ROTATIONAL DYNAMICS TUTORIAL.

SOLVED Q.

Pg 95 - 99

1, 3, 5, 8, 10, 13, 15, 19, 20

Pg 100 - 105

2, 4, 6, 7, 8, 9, 13

Pg 107 Comprehension.

Pg 110 - 113 5, 8, 10

UNSOLVED Q.

Pg 115 14, 15, 16, 19

Pg 116 - 125 2, 3, 7, 12, 15, 17, 18, 19, 21, 24
29, 32, 37, 42, 44, 45, 49, 51, 57, 59, 61
63, 64, 65, 69, 70

Pg 128 - 131 14, 17, 24, 26, 28, 29

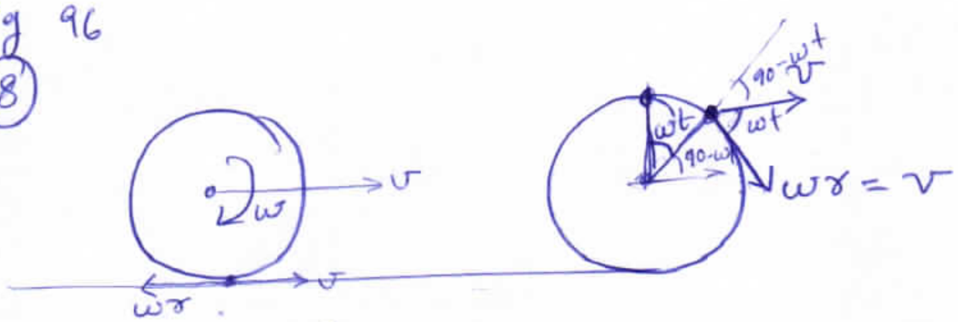
Pg 132 - 134 Comp 1, 2, 3, 4

Pg 135 Matrix 2, 3, 4

ROTATIONAL DYNAMICS TUTORIAL.

Pg 96

(8)



$v = \omega r$

$\omega = \frac{v}{r}$

$v_{net} = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos\theta}$

$= \sqrt{v^2 + v^2 + 2v\cdot v\cos\omega t}$

$= \sqrt{2v^2 + 2v^2\cos\omega t}$

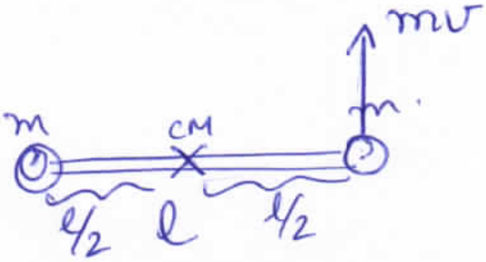
$= \sqrt{2v^2(1+\cos\omega t)}$

$= \sqrt{2v^2 \cdot 2\cos^2\frac{\omega t}{2}}$

$= 2v\cos\frac{\omega t}{2}$

$= 2v\cos\left(\frac{vt}{2r}\right)$

(15)



$\vec{L} = \vec{r} \times \vec{p}$

By COAM

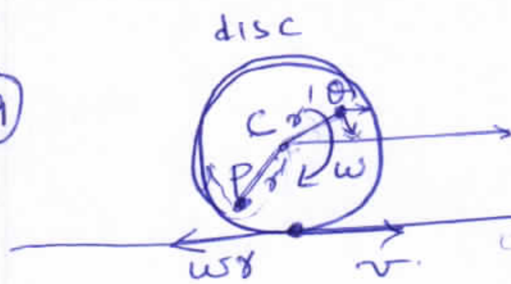
$= \frac{l}{2} \times mv = I_{CM}\,\omega$

$= \left[m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2\right]\omega$

$\frac{\cancel{m}vl}{\cancel{2}} = \frac{\cancel{m}l^{\cancel{2}}}{\cancel{2}}\omega$

$\boxed{\omega = \frac{v}{l}}$

(19) disc

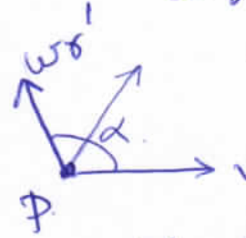


$v = \omega r$

v_Q $\sqrt{v^2 + (\omega r')^2 + 2v\omega r'\cos\theta}$

Q.

