QUADRATIC EQUATIONS

$f(x) = ax^2 + bx + c$ is a quadratic function.

$(a \neq 0)$

$a, b, c \in R$

$y = ax^2 + bx + c$

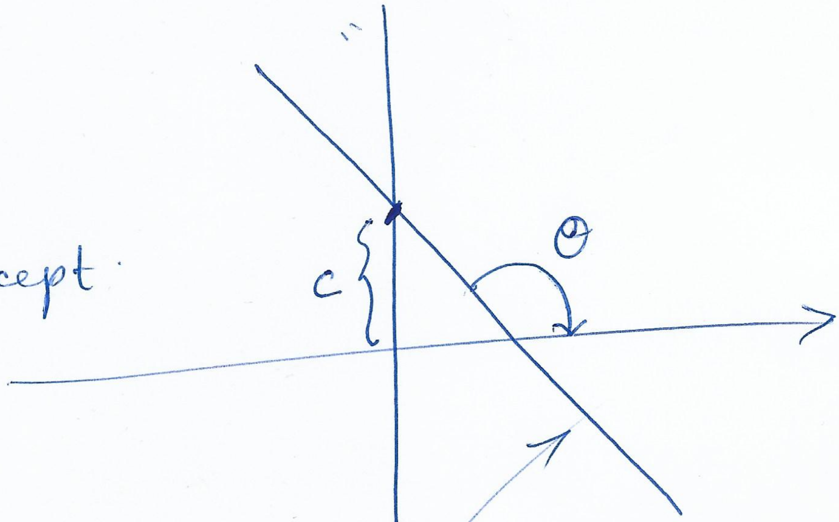
A B.

$x \longrightarrow f(x)$

$f(x) = y = mx + c$

Slope

y-intercept

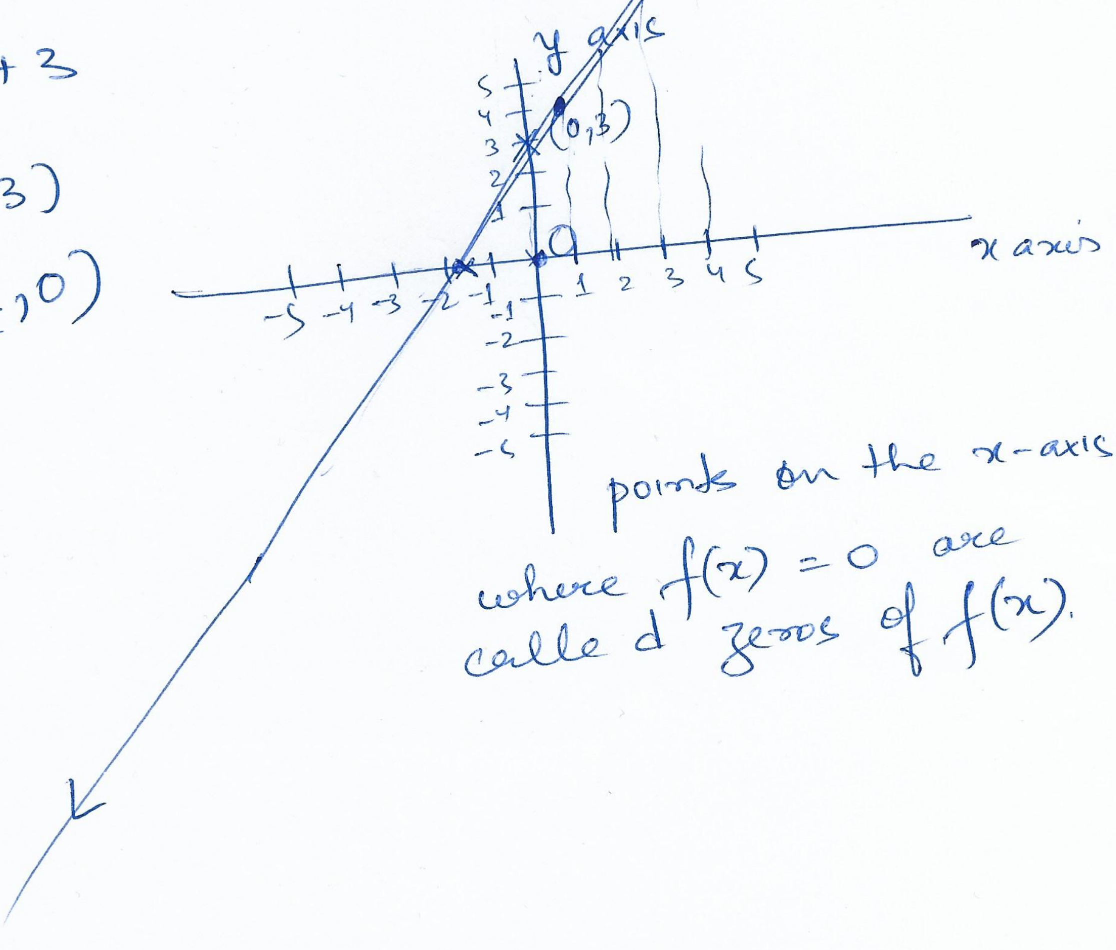


$m = \tan\theta$

$y = 2x + 3$

$(0,3)$

$\left(\frac{-3}{2}, 0\right)$



points on the x-axis where $f(x) = 0$ are called zeros of $f(x)$.

$y = ax^2 + bx + c$ Domain $= \mathbb{R}$.

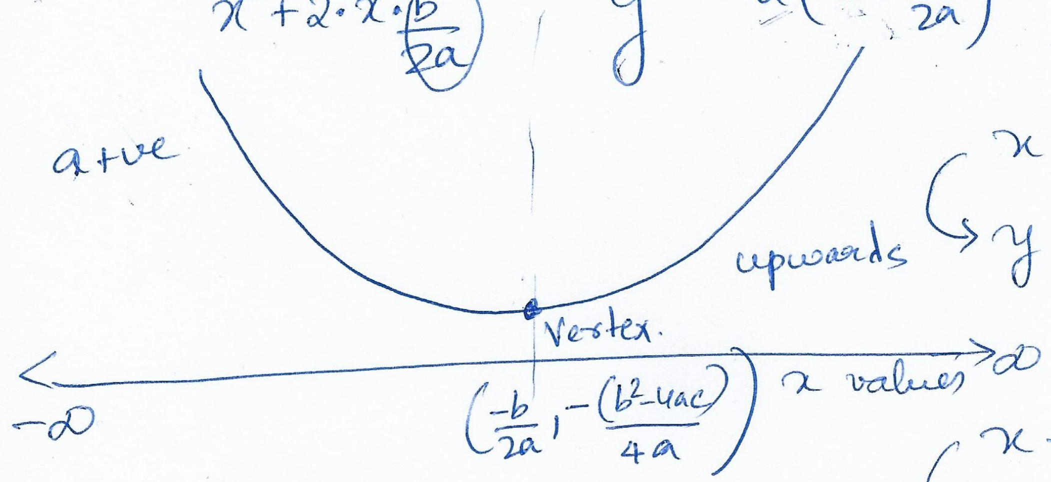
$y = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left(x^2 + 2\cdot\frac{b}{2a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right)$

