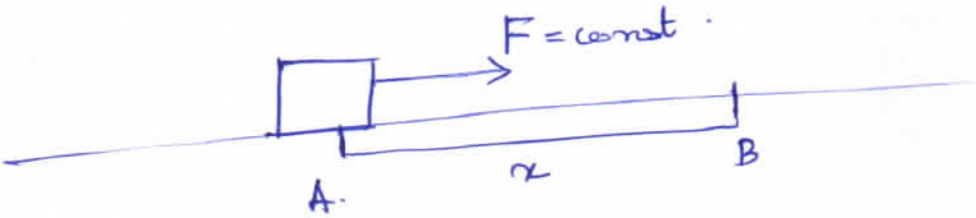
WORK, POWER & ENERGY

WORK

Work is said to be done when a force is applied on a body and a displacement takes place.



Work done by Force F = F × displacement

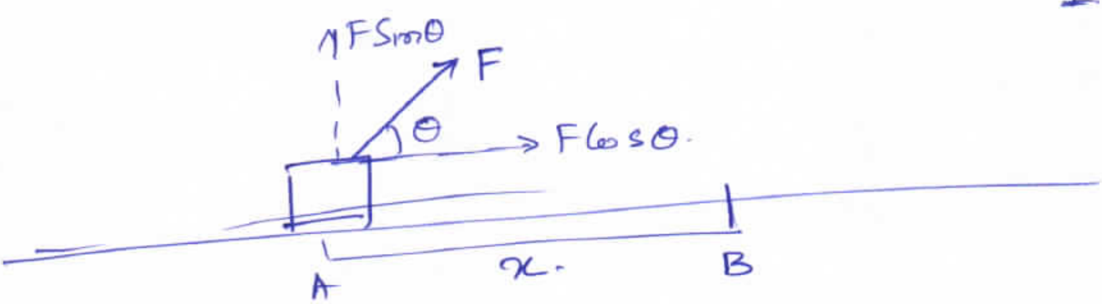
$= Fx$

(Unit of Work done is Nm

$1\ Nm = 1\ J$ ← Joule)

