Ray Optics & Optical Instruments

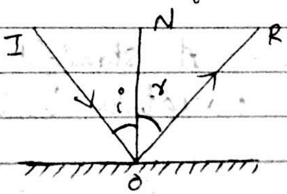
Optics :- It is the branch of physics that deals with the study of nature, production and propagation of light.

Two main branches

1. Ray / Geometrical Optics :- It concerns with the particle nature of light. It explains the formation of images in mirrors & lenses.
2. Wave / Physical Optics :- It concerns with the wave nature of light.

Reflection of light :-

When light is incident on a smooth polished surface, it bounce back to the same medium. This process is called reflection of light.



IO = incident ray.
OR = reflected ray.
ON = normal.
r = angle of reflection
i = angle of incidence

Laws of Reflection

1. Incident ray, reflected ray and the normal lies on the same plane.
2. Angle of incidence $\angle i$ = Angle of reflection $\angle r$.

Spherical Mirrors :-

It is a reflecting surface which forms a part of a sphere.

Two types of spheres are

1. **Concave mirror** :- A spherical Mirror in which the reflection of light takes place from the inner

Surface.

Convex mirror :- A spherical mirror in which the reflection of light takes place from the outer surface.

Important terms connected with Spherical Mirrors.

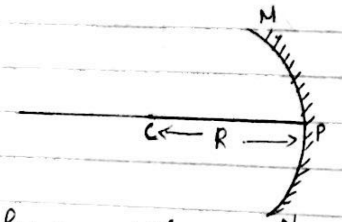
1. Pole (P) :- The centre of the spherical mirror.

2. Centre of curvature (C) :- It is the centre of the sphere of which mirror is a part.

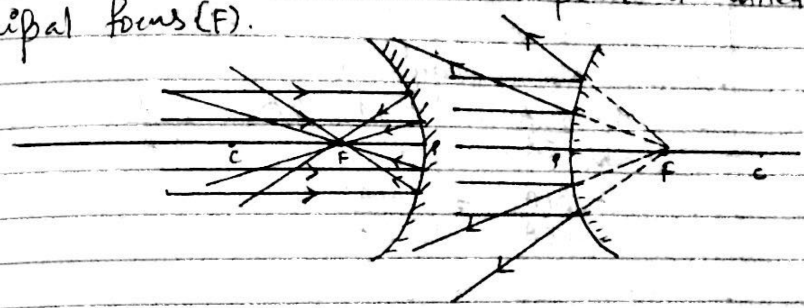
3. Radius of curvature (R) :- It is the radius of the sphere of which mirror is a part.

4. Principal axis :- A straight line passing through the centre of curvature and pole of the mirror.

5. Aperture :- It is the diameter of the spherical mirror.

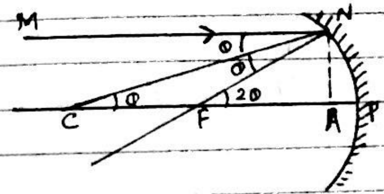


6. Principal focus :- When a number of rays incident parallel to the principal axis of a spherical mirror, after reflection it converges to the common point in the case of concave mirror and appears to diverge from the common point in the case of convex mirror. This common point is called Principal focus (F).



7. Focal length (f) - It is the distance between the principal focus and the pole of the mirror.

Relation between R and f in spherical Mirror.



The ray MN incident on the concave mirror at an angle of incidence θ. The ray is reflected making an angle θ with the normal along NF.

From the figure

$\angle MNC = \theta$

$\angle CNF = \theta$

$\angle NCF = \theta$

$\angle NFP = 2\theta$

NA be the ⊥r from N on the principal axis where A is very close to P.

In $\triangle NCA$

$$\tan\theta = \frac{NA}{CA} \quad \text{--- (1)}$$

and In $\triangle NFA$

$$\tan 2\theta = \frac{NA}{FA} \quad \text{--- (2)}$$

For small angle θ

$\tan\theta = \theta$ and $\tan 2\theta = 2\theta$

$\therefore$ the above equ's becomes

$\theta = \frac{NA}{CA}$ and $2\theta = \frac{NA}{FA}$

or $2 \times \frac{\cancel{NA}}{CA} = \frac{\cancel{NA}}{FA}$

$2 \times \frac{1}{CA} = \frac{1}{FA}$

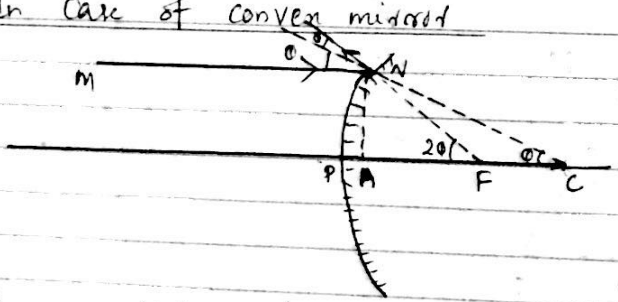
A is very close to P

$\therefore \frac{2}{CP} = \frac{1}{FP}$

$\frac{2}{R} = \frac{1}{f}$

or $f = \frac{R}{2}$

In case of convex mirror



The ray MN incident on the convex mirror at an angle θ. The ray reflected making an angle θ with the normal.
From the figure

$\angle NCP = \theta$

$\angle NFP = 2\theta$.

NA be the $\perp^r$ from N on the principal axis where A is close to P

In ΔNCA, $\tan\theta = \frac{NA}{CA}$ —①

ΔNAF $\tan 2\theta = \frac{NA}{FA}$ —②

For small angle θ $\tan\theta = \theta$ and $\tan 2\theta = 2\theta$.

equ's ①&②. $\theta = \frac{NA}{CA}$ and $2\theta = \frac{NA}{FA}$

$2 \times \frac{\cancel{NA}}{CA} = \frac{\cancel{NA}}{FA}$ or $\frac{2}{CA} = \frac{1}{FA}$.

$\frac{2}{R} = \frac{1}{f}$ or $R = 2f$

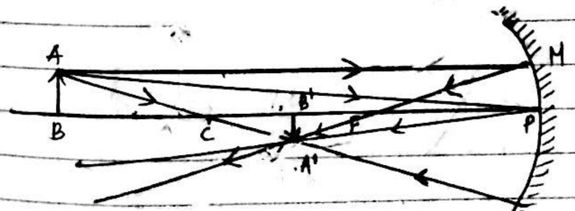
Mirror formula.

The mathematical relationship between object distance (u), image distance (v) and focal length (f) of a spherical mirror is

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.

Derivation of Mirror formula.

1. For a concave mirror – (Forms real image)



Consider an object AB placed on the principal axis beyond C of a concave mirror. A ray AM from the object incident parallel to the principal axis after reflection passes through the focus. Another ray AP incident on the pole of the mirror and is reflected along PB'. These two rays meet at B'. Therefore the image

$A'B'$ is formed which is real and inverted.
From the figure.

$PB = -u$

$PB' = -v$

$PC = -R$

$PF = -f$

Consider the similar Δ's

ΔABC and $\Delta A'B'C$

$$\frac{AB}{A'B'} = \frac{BC}{B'C}$$

$$= \frac{PB - PC}{PC - PB'}$$

$$= \frac{-u + R}{-R + v} \quad \text{——— ①}$$

ΔABP and $\Delta A'B'P$ are similar

$$\frac{AB}{A'B'} = \frac{AP}{B'P}$$

$$= \frac{-u}{-v}$$

$$= \frac{u}{v} \quad \text{——— ②}$$

From ① and ②

$$\frac{-u+R}{-R+v} = \frac{u}{v}$$

$$-uv + Rv = -uR + uv$$

$$Rv + Ru = 2uv$$

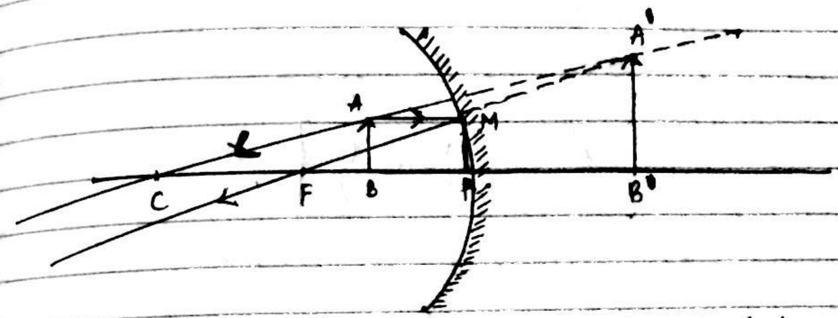
Dividing throughout by uvR

$$\frac{Rv}{uvR} + \frac{Ru}{uvR} = \frac{2uv}{uvR}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} \Rightarrow \frac{1}{u} + \frac{1}{v} = \frac{2}{2f}$$

or $\boxed{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$

2. For a concave mirror:- (forms Virtual image).



When an object AB is placed between the principal focus and Pole, a virtual erect image $A'B'$ is formed behind the mirror.
Using Cartesian sign convention

