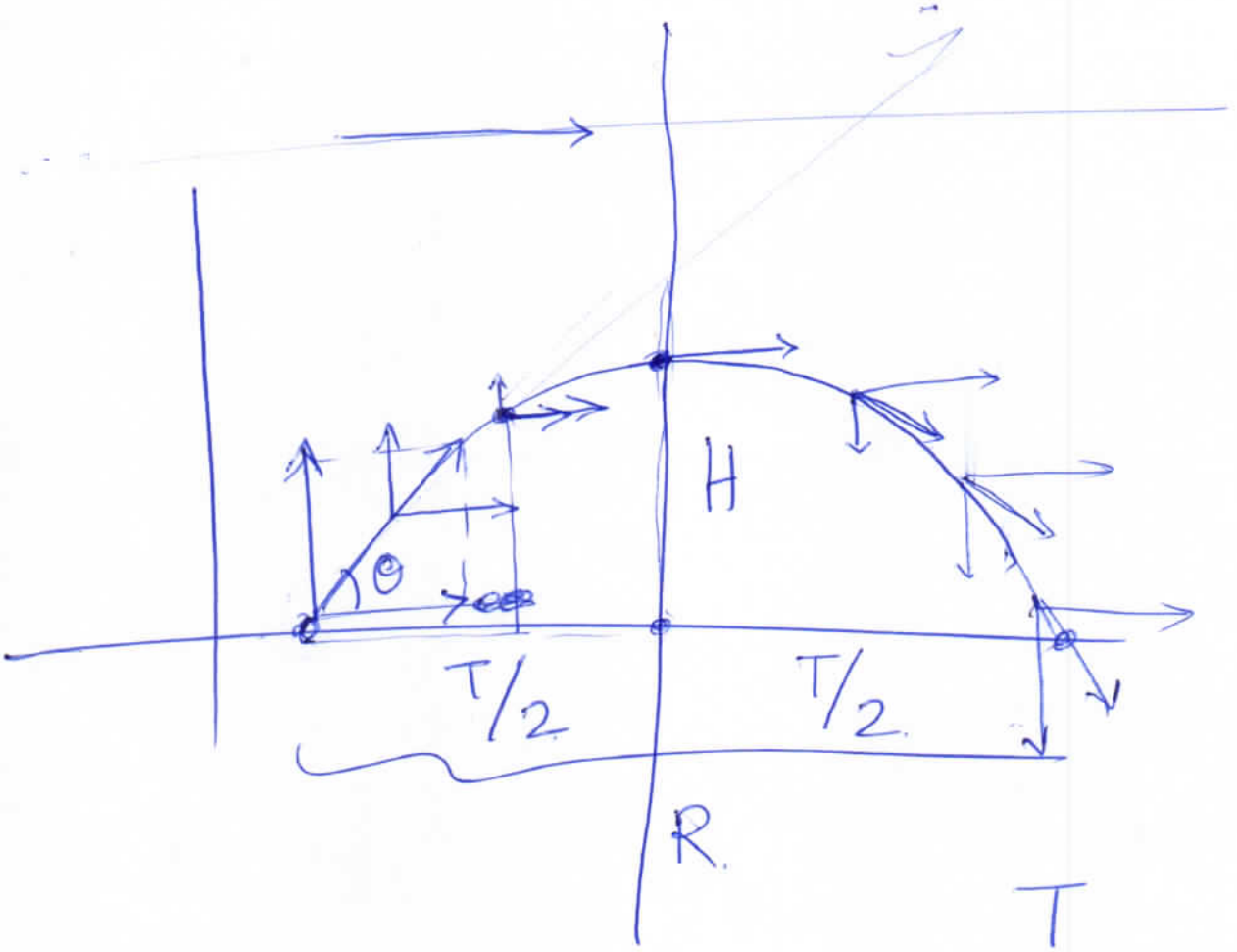
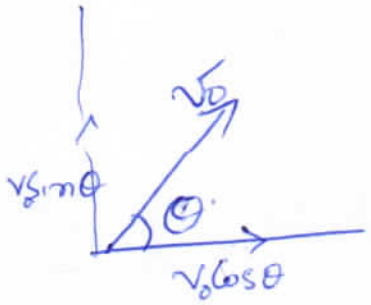
PROJECTILE MOTION





