VECTORS.

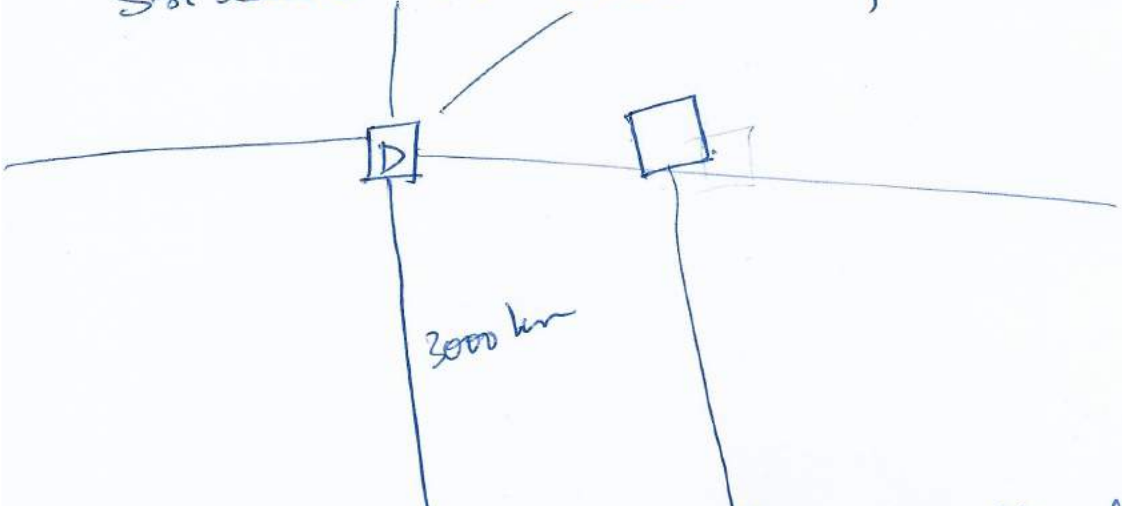
Scalar Quantity : Which does not need direction to be specified.

eg. Mass , time, temperature.

Vector Quantity : Which needs direction to be specified.

eg. position.

Sri lanka is 3000 km from Delhi



Sri lanka is 3000 km South of Delhi

$\vec{a}$ a

$\vec{v}$ v

magnitude: $|\vec{v}|$.

$\vec{v} = 30$ km/hr X

$\vec{v} = 30$ km/hr east.

$v = 30$ km/hr.

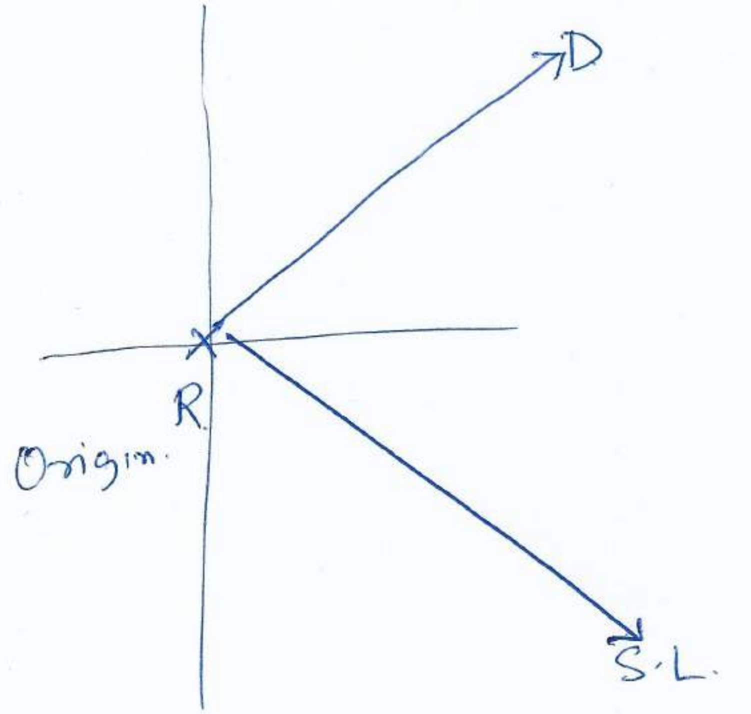
$\leftarrow a$

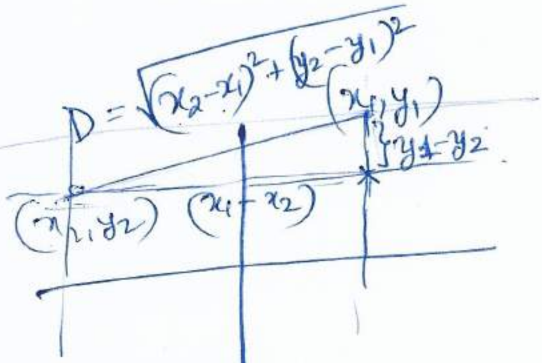
fixed —

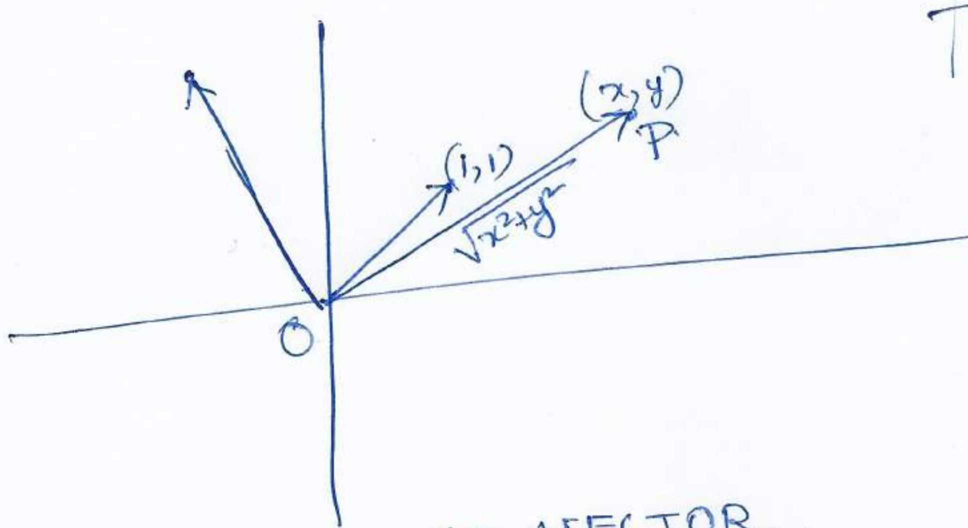
free —

POSITION VECTOR

Reference to a position.

