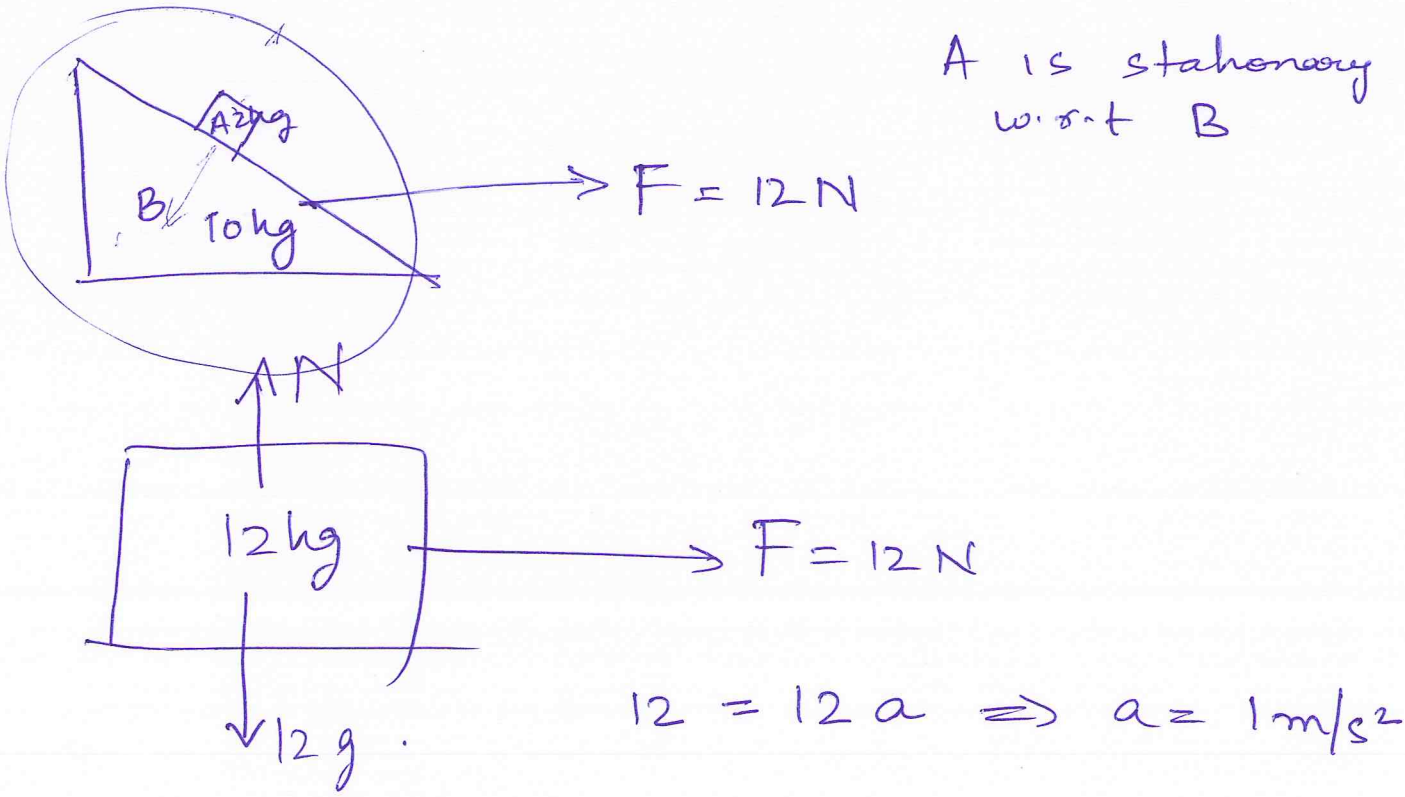
# Working with Newton's 2nd laws

1) First we have to decide the system on which the laws of motion are applied.

This system may be

- a single particle
- a block
- a combination of blocks
- blocks connected by string etc.

(Only restriction that all parts of the system should have same acceleration)




A is stationary w.r.t B

$12 = 12a \Rightarrow a = 1 m/s^2$

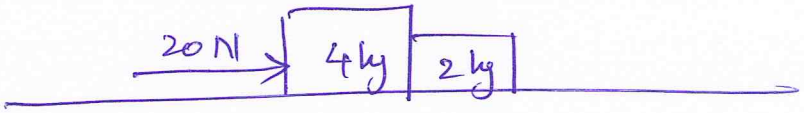
2) Once the system is decided we make a list of all force acting on the system & any force acting by one member of the system on the other is not considered.

3) Choose any two I axis (generally x, y) in the plane of the force.

$$\sum_i F_x = m a_x$$

$$\sum_i F_y = m a_y$$

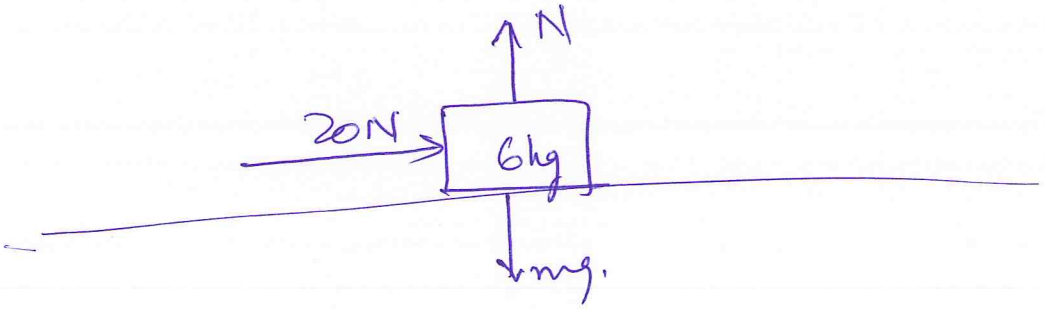
eg. Two blocks of mass 4kg & 2kg are placed side by side on a smooth horizontal surface. A horizontal force of 20N is applied on 4kg block as shown.




Find

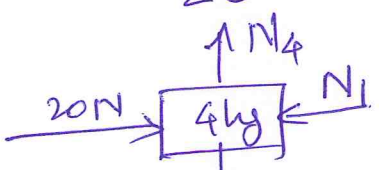
a) Acceleration of each block

b) Normal reaction between the blocks.

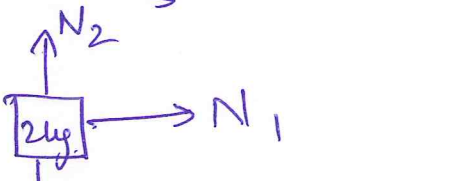



$$\sum_i F_x = 6a.$$

$$20 = 6a \implies a = \frac{10}{3}\ m/s^2$$



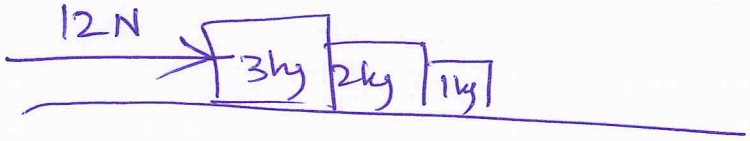

$20 - N_1 = 4a.$

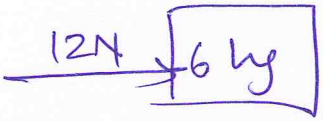



$N_1 = 2a.$

$N_1 = 2 \times 10/3 = \frac{20}{3}$

Q) Three blocks 3kg, 2kg & 1 kg are placed side by side as shown on a smooth floor. A horizontal force of 12 N is applied on 3kg block. Find Net Force acting on 2kg block.




