FUNCTIONS

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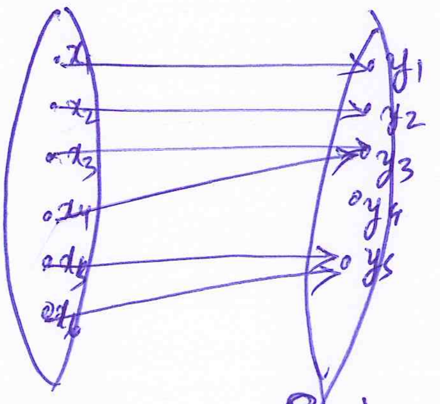
A TYPE OF RELATION

Def: A function $f : A \longrightarrow B$ is a relation which associates every element of set A to some unique element in set B.

A B



Domain Codomain

One – One
Many – One

Range $\subseteq$ Codomain (Surjective funct)

Range = Codomain (Onto)

Range $\subset$ Codomain (Into)

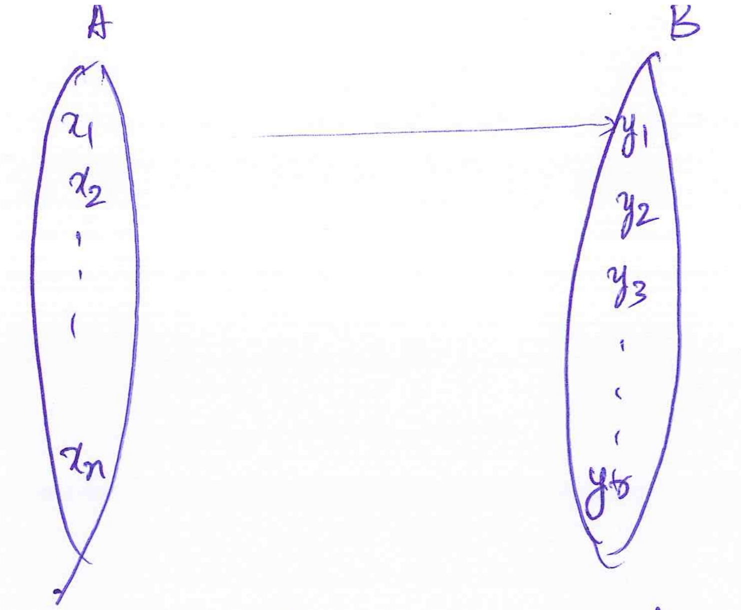
Domain = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$

Codomain = $\{y_1, y_2, y_3, y_4, y_5\}$

Range = $\{y_1, y_2, y_3, y_5\}$

$f(x_1) = y_1$

y_1 is image of x_1
x_1 is pre-image of y_1



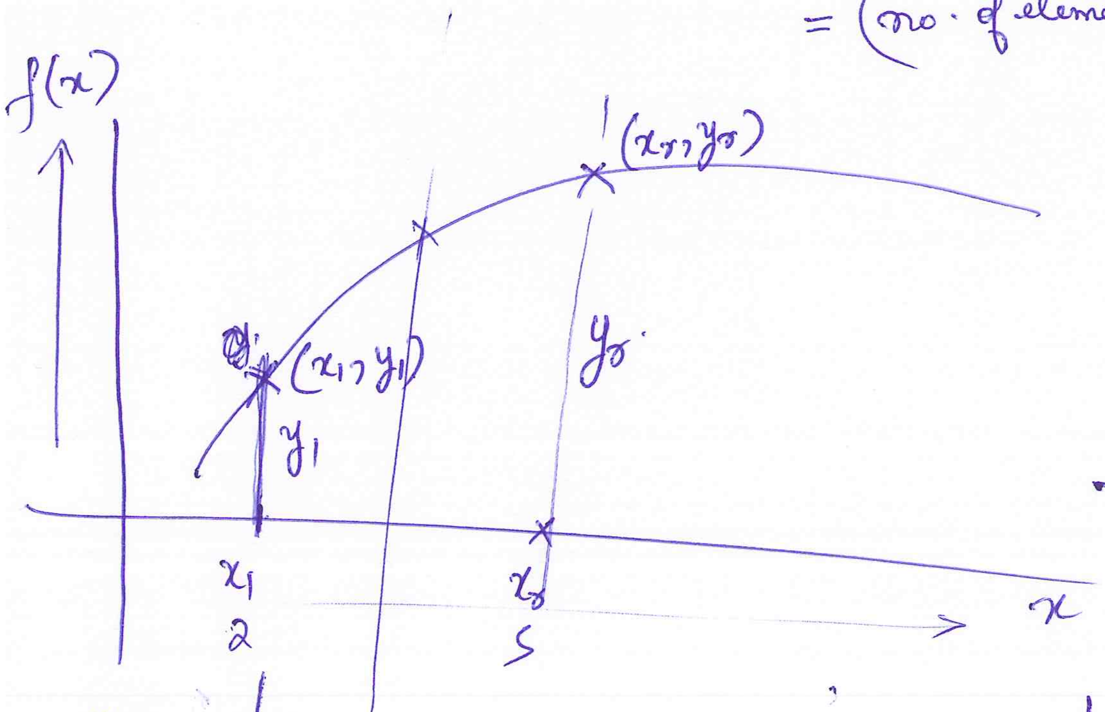
$x_1 \quad x_2 \quad x_3 \quad - - - - \quad x_n.$

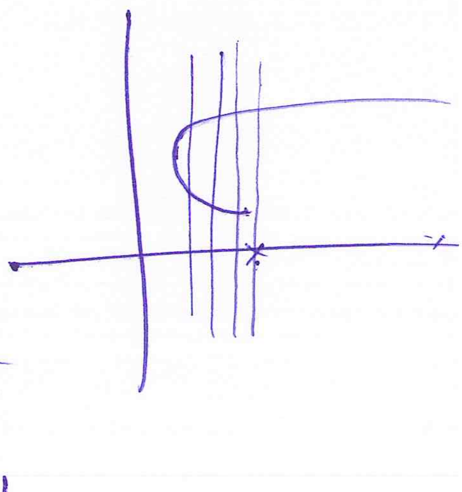
$r \quad r \quad r \quad \qquad \qquad r$