$$\vec{v_0} = v_0\cos\theta\, i + v_0 \sin\theta\, j$$

x direction

$u_x = v_0 \cos\theta$

$a_x = 0$

$v_x(t) = v_0 \cos\theta$

$s_x(t) = v_0 \cos\theta\,(t)$

y direction

$u_y = v_0 \sin\theta$

$a_y = -g$

$v_y(t) = v_0 \sin\theta - gt$

$s_y(t) = v_0 \sin\theta\,(t) - \frac{1}{2} g t^2$

$\vec{v}(t) = v_x i + v_y j$

$\vec{v}(t) = v_0 \cos\theta\, i + (v_0 \sin\theta - gt) j$

$t = T/2 \quad v_y = 0 \quad s_y = +H$

$v_y(t) = v_0 \sin\theta - gt$

$0 = v_0 \sin\theta - g\left(\frac{T}{2}\right)$

$$\Rightarrow \boxed{T = \frac{2 v_0 \sin\theta}{g}}$$

$s_y(t) = v_0 \sin\theta\, t - \frac{1}{2} g t^2$

$v_y^2 - u_y^2 = 2a s_y$

$0^2 - (v_0 \sin\theta)^2 = 2(-g) H$

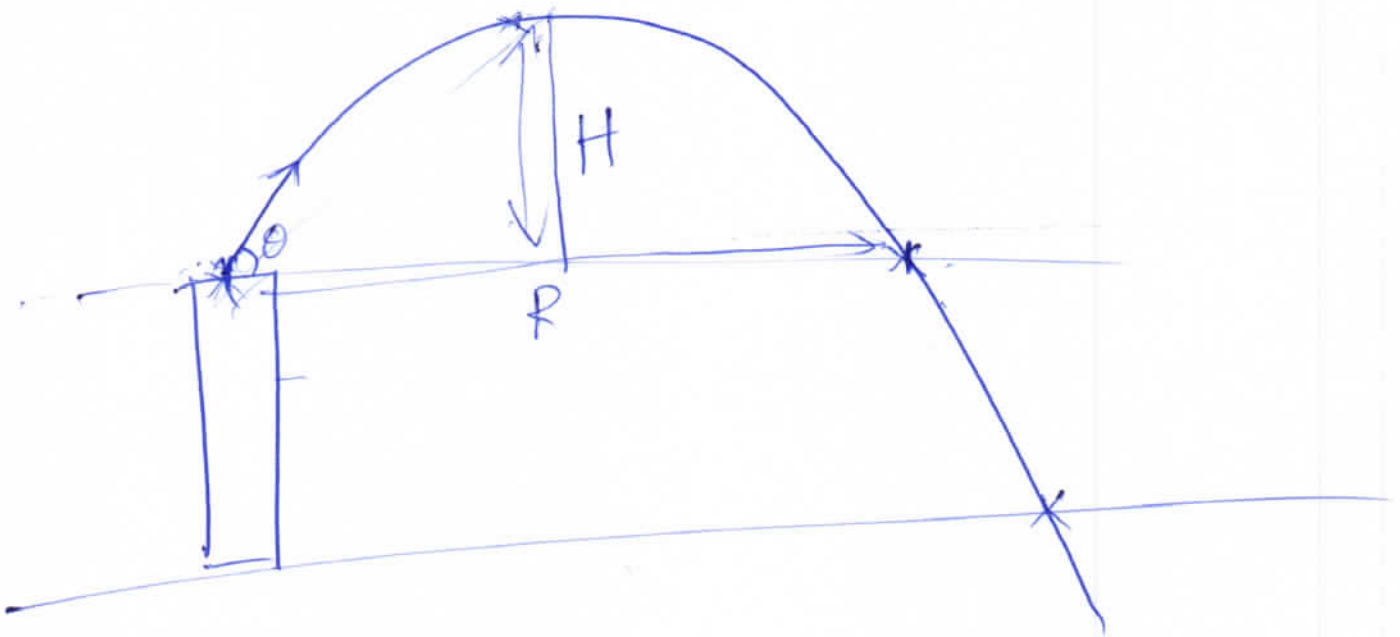
$$\boxed{H = \frac{v_0^2 \sin^2\theta}{2g}}$$

