CENTRE OF MASS & MOMENT OF INERTIA

TUTORIAL

SOLVED EX.

Pg 67 - 68

1, 3, 4, 7, 8

Pg 70-71

Comprehension & Match Type.

Pg 76 - 77

8, 9, 10

UNSOLVED EX.

Pg 79 - 82

4, 8, 10, 13, 17, 18, 22, 25, 28, 29, 31

Pg 83 - Comprehension

Pg 85 - Matrix.

g 79-82

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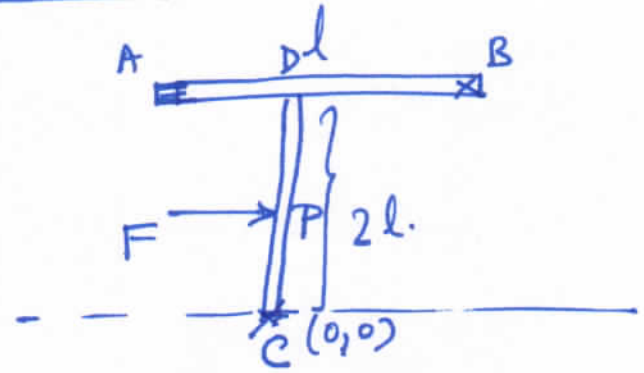


mass / unit length.

$m_{AB} = \lambda l$ — $cm_{AB} = (0, 2l)$

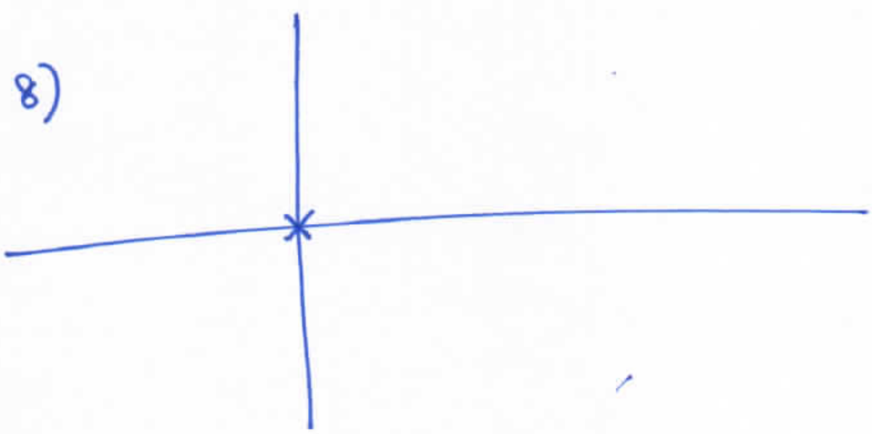
$m_{CD} = \lambda(2l)$ — $cm_{CD} = (0, l)$

$$x_{cm} = \frac{m_{AB}x_{AB} + m_{CD}x_{CD}}{m_{AB} + m_{CD}} = \frac{\lambda l \times 0 + 2\lambda l \times 0}{\lambda l + 2\lambda l} = 0$$

$$y_{cm} = \frac{m_{AB}y_{AB} + m_{CD}y_{CD}}{m_{AB} + m_{CD}} = \frac{\lambda l(2l) + 2\lambda l(l)}{\lambda l + 2\lambda l} = \frac{4\lambda l^2}{3\lambda l} = \frac{4l}{3}$$

$P\left(0, \frac{4l}{3}\right)$ (A)

8)



All NON-ZERO.