$BP = -u \qquad CP = -R = -2f$

$PB' = v$

$PF = -f$

Now ΔABC and $\Delta A'B'C$ are similar

$$\frac{AB}{A'B'} = \frac{CB}{CB'}$$

$$= \frac{CP - BP}{CP + PB'} = \frac{-2f + u}{-2f + v} \quad \text{——— ①}$$

ΔMPF and $\Delta A'B'F$ are similar

$$\frac{MP}{A'B'} = \frac{FP}{FB'}$$

$$\frac{AB}{A'B'} = \frac{FP}{FP + PB'}$$

$$= \frac{-f}{-f + v} \quad \text{——— ②}$$

from ① and ②

$$\therefore \frac{-2f + u}{-2f + v} = \frac{-f}{-f + v}$$

$$2f^2 - fu - 2fv + uv = 2f^2 - fv$$

or $$uv = fv + fu$$

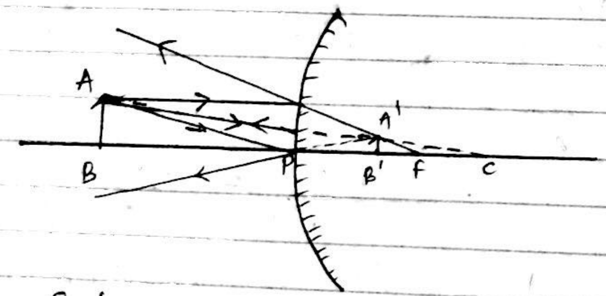
Dividing throughout by uvf

$$\frac{uv}{uvf} = \frac{fv}{uvf} + \frac{fu}{uvf}$$

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

3. Derivation of mirror formula for a convex mirror

Consider an object AB placed on the principal axis of a convex mirror of small aperture. After reflection a virtual and erect image A'B' is formed behind the mirror.



Using Cartesian sign convention

$BP = -u$

$PB' = -v$

$FP = f$

$PC = R = 2f$

Now

ΔABC and $\Delta A'B'C$ are similar

$$\frac{A'B'}{AB} = \frac{B'C}{BC} = \frac{PC - PB'}{BP + PC} = \frac{R - v}{-u + R} \quad (1)$$

$\therefore \Delta A'B'P \sim \Delta ABP$

$$\frac{A'B'}{AB} = \frac{PB'}{PB}$$

$$= \frac{+v}{-u} \quad (2)$$

$\therefore$ from (1) and (2)

$$\frac{R - v}{-u + R} = \frac{v}{-u}$$

$$-uR + uv = -uv + Rv$$

$$2uv = uR + Rv$$

Dividing both sides by uvR

$$\frac{2uv}{uvR} = \frac{uR}{uvR} + \frac{Rv}{uvR}$$

$$\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{2}{2f} = \frac{1}{v} + \frac{1}{u}$$

or $$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

Magnification. The ratio of height of the image to the height of the object is called linear magnification

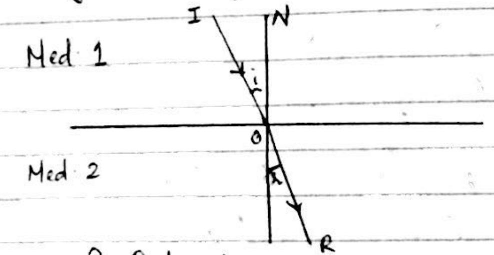
$$m = \frac{\text{Height of the image}}{\text{Height of the object}}$$

$$= \frac{h'}{h}$$

$$= \frac{-v}{u}$$

Refraction of light.

When light travels from one medium to another at the surface of seperation it bends. The bending of light is called refraction of light



Laws of Refraction.

1. The incident ray, the refracted ray and the normal lies on the same point.
2. The ratio of sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media.

ie $\frac{\sin i}{\sin r}$ = a Constant

$$\frac{\sin i}{\sin r} = n_{21}$$

where n_{21} is the refractive index of the second medium with respect to the first medium

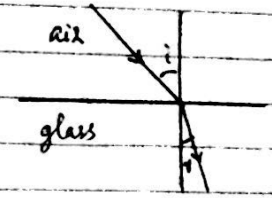
This law is called Snell's law.

Properties of refraction of light.

1. When a ray of light passes from an optically rarer medium to a denser medium it bends towards the normal.

In this case $\angle i > \angle r$.

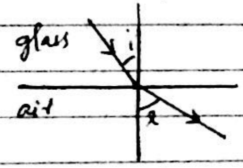
and $n_{21} > 1$



2. When a ray of light passes from an optically denser medium to rarer medium it bends away from the normal.

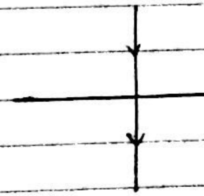
In this case $\angle r > \angle i$

and $n_{21} < 1$



3. A ray of light traveling along the normal passes undeflected.

ie $\angle i = \angle r = 0°$.



Refractive index interms of speed of light.

Refractive index of a medium may be defined as the ratio of speed of light in vacuum to the speed of light in that medium.

ie $n = \frac{c}{v}$ It is also called absolute refractive index.

Cause of refraction

The bending of light occurs due to the change in speed of light.

According to Snell's law

$$n_{21} = \frac{V_1}{V_2} = \frac{\sin i}{\sin r}$$

If $V_1 > V_2$

$\sin i > \sin r$

$i > r$, ray bends towards the normal.
The medium 2 is said to be denser medium.

If $V_1 < V_2$
$\sin i < \sin r$

$i < r$, ray bends away from the normal.
The medium 2 is said to be optically rarer medium.

If n_{21} is the refractive index of medium 2 with respect to medium 1 and n_{12} the refractive index of medium 1 with respect to medium 2

$$\therefore n_{12} = \frac{1}{n_{21}}$$

It follows that $n_{32} = n_{31} \times n_{12}$

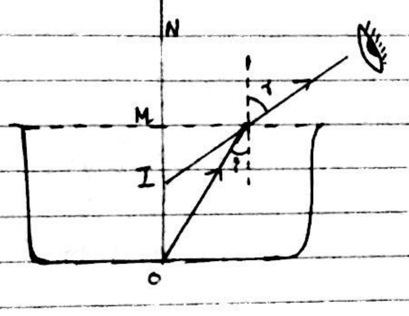
$$= \frac{V_1}{V_3} \times \frac{V_2}{V_1}$$

$$= \frac{V_2}{V_3} = n_{32}.$$

Practical Application of Refraction.

1. Real Depth and Apparent Depth.

Due to refraction of light the apparent depth of an object placed in denser medium is less than the real depth.



From figure.

OM = real depth
IM = apparent depth.
I = virtual image of O.

Clearly apparent depth IM is smaller than the real depth OM.
That is why an object placed at the bottom of water tank appears appears to be raised or appears shallower.

Refractive index $n = \frac{\text{real depth}}{\text{apparent depth}}$.

$$n = \frac{OM}{IM}$$

$$\therefore IM = \frac{OM}{n}$$

Normal Shift :- The height through which an object appears to be raised in a denser medium is called normal shift.

Thus normal shift = real depth − Apparent depth

$$= OM - IM$$

$$= OM - \frac{OM}{n}$$

$$= OM\left[1 - \frac{1}{n}\right]$$

But $OM = d$ is the real depth

$\therefore$ normal shift $S = d\left[1 - \frac{1}{n}\right]$

2. Advance sunrise and delayed sunset

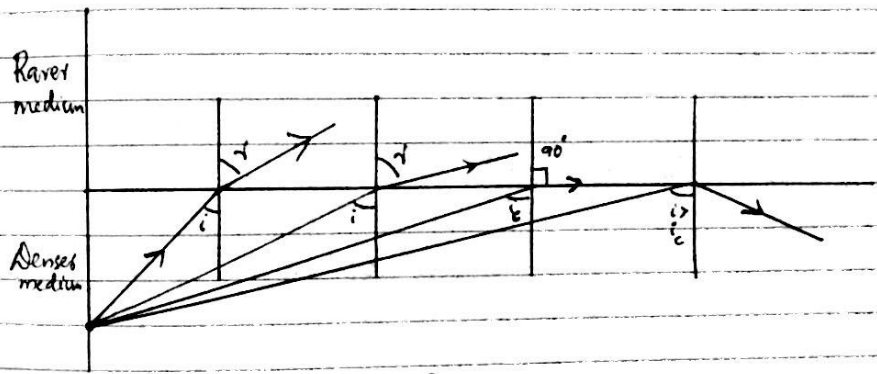
It is due to atmospheric refraction the sun is visible before actual sunrise and after actual sunset. At high altitude the density and refractive index of air layer decreases. The light ray starting from the sun travel from rarer to denser layers. They bent more and more towards the normal. This refracted ray reaches the observers eye. Due to this reason sun is visible 2 minute before the actual sun rise.

3. Apparent flattening (oval shape) of the sun at sunrise and sun set.

It is due to atmospheric refraction. At high altitude the density and refractive index of air layers decreases. The light ray starting from the sun travel from rarer to denser layers. They bent more and more towards the normal. This refracted ray reaches the observers eye.

Total internal reflection (TIR)

If light rays passes from an optically denser to a rarer medium it bends away from the normal. As the angle of incidence increases the angle of refraction also increases. At a particular value of angle of incidence the angle of refraction becomes 90°. This angle of incidence is called critical angle (i_c). If the angle of incidence is greater than the critical angle, the ray is totally reflected back. This phenomenon is called total internal reflection.



