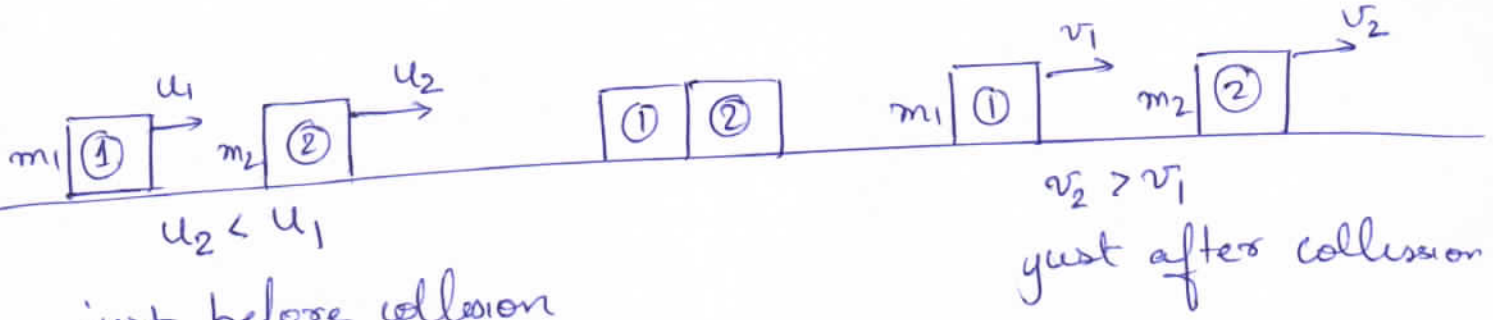
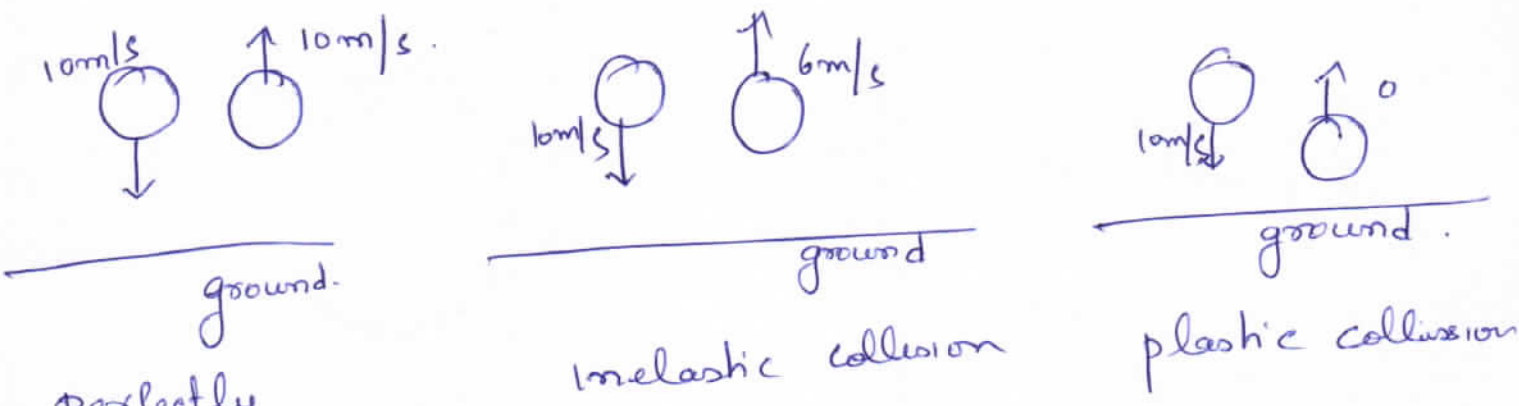
COLLISIONS

Momentum $= m\vec{v} = \vec{P}$

Impulse = Change in momentum $= \vec{P}_f - \vec{P}_i = \vec{I}$





Initial Momentum $P_i = m_1u_1 + m_2u_2$

Final Momentum $P_f = m_1v_1 + m_2v_2$

$\therefore$ Fnet external $= 0$ $\quad P_i = P_f$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

ALWAYS TRUE DURING A COLLISION

ELASTIC COLLISION

ii) $P_i = P_f$. $\Rightarrow$ $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ $\longrightarrow$ (1)

iii) K.E is conserved.

$K.E_i = K.E_f$.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \longrightarrow (2).$$

from (1) $m_1(u_1 - v_1) = m_2(v_2 - u_2)$

from (2) $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$

$\Rightarrow m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2)$

using (1) $u_1 + v_1 = u_2 + v_2$.

or $u_1 - u_2 = v_2 - v_1$

velocity of approach. velocity of separation.