10) $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 \cdots}{m_1 + m_2 + m_3 \cdots}$

$v_{cm} = \dfrac{m_1v_1 + m_2v_2 + m_3v_3 \cdots}{m_1 + m_2 + m_3 \cdots}$

$$v_{CM} = \frac{mv + Mv}{m+M} = v \checkmark$$

$$v_{CM} = ?$$

$$v_{CM} = \frac{m(v'+u) + Mv'}{M+m} \checkmark$$

$$Mv + mv = Mv' + m(v'+u)$$

$$v_{CM} = \frac{Mv + mv}{M+m} = v \checkmark$$

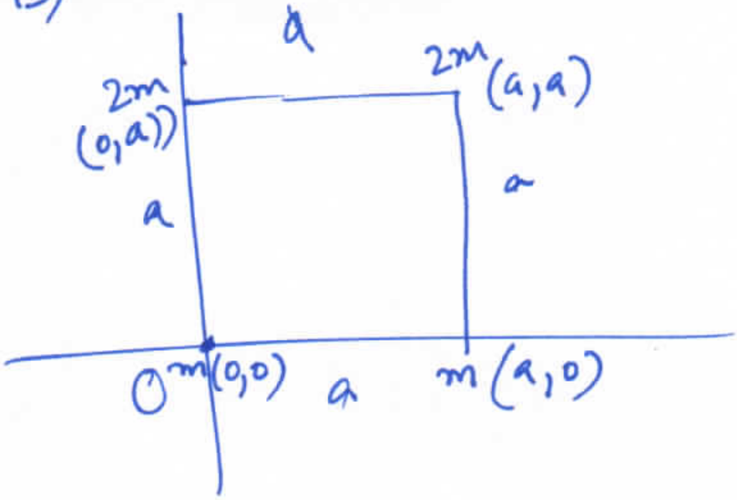
If no external force is acting on the system the ~~initial~~ final velocity of CM is same as the initial velocity.

A boat with a boy is standing stationary at the bank of the river. The boy starts walking towards the bank with a speed v relative to the ground. Assuming speed of water near the bank is zero. With what velocity will the boat move

$$v_{CM} = ? = 0 \quad \frac{m_b \times 0 + m_B \times 0}{m_b + m_B} = 0$$

$$u = -\frac{m_b}{M_B} v \Leftarrow v_{CM} = 0 = \frac{m_b \times v + m_B \times u}{m_b + m_B}$$

13)



$$x_{cm} = \frac{m\times 0 + 2m\times 0 + 2m\times a + m\times a}{m+2m+2m+m}$$

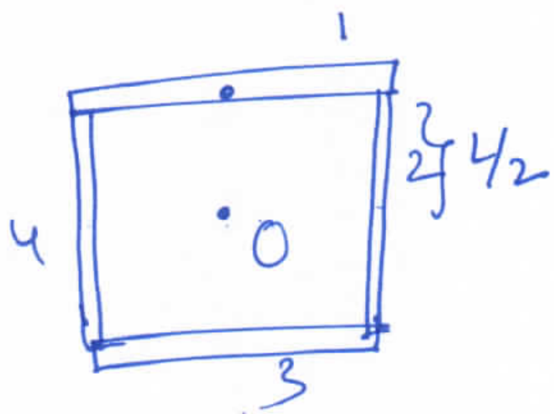
$$= \frac{3ma}{6m} = a/2$$

$$y_{cm} = \frac{m\times 0 + 2m\times a + 2m\times a + m\times 0}{6m}$$

$$= \frac{4\cancel{m}a}{6\cancel{m}} = \frac{2a}{3}$$

$\left(\frac{a}{2}, \frac{2a}{3}\right)$ (e)

(18)



(1) $\frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$

$= \frac{ML^2}{3}$

T. M. I $= 4\frac{ML^2}{3}$ (A)

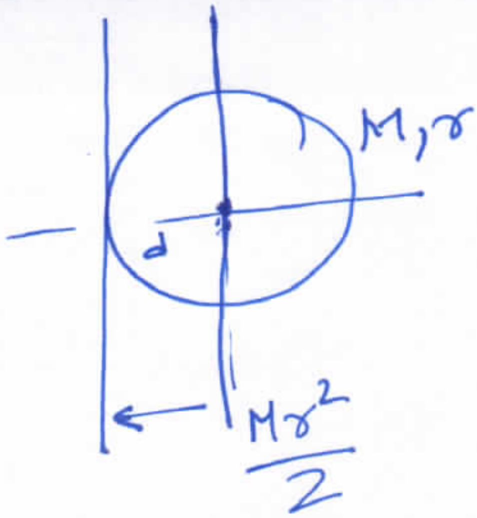
(18) $V/2$ Iron $V/2$ Alum.