$W.D = \vec{F} \cdot \vec{S}$

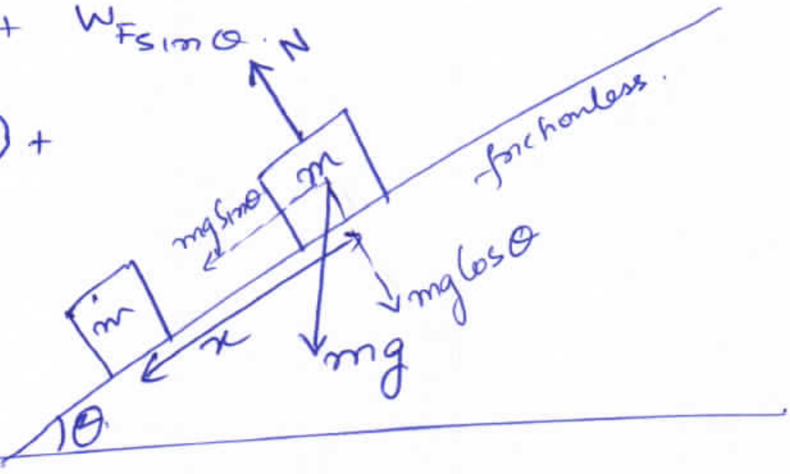
$= FS \cos\theta$



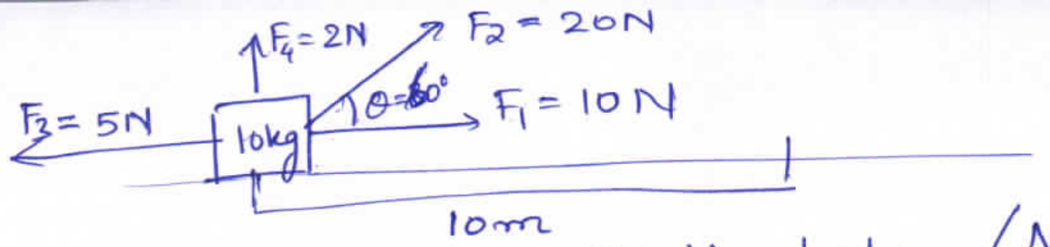
$W.D_{AB} = Fx \cos\theta.$

$W.D_{AB} = W_{F\cos\theta} + W_{F\sin\theta}$

$= (F\cos\theta \times x) +$



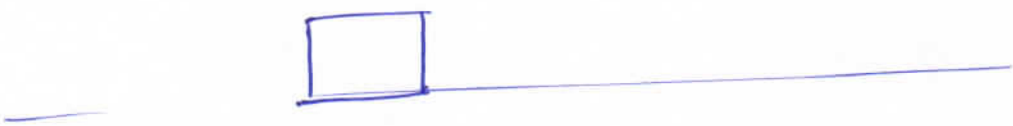
$W_{mg} = mg \sin\theta \times x$



i) Find what is work done on the body. (No friction)

ii) What is work done by the frictional force if $\mu_k = 0.2$

(i)



$$\cancel{|\vec{F_x}| = |\vec{F_1}| -}$$

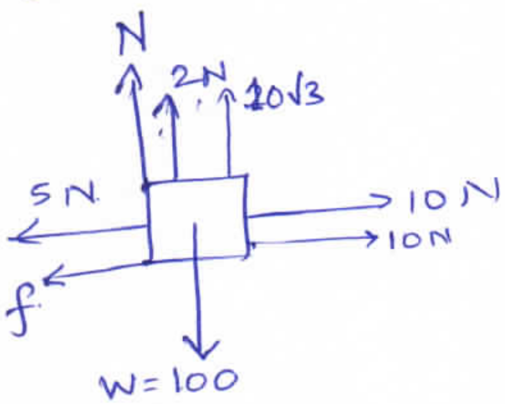
$$\vec{F_x} = \vec{F_1} + \vec{F_3} + \vec{F_2} \cos 60°$$

$$= 10\hat{i} - 5\hat{i} + 20\cos 60\,\hat{i}$$

$$= 15\hat{i}$$

$$W.D = 15 \times 10 = 150 \text{ J}$$

(ii)



$$f = \mu_k N$$

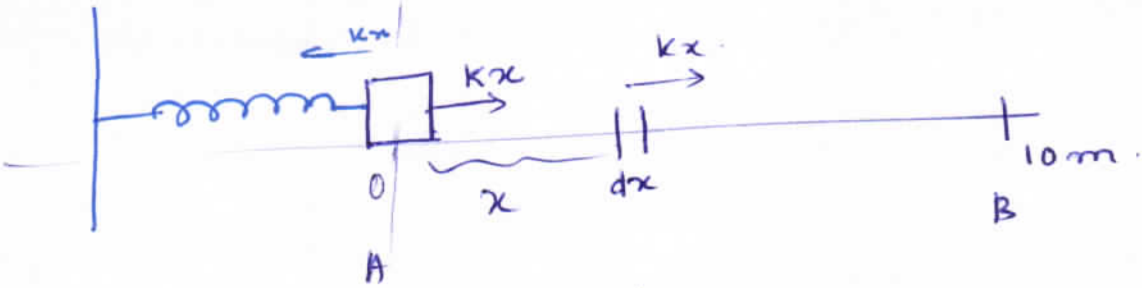
$$N + 2 + 10\sqrt{3} = 100$$

$$N = 98 - 10\sqrt{3}$$

$$W_f = -(\mu_k N)10$$

$$= -0.2(98 - 10\sqrt{3}) \times 10$$

$$= -2(98 - 10\sqrt{3}) \text{ J.}$$

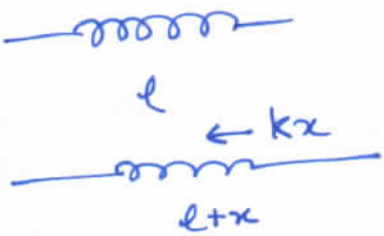


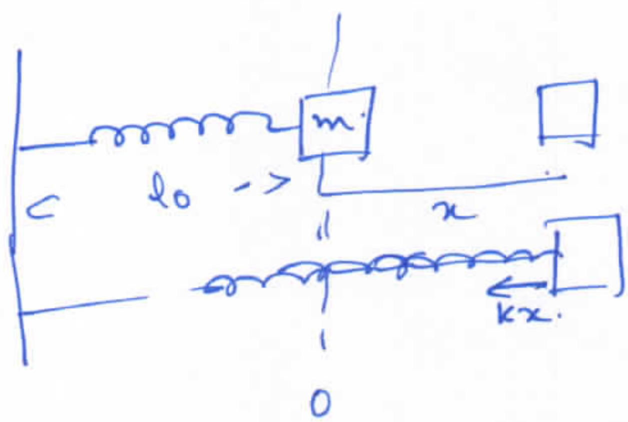
$$\int dW = \int_0^{10} kx\,dx$$

$$W = k\int_0^{10} x\,dx = k\left[\frac{x^2}{2}\right]_0^{10} \qquad \int x^n dx = \frac{x^{n+1}}{n+1}$$