$12 = 6a$

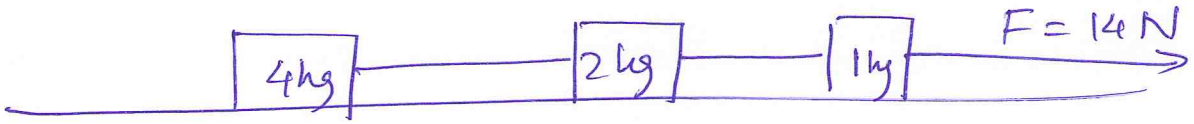
$a = 2 m/s^2$

$F_2 = 2a$

$= 2 \times 2 = 4$ N

$F_3 = 3a. = 6$ N

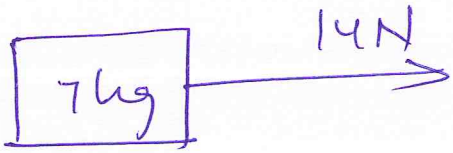
Q) In the arrangement shown. The strings are light & inextensible. The surface over which blocks are placed is smooth.



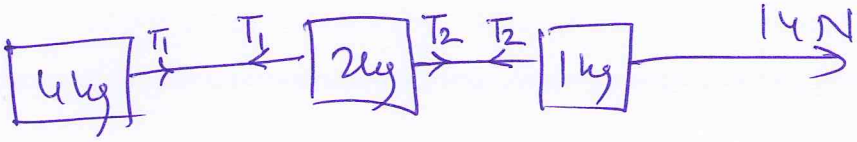

Find

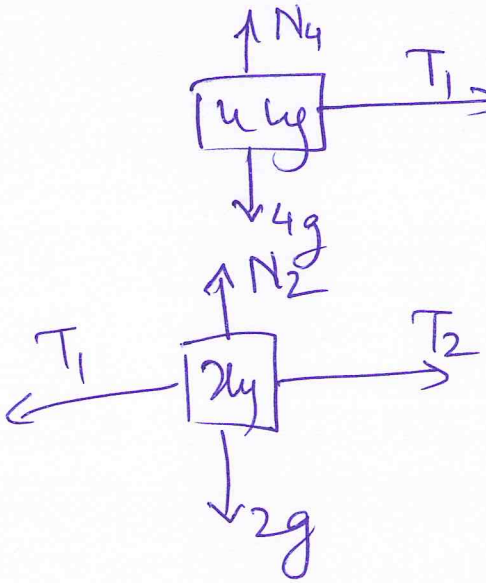
a) the acceleration of each block

b) the tension in each string.




$14 = 7a \qquad a = 2 m/s^2$




$T_1 = 4a.$

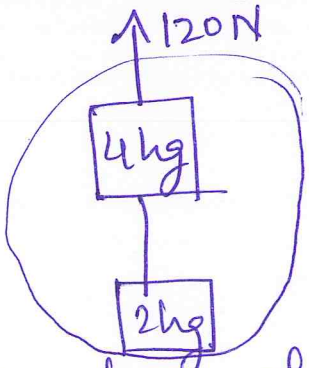
$T_1 = 4 \times 2 = 8N$

$T_2 - T_1 = 2a$

$T_2 = 2 \times 2 + 8$

$= 12N$

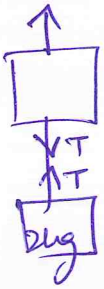
Q) Two blocks of mass 4kg & 2kg are attached by an inextensible light string as shown. Both the blocks are pulled vertically upwards by a Force F ($g = 10 m/s^2$)

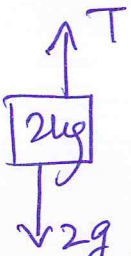



a) Find acceleration of blocks.

b) Tension in the string.


$120 - 60 = 6a \Rightarrow a = 10\,m/s^2$



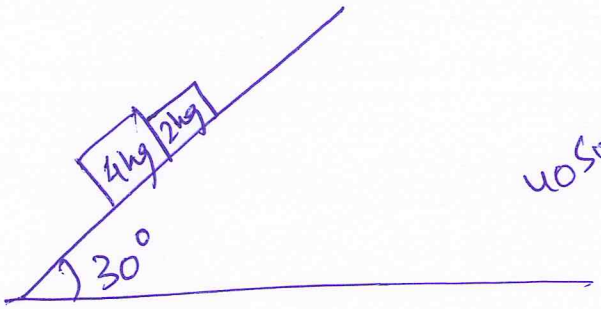



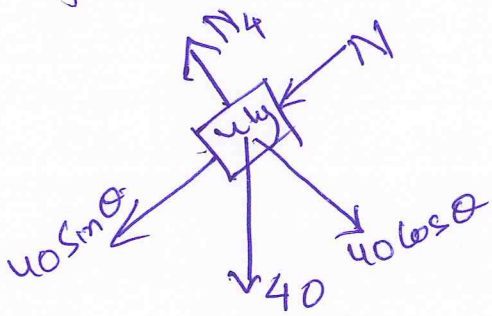

$T - 2g = 2a$

$T = 2g + 2a$

$= 40\,N$

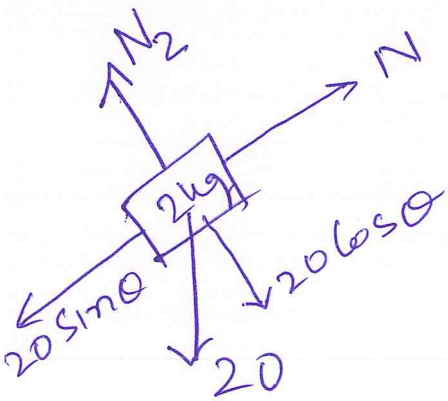
9) Two blocks of mass 2kg & 4kg are released from rest over a smooth inclined plane of inclination 30° as shown. What is the normal force between blocks




$40\sin\theta + N = 4a_4$

$20 + N = 4a_4$ ... (1)




$20\sin\theta - N = 2a_2$

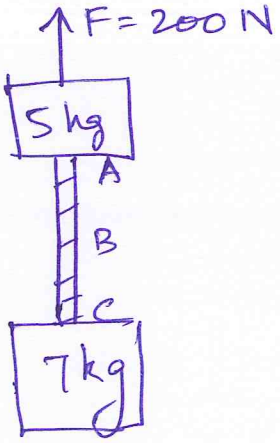
$10 - N = 2a_2$ ... (2)

If $N \neq 0 \Rightarrow a_4 = a_2 = a$

$30 = 6a \Rightarrow a = 5\,m/s^2$

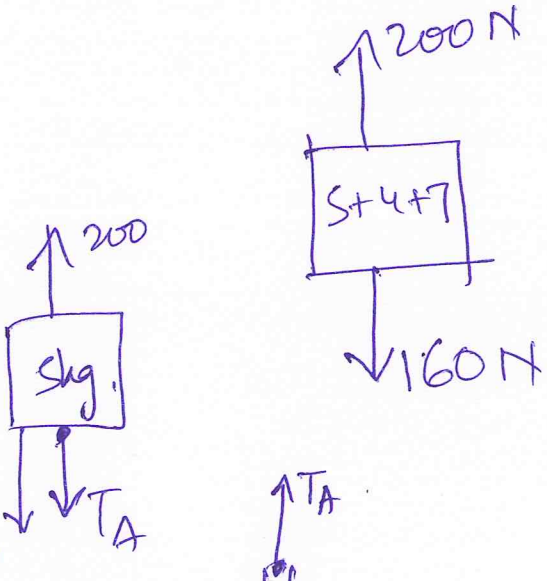
$N = 0$

Q) Two blocks shown are connected by a heavy uniform rope of Mass 4kg An upward force of 200N is applied as shown.



