UNITS & DIMENSIONS.

Physical Quantity – A characteristic which can be measured and describes various laws of Physics.

Fundamental Quantities

These are chosen and named as fundamental Quantities

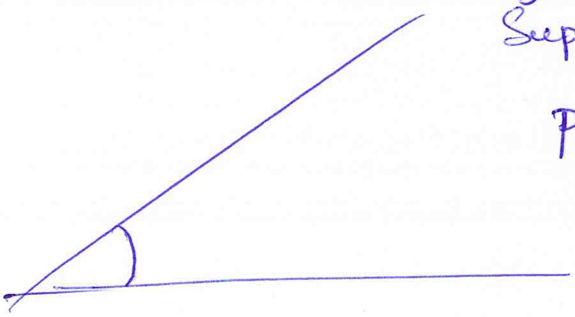
Derived Quantities.

These are derived from fundamental quantities.

Fundamental Quantity	SI Unit	Symbol.
1) Mass	Kilogram	kg
2) Length	metre	m
3) Time	second	s
4) Temperature	Kelvin	K
5) Amount of Substance	mol	mol
6) Electric Current.	Ampere	A
7) Luminous Intensity.	Candela	cd

Supplementary Quantities.

Plane Angle



Solid Angle.

Unit : Its a standared reference used for measurement of same physical quantity.

measurement = nu

numerical part · unit

System of Units

1) S.I Unit (System International d'Unite's).

2) CGS System.
length is centimetre, mass is grams, time is second

3) MKS system.
length is metre, mass is kilograms, time is second

4) FPS System: length is foot, mass pound.
time is second.

$n_1 u_1 = n_2 u_2$ { Measurement will not change if we change the units.

DIMENSIONS: of a physical quantity is the powers raised to the fundamental quantities.

Dimensions of fundamental Quantity

length	[L]
Mass	[M]
Time	[T]
Temperature	[K]
Electric Current	[A]
Amt. of substance	[mol]
luminous Intensity	[cd]

DIMENSIONS OF DERIVED QUANTITY.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{[M]}{[L^3]} = [M L^{-3}]$$

Find the dimensions of.

1) Velocity $= [LT^{-1}]$

2) Acceleration $= [LT^{-2}]$

3) Force $= [MLT^{-2}]$

4) Work done $= [ML^2T$

DIMENSION ANALYSIS:

1) Conversion from one unit to another.

$g = 9.8\ m/s^2 \xrightarrow{\text{CGS System}} cm/s^2$

$$9.8\ m/s^2 = 9.8\left(\frac{1m}{1s \times 1s}\right) = 9.8\left(\frac{100cm}{1s \times 1s}\right)$$

$$= 980\left(\frac{1cm}{1s \times 1s}\right)$$

$$\doteq 980\ cm/s^2$$

(2) To finding unit of a physical quantity.

$$\text{Velocity} = [LT^{-1}] = ms^{-1}$$

(3) To check correctness of a formula.

$$A = B + C^2$$ principle of Homogeneity of Dimensions.

$$\dim(A) = \dim(B) = \dim(C^2)$$

Ques If $v = a + bt + \dfrac{c}{d+t}$

The velocity of particle depends upon time 't' according to equation above then find dimensions of a, b, c & d.

$$\dim(a) = \dim(v) = [LT^{-1}]$$

$$\dim(d) = \dim(t) = [T]$$

$$\dim(bt) = \dim(v) = \dim(b)[T] = [LT^{-1}]$$

$$\Rightarrow \dim(b) = [LT^{-2}]$$

$$\dim\left(\frac{c}{d+t}\right) = \dim(v)$$

$$\frac{\dim(c)}{[T]} = \dim(v) \implies \dim(c) = [LT^{-1}][T] = [L]$$

④ Deriving a formula :

frequence (f) of a stretched string depends on tension (F) (Force) in the string, length (l) of the string, mass per unit length (μ) of string. Derive formula for frequency.

