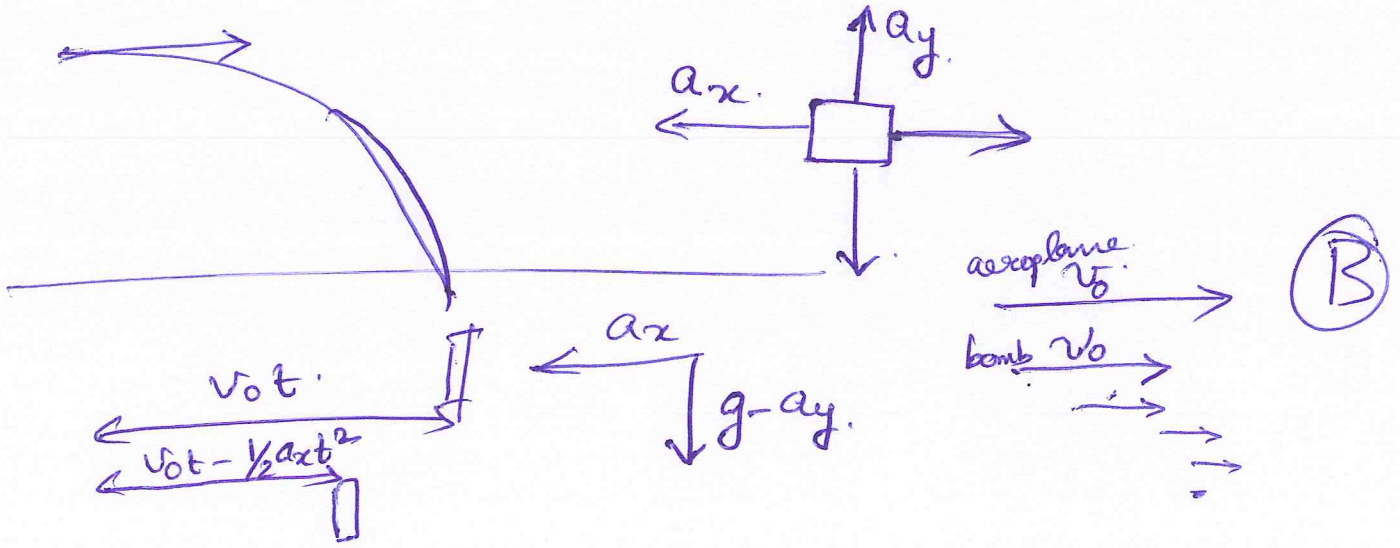
$$h = x - \frac{x^2}{40}$$

$A = 1 \qquad B = \frac{1}{40}$

$$\frac{A}{B} = \frac{1}{1/40} = 40$$

(D)

(5)



(B)

(7)

