TRIGONOMETRIC EQUATIONS.

$\sin\theta = \sin\alpha$

$\theta = n\pi + (-1)^n\alpha \quad ; \; n \in I \qquad \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos\theta = \cos\alpha.$

$\theta = 2n\pi \pm \alpha \quad ; \; n \in I \qquad \alpha \in [0, \pi]$

$\tan\theta = \tan\alpha$

$\theta = n\pi + \alpha \quad ; \; n \in I \qquad \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\sin^2\theta = \sin^2\alpha \qquad 2\sin^2\theta = 1 - \cos 2\theta$

$\frac{1-\cos 2\theta}{2} = \frac{1-\cos 2\alpha}{2}.$

$\cos 2\theta = \cos 2\alpha$

$2\theta = 2n\pi \pm 2\alpha$

$\theta = n\pi \pm \alpha \quad ; \; n \in I \qquad \alpha \in \left[0, \frac{\pi}{2}\right]$

$\cos^2\theta = \cos^2\alpha \qquad 2\cos^2\theta = 1 + \cos 2\theta$

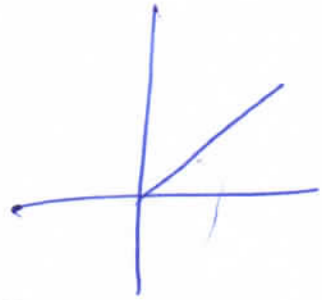
$\theta = n\pi \pm \alpha \quad ; \; n \in I \qquad \alpha \in [0, \pi/2]$

$\tan^2\theta = \tan^2\alpha$

$\theta = n\pi + \alpha \quad ; \; n \in I \qquad \alpha \in [0, \pi/2)$

(1) SIMULTANEOUS EQ.

$\sin\theta = -1/2$ & $\tan\theta = \frac{1}{\sqrt{3}}$



Step 1 Find θ

satisfying $0 \le \theta \le 2\pi$

$\sin\theta = -1/2$ -ve

$\tan\theta = \frac{1}{\sqrt{3}}$ +ve

3rd Quadrant.

$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

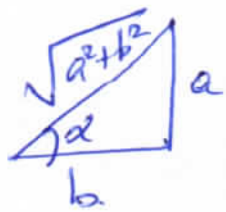
Step 2 Add $2n\pi$ to sol^n in step 1

to get general solution.

$\theta = 2n\pi + \frac{7\pi}{6}$

(2) SOLVING EQ OF TYPE

$a\cos\theta + b\sin\theta = c$



$\frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$

$\tan\alpha = \frac{a}{b}.$

$\sin\alpha\cos\theta + \cos\alpha\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$

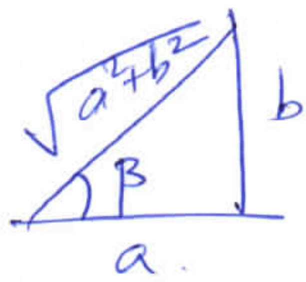
$\sin(\theta+\alpha) = \frac{c}{\sqrt{a^2+b^2}}$

$\alpha = \tan^{-1}\frac{a}{b}.$

$$-1 \le \frac{c}{\sqrt{a^2+b^2}} \le 1$$

$$-\sqrt{a^2+b^2} \le c \le \sqrt{a^2+b^2}$$

$$\frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$



$$\cos\beta\cos\theta + \sin\beta\sin\theta = \frac{c}{\sqrt{a^2+b^2}} \qquad \tan\beta = \frac{b}{a}.$$

$$\cos(\theta-\beta) = \frac{c}{\sqrt{a^2+b^2}}$$

$$\cos(\theta-\beta) = \cos\left(\cos^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)\right)$$

$$\theta - \beta = 2n\pi \pm \cos^{-1}\frac{c}{\sqrt{a^2+b^2}}$$

$$\theta = \beta + 2n\pi \pm \cos^{-1}\frac{c}{\sqrt{a^2+b^2}}$$

eg. $\sin\theta + \sqrt{3}\cos\theta = \sqrt{2}$

$$\frac{1}{\sqrt{1^2+(\sqrt{3})^2}}\sin\theta + \frac{\sqrt{3}}{\sqrt{1^2+(\sqrt{3})^2}}\cos\theta = \frac{\sqrt{2}}{\sqrt{1^2+(\sqrt{3})^2}} \Rightarrow \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta = \frac{1}{\sqrt{2}}$$

$\sin\alpha = \frac{1}{2} \qquad \alpha = 30^\circ = \pi/6$

$$\sin\alpha\sin\theta + \cos\alpha\cos\theta = \frac{1}{\sqrt{2}}$$

$$\cos(\theta-\alpha) = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right)$$

$$\theta-\alpha = 2n\pi \pm \frac{\pi}{4}$$

$$\theta = (2n\pi+\alpha) \pm \frac{\pi}{4}$$

$$= \left(2n\pi+\frac{\pi}{6}\right) \pm \frac{\pi}{4}$$

Q If α, β are distinct solutions of $a\cos\theta + b\sin\theta = c$

Find $\tan\left(\frac{\alpha+\beta}{2}\right)$, $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$

Q2 If the angles A & B of ΔABC satisfies eq's $3\sin A + 4\cos B = 6$ & $4\sin B + 3\cos A = 1$

find 3^{rd} angle C.