$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a^2}\right)\right]$

$(A+B)^2 = A^2 + 2AB + B^2$

$x^2 + 2\cdot x\cdot\frac{b}{2a}$

$y = a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a}\right)$



upwards

$x \longrightarrow \infty$

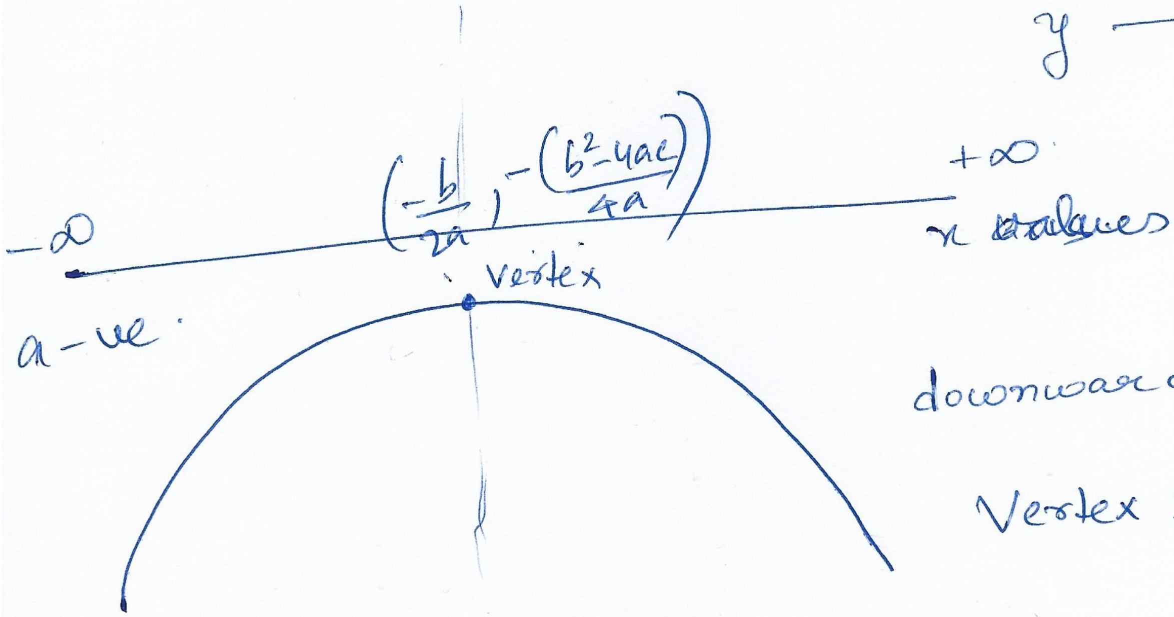
$y \longrightarrow \infty$ a+ve

$\longrightarrow -\infty$ a-ve

$x \longrightarrow -\infty$

$y \longrightarrow \infty$ a+ve

$y \longrightarrow -\infty$ a-ve



Discriminant

$D = b^2 - 4ac$

downward.

Vertex : $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

$$y = a\left(x+\frac{b}{2a}\right)^2 - \left(\frac{D}{4a}\right)$$

$\Downarrow$ zeros of quadratic function

$$y = 0$$

$$a\left(x+\frac{b}{2a}\right)^2 - \frac{D}{4a} = 0$$

$$a\left(x+\frac{b}{2a}\right)^2 = \frac{D}{4a}$$

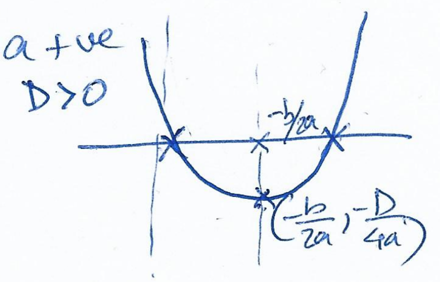
$$\left(x+\frac{b}{2a}\right)^2 = \frac{D}{4a^2}$$

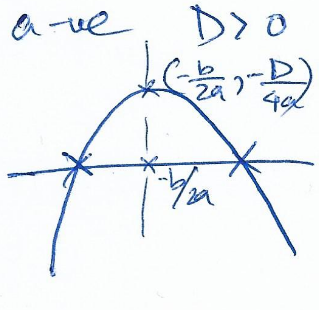
$$x+\frac{b}{2a} = \pm\frac{\sqrt{D}}{2a}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$ ✓ quadratic formula.

$D > 0 \quad (b^2 > 4ac)$

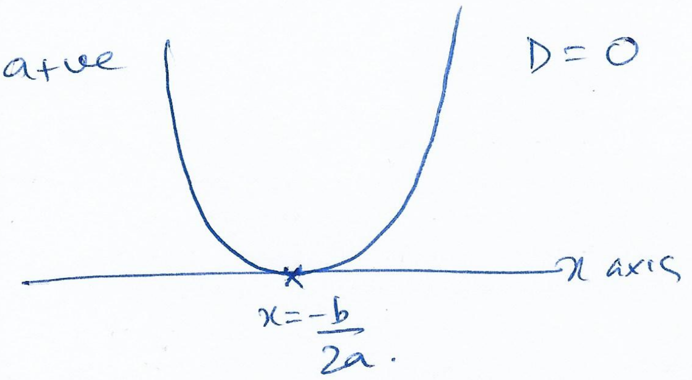
we will get two distinct values of x for which corresponding quadratic function turns 0