$$H = \frac{v_0^2 \sin^2\theta}{2g}$$

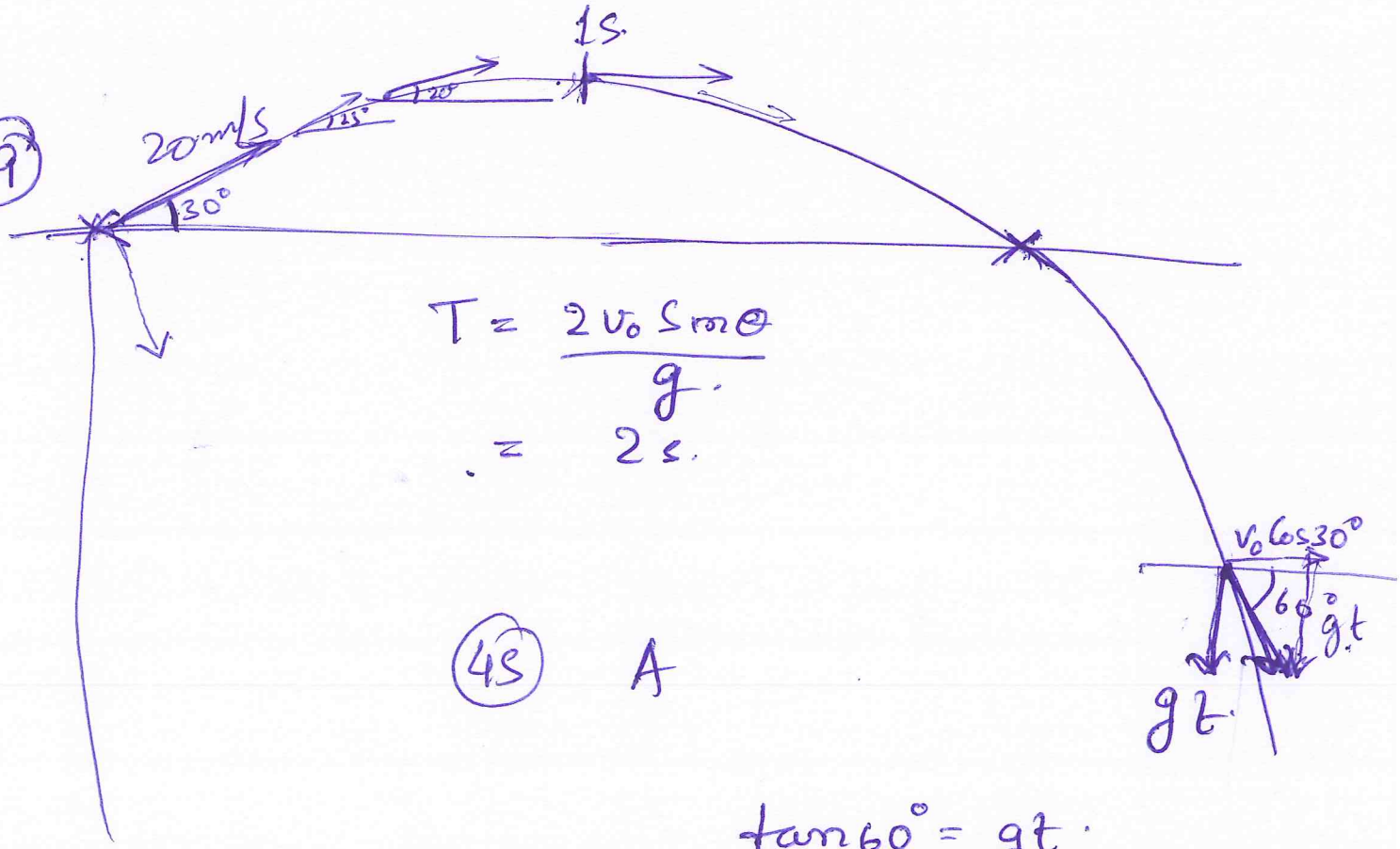
$$H_1 = H_2$$

$$\frac{v_1^2 \sin^2 45^\circ}{2g} = \frac{v_2^2 \sin^2 60^\circ}{2g}$$

$$\frac{v_1^2}{v_2^2} = \frac{\sin^2 60}{\sin^2 45} \Rightarrow \frac{v_1}{v_2} = \frac{\sin 60}{\sin 45} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$$

$\sqrt{\frac{3}{2}}$ (C)

(9)



$$T = \frac{2v_0 \sin\theta}{g}$$

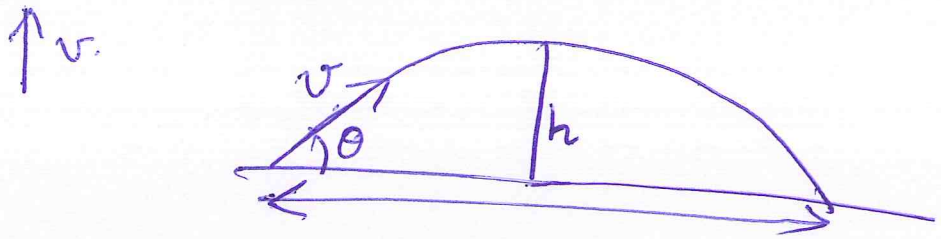
$$= 2s.$$

(45) A

$$\tan 60^\circ = \frac{gt}{v_0 \cos 30^\circ}$$

$$t = \frac{v_0 \cos 30^\circ \tan 60^\circ}{10} =$$

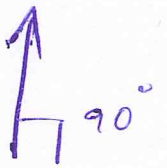
(10)