$$f = () F^a l^b \mu^c$$

$$[T^{-1}] = [MLT^{-2}]^a [L]^b [ML^{-1}]^c$$

$$[T^{-1}] = [M^{a+c} L^{a+b-c} T^{-2a}]$$

equating powers on both sides

$$-2a = -1 \implies a = 1/2$$

$$a + c = 0 \implies c = -a = -1/2$$

$$a + b - c = 0 \implies b = c - a = -1$$

$$f = K F^{1/2} l^{-1} \mu^{-1/2}$$

$$f = K \frac{1}{l} \sqrt{\frac{F}{\mu}}$$

LIMITATIONS TO DIMENSIONAL ANALYSIS

1) Two or more physical quantities can have the same dimensions.

eg. Torque $[ML^2T^{-2}]$ Work $[ML^2T^{-2}]$

2) While deriving a formula of a physical quantity if number of fundamental quantities involved is less than the number of factors on which the formula of a physical quantity depends then it is not possible to get the formula.

3) Getting numerical constants is not possible through Dimensional Analysis.

4) If the factors involved have same dimension then it is practically impossible to get the respective powers via dimensional analysis

$$X = A^a B^b C^c$$

Q) Find Dimensional formula of
a) Coefficient of viscosity 'n'
b) Charge 'q'
c) potential 'V'
d) Capacitance 'C' e) Resistance 'R'
If some equations relating them are as follows.

$$F = -\eta A\left(\frac{\Delta v}{\Delta l}\right) \longrightarrow \dim(\eta) = \frac{\dim(F)}{\dim(A)}\frac{\dim(\Delta l)}{\dim(\Delta v)} = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]} = [ML^{-1}T^{-1}]$$

$$q = It \longrightarrow \dim(q) = [TA]$$

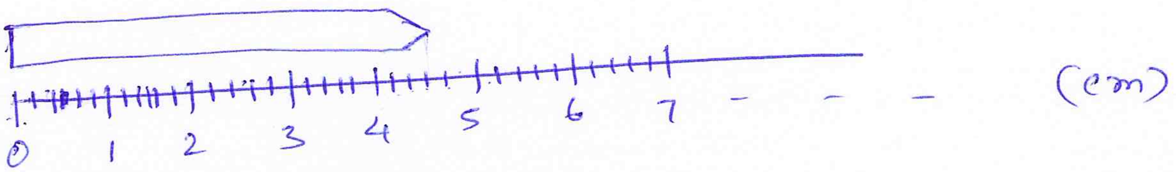
$$U = VIt \longrightarrow \dim(V) = \frac{\dim(U)}{\dim(I)\dim(t)} = \frac{[ML^2T^{-2}]}{[A][T]} = [ML^2T^{-3}A^{-1}]$$

$$q = CV \longrightarrow \dim(C) = \frac{\dim(q)}{\dim(V)} = \frac{[TA]}{[ML^2T^{-3}A^{-1}]} = [M^{-1}L^{-2}T^4A^2]$$

$$V = IR \longrightarrow \dim(R) = \frac{\dim(V)}{\dim(I)} = [ML^2T^{-3}A^{-2}]$$

x

SIGNIFICANT FIGURES.



4.61

Number of certain digits + 1st uncertain digit is the total ~~numfo~~ number of significant ~~digits~~ figures.

RULES FOR COUNTING SIGNIFICANT FIGURES.

1) All non-zero digits are significant

2345 4 SF

2) Zeros appearing between two non-zero digits is significant

2004 4 SF

3) If a number without decimal places have zeros at the end then those zeros are insignificant.

100	1 SF
1001	4 SF
101	3 SF
20000	1 SF

But for a measured quantity.

200 m	3 SF

4) Zeros occurring to the left of first non zero digit are insignificant.

0.0023	2 SF
0.1023	4 SF

5) In a number with decimal places the ~~to~~ zeros after last non zero digit are significant.

1400	2 SF
1.400	4 SF

6) Powers of 10 are not considered as significant digits.

$$2.7 \times 10^5 = 2 \text{ SF}$$

$$270000 = 2 \text{ SF}$$

$$\underline{2.70} \times 10^5 = 3 \text{ SF}$$

$$270000$$

7) Changes in unit of measurement should not change number of significant figures

2700 m   4 SF

↓ cm.

270000 cm.

