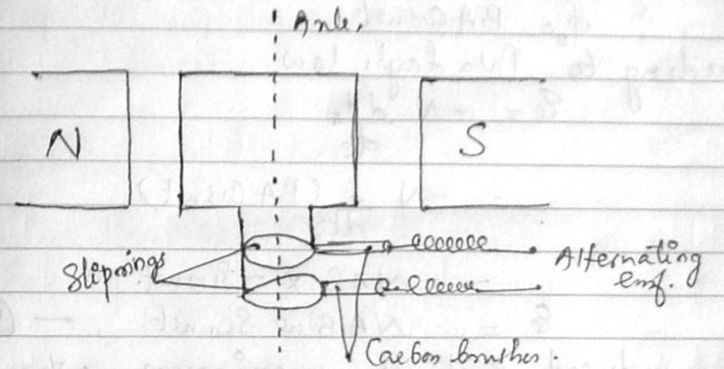
7 Alternating Current

AC generator

An a.c generator converts mechanical energy into electrical energy.

The basic element of an a.c generator are shown.



It consists of a coil mounted on a rotor shaft. The axis of rotation of the coil is perpendicular to the direction of the magnetic field. The coil called armature is mechanically rotated in the uniform magnetic field by some external means. The rotation of the coil causes the magnetic flux through it to change so an emf is induced in the coil. The ends of the coil are connected to an external circuit by means of slip rings and brushes.

Theory:

Consider a coil PQRS free to rotate in a uniform magnetic field $\vec{B}$. The axis of rotation of the coil is $\perp$ to the field $\vec{B}$. The flux through

through the coil, when its normal makes an angle θ with the field is given by

$$\phi_B = \vec{B} \cdot \vec{A}$$
$$= BA\cos\theta$$

where A is the area of the coil.
If the coil rotates with an angular velocity ω and turns through an angle θ in time 't'.
then $\theta = \omega t$

$$\therefore \phi_B = BA\cos\omega t$$

According to Faraday's law

$$\mathcal{E} = -N \frac{d\phi_B}{dt}$$
$$= -N \frac{d}{dt}(BA\cos\omega t)$$
$$= -NAB \times -\sin\omega t \cdot \omega$$
$$\mathcal{E} = NAB\omega \sin\omega t \quad \text{— (1)}$$

The induced emf is maximum, when $\sin\omega t = 1$
or $\omega t = 90°$
Maximum Emf $\mathcal{E}_m = NAB\omega$

$\therefore$ equ (1) becomes

$$\mathcal{E} = \mathcal{E}_m \sin\omega t \quad \text{— (2)}$$

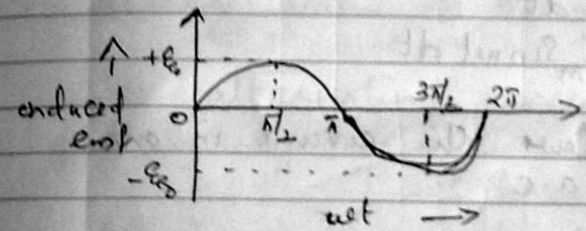
1. When $\theta = \omega t = 0°$, the plane of the coil is $\perp^r$ to $\vec{B}$
$\sin\omega t = \sin 0 = 0 \quad \therefore \mathcal{E} = 0$

2. When $\theta = \omega t = \pi/2$, the plane of the coil is $\parallel^{el}$ to $\vec{B}$
$\sin\omega t = \sin \pi/2 = 1 \quad \therefore \mathcal{E} = \mathcal{E}_0$

3. When $\theta = \omega t = \pi$, the plane of the coil is again $\perp^r$ to $\vec{B}$
$\sin\omega t = \sin\pi = 0 \quad \therefore \mathcal{E}_0 = 0$

4. When $\theta = \omega t = 3\pi/2$, the plane of the coil is again $\parallel^{el}$ to $\vec{B}$.
$\sin\omega t = \sin 3\pi/2 = -1 \quad \therefore \mathcal{E} = -\mathcal{E}_0$

5. When $\theta = \omega t = 2\pi$, the plane of the coil is $\perp^r$ to $\vec{B}$.
$\sin\omega t = \sin 2\pi = 0 \quad \therefore \mathcal{E}_0 = 0$



Thus the direction of the current changes periodically and therefore the current is called alternating current (a.c)

If 'ν' is the frequency of rotation of the coil
$\therefore \omega = 2\pi\nu$
equ (2) becomes

$$\mathcal{E} = \mathcal{E}_m \sin 2\pi\nu t$$

The frequency of rotation of generator in India is 50Hz, but in USA is 60Hz.

