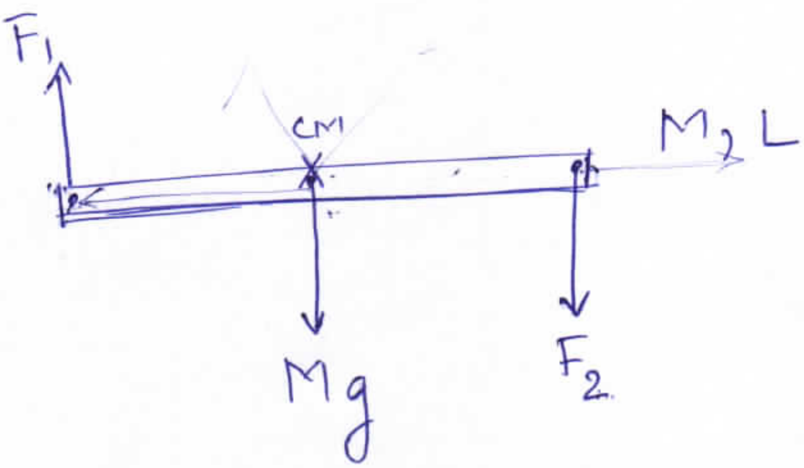
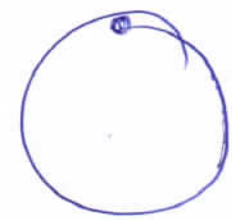
TORQUE (MOMENT OF A FORCE)

It's the cause which changes condition of rotational motion.





Equilibrium Condition

translational Eq

$$\sum F = 0$$

$$\sum \tau_{\text{any point}} = 0 \quad \text{Rotational Eq.}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$\vec{r}$: distance from point of rotation (reference about which torque is being calculated) to the point where the force is applied.

NOTE – ~~If Torque by a Force~~ If force is passing through point of rotation then torque due to it is ZERO

$$\vec{\tau}_{CM} = \vec{\tau}_{F_1} + \vec{\tau}_{Mg} + \vec{\tau}_{F_2}$$

$\frac{L}{2}(-\hat{i}) \times F_1(\hat{j})$ $0 \times Mg(-\hat{j})$ $\frac{L}{2}(\hat{i}) \times F_2(-\hat{j})$

$-\frac{LF_1}{2}\hat{k}$ 0 $-\frac{LF_2}{2}\hat{k}$

i j k

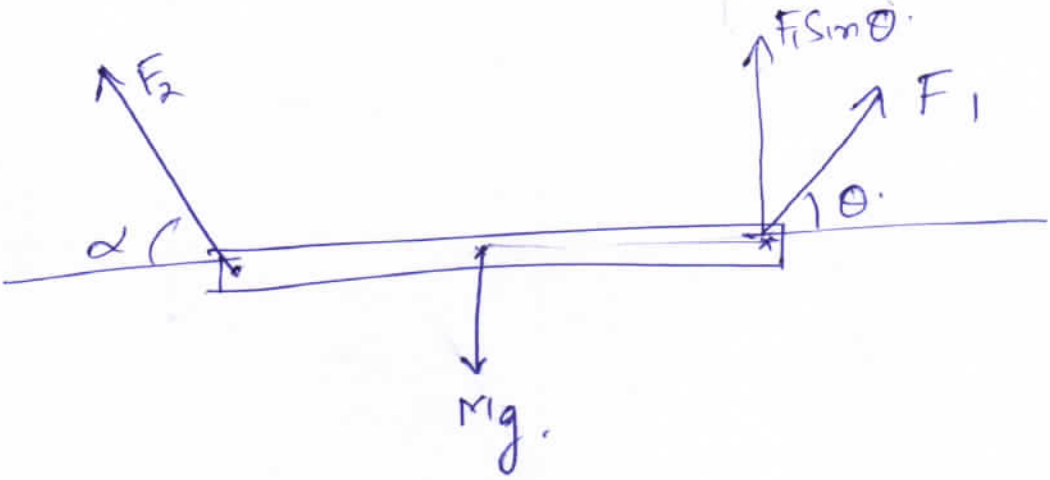
$$= -\frac{L}{2}(F_1 + F_2)\hat{k}$$

CLOCKWISE ROTATION ABOUT POINT OF ROTATION

—VE SIGN TO TORQUE (INTO THE ROTATION PLANE)