Necessary conditions for TIR

1. Light must travel from an optically denser to rarer medium

2. The angle of incidence must be greater than the critical angle (i_c) for the two media.

Relation between critical angle and refractive index.

According to Snell's law $\frac{\sin i}{\sin r} = n_{12}$

or $\frac{\sin i}{\sin r} = \frac{n_1}{n_2}$

If $i = i_c$, $r = 90°$

$$\frac{\sin i_c}{\sin 90} = \frac{n_1}{n_2}$$

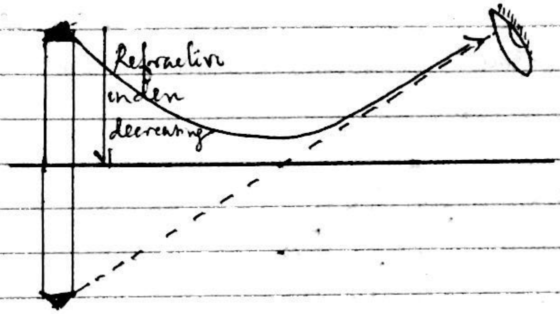
If the rarer medium is air $n_1 = 1$
and $n_2 = n$

$$\sin i_c = \frac{1}{n}$$

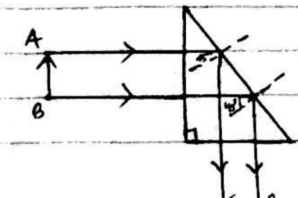
or $n = \frac{1}{\sin i_c}$

Application of TIR

1. Mirage: It is an optical illusion observed in desert or overhot extended surfaces like coal-tarred road due to which a travel sees a shimmering pond of water some distance ahead of him in which the surrounding objects like trees etc appears inverted.
Reason:- On a hot summer day, the surface of the earth becomes very hot. The layers of air near the earth are more heated than the higher ones. Hence the density increases as we move up. As the rays of light from a distant object travel towards earth through different layers they bent more and more away from the normal. A stage is reached when the angle of incidence becomes greater than the critical angle i_c and the ray is totally reflected. These rays reach the observer eye and he sees the inverted image.

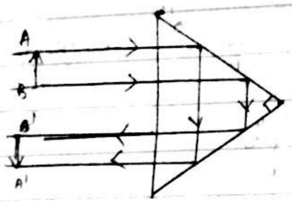


2. Total reflecting Prisms.
A right angled isosceles prism ie 45°-90°-45° is called a totally reflecting prisms. Whenever a ray falls normally on any face of such a prism it is incident on the inside face at 45°, ie an angle greater than the critical angle of the glass (42°) Hence this ray is always totally internally reflected. These prisms may be used in 3 ways.
(a) to deviate a ray through 90°.
A ray is incident normally on the hypotenuse at angle of 45°, greater than the critical angle. The light is totally internally reflected by the deviation of 90°.



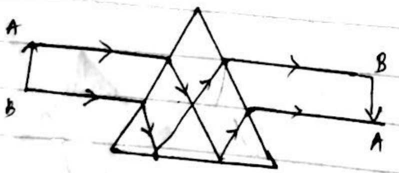
(b) To invert an image with deviation of rays through 180°.

Light is incident normally through the hypotenuse, it first suffer TIR from one of the faces and then from the other face. The final ray emerges through the hypotenuse deviated by 180°.



(c) To invert an image without deviation of light

Light enters through one of the short face at an angle. After refraction it is total internal reflected from the hypotenuse and refracted out of the other shorter face to become parallel to the incident ray. The ray do not suffer any deviation, it only becomes inverted.



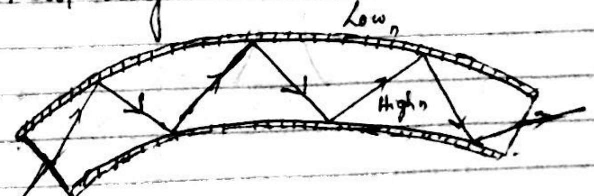
3. Brilliance / Sparkling of Diamond.

The brilliance of diamond is due to TIR. As the refractive index of diamond is very large, its critical angle is very small about 24.4°. The faces of diamond are so cut that the light entering the crystal suffers TIR repeatedly.

4. Optical fibres:-

Optical fibre make use of the phenomenon of total internal reflection. Optical fibres are fabricated with high quality glass or quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding.

When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out at the other end. Since light undergoes total internal reflection at each stage there is no appreciable loss in the intensity of the light signal. Optical fibres are fabricated such that light reflected at one side of inner surface strikes the other at an angle larger than the critical angle.



Uses of optical fibre.

1. Optical fibres are entensively used to tran[s]m[it] and receiving electrical signals in tele commun[ication]

2. Optical fibres can be used as light pipe to facilitate visnal enaminabes of interna[l] organs like esophagus stomach and intestines. This technique is called endoscofy.

3. It can be used as decorative lamps.

Spherical Lenses.

A lens is a piece of refract[ing] medium bounded by a spherical surfaces. It can be divided into two

1. conven lens (Converging lens)
2. Concave lens (diverging lens)

Conven lens.

This is a spherical lens, thicker at the centre than at the edges.

Concave lens.

This is a spherical lens, thinner at the centre than at the edges.

Definitions in connection with spherical lenses

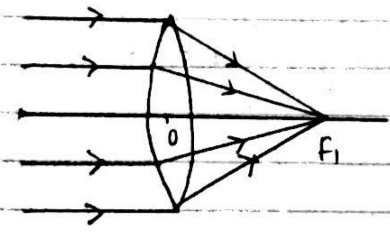
1. Optic centre: (O):- The centre of the lens is called the optic centre.

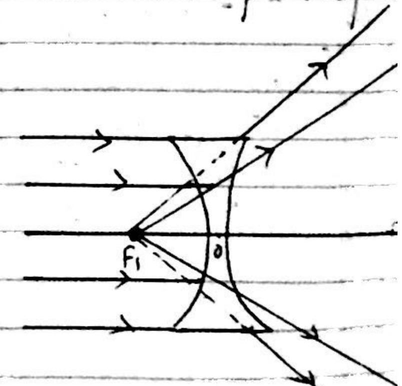
2. Centre of curvature (C):- The centre of curvature is the centre of the sphere of which lens is a part. A lens has 2 surfaces. So it has two centres of curvature (C_1 and C_2).

3. Radius of curvature (R):- It is the radius of the sphere of which lens is a part

4. Principal anis:- A straight line passing through two centres of curvature of the l[ens] is called the principal anis

5. Principal focus:- When a number of rays incident parallel to the principal anis after refraction, it converges to the common point in the case of conven lens and appears to diverge from a common point in the ca[se] of concave lens. This common point is ca[lled] principal focus. A lens has two principal focuses. (F_1 and F_2).





Focal length (f):- The distance between the principal focus and the optic centre is called focal length (f).

New Cartesian sign conventions for spherical lenses.

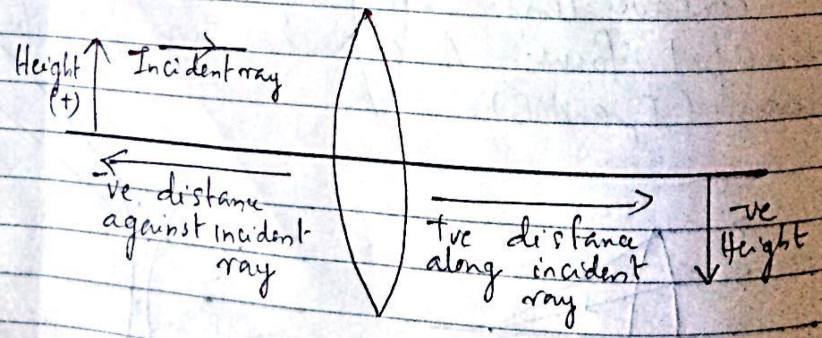
1. All distances are measured from the optic centre of the lens.

2. The distance measured in the same direction of the incident ray is taken as +ve.

3. The distance measured in the opposite direction of incident ray is taken as -ve.

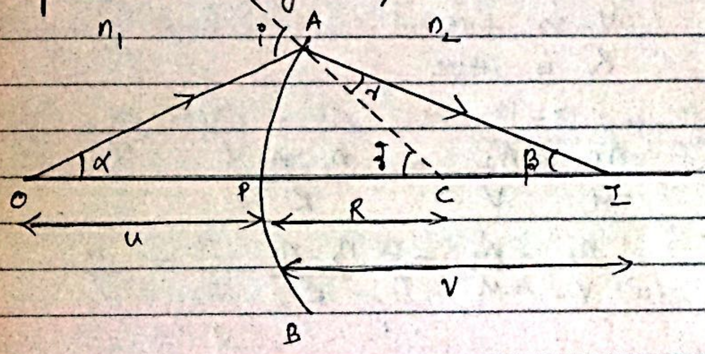
4. Height measured upward and perpendicular to the principal axis is taken as +ve.

5. Height measured downward and perpendicular to the principal axis is taken as -ve.



Refraction through a Spherical surface.

