TRIGONOMETRY TUTORIAL

Pg 13   1, 3, 5, 8, 13, 14

Pg 14   17, 18, 20, 26, 29, 30

Pg 15   6, 8, 9

Pg 18   Comp 3

Pg 19   1, 2

Pg 20   6, 7

Pg 13

(1) $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$

$\updownarrow$

$\cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)$

$$\frac{\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\sin\frac{\pi}{7}\times 2}{\sin\frac{\pi}{7}\times 2}$$

$$\frac{\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\sin\frac{2\pi}{7}\times 2}{2\sin\frac{\pi}{7}\times 2} = \frac{\cos\frac{3\pi}{7}\sin\frac{4\pi}{7}}{4\sin\frac{\pi}{7}}$$

$$= \frac{\cos\frac{3\pi}{7}\sin\frac{3\pi}{7}\times 2}{4\sin\frac{\pi}{7}\times 2}$$

$$= \frac{\cancel{\sin\frac{6\pi}{7}}}{8\cancel{\sin\frac{\pi}{7}}} = \frac{1}{8}$$

$\sin\theta = \sin\pi-\theta$

$\sin\frac{4\pi}{7} = \sin\left(\pi-\frac{4\pi}{7}\right)$

$= \sin\frac{3\pi}{7}$

$\sin(\pi-\theta)$

$= \sin\theta$

(3) $\theta+\phi=\alpha$ $\quad \sin\theta = k\sin\phi$

(c) then $\frac{k\sin\alpha}{1+k\cos\alpha}$.

$\phi = \alpha-\theta$

$\sin\phi = \sin(\alpha-\theta)$

$\frac{\sin\theta}{k} = \sin\alpha\cos\theta - \cos\alpha\sin\theta$

$\sin\theta\left(\frac{1}{k}+\cos\alpha\right) = \sin\alpha\cos\theta$.

$\tan\theta\left(\frac{1+k\cos\alpha}{k}\right) = \sin\alpha$

$\Rightarrow \tan\theta = \frac{k\sin\alpha}{1+k\cos\alpha}$

(5) $\tan x = \dfrac{m}{m+1}$ $\qquad$ $\cot y = (2m+1)$

$\tan y = \dfrac{1}{2m+1}$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$= \frac{\dfrac{m}{m+1} + \dfrac{1}{2m+1}}{1 - \dfrac{m}{m+1} \times \dfrac{1}{2m+1}}$$

$$= \frac{2m^2 + m + m + 1}{2m^2 + 3m + 1 - m}$$

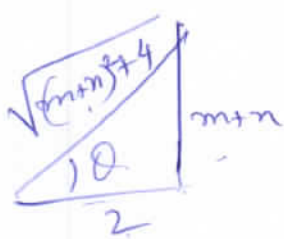
$\tan(x+y) = 1.$

$x+y = \dfrac{\pi}{4}$

(8) $\tan\theta + \sin\theta = m$ $\qquad$ $\tan\theta - \sin\theta = n$;

$\tan\theta = \dfrac{m+n}{2}$

$\sin\theta = \dfrac{m-n}{2}$



$\sin\theta = \dfrac{m+n}{\sqrt{(m+n)^2+4}}$

$$\frac{m-n}{2} = \frac{m+n}{\sqrt{(m+n)^2+4}} \quad \cancel{= \frac{m^2+n^2-2mn}{4} = \frac{m+n}{m^2+n^2}}$$

$$\frac{m-n}{m+n} = \frac{2}{\sqrt{(m+n)^2+4}}$$

$$\frac{(m-n)^2}{(m+n)^2} = \frac{4}{(m+n)^2+4}$$

$$\frac{(m-n)^2}{(m+n)^2-(m-n)^2} = \frac{4}{(m+n)^2+4-4}$$

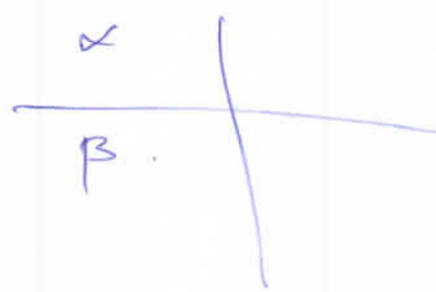
$$\frac{(m-n)^2}{4mn} = \frac{4}{(m+n)^2}$$

$$(m-n)^2(m+n)^2 = 16mn$$

$$(m^2-n^2)^2 = 16mn$$

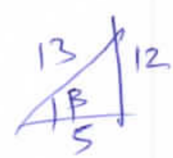
$$m^2-n^2 = 4\sqrt{mn}$$

D



13 $\frac{\pi}{2} < \alpha < \pi$ $\quad \pi < \beta < \frac{3\pi}{2}$

$\sin\alpha = \frac{15}{17}$ $\quad \tan\beta = \frac{12}{5}$



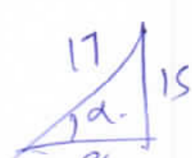
$$\sin(\beta-\alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha$$

$$= \frac{-12}{13}\left(\frac{-8}{17}\right) - \left(\frac{-5}{13}\right)\left(\frac{15}{17}\right)$$