$$R = \frac{u^2 \sin 2\theta}{g}$$

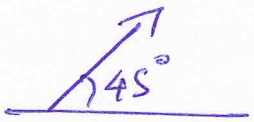
$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$\theta = 90^\circ$



$$h = \frac{u^2}{2g}$$

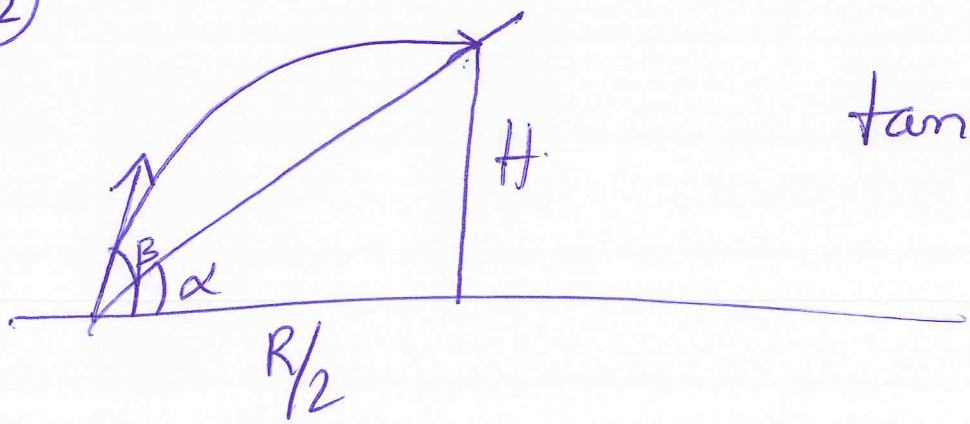
R_{MAX} $\quad \theta = 45^\circ$



$$R = \frac{u^2}{g} \times \sin 90^\circ$$

$$= 2h$$

(12)



$$\tan\alpha = \frac{H}{R/2}$$

$$= \frac{\frac{u^2 \sin^2\beta}{2g}}{\frac{u^2 \sin 2\beta}{2g}}$$

$$= \frac{\tan\beta}{2}$$

(A)

