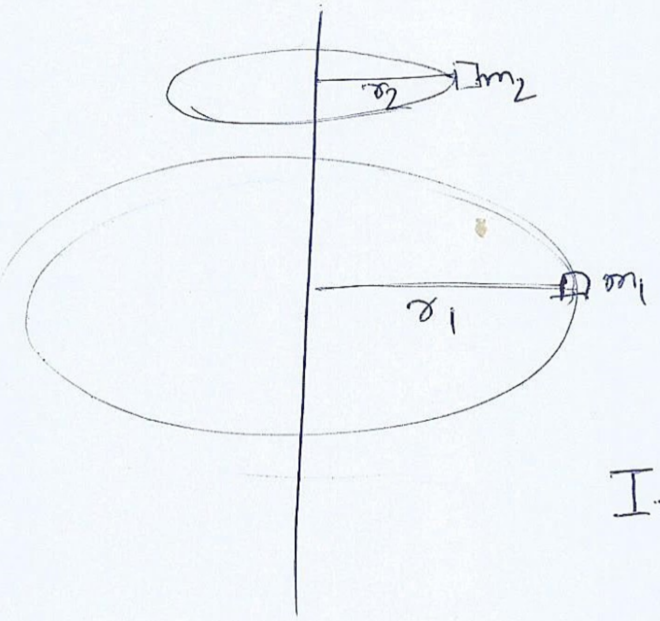
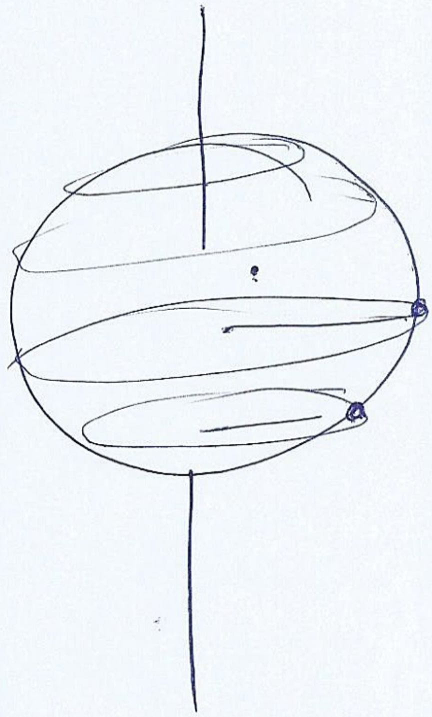
MOMENT OF INERTIA