APB is a convex surface which seperates two media of refractive index n_1 and n_2. A point object O is placed at a distance 'u' from the surface a ray OA incident at an angle i, after refraction it bends towards the normal and an image I is formed making an angle r with the normal. α, β and γ are the angles made with the principal axis by O, I and C



In ΔAOC

$$i = \alpha + \gamma \quad \text{— ①}$$

In ΔAIC

$$\gamma = r + \beta$$

$$\therefore r = \gamma - \beta \quad \text{— ②}$$

According to Snell's law

$$\frac{\sin i}{\sin r} = n_{21} = \frac{n_2}{n_1}$$

for small aperature i and r are small

$$\therefore \sin i = i$$

$$\sin r = r$$

$\therefore$ the above equation becomes

$$\frac{i}{r} = \frac{n_2}{n_1}$$

$n_1 i = n_2 r$

$n_1(\alpha+\gamma) = n_2(\gamma-\beta)$

$$n_1\left(\frac{AP}{u}+\frac{AP}{R}\right) = n_2\left[\frac{AP}{R}-\frac{AP}{V}\right]$$

$$\frac{n_1}{u}+\frac{n_1}{R} = \frac{n_2}{R}-\frac{n_2}{V}$$

$$\frac{n_1}{u}+\frac{n_2}{V} = \frac{n_2-n_1}{R}$$

Using new cartesian sign convention

u is -ve

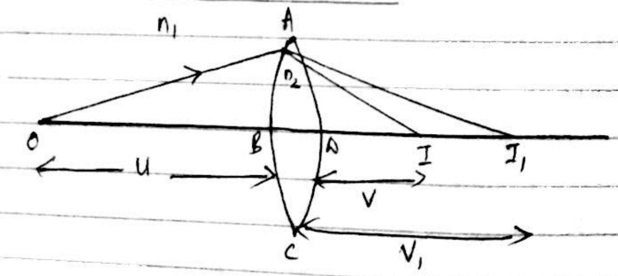
V is +ve

R is +ve

$$\therefore \frac{n_1}{-u}+\frac{n_2}{V} = \frac{n_2-n_1}{R}$$

ie $$\frac{n_2}{V}-\frac{n_1}{u} = \frac{n_2-n_1}{R}$$

Lens maker's formula.



Consider a convex lens of refractive index n_2 placed in a medium of refractive index n_1 consisting of two spherical surfaces ABC and ADC of radius of curvature R_1 and R_2. A point object O is placed on the principal axis in the medium of refractive index n_1. An image I_1 is formed at a distance V_1 from the first surface

ie $$\frac{n_2}{V_1}-\frac{n_1}{u} = \frac{n_2-n_1}{R_1} \quad \text{—— (1).}$$

The image I_1 act as a virtual object for the second surface the final image I is formed on the same side at a distance V from the surface

$$\therefore \frac{n_1}{V}-\frac{n_2}{V_1} = \frac{n_1-n_2}{R_2} \quad \text{—— (2).}$$

Adding (1) and (2)

$$\cancel{\frac{n_2}{V_1}}-\frac{n_1}{u}+\frac{n_1}{V}-\cancel{\frac{n_2}{V_1}} = \frac{n_2-n_1}{R_1}+\frac{n_1-n_2}{R_2}$$

$$\therefore \frac{n_1}{V}-\frac{n_1}{u} = n_2-n_1\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\cancel{n_1}\left[\frac{1}{V}-\frac{1}{u}\right] = \cancel{n_1}\left[\frac{n_2}{n_1}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\frac{1}{V}-\frac{1}{u} = \left[\frac{n_2}{n_1}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\frac{1}{V}-\frac{1}{u} = \left[n_{21}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad \text{—— (3)}$$

This is Lens maker's formula.

When an object is at infinity. the image is formed at the focus ie $u = \infty$

$v = f$

∴ the Lens maker's formula becomes

$$\frac{1}{f} - \frac{1}{\infty} = \left[{}_{21}n - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = \left[{}_{21}n - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{--- (4)}$$

Compairing (3) and (4)

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

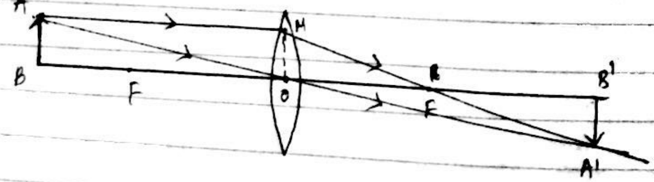
This is thin Lens formula.

Thin lens formula.

It is a mathematical relationship between object distance 'u', image distance 'v' and focal length 'f' of a spherical lens.

The relation is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Derivation of thin lens formula for a convex lens when it forms a real image.



Consider an object AB placed ⊥r to the principal axis of a convex lens beyond F. A real, inverted and magnified image A'B' is formed on the other side of the lens.

From the figure $\Delta ABO \sim \Delta A'B'O$

$$\frac{AB}{A'B'} = \frac{BO}{B'O} = \frac{-u}{v} \quad \text{--- (1)}$$

$\Delta MOF \sim \Delta A'B'F$

$$\frac{MO}{A'B'} = \frac{OF}{B'F}$$

$$\frac{AB}{A'B'} = \frac{OF}{OB' - OF}$$

$$= \frac{f}{v - f} \quad \text{--- (2)}$$

$$\therefore -\frac{u}{v} = \frac{f}{v - f}$$

$$-u(v - f) = fv$$

$$-uv + uf = fv$$

Dividing through out by uvf

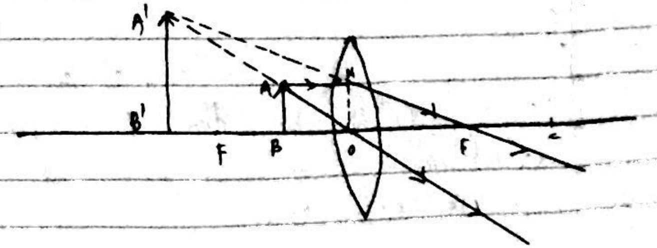
$$\therefore \frac{-uv}{uvf} + \frac{uf}{uvf} = \frac{fv}{uvf}$$

$$-\frac{1}{f} + \frac{1}{v} = \frac{1}{u}$$

$$\text{or} \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Derivation of thin lens formula for a convex lens when it forms a virtual image.

When an object AB is placed between the optical centre and the focus F of a convex lens, the image A'B' formed by the convex lens is virtual erect and magnified.



From the figure A'B'O and ABO are similar

$$\frac{A'B'}{AB} = \frac{B'O}{BO}$$

$$= \frac{-v}{-u} = \frac{v}{u} \quad \text{— (1)}$$

ΔA'B'F and MOF are similar

$$\frac{A'B'}{MO} = \frac{B'F}{OF}$$

$$= \frac{B'O + OF}{OF}$$

$$\frac{A'B'}{AB} = \frac{-v+f}{f} \quad \text{— (2)}$$

from (1) and (2) $\frac{v}{u} = \frac{-v+f}{f}$

$$vf = -uv + uf$$

Dividing throughout by uvf

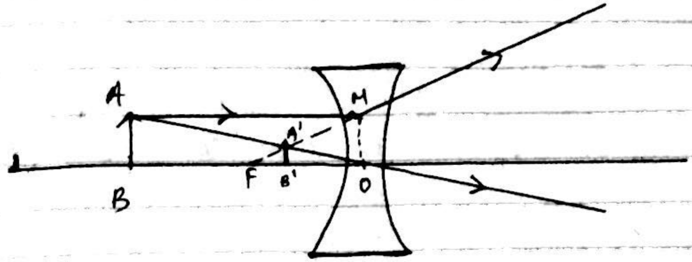
$$\frac{vf}{uvf} = -\frac{uv}{uvf} + \frac{uf}{uvf}$$

$$= \frac{1}{u} = -\frac{1}{f} + \frac{1}{v}$$

or $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Derivation of thin lens formula for a concave lens.

AB is an object placed perpendicular to the principal axis. A virtual erect and diminished image A'B' is formed due to refraction.



From the figure ΔABO and ΔA'B'O are similar

$$\frac{A'B'}{AB} = \frac{B'O}{BO}$$

$$= \frac{-v}{-u} = \frac{v}{u} \quad \text{— (1)}$$

Δ A'B'F and MOF are similar

$$\frac{A'B'}{MO} = \frac{FB'}{FO}$$

$$= \frac{FO - B'O}{FO}$$

$$\frac{A'B'}{AB} = \frac{-f+v}{-f} \quad \text{— (2)}$$

from (1) and (2)

$$\frac{v}{u} = \frac{-f+v}{-f}$$

$$-vf = -uf + uv$$

Deviding throughout by uvf

$$\frac{-vf}{uvf} = \frac{-uf}{uvf} + \frac{uv}{uvf}$$

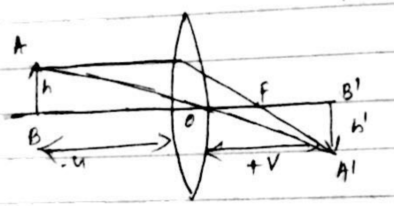
$$-\frac{1}{u} = -\frac{1}{v} + \frac{1}{f}$$

or $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Linear magnification: It is defined as the ratio of size of the image to the size of the object.

i.e. $m = \dfrac{\text{Size of the image}}{\text{Size of the object}}$

$= \dfrac{h'}{h}$



ΔAOB and $\Delta A'OB'$

$$\frac{AB}{A'B'} = \frac{BO}{B'O}$$

or $$\frac{A'B'}{AB} = \frac{B'O}{BO}$$

$$\frac{-h'}{h} = \frac{v}{-u}$$

or $\dfrac{h'}{h} = \dfrac{v}{u} = m$: This is magnification

Power of a lens.