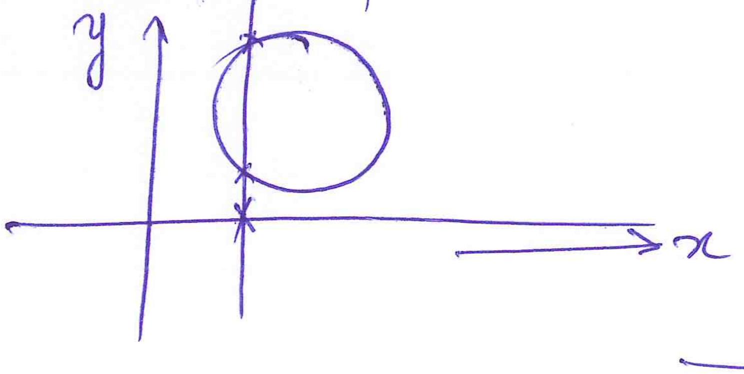
n elements. $\qquad$ r elements.

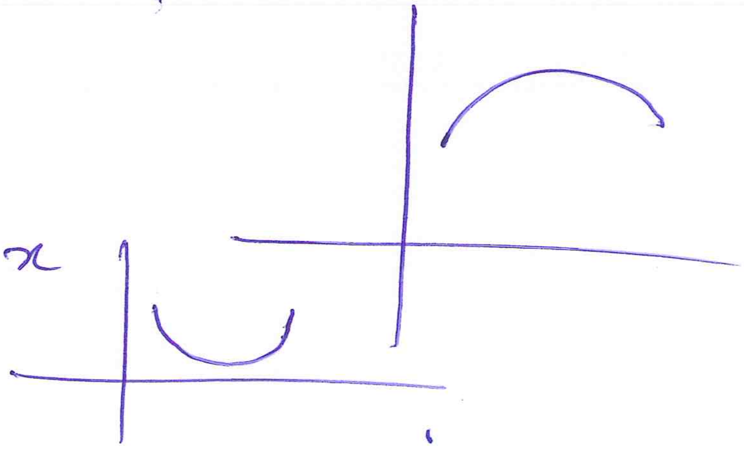
How many functions are possible from $A \longrightarrow B$ $= \boxed{r^n}$..

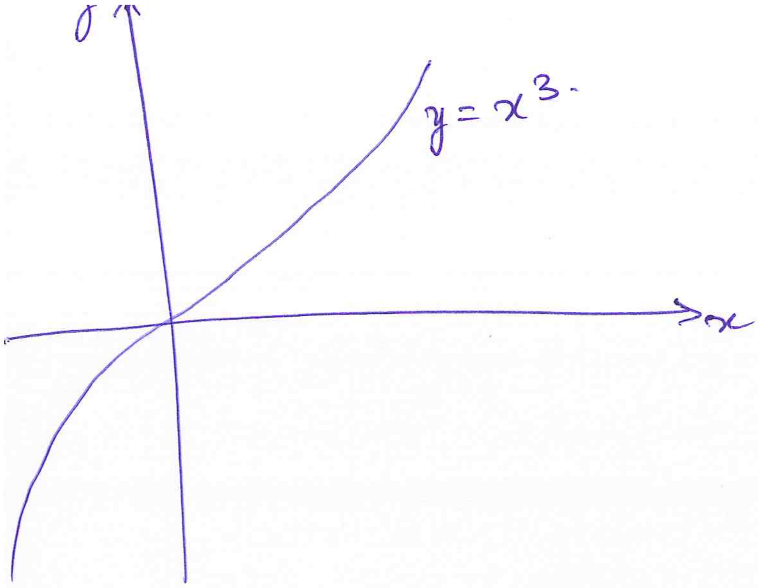
$= (\text{no. of elements in codomain})^{(\text{no. of elements in domain})}$