(14) $v_0 \longrightarrow 2\%$

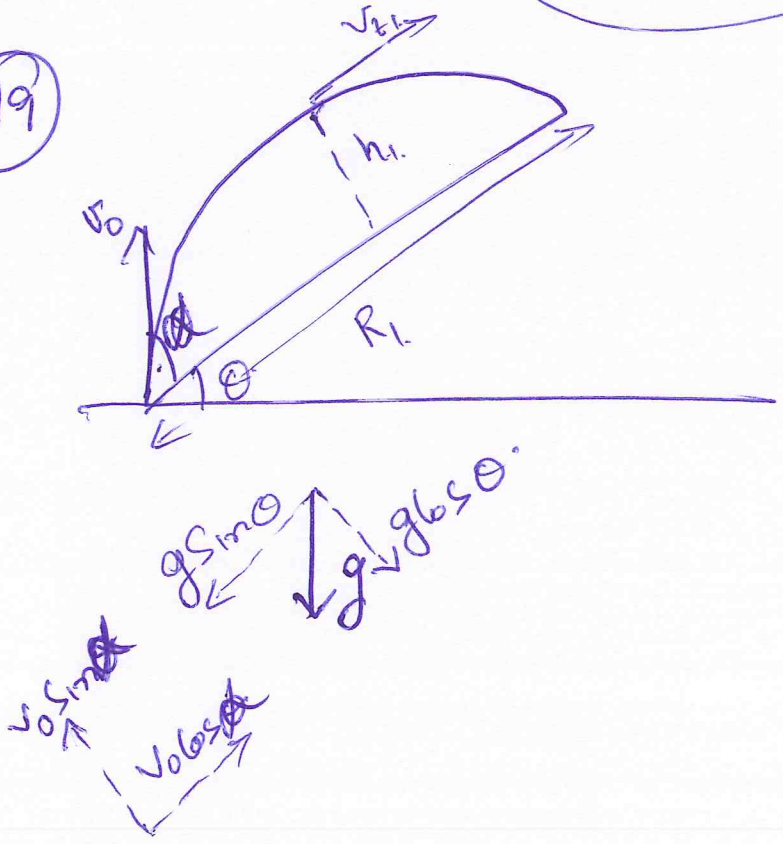
$$T = \frac{2v_0 \sin\theta}{g}$$

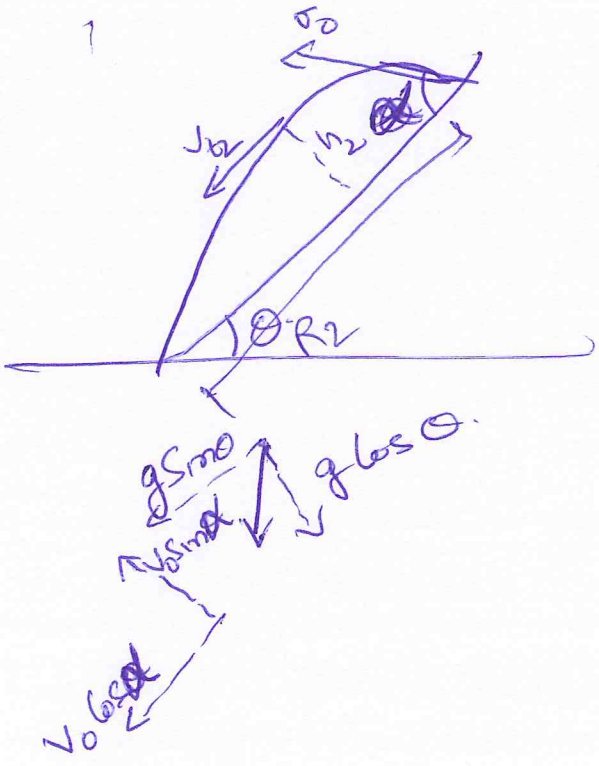
$$dT = \frac{2\,dv_0 \sin\theta}{g}$$

$\frac{dT}{T} \times 100\%$ $\quad$ $\frac{dv_0}{v_0} \times 100\%$

2%

(19)





$v = u + at$

$0 = v_0 \sin\alpha - g\cos\theta\, t$

$t_1 = \frac{v_0 \sin\alpha}{g\cos\theta}$

$t_2 = \frac{v_0 \sin\alpha}{g\cos\theta}$

Block

$v^2 - u^2 = 2as$.

$0^2 - (v_0 \sin\alpha)^2 = 2(-g\cos\theta)\, h_1$

$h_1 = \dfrac{v_0^2 \sin^2\alpha}{2g\cos\theta}$

$h_2 = \dfrac{v_0^2 \sin^2\alpha}{2g\cos\theta}$

(D) $v_{t_2} > v_{t_1}$

X

$h = 0 \times t + \frac{1}{2} g \cos\theta\, t^2$.

$\dfrac{v_0^2 \sin^2\alpha}{2g\cos\theta} = \frac{1}{2} g \cos\theta\, t^2$

$t = \dfrac{v_0 \sin\alpha}{g\cos\theta}$.