Power of a lens is a measure of the convergence and divergence which a lens introduces in the light falling on it.

It is defined as the reciprocal of focal length

$$P = \frac{1}{f}.$$

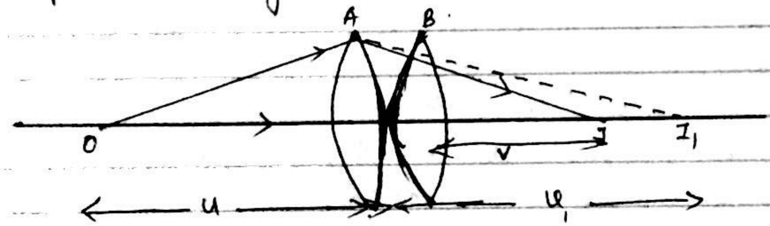
Its S-I unit is $\dfrac{1}{m} = m^{-1}$ or dioptre (D).

1 Dioptre is defined as the power of a lens of focal length 1 metre.

Power of a lens is +ve, the lens becomes converging (convex lens). It is -ve, the lens becomes diverging (concave lens).

Combination of thin lenses in contact.

Consider two lenses A & B of focal length f_1 and f_2 placed in contact with each other. Let the object be placed at a point O beyond the focus of first lens A.



The first lens produces an image at I_1. This image I_1 act as a virtual object for the second lens producing the final image at I

For the image formed by the first lens A

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \quad \text{---} (1)$$

For the image formed by the second lens B

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \text{---} (2)$$

(1) + (2) =>

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{---} (3)$$

If the two lens-systems is regarded as equ- to a single lens of focal length f we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

∴ equ:(3) becomes

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

For 'n' lenses

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \cdots + \frac{1}{f_n}$$

In terms of power

$$P = P_1 + P_2 + \cdots + P_n.$$

Combination of lenses helps to obtain diverging or converging lenses of desired magnification. It also enhances sharpness of the image. Since the image formed by the first lens becomes the object for the second. implies that the total magnification 'm' of the combination is a product of magnification of individual lens

$$m = m_1 m_2 m_3 \cdots$$

Such a system of combination of lenses is commonly used in designing lenses for cameras, microscopes and telescopes etc.

Refraction through a glass prism

ABC is the outline of a glass prism. AB and AC are the faces of the prism and BC is the base of the prism. A is the angle of the prism

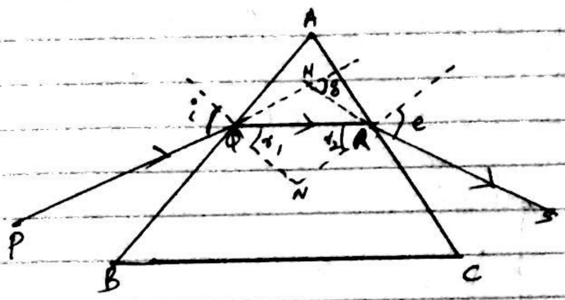


Figure shows the refraction of light through a triangular glass prism. PQ is the incident ray incident at an angle of incidence i. QR is the refracted ray refracted at an angle r_1, r_2 and RS is the emergent ray and the angle of emergence is e

The angle between the direction of the incident ray PQ and the emergent ray RS is called the angle of deviation (δ)

In the quadrilateral AQNR

$$\angle A + \angle QNR = 180°$$

From the triangle QNR

$$r_1 + r_2 + \angle QNR = 180°$$

Comparing the above equations

$$\angle A + \angle QNR = r_1 + r_2 + \angle QNR$$

$$\therefore \angle A = r_1 + r_2 \quad \text{——} (1)$$

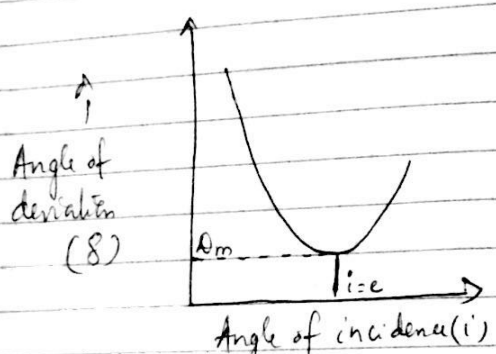
From the $\triangle^{le}$ QMR

$$\delta = (i - r_1) + (e - r_2)$$

$$= i + e - (r_1 + r_2)$$

$$= i + e - A \quad \text{——} (2)$$

The angle of deviation depends on the angle of incidence. As the angle of incidence increases angle of deviation decreases reaches a minimum value D_m and then increases.



When $\delta = D_m$

$i = e$ which implies $r_1 = r_2 = r$

$\therefore$ equ (1) becomes

$$A = r + r = 2r$$

$$\text{or} \quad r = A/2 \qquad \text{——} \ (3)$$

equ(2) becomes

$$D_m = i + i - A$$

$$= 2i - A$$

$$\text{or} \quad 2i = A + D_m$$

$$i = \frac{A + D_m}{2} \qquad — (4)$$

The refractive index of the prism is

$$n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin(A/2)}$$

Deviation produced by a small angle

Refractive index of the prism

$$n = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin(A/2)}$$

For a small angle prism

ie a thin prism, D_m is very small

$$\therefore \quad n = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin(A/2)} \simeq \frac{\left(\frac{A + D_m}{2}\right)}{(A/2)}$$

$$n = \frac{A + D_m}{A}$$

$$nA = A + D_m$$

$$\text{or} \quad D_m = nA - A$$

$$= (n-1)A$$

It implies that, thin prisms do not deviate light much.

Dispersion of White light

The phenomenon of splitting of white light into its component colours on passing through a refracting medium is called dispersion of light. The pattern of the coloured bands obtained on the screen is call[ed] Spectrum (VIBGYOR).

Cause of dispersion. White light consists of a continuous range of wavelengths (400 - 700nm). Dispersion takes place because the refractive index of the refracting medium is different for different wavelengths.

The refractive index 'n' of a material of wavelength λ is given by Cauchy's relation

$$n = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$$

where a b and c are constants. the values of which depend on the nature of the material.

For a small angled prism

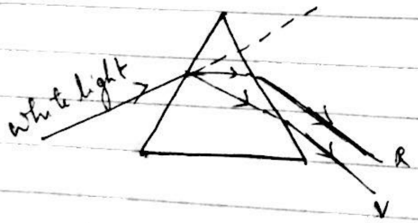
$$\cancel{D} \quad D_m = (n-1) A$$

Now $\lambda_{red} > \lambda_{violet}$

$n_{red} < n_{violet}$ and hence

$$D_{red} < D_{violet}$$

Thus the red colour is deviated the least and the violet is deviated the most.



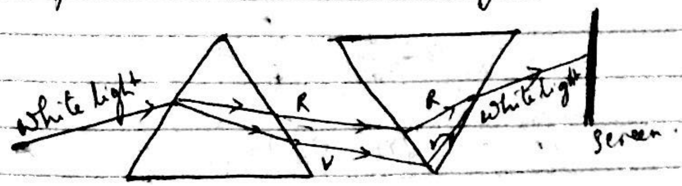
Newton's Classic experiment of dispersion of white light.

Take two prisms of the same glass material and of same refracting angle place the second prism upside down with respect to the first prism. Allow a narrow beam of white light to fall on the I prism and observe the emergent beam from the second prism on a screen. Clearly, the first prism disperses the white light into its component colours, which are then recombined by the inverted prism in to white light. This proves that white light itself consists of different colours which are just dispersed by the prism.

Chromatic aberration

This is the defect of lens. When white light is passed through a lens it disperse by which red colour will focus near to the lens and violet will focus at another point far from the lens. So the focus is not clear. This defect of lens is called chromatic aberration. This can be minimised by using thin lens.

Schematic diagram of Newton's Classic experi on dispersion of white light.



Angular dispersion.

The deviation produced by a prism

~~Dispersion~~ $D = (n-1) A$

For violet $D_V = (n_V - 1) A$

For red $D_R = (n_R - 1) A$

~~when D_V, D_R and D be the angles of deviation for violet, red and ...~~

Angular dispersion $= D_V - D_R$

$= (n_V - 1) A - (n_R - 1) A$

$= (n_V - n_R) A$

ie angular seperation between the two entire colours in the spectrum is called angular dispersion.

Dispersive Power. (ω)

It is the ability of the prism material to cause dispersion. It is defined as the ratio of the angular dispersion to the mean deviation.

$$\omega = \frac{\text{Angular dispersion}}{\text{Mean deviation}}$$

$$= \frac{D_V - D_R}{D}$$

$$= \frac{(n_V - n_R) A}{(n-1) A}$$

$$\omega = \frac{(n_V - n_R)}{}$$

Rainbow

The rainbow is the natural spectrum of light produced by refraction, dispersion and total internal reflection of sunlight by spherical rain drops.