$$= k\left(\frac{10^2}{2} - \frac{0^2}{2}\right)$$

$$= 50K$$





$F = -kx$

Spring force.

$$W = \int_0^x -kx\,dx = \underline{-\frac{1}{2}kx^2}$$

Work done by Spring force $= -\frac{1}{2}kx^2$

<u>k = spring constant</u>. (N/m)

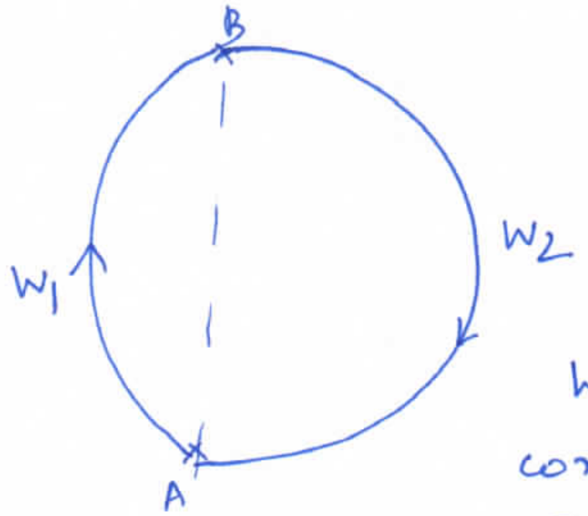
CONSERVATIVE FORCE | NON - CONSERVATIVE FORCE

Conservative force → gravity

Non-conservative force → Frictional Force.

If a body is under the action of a force that does no net work during any closed loop then the force is conservative.

If work is done, then the force is non conservative.



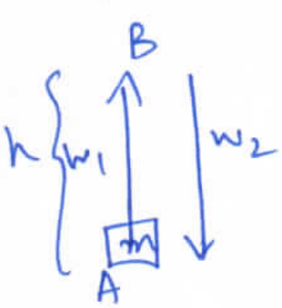
If the force is conservative

$$W_1 + W_2 = 0$$

$$W_2 = -W_1$$

Work done by a conservative force is pathindependent

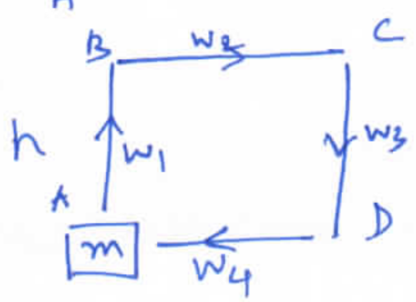
Gravity.



$$w_1 = -mg \times h = -mgh.$$

$$w_2 = -mg \times (-h) = mgh.$$

$$w_1 + w_2 = 0$$

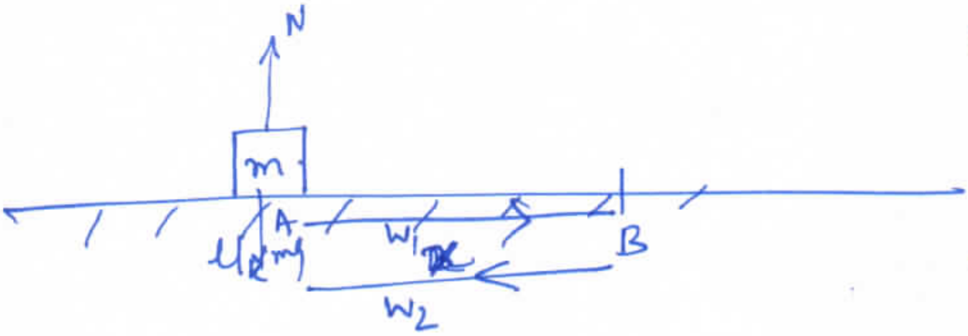


$$w_1 = -mgh. \qquad w_3 = mgh.$$

$$w_2 = 0 \qquad w_4 = 0$$

$$w_1 + w_2 + w_3 + w_4 = 0$$

Frictional force (μ_k)