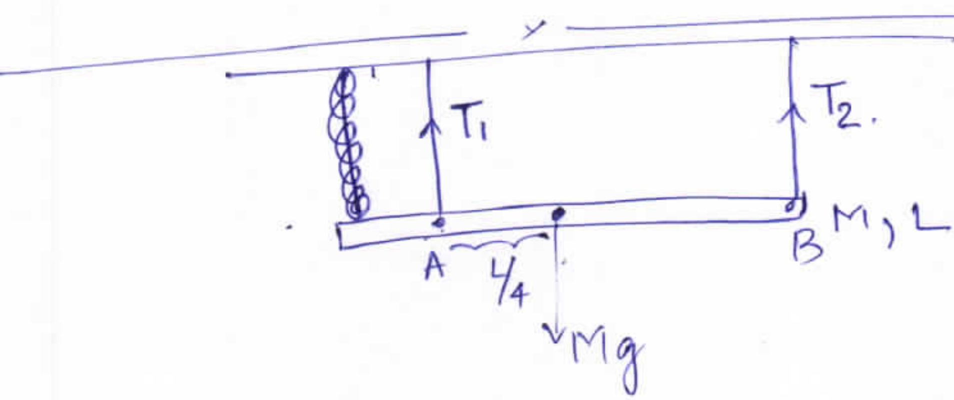
ANTICLOCKWISE

+VE SIGN TO TORQUE (OUT OF THE PLANE OF ROTATION)



like $\vec{F} = m\vec{a}$

$\vec{\tau} = I\vec{\alpha}$



Find T_1 & T_2. if body is in equilibrium.

for TE $T_1 + T_2 = Mg$ —— (1)

$T_1 = \frac{2Mg}{3}$

for RE $\tau_A = \tau_{T_1} + \tau_{Mg} + \tau_{T_2}$

$0 = 0 - Mg \times \frac{L}{4} + T_2 \times \frac{3L}{4}$

$T_2 = \frac{Mg}{3}$.

ANGULAR MOMENTUM

$$\vec{P} = m\vec{v}$$

$$\vec{L} = I\vec{\omega}$$

direction of $\vec{\omega}$ & $\vec{L}$ same

$$= mr^2 \times \omega$$

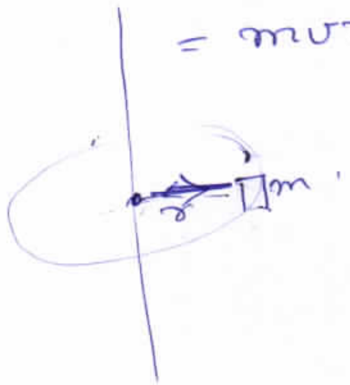
$$= mr^2 \times \frac{v}{r}$$

$$= mvr = m\vec{v}\,\vec{r}$$

$$= \vec{p}\,\vec{r}$$

$$= \vec{r} \times \vec{p}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

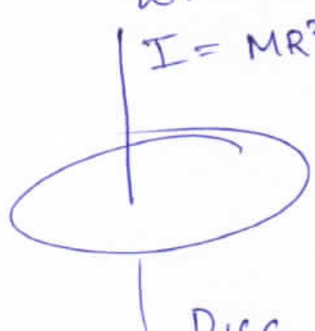


eg 1 A ring rotating with angular velocity ω about its natural axis. What is its angular momentum.

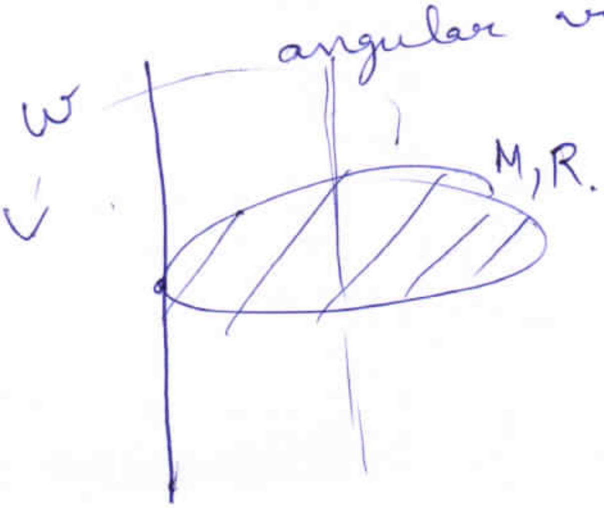
$I = MR^2$

$$\vec{L} = I\vec{\omega}$$

$$\vec{L} = MR^2\omega$$



eg 2 ~~Ring~~ Disc about tangential axis parallel to natural axis rotating with ω angular velocity.