Q3 If $0 < x < \pi/2$ & $\sin(x+28^\circ) = \cos(3x-78^\circ)$ find x?

P.T. $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

$\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$\sin 2\theta = \frac{2\sin\theta\cos\theta\,(\sec^2\theta)}{(\sec^2\theta)}$$

$$= \frac{2\,\frac{\sin\theta}{\cos\theta}}{1+\tan^2\theta}$$

$$= \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \frac{(\cos^2\theta - \sin^2\theta)\sec^2\theta}{\sec^2\theta}$$

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

Ans 1

$$a\cos\alpha + b\sin\alpha = c$$

$$a\cos\beta + b\sin\beta = c$$

$$a(\cos\alpha - \cos\beta) + b(\sin\alpha - \sin\beta) = 0$$

$$a\left(2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)\right) + b\left(2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right) = 0$$

$-\sin\left(\frac{\alpha-\beta}{2}\right)$

$$\sin\left(\frac{\alpha-\beta}{2}\right)\left[-2a\sin\left(\frac{\alpha+\beta}{2}\right) + 2b\cos\left(\frac{\alpha+\beta}{2}\right)\right] = 0$$

$\because \alpha \neq \beta \qquad \sin\left(\frac{\alpha-\beta}{2}\right) \neq 0$

$$-2a\sin\left(\frac{\alpha+\beta}{2}\right) + 2b\cos\left(\frac{\alpha+\beta}{2}\right) = 0$$

$$\cancel{\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a}}$$

$$2a\sin\left(\frac{\alpha+\beta}{2}\right) = 2b\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)} = \frac{2b}{2a}$$

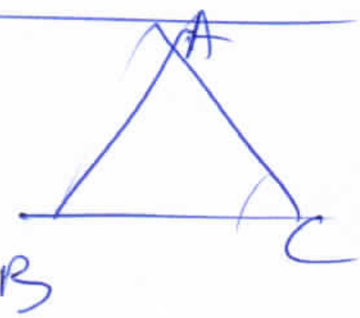
$$\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a}$$

$$\tan C = b/a \qquad C = \frac{\alpha+\beta}{2}$$

$\sin 2C$	$\cos 2C$	$\tan 2C$
$= \dfrac{2\tan C}{1+\tan^2 C}$	$= \dfrac{1-\tan^2 C}{1+\tan^2 C}$	$= \dfrac{2\tan C}{1-\tan^2 C}$
$= \dfrac{2\,b/a}{1+b^2/a^2}$	$= \dfrac{1-\left(\frac{b}{a}\right)^2}{1+\left(b/a\right)^2}$	$= \dfrac{2\,b/a}{1-\left(\frac{b}{a}\right)^2}$
$= \dfrac{2ab}{a^2+b^2}$	$= \dfrac{a^2-b^2}{a^2+b^2}$	$= \dfrac{2ab}{a^2-b^2}$

Ans2

$$C = \pi - (A+B)$$



$$(3\sin A + 4\cos B)^2 = 36$$
$$(4\sin B + 3\cos A)^2 = 1.$$

$$9\sin^2 A + 16\cos^2 B + 24\sin A\cos B + 16\sin^2 B + 9\cos^2 A + 24\cos A\sin B = 37$$

$$9(\sin^2 A + \cos^2 A) + 16(\sin^2 B + \cos^2 B) + 24(\sin A\cos A + \cos A\sin B) = 37$$

$$25 + 24(\sin(A+B)) = 37$$

$$24\sin(A+B) = 12$$

$$\sin(A+B) = \tfrac{1}{2} \qquad A+B = \frac{\pi}{6},\ \frac{5\pi}{6}$$

$A+B = 30^\circ \mid 150^\circ$

$C = 150^\circ / 30^\circ$

If $C = 150^\circ$

$A \leq 30^\circ$ $\quad$ $3\sin 30^\circ + 4\cos 0$

$B = 0^\circ$ $\quad$ $= 3\times\frac{1}{2} + 4$

$= 5.5$

Only solⁿ $C = 30^\circ$

Ans3

If $0 < x < \pi/2$ $\quad$ $\sin\theta = \cos(90-\theta)$

$\sin(x+28^\circ) = \cos(3x-78^\circ)$

$\cos(90^\circ-(x+28^\circ)) = \cos(3x-78^\circ)$

$\cos(62^\circ - x) = \cos(3x-78^\circ)$

$62^\circ - x = 2n\pi \pm (3x-78^\circ)$

$62^\circ - x = 360^\circ(n) + (3x-78^\circ)$

$4x = -360^\circ(n) + 140^\circ$ $\mid$ $n=0$ $\quad$ $\underline{\underline{35^\circ}}$