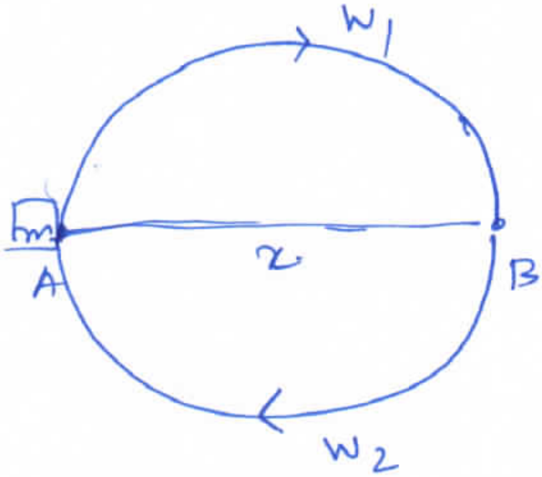


$$W_1 = -\mu_k mg\, x$$

$$W_2 = \mu_k mg\, (-x)$$

$$W_1 + W_2 = -2\mu_k mg\, x$$

$$\neq 0$$



$$W_1 = -\mu_k mg \left(\frac{\pi x}{2}\right)$$

$$W_2 = (\mu_k mg)\left(-\frac{\pi x}{2}\right)$$

$$W_1 + W_2 = -\pi \mu_k mg\, x$$

Work done by Non conservative forces are path dependent.

Power : Any agent who can do work is said said to have power.

power = Rate of doing Work.

$$= \frac{W.D}{\text{time taken}} = \frac{W}{t} = \frac{J}{s} = Js^{-1}$$

$$= \text{Watt}$$

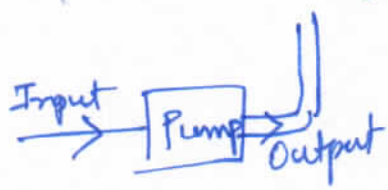
$$\text{Power} = \frac{W}{t} = \frac{\vec{F}\cdot\vec{s}}{t} = \vec{F}\cdot\vec{v}$$

Efficiency of a Machine

Input Power $\rightarrow$ [Machine] $\rightarrow$ output power

$$= \frac{\text{Output Power}}{\text{Input power}} < 1$$

Q) A pump on ground floor can pump water to fill a tank of volume $30\,m^3$ in 15 min. If the tank is 40 m above the ground & Efficiency of pump is 30%. How much electric power is consumed by the pump?



$$m = V \times d = 30 \times 1000$$

$$W = mg \times 40 = 30 \times 1000 \times 10 \times 40$$

$$\underset{(\text{Output})}{\text{Power of pump}} = \frac{W}{t} = \frac{30 \times 1000 \times 10 \times 40}{15 \times 60} = \frac{4}{3} \times 10^4 \text{ Watt}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \Rightarrow \text{Input} = \frac{\text{Output}}{\text{Efficiency}} = \frac{\frac{4}{3}\times 10^4}{0.3} = \frac{4}{9}\times 10^5 \text{ Watt}$$

$$= \frac{400}{9} \text{ KW}$$

Q A train 100 metric ton is running on a level track with uniform speed of 72 km/hr. If frictional resistance amounts to 0.5 kg/metric ton. Find power of engine.

$P = \vec{F} \cdot \vec{v}$ $\quad\quad$ $\vec{v} = \cancel{72}^{4} \times \frac{5}{\cancel{18}} = 20\ m/s.$

$$\vec{F} = 50\ kgf = 500\ N$$

$$P = 500 \times 20 = 10000\ W = 10\ KW$$

ENERGY : Anything which has capacity to do work is said to have Energy.

Chemical Energy : Energy due to chemical reaction.

Mechanical Energy : Due to virtue of position or motion.

Sound Energy

Electrical Energy.

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CONSERVATION OF ENERGY

Energy can neither be created Nor destroyed
It can only change its form from one to another.