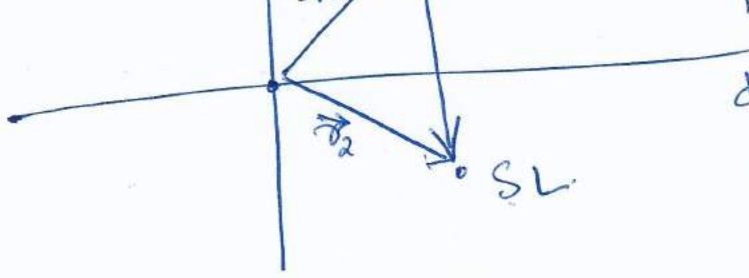






$\vec{r} = \overrightarrow{OP}$ ← position vector of point P.

DISPLACEMENT VECTOR



if I am going from point A to point B.

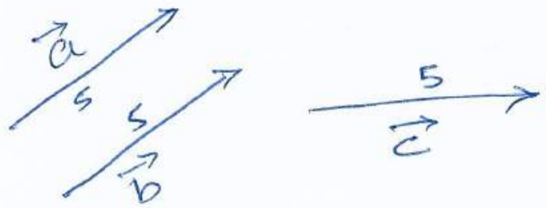
displacement vector = $\overrightarrow{AB}$

= Difference of p.v B & p.v A

$= \vec{r}_b - \vec{r}_a$

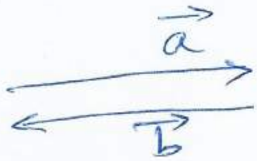
EQUAL VECTORS

Having same magnitude & direction.



NEGATIVE VECTOR

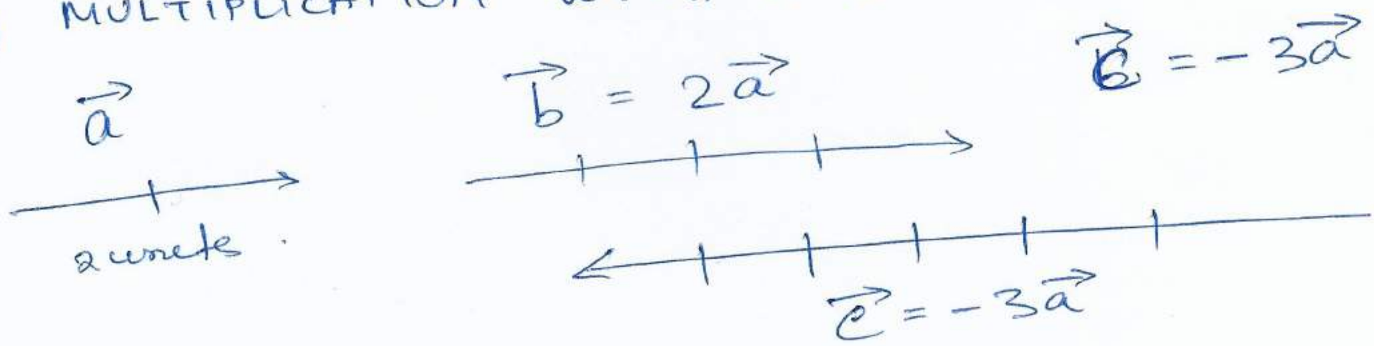
$\vec b = -\vec a$



$\vec b$ is said to be negative of $\vec a$ if direction of $\vec b$ is opposite of vector $\vec a$ & $|\vec a| = |\vec b|$

ALGEBRA OF VECTORS.

(1) MULTIPLICATION WITH A SCALAR



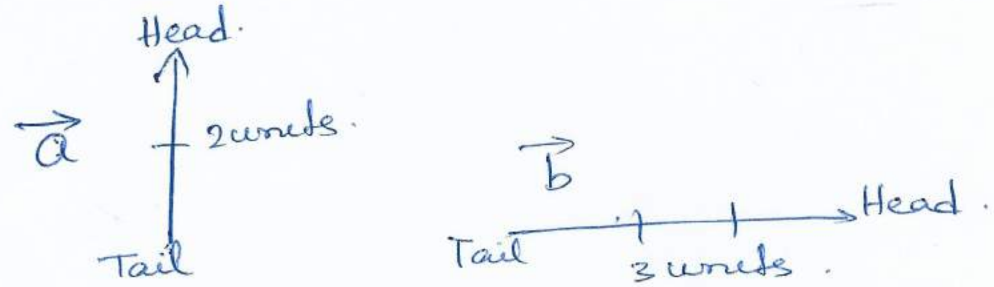
② ADDITION | SUBTRACTION OF VECTORS

a) → TRIANGLE LAW OF ADDITION

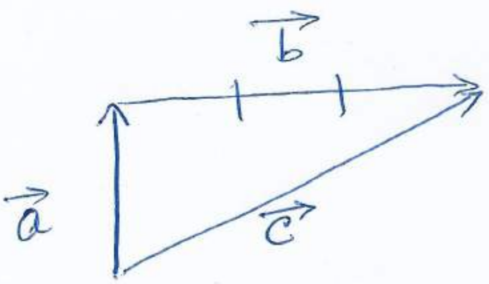
ADDITION

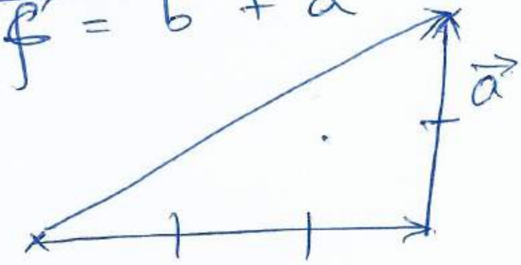
$\vec{a} + \vec{b}$ $\quad$ $\vec{a} - \vec{b}$

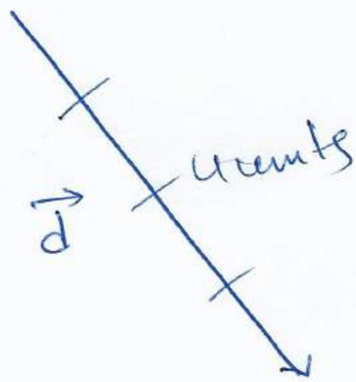
$\vec{a} + (-\vec{b})$



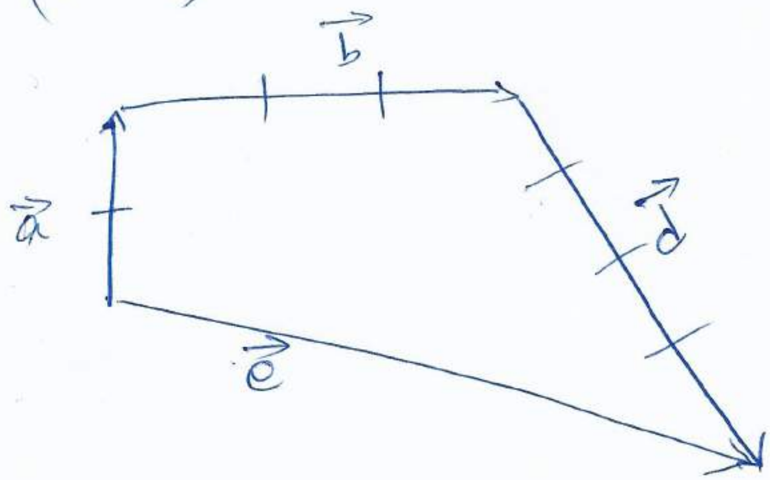
$\vec{c} = \vec{a} + \vec{b}$ $\qquad$ $\vec{f} = \vec{b} + \vec{a}$