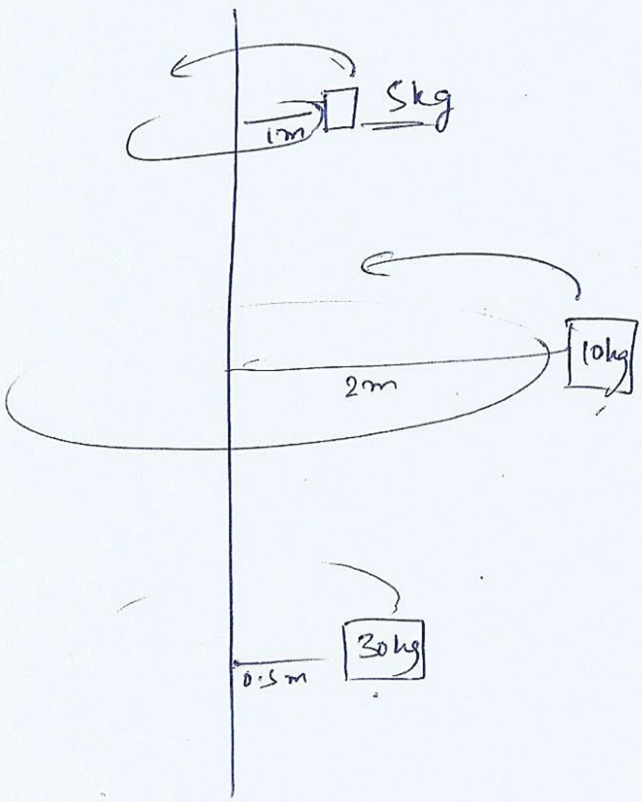


$$I = mr^2$$

$$I_{TOT} = I_1 + I_2$$

$$= m_1 r_1^2 + m_2 r_2^2$$



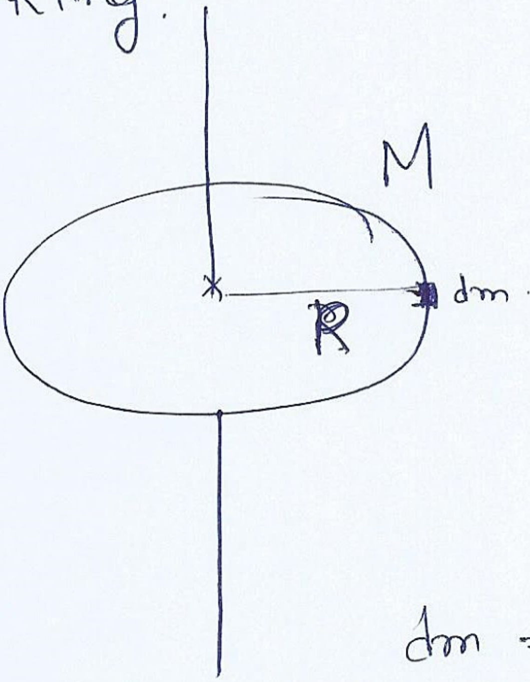


$$I = 30 \times 0.5^2 + 10 \times 2^2 + 5 \times 1^2$$

$$= 52.5 \ kg\,m^2$$

lets find Moment of Inertia of rigid bodies

① Ring.



$$\int dI = \int dm\, R^2$$

$$I = MR^2$$

$$dm = \frac{dx}{2\pi R} \times M.$$

$$\int dI = \int_0^{2\pi R} \left(\frac{dx\, M}{2\pi R}\right) \times R^2$$

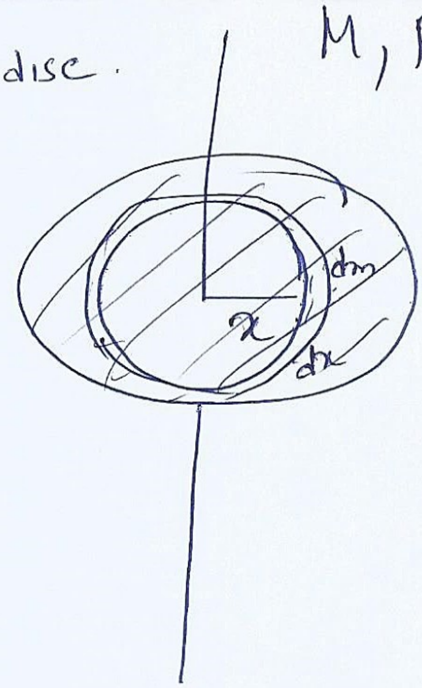
$$I = \left.\frac{x\, R^2 M}{2\pi R}\right]_0^{2\pi R.}$$

$$= 2\pi R \left(\frac{R^2 M}{2\pi R}\right) - \underbrace{0\left(\frac{R^2 M}{2\pi R}\right)}_{0}$$

$$\downarrow$$

$$MR^2$$

2) disc. $M, R.$



$$dm = \frac{M}{\pi R^2} \times 2\pi x \, dx$$

$$dI = dm \, x^2$$

$$I = \int_0^R dm \, x^2$$

$$= \int_0^R \frac{M}{\cancel{\pi} R^2} 2\cancel{\pi} x \, dx \cdot x^2$$

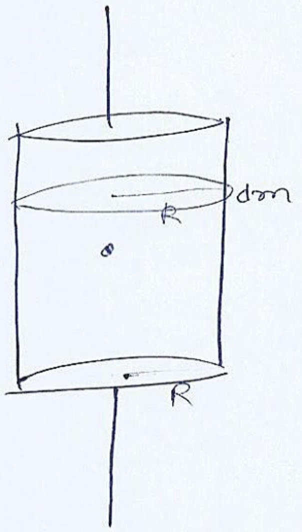
$$= \frac{2M}{R^2} \int_0^R x^3 \, dx$$

$$= \frac{2M}{R^2} \left. \frac{x^4}{4} \right]_0^R$$

$$= \frac{2M}{R^2} \left(\frac{R^4}{4} - \frac{0^4}{4} \right)$$

$$= \frac{\cancel{2}M}{\cancel{R^2}} \times \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} = \frac{MR^2}{2}$$