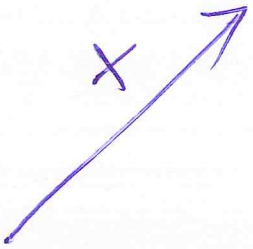
$T_1 = T_2 = \dfrac{2v_0 \sin\alpha}{g\cos\theta}$ (B) ✓

$x = R_1$

$a = -g\sin\theta$

T

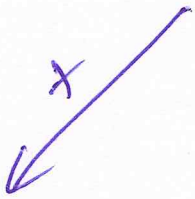
$v_0\cos\theta$



$$R_1 = v_0\cos\theta\, T - \frac{1}{2}g\sin\theta\, T^2$$

$x = R_2$

$a = g\sin\theta$



$v_0\cos\theta$

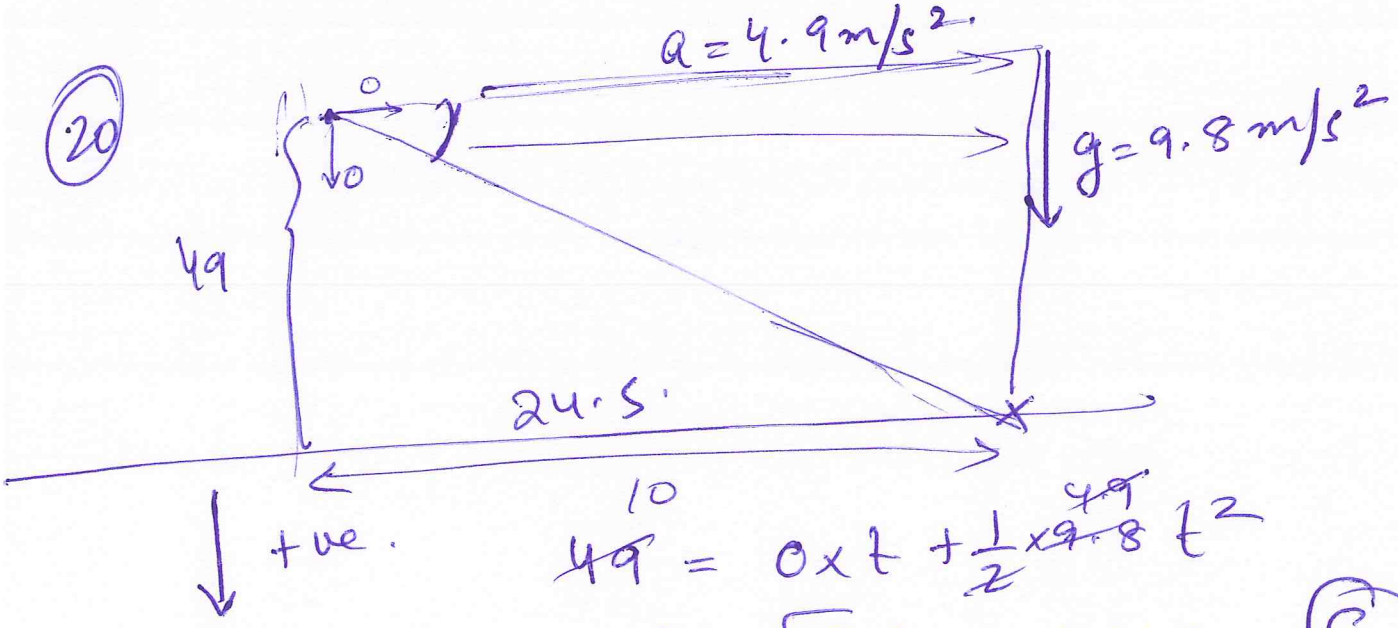
T.

$$R_2 = v_0\cos\theta\, T + \frac{1}{2}g\sin\theta\, T^2$$

(C) ✓

$$R_2 - R_1 = g\sin\theta\, T^2$$

(20)



$$49 = 0 \times t + \frac{1}{2} \times 9.8\, t^2$$

$t = \sqrt{10}$ s. $= 3.16$ s

(C)