$0 < \theta < 90$



$90 < \alpha < 180$

v_P $\sqrt{v^2 + (\omega r')^2 + 2v\omega r'\cos\alpha}$

$v_P < v_Q$

$v_C = v$

$v_C^2 = v^2$

$v_P^2 = v^2 + (\omega r')^2 + 2v\omega r'\cos\alpha$

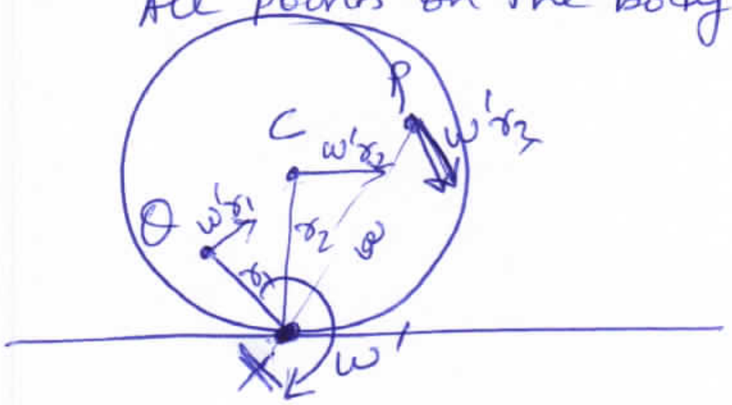
$v_P^2 = v^2 + (\omega r')^2 + 2v\omega r'\cos\alpha.$

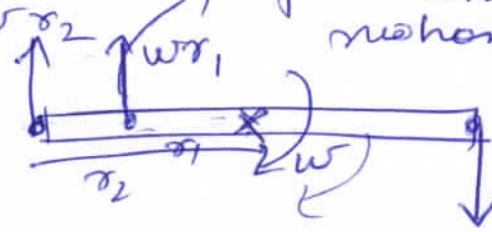
$v_P^2 - v_C^2 = (\omega r')^2 + 2v\omega r'\cos\alpha.$

$\omega r'(\omega r' + 2v\cos\alpha)$

$$v_p^2 - v_c^2 = \omega r'(\omega r' + 2v\cos\alpha)$$

∴ a point X has zero velocity. All points on the body can be assumed to be in pure rotational motion about X





$$V_p = \omega' r_3$$
$$V_c = \omega' r_2$$
$$V_Q = \omega' r_1$$

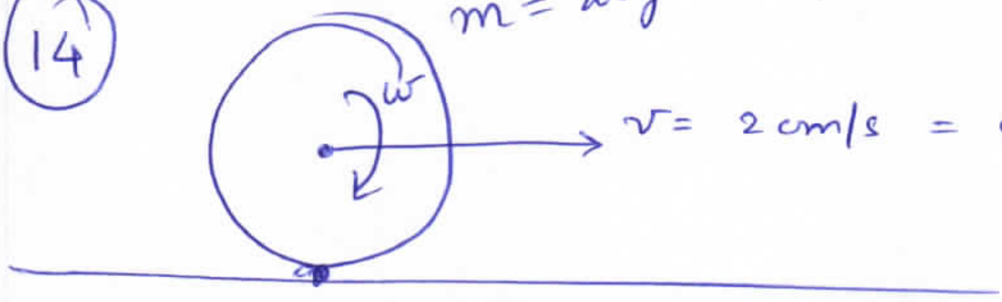
$$r_3 > r_2 > r_1$$
$$\omega' r_3 > \omega' r_2 > \omega' r_1$$
$$V_p > V_c > V_Q$$

$$\omega' r = v$$
$$\omega' = \frac{v}{r}$$

Fig 115

(14)

$m = 200g = 0.2kg$

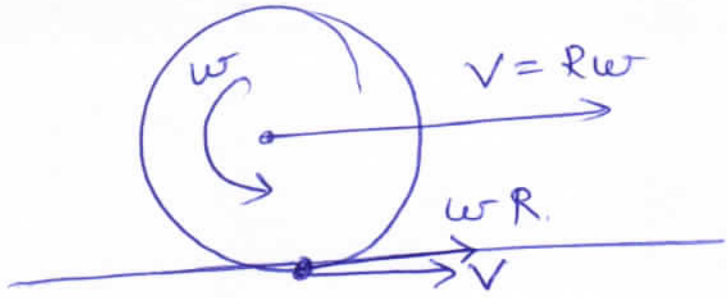


$v = 2\,cm/s = 0.02\,m/s$

$v = \omega r$

$$K.E = \text{translational K.E} + \text{Rotational K.E}$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{2}{5}mr^2\left(\frac{v}{r}\right)^2$$
$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$
$$= \frac{7}{10}mv^2 = \frac{7}{10}\times 0.2\times 4\times 10^{-4}$$
$$= 5.6\times 10^{-5}\,J$$