$x = -90^\circ(n) + 35^\circ$

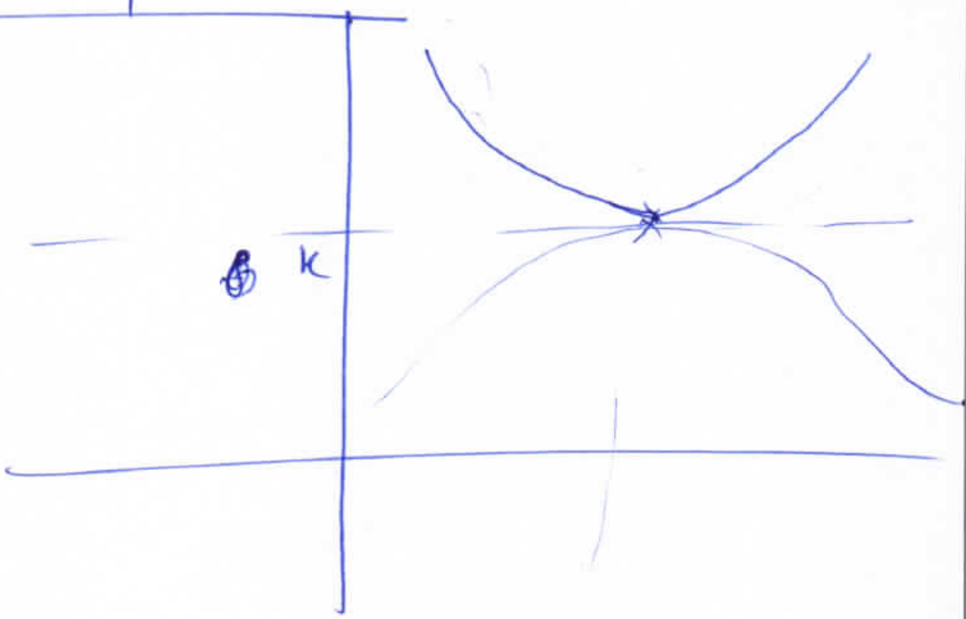
$62^\circ - x = 360^\circ(n) - (3x - 78^\circ)$

$2x = 360^\circ(n) + 16^\circ$

$x = 180^\circ(n) + 8^\circ \quad \Big| \quad n = 0 \quad \underline{\underline{8^\circ}}$

Boundary condition problems

$$\left.\begin{aligned} f(x) &\le k \\ g(x) &\ge k \end{aligned}\right\}$$



$f(x) = k$
$\& \; g(x) = k$

$\sin^6 x = 1 + \cos^4 3x$

$-1 \le \sin x \le 1$

$0 \le \sin^2 x \le 1$

$0 \le \sin^6 x \le 1$

$\sin^6 x = 1$

$\& \; 1 + \cos^4 3x = 1$

$-1 \le \cos x \le 1$

$0 \le \cos^2 x \le 1$

$0 \le \cos^4 3x \le 1$

$1 \le (1 + \cos^4 3x) \le 2$

$\sin^6 x = 1$

$\sin^2 x = 1$

$1 - \cos^2 x = 1$

$\cos^2 x = 0 = \cos^2 \frac{\pi}{2}$

$\left. x = n\pi \pm \frac{\pi}{2} \right\} n \in I \Rightarrow \underline{\underline{\frac{\pi}{2}(2n \pm 1)}}$

$\left(\pi/2, \frac{3\pi}{2},\right.$

$$1 + \cos^4 3x = 1$$

$$\cos^4 3x = 0$$

$$\cos^2 3x = 0 = \cos^2 \pi/2.$$

$$3x = n\pi \pm \pi/2$$

$$x = \frac{n\pi}{3} \pm \frac{\pi}{6} \quad ; n \in I.$$

$$x = \frac{\pi}{6}(2n \pm 1) \text{ --- } \left(\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \ldots\right)$$

$$x = \frac{\pi}{2}(2n + 1) \quad \text{Sol}^n.$$

$$\left.\begin{array}{l} y = n \\ y = 3n \end{array}\right] \quad \begin{array}{l} (1, 2, 3, 4, 5 \ldots 6) \\ (3, 6, 9 \ldots) \\ y = 3n. \end{array}$$

Q $\sin^2 x + \cos^2 y = 2\sec^2 z$

L.H.S ≤ 2

R.H.S $\geq 2.$

$\underline{\sin^2 x} + \underline{\cos^2 y} = 2 \qquad 2\sec^2 z = 2.$

$\sin^2 x = 1 \Rightarrow \sin^2 x = \sin^2 \pi/2 \qquad \sec^2 z = 1.$

$x = n_1\pi \pm \pi/2.$

$\cos^2 y = 1. \qquad \cos^2 z = 1$

$\cos^2 y = \cos^2 0 \qquad y = n_2\pi \qquad z = n_3\pi.$