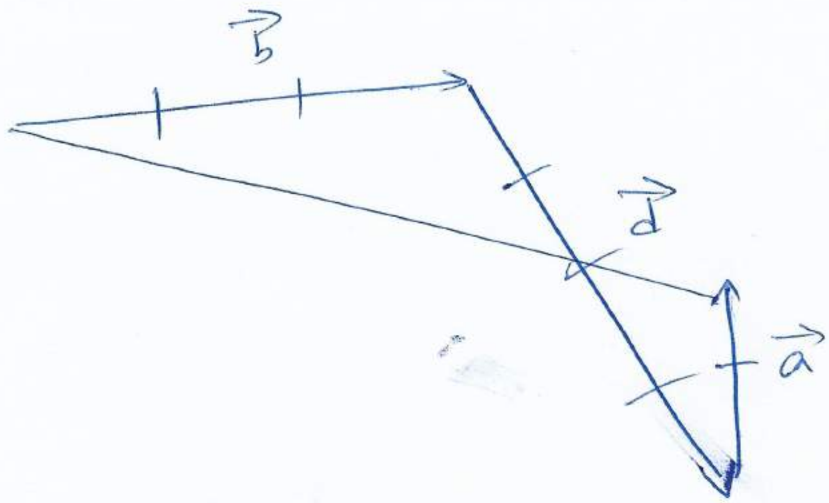




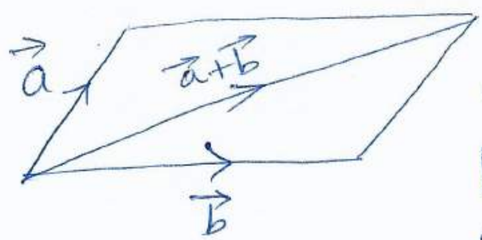
$\vec{e} = (\vec{a} + \vec{b}) + \vec{d}$ $\quad$ or $\quad$ $\vec{a} + (\vec{b} + \vec{d})$



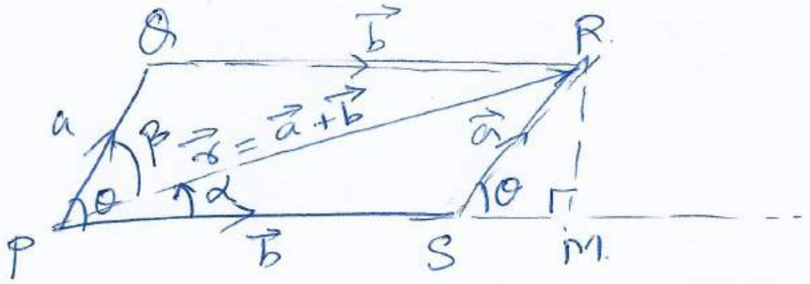
$$\vec{g} = \vec{b} + \vec{d} + \vec{a}$$



PARALLELOGRAM LAW OF ADDITION



If 2 vectors are represented as adjacent sides of a parallelogram then the diagonal through there intersection represents the resultant.



$RS = |\vec{a}|$ $\qquad$ $PS = |\vec{b}|$

Δ RSM. $\qquad \dfrac{RM}{RS} = \sin\theta \implies RM = RS \sin\theta$

$$= |\vec{a}| \sin\theta$$

$$\frac{SM}{RS} = \cos\theta \implies SM = RS\cos\theta$$

$$= |\vec{a}| \cos\theta$$

In Δ RMP.

$$RP^2 = RM^2 + PM^2$$

$$|\vec{r}|^2 = (|\vec{a}|\sin\theta)^2 + (|\vec{b}| + |\vec{a}|\cos\theta)^2$$

$$= |\vec{a}|^2\sin^2\theta + |\vec{a}|^2\cos^2\theta + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

$$= |\vec{a}|^2(\sin^2\theta + \cos^2\theta) + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{r}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

$$\boxed{|\vec{r}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta}}$$

θ = angle between $\vec{a}$ & $\vec{b}$

In Δ RMP

$$\boxed{\tan\alpha = \frac{RM}{PM} = \frac{|\vec{a}|\sin\theta}{|\vec{b}| + |\vec{a}|\cos\theta}}$$

α is angle made by resultant $\vec{r}$ with vector $\vec{b}$

$$\tan\beta = \frac{|\vec{b}|\sin\theta}{|\vec{a}| + |\vec{b}|\cos\theta}$$

β is angle made by resultant $\vec{r}$ with vector $\vec{a}$

Some Special Cases.

i) If two vectors are in same direction

$\theta = 0^\circ$

$\cos\theta = 1$

$\vec{r} = \vec{a} + \vec{b}$

$|\vec{r}| = \sqrt{a^2 + b^2 + 2ab(1)} = \sqrt{(a+b)^2} = a+b.$

ii) If two vectors are in opposite direction.

$\theta = 180^\circ$

$\cos 180^\circ = -1.$

$\vec{r} = \vec{a} + \vec{b}$

$|\vec{r}| = \sqrt{a^2 + b^2 + 2ab(-1)} = \sqrt{(a-b)^2} = a-b$

iii) If two vectors are $\perp$ to each other.