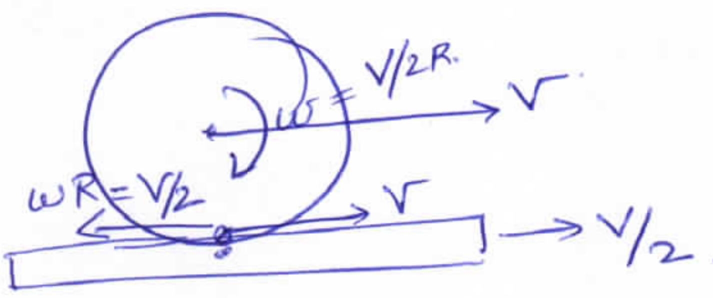
(15) A)



$v + \omega R = 2v \neq 0$

forward slip.

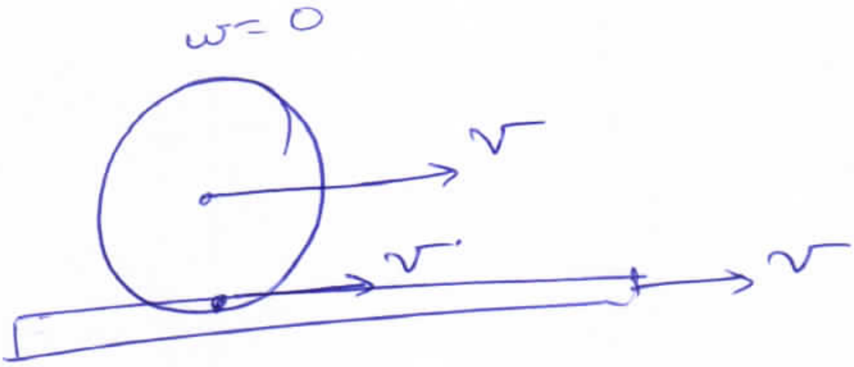
B)



pure rolling

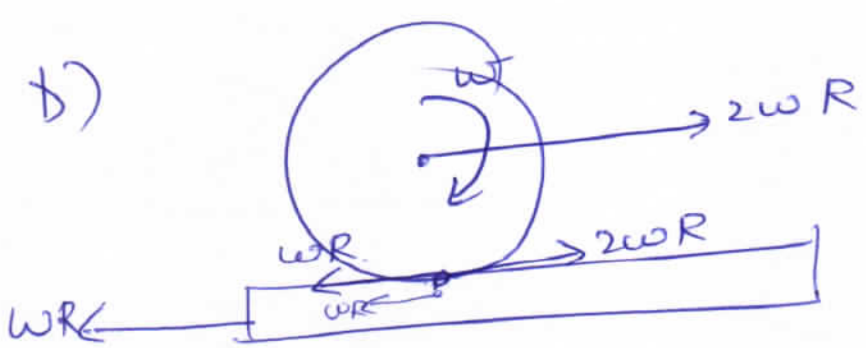
$v_{rel} = \frac{v}{2} - \frac{v}{2} = 0$

C)



pure rolling.

D)



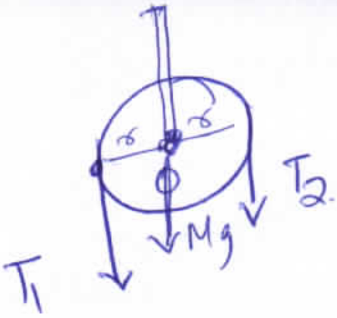
$\rightarrow \omega R$

$2\omega R \rightarrow$

forward slip.

16) $\underline{\tau_{CM} = I\alpha}$ if body rotates about an axis passing through CM.