If R is the resistance of the coil, then the current in the circuit $I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_m \sin\omega t}{R} = I_m \sin\omega t$

where $I_m = \frac{\mathcal{E}_m}{R}$ is the maximum value of a.c

Average value of a.c over one complete cycle.

The alternating current at any instant 't' is given by

$$I = I_m \sin \omega t$$

At small time dt, current I remains a constant then the amount of charge flows through the circuit

$$dq = I\,dt$$
$$= I_m \sin \omega t\, dt.$$

Total charge that flows the circuit in one complete cycle of a.c

$$q = \int_0^T dq$$

$$= \int_0^T I_m \sin\omega t\, dt = I_m \int_0^T \sin\omega t\, dt.$$

$$= I_m \left(-\frac{\cos\omega t}{\omega}\right)_0^T$$

$$= \frac{I_m}{\omega}\left[-\cos\frac{2\pi}{T}t\right]_0^T$$

$$= \frac{I_m}{\omega}\left[\cos\frac{2\pi}{T}t\right]_T^0$$

$$= \frac{I_m}{\omega}\left[\cos 0 - \cos\frac{2\pi}{T}\times T\right]$$

$$= \frac{I_m}{\omega}\left[\cos 0 - \cos 2\pi\right]$$

$$= \frac{I_m}{\omega}\left[1-1\right] = 0$$

Average value of a.c over one complete cycle of a.c

$$I_{ave} = \frac{q}{T} = \frac{0}{T} = \underline{\underline{0}}.$$

Thus average value of ac over a complete cycle of ac is zero

Average value of ac over half cycle.

The alternating current at any instant 't' is given by

$$I = I_m \sin\omega t.$$

At small time dt current I remains a constant, then the amount of charge flows through the circuit

$$dq = I\,dt$$
$$= I_m \sin\omega t\, dt.$$

Total charge that flows the circuit in the half cycle

$$q = \int_0^{T/2} dq$$

$$= \int_0^{T/2} I_m \sin\omega t\, dt = I_m \int_0^{T/2} \sin\omega t\, dt$$

$$= I_m \left(-\frac{\cos\omega t}{\omega}\right)_0^{T/2}$$

$$= \frac{I_m}{\omega}\left[\cos\omega t\right]_{T/2}^{0}$$

$$= \frac{I_m}{\omega}\left(\cos\frac{2\pi}{T}t\right)_{T/2}^{0}$$

$$= \frac{I_m}{\omega}\left[\cos 0 - \cos\frac{2\pi}{T}\cdot\frac{T}{2}\right]$$

$$= \frac{I_m}{\omega}\left[\cos 0 - \cos\pi\right]$$

$$= \frac{I_m}{\omega}\left[1 - -1\right] = \frac{2I_m}{\omega}$$

$$= \frac{2 I_m}{2\pi/T}$$

$$= \frac{I_m \times T}{\pi}$$

The average value of ac over one half cycle is

$$I_{av} = \frac{\text{charge}}{\text{time}} = \frac{q}{T/2} = \frac{2q}{T}$$

$$= \frac{2\,I_m T}{\pi \times T}$$

$$I_{av} = \frac{2}{\pi} \times I_m$$

$$= 0.637\, I_m$$

Thus the average value of a.c is $\frac{2}{\pi}$ or 0.637 times its peak value.
Thus similar relation can be proved for the alternating emf which is

$$\mathcal{E}_{av} = \frac{2}{\pi}\mathcal{E}_m$$

$$= 0.637\,\mathcal{E}_m$$

Root mean square (RMS) or Virtual or Effective value of a.c.