MECHANICAL ENERGY

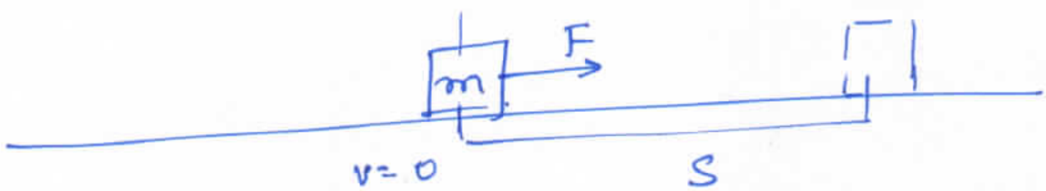
POTENTIAL ENERGY

(If body has the capacity to do work by virtue of its position it is said to have potential Energy (P.E.)

KINETIC ENERGY

If body has capacity to do work by virtue of its motion, it is said to have Kinetic Energy (K.E.)

$$K.E = \frac{1}{2} m v^2$$



W.D by F $\Rightarrow$ Fs

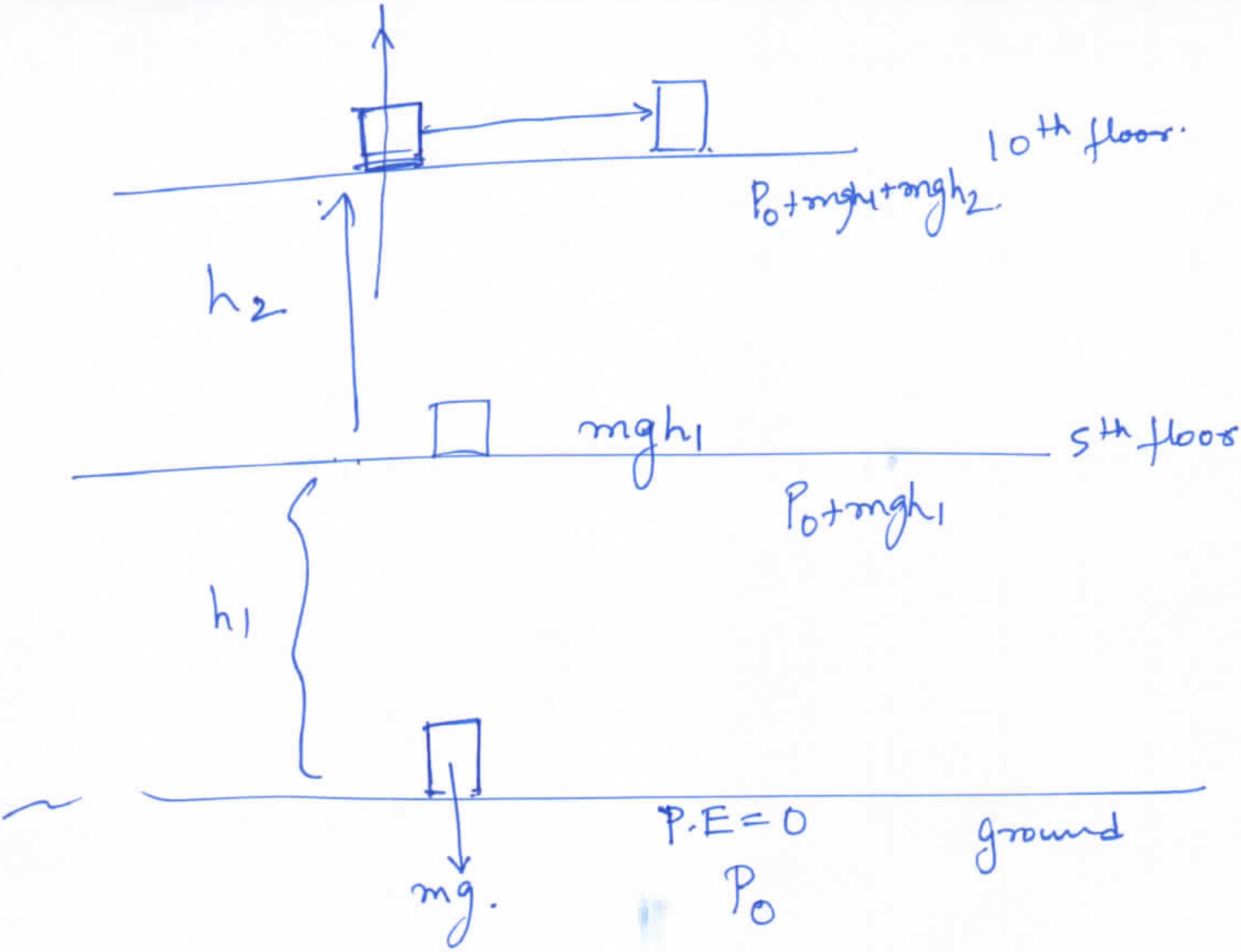
$$a = \frac{F}{m}.$$

$$v^2 - 0^2 = 2as$$

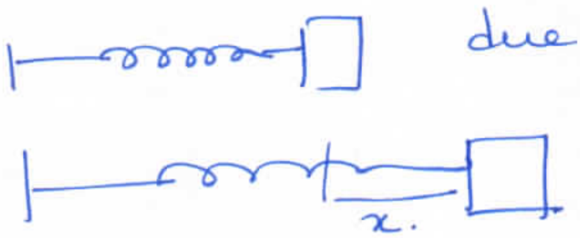
$$v^2 = 2\frac{F}{m}s.$$

$$\frac{1}{2} m v^2 = Fs.$$

Potential Energy : Work done on the body to change its position, size or configuration converts into its potential Energy



Potential Energy



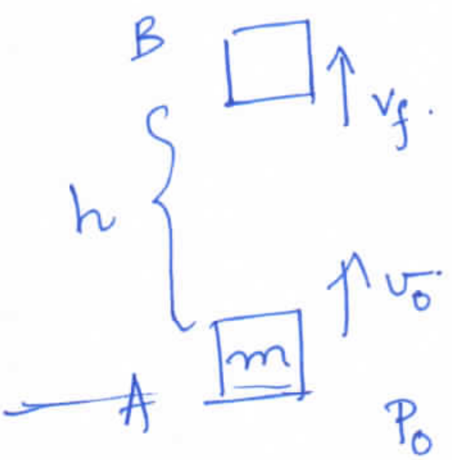
due to change in size & configuration.

$P \cdot E = \frac{1}{2} kx^2$ for spring.

CONSERVATION OF MECHANICAL ENERGY.

In a process where there is no exchange of energy with any different form the mechanical energy can be conserved.

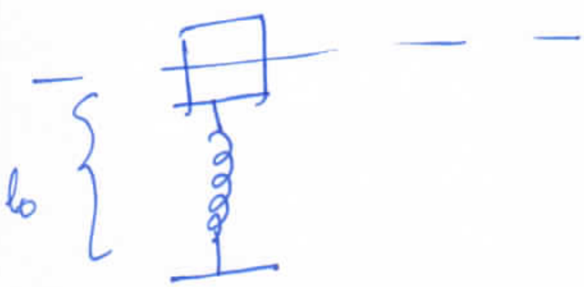
$$\underline{K.E + P.E = \text{const}}$$