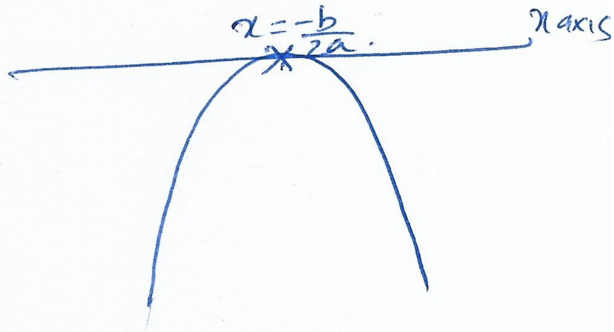


$D = 0 \quad (b^2 = 4ac)$

a+ve $\qquad D = 0$



a−ve. $\qquad D = 0$



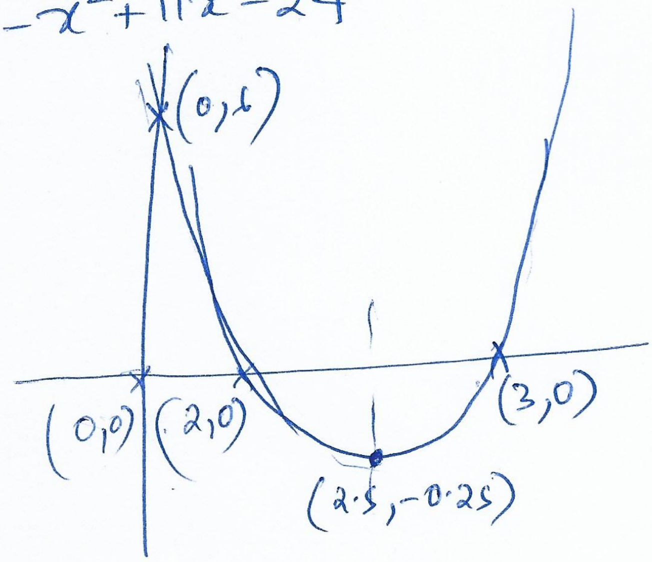
$y = ax^2 + bx + c$

$x = 0$ graph cuts y axis

$y = c$ when $x = 0$

lets draw graph of ① $y = x^2 - 5x + 6$

② $y = -x^2 + 11x - 24$



① $y = x^2 - 5x + 6$.

$a = 1 \quad b = -5 \quad c = 6$

$V = \left(\frac{-b}{2a}, \frac{-D}{4a}\right) = \left(\frac{5}{2}, -\frac{1}{4}\right)$

$x = \frac{-b \pm \sqrt{D}}{2a} = 2, 3$

$x = 0 \quad y = 6$

If α & β are roots of quadratic eq.

$ax^2 + bx + c = 0$.

$\alpha + \beta = \frac{-b}{a} \Leftarrow \frac{\alpha + \beta}{\cancel{2}} = \frac{-b}{\cancel{2}a}$.

(2) $y = -x^2 + 11x - 24$

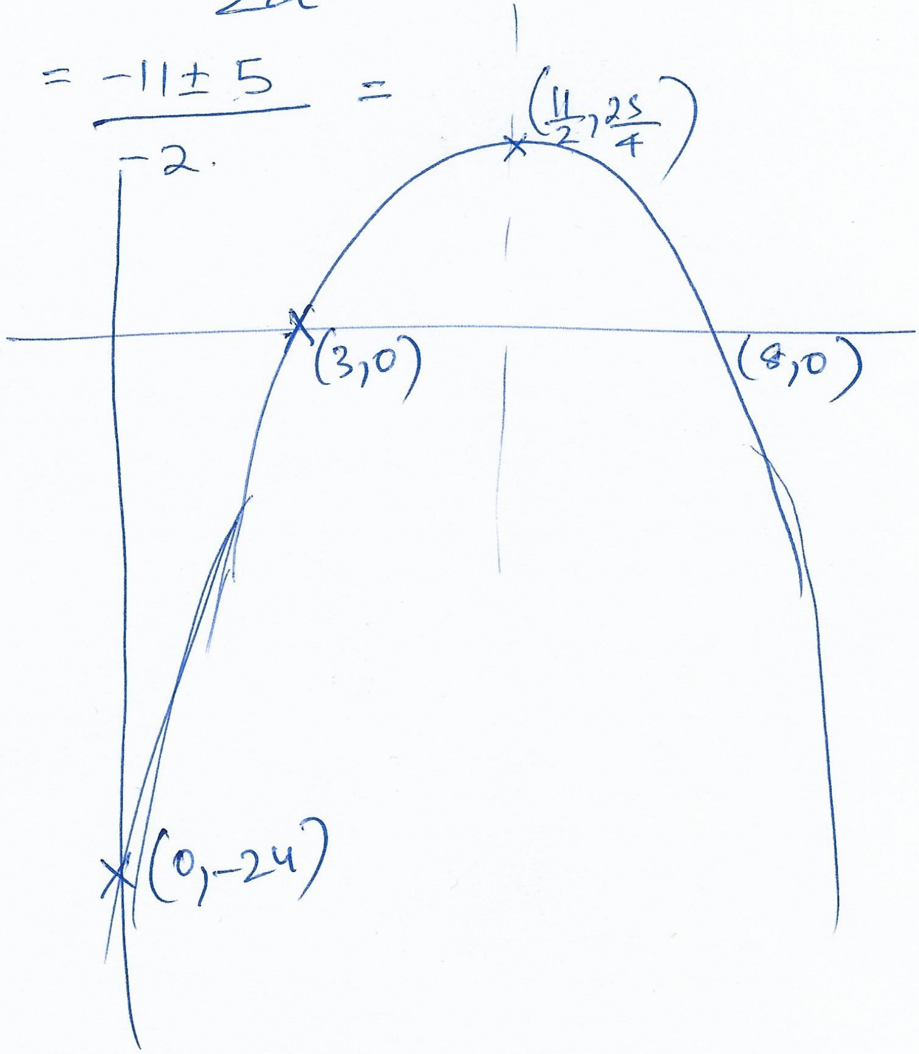
$a = -1 \quad b = 11 \quad c = -24.$

$D = b^2 - 4ac = 25$

$V = \left(\frac{-b}{2a}, \frac{-D}{4a}\right) = \left(\frac{11}{2}, \frac{25}{4}\right)$