It is defined as the value of a direct current which produces the same heating effect in a given resistor as is produced by the given alternating current when passed for the same time.
Suppose an alternating current $I = I_m \sin\omega t$ be passed through a circuit of resistance R. The amount of heat produced in same time dt will be

$$dH = I^2 R\,dt \quad \text{—— (1)}$$

Heat produced in one complete cycle will be

$$H = \int_0^T dH$$

$$= \int_0^T I^2 R\,dt$$

If I_{rms} be the r.m.s or effective value of a.c Then heat produced in time T must be

$$H = I_{rms}^2 R T \quad \text{—— (2)}$$

$$\therefore I_{rms}^2 R T = \int_0^T I^2 R\,dt$$

$$I_{rms}^2 T = \int_0^T I^2\,dt$$

$$I_{rms}^2 = \frac{1}{T}\int_0^T I^2\,dt \quad \text{—— (3)}$$

Now $\int_0^T I^2\,dt = \int_0^T I_m^2 \sin^2\omega t\,dt = I_m^2 \int_0^T \sin^2\omega t\,dt$

$$= \frac{I_m^2}{T}\int_0^T\left(\frac{1-\cos 2\omega t}{2}\right)dt$$

$$= \frac{I_m^2}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\right]$$

$$= \frac{I_m^2}{2}\left[(t)_0^T - \left(\frac{\sin 2\omega t}{2\omega}\right)_0^T\right]$$

$$= \frac{I_m^2}{2}\left[(T-0) - \left[\frac{\sin 2\omega . T}{2\omega} - \sin 0\right]\right]$$

$$= \frac{I_m^2}{2}\left[T - \frac{1}{2\omega} \times \sin 2\omega . T\right]$$

$$= \frac{I_m^2}{2}\left[T - \frac{1}{2\omega}\sin 2 . \frac{2\pi}{T} . T\right]$$

$$= \frac{I_m^2}{2}\left[T - \frac{1}{2\omega}\sin 4\pi\right]$$

$$= \frac{I_m^2}{2}[T-0]$$

$$= \frac{I_m^2 T}{2}$$

$\therefore$ equ (3) becomes

$$I_{rms}^2 = \frac{1}{T} \times \frac{I_m^2 T}{2}$$

$$I_{rms}^2 = \frac{I_m^2}{2}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{1}{\sqrt{2}} I_m$$

$$\therefore I_{rms} = 0.707\, I_m$$

Root mean square value of an alternating emf.

RMS value of an alternating emf is defined as that value of steady voltage that produces the same amount of heat in a given resistance as is produced by the given alternating emf when applied to the same resistance for the same time.

Suppose an alternating emf V applied to a resistance R is given by

$$V = V_m \sin \omega t.$$

Heat produced in a small time dt will be

$$dH = I^2 R\, dt.$$

$$= \frac{V^2}{R^2} . R\, dt$$

$$= \frac{V^2}{R} dt$$

$$= \frac{V_m^2 \sin^2 \omega t}{R} dt$$

Heat produced in time T.

$$H = \int_0^T dH = \int_0^T \frac{V_m^2 \sin^2 \omega t}{R} dt.$$

$$= \frac{V_m^2}{R}\int_0^T \sin^2 \omega t\, dt$$

$\int_0^T \sin^2 \omega t\, dt = T/2$ proved before.

$$= \frac{V_m^2}{R} \times \frac{T}{2}$$

$$H = \frac{V_m^2 T}{2R}.$$

If V_{rms} is the rms value of alternating emf.

then $H = \frac{V_{rms}^2}{R} T$

$\therefore \frac{V_{rms}^2 T}{R} = \frac{V_m^2 T}{2R}$

$\therefore V_{rms}^2 = \frac{V_m^2}{2}$

$V_{rms} = \frac{V_m}{\sqrt{2}}$

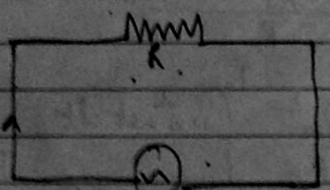
$V_{rms} = \frac{1}{\sqrt{2}} \cdot V_m$

$V_{rms} = 0.707\, V_m$

A.C circuit containing resistor only.

Consider an a.c circuit containing resistor R only connected to a source of alternating emf.