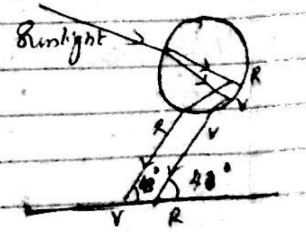
The conditions for observing a rain bow is that when the sun shines on rain drops, during or after a shower. The sun should be shining in one part of the sky while it is raining in the opposite part of the sky.

Rain bow are of two types

1. The inner brighter rain bow is called primary rain bow
2. The outer fainter rain bow is called secondary rain bow.

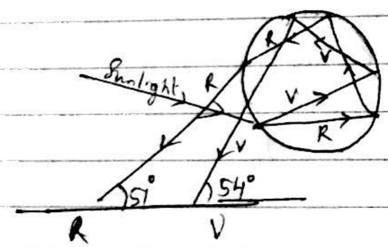
Primary rain bow.

The primary rainbow is formed by rays which undergo one internal reflection and two refractions and finally emerge from the rain drops at minimum deviation. The red rays emerge at 43° and violet rays emerge at another angle of 41°. The parallel beam of sun light getting dispersed at these angles produces a cone of rays at the observers eye. Thus the rain bow is seen as a colourful arc with its inner edge violet and outer edge red in colour.



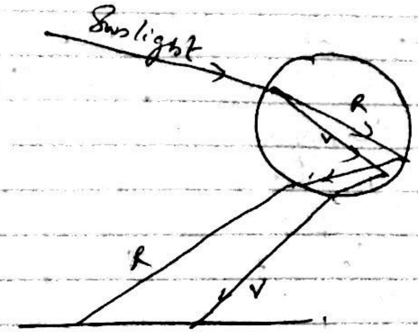
Secondary rain bow.

The secondary rainbow is formed by the rays which undergo two internal reflections and two refractions before emerging from the water drops at minimum deviation. Due to two internal reflections, the sequence of colour is secondary rainbow is opposite to that of the primary rainbow. Here the inner red rays emerge from the water drops at 51° and outer violet rays emerge at angle of 54°



Formation of Rain bow.

Sunlight is first refracted as it enters a raindrop which causes the different wavelengths of white light to separate. Longer wavelength of light (red) are bent the least while the shorter wavelength (violet) are bent the most. These components rays strike the inner surface of the water drop and get internally reflected if the angle between the refracted ray and normal to the drop surface is greater than the critical angle 48°. The reflected light is refracted again as it comes out of the drop.



Scattering of light.

This is the phenomenon in which light is deflected from its path due to its interaction with the particles of medium, like dust, water droplets etc.

~~Bhm~~ Rayleigh's law of scattering.

According to Rayleigh's law the intensity of the light scattered is inversely proportional to the 4th power of its wavelength

$$I \propto \frac{1}{\lambda^4}$$

Blue colour of sky.

It is due to scattering of sunlight by the molecules of the atmosphere. According to Rayleigh's law of scattering, light at the shorter wavelength region scattered more than light at longer wavelength. This scattered light enters our eyes. That is from the region violet, indigo and blue. Here the most predominant colour is blue. So sky appears blue.

Reddishness at sunset and Sunrise.

When the Sun is near the horizon at sunset or Sunrise. The light rays have to travel longer distances in the atmosphere. In accordance with Rayleigh's law of Scattering, the lower wave length in the blue region are almost completely scattered away. The higher wavelengths are least scattered and reach our eyes. Here the most predominant colour is red. Hence sun appears red during sunrise and sun set.

Clouds appear white

Larger particles like raindrops dust etc donot scatter light in accordance with Rayleigh's law. In clouds the size of the water droplets is large so all the colours are scattered equally. Hence clouds appears white.

Danger signals are red.

According to Rayleigh, the intensity of scattered light $I \propto \frac{1}{\lambda^4}$. In the visible spectrum red colour has the largest wavelength, it is scattered least. It covers longer distances without any loss of intensity. Hence danger signals are red.

Optical Instruments

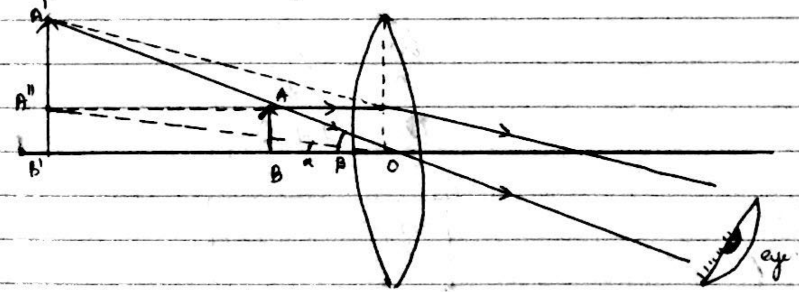
Simple Microscope (Magnifying glass).

A simple microscope is a convex lens of short focal length held close to the eye

Working

(i) When the final image is formed at the least distance of distinct vision.

When an object AB is placed between the focus and the optic centre of a convex lens, a virtual erect and magnified image A'B' is formed on the same side of lens.



Magnifying power :- Magnifying power of a simple microscope is defined as the ratio of angle subtended by the image and the object at the eye when both are at the least distance of distinct vision

ie $m = \frac{\beta}{\alpha}$

$= \frac{\tan \beta}{\tan \alpha} = \frac{AB/OB}{A''B'/OB'}$

$$m = \frac{AB}{OB} \times \frac{OB'}{A''B'}$$

$$= \frac{AB \times OB'}{OB \times AB}$$

$$= \frac{OB'}{OB}$$

$$= \frac{-D}{-x} = \frac{D}{x}.$$

Using lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-D} + \frac{1}{x} = \frac{1}{f}$$

$$\frac{D}{-D} + \frac{D}{x} = \frac{D}{f}$$

$$-1 + \frac{D}{x} = \frac{D}{f}$$

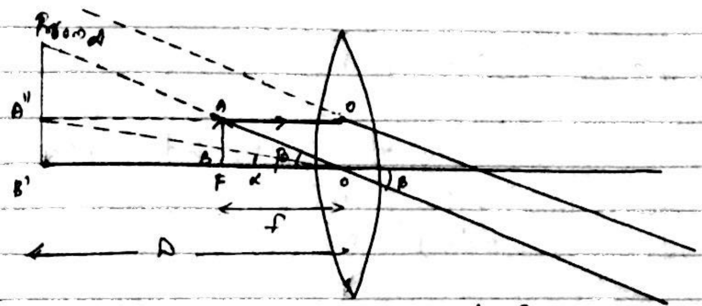
$$\frac{D}{x} = 1 + \frac{D}{f}$$

$$\boxed{m = 1 + \frac{D}{f}}$$

Shorter the focal length of a convex lens the greater is its magnification.

(ii) <u>When the final image is formed at infinity</u>

When the object is placed at the focus of the convex lens, the image is formed at infinity.



$m = \frac{\tan\beta}{\tan\alpha}$ $m = \frac{\tan\beta}{\tan\alpha}$.

Magnifying power $m = \dfrac{AB/OB}{A'B'/OB'}$

$$= \frac{AB}{OB} \times \frac{OB'}{AB}$$

$$= \frac{OB'}{OB}$$

$$= \frac{-D}{-f}$$

$$= \frac{D}{f}.$$

<u>Uses of simple microscope</u>

* Watch makers and jewellers use a magnifying glass for having a magnified view of small objects
* In magnifying the printed letters in a book.
* Magnifying glass is used in science lab to read vernier calipers

(P.T.O)

Compound Microscope.

A compound microscope is an optical device used to see magnifying images of tiny objects.
It consists of two convex lenses of shorter focal length arranged co-axially at the ends of two sliding metal tubes.

1 Objective :- It is a convex lens of very short focal length f_o and small aperture positioned near the object.

2. Eyepiece :- It is a convex lens of comparatively larger focal length f_e and larger aperture than objective positioned near the eye.

Working.

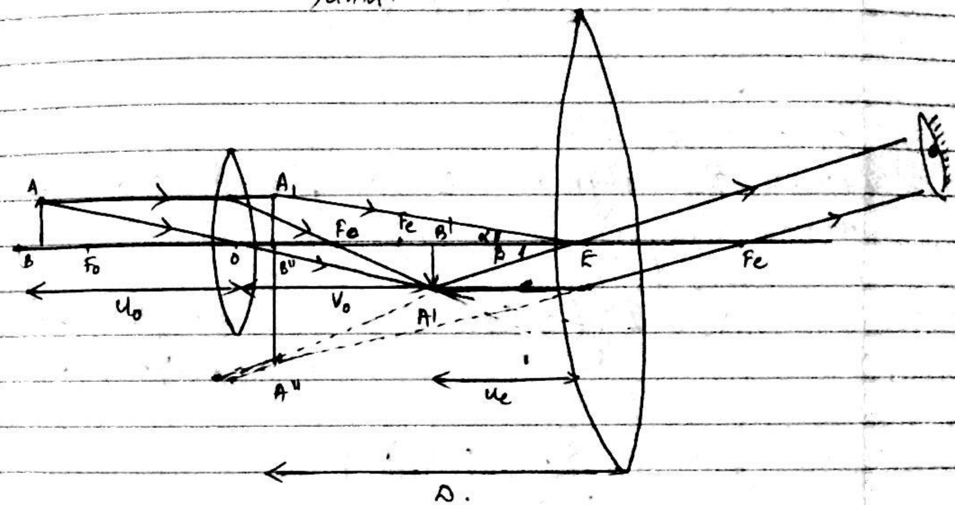
1 When the final image is formed at the least distance of distinct vision.