Q. 19



$$T_1 r - T_2 r$$

$$a = \alpha r$$

$$Mg - T_1 = Ma$$

$$T_2 - mg = ma$$

$$T_1 r - T_2 r = I\alpha$$

$$T_1 - T_2 = I\frac{\alpha}{r}$$

$$T_1 - T_2 = I\frac{a}{r^2}$$

$$Mg + \underline{T_2 - T_1} - mg = (M+m)a$$

$$Mg + \left(-\frac{Ia}{r^2}\right) - mg = (M+m)a$$

$$\left(M + m + \frac{I}{r^2}\right)a = (M-m)g$$

$$a = \frac{(M-m)g}{M+m+\frac{I}{r^2}}$$

$$Mg - Ma = T_1$$

$$M(g-a) = T_1$$

$$M\left[g - \frac{(M-m)g}{M+m+\frac{I}{r^2}}\right] = T_1$$

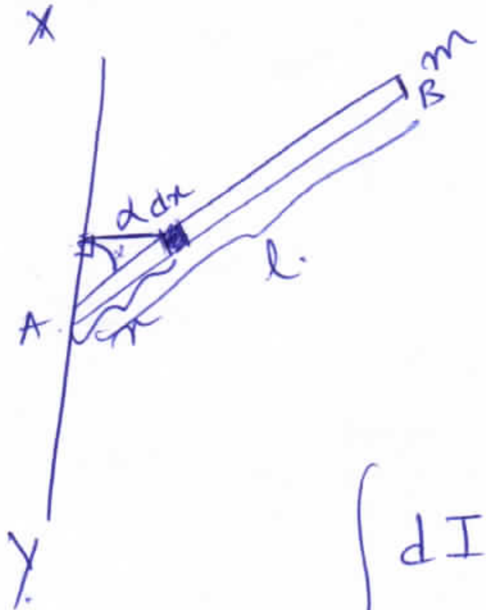
$$Mg\left[1 - \frac{M-m}{M+m+\frac{I}{r^2}}\right] = T_1$$

$$T_1 = M\left[\frac{\cancel{Mg} + mg + \frac{Ig}{r^2} - \cancel{Mg} + mg}{M+m+\frac{I}{r^2}}\right]$$

$$T_1 = \frac{M\left[2m + \frac{I}{r^2}\right]g}{M+m+\frac{I}{r^2}}$$

Pg 116-125

(2)



$$m' = \frac{m}{l}\,dx$$

$$\frac{p}{x} = \sin\alpha$$

$$p = x\sin\alpha$$

$$\int dI = m' p^2 = \int_0^l \frac{m}{l}\,dx\; x^2 \sin^2\alpha$$

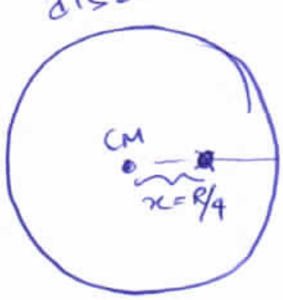
$$I = \frac{m}{l}\sin^2\alpha \int_0^l x^2\,dx$$

$$= \frac{m}{l}\sin^2\alpha \left.\frac{x^{2+1}}{2+1}\right]_0^l$$

$$= \frac{m\sin^2\alpha}{l} \times \frac{l^3}{3} = \frac{ml^2}{3}\sin^2\alpha$$

disc

(3)



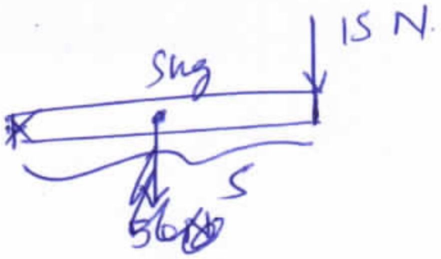
$$I_{AXIS} = I_{CM} + Mx^2$$

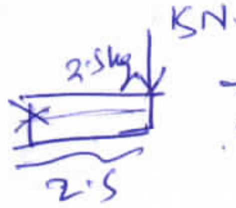
$$= \frac{MR^2}{2} + M\left(\frac{R}{4}\right)^2$$

$$= \frac{MR^2}{2} + \frac{MR^2}{16} = \frac{9MR^2}{16}$$

(C)

(7)





$\tau = I\alpha$

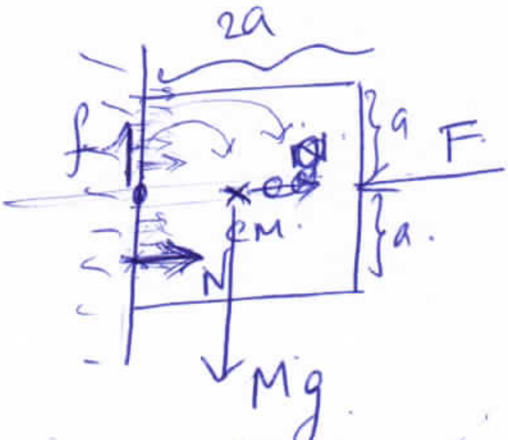
$\alpha = \frac{\tau}{I}$

$$\frac{15\times \cancel{5}}{\frac{\cancel{5}\times 5^2}{3}} < \frac{15\times 2.5}{\frac{2.5\times 2.5^2}{3}}$$