$$\vec{L} = \vec{I}\omega$$

$$= \left(\frac{MR^2}{2} + MR^2\right) \times \omega$$

$$= \frac{3MR^2}{2}\omega$$

$\Delta\vec{P} = 0$ when $\vec{F}_{net} = 0$ conservation of linear momentum.

$\Delta\vec{L} = 0$ when $\vec{\tau}_{net} = 0$ conservation of angular momentum.

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{P}}{dt}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}_{net}$$

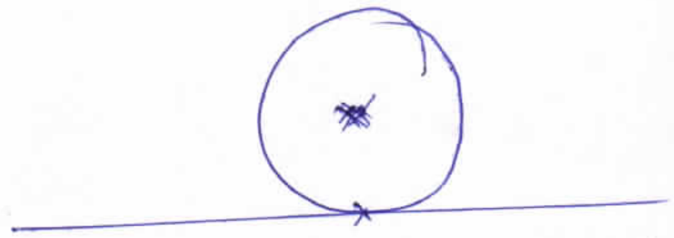
$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

If $\vec{\tau}_{net} = 0 \Rightarrow \dfrac{d\vec{L}}{dt} = 0$

$\Downarrow$

$\vec{L}$ = constant.

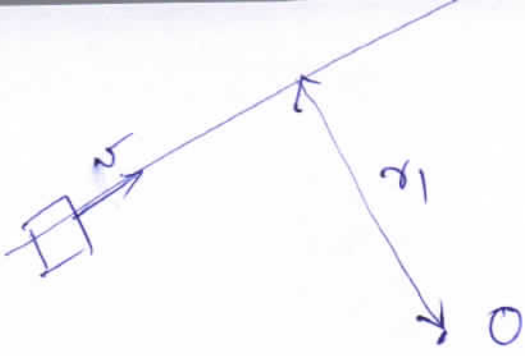
Will it possess angular momentum?



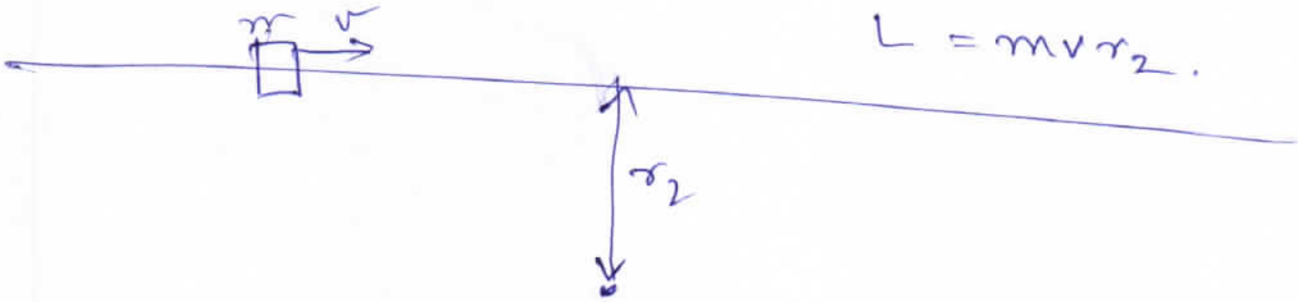
$$\vec{L} = mvr$$

angular momentum of a body performing translational motion about point O other than the centre of mass.

r = ⊥ (to velocity) length from point of ~~the~~ reference.