$x = \frac{-b \pm \sqrt{D}}{2a} = 3, 8$

$= \frac{-11 \pm 5}{-2} =$

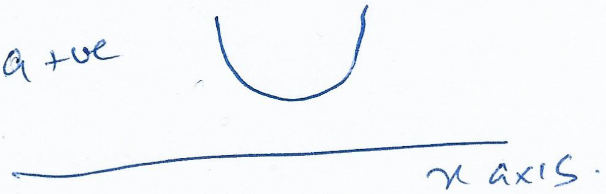


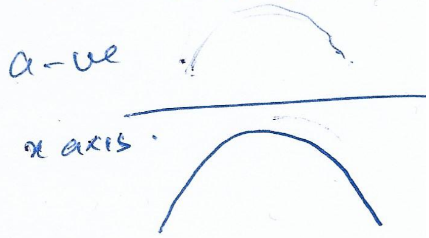
$x = 0$

$y = -24$

(3) If $D < 0$

No real roots. $\Rightarrow$ No intersection with x-axis





Q1 Draw graph of $y = -x^2 + x - 1$

Q2 If $a, b \in R$ $\quad a \neq 0$

if $ax^2 - bx + 1 = 0$ has Imaginary roots

then prove that $a + b + 1 > 0$

Q3 If $c > 0$ & $ax^2 + 2bx + 3c = 0$ does not have any real roots then

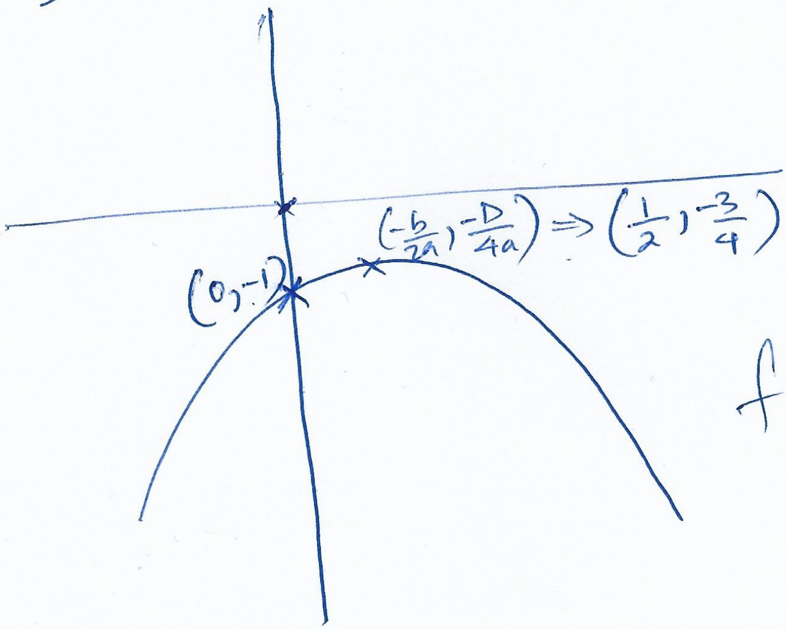
prove i) $a - 2b + 3c > 0$ ii) $a + 4b + 12c > 0$

Ans.

(1) $y = -x^2 + x - 1$

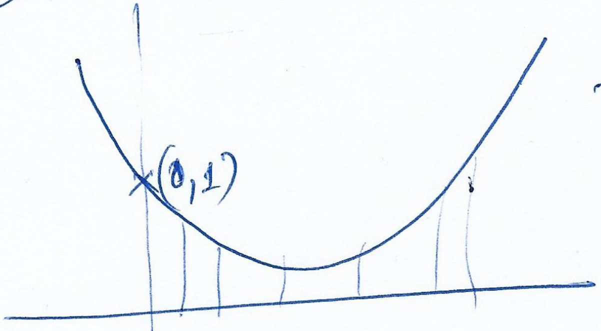
$a = -1 \quad b = 1 \quad c = -1$

$D = 1 - 4 = -3$ $\quad D < 0 \Rightarrow$ Imaginary roots.

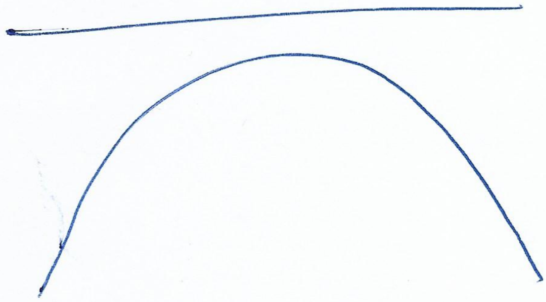


$f(x) =$ ______

(2) $ax^2 - bx + 1 = 0$ Imaginary roots.



$f(0) = 1.$



$f(x) = ax^2 - bx + 1$

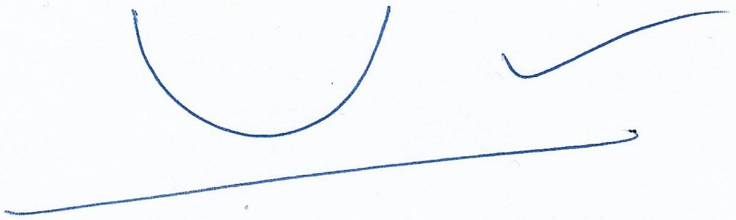
$\forall x \quad f(x) > 0$

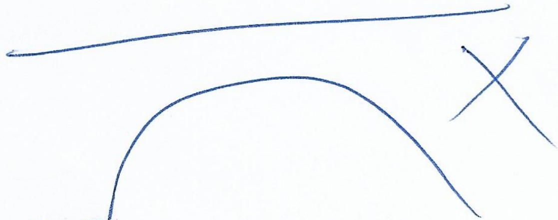
$a + b + 1$

$x = -1 \quad f(-1) > 0$

$a + b + 1 > 0$

(3) $f(0) = 3c \Rightarrow f(0) > 0$





$\forall x \quad f(x) = ax^2 + 2bx + 3c > 0$

i) $a - 2b + 3c$

$f(-1) = a - 2b + 3c > 0$

ii) $a + 4b + 12c$

$f(\frac{1}{2}) = \frac{a}{4} + \frac{b}{} + 3c = \frac{a + 4b + 12c}{4} > 0$

Q) Find 'a' for which

$(a-4)x^2 - 2ax + 2a - 6 < 0$ is true for all real values of x

i) $a - 4 = 0 \longrightarrow$ linear eq.

$a = 4.$

$-2(4)x + 2(4) - 6 < 0$

$\underline{x > \frac{1}{4}}$ × $a \neq 4.$

(∵ exp (inequality) holds true for all x)

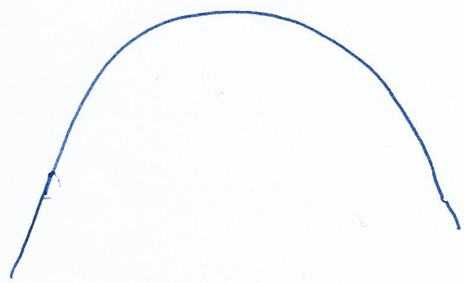
ii) $a \neq 4$

$\underline{a < 4}$ $\underline{a > 4}$ ×

$f(0) = 2a - 6 < 0$

$\underline{a < 3.}$

$\Rightarrow a - 4 < 0$



$D < 0$

$(-2a)^2 - 4(2a-6)(a-4) < 0$

$\cancel{4}a^2 - \cancel{4}(2a^2 - 14a + 24) < 0$

$-a^2 + 14a - 24 < 0 \Rightarrow \underline{a^2 - 14a + 24 > 0}$

Solving an inequality of higher order (Wavy-curve Method)

$f(x) < 0$ $\qquad$ $f(x) > 0$

$$f(x) = ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + \cdots$$

$$f(x) < 0$$

1st Step.

Make a +ve ← coefficient of highest power of x.

2nd Step

factorize $f(x)$ into linear factors.

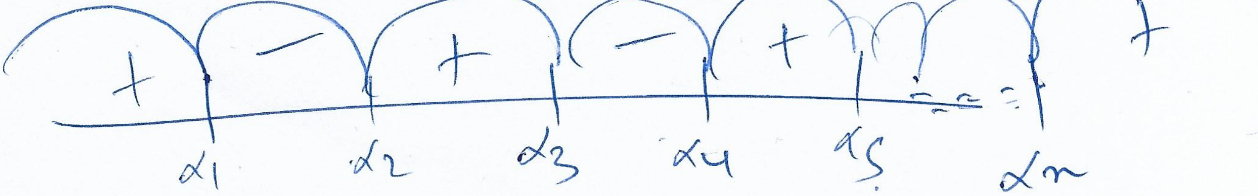
$$f(x) = (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\cdots$$

3rd Step.

arrange $\alpha_1, \alpha_2 \cdots \alpha_n$ in ascending order.

