COMPLEX NUMBER TUTORIAL.

SOLVED EX.

Pg 56-80 — 3, 6, 8, 11, 12, 13.

Comprehension.

UNSOLVED

Pg 62-66 — 2, 4, 8 (Section A)

Section B → 1, 3, 5

11, 12, 14, 18, 20, 25, 23

27, 30, 33, 38, 40, 44.

Pg 66
Section C → 6, 8, 13, 16, 18

Pg 69-71 — Comp-1., Comp-3, Comp 4.

Pg 73 — 4

Pg 62

2) $(i+1)^8 + (1-i)^8$

$$\left(\sqrt{2}e^{i\pi/4}\right)^8 + \left(\sqrt{2}e^{-i\pi/4}\right)^8$$

$$= 2^4 e^{i(2\pi)} + 2^4 e^{-i2\pi}$$

$$= 2^4\left[e^{i2\pi} + e^{-i2\pi}\right] = 2^4\left[2\cos 2\pi\right] = 32$$

4) $$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

$$\frac{4x + 2ix - 6i - 2 + 6y + 3y - 9iy + 2iy + 3i - 1}{10} = i$$

$$\frac{4x + 9y - 3}{10} + i\left(\frac{2x - 7y - 3}{10}\right) = 0 + i$$

$4x + 9y - 3 = 0$ $\quad\quad$ $\frac{2x - 7y - 3}{10} = 1.$

$4x + 9y - 3 = 0$ $\quad\quad$ $2x - 7y - 13 = 0 \quad \times 2$

$4x - 14y - 26 = 0$

$23y = -23 \quad y = -1.$

$x = 3$

(8)

$$a^2+b^2=1 \qquad \frac{1+a+ib}{1+a-ib}$$

$$a^2-(ib)^2=1$$

$$(a+ib)(a-ib)=1.$$

$$a+ib=\frac{1}{a-ib}$$

$$a+ib+1=\frac{1}{a-ib}+1$$

$$\frac{a+ib+1}{1+a-ib}=\frac{1}{a-ib}=\frac{a+ib}{a^2+b^2}=a+ib$$

(9)

$$\sin\frac{6\pi}{5}+i\left(1+\cos\frac{6\pi}{5}\right)$$

$$-\sin\frac{\pi}{5}+i\cancel{+}i\cos\frac{\pi}{5}$$

$$i-\left(\sin\frac{\pi}{5}+i\cos\frac{\pi}{5}\right)$$

$$i-\left(\cos\frac{3\pi}{10}+i\sin\frac{3\pi}{10}\right)$$

$$\pi-\tan^{-1}\left|\frac{1+\cos\frac{6\pi}{5}}{\sin\frac{6\pi}{5}}\right|$$

$$\pi-\tan^{-1}\left|\frac{1-\cos 36^\circ}{-\sin 36^\circ}\right|$$

$$\pi-\tan^{-1}\left|\frac{\cancel{2}\sin^2 18^\circ}{\cancel{2}\sin 18\cos 18}\right| \Rightarrow \pi-\tan^{-1}\tan 18$$

$$\Rightarrow \pi-18^\circ=162^\circ$$

Section B

1)

$$\left(\frac{1+i}{1-i}\right)^{4n+1}$$

$$\left(\frac{\cancel{\sqrt{2}}e^{i\pi/4}}{\cancel{\sqrt{2}}e^{-i\pi/4}}\right)^{4n+1} = \left(e^{i\pi/4+i\pi/4}\right)^{4n+1}$$

$$= \left(e^{i\pi/2}\right)^{4n+1}$$

$$= i^{4n+1} = i^{4n}\cdot i$$

$$= i$$

3) $$\frac{3+2i\sin\theta}{1-2i\sin\theta} = x+i0$$

$$3+2i\sin\theta = x-2xi\sin\theta.$$

$$x = 3.$$

$$2\sin\theta = -2x\sin\theta.$$

$$2\sin\theta = -6\sin\theta.$$

$$8\sin\theta = 0$$

$$\theta = n\pi \quad (C)$$

11) $\frac{2z_1}{3z_2} = ki$ $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$