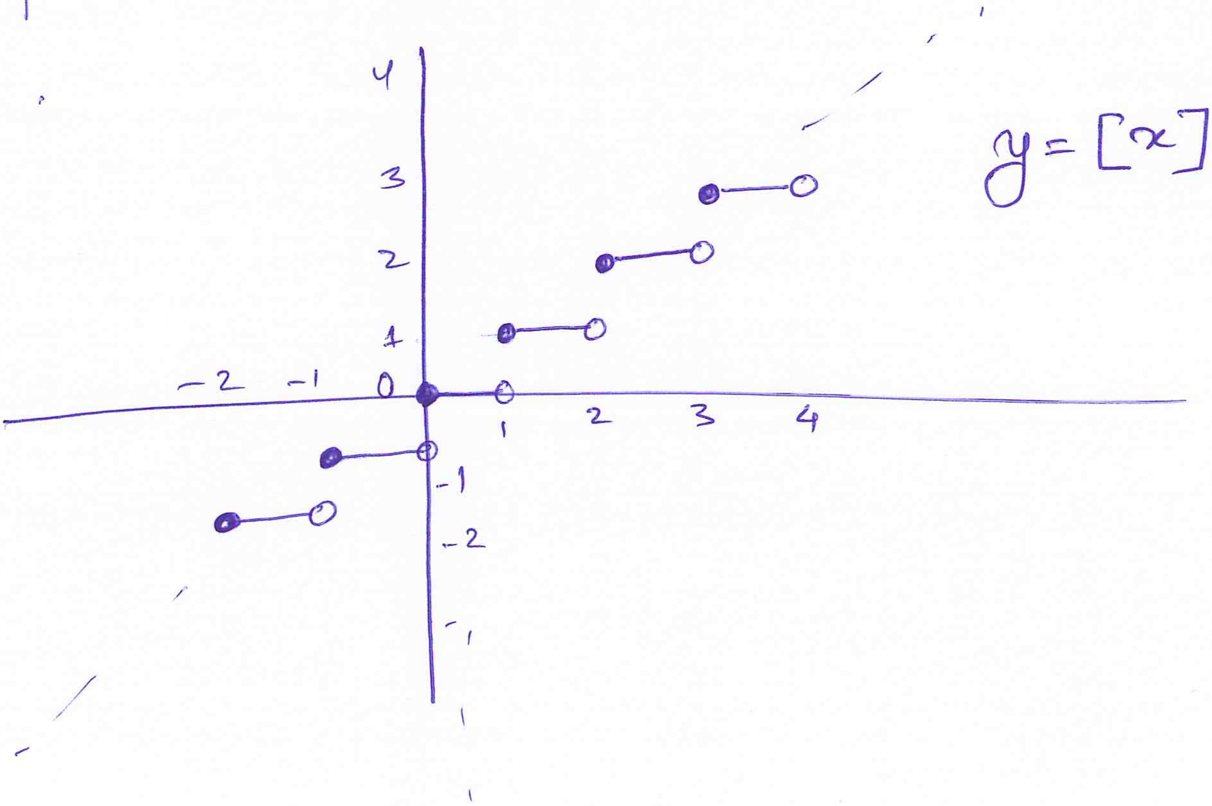




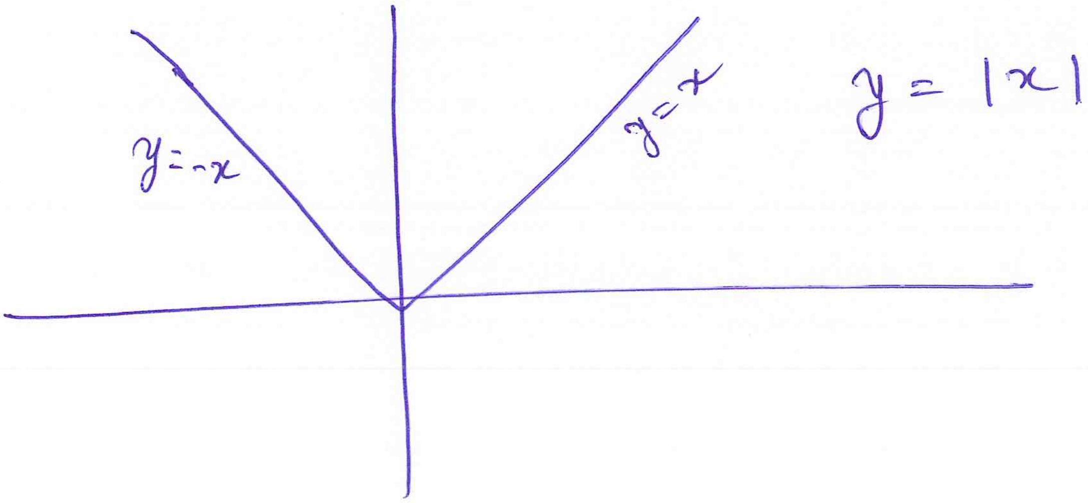




y
$y = x^3$.
x



$y = [x]$
4
3
2
1
0
-2
-1
1
2
3
4
-1
-2



$y = |x|$
$y = -x$
$y = x$

Finding Domain of Any function :

$f(x)$ f of x

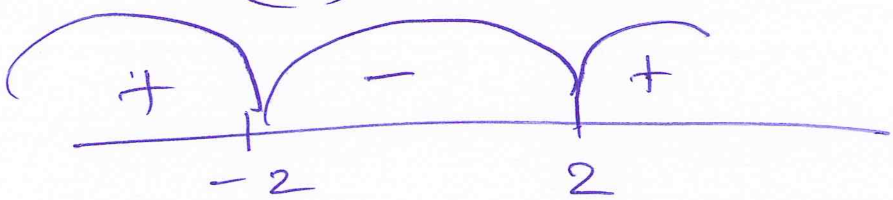
The domain of $y = f(x)$ is the set of all real x for which $f(x)$ is defined.

i) Expression under root :

$f(x) = y = \sqrt{x^2 - 4}$

$x^2 - 4 \geq 0$

$(x-2)(x+2) \geq 0$



$x \geq 2$

or

$x \leq -2$

$(-\infty, -2] \cup [2, \infty)$

ii) Denominator $\neq 0$

$f(x) = \dfrac{x^2 - 4}{x+2}$

$x + 2 \neq 0$

$x \neq -2$

$\mathbb{R} - \{-2\}$

$(-\infty, -2) \cup (-2, \infty)$

iii) if $y = f(x) \Rightarrow$ Domain D_1

$y = g(x) \Rightarrow$ Domain D_2.

Domain of $f(x) \pm g(x)$ $\quad D_1 \cap D_2$

Domain of $f(x) \times g(x)$ $\quad D_1 \cap D_2$

Domain of $\dfrac{f(x)}{g(x)} \Rightarrow D_1 \cap D_2 - \{g(x)=0\}$

eg. find domain

1) $y = \dfrac{\sqrt{x^2-9}}{x-4}$

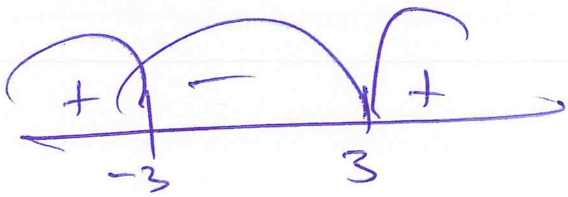
2) $y = \dfrac{x}{\sqrt{x^2-5x+6}}$

3) $y = \sqrt{4-x^2} + \sqrt{x^2-9}$

Ans 1

$x^2 - 9 \geq 0$

$(x-3)(x+3) \geq 0$



$x \geq 3 \quad$ or $x \leq -3$

$x \neq 4$

$(-\infty, -3] \cup [3, \infty)$

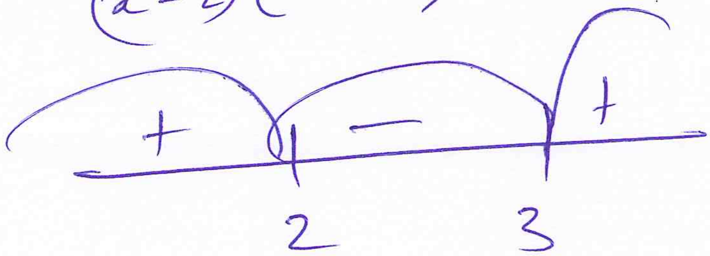
$(-\infty, -3] \cup [3, 4) \cup (4, \infty)$