Substitute $v_2 = u_1 - u_2 + v_1$ in (1)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)$

$m_1u_1 + m_2u_2 = v_1(m_1 + m_2) + m_2u_1 - m_2u_2$

$$\boxed{u_1\frac{(m_1 - m_2)}{m_1 + m_2} + u_2\frac{(2m_2)}{m_1 + m_2} = v_1}$$

$$\boxed{u_2\frac{(m_2 - m_1)}{m_1 + m_2} + u_1\frac{(2m_1)}{m_1 + m_2} = v_2.}$$

Case i) If $m_1 = m_2$. (masses are same)

$$v_1 = u_2$$
$$v_2 = u_1$$

} velocities are interchanged.

Case ii) $m_2 >> m_1$

$$v_1 = \frac{u_1\left(\frac{m_1}{m_2} - \frac{m_2}{m_2}\right)}{\frac{m_1}{m_2} + \frac{m_2}{m_2}} + \frac{u_2\left(2\frac{m_2}{m_2}\right)}{\frac{m_1}{m_2} + \frac{m_2}{m_2}}$$

$$v_1 = -u_1 + 2u_2$$

~~Case iii)~~ $v_2 = u_2$

Case iii) $m_1 >> m_2$

$$v_1 = u_1$$
$$v_2 = -u_2 + 2u_1$$

ii) Inelastic Collision

a) Momentum is conserved.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

b) coefficient of restitution (e) = $\frac{\text{velocity of separation}}{\text{velocity of approach}}$

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

$0 \leq e < 1$

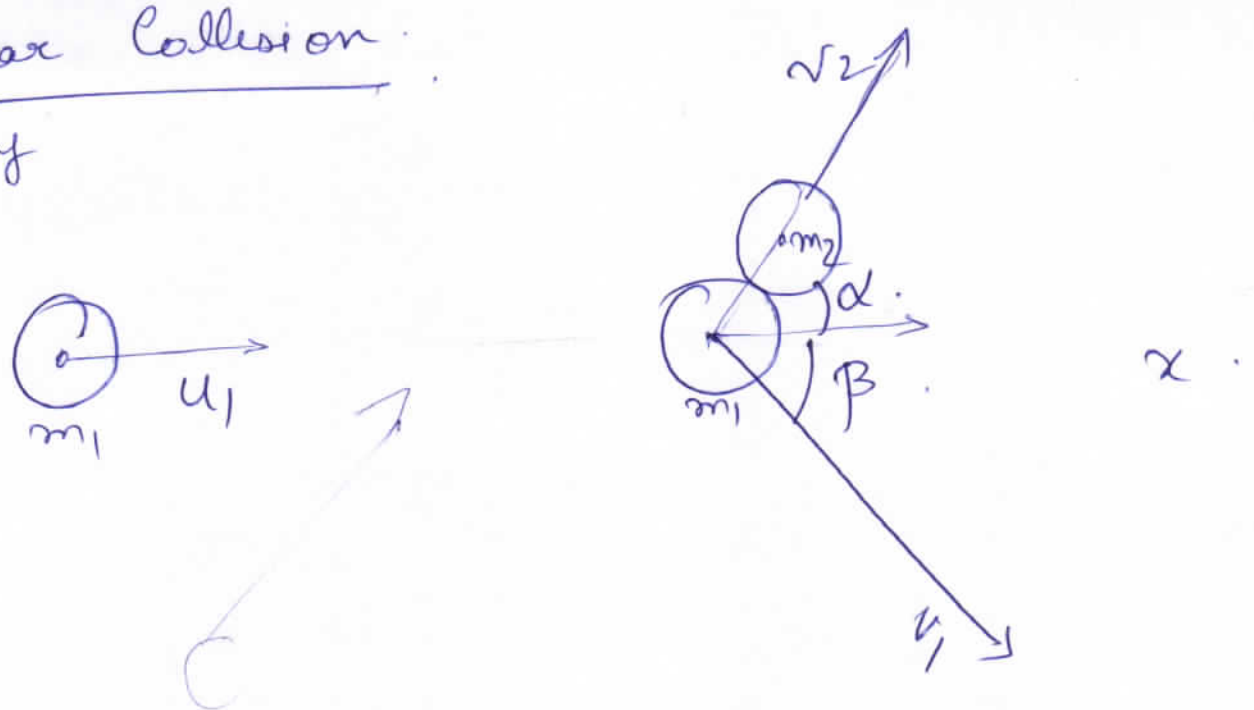
plastic collision

There is always loss in K.E

loss in K.E = K.E$_i$ − K.E$_f$.