$$\frac{z_1}{z_2} = \frac{\frac{3k}{2} i}{1.}$$

$$\frac{z_1 - z_2}{z_1 + z_2} = \frac{-1 + \frac{3k}{2} i}{1 + \frac{3k}{2} i}$$

$$\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = \left|\frac{-1 + \frac{3k}{2} i}{1 + \frac{3k}{2} i}\right| = \sqrt{\frac{\cancel{1 + \frac{9k^2}{4}}}{\cancel{1 + \frac{9k^2}{4}}}} = 1$$

(12)

$$\left|z + \frac{2}{z}\right| = 2 \qquad |z|_{MAX} = ?$$

let $z = re^{i\theta}$ $\quad$ $\frac{2}{z} = \frac{2}{re^{i\theta}} = \frac{2}{r} e^{-i\theta}.$

$|z| = r$

$$\left|re^{i\theta} + \frac{2}{r} e^{-i\theta}\right| = 2$$

$$\left|r(\cos\theta + i\sin\theta) + \frac{2}{r}(\cos\theta - i\sin\theta)\right| = 2$$

$$\left|\cos\theta\left(r + \frac{2}{r}\right) + i\sin\theta\left(r - \frac{2}{r}\right)\right| = 2$$

$$\left\{\cos\theta\left(r+\frac{2}{r}\right)\right\}^2+\left\{\sin\theta\left(r-\frac{2}{r}\right)\right\}^2=4$$

$$\left(r^2+\frac{4}{r^2}+4\right)\cos^2\theta+\left(r^2+\frac{4}{r^2}-4\right)\sin^2\theta=4$$

$$r^2+\frac{4}{r^2}+4\cos 2\theta=4$$

$$r^2+\frac{4}{r^2}=4(1-\cos 2\theta)$$

$$r^2+\frac{4}{r^2}=\cancel{8\cos}\ 8\sin^2\theta.$$

$$r^2+\frac{4}{r^2}\le 8. \qquad r^2=t.$$

$$r^2+\frac{4}{r^2}-8\le 0 \Rightarrow r^4-8r^2+4\le 0$$

$$\Rightarrow t^2-8t+4\le 0$$

$$\cancel{\left(r+\frac{2}{r}\right)^2-2^2\le 0}$$

$$\cancel{\left(r-\frac{2}{r}+2\right)\left(r-\frac{2}{r}-2\right)\le 0}$$

$$t=\frac{+8\pm\sqrt{64-4\times 4}}{2}=\frac{8\pm\sqrt{48}}{2}=\frac{8\pm 4\sqrt{3}}{2}=4\pm 2\sqrt{3}.$$