$$= \frac{96+75}{13\times17} = \frac{171}{13\times17} = \frac{171}{221}$$

C

$\sin\beta = -\frac{12}{13}$



$\cos\alpha = -\frac{8}{17}$

(14) $m \tan(\theta - 30^\circ) = n \tan(\theta + 120^\circ)$

$$\frac{m+n}{m-n}.$$

$$\frac{m}{n} = \frac{\tan(\theta+120^\circ)}{\tan(\theta-30^\circ)}$$

$$\frac{m+n}{m-n} = \frac{\tan(\theta+120^\circ) + \tan(\theta-30^\circ)}{\tan(\theta+120^\circ) - \tan(\theta-30^\circ)}$$

$$= \frac{\dfrac{\sin(\theta+120^\circ)}{\cos(\theta+120^\circ)} + \dfrac{\sin(\theta-30^\circ)}{\cos(\theta-30^\circ)}}{\dfrac{\sin(\theta+120^\circ)}{\cos(\theta+120^\circ)} - \dfrac{\sin(\theta-30^\circ)}{\cos(\theta-30^\circ)}}$$

$$= \frac{\sin(\theta+120^\circ+\theta-30^\circ)}{\sin(\theta+120^\circ-(\theta-30^\circ))}$$

$$= \frac{\sin(90^\circ+2\theta)}{\sin 150^\circ}$$

$$= \underline{2\cos 2\theta}$$ (A)

Pg 14

(17) $\sin^4\left(\frac{\pi}{8}\right) + \sin^4\left(\frac{3\pi}{8}\right) + \sin^4\left(\frac{5\pi}{8}\right) + \sin^4\left(\frac{7\pi}{8}\right)$

$\sin^4\left(\frac{5\pi}{8}\right) \downarrow \sin^4\frac{3\pi}{8}$ $\qquad$ $\sin^4\left(\frac{7\pi}{8}\right) \downarrow \sin^4\frac{\pi}{8}$

$2\left(\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8}\right)$

$2\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}\right)$

$\sin\frac{3\pi}{8} = \cos\left(\frac{\pi}{2} - \frac{3\pi}{8}\right)$

$= \cos\frac{\pi}{8}$

$2(a^4 + b^4)$

$(a^2+b^2)^2 = \underline{a^4+b^4} + 2a^2b^2$

$(a^2+b^2)^2 - 2a^2b^2 = a^4+b^4$

$1^2 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8} = \sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}$

$1 - 2\left(\frac{2\sin\frac{\pi}{8}\cos\frac{\pi}{8}}{2}\right)^2$

$1 - 2\left(\frac{\sin\frac{2\pi}{8}}{2}\right)^2$

$1 - \cancel{2}\,\frac{\sin^2\frac{\pi}{4}}{\cancel{4}\,2} = \frac{3}{4}$

$2\left(\frac{3}{4}\right) = \frac{3}{2}$

(18) $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3.$ $\quad \theta \in R$

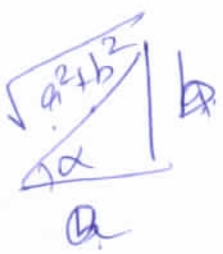
$5\cos\theta + 3\cos\theta\cos\frac{\pi}{3} - 3\sin\theta\sin\frac{\pi}{3} + 3$

$5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\dfrac{\frac{13}{2}}{\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}}\cos\theta - \dfrac{\frac{3\sqrt{3}}{2}}{\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}}\sin\theta + 3$ $\quad \dfrac{a\cos\theta}{\sqrt{a^2+b^2}} + \dfrac{b\sin\theta}{\sqrt{a^2+b^2}} = \sin(\theta \pm \alpha)$

$\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}\left(\cos\alpha\cos\theta - \sin\alpha\sin\theta\right) + 3.$



$7\cos(\theta + \alpha) + 3.$

$-7 + 3 = -4$ min

$7 + 3 = 10$ max.

$\sqrt{\frac{169 + 27}{4}}$

$\sqrt{\left(\frac{14^2}{2^2}\right)} = \frac{14}{2} = 7$

(B)

20

$$\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ$$

$$\frac{1}{4}\left\{2\cos 6^\circ \cos 66^\circ\right\}\left\{2\cos 42^\circ \cos 78^\circ\right\}$$

$$\frac{1}{4}\left\{\cos 72^\circ + \cos 60^\circ\right\}\left\{\cos 120^\circ + \cos 36^\circ\right\}$$

$$\frac{1}{4}\left\{\cancel{\cos}\,\sin 18^\circ + \frac{1}{2}\right\}\left\{-\frac{1}{2} + \sin 54^\circ\right\}$$

$$\frac{1}{4}\left\{\frac{\sqrt{5}-1}{4} + \frac{1}{2}\right\}\left\{-\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right\}$$

$$\frac{1}{4}\left\{\frac{\sqrt{5}+1}{4}\right\}\left\{\frac{\sqrt{5}-1}{4}\right\}$$

$$\frac{1}{16\times\cancel{4}} \times \cancel{4} = \frac{1}{16}$$

(26) $A = 133^\circ \qquad 2\cos\left(\frac{A}{2}\right) = ?$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos A = \sqrt{\frac{1+\cos 2A}{2}}$$