$R = ? \quad s_x(t) = v_0 \cos\theta\, t$

$R = v_0 \cos\theta\, (T)$

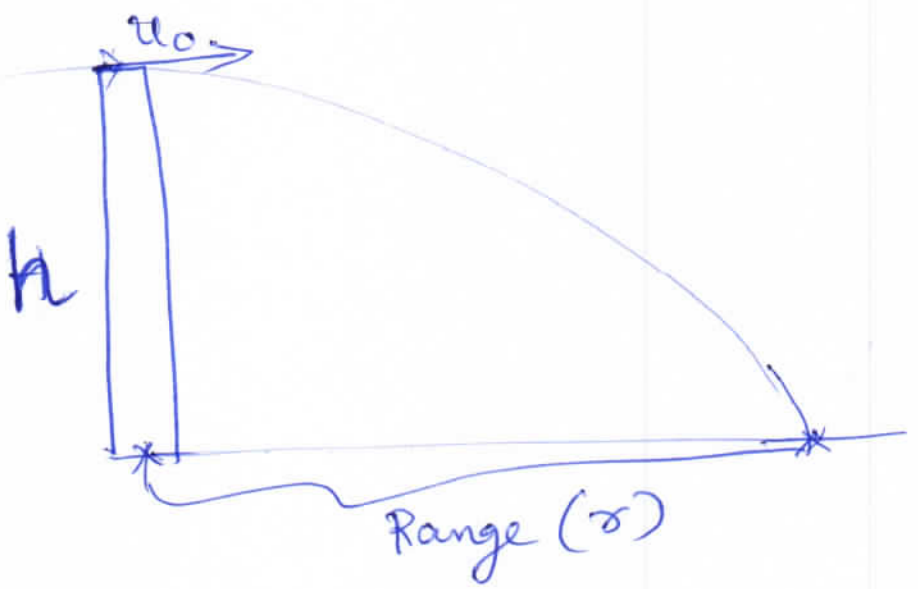
$= v_0 \cos\theta \left(\frac{2 v_0 \sin\theta}{g}\right)$

$= \frac{2 v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$



$$\vec{v}(t) = v_0\cos\theta\, i + (v_0\sin\theta - gt)\, j$$

$$s(t) = (v_0\cos\theta\, t)\, i + \left(v_0\sin\theta\, t - \frac{1}{2}gt^2\right) j$$



$$r = u_0 t$$

$$r = u_0\sqrt{\frac{2h}{g}}$$

$$-h = 0t + \frac{1}{2}(-g)t^2$$

$$h = \frac{1}{2}gt^2 \qquad t = \sqrt{\frac{2h}{g}}$$

Q1 Find the angle of projection of a projectile for which horizontal ~~range~~ range & Max Height attained are equal.

$$\frac{v_0^2 \sin 2\theta}{g} = \frac{v_0^2 \sin^2\theta}{2g} \Rightarrow 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$

$$4 = \tan\theta$$

$$\theta = \tan^{-1}(4)$$

Q2 There are two angles of projections for which the horizontal range is same. Show that the sum of maximum heights for these two angles is independent of the angle of projection.

Q3 A projectile is fired horizontally with a velocity of 100 m/s from top of a hill 500 m high.

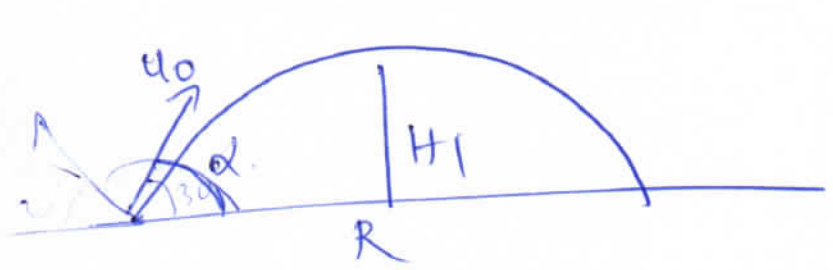
a) Find time taken by projectile to reach the ground

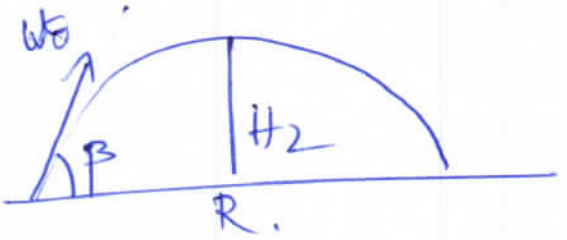
b) velocity with ~~wd~~ which projectile hits the ground ($g = 10 m/s^2$)

Q4) What is average velocity of a particle projected from the ground with speed 'u' at an angle 'α' with the vertical over a time interval from beginning till it strikes the ground again.