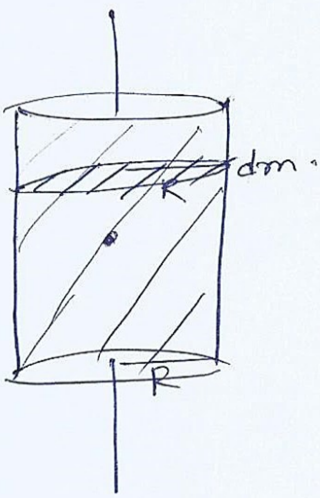
(3) Hollow Cylinder of radius R & Mass M.



$$\int dI = \int dm\, R^2$$

$$I = MR^2$$

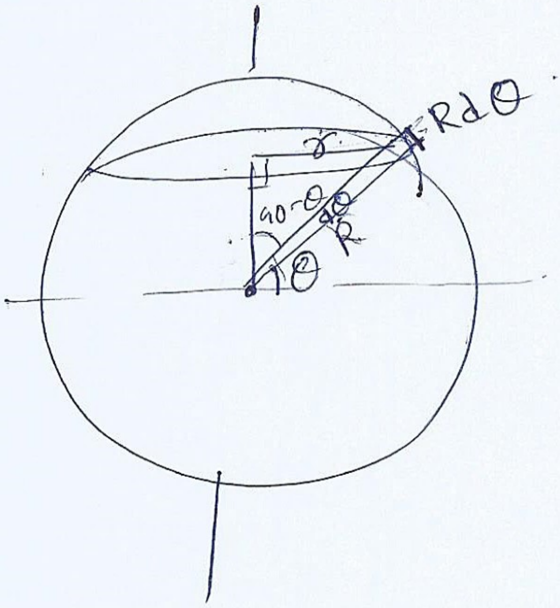
(4) Solid Cylinder of radius R & Mass M.



$$\int dI = \int \frac{dm\, R^2}{2}$$

$$I = \frac{MR^2}{2}$$

⑤ Hollow Sphere of radius R & Mass M.



$$dm = \frac{M}{4\pi R^2} \times 2\pi r R d\theta.$$

$$= \frac{M}{\cancel{4\pi R^2}_{2}} \cancel{2\pi} \cancel{R} \cos\theta \cancel{R} d\theta.$$

$$= \frac{M}{2} \cos\theta \, d\theta.$$

$$\frac{r}{R} = \sin(90-\theta) \Rightarrow r = R\cos\theta.$$

$$\int dI = \int dm \, r^2$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{M}{2} \cos\theta \, d\theta \, (R\cos\theta)^2$$

$$= \int_{-\pi/2}^{\pi/2} \frac{MR^2}{2} \cos^3\theta \, d\theta.$$

$$= \frac{MR^2}{2} \int_{-\pi/2}^{\pi/2} (1-\sin^2\theta) \underline{\cos\theta \, d\theta}$$

$\sin\theta = t$

$\cos\theta d\theta = \underline{dt}$

$$= \frac{MR^2}{2} \int_{-1}^{1} (1-t^2) dt$$

$$= \frac{MR^2}{2}\left[t - \frac{t^3}{3}\right]_{-1}^{1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$= \frac{MR^2}{2}\left(\left(1 - \frac{1^3}{3}\right) - \left(-1 - \frac{(-1)^3}{3}\right)\right)$$

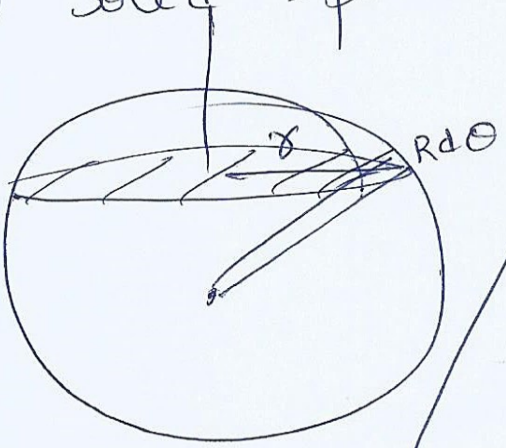
$t^0 \qquad \frac{t^{0+1}}{0+1} = t$

$$= \frac{MR^2}{2}\left[\frac{2}{3} - \left(\frac{-2}{3}\right)\right]$$

$t^2 \qquad \frac{t^{2+1}}{2+1} = \frac{t^3}{3}.$

$$= \frac{MR^2}{\cancel{2}}\left[\frac{4^2}{3}\right] = \boxed{\frac{2MR^2}{3}}$$