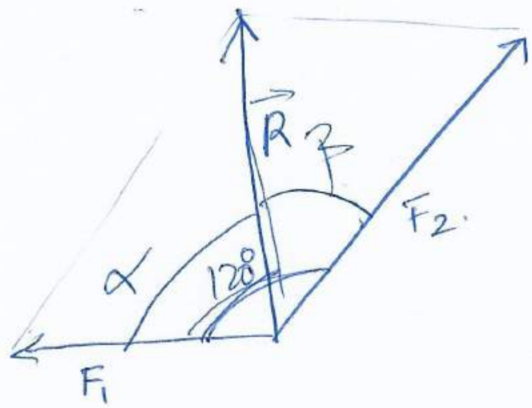
$\theta = 90^\circ \qquad \cos 90^\circ = 0$

$\vec{r} = \vec{a} + \vec{b}$

$|\vec{r}| = \sqrt{a^2 + b^2 + 2ab(0)} = \sqrt{a^2 + b^2}$

Q) If two forces $|\vec{F_1}| = 5N$ & $|\vec{F_2}| = 20N$ are at 120° to each other.

find $\vec{R} = \vec{F_1} + \vec{F_2}$

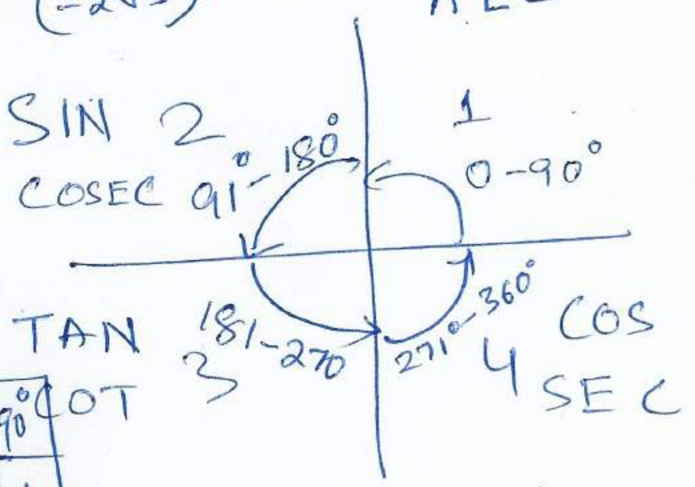


$$|\vec{R}| = \sqrt{5^2+20^2+2(5)(20)\left(\frac{-1}{2}\right)}$$
$$= \sqrt{325}$$
$$= 5\sqrt{13}\ \text{N}$$

$$\tan\alpha = \frac{|\vec{F_2}|\sin\theta}{|\vec{F_1}| + |\vec{F_2}|\cos\theta} = \frac{20\times\frac{\sqrt{3}}{2}}{5+20\times\left(-\frac{1}{2}\right)}$$
$$= \cancel{20x}\ \frac{10\sqrt{3}}{-5}$$

$$\tan\alpha = -2\sqrt{3}$$
$$\alpha = \tan^{-1}(-2\sqrt{3})$$

Sin Cosec
Cos Sec
tan Cot



θ	0°	30°	45°	60°	90°
Sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

$0° \le \theta \le 90°$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(90° - \theta) = \cos\theta$
$\cos(90° - \theta) = \sin\theta$
$\tan(90° - \theta) = \cot\theta$

$\cos 120° \longrightarrow$ (0-90°)

$\sin(90° + \theta) = +\cos\theta$
$\cos(90° + \theta) = -\sin\theta$
$\tan(90° + \theta) = -\cot\theta$
$\sec(90° + \theta) = -\operatorname{cosec}\theta$
$\operatorname{cosec}(90° + \theta) = +\sec\theta$
$\cot(90° + \theta) = -\tan\theta$

$\sin(270^\circ + \theta) = -\cos\theta$

$\cos(270^\circ + \theta) = +\sin\theta$

$\tan(270^\circ + \theta) = -\cot\theta$

$\cot(270^\circ + \theta) = -\tan\theta$

$\sec(270^\circ + \theta) = +\operatorname{cosec}\theta$

$\operatorname{cosec}(270^\circ + \theta) = -\sec\theta$

$\sin(270^\circ - \theta) = -\cos\theta$

$\cos(270^\circ - \theta) = -\sin\theta$

$\tan(270^\circ - \theta) = +\cot\theta$

$\cot(270^\circ - \theta) = +\tan\theta$

$\sec(270^\circ - \theta) = -\operatorname{cosec}\theta$

$\operatorname{cosec}(270^\circ - \theta) = -\sec\theta$

$\sin(180^\circ - \theta) = +\sin\theta$

$\cos(180^\circ - \theta) = -\cos\theta$

$\tan(180^\circ - \theta) = -\tan\theta$

$\cot(180^\circ - \theta) = -\cot\theta$

$\sec(180^\circ - \theta) = -\sec\theta$

$\operatorname{cosec}(180^\circ - \theta = +\operatorname{cosec}\theta$

$\sin(180^\circ + \theta) = -\sin\theta$

$\cos(180^\circ + \theta) = -\cos\theta$

$\tan(180^\circ + \theta) = +\tan\theta$

$\cot(180^\circ + \theta) = +\cot\theta$

$\sec(180^\circ + \theta) = -\sec\theta$

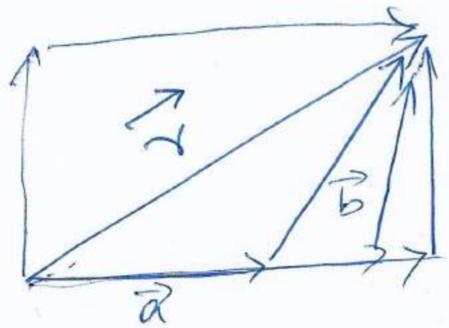
$\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$

$\tan 855^\circ = \tan 135^\circ = \tan(90^\circ + 45^\circ) = -\cot 45^\circ = -1$

$\sec 120^\circ = \sec(90^\circ + 30^\circ) = -\operatorname{cosec} 30^\circ = -2$

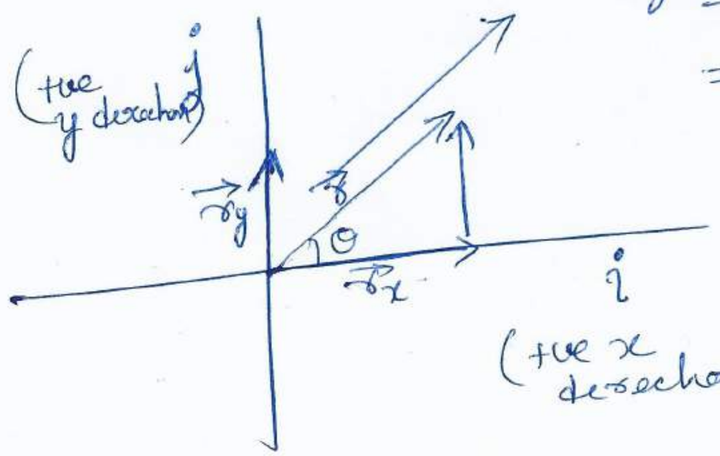
$\sin 570^\circ = \sin 210^\circ = \sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$