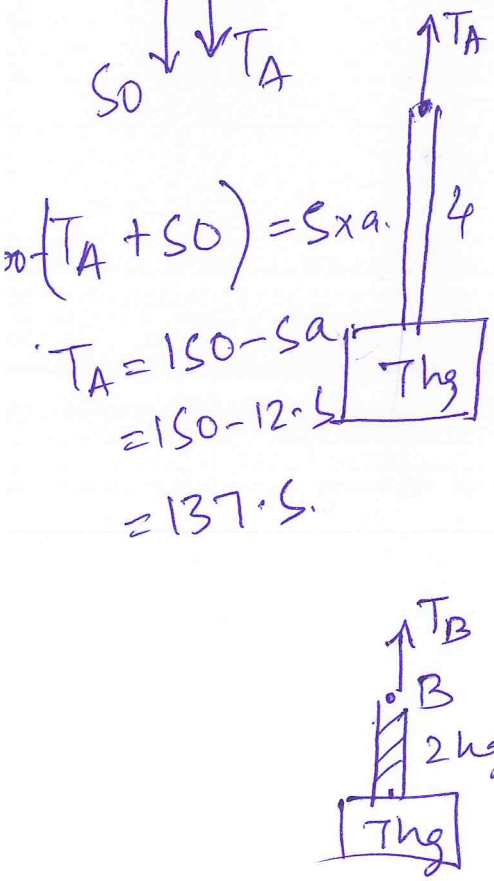

a) What is acceleration of system.

b) Tension at A, B




$200 - 160 = 16a$.

$a = 2.5 m/s^2$




$50 - (T_A + 50) = 5 \times a$.

$T_A = 150 - 5a$

$= 150 - 12.5$

$= 137.5$.

$T_A - 110 = 11a$

$T_A = 110 + 11a$

$= 11(10 + 2.5)$

$= 11(12.5)$

$= 137.5\ N$.

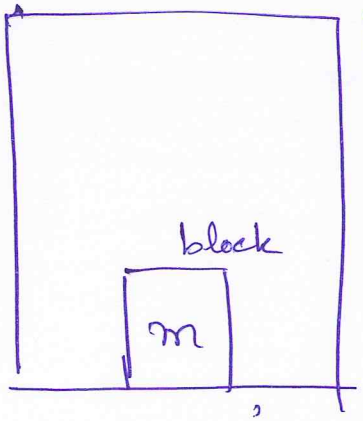



$T_B - 90 = 9a$.

$T_B = 9(10 + 2.5)$

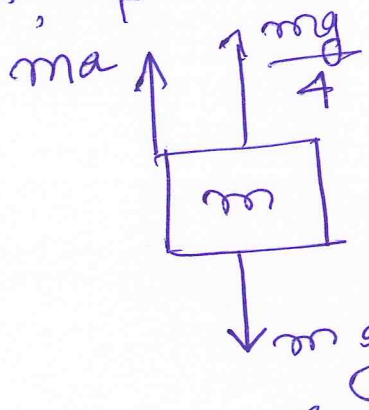
$= 9 \times 12.5 = 112.5\ N$

Q) What should be the acceleration of the box shown so that the block of mass 'm' exerts a force $\frac{mg}{4}$ on the floor of box.




NON-INERTIAL FRAME ←

Inertial frame.




$mg = \frac{mg}{4} + ma.$

$a = \frac{3g}{4}$

$= 7.5\ m/s^2$

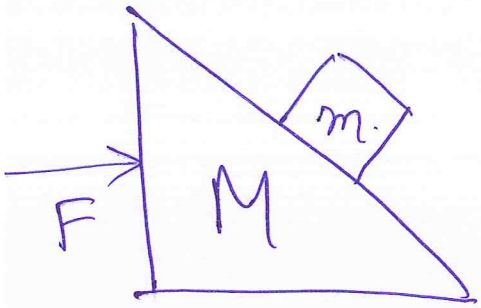
Pseudo Force (Just to balance force so that Newton's laws are applicable)

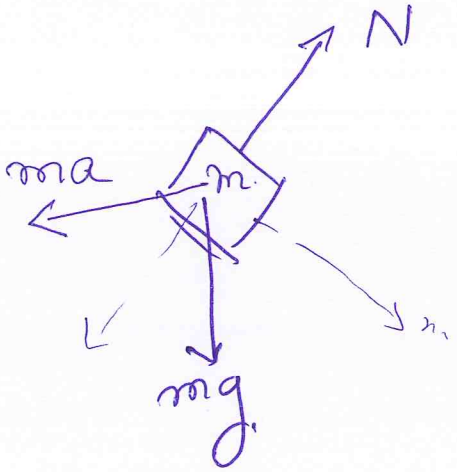
opposite to direction of acceleration of Non Inertial frame

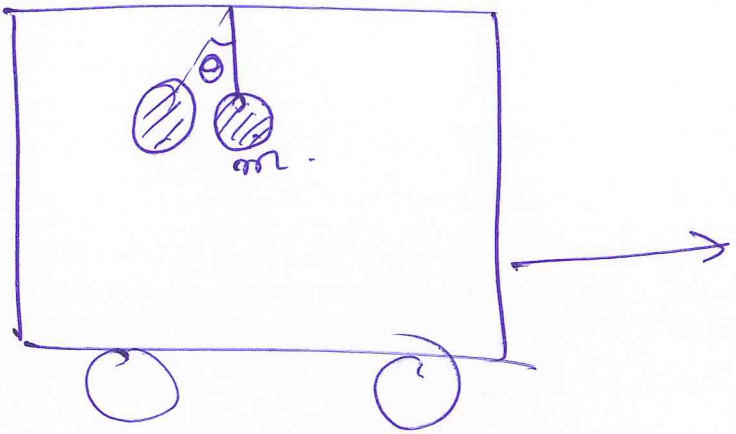
magnitude = ma

m is mass of object/body on which pseudo force is being applied

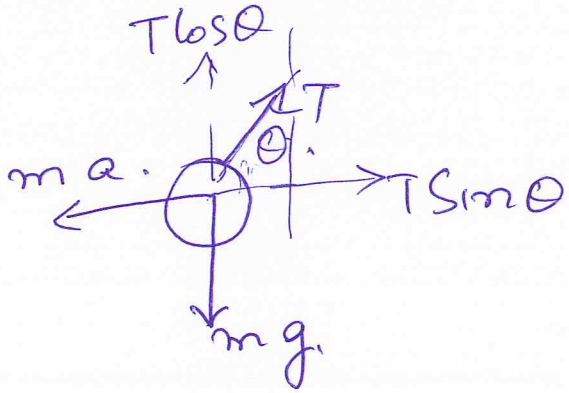
a is acceleration of non-inertial frame.