(6) Solid Sphere of radius R, Mass M.



$$dm = \frac{M}{\frac{4}{3}\pi R^3} \times (\pi r^2 R\, d\theta)$$

$$dm = \frac{3M}{4}\cos^2\theta\, d\theta$$

$r = R\cos\theta$

$$\int dI = \int \frac{dm\, r^2}{2}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{3M}{4}\cos^2\theta\, d\theta \times \frac{(R\cos\theta)^2}{2}$$

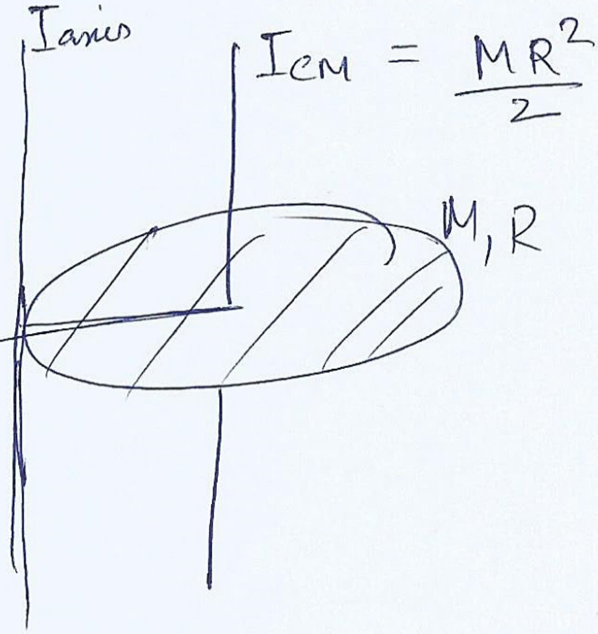
$$= \frac{3MR^2}{8}\int_{-\pi/2}^{\pi/2}\cos^4\theta\, d\theta = \frac{2}{5}MR^2$$