$(-\infty, -3] \cup [3, \infty) - \{4\}$

2) $x^2 - 5x + 6 > 0$

$x^2 - 2x - 3x + 6 > 0$

$(x-2)(x-3) > 0$



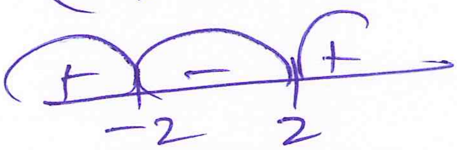
$(-\infty, 2) \cup (3, \infty)$

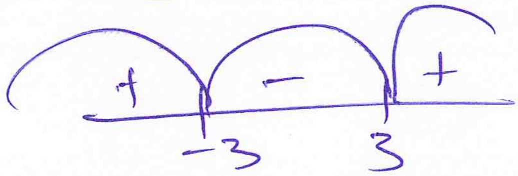
$\mathbb{R} - [2, 3]$

3) $4 - x^2 \geq 0 \quad \cap \quad x^2 - 9 \geq 0$

$x^2 - 4 \leq 0 \qquad (x-3)(x+3) \geq 0$

$(x-2)(x+2) \leq 0$





$[-2, 2] \quad \cap \quad (-\infty, -3] \cup [3, \infty)$

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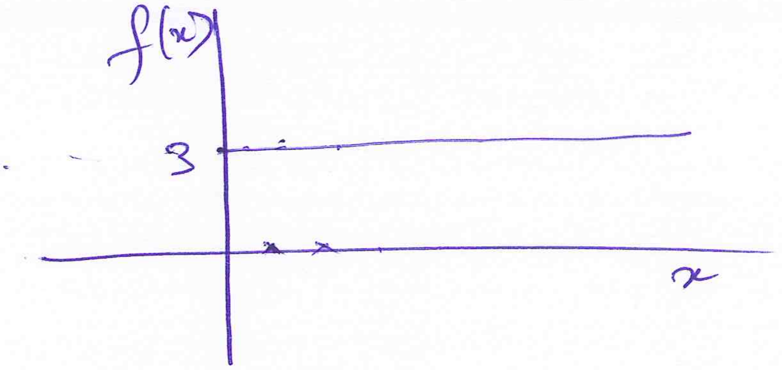
Types of Functions.

i) Constant Function:

$\Rightarrow$ range is a singleton set.



$f(x) = 3$

ii) Polynomial Functions

A function of the form

$y = f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots a_{n-1} x + a_n$

where $a_0, a_1, a_2 \cdots a_n$ are real constants

& n is a non-negative integer

If $a_0 \neq 0$ then n is the degree of the polynomial function.

eg. $f(x) = 3x^2 + 5$ ✓ $\qquad f(x) = x^{100} + 2x^{55} + 3$ ✓

$f(x) = x + \frac{1}{x}$ $\qquad f(x) = \sqrt{x}$

$x^1 + x^{-1}$ $\qquad = x^{1/2}$

if degree of polynomial is odd the ~~domain~~ range is set of real numbers.

3 degree

3 Real roots

1 real root

$f(x) = -x^3$

Q If $f(x) \times f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

prove that $f(x)$ is $1 + x^n$

or $1 - x^n$

$$f(x) \times f(1/x) - f(x) = f(1/x)$$

$$f(x)\left[f(1/x) - 1\right] = f(1/x)$$

$$f(x) = \frac{f(1/x)}{f(1/x) - 1} \quad \text{——} \ (1)$$

$$f(1/x) = \frac{f(x)}{f(x) - 1} \quad \text{——} \ (2)$$

$$\frac{f(x)}{f(1/x)} = f(x) - 1 = \frac{1}{f(1/x) - 1}$$

$$\left[f(x)-1\right]\left[f(1/x)-1\right] = 1$$

$$\Rightarrow \quad f(x) - 1 = \pm x^n$$

$$\Rightarrow \quad f(x) = 1 + x^n$$

or

$$1 - x^n$$

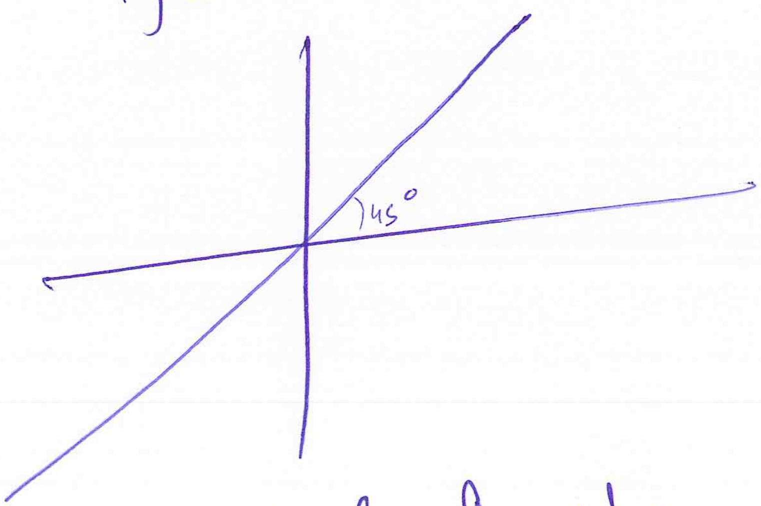
iii) <u>Algebraic function.</u> $\begin{cases} \text{polynomial} \\ \text{Non-polynomial} \end{cases}$