2.700×10^5 cm.   4 SF

2700   2 SF

↓

2.7×10^3 2 SF

8) Find the number of significant figures for the following

1) 1245 (4)   2) 6024.7 (5)   3) 0.071 (2)   4) 4100 cm (4)

5) 220 (2)   6) 2.450 (4)   7) 1.9×10^{-5} (2)

8) 2.80×10^6 (3)

ALGEBRIC OPERATIONS. (EFFECT ON RESULT'S SIGNIFICANT FIGURES)

2.1 + 1.45

$$\begin{array}{r} 2.1 \\ 1.45 \\ \hline 3.55 \end{array}$$

A N

2.1

A A N

1.45

3.5 5

A N N

2.

1) Addition | Subtraction.

$\underbrace{10.1}_{\text{1 SF After decimal}} + \underbrace{9.245}_{\text{3 SF After decimal}} + \underbrace{0.45}_{\text{2 SF After decimal}}$

From the list of measured values added/subtract let 'n' be the number of significant figures after decimal place and least among them.

The result will have 'n' significant figures after decimal place.

Here n = 1

$$\begin{array}{r} 10.1 \\ 9.245 \\ 0.45 \\ \hline 19.795 \\ \hline \end{array}$$

$19.8 \leftarrow 19.795$

(2) Multiplication | Division.

From list of measured values to multiplied or divided let 'n' be the least number of significant ~~digits~~ figures among them. Then the result will have 'n' number of significant figures.

$$10.1 + \frac{\overset{(2)}{2.1} \times \overset{(3)}{3.25}}{\underset{(3)}{10.0}} = \frac{6.825}{10.0} = 0.68|25$$

$$= 0.68$$

$$10.\underline{1} + 0.\underline{68} = 10.78 = 10.8$$

~~10.1 + 0.6825.~~

RULES FOR ROUNDING OFF

ROUND IT TO 3 SF $\quad$ 10.6|65

cut off place

$10.665 = 10.7$

$10.5|51 = 10.6$

$10.6|49 = 10.6$

$10.500 = 10.5.$

$10.6500 = 10.7$

$10.6|5 = 10.6$

$10.75 = 10.8$

1) If number lying to right of cut off digit is less than 5 then cut off digit is retained however if it is more than 5 then cut off digit is increased by 1.

2) If the cut off digit is followed by

a) 5 and other digits then ~~number~~ cut off digit increased by 1.

b) only 5.
then cut off digit is changed to even.

Round to 2SF

1) 1.25 (1.2) 2) 1.452 (1.5) 3) 3.55 (3.6)

4) 62.5 (62) 5) 72.51 (73) 6) 73.5 (74)

ERROR ANALYSIS.

The measured value of a physical quantity is usually different from true value since result is determined experimentally & contains some uncertainty. This uncertainty is called the error.

ERRORS

SYSTEMATIC ERRORS.

RANDOM ERRORS.

Due to instruments used, imperfection in experimental technique or carelessness of person.

1) Least Count

2) Zero Error.

1) ABSOLUTE ERROR: Difference between true value & measured value of a quantity.

~~If the~~ If $x_1, x_2, x_3 - - - x_n$ are n readings.

$$x_m = \frac{x_1 + x_2 + x_3 + - - - + x_n}{n}$$

mean of all readings.

$e_1 = |\text{true value} - x_1|$ error in measurement

If true value is not given we will use 'x_m' inste

$e_1 = |x_m - x_1|$

$e_2 = |x_m - x_2|$

$\vdots$

$e_n = |x_m - x_n|$

MEAN ABSOLUTE ERROR

$$e_m = \frac{e_1 + e_2 + e_3 + \cdots e_n}{n}$$

RELATIVE ERROR $= \dfrac{\text{MEAN ABSOLUTE ERROR}}{\text{MEAN OF MEASUREMENTS}}$

$$= \frac{e_m}{x_m}$$

PERCENTAGE ERROR $= \dfrac{e_m}{x_m} \times 100\%$

Q
The diameter of wire is measured by screw guage was found to be

2.620, 2.625, 2.628, 2.626 cm.

Calculate:

a) Mean Value of diameter

2.625

b) Mean absolute error.

$0.00225 = \dfrac{0.00500 + 0.000 + 0.00300 + 0.00}{4}$

COMBINATION OF ERRORS.