$\theta = ?$



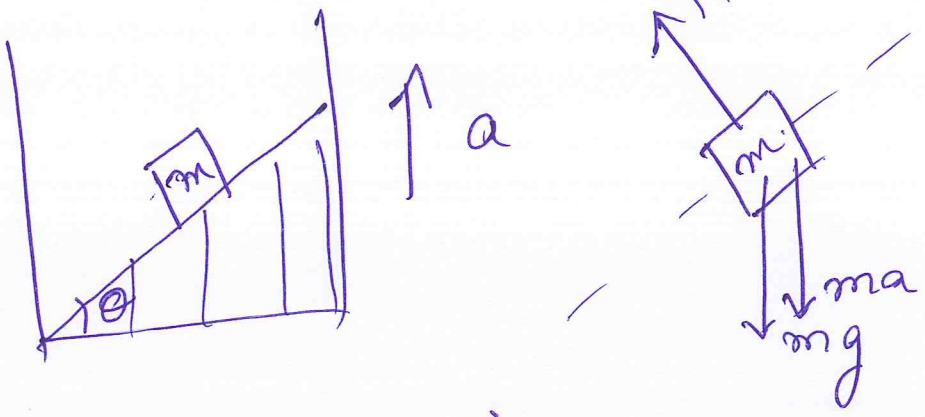

$T\cos\theta = mg$

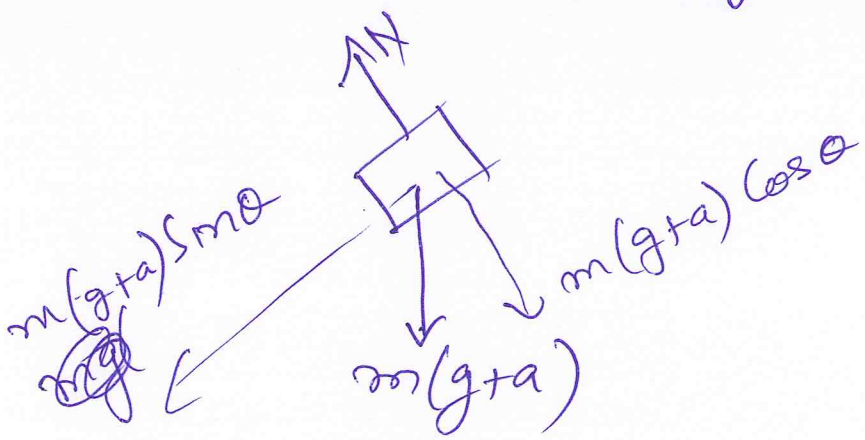
$T\sin\theta = ma$

$\tan\theta = \frac{a}{g}$

$\theta = \tan^{-1}\frac{a}{g}$

Q) A wedge is fixed to an elevator moving upwards with an acceleration 'a'. A block of mass m is placed over the wedge. Find acceleration of block w.r.t wedge (Neglect friction)