$V = V_m \sin wt$ —— (1)



$V = V_m \sin wt$

Let I be the current in the circuit at instant t

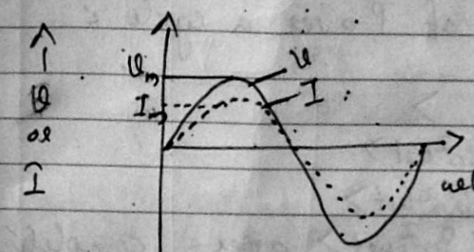
Current $I = \frac{V}{R}$

$I = \frac{V_m \sin wt}{R}$

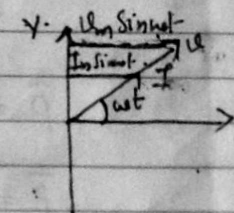
$I = I_m \sin wt$ — (2) where $I_m = \frac{V_m}{R}$ is the maximum or peak value of a.c

From (1) & (2) it is clear that the voltage and current are in same phase.

The graph showing the variation of V and I versus wt is



The phasor diagram for a resistive a.c circuit is



Phasors & Phasor diagram.

In order to show the phase relationship between voltage and current in an a.c circuit we use the notion of phasors. A phasor is a vector which rotates about the origin with angular velocity w. The magnitude of phasors V and I represent the amplitudes or the peak values V_m and I_m of these quantities.

A diagram that represents alternating current and voltage of the same frequency as rotating vectors along with proper phase angle between them is called a phasor diagram.

Power consumed in a resistive circuit.

The instantaneous power dissipated in the resistor is

$$P = I^2 R$$
$$= (I_m \sin \omega t)^2 R$$
$$= I_m^2 \sin^2 \omega t \, R.$$

The average value of Power a cycle is

$$\bar{P} = \langle I^2 R \rangle$$
$$= \langle I_m^2 R \sin^2 \omega t \rangle$$
$$= I_m^2 R \langle \sin^2 \omega t \rangle$$

Average value of $\langle \sin^2 \omega t \rangle$ over a complete cycle is

$$\langle \sin^2 \omega t \rangle = \frac{1}{2}.$$

$$\therefore \bar{P} = I_m^2 R \times \frac{1}{2}$$

$$\bar{P} = \frac{I_0^2 R}{2}$$

$$\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} = \frac{1}{2} - \frac{\cos 2\omega t}{2}$$
$$\int_0^T \sin^2 \omega t = \frac{1}{2}(t)_0^T - \frac{1}{2\omega}(\sin 2\omega t)_0^T$$
$$= \frac{1}{2}$$

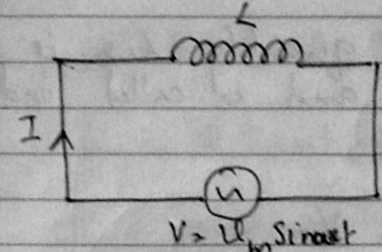
RMS value of current or effective value of current.

$$I_{rms} = \frac{I_m}{\sqrt{2}}.$$
$$I_{rms}^2 = \frac{I_m^2}{2}$$
$$\therefore \bar{P} = I_{rms}^2 R.$$

A.C circuit containing inductor only.

Consider an a.c circuit containing inductor L only connected to a source of alternating emf

$$v = V_m \sin \omega t \quad \text{— (1)}$$



The alternating current flows through the inductor, a back emf $-L\frac{dI}{dt}$ is set up.

$$\therefore \text{Net emf } v - L\frac{dI}{dt}$$

But this emf must be zero because there is no resistance in the circuit.

$$v - L\frac{dI}{dt} = 0$$
$$v = L\frac{dI}{dt}$$
$$V_m \sin \omega t = L\frac{dI}{dt}$$
$$\therefore dI = \frac{V_m}{L} \sin \omega t \, dt$$

Integrating

$$\int dI = \int \frac{V_m}{L} \sin \omega t \, dt$$
$$I = -\frac{V_m}{L} \frac{\cos \omega t}{\omega}$$
$$= \frac{V_m}{L\omega} \times -\cos \omega t \quad \text{— (2)}$$

But $-\cos\omega t = \sin(\omega t - \pi/2)$
we have eqn (a) becomes.