Components of vectors



A vector $\vec{r}$ represented as sum of 2 vectors $\vec{r_1}$ & $\vec{r_2}$ $\perp$ to each other. then $\vec{r_1}$ & $\vec{r_2}$ are said to be components of $\vec{r}$

$$\vec{r} = \vec{r_1} + \vec{r_2}$$
$$= \vec{r_x} + \vec{r_y}$$



$$\frac{|\vec{r_x}|}{|\vec{r}|} = \cos\theta \qquad \frac{|\vec{r_y}|}{|\vec{r}|} = \sin\theta$$

$$|\vec{r_x}| = |\vec{r}|\cos\theta \qquad |\vec{r_y}| = |\vec{r}|\sin\theta$$

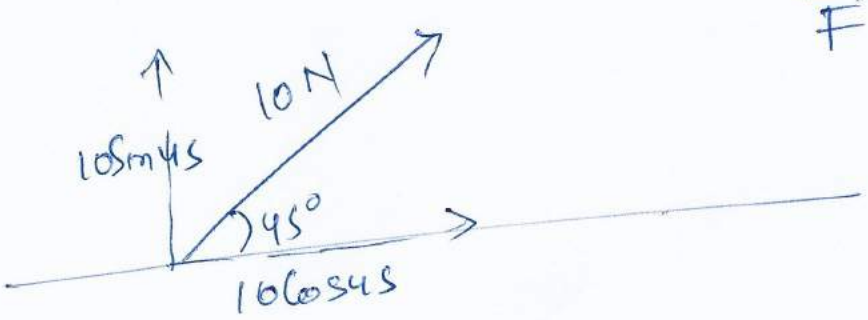
$$\vec{r} = \underbrace{|\vec{r_x}|\, i} + \underbrace{|\vec{r_y}|\, j}$$

$$\vec{r} = r\cos\theta\, i + r\sin\theta\, j$$

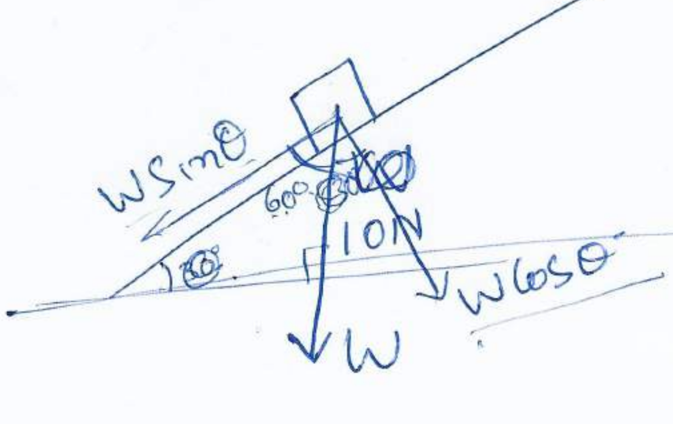
θ angle made by $\vec{r}$ with the x axis

Q) Resolve a Force 10N horizontally & Vertically which angle 45° with horizontal

Q) Resolve a Weight 10N in two directions which are ||l & ⊥ to a slope inclined at 30° to the horizontal.



$$\vec{F} = 10\cos 45\, i + 10\sin 45\, j$$
$$= 5\sqrt{2}\, i + 5\sqrt{2}\, j$$



PRODUCT OF VECTORS

SCALAR PRODUCT (DOT PRODUCT) $\bullet$

Here we get a result as scalar quantity.

$\vec{a} \cdot \vec{b}$ = a scalar qty $= |\vec{a}||\vec{b}|\cos\theta$

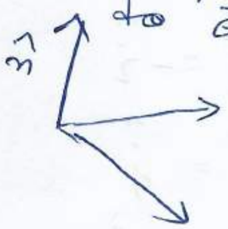
VECTOR PRODUCT (CROSS PRODUCT) $\times$

Here we get result as vector quantity

$\vec{a} \times \vec{b}$ = a vector qty

magnitude: $|\vec{a}||\vec{b}|\sin\theta$

direction: $\hat{n}$ — a direction perpendicular to $\vec{a}$ & $\vec{b}$



i) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ commutative

ii) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ distributive.

iii) $\vec{a} \cdot \vec{a} = |\vec{a}|^2 \quad (\because \cos\theta = 1)$

iv) $\hat{i} \cdot \hat{i} = 1 \qquad \hat{i} \cdot \hat{j} = ? \quad \hat{i} \cdot \hat{k} = ?$

$\hat{j} \cdot \hat{j} = 1$

$\hat{k} \cdot \hat{k} = 1$

v) $\hat{i} \cdot \hat{j} = 0$

$\hat{j} \cdot \hat{k} = 0$

$\hat{i} \cdot \hat{k} = 0$

vi) $(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}) \cdot (a_2\hat{i} + b_2\hat{j} + c_2\hat{k}) = a_1a_2 + b_1b_2 + c_1c_2$

Q. Work done by a $\vec{F}$ on a body $W = \vec{F} \cdot \vec{s}$ where $\vec{s}$ is the displacement of body. Given $\vec{F} = (2\hat{i} + 3\hat{j} + 4\hat{k})$N a body is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j} + \hat{k})$m to $\vec{r}_2 = (\hat{i} + \hat{j} + \hat{k})$m. Find work done by this Force.

Q. Prove than $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ is perpendicular to $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

properties of cross product

i) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

(ii) magnitude of $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta$

If $\sin\theta = 0$ $\quad \vec{A} \times \vec{B} = 0$

$\hat{i} \times \hat{i} = 0 \qquad \hat{j} \times \hat{j} = 0 \qquad \hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{i} = -\hat{i} \times \hat{j} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{k} = -\hat{j}$

If $\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = (b_1c_2 - c_1b_2)\hat{i} - (a_1c_2 - c_1a_2)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Q Find a unit vector $\perp$ to $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$

& $\vec{B} = \hat{i} - \hat{j} + \hat{k}$ both.

$$\vec{C} = \vec{A} \times \vec{B} \qquad (4\hat{i} - \hat{j} - 5\hat{k})$$

Q Show $\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$ is parallel to vector $\vec{B} = 3\hat{i} - 3\hat{j} + 6\hat{k}$.

If $\vec{A}$ is $\parallel$ to $\vec{B}$

the $\vec{A} = m\vec{B}$

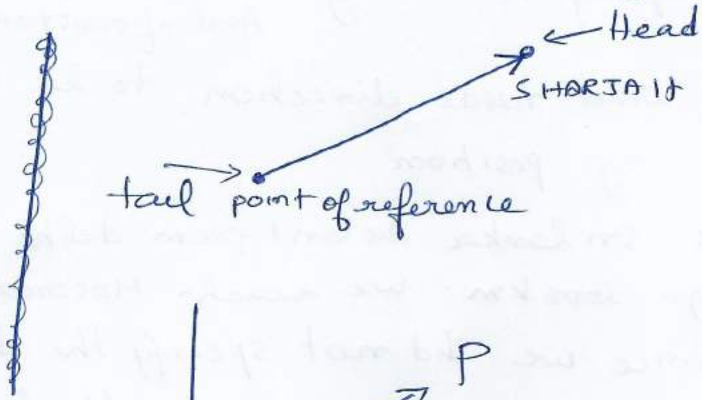
$$\hat{i} - \hat{j} + 2\hat{k} = \frac{1}{3}(3\hat{i} - 3\hat{j} + 6\hat{k})$$