$K.E = \frac{1}{2}mv_f^2 \qquad P.E = P_0 + mgh$

$K.E = \frac{1}{2}mv_0^2 \qquad P.E = P_0$

$$P_0 + \frac{1}{2}mv_0^2 = (P_0 + mgh) + \frac{1}{2}mv_f^2$$



mass m with velocity v comes & hits mass 2m & instantly comes to rest. Find maximum compression in spring If spring constant is k.

Initial Energy. Final Energy (when m hits 2m mass)

$$\frac{1}{2} m v_0^2 = \frac{1}{2} \times 2m\, v'^2 + 0$$

$$v' = \frac{v_0}{\sqrt{2}}$$

Initial Energy (when 2m starts moving) Final Energy (at max compression)

$$\frac{1}{2} \times 2m \left(\frac{v_0}{\sqrt{2}}\right)^2 = \frac{1}{2} \times 2m (0)^2 + \frac{1}{2} k x_{max}^2$$

$$x_{max} = v_0 \sqrt{\frac{m}{k}}$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} k \left(\frac{x_{max}}{2}\right)^2 + \frac{1}{2}(2m) v^2$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} k \left(\frac{v_0}{2}\sqrt{\frac{m}{k}}\right)^2 + \frac{1}{2}(2m) v^2$$

$$\frac{1}{2} \cancel{m} v_0^2 - \frac{1}{8} \cancel{m} v_0^2 = \cancel{m} v^2$$

$$\frac{3}{8} v_0^2 = v^2 \Rightarrow v = \sqrt{\frac{3}{8}}\, v_0$$