$$\left\{t-(4+2\sqrt{3})\right\}\left\{t-(4-2\sqrt{3})\right\}\le 0$$

$+$ $\quad 4-2\sqrt{3}$ $\quad -$ $\quad 4+2\sqrt{3}$ $\quad +$

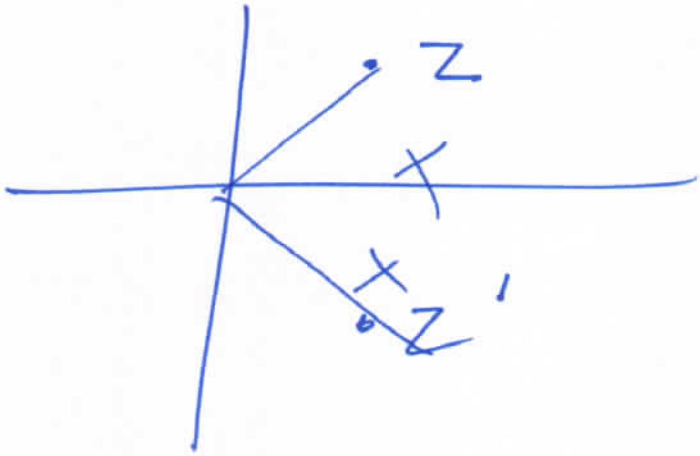
$$4-2\sqrt{3}\le r^2\le 4+2\sqrt{3}$$

$$r=\sqrt{4+2\sqrt{3}}=\sqrt{(\sqrt{3})^2+1^2+2\sqrt{3}}=\sqrt{(\sqrt{3}+1)^2}=\sqrt{3}+1$$

(14) $|z| = |z^c|$ ✓

$$z \cdot z^c = |z|^2 = |z^c|^2$$ ✓

$$(z_1 + z_2)^c = z_1^c + z_2^c$$ ✓



$\arg z \neq \arg z^c$

(18) α, β are complex $\quad |\beta| = 1 \;\downarrow\; \beta\beta^c = 1$

$$\left|\frac{\beta - \alpha}{1 - \alpha^c \beta}\right| = x$$

$$x^2 = \frac{|\beta - \alpha|^2}{|1 - \alpha^c \beta|^2} = \frac{(\beta - \alpha)(\beta - \alpha)^c}{(1 - \alpha^c\beta)(1 - \alpha^c\beta)^c}$$

$$= \frac{(\beta - \alpha)(\beta^c - \alpha^c)}{(1 - \alpha^c\beta)(1 - \alpha\beta^c)}$$

$$= \frac{\beta\beta^c + \alpha\alpha^c - \alpha\beta^c - \alpha^c\beta}{1 + \alpha\alpha^c\beta\beta^c - \alpha\beta^c - \alpha^c\beta}$$

$$\frac{1 + \alpha\alpha^c - \cancel{\alpha\beta^c} - \alpha^c\beta}{1 + \alpha\alpha^c - \cancel{\alpha\beta^c} - \alpha^c\beta} = 1 \Longleftarrow$$

$\underline{\underline{x = 1}}$

(20) $\operatorname{Re}\left(\frac{z-8i}{z+6}\right) = 0$

$z = x+iy$

$$z' = \frac{x+iy-8i}{x+iy+6} = \frac{(x)+i(y-8)}{(x+6)+i(y)}$$

$$\Rightarrow \frac{\{x+i(y-8)\}\{(x+6)-iy\}}{(x+6)^2+y^2}$$

$$\Rightarrow \frac{(x+6)x+y(y-8)+i(\quad)}{(x+6)^2+y^2}.$$

$$\operatorname{Re} z' = \frac{(x+6)x+y(y-8)}{(x+6)^2+y^2} = 0$$

$$\Rightarrow x^2+y^2+6x-8y = 0$$

(38) i^i

$$i = re^{i\theta} = 1e^{i\pi/2} = e^{i(2n\pi+\frac{\pi}{2})}$$

$$\left(e^{i\pi/2}\right)^i = e^{i^2\pi/2} = e^{-(\pi/2+2n\pi)}$$

$$= e^{-\frac{\pi}{2}(1+4n)}$$ (D)

Pg 62

5 $\left|\dfrac{(p+i)^2}{2p-i}\right| = |\mu + i\lambda| \qquad \mu^2+\lambda^2 = ?$

$$\frac{|p+i|^2}{|2p-i|} = \sqrt{\mu^2+\lambda^2} = \frac{\left(\sqrt{p^2+1}\right)^2}{\sqrt{(2p)^2+1^2}}$$