Theorem's

1) Theorem of $||^{le}$ axis

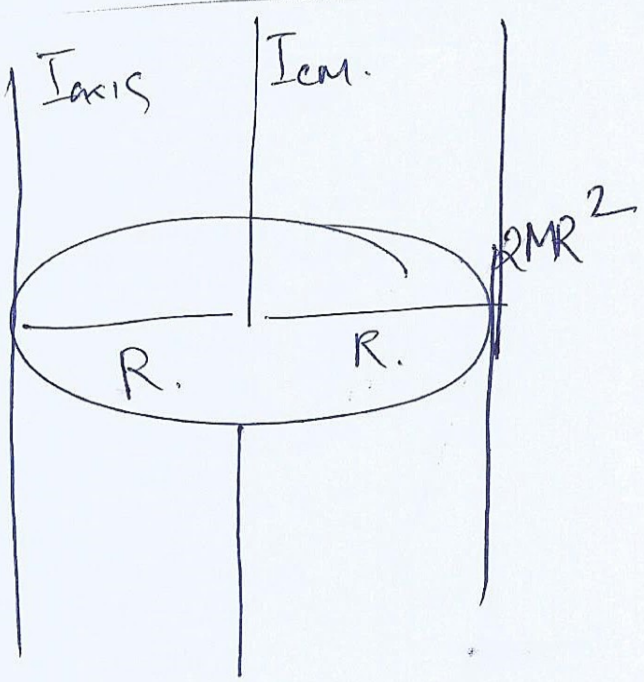
$$I_{||axis} = I_{CM} + Mx^2$$

distance between CM axis & || axis



$$I_{axis} = \frac{MR^2}{2} + MR^2$$

$$= \frac{3MR^2}{2}.$$



$$2MR^2 = I_{CM} + MR^2$$

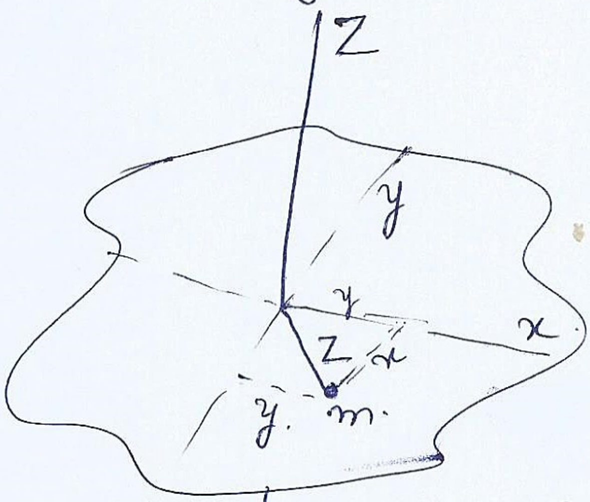
$$I_{CM} = MR^2$$

$$I_{axis} = I_{CM} + MR^2$$

$$= MR^2 + MR^2$$

$$= 2MR^2$$

2) Theorem of $\perp$ axis

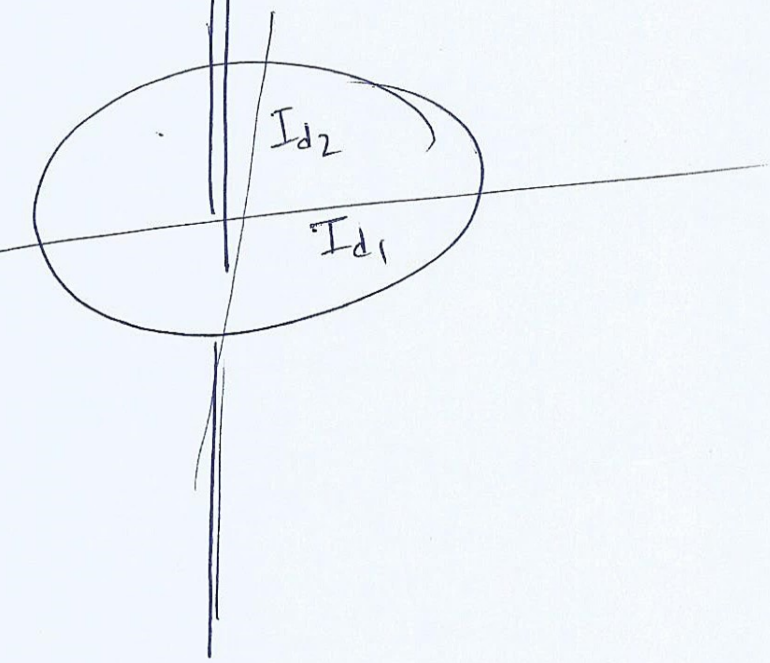


mz^2.