②

R





$$R = \frac{u_0^2 \sin 2\alpha}{g} = \frac{u_0^2 \sin 2\beta}{g}.$$

$$H_1 = \frac{u_0^2 \sin^2\alpha}{2g} \qquad H_2 = \frac{u_0^2 \sin^2\beta}{2g}.$$

$$H_1 + H_2 = \frac{1}{2g}\left(u_0^2 \sin^2\alpha + u_0^2 \sin^2\beta\right)$$

$$= \frac{1}{2g}\left(\frac{Rg}{\sin 2\alpha}\sin^2\alpha + \frac{Rg}{\sin 2\beta}\sin^2\beta\right)$$

$$= \frac{R}{2}\left(\frac{\tan\alpha}{2} + \frac{\tan\beta}{2}\right)$$

(B) $\sin 2\alpha = \sin 2\beta$.

$$2\alpha = n\pi + (-1)^n 2\beta.$$

$$\alpha = \frac{n\pi}{2} + (-1)^n \beta$$

$$\alpha = \frac{\pi}{2} - \beta$$

$$H_1 = \frac{u_0^2 \sin^2\alpha}{2g}$$

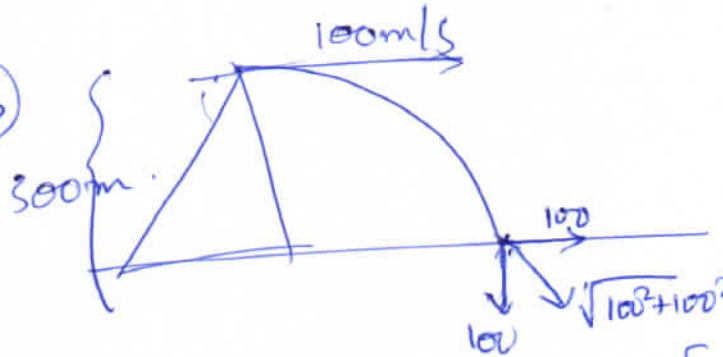
$$H_2 = \frac{u_0^2 \sin^2\beta}{2g}$$

$$H_1 + H_2 = \frac{u_0^2 \sin^2\alpha}{2g} + \frac{u_0^2 \sin^2\beta}{2g}$$

$$= \frac{2u_0^2 \sin^2\alpha}{2g} + \frac{u_0^2 \cos^2\alpha}{2g}$$

$$= \frac{u_0^2}{2g}$$

(3)



$= 100\sqrt{2}$ m/s.

$$v_y^2 - u_y^2 = 2a_y s_y$$

$$v_y^2 - 0^2 = 2(-g)(-500)$$

$$v_y^2 = 2 \times 5000$$

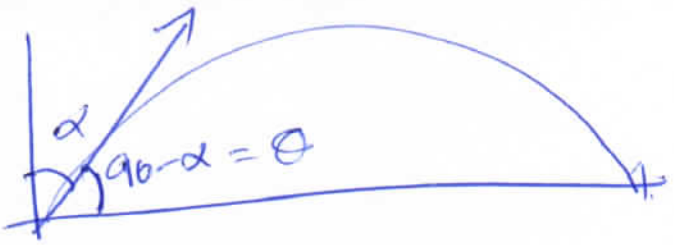
$$v_y = 100$$

$$\vec{v_f} = 100\hat{i} - 100\hat{j}$$

$$\sqrt{100^2+100^2} = 100\sqrt{2}$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 500}{10}} = 10s$$

(4)



Avg Velocity = $\frac{\text{Total displacement}}{\text{total time}}$.

$= \frac{R}{T}$

$= \frac{v_0^2 \sin 2\theta}{g\left(\frac{2v_0 \sin\theta}{g}\right)}$

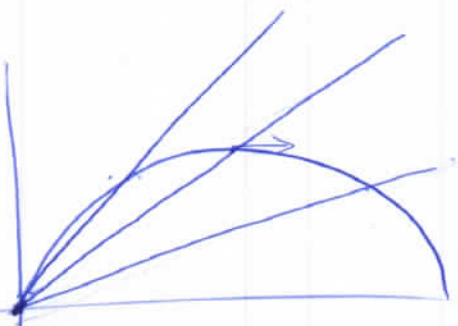
$= \frac{2\sin\theta\cos\theta\,(v_0)}{2\sin\theta}$

$= v_0 \sin\alpha$

$\vec{v}_{avg} = v_0 \sin\alpha \cdot \hat{i}$

Projectile Motion on Inclined Plane.