$$f(x) = x^3 + 4x + 3$$

$$f(x) = x^3 + 4\sqrt{x} + 1$$

iv) <u>Identity function.</u>

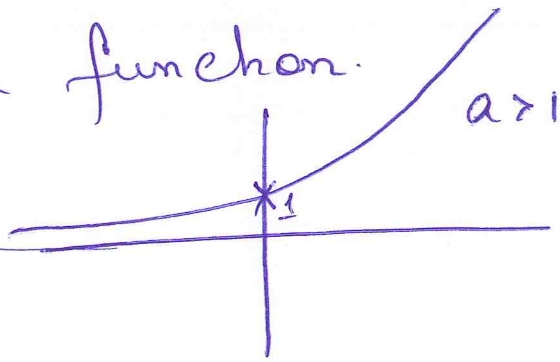
$$f(x) = x \quad \forall \; x \in R$$



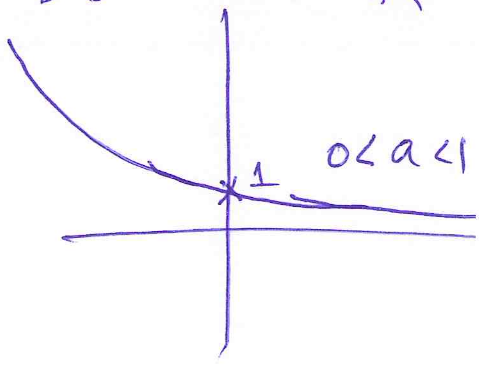
Domain $\mathbb{R}$.

v) Exponential function.