$$mZ^2 = mx^2 + my^2$$

$$I_z = I_x + I_y$$

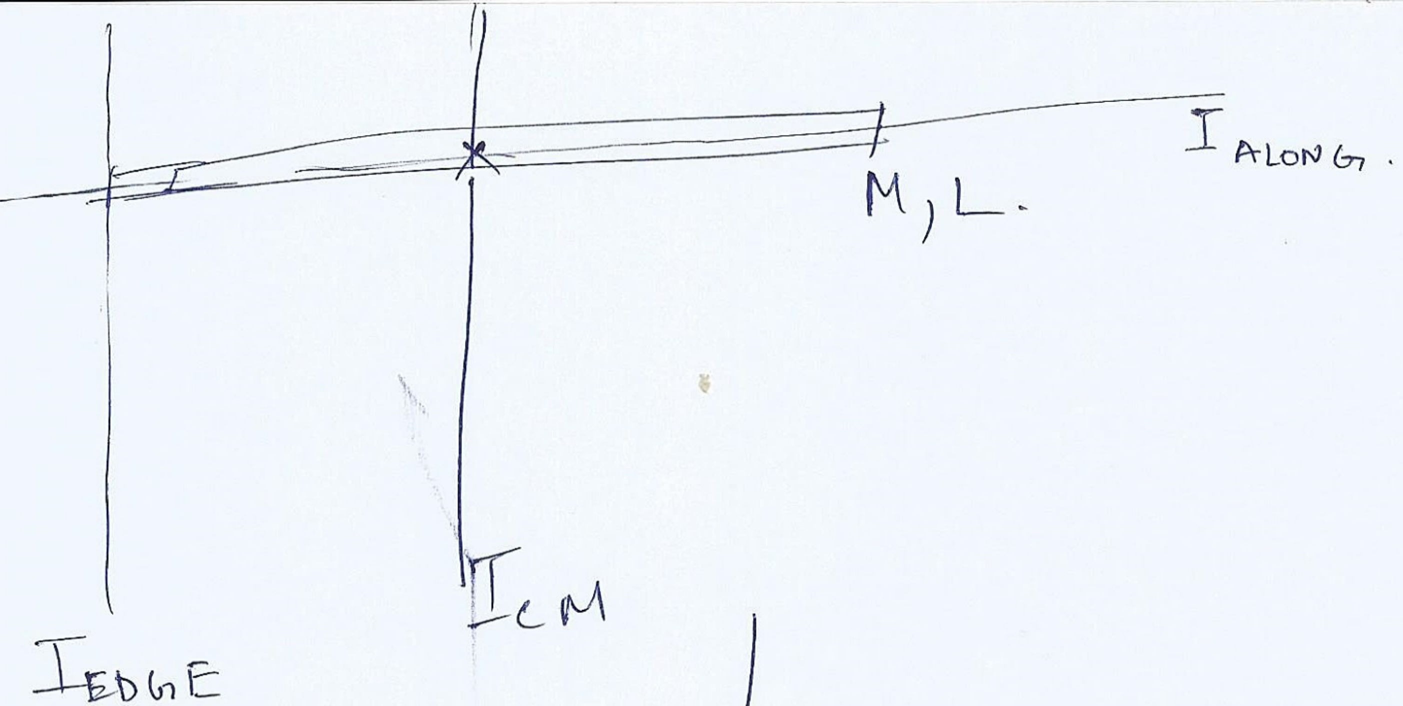
$$I_{cm} = MR^2$$

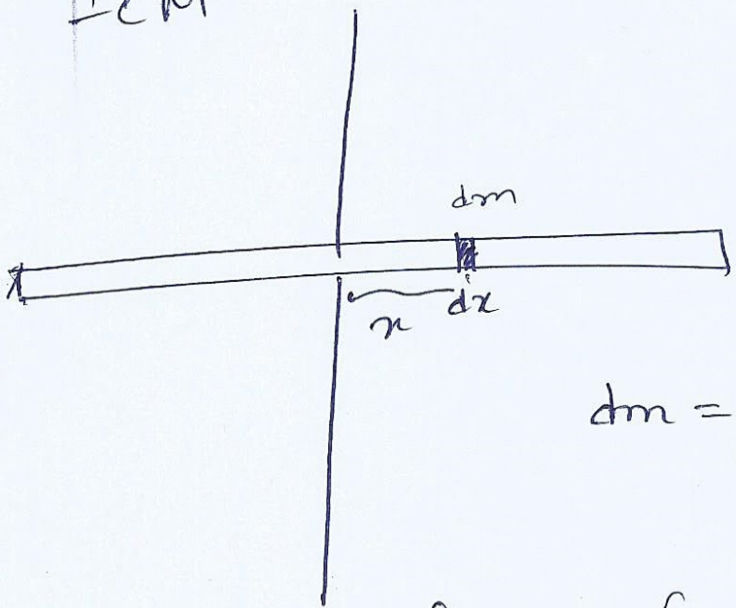


$$I_{d_1} + I_{d_2} = I_{cm} = MR^2$$

$$2I_d = MR^2$$

$$I_d = \frac{MR^2}{2}.$$





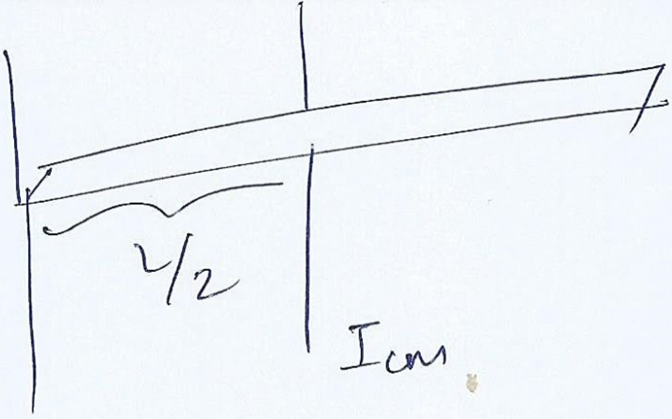
$$dm = \frac{M}{L} \times dx$$

$$\int dI = \int dm\, x^2$$

$$I = \int_{-L/2}^{L/2} \frac{M}{L}\, dx\, x^2$$

$$= \frac{M}{L} \left. \frac{x^3}{3} \right]_{-L/2}^{L/2}$$