// to incline	⊥ to incline
$u_{\parallel} = v_0\cos\theta$	$u_{\perp} = v_0\sin\theta$
$a_{\parallel} = -g\sin\alpha$	$a_{\perp} = -g\cos\alpha$



let T be time of flight on incline
↘ time when particle hits the incline.

$s_{\perp} = 0 \qquad t = T$

$s = ut + \frac{1}{2}at^2$

$s_{\perp} = u_{\perp}t + \frac{1}{2}a_{\perp}t^2$

$0 = (v_0\sin\theta)T + \frac{1}{2}(-g\cos\alpha)T^2$

$$T = \frac{2v_0\sin\theta}{g\cos\alpha}$$

$s_{11} = ?$ $\quad t = T$

$$s = ut + \frac{1}{2}at^2$$

$$s_{11} = u_{11}t + \frac{1}{2}a_{11}t^2$$

$$s_{11} = (v_0\cos\theta)(T) + \frac{1}{2}(-g\sin\alpha)T^2$$

$$= (v_0\cos\theta)\left(\frac{2v_0\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha\left(\frac{4v_0^2\sin^2\theta}{g^2\cos^2\alpha}\right)$$

$$= \frac{2v_0^2\sin\theta}{g}\left[\frac{\cos\theta}{\cos\alpha} - \frac{\sin\theta\tan\alpha}{\cos\alpha}\right]$$

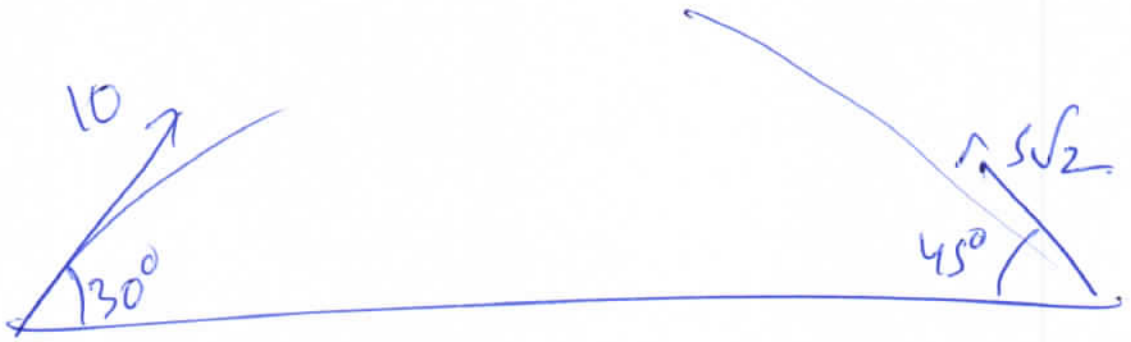
$$= \frac{2v_0^2\sin\theta}{g}\left[\frac{\cos\theta}{\cos\alpha} - \frac{\sin\theta\sin\alpha}{\cos^2\alpha}\right]$$

$$= \frac{2v_0^2\sin\theta}{g}\left[\frac{\cos\theta\cos\alpha - \sin\theta\sin\alpha}{\cos^2\alpha}\right]$$

$$= \boxed{\frac{2v_0^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}}$$

Q1 Two particles A & B are projected from ground at $t=0$, towards each other with speeds 10 m/s & $5\sqrt{2}$ m/s at angles 30° & 45° with horizontal from the two points seperated by a distance of 15 m. Will they collide (If yes find time when they collide)

Ans 1



$10\cos 30\, t + 5\sqrt{2}\cos 45\, t = 15.$

$\left(\frac{10\sqrt{3}}{2} + 5\sqrt{2} \times \frac{1}{\sqrt{2}}\right) t = 15$

$t = \frac{15}{5(1+\sqrt{3})} = \frac{3}{\sqrt{3}+1}\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$= \frac{3}{2}(\sqrt{3}-1)$