4th Step

Mark $\alpha_1, \alpha_2 \cdots \alpha_n$ on number line

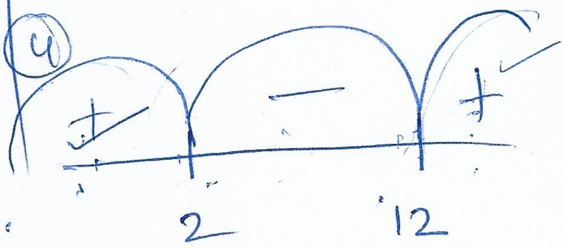


① $a^2 - 14a + 24 > 0$

② $(a-2)(a-12) > 0$

③ $2, 12$.

④



$a > 12$

or $a < 2$

Consider the quadratic function $f(x) = ax^2+bx+c$

corresponding quad. eq $\Rightarrow$ $ax^2+bx+c=0$

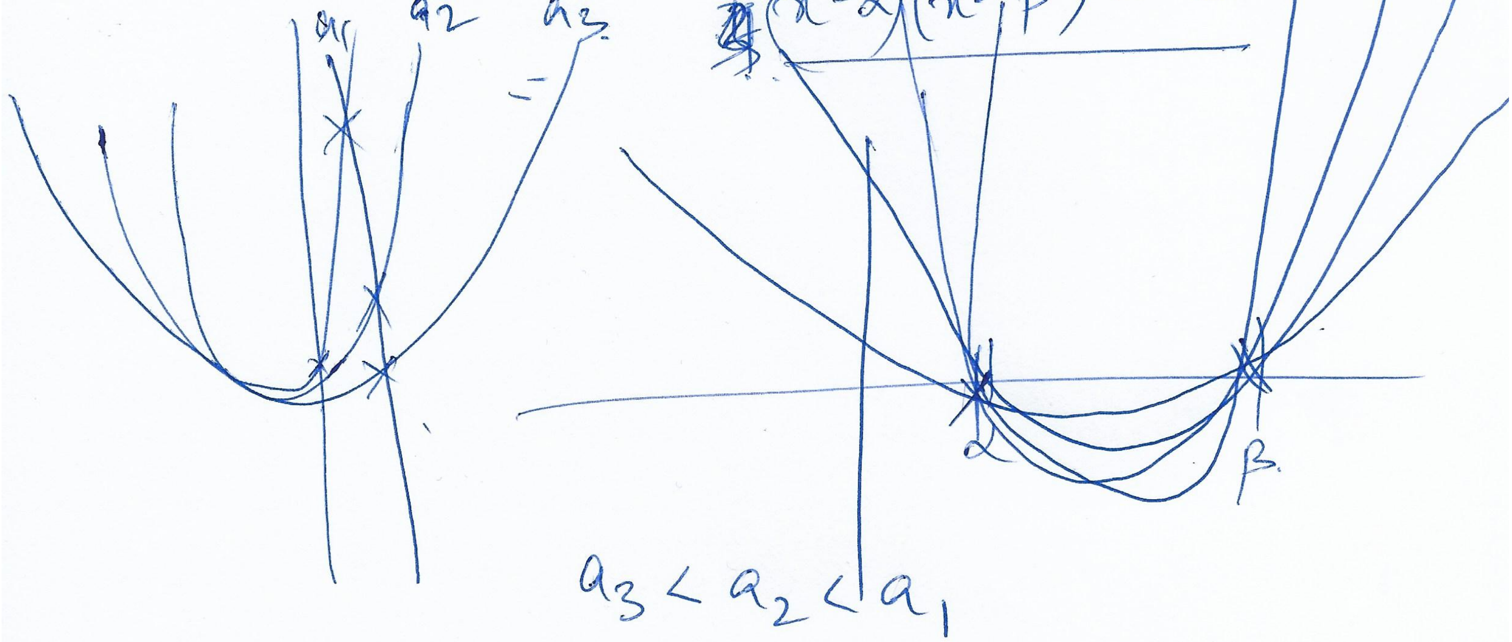
let α, β be the roots of this eq.

$$f(x) = a(x-\alpha)(x-\beta)$$

$$a(x-\alpha)(x-\beta) = 0$$

$$\Downarrow$$

$$\cancel{a}(x-\alpha)(x-\beta) = 0$$



$$a_3 < a_2 < a_1$$

$$f(x) = ax^2 - ax(\alpha+\beta) + a\alpha\beta$$

$$b = -a(\alpha+\beta) \Rightarrow \alpha+\beta = -\frac{b}{a}$$

$$c = a\alpha\beta \Rightarrow \alpha\beta = \frac{c}{a}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$f(x) = ax^2 + bx + c$

a, b, c are rational.

Irrational.

$x = p + \sqrt{q}$

other root has to be

$x = p - \sqrt{q}$

$x^2 = 25$ ①

$x = \sqrt{25}$ ②

$x = 5.$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{D}}{2a}.$$

$$= -\frac{b}{2a} + \frac{\sqrt{D}}{2a}$$

$$-\frac{b}{2a} - \frac{\sqrt{D}}{2a}.$$

Common roots

$Q_1 : ax^2 + bx + c = 0$

$Q_2 : px^2 + qx + r = 0$

① Q_1 & Q_2 have 2 roots in common.

$x^2 - 5x + 6 = 0$

$k(x^2 - 5x + 6) = 0$

$ax^2 + bx + c = 0$

$k(ax^2 + bx + c) = 0$

$kax^2 + kbx + kc = 0$

$ka = p$

$kb = q$

$kc = r.$

$$k = \boxed{\frac{p}{a} = \frac{q}{b} = \frac{r}{c}}$$

2) One root in common.

$Q_1 : ax^2+bx+c=0$ $\quad$ $Q_2 : px^2+qx+r=0$

let α be the common root.