$$\underline{f(x) = a^x}$$

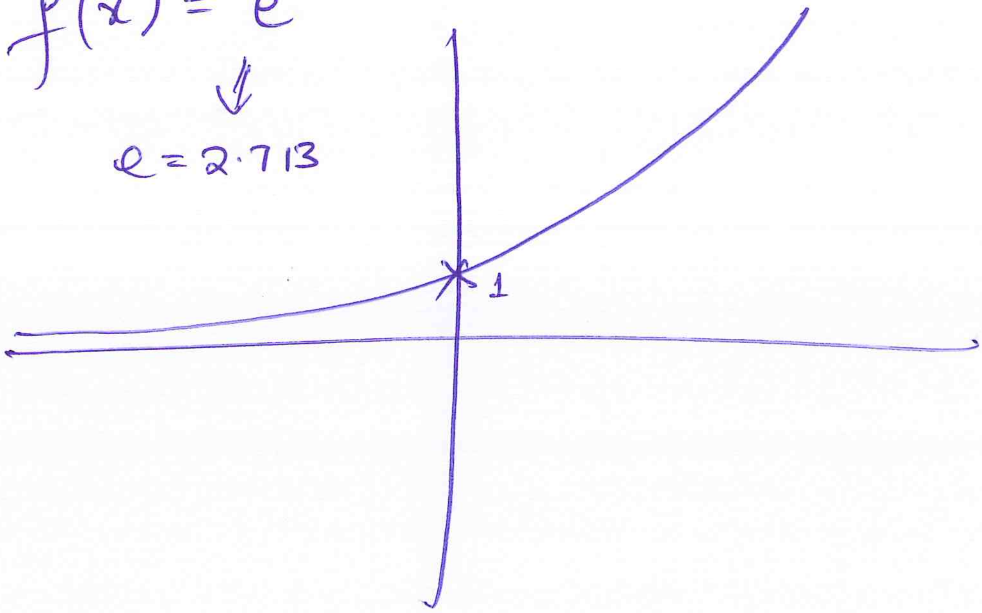


Domain $\mathbb{R}$



$f(x) = e^x$

$e = 2.713$

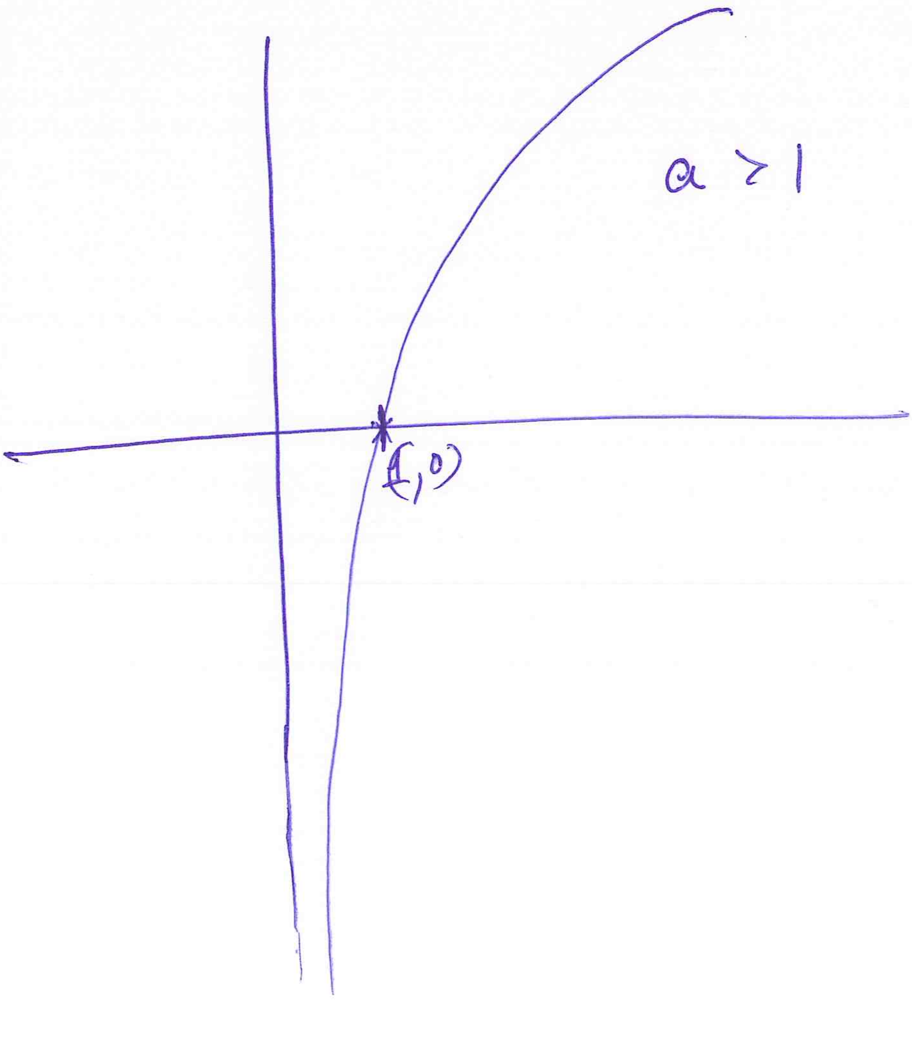


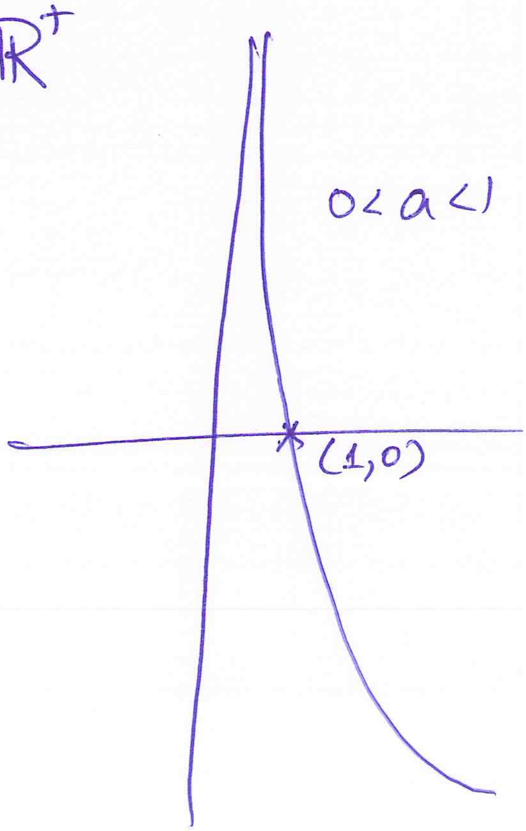
vi) <u>Logarithmic function :</u>

$f(x) = \log_a(x)$ $\quad a > 0 \quad (a \neq 1)$

Domain $(0, \infty)$

$\mathbb{R}^+$





Some properties of logarithmic function.

i) Domain ≥ 0 $(0, \infty)$

ii) base $\underline{a > 0 \ \& \ a \neq 1}$

$f(x) = \log_a x$ $\log_a 1 = 0$

$= \dfrac{\log_{10} x}{\log_{10} a}$

$\log_2 3 = \dfrac{\log_{10} 3}{\log_{10} 2}$

iii) $\log_a (x_1 x_2) = \log_a x_1 + \log_a x_2$

eg $\log_3 2x = \log_3 2 + \log_3 x$

iv) $\log_a (x^n) = \log_a (x \cdot x \cdot x \cdots x) = \log_a x + \log_a x + \cdots$ n times

$= \underline{n} \log_a x$

v) $\log_a \left(\dfrac{x_1}{x_2}\right) = \log_a x_1 - \log_a x_2$

vi) $\log_a 1 = 0$

vii) $\log_a a = \dfrac{\log_z a}{\log_z a} = 1$

4) $2\log_3 x + \log_3(x^2-3) = \log_3(0.5) + 5$

Ans 1

$$2 \cdot 3^{\log_4 x} + 3^{\log_4(x)} = 27$$

$$\cancel{3} \cdot 3^{\log_4(x)} = \cancel{27}\ 9$$

$$3^{\log_4(x)} = 3^2$$

$$\log_4(x) = 2$$

$$x = 4^2 = 16$$

Ans 2

$$\log_x 10 = y$$

$$y^3 - 6y^2 + 11y - 6 = 0$$

$$y = 1, 2, 3$$

$\log_x 10 = 1 \Rightarrow 10 = x^1$ or $x = 10$

$\log_x 10 = 2 \Rightarrow 10 = x^2$ or $x = \sqrt{10}$

$\log_x 10 = 3 \Rightarrow 10 = x^3$ or $x = \sqrt[3]{10}$

Ans 3

$$x^2 + 5x = 2x^2 - 2x - 8$$

$$\Rightarrow x^2 - 7x - 8 = 0$$

$$\Rightarrow x^2 - 8x + x - 8 = 0$$

$$\Rightarrow x(x-8) + 1(x-8) = 0 \Rightarrow x = \cancel{-1},\ 8$$

$$x^2 + 5x > 0$$

$x^2 - 6x + 8 > 0, \neq 1$ & $2x^2 - 2x - 8 > 0, \neq 1$

viii) $x_1^{\log_a(x_2)} = x_2^{\log_a(x_1)}$

$\log_a z = \log_a y$

$z = y.$

$Z = x_1^{\log_a(x_2)}$

$\log_a Z = \log_a\left(x_1^{\{\log_a(x_2)\}}\right)$

$\log_a Z = \{\log_a(x_2)\}\{\log_a(x_1)\}$

$= \log_a(x_1) \log_a(x_2)$

$\log_a Z = \log_a (x_2)^{\{\log_a(x_1)\}}$

$Z = x_2^{\log_a(x_1)}$

ix) $(a)^{\log_a(x)} = (x)^{\log_a(a)} = x^1 = x$

x) $\log_a x = b$

$x = a^b$

Q Solve the following equations

1) $2x^{\log_4(3)} + 3^{\log_4(x)} = 27$

2) $\left(\log_x(10)\right)^3 - 6\left(\log_x(10)\right)^2 + 11\log_x(10) - 6 = 0$