1) Addition | Subtraction. (Absolute errors are added)

$$R = X \pm Y$$

$$R \pm \Delta R = (X \pm \Delta X) \pm (Y \pm \Delta Y)$$

$$= (X \pm Y) \pm (\Delta X + \Delta Y)$$

$$\cancel{R(X \pm Y)} \pm \Delta R = \cancel{(X \pm Y)} \pm (\Delta X + \Delta Y)$$

$$\underline{\pm \Delta R} = \pm (\underline{\Delta X} + \underline{\Delta Y})$$

~~2) MULTIPLICATION / DIVISION.~~

$\downarrow$

$$\cancel{Z = A \cdot B}$$

Subtraction.

$$R = X - Y$$

$$R \pm \Delta R = (X \pm \Delta X) - (Y \pm \Delta Y)$$

$$R \pm \Delta R = X \pm \Delta X - Y \pm \Delta Y$$

$$\pm \Delta R = \pm (\Delta X + \Delta Y)$$

2) Multiplication | Division. (Relative Errors are added up) or Percentage Errors are Added Up.

$Z = A \times B$

$Z \pm \Delta Z = (A \pm \Delta A) \times (B \pm \Delta B)$

$= A \times B \pm (\Delta A)B \pm (\Delta B)A \pm \underbrace{\Delta A \, \Delta B}_{\to 0}$

$$\frac{\Delta Z}{Z} = \frac{A \Delta B}{Z} + \frac{B \Delta A}{Z}$$

$$\frac{\Delta Z}{Z} = \frac{\Delta B}{B} + \frac{\Delta A}{A}.$$

$$\frac{\Delta Z}{Z} \times 100\% = \frac{\Delta B}{B} \times 100\% + \frac{\Delta A}{A} \times 100\%$$

$Z_1 = A^2 B^3$

$= A \times A \times B \times B \times B$

$Z = A^n B^m$

$\frac{\Delta Z}{Z} \times 100\% = n \frac{\Delta A}{A} \times 100\% + m \frac{\Delta B}{B} \times 100\%$

$$\frac{\Delta Z_1}{\Delta Z} \times 100\% = \frac{\Delta A}{A} \times 100\% + \frac{\Delta A}{A} \times 100\% + 3\frac{\Delta B}{B} \times 100\%$$

$$= 2\frac{\Delta A}{A} \times 100\% + 3\frac{\Delta B}{B} \times 100\%$$

$Z = \frac{A^n}{B^m} \Rightarrow \frac{\Delta Z}{Z} \times 100\% = n \frac{\Delta A}{A} \times 100\% + m \frac{\Delta B}{B} \times 100\%$

Q1 If side of square is $3 \pm 2\%$

Find % error in Area of square.

Q2 If $y = x^3 z^2$

error in x is 1%

error in z is 2%

What is error in y.

Q3 If

$A = 1.0\,m \pm 0.2\,m$

$B = 2.0\,m \pm 0.2\,m \Rightarrow 2.0\,m \pm 10\%$

$C = 2.5\,m/s \pm 0.5\,m/s \Rightarrow 2.5\,m/s \pm 20\%$

$D = 0.10\,s \pm 0.01\,s \Rightarrow 0.10\,s \pm 10\%$

Find

1) $A + B$

2) $B - A$

3) $C \times D$

4) B/D

5) $3 \times A$

Ans 1 $S = 3 \pm 2\%$

$$A = S^2$$

$$\% \text{ error in } A = 2(\% \text{ error in } S)$$
$$= 2 \times (2\%)$$
$$= 4\%$$

Ans 2 $y = x^3 z^2$

$$\% \text{ error in } y = 3(\% \text{ error in } x) + 2(\% \text{ error in } y)$$
$$= 3(1\%) + 2(2\%)$$
$$= 7\%$$

Ans 3

(1) $A+B = 3.0\,m \pm (0.4\,m)$

(2) $B-A = 1.0\,m \pm (0.4\,m)$

(3) $C \times D = 0.25\,m \pm 30\%$
$= 0.25\,m \pm 0.08\,m$

4) $B/D = 20\,m/s \pm 20\%$
$= 20\,m/s \pm 4\,m/s$

5) $3A = 3.0\,m \pm 0.6\,m.$