$L = mvr_1$

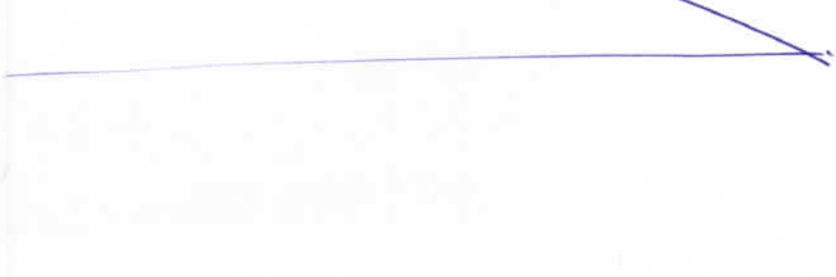


$L = mvr_2$.

$$\boxed{\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = \frac{d(I\vec{\omega})}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha}}$$

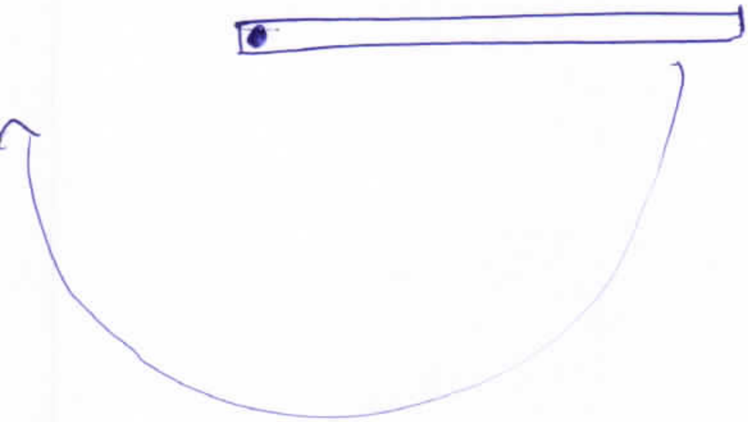
TYPES OF MOTION

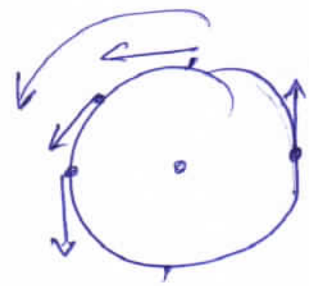
1)



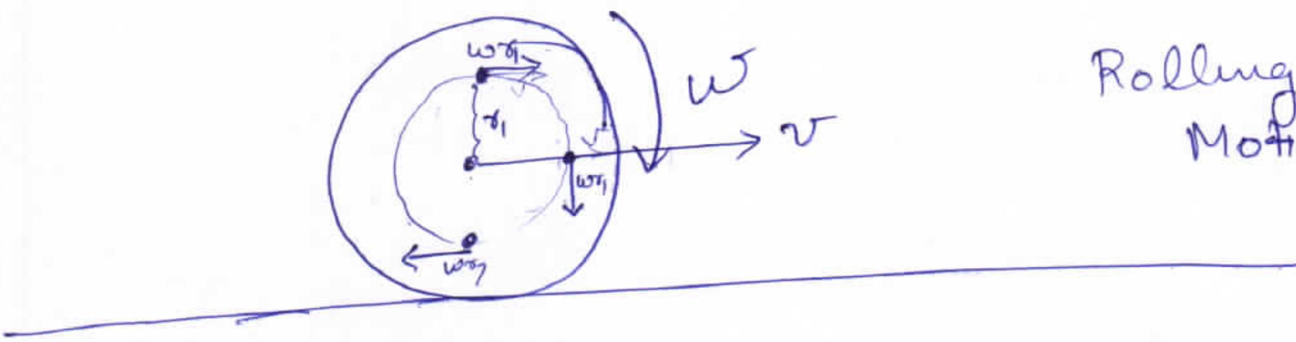
pure slipping
purely translational motion.

2) purely rotational motion.