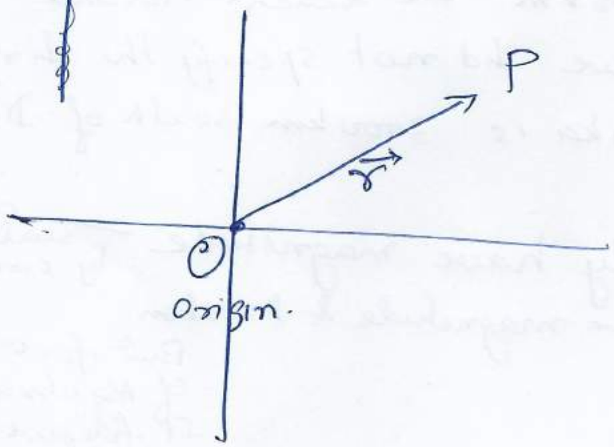
$$\vec{A} = \frac{1}{3}\vec{B}$$

POSITION VECTOR

point of reference to the position.

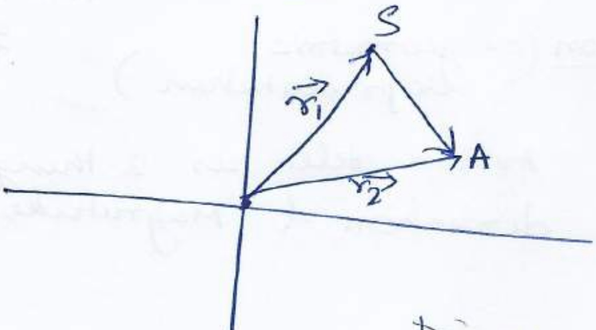
eg. POSITION VECTOR OF SHARJAH from here





$OP \rightarrow$ Magnitude
$\overrightarrow{OP}$ gives direction.

DISPLACEMENT VECTOR



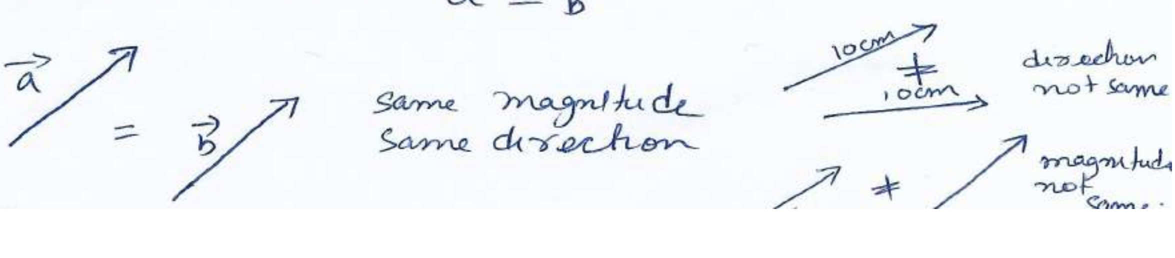
goes from Sharjah (S) to Abu Dhabi (A)

$\overrightarrow{SA}$ = displacement vector.

Displacement Vector = Difference between two position vectors.

EQUAL VECTORS

If magnitudes & directions are same for two vectors $\vec{a}$ & $\vec{b}$ then $\vec{a} = \vec{b}$



VECTOR

A physical quantity should be described with or without direction.

<u>Scalar quantity</u> : which does not need direction to be specified eg. Mass, temperature, time, Electric current, Amt. of substance, Volume.

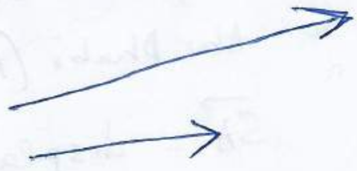
<u>Vector quantity</u> : Which needs direction to be specified.
eg. position

How far is Srilanka distant from delhi – let say 3000km
But If we go 3000 km We reach Moscow
Why? Since we did not specify the direction.
We say Srilanka is 3000 km south of Delhi.

Scalar Quantity only have magnitude ← operations on scalar qty are done by simple rule of Algebra.
Vector Qty have magnitude & Direction.
But for vectors all rules of Algebra is not followed
It follows laws of Vector Algebra

A Vector qty is represented by $\vec{a}$ ← arrow represents that its a vector qty.

A <u>visual representation</u> (or geometric representation)



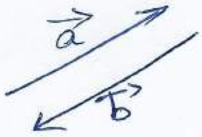
Arrow tells us 2 things
direction & Magnitude (length of arrow)

<u>Magnitude of vector quantity</u> $\vec{a}$ is represented by $|\vec{a}|$
and just gives the value of vector quantity. is a scalar
∴ it is scalar it will follow laws of Algebra.

NEGATIVE VECTOR

If direction of vector $\vec{b}$ is opposite of vector $\vec{a}$ & $|\vec{a}| = |\vec{b}|$ then $\vec{b}$ is said to be negative of $\vec{a}$

$$\vec{b} = -\vec{a}$$



OPERATIONS ON VECTORS

→ MULTIPLICATION WITH A SCALAR

If $\vec{a} = 3$ m east.

How will

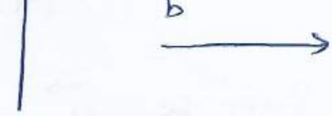
$2\vec{a}$ be represented.

$2\vec{a} = 6$ m east.

So magnitude is multiplied by scalar & direction remains same.

→ ADDITION OF VECTORS

Suppose there are two quantities $\vec{a}$ ↑ $\vec{b}$

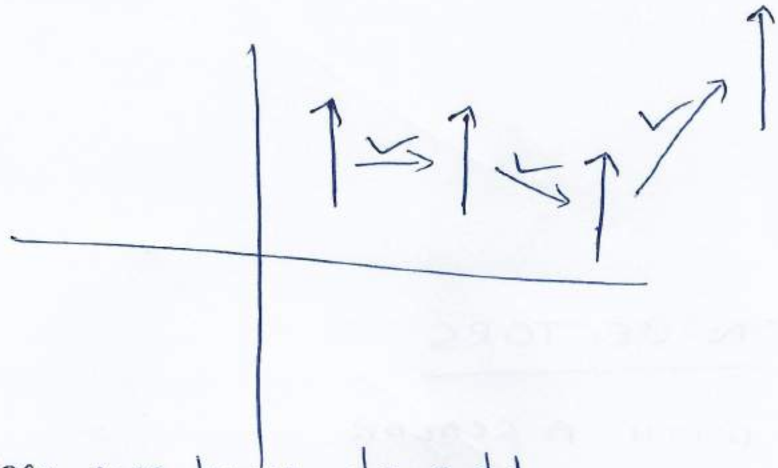


What should be fundamental rule for addition of vectors (their nature should be same)

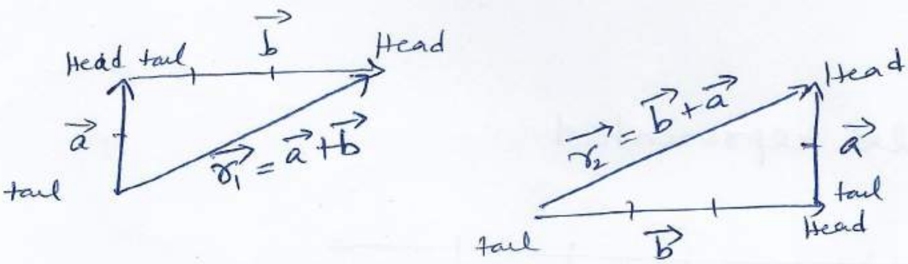
eg. cannot add displacement with velocity

Geometric addition.