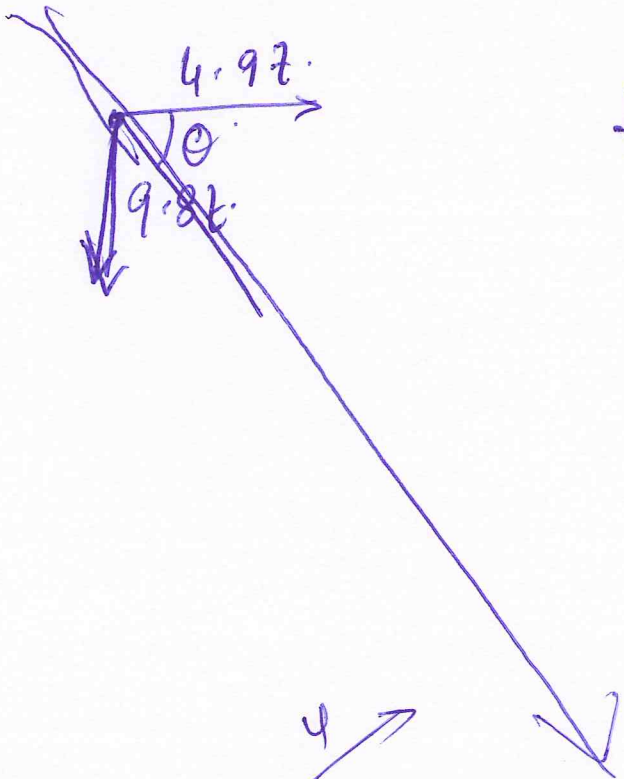
$$x = 0 \times t + \frac{1}{2} \times 4.9 \times 10^5.$$

$$x = 24.5\ m.$$

$$\tan\theta = \frac{49}{24.5} = 2.$$

ACD

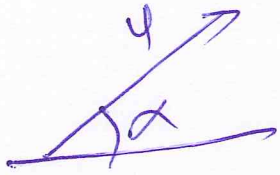
$\theta > 45^\circ$ (D)

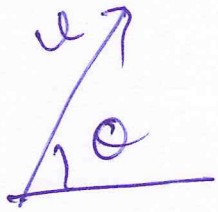


$$\tan\theta = \frac{9.8t}{4.9t}$$

$$\tan\theta = 2$$

Comp 1





$$R = \frac{u^2 \sin 2\alpha}{g} = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\alpha = \sin 2\theta$$

$$2\theta = n\pi + (-1)^n 2\alpha$$

$$\theta = \frac{n\pi}{2} + (-1)^n \alpha. \qquad \theta = \frac{\pi}{2} - \alpha.$$

$$h_1 = \frac{u^2 \sin^2\alpha}{2g}$$

$$h_2 = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \cos^2\alpha}{2g}$$

$$h_1 h_2 = \frac{(u^2)^2 \sin^2\alpha \cos^2\alpha}{2^2 g^2} \times \frac{2^2}{2^2}$$

$$h_1 h_2 = \frac{(u^2)^2 \sin^2 2\alpha}{(2^2)^2 g^2}$$

$$\sqrt{h_1 h_2} = \frac{u^2 \sin 2\alpha}{4g} = \frac{R}{4}$$

$$R = 4\sqrt{h_1 h_2} \qquad (A)$$

(25) $$t_1 = \frac{2u\sin\alpha}{g} \qquad t_2 = \frac{2u\sin\theta}{g} = \frac{2u\sin(90-\alpha)}{g} = \frac{2u\cos\alpha}{g}$$

$$\frac{t_1}{t_2} = \frac{2u\sin\alpha/g}{2u\cos\alpha/g} = \tan\alpha. \qquad (C)$$

(26) $$h_1/h_2 = \frac{u^2\sin^2\alpha/2g}{u^2\cos^2\alpha/2g} = \tan^2\alpha. \qquad (C)$$