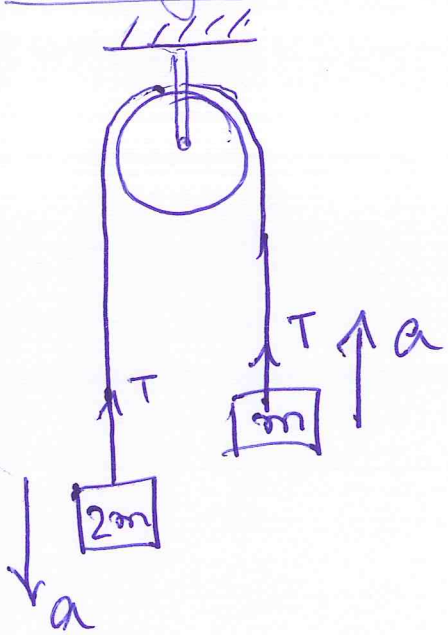



$$m(g+a)\sin\theta = m a_{net}$$

$$a_{net} = (g+a)\sin\theta.$$

# Pulleys





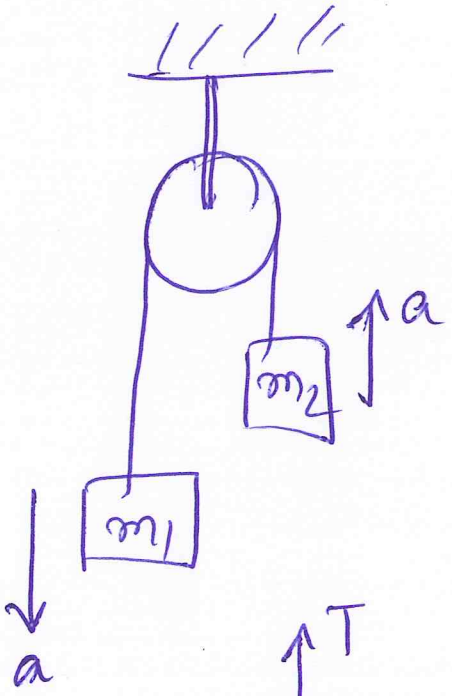
$2mg - T = 2ma$ — ①

$T - mg = ma$ — ②

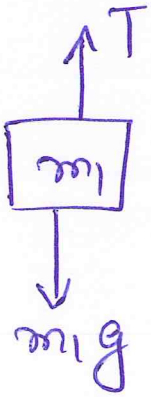
① + ②

$2mg - mg = 2ma + ma$

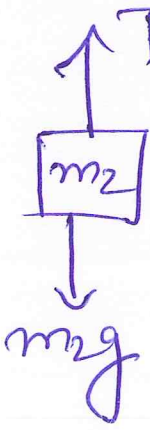
$a = \frac{g}{3}$




$m_1 > m_2$




$m_1 g - T = m_1 a$



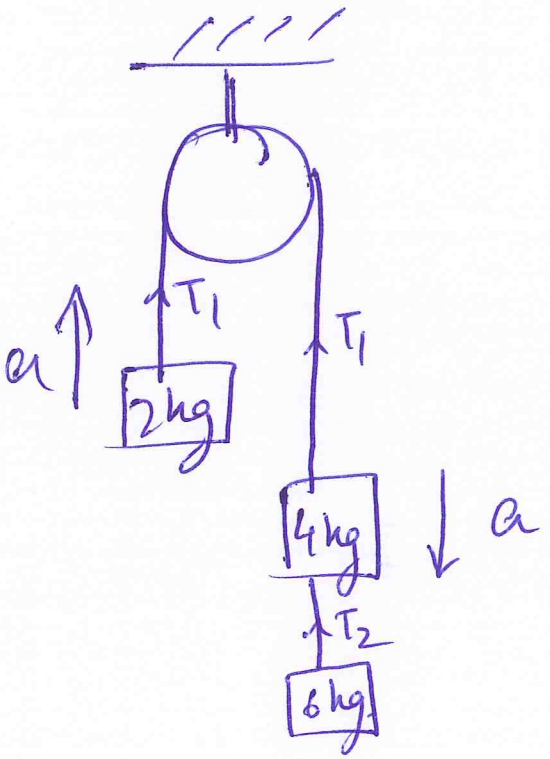

$T - m_2 g = m_2 a$

$m_1 g - m_2 g = m_1 a + m_2 a$

$a = \frac{(m_1 - m_2) g}{m_1 + m_2}$

$$T = \frac{2m_1m_2g}{m_1+m_2}$$

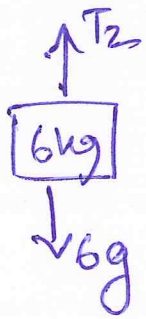
Q) In the system shown, pulley is smooth & String is massless & inextensible. Find acceleration $a$, Tensions $T_1$ & $T_2$




$$a = \left(\frac{m_2-m_1}{m_1+m_2}\right)g$$

$$= \left(\frac{10-2}{10+2}\right)g = \frac{2g}{3}\ m/s^2$$

$$T_1 = \frac{2m_1m_2g}{m_1+m_2} = \frac{2\times2\times10\,g}{12} = \frac{10g}{3}$$

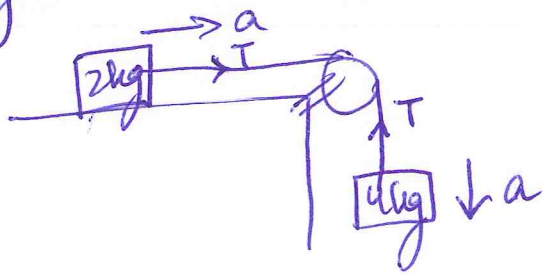


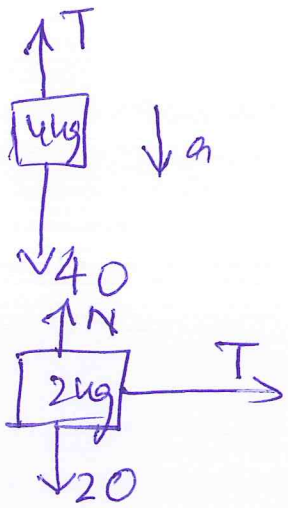

$$6g - T_2 = 6a$$

$$T_2 = 6g - 6a.$$

$$= 6\left(g - \frac{2g}{3}\right) = 2g\ N$$

Q) In the system shown all surfaces are smooth. String is massless & inextensible. Find acceleration 'a' of the system & Tension 'T' in string ($g = 10\ m/s^2$)





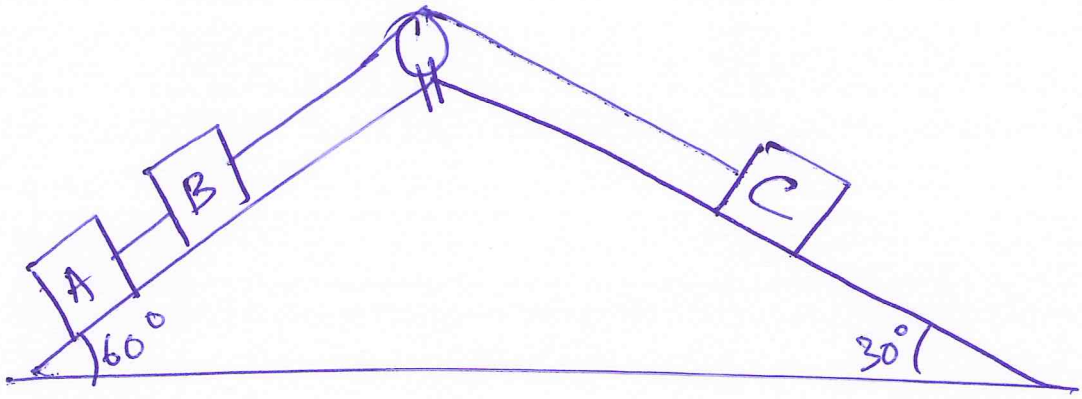

$40 - T = 4a$

$T = 2a$

$\rightarrow a = \frac{20}{3} m/s^2$

$\rightarrow T = \frac{40}{3} N$

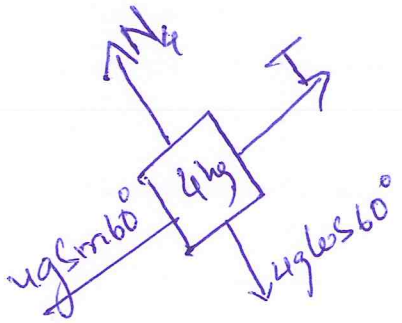
Q3 In the figure mass of A, B, C are 1kg, 3kg, 2kg respectively

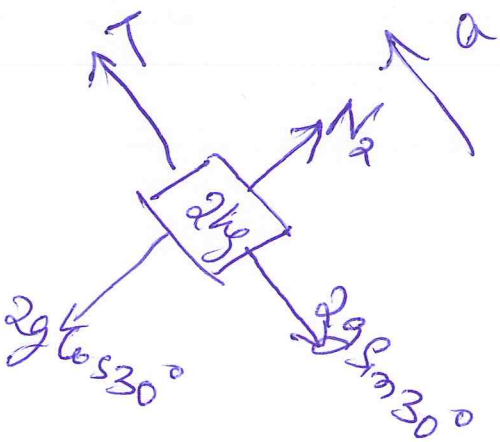



Find

a) Acceleration of system.

b) tension in the string (Neglect friction)




$40\frac{\sqrt{3}}{2} - T = 4a$

$T - 20 \times \frac{1}{2} = 2a$

$$20\sqrt{3} - T = 4a$$

$$+ \quad T - 10 = 2a$$

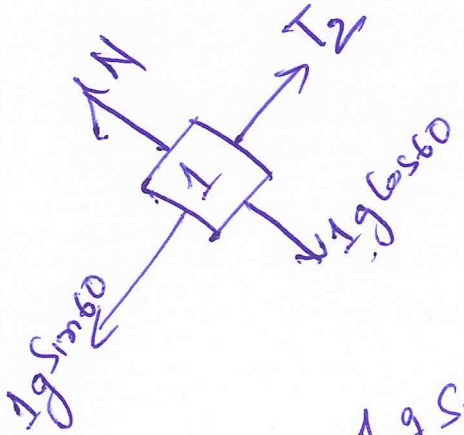
$$20\sqrt{3} - 10 = 6a \implies a = \frac{10(2\sqrt{3}-1)}{6} \text{ m/s}^2$$

$$T = 2a + 10$$

$$= \frac{20(2\sqrt{3}-1)}{6} + 10$$

$$= \frac{20}{6}(2\sqrt{3} - 1 + 3) = \frac{\cancel{20}^{10}}{\cancel{6}_{3}}(2\sqrt{3}+2)$$