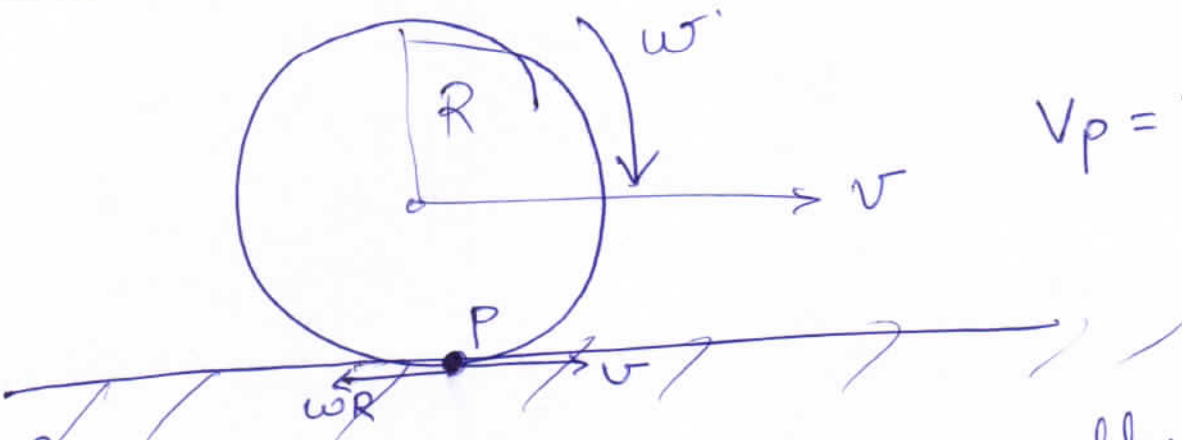


3) Translational + Rotational.



Rolling Motion.

Pure Rolling



$V_P = v - \omega R$

for pure rolling

$V_{PS} = 0$

$V_P - V_S = 0$

or $V_P = V_S$.