$\rho_i > \rho_a$

M.I ↑ as distance of mass from axis ↑

(B)

(22)



$$I_{AXIS} = Mr^2 = I_d + I_d$$

$$Mr^2 = 2I_d$$

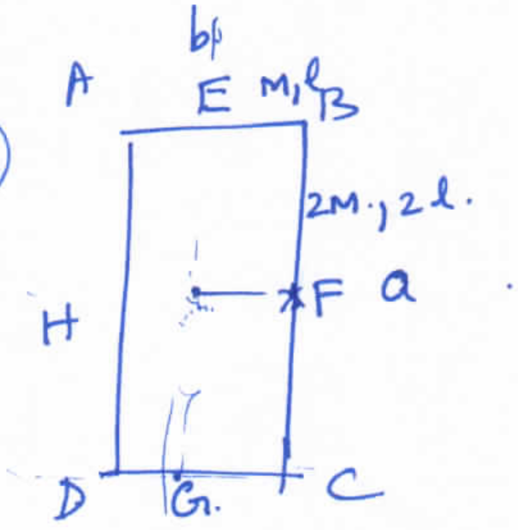
$$I_d = \frac{Mr^2}{2}$$

$$I_T = I_c + Md^2$$

$$= \frac{Mr^2}{2} + Mr^2$$

$$= \frac{3Mr^2}{2} \quad \text{(B)}$$

(25)



$$a = 2b.$$

$$I. = I_{EG} + I_{HF}$$

$$= I_{DB} + I_{AC} \times$$

$$I_{EG} = 2(B\,C + D\,C)$$

$$= 2\left(2M\left(\frac{l}{2}\right)^2 + \frac{M(l)^2}{12}\right) \quad \text{(D)}$$

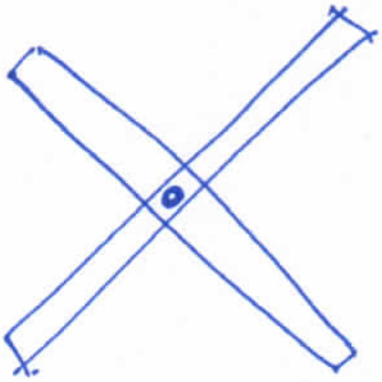
$$= 2\left(\frac{Ml^2}{2} + \frac{Ml^2}{12}\right) = \frac{14Ml^2}{12} = \frac{7Ml^2}{6}$$

$$I_{HF} = 2(BC + DC)$$

$$= 2\left(\frac{2M(2l)^2}{12} + Ml^2\right) = 2\left(\frac{2Ml^2}{3} + Ml^2\right)$$

$$= 10Ml^2/3$$

(28)



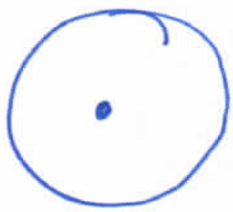
$$\frac{Ml^2}{12} + \frac{Ml^2}{12}$$

$$= \frac{Ml^2}{6}.$$

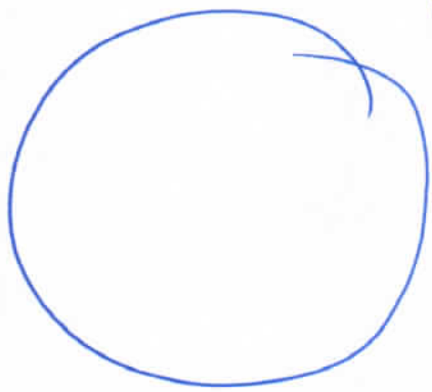
$M \propto 2\pi R$

$M' \propto 2\pi n R$

(29)



R.