Case II ✓

(B)

(12)

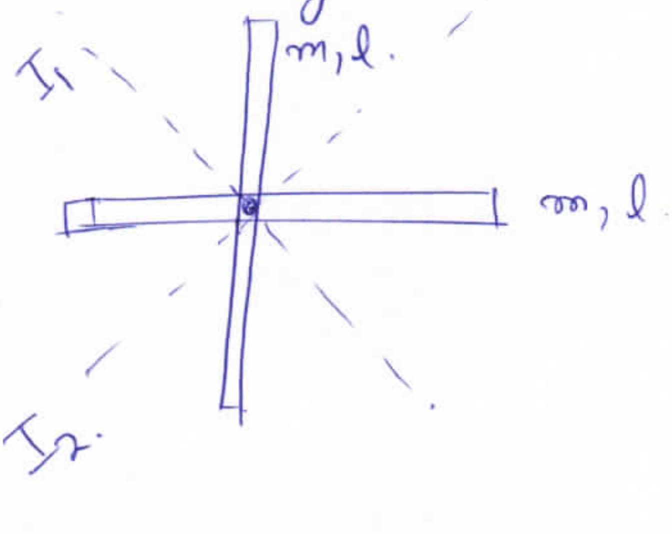


$f = Mg$ ✓

$F = N$

D ✗

(15)



$$I_{CM} = \frac{ml^2}{12} + \frac{ml^2}{12}$$

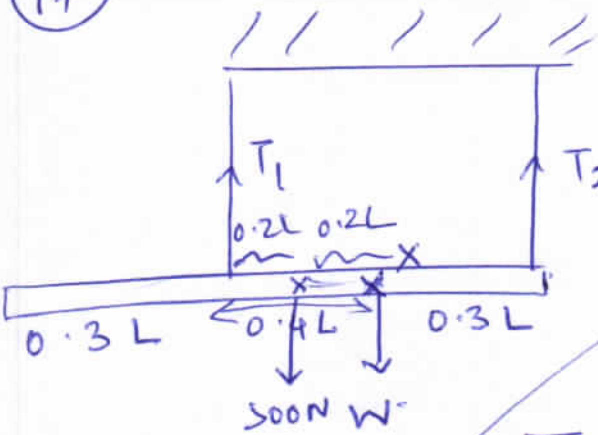
$$= \frac{ml^2}{6}$$

$$I_1 + I_2 = I_{CM}$$

$$2I = I_{CM}$$

$$I = \frac{I_{CM}}{2} = \frac{\frac{ml^2}{6}}{2} = \frac{ml^2}{12}$$ (C)

(17)



$$T_1 + T_2 = 500 + W$$

$$2T = 500 + W$$

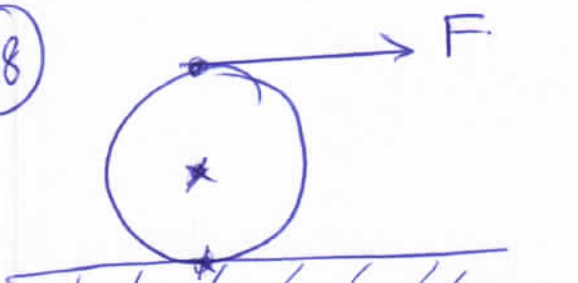
$$\Sigma_x = T_2 \times 0.3L - T_1 \times 0.4L + 500 \times 0.2L$$

$$= 0 = -0.1TL + 500 \times 0.2L$$

$$T = \frac{500 \times 0.2\cancel{L}}{0.1\cancel{L}} = 1000\ N$$

$$2 \times 1000 = 500 + W \Rightarrow W = 1500N$$

(18)



$v = \omega R$, $a = \alpha R$

$f = 0$?

f.

$$F \times R = I\alpha.$$

$$FR + fR = I\alpha.$$

$$F + f = I\alpha/R = \frac{MR^{\cancel{2}}\alpha}{2\cancel{R}}$$

$$F - f = Ma.$$

$$F + f = \frac{M}{2}a.$$

$$F - f = Ma.$$

$$2F = \frac{3}{2}Ma \Rightarrow a = \frac{4F}{3M}$$