$= \frac{3}{2}(0.732)$

$= 3(0.366)$

$= 3\left(\frac{3}{10} + \frac{66}{1000}\right)$

$= 3\left(\frac{3}{10} + \frac{0.66}{10}\right)$

$= 3\left(\frac{3}{10} + \frac{2}{30}\right)$

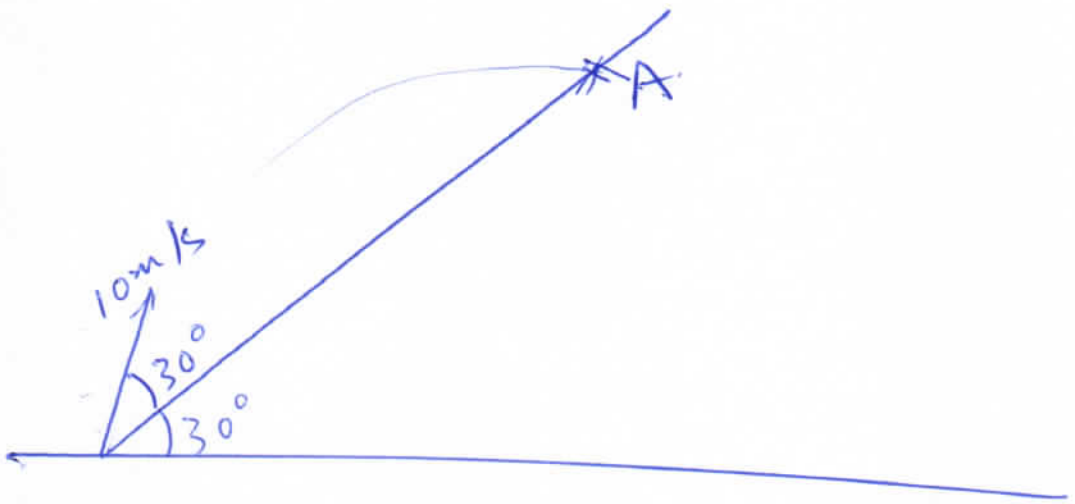
$= 3\left(\frac{11}{30}\right) = \frac{11}{10}\ s$

$S_{y_1} = 10\sin 30\left(\frac{11}{10}\right) - \frac{1}{2}(10)\left(\frac{11}{10}\right)^2$ $S_{y_2} = 5\sqrt{2}\sin 45\left(\frac{11}{10}\right) - \frac{1}{2}\times 10\left(\frac{11}{10}\right)^2$

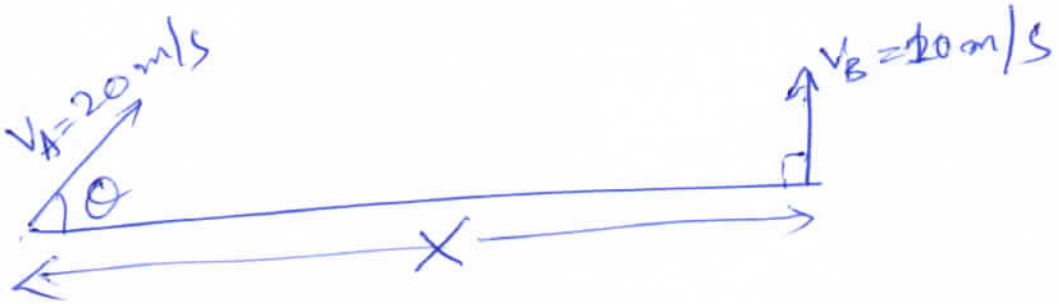
$v_1 \sin\theta_1 = v_2 \sin\theta_2$. $v_1 \sin\theta_1 = v_2 \sin\theta_2$

collide

Q2 A particle is projected along an incline plane as shown. What is speed of particle when it collides at point A



Q3 Two particles A & B are projected simultaneously in the directions shown with velocities $V_A = 20\,m/s$ & $V_B = 10\,m/s$, respectively. They collide in air after 0.5 s. Find a) θ b) X



To collide $V_A \sin\theta = V_B \sin 90°$

$20 \sin\theta = 10$

$\theta = 30°$

$X = V_A \cos\theta\,(0.5) = 20 \frac{\sqrt{3}}{2} \times \frac{1}{2} = 5\sqrt{3}$

Ans2

$$T = \frac{2V_0 \sin\theta}{g\cos\alpha}$$

$$= \frac{2 \times 10 \sin 30}{10 \times \sqrt{3}/2} = \frac{2}{\sqrt{3}}\ s.$$

$$v = 10\cos 60\, i + \left(10\sin 60 - 10 \times \frac{2}{\sqrt{3}}\right) j$$

$$= 5i + \left(\frac{10\sqrt{3}}{2} - \frac{20}{\sqrt{3}}\right) j$$

$$= 5i + \left(-\frac{10}{2\sqrt{3}} j\right)$$

$$= 5i - \frac{5}{\sqrt{3}} j$$

$$\sqrt{5^2 + \frac{5^2}{3}}$$

$$= 5\sqrt{1 + \frac{1}{3}} = \frac{10}{\sqrt{3}}$$