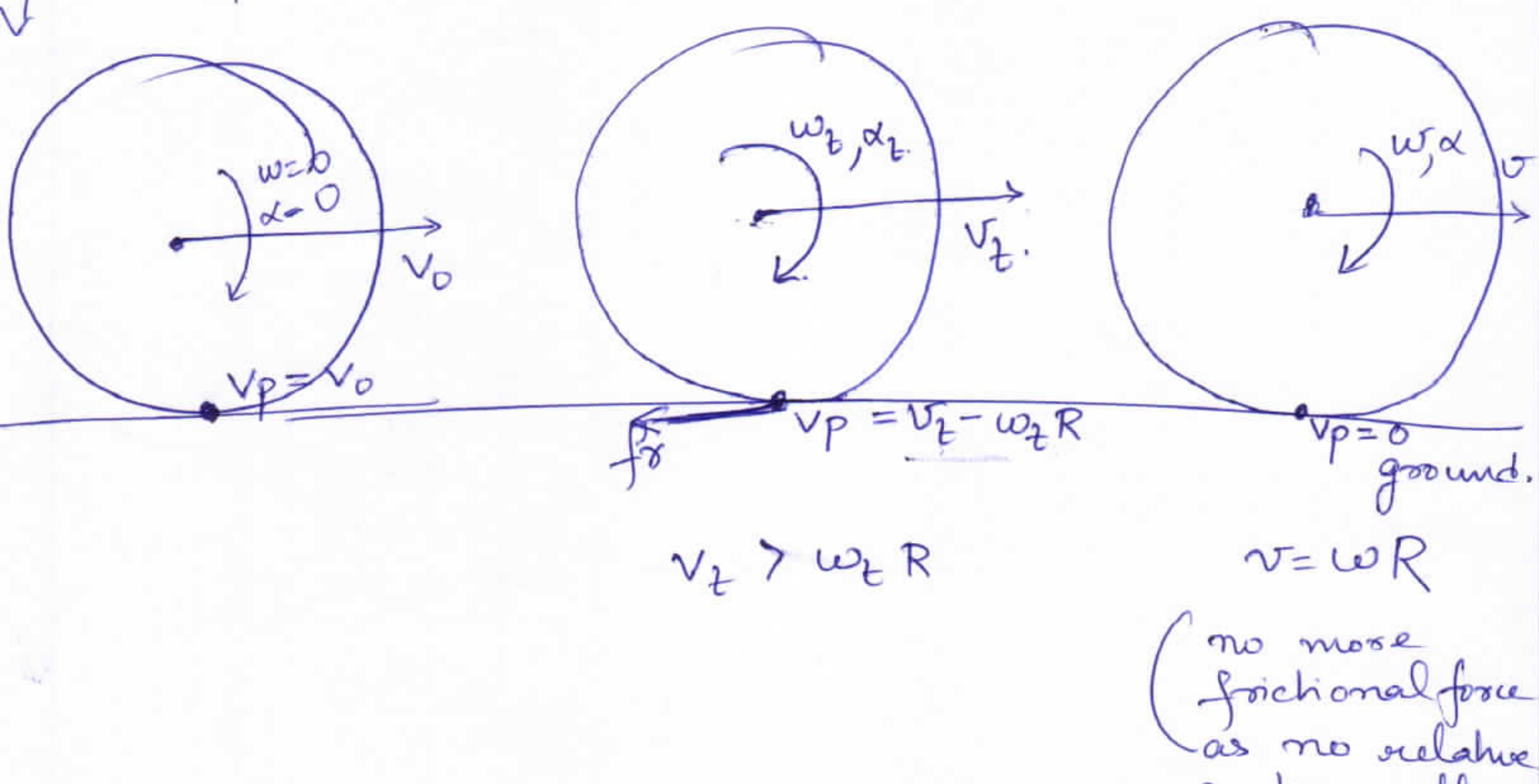
$V_p = V_s$ (Condition for pure rolling

when surface is ground $V_s = 0$

$$V_p = 0 \Rightarrow v - \omega R = 0$$

$$\text{or} \quad \left.\begin{aligned} v &= \omega R \\ a &= \alpha R \end{aligned}\right\} \checkmark$$

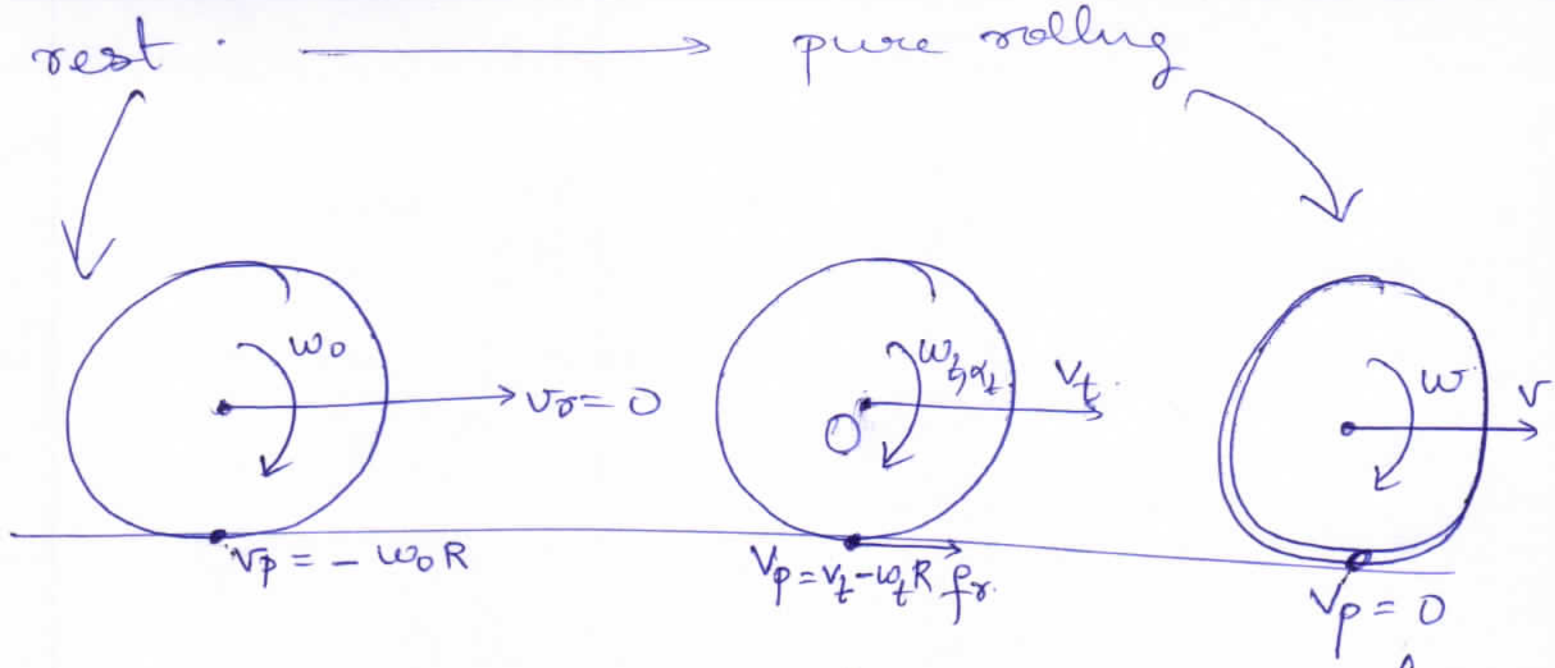
SLIPPING ⟶ PURE ROLLING.