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

Angular Collision



P_x.

$$m_1 u_1 = m_1 v_1 \cos\beta + m_2 v_2 \cos\alpha$$

P_y

$$0 = -m_1 v_1 \sin\beta + m_2 v_2 \sin\alpha$$

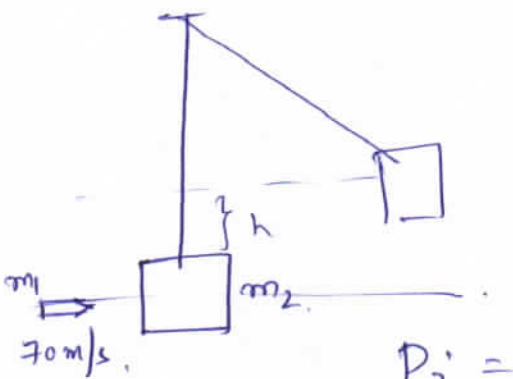
If Elastic collision

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

If Inelastic

$$e = \frac{\vec{v_2} - \vec{v_1}}{\vec{u_1}}$$

Q A bullet of mass 0.012 kg moving horizontally with speed 70 m/s strikes a block of wood of mass 0.4 kg and instantly comes to rest w.r.t block. The block is suspended from ceiling by means of a thin wire. Calculate the height to which block will rise & also estimate the heat produced.



$P_i = 0.012 \times 70 + 0.4 \times 0.$

$P_f = (0.412)v$

$P_i = P_f \Rightarrow v \approx 2$ m/s.

T.E i

$\frac{1}{2}mv^2 + 0 = 0 + mgh.$

$\frac{1}{2}mv^2 = mgh \Rightarrow h = \frac{v^2}{2g}$

$= \frac{4}{20} = \underline{0.2m.}$

Loss in K.E = Heat Energy

$\left(\frac{1}{2} \times 0.012 \times 70^2 + 0\right) - \left(\frac{1}{2} \times 0.412 \times 2^2\right)$

COLLISIONS TUTORIAL.

SOLVED EX.

Pg. 26-28 1 2 5 6 10

Pg 31 linked comp.

Pg 34 - 38 3, 5, 6, 7, 8, 10

H.W

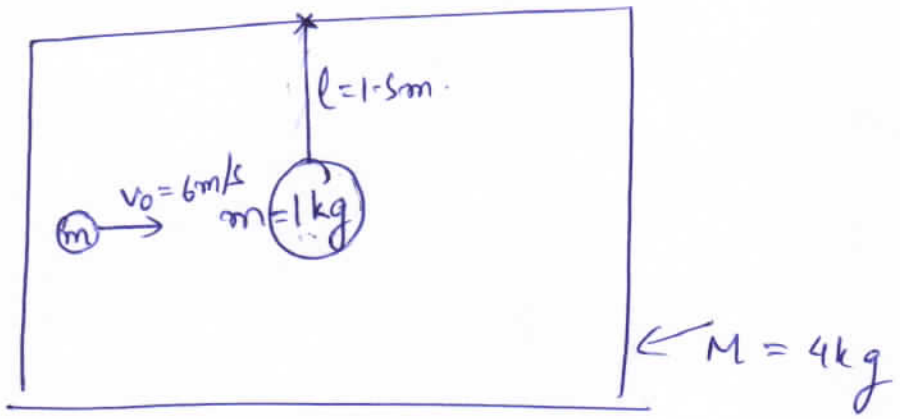
Pg 39 7, 13

Pg 40 – 45 4, 7, 8, 14, 15, 16, 18, 26, 29, 32
35, 40, 41, 44, 46, 47, 48.

Pg 47 Comp 1.

Pg 48 Comp 3.

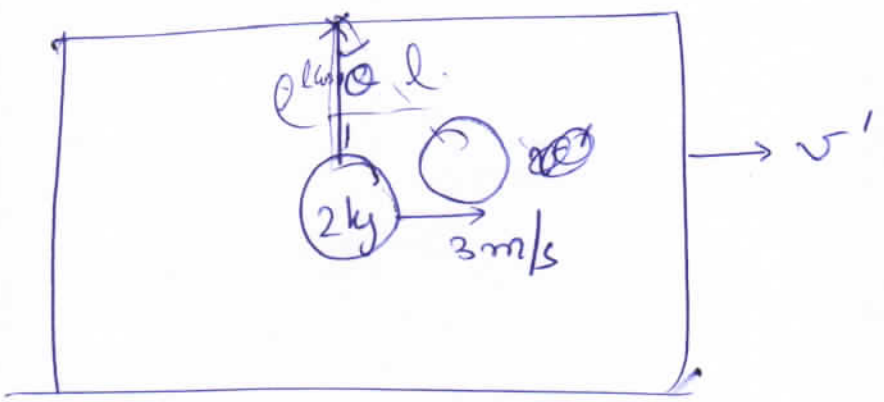
Linked Comprehension Pg 31



Collision 1 (Plastic 1 kg & 1 kg)

$1 \times 6 = (1+1) \times v$

$v = 3\ m/s.$



$2 \times 3. = (2+4) v'$

$v' = 1\ m/s.$

1) → A.

2) → C.

$\frac{1}{2} \times 2 \times 3^2 = 2 \times 10 \times 1.5 (1 - \cos\theta) + \frac{1}{2}(2+4) \times 1^2$

$9 = 30(1 - \cos\theta) + 3.$

$\frac{6}{30} = 1 - \cos\theta.$

$\cos\theta = \frac{4}{5} = 0.8.$ $\quad \sin\theta = \frac{3}{5} \Rightarrow \theta = 37°$

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