When dealing with vectors we have one advantage that we can move it to any position in space without changing its magnitude & direction.



Suppose we have to add

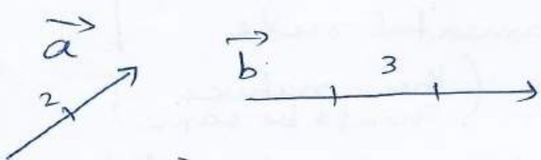


Doesn't matter if we start with $\vec{a}$ or $\vec{b}$

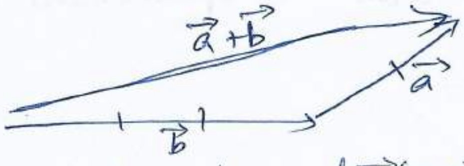
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

This method is called Triangle law of addition.
($\because$ a triangle is formed)

Suppose $\vec{a}$



$\vec{a} + \vec{b}$

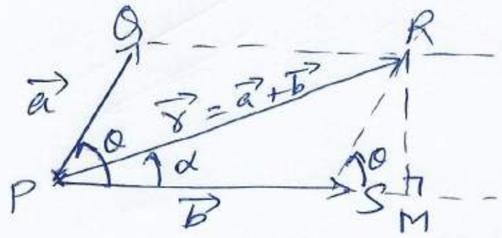


magnitude is larger for $\vec{a} + \vec{b}$ (Even though magnitude of $\vec{a}$ & $\vec{b}$ here is same)

* So magnitude & direction of $\vec{r}(\vec{a}+\vec{b})$ depends on angle between the vectors.

Parallelogram law of Addition of Vectors

If two vectors are represented by 2 adjacent sides of a parallelogram then the diagnol through their intersection represents their resultant (addition)



PR is diagnol passes through intersection of $\vec{a}$ & $\vec{b}$

$RS = |\vec{a}| = a$

$PS = |\vec{b}| = b$

In Δ RSM

$\frac{RM}{RS} = \sin\theta \quad \therefore RM = RS \cdot \sin\theta$
$= a\sin\theta$

$\frac{SM}{RS} = \cos\theta \quad \therefore SM = RS \cdot \cos\theta$
$= a\cos\theta$

In Δ PRM

$PR^2 = RM^2 + PM^2$
$= (a\sin\theta)^2 + (PS + SM)^2$
$= a^2\sin^2\theta + (b + a\cos\theta)^2$
$= a^2\sin^2\theta + b^2 + a^2\cos^2\theta + 2ab\cos\theta$

$r^2 = a^2 + b^2 + 2ab\cos\theta$

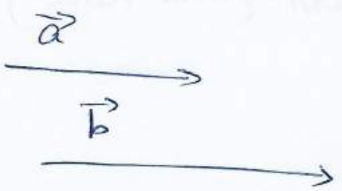
$|\vec{r}| = r = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

Direction of resultant.

$\tan\alpha = \frac{RM}{PM} = \frac{a\sin\theta}{b + a\cos\theta}$ (α is angle made with $\vec{b}$)

Special Cases

i) two vectors in same direction.



$\cos\theta = 1$

$\because \theta = 0^\circ$

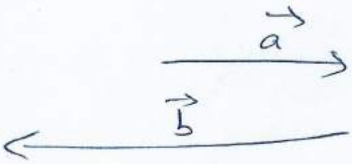
$\vec{r} = \vec{a} + \vec{b}$

$|\vec{r}| = \sqrt{a^2 + b^2 + 2ab \cdot 1}$

$= a + b = |\vec{a}| + |\vec{b}|$

ii) two vectors in opposite direct.

$\theta = 180^\circ \quad \cos\theta = -1$



$\vec{r} = \vec{a} + \vec{b}$

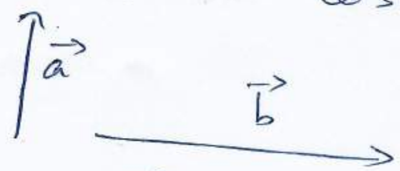
$|\vec{r}| = \sqrt{a^2 + b^2 + 2ab \cdot (-1)}$

$= \sqrt{a^2 + b^2 - 2ab}$

$= a - b = |\vec{a}| - |\vec{b}|$

iii) two vectors are $\perp$ to each other.

$\theta = 90^\circ \quad \cos\theta = 0$



$\vec{r} = \vec{a} + \vec{b}$

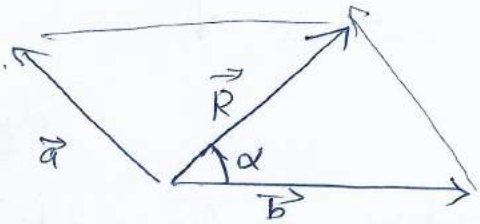
$|\vec{r}| = \sqrt{a^2 + b^2 + 2ab \cdot 0} = \sqrt{a^2 + b^2}$