$$= \frac{20}{3}(\sqrt{3}+1) \text{ N}$$




$$1g\sin 60 - T_2 = 1a.$$

$$T_2 = g\sin 60 - a.$$

$$= \frac{10\sqrt{3}}{2} - \frac{10}{6}(2\sqrt{3}-1)$$

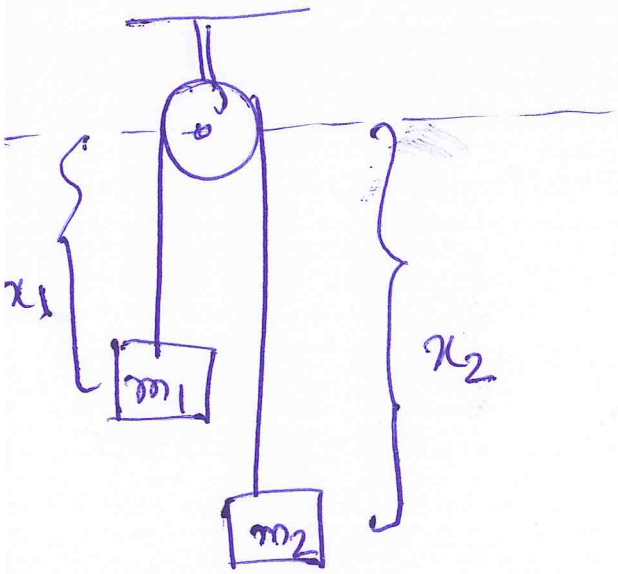
$$= 5\sqrt{3} - \frac{10}{3}\sqrt{3} + \frac{5}{3}$$

$$= \frac{5\sqrt{3}}{3} + \frac{5}{3} = \frac{5}{3}(\sqrt{3}+1) \text{ N}$$

# Constraint Equation.

↓

derives Relationship between accelerations.




$$x_1 + x_2 = l = \text{const}$$

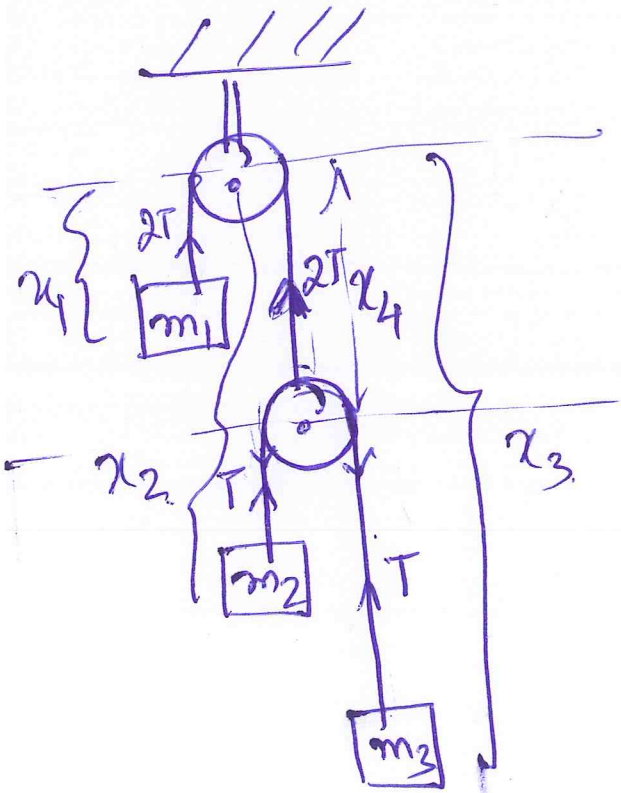
$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

$$v_1 + v_2 = 0$$

$$\frac{dv_1}{dt} + \frac{dv_2}{dt} = 0$$

$$a_1 + a_2 = 0$$

$$a_2 = -a_1$$




$$x_1 + x_4 = \text{const} = l_1$$

$$a_1 + a_4 = 0$$

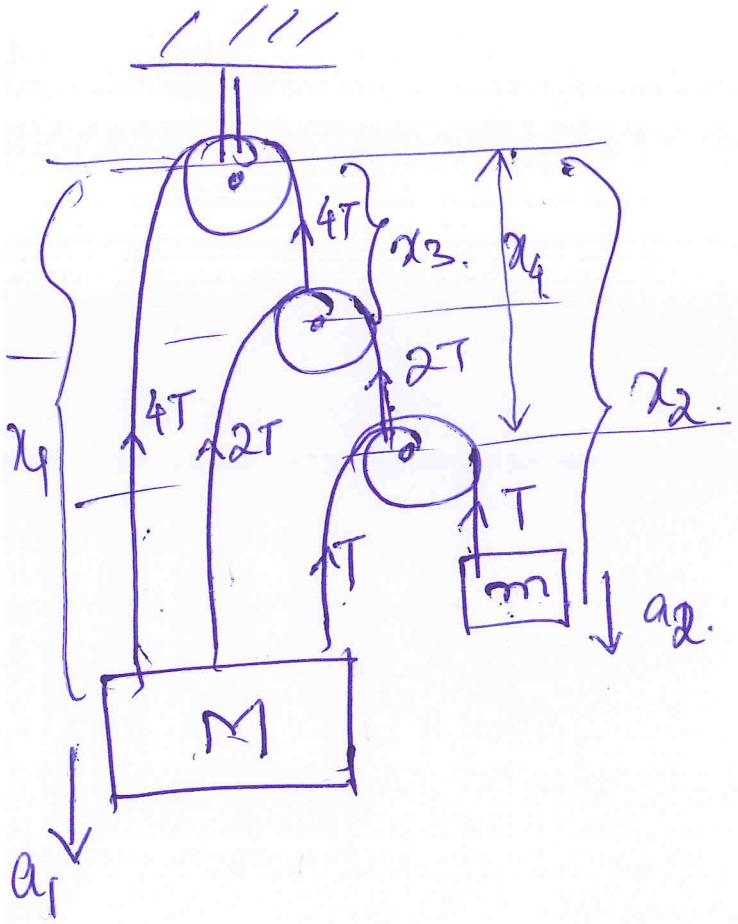
$$(x_2 - x_4) + (x_3 - x_4) = l_2 = \text{const}$$