3) $\log_{(x^2-6x+8)}\left(\log_{(2x^2-2x-8)}(x^2+5x)\right) = 0$

4) $\log_3 x^2 + \log_3(x^2-3) = \log_3(0.5) + (\log_3 8)$

$$\log_3(x^2(x^2-3)) = \log_3(0.5 \times 8)$$

$$\log_3(x^2(x^2-3)) = \log_3 4$$

$$x^2(x^2-3) = 4$$

$$x^4 - 3x^2 - 4 = 0$$

$$(x^2-4)(x^2+1) = 0$$

$$x = \pm 2 \qquad \Rightarrow x = +2$$

Find Domain :

1) $f(x) = \dfrac{x^2-3x+5}{(x-1)(x-2)}$

2) $f(x) = \dfrac{1}{2-\sin 2x}$

3) $f(x) = a^x - 1 \qquad (a \neq 1, a > 0)$

4) $y = f(x)$ is given by $10^y + 10^x = 10$

5) $f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+3}$

6) $f(x) = \log_{10}\left\{ \log_{|\sin x|}(x^2-8x+23) - \dfrac{3}{\log_2|\sin x|} \right\}$

1) $x-1 \neq 0$ $x \neq 1$

$x-2 \neq 0$ $x \neq 2$

$$x \in \mathbb{R} - \{1, 2\}$$

2) $2 - \sin 2x \neq 0$

$\sin 2x \neq 2$ $\forall x$.

$$x \in \mathbb{R}$$

3) $a^x - 1$ $x \in \mathbb{R}$.

4) $y = f(x)$ $10^y + 10^x = 10$

$$10^y = 10 - 10^x$$

$$\log_{10} 10^y = \log_{10}(10 - 10^x)$$

$$y \log_{10} 10 = \log_{10}(10 - 10^x)$$

$$y = \log_{10}(10 - 10^x)$$

$$10 - 10^x > 0$$

$$10^x < 10^1$$

$x \in (-\infty, 1)$ $x < 1$

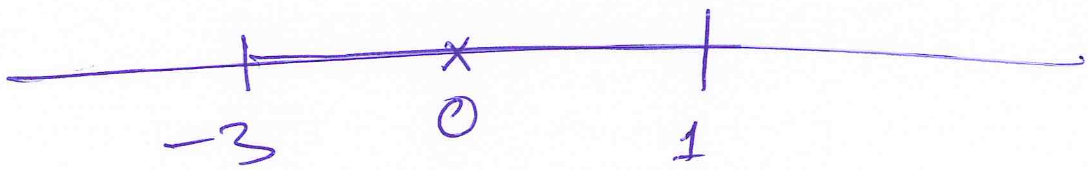
5)

$1-x>0 \qquad\qquad x+3 \geq 0$

$x<1 \qquad\qquad x \geq -3.$

$1-x \neq 1.$

$x \neq 0$



$[-3,0) \cup (0,1)$

6) $\log_{|\sin x|}(x^2-8x+23) - 3\log_{|\sin x|} 2 > 0$ $\quad \cancel{\frac{1}{\log_b a} = \log_a b}$

$\log_{|\sin x|}\left(\frac{(x^2-8x+23)}{8}\right) > 0$

$\frac{x^2-8x+23}{8} < |\sin x|^0 \qquad \log_a b = c$

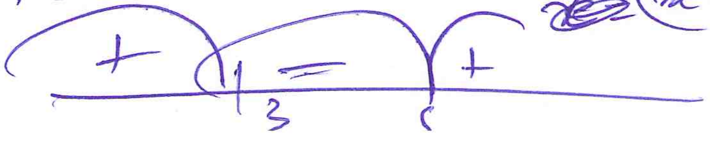
$b = a^c$

$\frac{x^2-8x+23}{8} < 1$

$x^2-8x+23 < 8 \Rightarrow x^2-8x+15<0$

$(x-3)(x-5)<0$

$x \in (3,5)$



$$|\sin x| \neq 1.$$

$$x \neq \pi/2,\ \frac{3\pi}{2},\ \frac{5\pi}{2}$$

$$|\sin x| > 0$$

$$x \neq \pi, 2\pi, 3\pi \ldots$$

$$\underset{3}{|} \quad \underset{\pi}{|} \quad \underset{\frac{3\pi}{2}}{|} \quad \underset{5}{|}$$

$$(3, \pi) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 5\right)$$

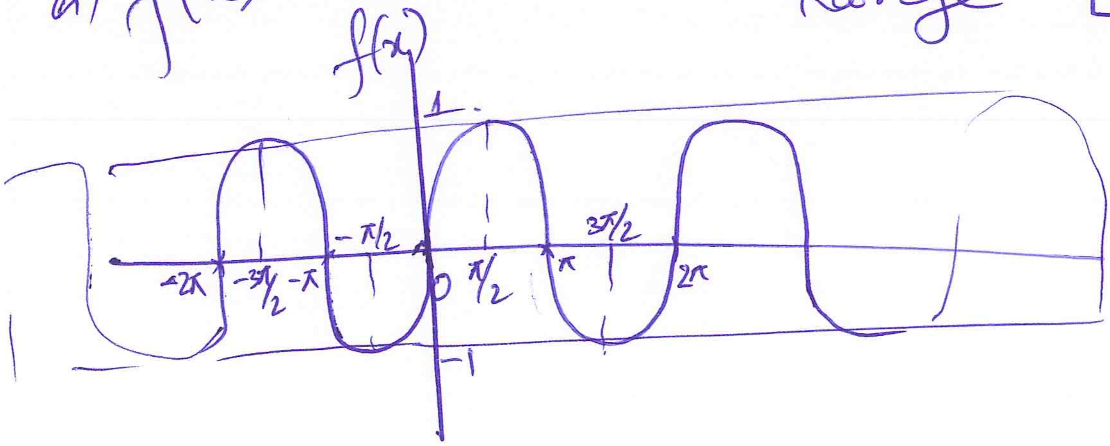
x

vii) Trigonometric functions:

a) $f(x) = \sin x$.