$V_t > \omega_t R$

$v = \omega R$

(no more frictional force as no relative motion with surface)



$v_t < \omega_t R$

$f_r = 0$

$v = \omega R$

$$\tau_0 = f_r \times R$$

$$\tau_0 = I_0 \alpha$$

$$f_r \times R = I_0 \alpha$$

$$\alpha = \frac{f_r R}{I_0}$$

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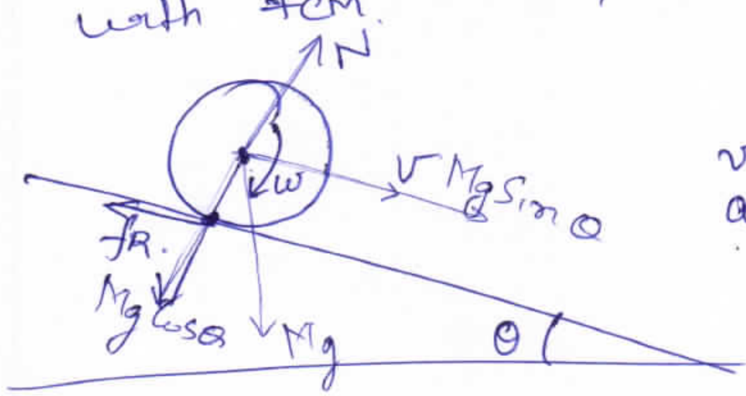
find linear acceleration 'a' for the body with $I_{CM} = I$; radius R & Mass M. if body is purely rolling.



$v = \omega R$.

$a = \alpha R$.

$$\tau = f_R \times R = I\alpha.$$

$$f_R = \frac{I\alpha}{R} \quad \text{—— (1)}$$

$$= \frac{Ia}{R^2}$$

$$N = Mg\cos\theta$$

$$Mg\sin\theta - f_R = Ma$$

$$Mg\sin\theta - \frac{Ia}{R^2} = Ma$$

$$Mg\sin\theta = \left(M + \frac{I}{R^2}\right) a$$

$$a = \frac{Mg\sin\theta}{M + \frac{I}{R^2}}$$

$$\boxed{a = \frac{g\sin\theta}{\left(1 + \frac{I}{MR^2}\right)}}$$

$$f_R \le (f_R)_{MAX}$$

$$\frac{Ia}{R^2} \le \mu Mg\cos\theta$$

$$\frac{I\, g\sin\theta}{R^2\left(1 + \frac{I}{MR^2}\right)} \le \mu Mg\cos\theta$$

$$\boxed{\mu \ge \frac{\tan\theta}{1 + \frac{MR^2}{I}}}$$

Condition for pure rolling on inclined plane.

Total K·E of a body during Rotational + Translational motion.

$$T.E = \underbrace{\frac{1}{2} m v_{cm}^2}_{\text{translational K·E}} + \underbrace{\frac{1}{2} I \omega^2}_{\text{rotational K·E}}$$

Angular Momentum of body Executing Combined Translatory & Rotatory motion.

$$\vec{L}_{cm} + m(\vec{r} \times \vec{v})$$

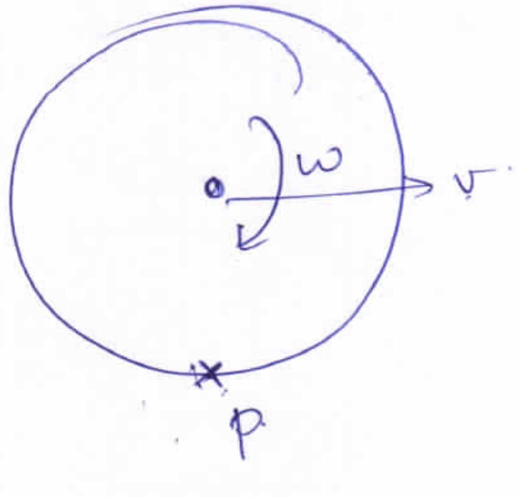
$$\vec{L}_p = I_{cm}\vec{\omega}_{cm} + \vec{r} \times m\vec{v}_{cm}$$

angular velocity about CM.