nR

$M' = nM$

$I_1 = MR^2$

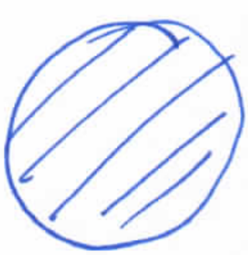
$I_2 = M'(nR)^2$

$= nMn^2R^2 = n^3MR^2$

$$\frac{I_1}{I_2} = \frac{1}{8} = \frac{\cancel{MR^2}}{n^3\cancel{MR^2}}$$

$n = 2$

(31)

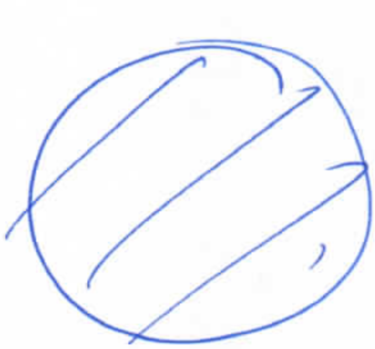


R, t

I_x

$M_x = \rho \times \pi R^2 t$

$I_x = \frac{M_x R^2}{2}$



$4R,\ t/4$

I_y

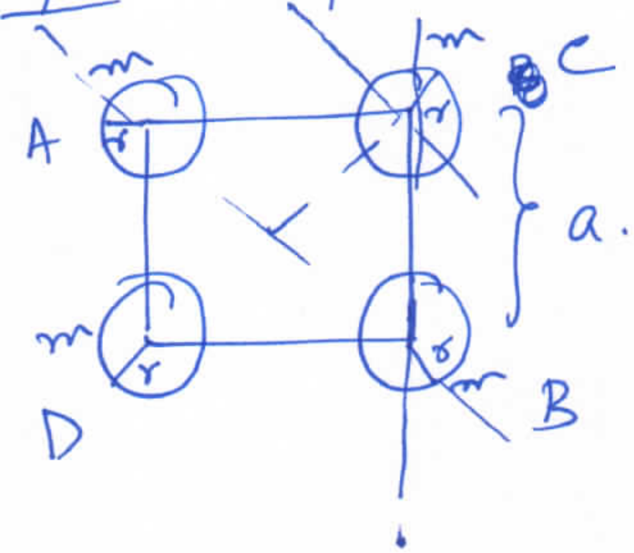
(A)

$M_y = \rho \times \pi(4R)^2 \times t/4$

$I_y = \frac{M_y (4R)^2}{2}$

$$\frac{I_x}{I_y} = \frac{\frac{M_x R^2}{2}}{\frac{16 M_y R^2}{2}} = \frac{M_x}{16 M_y} = \frac{1}{64}$$

Pg 83 Comprehension



(1) $\frac{2}{5} m r^2$ (B)

(2) $\frac{2}{5} m r^2 + m\left(\frac{a}{\sqrt{2}}\right)^2$

$= \frac{2}{5} m r^2 + \frac{m a^2}{2}$

$= m\left(\frac{2}{5} r^2 + \frac{a^2}{2}\right)$

(D)

(3) $\frac{2}{5} m r^2 + m a^2$ (B)

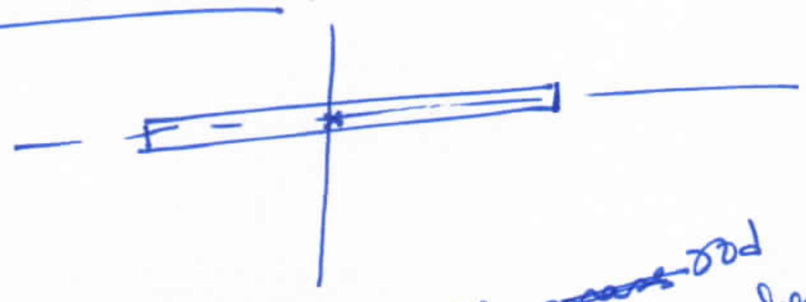
(4) $\frac{2}{5} m r^2$

(5) 2 (Q1 + Q2)

(6) 2 (Q3 + Q4)

Pg 85 Matrix.

