TRIGONOMETRIC EQUATIONS.

Equation involving one or more than one trigonometric ratio of an unknown angle.

eg. $\sin\theta = \frac{1}{2}$

$\theta = 30^\circ$ or $\frac{\pi}{6}$

$\theta = 180^\circ - 30^\circ$ ✓
$= 150^\circ$ or $\frac{5\pi}{6}$

$\theta = 360^\circ + 30^\circ$ ✓

$\theta = 510^\circ = 540^\circ - 30^\circ$

$\sin\theta = \sin\alpha$ $\qquad \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

principal soln.

$\sin\theta = \sin\frac{\pi}{6}$

$\theta = n\pi - \alpha$ $\quad$ n is odd.

$\theta = n\pi + \alpha$ $\quad$ n is even.

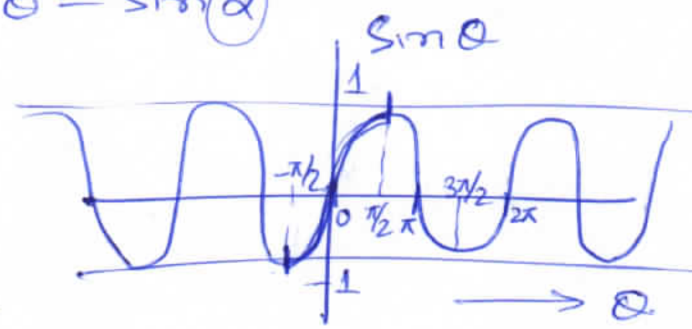
$\sin\theta = \sin\alpha$ $\quad n \in I$

$\theta = n\pi + (-1)^n\alpha$ ⟵ general solution

$\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\theta = -\frac{1}{2}$ $\qquad$ $\sin\theta = \sin(\alpha)$

$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$



$$\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$$

$n=0$ $\quad -\pi/6$

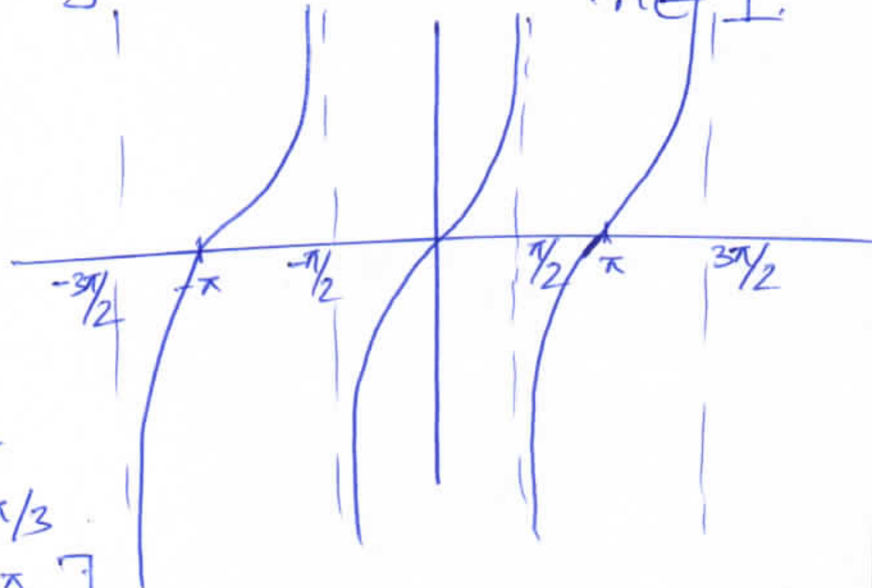
$n=1$ $\quad \frac{7\pi}{6}$ $\qquad \sin\left(\frac{7\pi}{6}\right) = \sin\left(\pi+\frac{\pi}{6}\right) = -\sin\frac{\pi}{6}$

$n=2$ $\quad \frac{11\pi}{6}$ $\qquad \sin\left(\frac{11\pi}{6}\right) = \sin\left(2\pi-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6}$

$\tan\theta = \tan\alpha.$ $\qquad \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\left.\begin{aligned}\theta &= 2n\pi + \alpha. \\ &= (2n+1)\pi + \alpha.\end{aligned}\right\} \longrightarrow \boxed{\theta = n\pi + \alpha.}$$