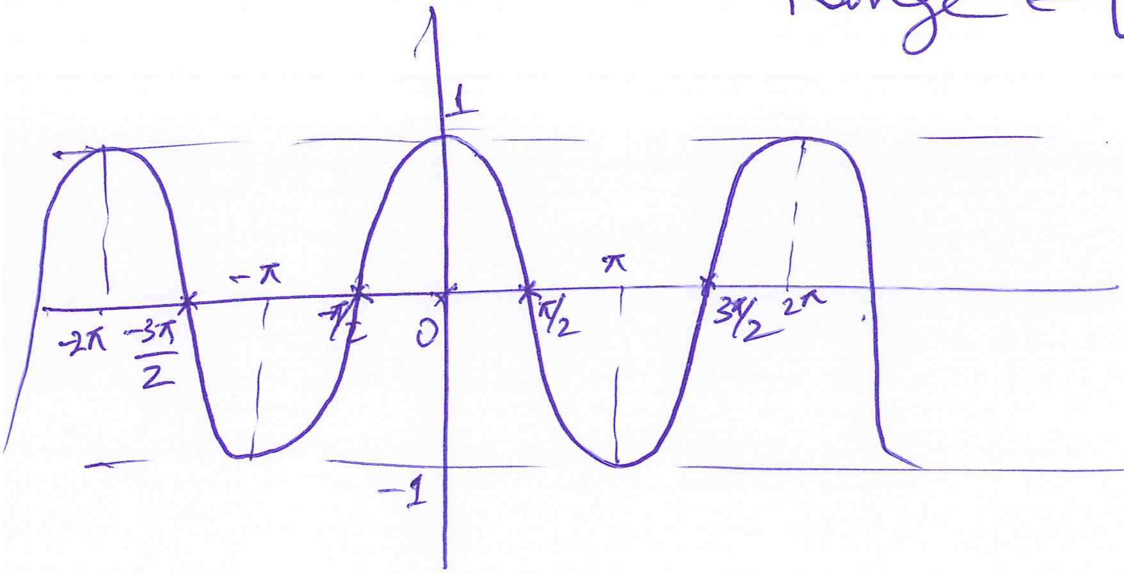
Domain $\in \mathbb{R}$.

Range $[-1, 1]$



b) $f(x) = \cos x$ Domain $\in \mathbb{R}$

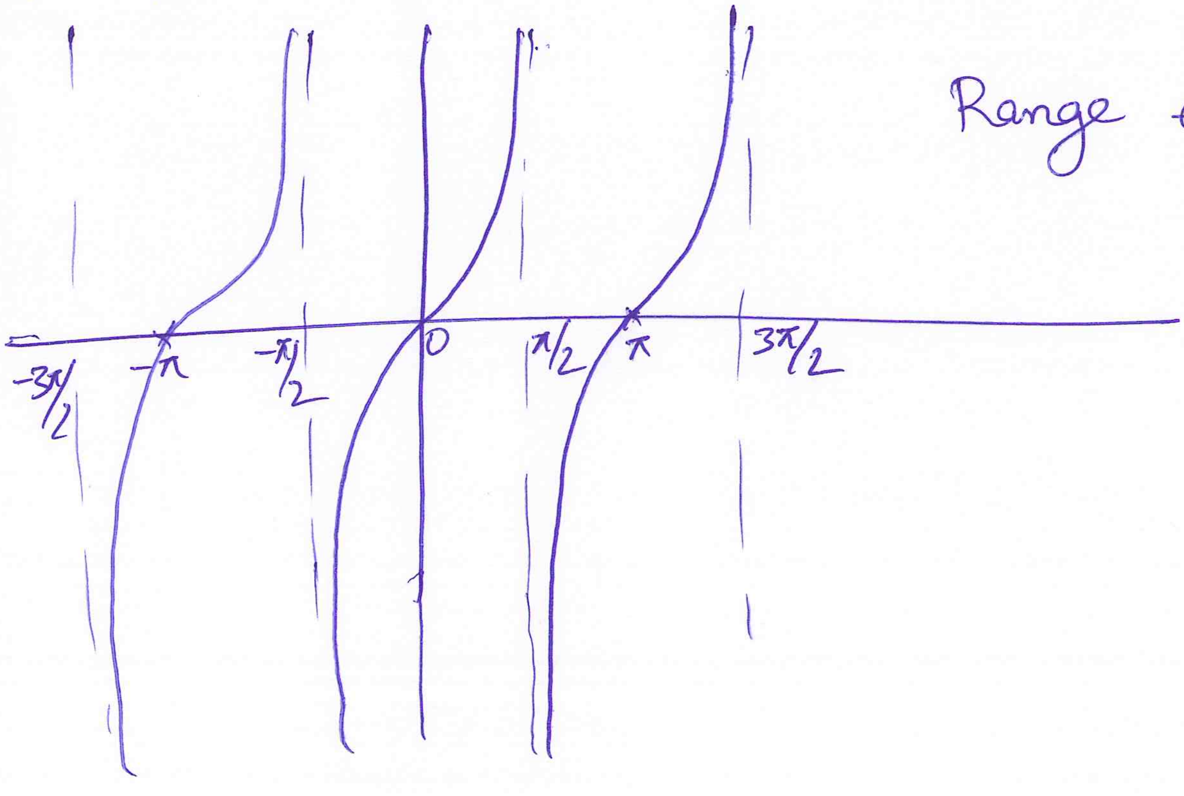
Range $\in [-1, 1]$



c) $f(x) = \tan x$. Domain $\mathbb{R} - \{(2n+1)\frac{\pi}{2}\ n \in Z\}$

Range $\in \mathbb{R}$

$(-\infty, \infty)$



d) $f(x) = \operatorname{Cosec} x = \frac{1}{\sin x}$ Domain $\in \mathbb{R} - \{n\pi\ n \in Z\}$

Range. $\mathbb{R} - (-1, 1)$

e) $f(x) \sec x = \frac{1}{\cos x}$ Domain $\in \mathbb{R} - \{\frac{(2n+1)\pi}{2}\ n \in Z\}$

Range $\mathbb{R} - (-1, 1)$

f) $f(x) = \cot x = \frac{1}{\tan x}$ Domain. $\in \mathbb{R} - \{n\pi, n \in Z\}$

Range $\mathbb{R}$