(A)

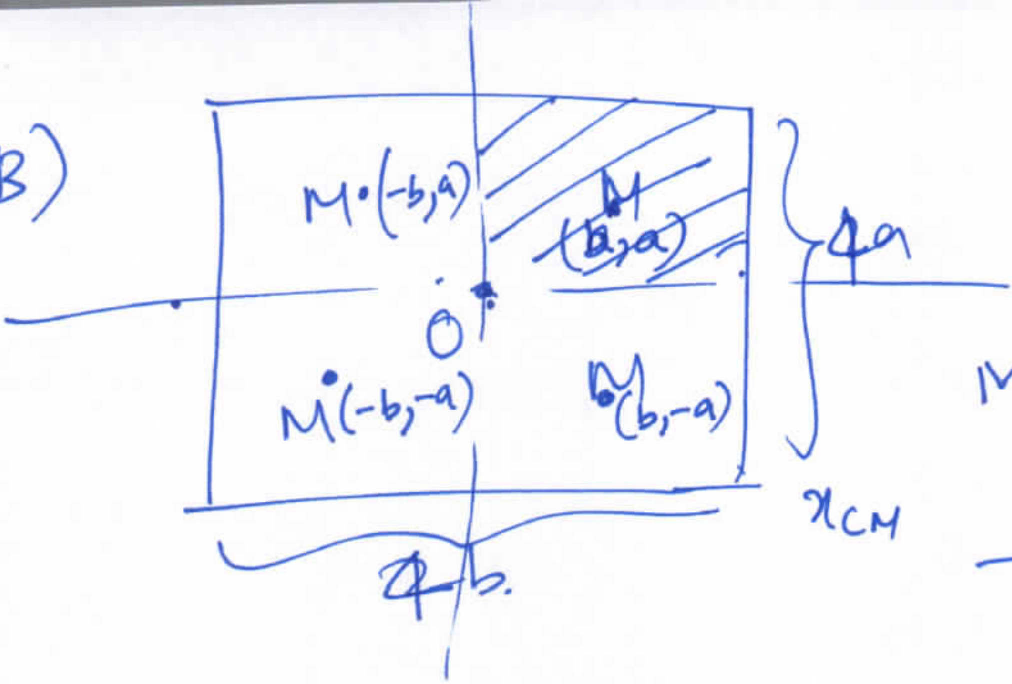
(A) 

$\because$ Mass Not Uniform

$\therefore$ CM cannot be at center of rod. (which is origin here)

And since the ~~mass~~ rod is along x axis CM will be ~~somewhere~~ on x axis

B)



3rd quad..