$$\cos \frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$$

$2\cos\frac{133}{2} = 2\cos 66.5 < 1$

A) $-\sqrt{1+\sin A} - \sqrt{1-\sin A}$ X $-$ve

B) $-\sqrt{1+\sin A} + \sqrt{1-\sin A}$ X $-$ve

C) $\sqrt{1+\sin A} - \sqrt{1-\sin A}$ possible $+$ve

D) $\sqrt{1+\sin A} + \sqrt{1-\sin A}$ $+$ve > 1 X

(29) $A = \sin^2\theta + \cos^4\theta$

$\cos^2\theta = \left(\frac{1+\cos 2\theta}{2}\right)$

$= \sin^2\theta + (\cos^2\theta)^2$

$= \frac{1-\cos 2\theta}{2} + \left(\frac{1+\cos 2\theta}{2}\right)^2$

$= \cancel{1+\cos}\ \frac{1}{4}\left(\cos^2 2\theta + 2\cos 2\theta + 1\right) + \frac{1}{4}\left(2 - 2\cos 2\theta\right)$

$A = \frac{1}{4}\left(\cos^2 2\theta + 3\right)$

$$A = \frac{1}{4}\cos^2 2\theta + \frac{3}{4}$$

$$A_{min} = \frac{3}{4} \qquad A_{max} = 1$$

$$\frac{3}{4} \le A \le 1 \qquad B$$

30 $A + B + C = \pi.$

$$X = \tan^2\left(\frac{A}{2}\right) + \tan^2\frac{B}{2} + \tan^2\left(\frac{C}{2}\right)$$

$\text{Min}(X)$

$$X = \tan^2\left(\frac{A}{2}\right) + \tan^2\frac{B}{2} + \tan^2\left(\frac{\pi}{2} - \frac{(A+B)}{2}\right)$$

$$= \tan^2\left(\frac{A}{2}\right) + \tan^2\left(\frac{B}{2}\right) + \cot^2\left(\frac{A+B}{2}\right)$$

$$= \tan^2\left(\frac{A}{2}\right) + \tan^2\left(\frac{B}{2}\right) + \frac{1}{\tan^2\left(\frac{A+B}{2}\right)}$$

$\cancel{= \tan^2\left(\frac{A}{2}\right) + \tan^2 B}$ $\quad x = \tan\frac{A}{2} \quad y = \tan\frac{B}{2}.$

$$x^2 + y^2 + \frac{1}{\left(\frac{x+y}{1-xy}\right)^2}$$

$$\cancel{x = x^2 + y^2 \mp \frac{1 + x^2y^2 - 2xy}{x^2 + y^2 + 2xy}}$$

A.M $\geq$ G.M

$$x^2 \quad y^2 \quad z^2$$

$$\frac{x^2 + y^2 + z^2}{3} \geq (x^2y^2z^2)^{1/3}$$

$$x^2 + y^2 + z^2 \geq 3(x^2y^2z^2)^{1/3}$$

$$3x^2 = 3x^2 \qquad x^2 = y^2 = z^2$$

$$\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$$

$$= 3\tan^2\frac{A}{2}$$

$$= 3 \times \frac{1}{3} = 1$$

Pg 15 6

$$\tan\frac{x}{2} = \operatorname{cosec} x - \sin x$$

$$= \frac{1}{\sin x} - \sin x$$

$$\tan\frac{x}{2} = \frac{1-\sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\tan\frac{x}{2} = \frac{\left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)^2}{\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}} = \frac{\dfrac{1+\tan^4\frac{x}{2}-2\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{2\tan\frac{x}{2}}$$

$$2\tan^2\frac{x}{2} = \frac{\tan^4\frac{x}{2} - 2\tan^2\frac{x}{2} + 1}{1 + \tan^2\frac{x}{2}}$$

$$2\tan^4\frac{x}{2} + 2\tan^2\frac{x}{2} = \tan^4\frac{x}{2} - 2\tan^2\frac{x}{2} + 1$$

$$\tan^4\frac{x}{2} + 4\tan^2\frac{x}{2} - 1 = 0$$

$$\tan^2\frac{x}{2} = \frac{-4 \pm \sqrt{16 - 4 \cdot 4}}{2}.$$

$$= \frac{-4 \pm \sqrt{20}}{2}$$

$$\tan^2\frac{x}{2} = -2 \pm \sqrt{5}$$

$$= -2 + \sqrt{5}$$

$(\sqrt{5} - 2)$

B) C) ✓

(8) $\sin A + \sin B + \sin C = \cos A + \cos B + \cos C = 0$

$\sin A + \sin B + \sin C = 0$

$\sin^2 A + \sin^2 B + \sin^2 C = -2(\sin A \sin B + \sin B \sin C + \sin A \sin C)$

$\sin A + \sin B = -\sin C.$

$\sin^2 A + \sin^2 B + 2 \sin A \sin B = \sin^2 C$

$-2 \sin A \sin B = \sin^2 A + \sin^2 B - \sin^2 C$