$$x_2 + x_3 - 2x_4 = l_2 = \text{const}$$

$$a_2 + a_3 - 2a_4 = 0$$

$$a_2 + a_3 - 2(-a_1) = 0 \Rightarrow 2a_1 + a_2 + a_3 = 0$$

$$a_3 + a_2 + 2a_1 = 0$$




$$x_1 + x_3 = l_1$$

$$a_1 + a_3 = 0$$

$$(x_1 - x_3) + (x_4 - x_3) = l_2$$

$$a_1 - a_3 + a_4 - a_3 = 0$$

$$a_1 + a_4 = 2a_3$$

$$a_1 + a_4 = -2a_1$$

$$3a_1 + a_4 = 0$$

$$(x_1 - x_4) + (x_2 - x_4) = 0$$

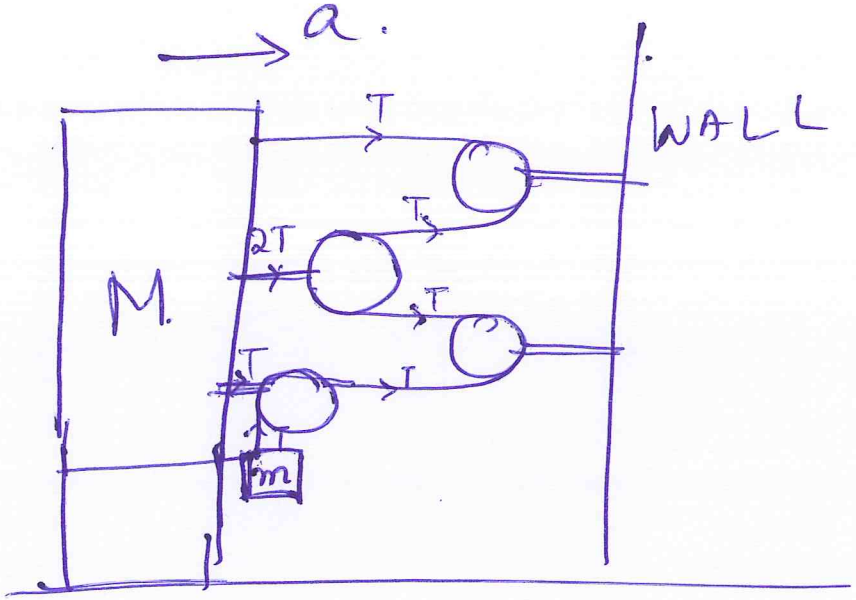
$$a_1 - a_4 + (a_2 - a_4) = 0$$

$$a_1 + a_2 - 2a_4 = 0$$

$$a_1 + a_2 - 2(-3a_1) = 0$$

$$7a_1 + a_2 = 0$$

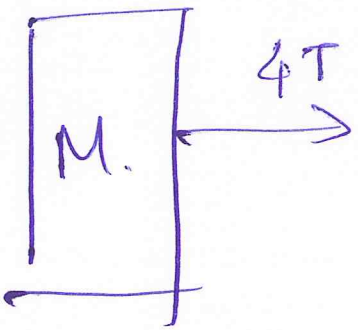
$$7a_1 + a_2 = 0$$



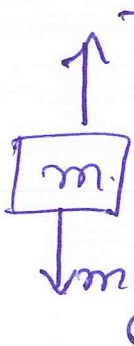

$4T \qquad T$

$4a + 1a' = 0$

$a' = -4a.$




$4T = (M+m)a.$



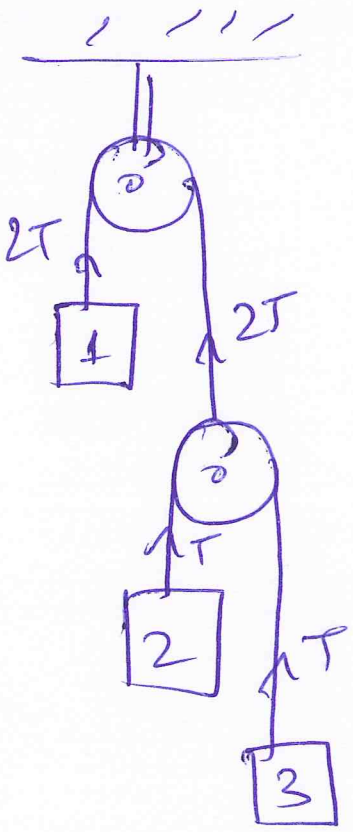

$mg - T = m(4a)$

$mg - \frac{(M+m)a}{4} = 4ma.$

$mg = \frac{(17m+M)a}{4}$

$a = \frac{4mg}{17m+M}$

Q) At certain moment of time, velocities of 1 & 2 both are 1 m/s upwards. Find velocity of 3 at that moment.



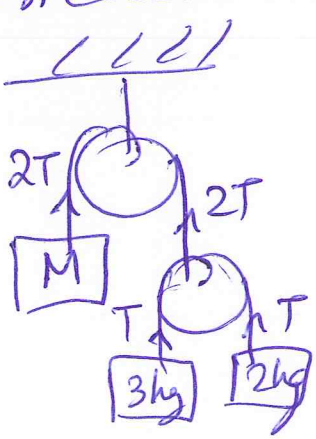

$2a_1 + a_2 + a_3 = 0$

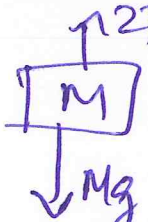
↑ +ve $\quad 2v_1 + v_2 + v_3 = 0$

$2(1) + 1 + v_3 = 0$

$v_3 = -3$ m/s

2) Find the mass M so that it remains at rest in the figure below. Both the pulley & string are light & frictionless is absent.



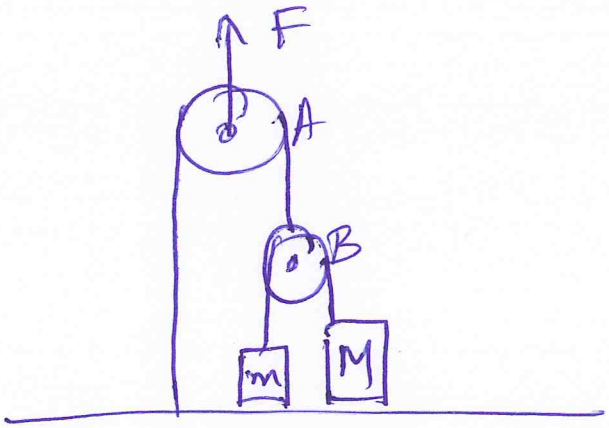

$T = \frac{2 \times 2 \times 3}{(2+3)} g \Rightarrow \frac{120}{5} = 24$ N

$2T = Mg$

$\Rightarrow M = \frac{2T}{g} = \frac{2 \times 24}{10} = 4.8$ kg

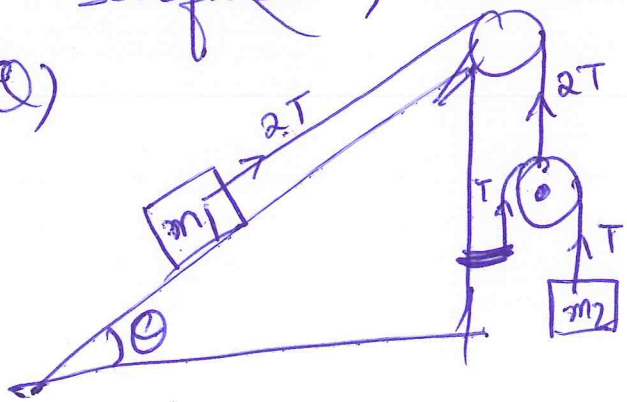
Q) Two blocks of mass $m = 5kg$ & $M = 10kg$ are connected by a string passing over a pulley B as shown. Another string connects the center of pulley B to the floor and passes over another pulley A as shown.