$a\alpha^2+b\alpha+c=0$ $\quad$ & $\quad$ $p\alpha^2+q\alpha+r=0$

$\alpha^2 \quad , \quad \alpha.$

$a\alpha^2+b\alpha+c=0 \; — \; p$

$p\alpha^2+q\alpha+r=0 \; — \; a.$

$pa\alpha^2+pb\alpha+pc=0$

$\ominus \; ap\alpha^2+aq\alpha+ar=0$

$(pb-aq)\alpha = ar-pc$

$$\alpha = \frac{ar-pc}{pb-aq}.$$

$$a\alpha^2+b\left(\frac{ar-pc}{(pb-aq)}\right)+c=0$$

$$a\alpha^2 = \frac{aqc+p\cancel{b}c-abr-\cancel{b}pc}{pb-aq}.$$

$$\cancel{a}\alpha^2 = \frac{\cancel{a}(qc-br)}{pb-aq}$$

$a\alpha^2+b\alpha+c=0$

$p\alpha^2+q\alpha+r=0$

$$\frac{\alpha^2}{qc-br} = \frac{\alpha}{ar-pc} = \frac{1}{pb-aq}. \Rightarrow \frac{\alpha^2}{br-qc} = \frac{\alpha}{-(ar-pc)} = \frac{1}{aq-pb}$$

Q7) Find value of k if the equation

$3x^2-2x+k=0$ & $6x^2-17x+12=0$

has a common root.

let α be common root.

$3\alpha^2-2\alpha+k=0$

$6\alpha^2-17\alpha+12=0$

$$\frac{\alpha^2}{-2\times12-(-17)k} = \frac{\alpha}{-(3(12)-6k)} = \frac{1}{3(-17)-(6)(-2)}$$

$$\frac{\alpha^2}{17k-24} = \frac{\alpha}{6k-36} = -\frac{1}{39}.$$

$$\alpha^2 = -\frac{(17k-24)}{39} \qquad \alpha = -\frac{(6k-36)}{39}$$

$$\left(-\frac{(6k-36)}{39}\right)^2 = -\frac{(17k-24)}{39}$$

$$\frac{\cancel{36}^{12}(k-6)^2}{\cancel{39^2}_{13}} = \frac{24-17k}{\cancel{39}}$$

$12k^2-144k+432=312-221k.$

$12k^2+77k+120=0$

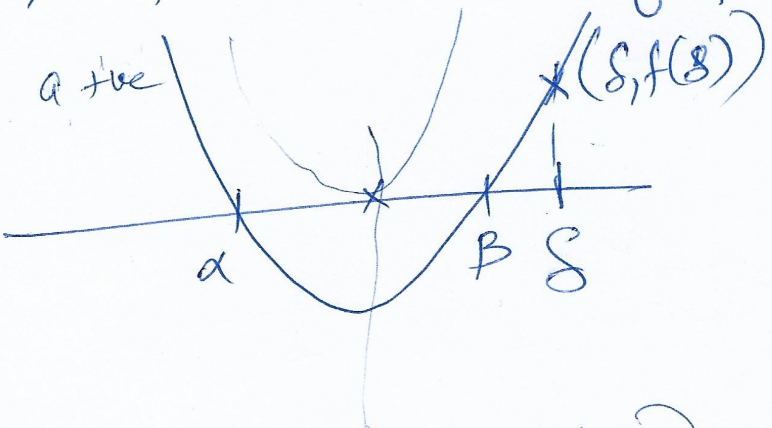
$\Rightarrow (4k+15)(3k+8)=0$ $\qquad \Rightarrow k=-\frac{15}{4}$ or $k=-\frac{8}{3}$

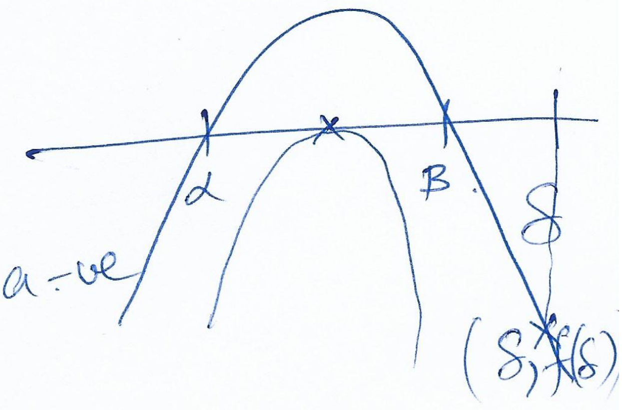
location of roots.

If $f(x) = ax^2+bx+c$ & $a \neq 0$; $a, b, c \in R$.

let $\delta_1, \delta_2, \delta_3 \in R$ s.t $\delta_1 < \delta_2$

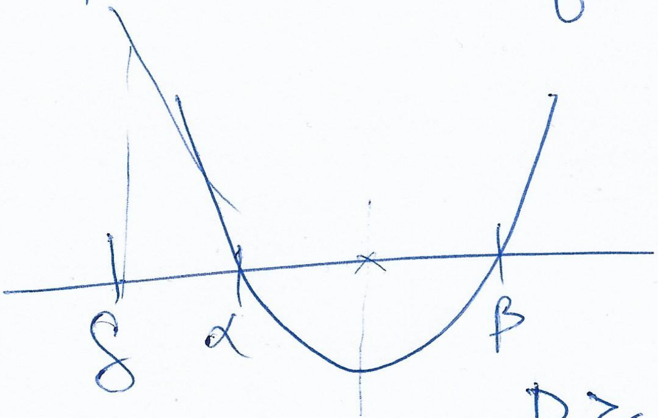
i) If both roots of $f(x)$ is less δ

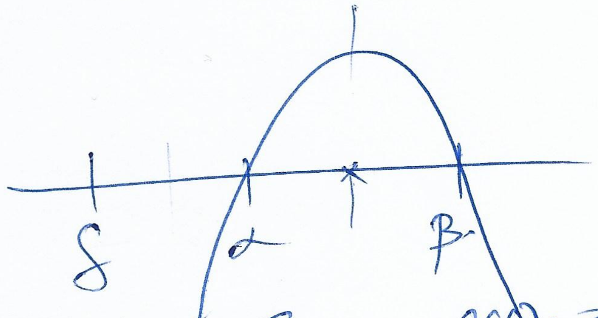




$$\left.\begin{array}{l} D \geq 0 \checkmark \\ \dfrac{-b}{2a} < \delta \checkmark \\ af(\delta) > 0 \end{array}\right\} \longrightarrow$$

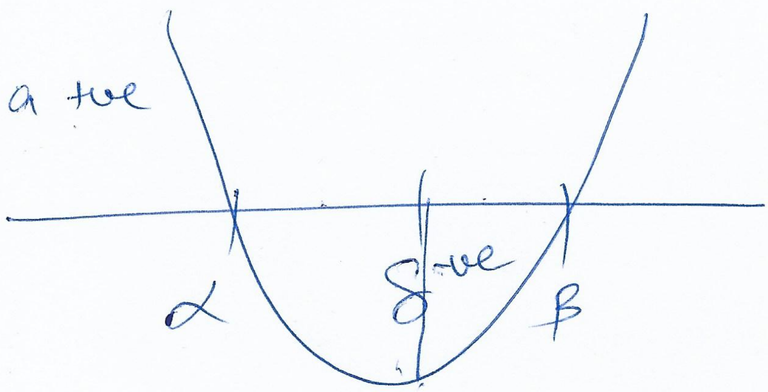
② If both roots of $f(x)$ is greater than δ.