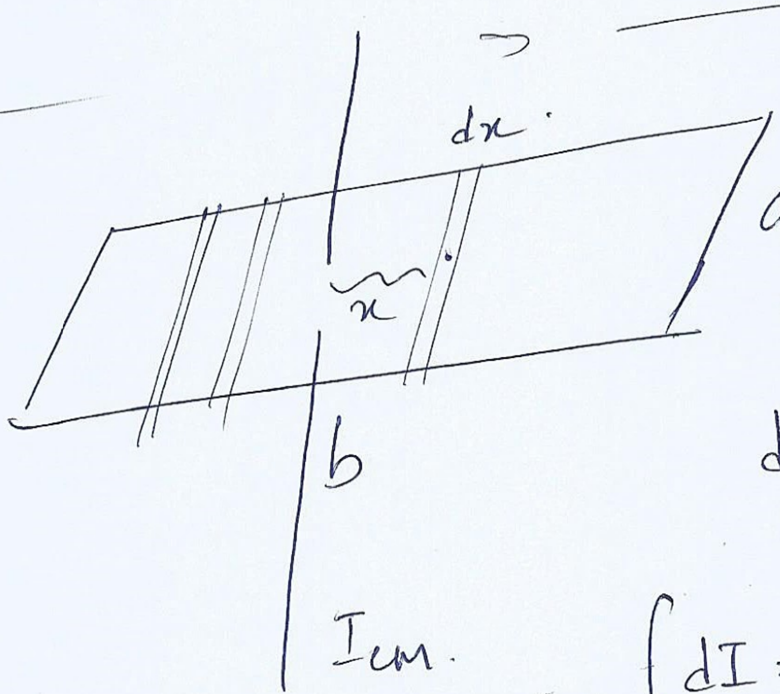
$$I_{CM} = \frac{ML^2}{12}$$



$$I_{EDGE} = I_{cm} + M\left(\frac{L}{2}\right)^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$$

$I_{ALONG} = 0$



$$dm = \frac{M}{\cancel{a}b} \times \cancel{a}\,dx$$

$$dI = \frac{dm\,a^2}{12} + dm\,x^2$$

$$dI = \frac{M\,dx\,a^2}{12b} + \frac{M\,dx\,x^2}{b}$$

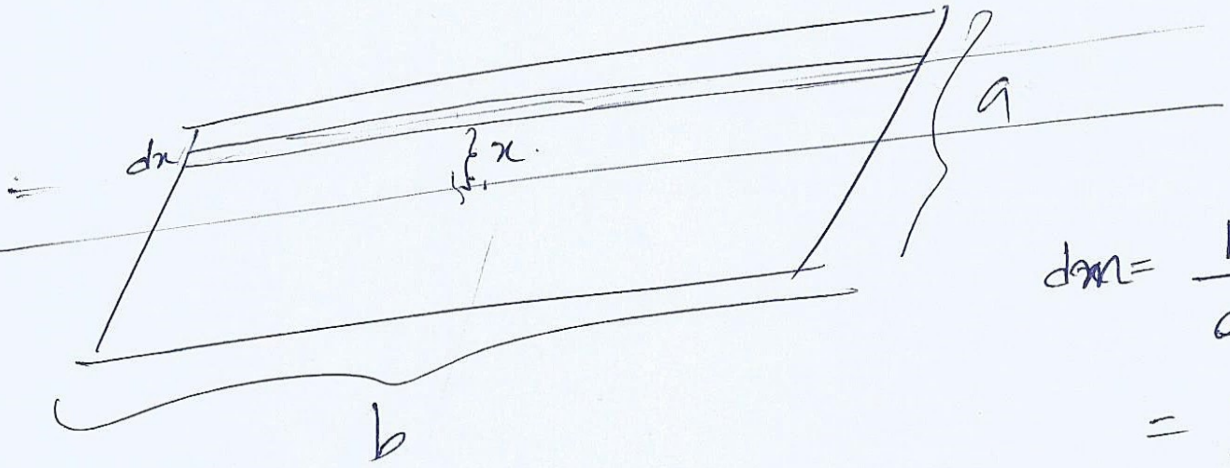
$$\int dI = \frac{M}{b}\int_{-b/2}^{b/2}\left(\frac{a^2}{12}dx + x^2 dx\right)$$