An upward force F is applied at the center of pulley A. Both the pulleys are massless. Find accelerations of m & M, if F is a) 100 N b) 300 N c) 500 N

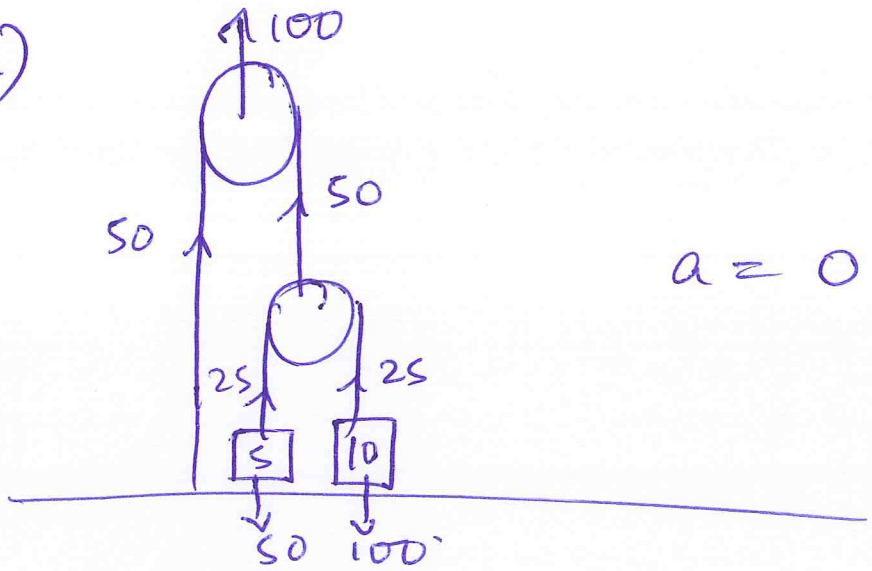



Q) Consider the situation shown in figure. The block B moves on a frictionless surface, while coefficient.

Q) Find accelerations of $m_2$ if $m_2$ is $\eta m_1$ Pulleys are massless & frictionless

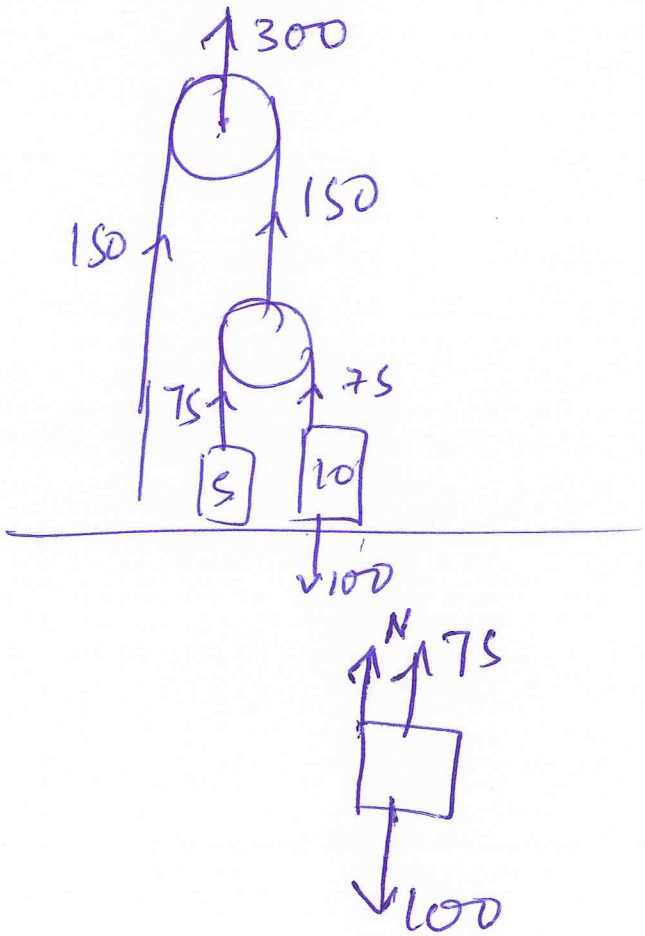


a)




$a = 0$

b)




$75 - 50 = 5a$

$a = 5\ m/s^2$

Shg $a = 5\ m/s^2$

Cothg $a = 0$




$100 - 75 - N = 10 \times 5$

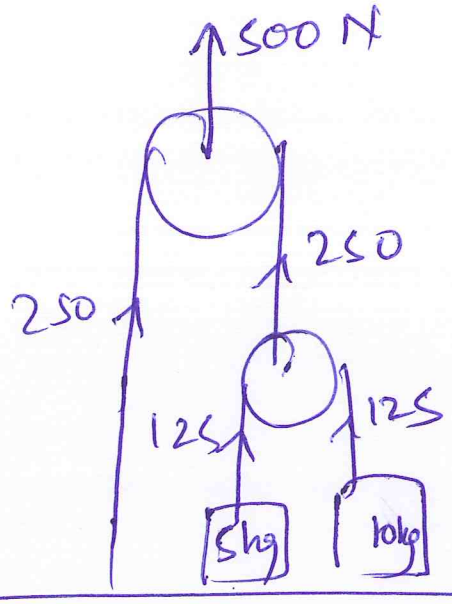
$25 - N = 50.$

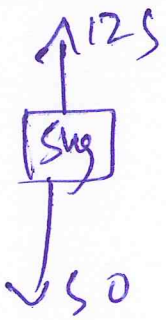
$N = -25.$ X

$100 - 75 - N = 0$

$N = 25\ N.$

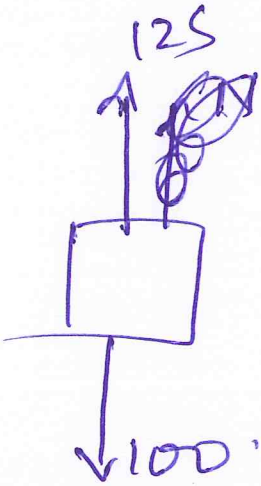
(8)




$125 - 50 = 5a_5$

$a_5 = 15\ m/s^2$



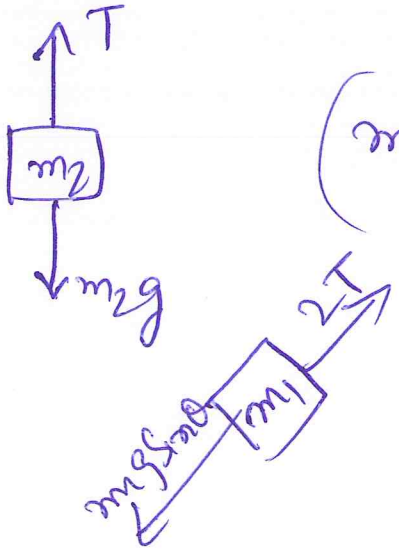

$125 - 100 = 10\, a_{10}$

$a_{10} = \frac{25}{10} = 2.5\ m/s^2$

$2a_1 + a_2 = 0$

$a_2 \downarrow \quad a_1 \rightarrow$

$a \quad a/2$




$(m_2 g - T = m_2 a) \times 2$

$2T - m_1 g \sin\theta = m_1 \frac{a}{2}$

$2m_2 g - m_1 g \sin\theta = 2m_2 a + \frac{m_1 a}{2}$

$a = \frac{2g(2m_2 - m_1 \sin\theta)}{4m_2 + m_1}$