Q. Two forces $|\vec{F_1}| = 5\text{ N}$ & $|\vec{F_2}| = 20\text{ N}$ are at $120°$ to each other. find $\vec{R} = \vec{F_1} + \vec{F_2}$

$$R = \sqrt{5^2 + 20^2 + 2\times5\times20\cos120}$$

$$= \sqrt{5^2+20^2+2\times5\times20\left(-\tfrac{1}{2}\right)}$$

$$|\vec{R}| = R = 18.03\text{ N}$$



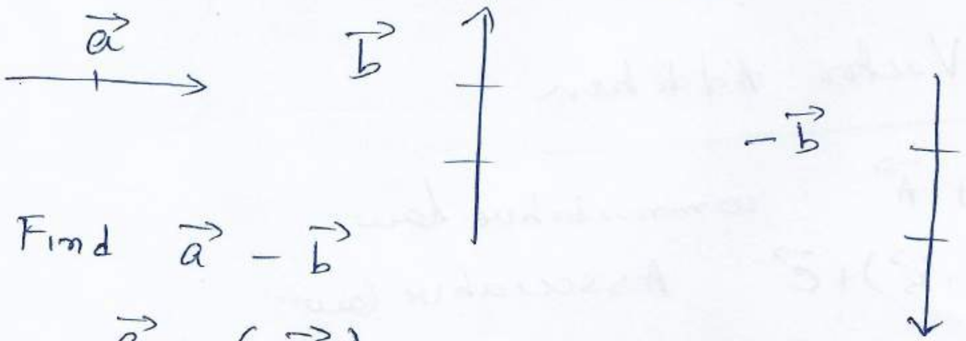
Direction of Resultant

$$\tan\alpha = \frac{a\sin\theta}{b + a\cos\theta} \qquad (\alpha \text{ is with } \vec{b})$$

$$= \frac{5\sin120°}{20+5\cos120°} = \frac{\frac{5\sqrt{3}}{2}}{20 - 5\times\frac{1}{2}} = \frac{\frac{\cancel{5}\sqrt{3}}{\cancel{2}}}{{}_{7}\cancel{35}/\cancel{2}} = \frac{\sqrt{3}}{7}$$

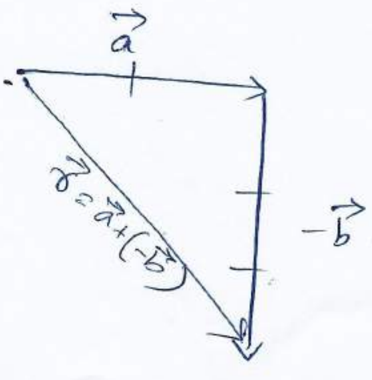
$$\alpha = \tan^{-1}\frac{\sqrt{3}}{7}$$

SUBTRACTION OF VECTORS

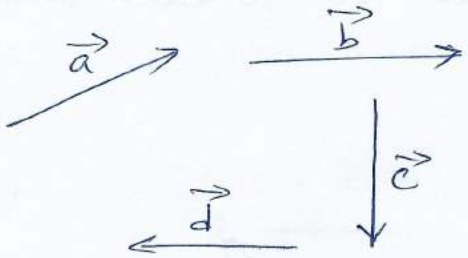


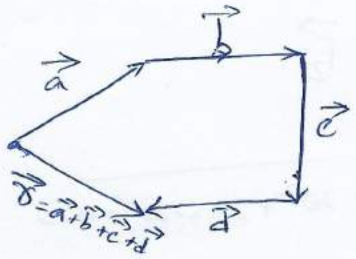
Find $\vec{a} - \vec{b}$

$\vec{a} + (-\vec{b})$



More than two vectors

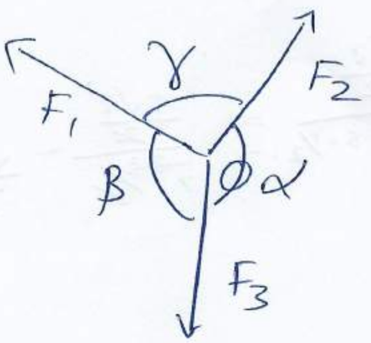




Polygon law

If vectors to be added are represented by sides are polygon the resultant $\vec{r}$ is drawn from tail of first to the head of the last vector.

Lami's Theorem



Three forces F_1, F_2 & F_3 are acting on a point 'O' which is in equilibrium (Net Force is 0)

Then $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$

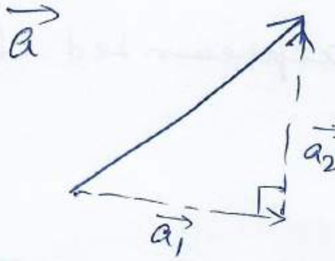
Properties of Vector Addition

1) $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ commutative law

2) $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$ Associative law.

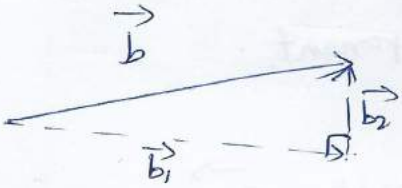
Components of vector

Consider any vector

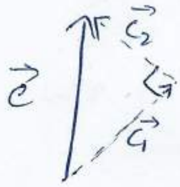


$$\vec{a} = \vec{a_1} + \vec{a_2}$$

Can be represented by sum of 2 vectors ($\perp$ to each other)



$$\vec{b} = \vec{b_1} + \vec{b_2}$$ ($\vec{b_1}$ & $\vec{b_2}$ are $\perp$ to each other)

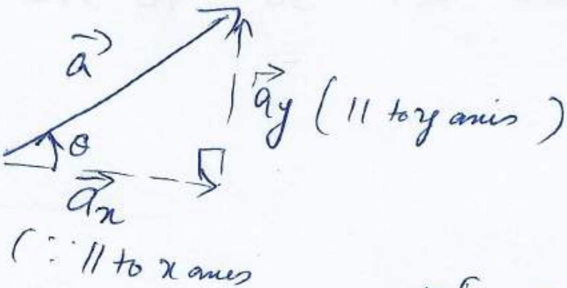


$$\vec{c_1} + \vec{c_2} = \vec{c}$$

but if I restrict condition of c_1 & c_2 that they should be ~~along~~ parallel to coordenate axis

$\vec{c}$  $= \vec{c} = \vec{c_2}$

So



$$\vec{a} = \underbrace{\vec{a_x}}_{x \text{ component of } \vec{a}} + \underbrace{\vec{a_y}}_{y \text{ component of } \vec{a}}$$

If θ is angle made by $\vec{a}$ with x-axis

$|\vec{a_x}| = |\vec{a}| \cos\theta$

$|\vec{a_y}| = |\vec{a}| \sin\theta$

direction along x

is represented by $\hat{i}$

direction along y is represented by $\hat{j}$

direction along z is represented by $\hat{k}$

$$\vec{a} = \vec{a_x} + \vec{a_y}$$

$$= \underbrace{\underbrace{|a_x|}_{\text{magnitude}}\ \underbrace{\hat{i}}_{\text{direction}}}_{x \text{ component}} + \underbrace{\underbrace{|a_y|}_{\text{mgn}}\ \underbrace{\hat{j}}_{\text{direction}}}_{y \text{ component}}$$

breaking $\vec{a}$ into $\vec{a_x}$ and $\vec{a_y}$ ($\perp$ to each other)

If θ is angle made $\vec{a}$ with $\vec{a_x}$ then

$$\vec{a} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$$

Q. Resolve a Force 10 N horizontally & vertically and which makes an angle of 45° with horizontal

Q. Resolve a weight 10 N in two directions which are parallel and perpendicular to a slope inclined at 30° to the horizontal.