distance of CM from P

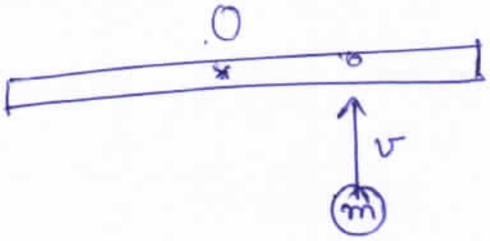
$\vec{v}$ = velocity of CM.

suppose P is point of reference.



$L_{cm} + L_p$

$I_c\omega_c + mvR$.



lying on a frictionless surface.

A small mass m moving with velocity v hits the rod & gets stuck to it at a distance of $L/4$ from its CM

By conservation of linear momentum!

Initial $\vec{p}$

$$mv = (M+m)v' \Rightarrow v' = \frac{mv}{M+m}.$$

Conservation of angular momentum. about O

$$mv\left(\frac{L}{4}\right) + 0 = (m+M)v'(r) + I\omega$$

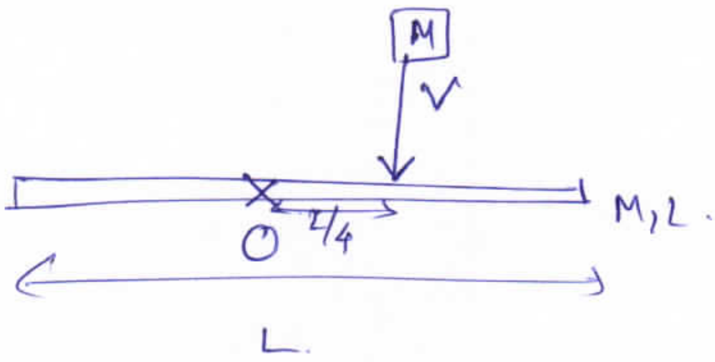
distance b/w O & CM.

0

$$mv\left(\frac{L}{4}\right) = \left\{\frac{ML^2}{12} + m\left(\frac{L}{4}\right)^2\right\}\omega$$

Pg 97 (11)

HINGED at O.



About O

COAM.

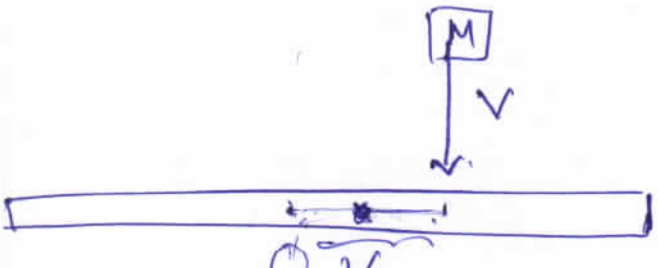
$$0 + MV\left(\frac{L}{4}\right)$$

$$= I\omega$$

$$= \Big\{\underbrace{\frac{ML^2}{12}}_{\text{rod about } O} + \underbrace{M\left(\frac{L}{4}\right)^2}_{\text{insect about } O}\Big\}\omega$$

$$\omega = \frac{12}{7}\frac{V}{L}$$

NOT HINGED.



new CM of insect + ROD

$$\frac{M(0) + M\left(\frac{L}{4}\right)}{M+M}$$

$$= \frac{L}{8} \text{ from } O$$

COAM about new CM

$$MV\left(\frac{L}{4} - \frac{L}{8}\right) + \underset{\text{rod}}{0}$$

$$= I\omega$$

$$= \left\{\frac{ML^2}{12} + M\left(\frac{L}{8}\right)^2 + M\left(\frac{L}{8}\right)^2\right\}\omega$$

$$\cancel{M}V\frac{\cancel{L}}{8} = \cancel{M}\left(\frac{L^{\cancel{2}}}{12} + \frac{L^{\cancel{2}}}{32}\right)\omega$$

$$\frac{V}{8} = \left(\frac{8L + 3L}{4(24)}\right)\omega$$

$$\omega = \frac{12}{11}\frac{V}{L}$$