$$I = \frac{M}{b}\left[\frac{a^2}{12}x + \frac{x^3}{3}\right]_{-b/2}^{b/2}$$

$$I = \frac{M}{b}\left[\frac{a^2}{12}b + \frac{b^3}{12}\right]$$

$$= \frac{Ma^2}{12} + \frac{Mb^2}{12}$$

$$= \frac{M}{12}\left(a^2+b^2\right)$$



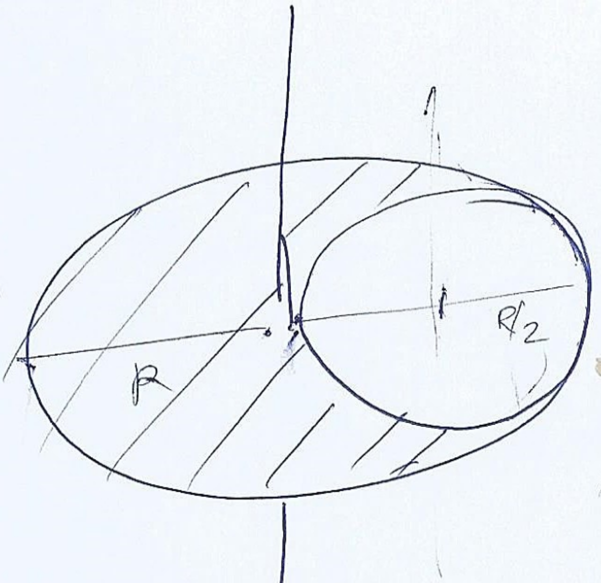
$$dm = \frac{M}{a\cancel{b}} \times \cancel{b}\,dx$$

$$= \frac{M\,dx}{a}$$

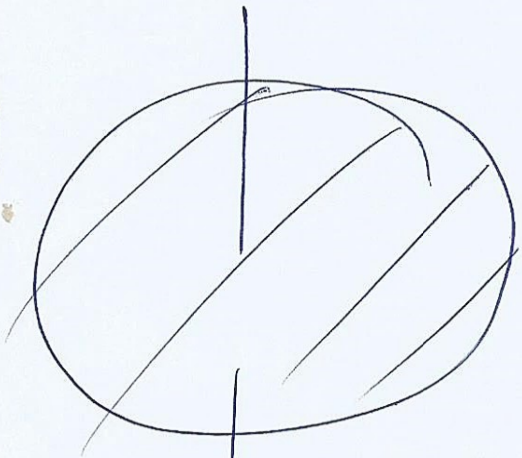
$$dI = 0 + dm\,x^2$$

$$\int dI = \int_{-a/2}^{a/2} \frac{M\,dx}{a} x^2$$

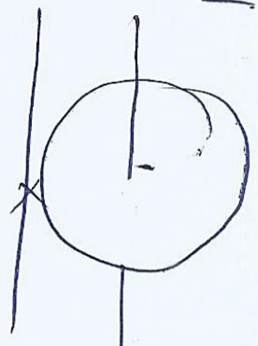
$$= \frac{Ma^2}{12}$$



M, R.



$-\frac{M}{4}, \frac{R}{2}$



$x + \frac{x}{4}$ M.

$$\frac{MR^2}{2} + \frac{-\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \left(-\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$$

$x \propto R^2$

$y \propto \left(\frac{R}{2}\right)^2$

$y = \frac{x}{4}$.

$x - \frac{x}{4} = M$

$x = \frac{4M}{3}$ $\quad y = \frac{M}{3}$