(27) $$h_1 + h_2 = u^2\sin^2\alpha/2g + u^2\cos^2\alpha/2g = u^2(\sin^2\alpha+\cos^2\alpha)/2g = \frac{u^2}{2g} \qquad (D)$$

$$\vec{r} = at\,\hat{i} + (bt - ct^2)\,\hat{j}$$

$x = at$. $\qquad x = u\cos\theta\, t$

$y = bt - ct^2$ $\qquad y = u\sin\theta\, t - \frac{1}{2}gt^2$

$$\begin{cases} u\cos\theta = a & \longrightarrow (1) \\ u\sin\theta = b & \longrightarrow (2) \end{cases}$$

$$c = g/2 \qquad \longrightarrow (3)$$

$(1)^2 + (2)^2$

$$u^2\cos^2\theta + u^2\sin^2\theta = a^2 + b^2$$

$$u^2(1) = a^2 + b^2$$

$$u = \sqrt{a^2 + b^2} \qquad (C)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{u\sin\theta}{u\cos\theta} = \frac{b}{a}$$

$$\tan\theta = \frac{b}{a} \qquad (C)$$

$$T = \frac{2u\sin\theta}{g} = \frac{2b}{2c} = \frac{b}{c} \qquad (A)$$

$g = 2c$ (C)

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{(u\sin\theta)^2}{2g} = \frac{b^2}{4c}$$

(C)

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{u^2\, 2 \sin\theta \cos\theta}{g}$$

$$= \frac{2u\sin\theta\; u\cos\theta}{g}$$

$$= \frac{2ab}{2c} = \frac{ab}{c}$$

(A)

Pg 106

(40) $h_{MAX} = \frac{u^2}{g(1+\sin\alpha)}$ α is angle of inclined plane.

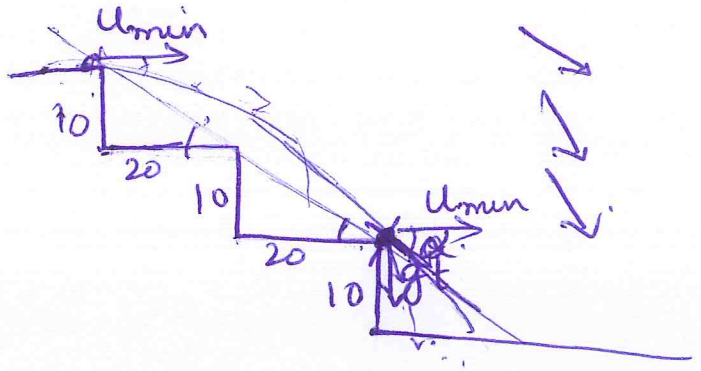
$$= \frac{u^2}{g\left(1+\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{R}{\left(1+\frac{1}{\sqrt{2}}\right)}$$

$R = \frac{u^2}{g}$.

$p = 1$

(43)



$$\tan\theta = \frac{1}{2}$$

$$\tan\alpha = \frac{gt}{u_{min}}$$

$$\frac{+20}{100} = 0 + \frac{1}{2} \times 10 t^2$$

$$t = \frac{2}{10}\,s = \frac{1}{5}\,s.$$

$$\tan\alpha = \frac{1}{2} = \frac{gt}{u_{min}}$$

$$u_{min} = \frac{2gt}{g}$$

$$u_{min} = 2 \times 10 \times \frac{40}{100 u_{min}}$$

$$u_{min} \times t = \frac{40}{100}$$

$$u_{min}^2 = \frac{2 \times 400}{100}$$

$$t = \frac{40}{100\, u_{min}}$$

$$u_{min} = 2\sqrt{2}$$

$$\frac{2}{100 \times 2\sqrt{2}} = \frac{\sqrt{2}}{10}$$

$$u_{min} \times \frac{1}{5} = \frac{40}{100}$$

$$u_{min} = 2\ m/s.$$