$n \in I.$



eg. $\tan\theta = \sqrt{3}$

$\tan\theta = \tan\frac{\pi}{3}$

$\theta = n\pi + \frac{\pi}{3}$

$n=-2$ $\quad -5\pi/3$

$n=-1$ $\quad -\frac{2\pi}{3}$

$n=0$ $\quad \frac{\pi}{3}$

$n=1.$ $\quad 4\pi/3$

n

$\tan\frac{2\pi}{3} = \tan\left(\pi-\frac{\pi}{3}\right)$

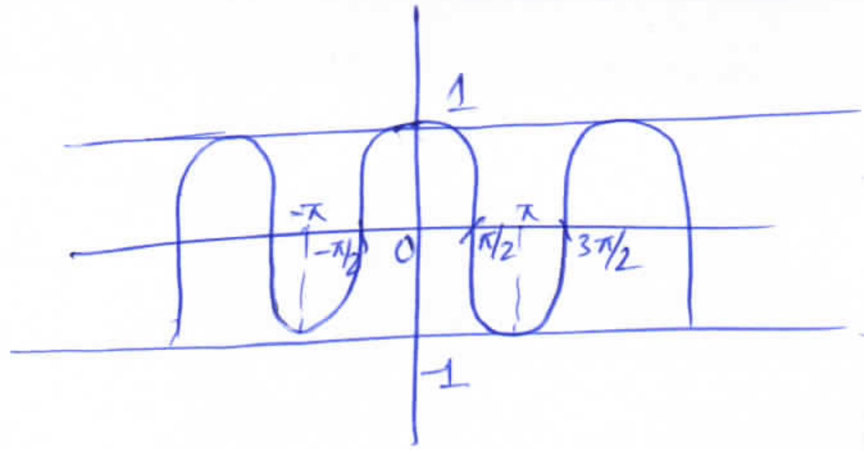
$= -\tan\frac{\pi}{3}$

$= -\sqrt{3}$

$\cos\theta = \cos\alpha$

$\cos\theta = -\frac{1}{2}$

$\cos\theta = \cos\frac{2\pi}{3}$



$\cos\theta = \cos\alpha.$ $\quad \alpha \in [0, \pi]$

$$\boxed{\theta = 2n\pi \pm \alpha \qquad n \in I}$$

$n = 0 \quad +\frac{2\pi}{3}$

$\qquad\quad -\frac{2\pi}{3}.$

$\cos\left(\frac{2\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$\cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

$n = 1. \quad \frac{8\pi}{3}.$

$\qquad\quad \frac{4\pi}{3}.$

$\cos\left(\frac{8\pi}{3}\right) = \cancel{\cos\left(2\pi + \frac{2\pi}{3}\right)}$

$= \cos\left(3\pi - \frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3} = -\frac{1}{2}$

Q) $\cos x \cos 2x \cos 3x = \frac{1}{4} \qquad x \in (0, \pi/4)$

$4\cos x \cos 2x \cos 3x - 1 = 0$

$(2\cos x \cos 2x)(2\cos 3x) - 1 = 0$

$(\cos(3x) + \cos x)(2\cos 3x) - 1 = 0$

$2\cos^2 3x + \underbrace{2\cos x \cos 3x}_{\cos 4x + \cos 2x} - 1 = 0$

Q) $\cos x \cos 2x \cos 3x = \frac{1}{4}$ $\quad x \in \left(0, \frac{\pi}{4}\right)$

$$4\cos x \cos 2x \cos 3x - 1 = 0$$

$$(2\cos x \cos 3x)(2\cos 2x) - 1 = 0$$

$$(\cos 4x + \cos 2x)(2\cos 2x) - 1 = 0$$

$$2\cos 2x \cos 4x + \underline{2\cos^2 2x - 1} = 0$$

$$2\cos 2x \cos 4x + \underline{\cos 4x} = 0$$

$$\cos 4x (2\cos 2x + 1) = 0$$

$\cos 4x = 0 \longrightarrow \cos 4x = \cos \pi/2$

$4x = 2n\pi \pm \frac{\pi}{2}$

$x = \frac{2n\pi}{4} \pm \frac{\pi}{8}$

$n \in I$

$n = 0 \quad \pm \pi/8$

$n = 1 \quad \frac{\pi}{2} + \frac{\pi}{8}$

$\frac{\pi}{2} - \frac{\pi}{8}$

$2\cos 2x + 1 = 0$

$\downarrow$

$2\cos 2x = -1$

$\cos 2x = -1/2$

$\cos 2x = \cos \frac{2\pi}{3}$

$2x = 2n\pi \pm \frac{2\pi}{3}$

$x = n\pi \pm \frac{\pi}{3} \quad n \in I.$

$\pm \pi/3 > \pi/4.$

$\pi + \pi/3 > \pi/4$

$\pi - \pi/3 > \pi/4$

$$\boxed{x = \frac{\pi}{8}}$$

iv) $\sin^2\theta = \sin^2\alpha.$

$1 - 2\sin^2\theta = \cos 2\theta.$

$\frac{1-\cos 2\theta}{2} = \sin^2\theta.$

$$\frac{1-\cos 2\theta}{2} = \frac{1-\cos 2\alpha}{2}$$

$$\cos 2\theta = \cos 2\alpha. \longrightarrow 2\alpha \in [0, \pi]$$

$\alpha \in [0, \frac{\pi}{2}]$

$$2\theta = 2n\pi \pm 2\alpha.$$

$$\boxed{\theta = n\pi \pm \alpha.}$$

$\alpha \in [0, \pi/2]$

v) $\cos^2\theta = \cos^2\alpha.$

$2\cos^2\theta - 1 = \cos 2\theta$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$\frac{1+\cos 2\theta}{2} = \frac{1+\cos 2\alpha}{2}$$

$$\cos 2\theta = \cos 2\alpha$$

$\cos\theta' = \cos\alpha'$

$\theta' = 2n\pi \pm \alpha'$

$2\theta = 2n\pi \pm 2\alpha.$

$$2\theta = 2n\pi \pm 2\alpha$$

$$\theta = n\pi \pm \alpha$$

$\alpha \in [0, \pi/2]$

vi) $\tan^2\theta = \tan^2\alpha$

$$\theta = n\pi \pm \alpha \quad n \in I.$$

$\alpha \in [0, \frac{\pi}{2})$