$$\sqrt{\mu^2+\lambda^2} = \frac{p^2+1}{\sqrt{(4p^2+1)}}$$

$$\mu^2+\lambda^2 = \frac{(p^2+1)^2}{4p^2+1}$$

Pg 66

1 $(x+iy)^{1/5} = a+ib \qquad u = \dfrac{x}{a} - \dfrac{y}{b}.$

$$\begin{aligned}
(x+iy) &= (a+ib)^5 \\
&= \{(a+ib)^2\}^2 (a+ib) \\
&= (a^2-b^2+2abi)^2 (a+ib) \\
&= (a^4+b^4-2a^2b^2-4a^2b^2+4a^3bi-4ab^3i)(a+ib) \\
&= a^5+ia^4b+ab^4+ib^5-6a^3b^2-6a^2b^3i \\
&\quad +4a^4bi-4a^3b^2-4a^2b^3i+4ab^4
\end{aligned}$$

$$x+iy = \left(a^5 - 10a^3b^2 + 5ab^4\right) + i\left(5a^4b + b^5 - 10a^2b^3\right)$$

$$\frac{x}{a} = a^4 - 10a^2b^2 + 5b^4$$

$$\frac{y}{b} = 5a^4 + b^4 - 10a^2b^2$$

$$\frac{x}{a} - \frac{y}{b} = a^4 - b^4 + 5(b^4 - a^4)$$

$$= -4(a^4 - b^4) = u$$

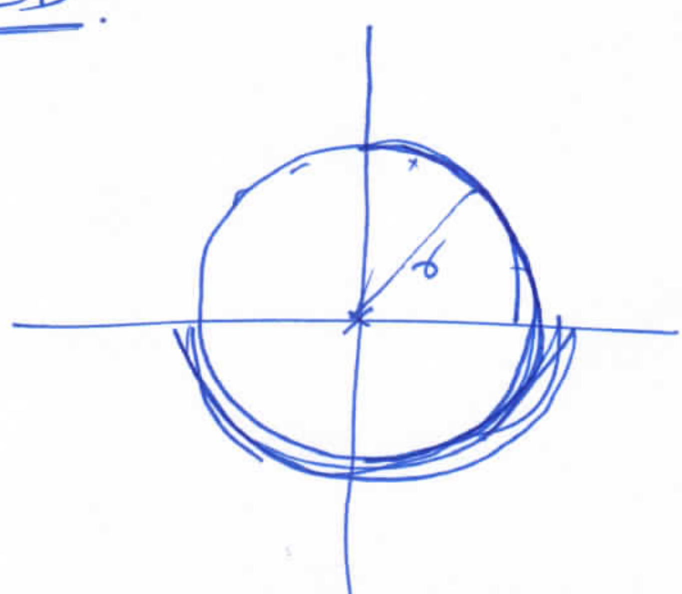
$$= -4(a^2+b^2)(a^2-b^2)$$

$$u = -4(a+ib)(a-ib)(a-b)(a+b)$$

<u>ABCD</u>.

(6) $z_1 \neq z_2$

$|z_1| = |z_2|$



$$\frac{z_1+z_2}{z_1-z_2}.$$

A

$$z_1 = a^2 + ib \qquad = a^4 + b^2 = c^2 + d^4$$
$$z_2 = c - id^2 \qquad |z_1|^2 = |z_2|^2$$

$$\frac{a^2+c+i(b-d^2)}{a^2-c+i(b+d^2)}$$

$$\frac{\{a^2+c+i(b-d^2)\}\{(a^2-c)-i(b+d^2)\}}{+\text{ve quantity.}}$$

$$(a^4-c^2+b^2-d^4)$$
$$+i\,(\cancel{a^2b}-a^2d^2-bc+\cancel{cd^2}$$
$$-\cancel{a^2b}-bc-a^2d^2-\cancel{cd^2})$$

$$\{(a^4+b^2)-(d^4+c^2)\}+i\{-2a^2d^2-2bc\}$$

$$\frac{\{(a^4+b^2)-(d^4+c^2)\}-2i\{a^2d^2+bc\}}{+\text{ve real.}}$$

$$\frac{0-2i(a^2d^2+bc)}{+\text{ve real.}}$$

$$a^2d^2 = -bc$$