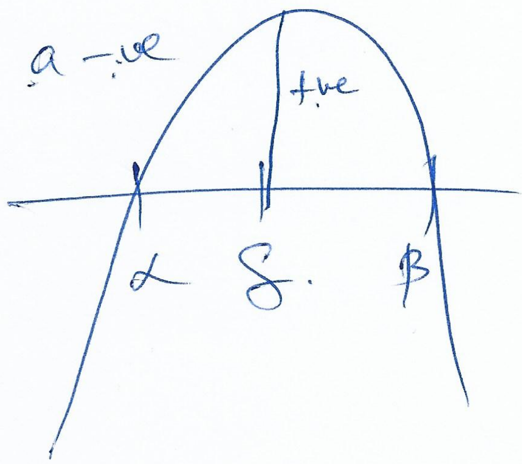




$D \geq 0$; $\dfrac{-b}{2a} > \delta$, $af(\delta) > 0$

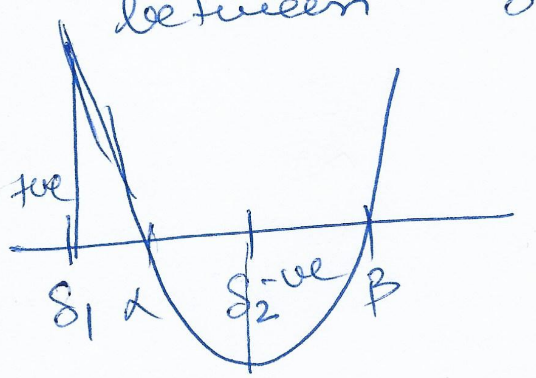
③ If δ lies between roots of $f(x)$

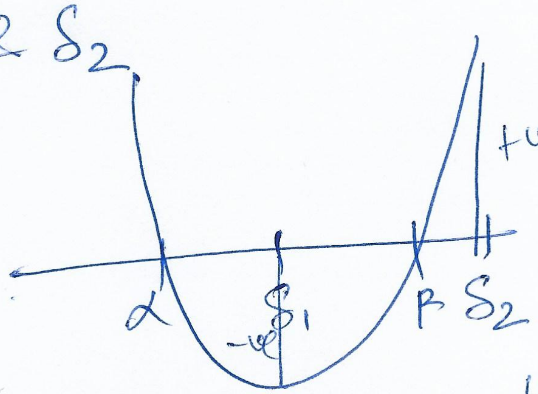




$$\left.\begin{array}{l} D > 0 \\ af(\delta) < 0 \end{array}\right\}$$

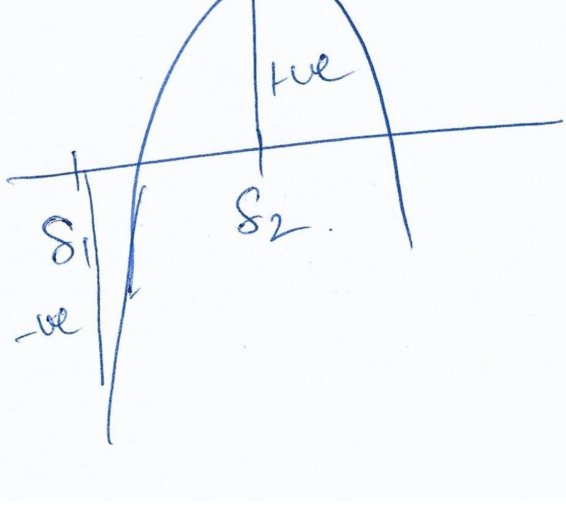
④ If exactly one root of $f(x)$ lies between δ_1 & δ_2

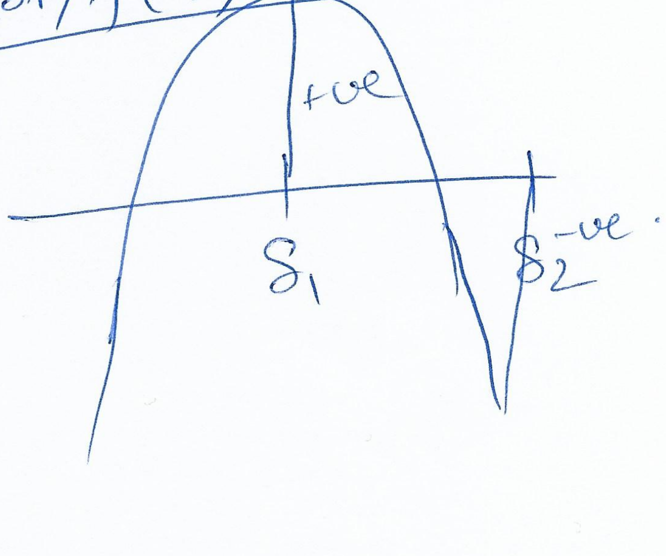




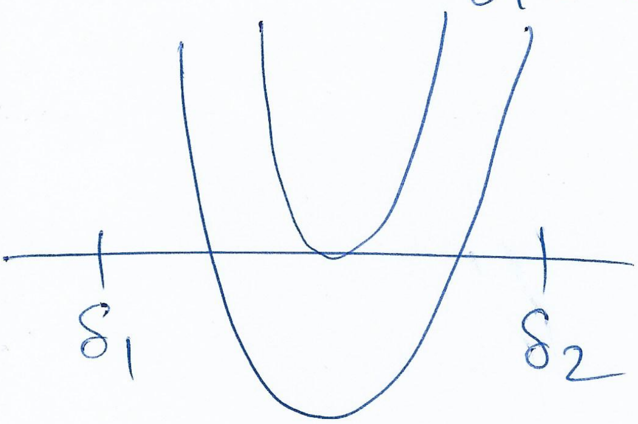
$D > 0$

$f(\delta_1) f(\delta_2) < 0$

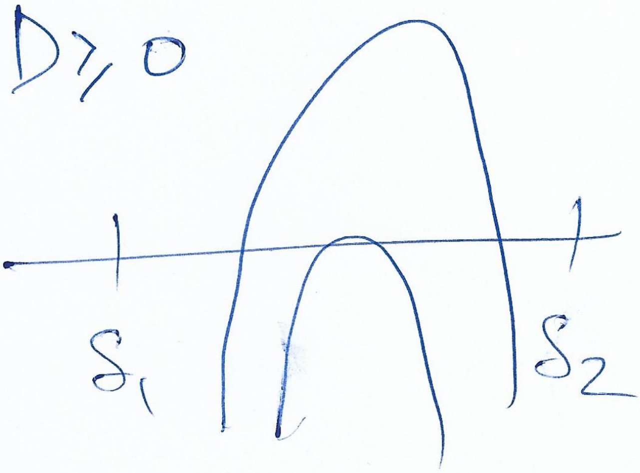




(5) If both roots of $f(x)$ lies between δ_1 & δ_2.