When the object AB is placed slightly larger than the focal length of the objective. The objective forms a real and inverted magnified image A'B'. It lies in between the optic centre and focus of the eye piece. So the magnified image A"B" is formed.

Magnifying power of a compound microscope is defined as the ratio of the angle subtended at the eye by the virtual image to the angle subtended at the eye by the object when both are at least distance of distinct vision from the eye

$$m = \frac{\beta}{\alpha}$$

$$= \frac{\tan\beta}{\tan\alpha}$$



$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$$

$$= \frac{\frac{A''B''}{EB''}}{\frac{A_1B''}{EB''}} = \frac{A''B''}{A_1B''}$$

$$= \frac{A''B''}{A'B'} \times \frac{A'B'}{A_1B''}$$

$$= m_e \times m_o \quad (1)$$

$m_o = \frac{v_o}{u_o}$

As the eyepiece act as a simple microscope

$m_e = 1 + \frac{D}{f_e}$

∴ equ (1) becomes

$$m = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$$

Special Cases

(i) As the object AB is placed close to the focus f_o

$u \simeq f_o$

$v_o = L$ (where L is the length of microscope tube)

$$\therefore \; m = \frac{L}{-f_o}\left(1+\frac{D}{f_e}\right)$$

(ii) When the final image is formed at infinity

$$m = \frac{v_o}{u_o}\left(\frac{D}{f_e}\right)$$

Astronomical telescope (Refracting telescope)

It is a refracting type telescope used to see heavenly bodies.

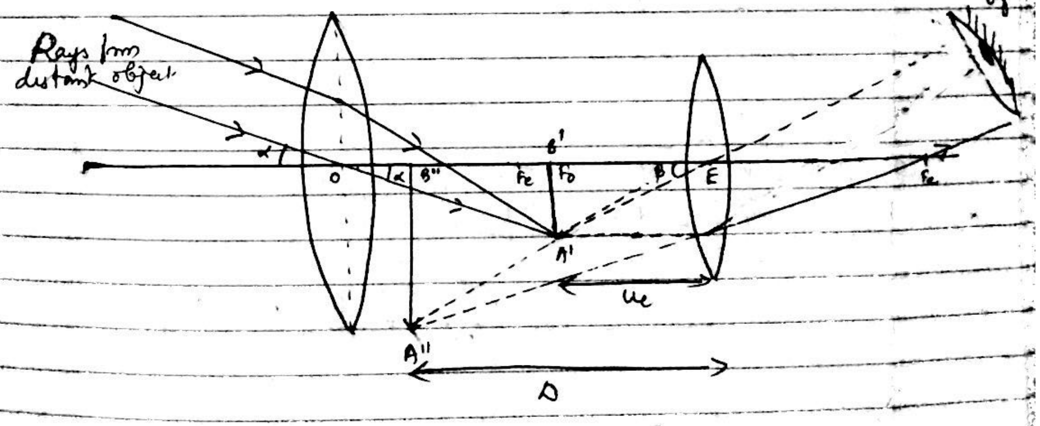
It consists of two converging lenses mounted co-axially at the outer ends of two sliding tubes.

1. Objective:- It is a convex lens of large focal length and much larger aperture.

2. Eyepiece:- It is a convex lens of small focal length and small aperture

Working:- When the final image is formed at the least distance of distinct vision.

The parallel beam of light coming from the distant objects falls on the objective. The objective focuses the beam in its focal plane and forms a real and inverted image A'B'. This image A'B' acts as the object of eyepiece. The eyepiece magnifies this image so that final image A''B'' is magnified and inverted with respect to the object at the least distance of distinct vision.



Magnifying power:- It is defined as the ratio of angle subtended at the eye by the final image formed at the least distance of distinct vision to the angle subtended at the eye by the object at infinity

$$m = \frac{\beta}{\alpha}$$

$$= \frac{\tan\beta}{\tan\alpha}$$

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$$

$OB' = f_0$ = focal length of the objective

$B'E = -u_e$ = object distance of eyepiece

$$m = \frac{-f_0}{u_e} \quad \text{——} \; (1)$$

Again for eye piece $u = -u_e$

$V = -D$

$$\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

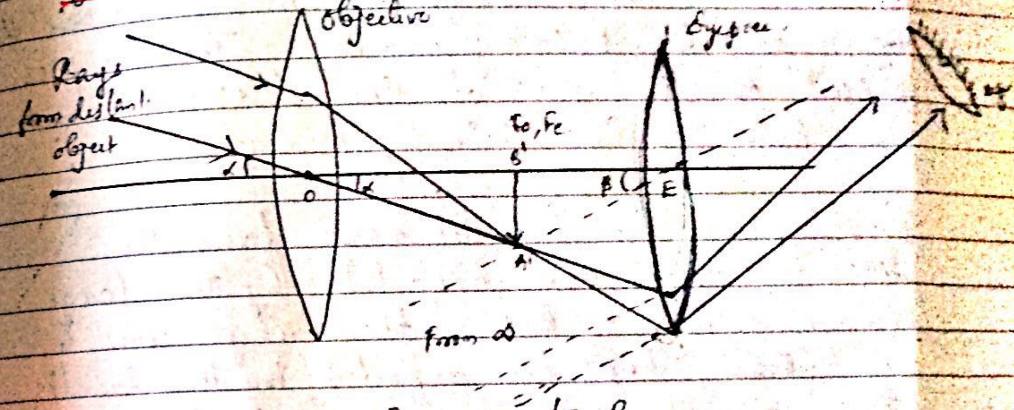
or $$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

Hence equ. (1) becomes

$$m = \frac{-f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Clearly for large magnifying power $f_0 >> f_e$. The negative sign indicates that the final image formed is real and inverted.

Special cases:- In normal adjustment the final image is formed at infinity.



$$\therefore m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$$

$$= \frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$$

$OB' = f_0$ = focal length of the objective

and $B'E = -f_e$ = focal length of the eyepiece

$$m = \frac{-f_0}{f_e}$$

Clearly for large magnifying power $f_0 >> f_e$ the negative sign indicates that the final image is real and inverted.

Cassegrain reflecting telescope.

It consists of a large concave paraboloidal mirror having a hole at its centre. There is a small convex (secondary mirror) near the focus of the primary mirror. The eye piece is placed on the axis of the telescope near the hole of the primary mirror.

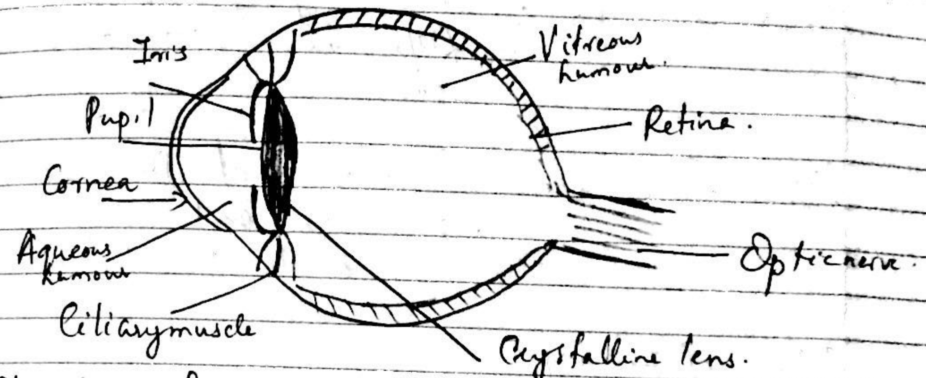
Working:- The parallel rays from the distant object are reflected by the large concave mirror. Before these rays come to the focus, they are reflected by the small convex mirror and are converged to the eyepiece. The final image is inverted with respect to the object.

Advantages of reflecting-type telescope over a refracting type telescope.

1. No chromatic aberration, because mirror is used
2. Spherical aberration can be removed by using paraboloidal mirror
3. Higher resolution can be obtained by using mirror of larger aperture.
4. The image is bright, because there is no loss of light due to refraction.

Human Eye.

It is the most valuable and sensitive sense organ. It is a remarkable optical instrument.



Structure of eye.

Cornea:- It is a transparent membrane on the front portion of the eyeball through which light enters the eye.

Iris and Pupil:- Iris is an opaque circular diaphragm having a small central hole called the pupil. Under the muscular action of the iris, the size of the pupil becomes smaller in bright light and larger in dim light.

Eye lens:- It is a double convex lens. It is composed of fibrous jelly like material. The lens is held in position by means of ciliary muscles.