$$x_{CM} = \frac{M(-b) + \cancel{M(-b)} + \cancel{M(b)}}{3M}$$

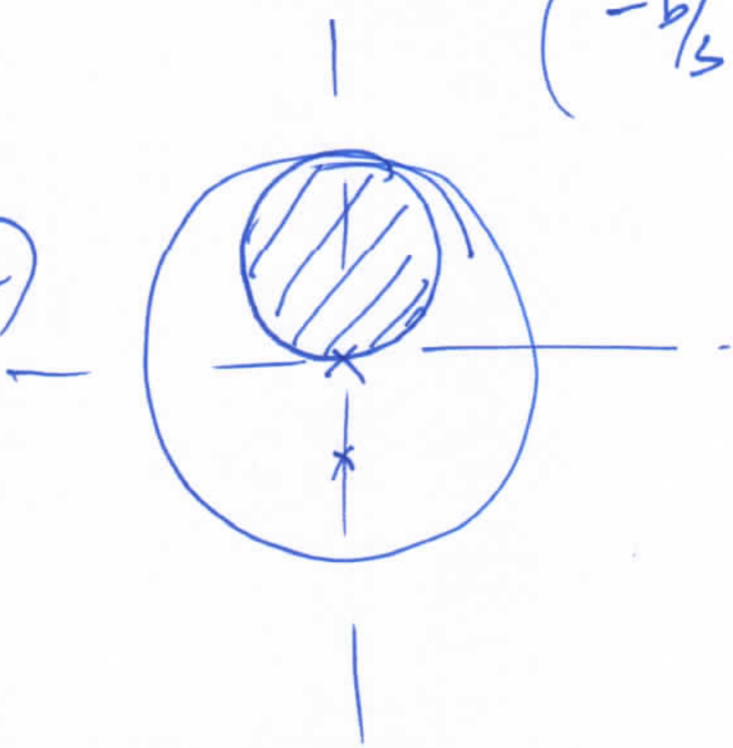
$$= -b/3$$

(S) $$y_{CM} = \frac{M(a) + M(-a) + M(-a)}{3M}$$

$$= -a/3$$

$\left(-b/3, -a/3\right)$ 3rd quadrant.

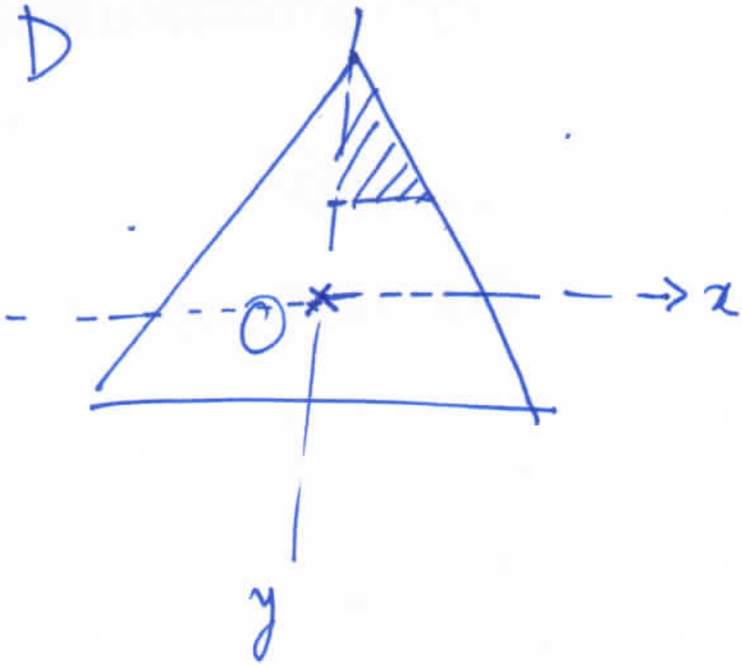
c)



(A) y axis

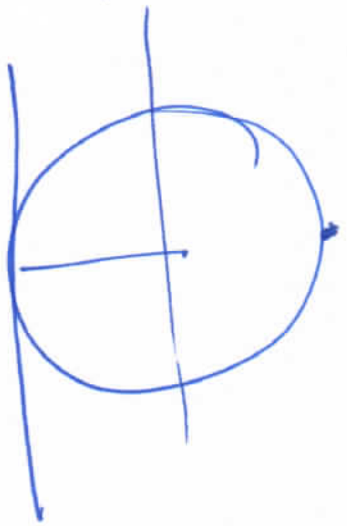
$-y$.

D



$\longrightarrow$ S

2



$$I_{axis} = \frac{MR^2}{2}$$

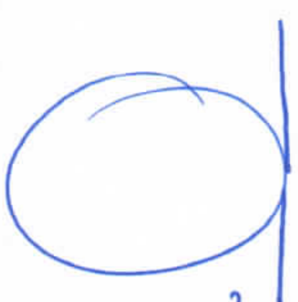
$$I_d = \frac{MR^2}{4}$$

$$I_T = \frac{MR^2}{4} + MR^2$$

$$= \frac{5MR^2}{4}$$

$$\cancel{M}k^2 = \frac{5\cancel{M}R^2}{4} \Rightarrow k = \frac{R}{2}\sqrt{5}.$$

A $\longrightarrow$ S



$$\frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

$$Mk^2 = \frac{3}{2}MR^2$$

$$k = \sqrt{\frac{3}{2}}\,R.$$

A $\longrightarrow$ R.