$D \geq 0$



$$\delta_1 < \frac{-b}{2a} < \delta_2$$

$$f(\delta_1) f(\delta_2) > 0$$

Basic properties of Inequalities.

If $a < b$ & $b < c \implies a < c$

If $a < b \implies a + c < b + c$

If $a < b \implies a - c < b - c$

If $a < b$. $ac < bc$ if $c > 0$

$ac > bc$ if $c < 0$

If $a < b$ $\frac{a}{c} < \frac{b}{c}$ if $c > 0$

$\frac{a}{c} > \frac{b}{c}$ if $c < 0$

$f(x) \leq 0$ $f(x) \geq 0$

If Inequality is of the form

$$\frac{P(x)Q(x)}{Q(x)} \geq 0 \quad \leq 0$$

$$(Q(x))$$

$$\Leftarrow \frac{P(x)\,Q(x)}{\{Q(x)\}^2} \geq 0 \quad \leq 0$$

Q Find the values of x for which the following inequality holds $\dfrac{8x^2+16x-51}{(2x-3)(x+4)} > 3$

Q) Solve : $\dfrac{2x}{2x^2+5x+2} > \dfrac{1}{x+1}$

Q) Find all values of $m \in R$ so that both roots of the equation

$x^2 - 6mx + 9m^2 - 2m + 2 = 0$ exceeds 3

Ans
Q1

$$\frac{8x^2+16x-51}{(2x-3)(x+4)} - 3 > 0$$

$$\frac{(8x^2+16x-51) - 3(2x-3)(x+4)}{(2x-3)(x+4)} > 0$$

$$\frac{2x^2+x-15}{(2x-3)(x+4)} > 0$$

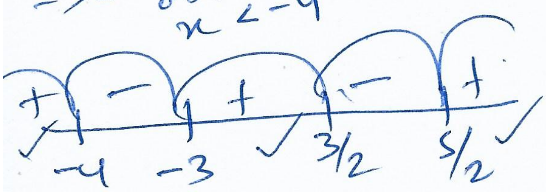
$x \in (-\infty,-4) \cup (-3, 3/2) \cup (5/2, \infty)$

$$\frac{2x^2+6x-5x-15}{(2x-3)(x+4)} > 0$$

$$\frac{2x(x+3)-5(x+3)}{(2x-3)(x+4)} > 0$$

$x > 5/2$
or
$-3 < x < 3/2$
or
$x < -4$

$$(2x-3)(x+4)(2x-5)(x+3) > 0$$



Ans 2

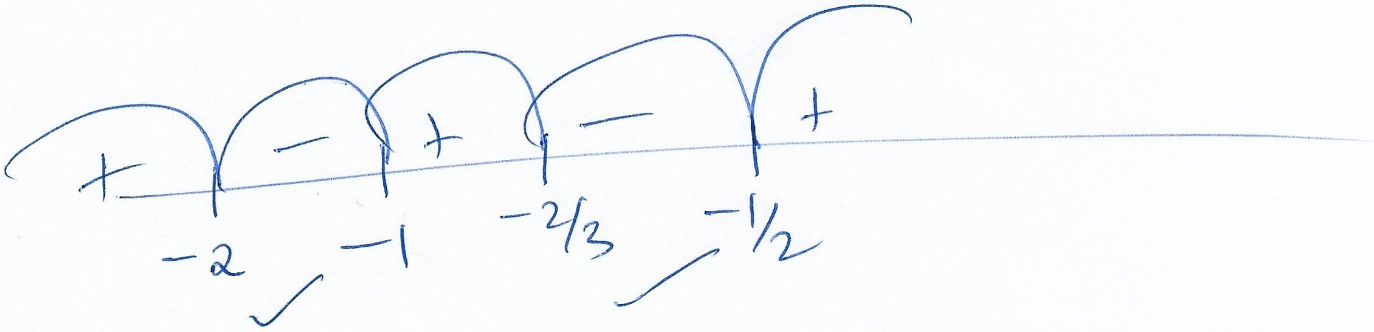
$$\frac{2x}{2x^2+5x+2} - \frac{1}{x+1} > 0$$

$$\frac{2x^2+2x-(2x^2+5x+2)}{(2x^2+5x+2)(x+1)} > 0$$

$$\frac{(-3x-2)}{(2x^2+4x+x+2)(x+1)} > 0$$

$$\frac{-3\left(x+\frac{2}{3}\right)}{(2x+1)(x+2)(x+1)} > 0 \qquad \cancel{2\left(x+\tfrac{1}{2}\right)}$$

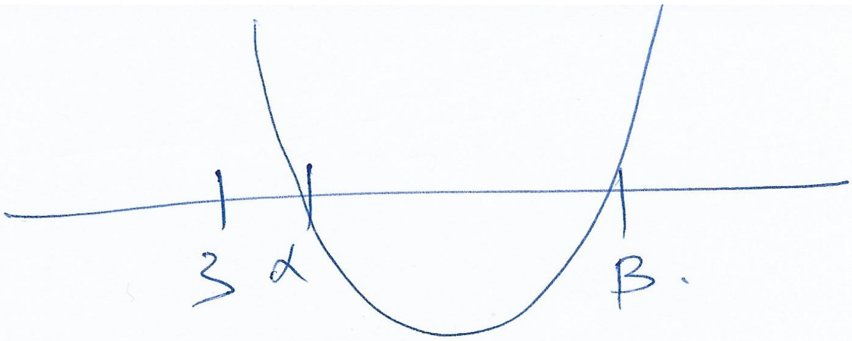
$$\frac{3\times 2\left(x+\frac{1}{2}\right)(x+1)(x+2)\left(x+\frac{2}{3}\right)}{\{(2x+1)(x+2)(x+1)\}^2} < 0$$



$-2 < x < -1$ or $-\frac{2}{3} < x < -\frac{1}{2}$

$(-2, -1) \cup \left(-\frac{2}{3}, -\frac{1}{2}\right)$

Ans 3



$f(3) > 0 \rightarrow 3^2 - 18m + 9m^2 - 2m + 2 > 0$

$9m^2 - 20m + 11 > 0$

$\frac{6m}{2} > 3 \Leftarrow \frac{-b}{2a} > 3.$

$m > 1$

$D \geq 0$

$b^2 - 4ac \geq 0$

$36m^2 - 4(9m^2 - 2m + 2)(1) \geq 0$

$8m - 8 \geq 0$

$m \geq 1$

$m > 1$

$m > \frac{11}{9}.$

$9m^2 - 20m + 11 > 0$

$9m^2 - 9m - 11m + 11 > 0$

$9m(m-1) - 11(m-1) > 0$

$(9m - 11)(m - 1) > 0$

$9\left(m - \frac{11}{9}\right)(m - 1) > 0$

$+ \quad 1 \quad - \quad \frac{11}{9} \quad +$

$m > 11/9.$

$m < 1$