Aquous humour:- It is a watery salty fluid that fills the space between the cornea and the iris.

Retina :- It is a delicate membrane on the inner side of the back wall of the eye It act as a screen.

Vitreous humour :- It is a jelly like fluid that fills the space between the retina and the eye lens.

Optic nerve :- The function of the optic nerve is to send signals from the retina to the brain

Action of the eye:

A ray from the object enter the eye, they suffer refraction, a real and inverted image is formed on the retina The light sensitive cells of retina get activated and generate electrical signals that are sent to the brain through the optic nerve. Brain translates the inverted image into an erect image

Accomadation

It is the ability or property of the eyelens due to which it can change its curvature or focal length so that images of objects at various distances can be formed on the retina by the action of ciliary muscle

When the ciliary muscles are relaxed the eyelens is thin and its focal length increases so we have to see distant objects clearly.

When the ciliary muscles are contracted the eye lens is thick its focal length decreases, so we can see nearer objects clearly.

Range of normal human eye.

A normal eye can see the object situated any where between infinity and 25cm. This is the range of normal vision

Least distance of distinct vision.

The minimum distance from the eye at which the eye can see objects clearly without any strain is least distance of distinct vision. It is denoted by D. For a normal eye its value is 25cm.

Persistance of vision.

The phenomenon of the continuation of the impression of any image on the retina for about $\frac{1}{16}$ of a second even after the removal of object is called persistance of vision.

Defects of vision.

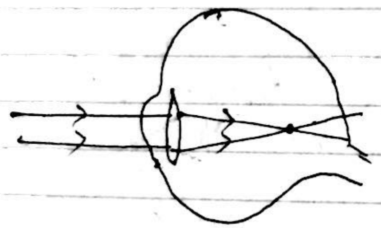
1. Myopia / (Short sight)

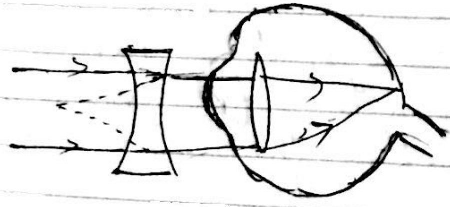
A person with myopic eye can see nearby objects clearly but cannot see distant objects

Causes
1. The eye ball gets elongated
2. The focal length of the eye lens becomes too small.

Reason:- From the above causes the rays from the distant object do not meet at the retina, but at a point infront of the retina

Correction: It can be corrected by using a concave lens of appropriate focal length.





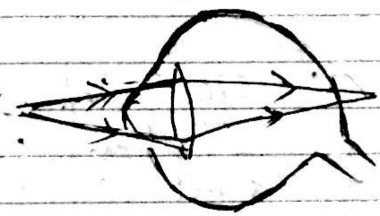
Hypermetropia (Far sight)

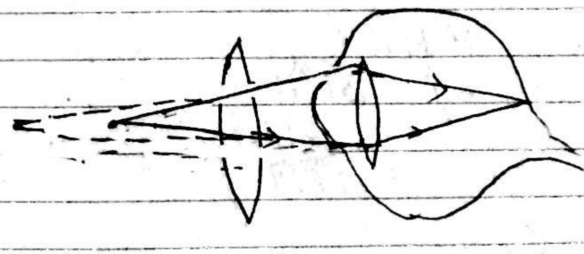
A person with hypermetropic eye can see distant objects clearly but cannot see nearby objects.

Causes:-
1. The eye ball becomes too small
2. The focal length of the eye lens become too large.

Reason:- From the above causes the rays from the nearby object meet at a point behind the retina

Correction:- It can be corrected by using a convex lens of appropriate focal length.

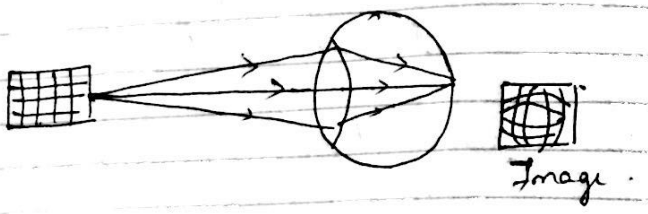


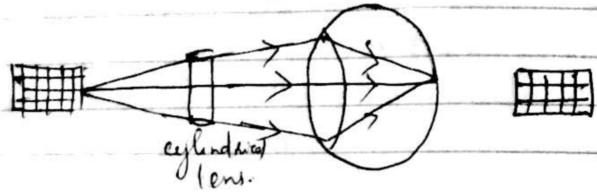


Astigmatism:- It is a defect of vision in which a person cannot simultaneously see both the horizontal and vertical views of an object with the same clarity

Cause:- This defect occurs when the cornea is not spherical in shape

Correction :- It can be corrected by a lens whose surface is cylindrical.

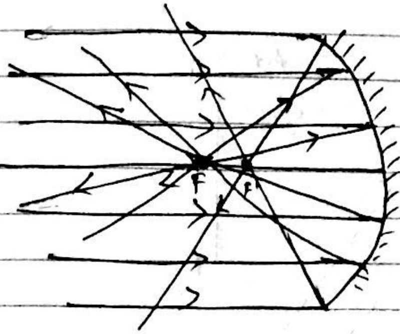




Presbyopia :-

This defect is similar to hypermetropia. That is a person having this defect cannot see nearby objects but can see distant objects without any difficulty. This defect differs from hypermetropia in the cause by which it is produced. It usually occurs in elderly people due to the loss of flexibility of the eye lens and stiffening of the ciliary muscles. It can be corrected by using a convex lens of appropriate focal length.

Spherical aberration :- The inability of a spherical mirror of large aperture to bring all the rays of wide beam of light falling on it to focus at a single point is called spherical aberration.



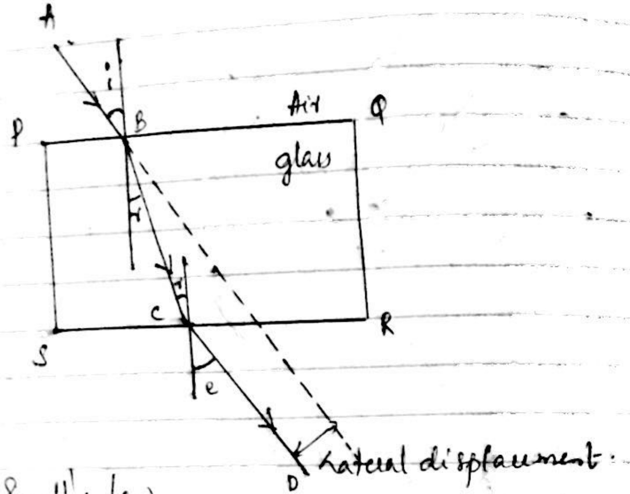
Paraxial rays are focussed at the principal focus F while the marginal rays meet the principal axis at a point close to the pole (F'). The results is that image is blurred.

Spherical aberration can be reduced by

1. Using spherical mirrors of small aperture
2. Using paraboloidal mirror
3. Using stops which cut the marginal rays.

Refraction through a rectangular glass slab.

Consider a glass slab PQRS. A ray AB is incident on the face PQ at an angle of incidence i. On entering the glass slab, it bends towards the normal and travels along BC at an angle of refraction r. The refracted ray BC is incident on face SR at an angle of incidence r. The emergent ray CD bends away from the normal at an angle of emergence e.



Using Snell's law
For the face PQ

$$\frac{\sin i}{\sin r} = n_{ga} \quad \text{——— (1)}$$

For the face SR

$$\frac{\sin r}{\sin e} = n_{ag} = \frac{1}{n_{ga}} \quad \text{——— (2)}$$

Multiplying (1) and (2)

$$\frac{\sin i}{\sin r} \times \frac{\sin r}{\sin e} = n_{ga} \times \frac{1}{n_{ga}}$$

or $$\frac{\sin i}{\sin e} = 1$$

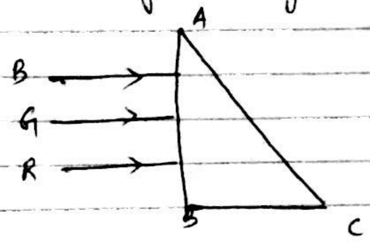
$$\sin i = \sin e$$

$$i = e$$

Angle of incidence = Angle of emergence.

The perpendicular distance between the incident and emergent rays, when light is incident obliquely on a refracting slab with parallel faces is called lateral shift or lateral displacement.

Qu: Three rays of light - red (R) green (G) and blue (B) are incident on the face AB of right angled prism ABC. The refractive indices of the material of the prism for red, green and blue are 1.39, 1.44 and 1.47 respectively. Out of the three which colour will emerge out of face AC. Justify your answer. Trace the path of the rays through the prism.



Ans: As light is incident normally on face AB, so no refraction occurs at face AB. Light is incident on face AC at $i = 45°$. The face will not transmit light for which

$$i > i_c \quad \text{or} \quad \sin i > \sin i_c$$

$$\sin 45 > \frac{1}{n}$$

$$\text{or} \quad n > \sqrt{2} = 1.414$$

As $n_R < n$ while n_G and $n_B > n$, so only red colour will be transmitted through face AC while green and blue rays will suffer TIR