$$I = I_m \sin(\omega t - \pi/2) \quad \text{———} \ (2)$$

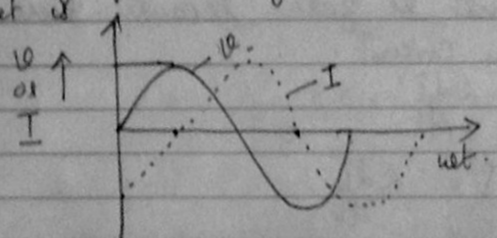
where $I_m = \frac{V_m}{L\omega}$ is the maximum value of current. The quantity $L\omega$ is analogous to the resistance and is called inductive reactance X_L

$$X_L = L\omega$$

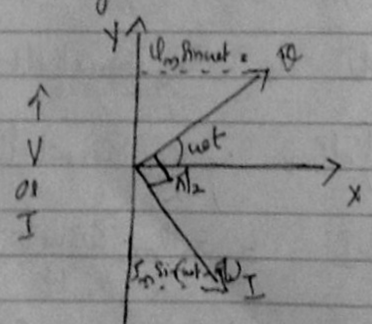
$$\therefore I_m = \frac{V_m}{L\omega} = \frac{V_m}{X_L}.$$

From (1) and (2) it is clear that in an inductive ac circuit, the voltage leads the current by $\pi/2$ or the current lags the voltage by $\pi/2$.

The graph showing the variation V and I versus ωt is



The Phasor diagram for an inductive a.c circuit is



Average power consumed in an inductive circuit.

The instantaneous power supplied to the inductor is

$$P_L = IV$$
$$= I_m \sin(\omega t - \pi/2) \cdot V_m \sin\omega t.$$
$$= I_m - \cos\omega t \cdot V_m \sin\omega t.$$
$$= -I_m V_m \frac{2\sin\omega t \cos\omega t}{2}$$
$$= \frac{-I_m V_m}{2} \sin 2\omega t$$

So the average power over a complete cycle is

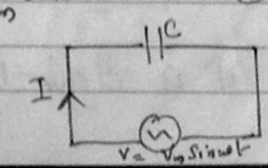
$$\overline{P}_L = \left\langle \frac{-I_m V_m}{2} \sin 2\omega t \right\rangle$$
$$= \frac{-I_m V_m}{2} \langle \sin 2\omega t \rangle$$
$$= \frac{-I_m V_m}{2} \times 0 = 0$$

Since the average of $\langle \sin 2\omega t \rangle$ over a complete cycle is zero.
Thus the average power supplied to an inductor over one complete cycle is zero.

A.C circuit containing capacitance Only.

Consider a a.c circuit containing capacitor C only, connected to a source of emf

$$V = V_m \sin\omega t \quad \text{———} \ (1)$$



Let q be the charge on the capacitor at any time t.
The instantaneous voltage v across the capacitor is

$$V = \frac{q}{C}$$

$$v_m \sin \omega t = \frac{q}{C}$$

$$q = v_m C \sin \omega t.$$

But current $I = \frac{dq}{dt}$.

$$= \frac{d}{dt}(v_m C \sin \omega t)$$

$$= v_m C \frac{d}{dt}(\sin \omega t)$$

$$= v_m C \cos \omega t \cdot \omega.$$

$$= \frac{v_m}{1/C\omega} \cos \omega t. \quad \text{—— (a)}$$

But $\cos \omega t = \sin(\omega t + \pi/2)$

$\therefore$ equ (a) becomes $I = I_m \sin(\omega t + \pi/2)$ —— (2)

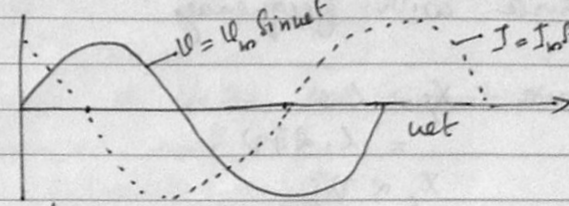
where $I_m = \frac{v_m}{1/C\omega}$ is the maximum value of current. The quantity $\frac{1}{C\omega}$ is analogous to the resistance and is called capacitive reactance X_C

ie $X_C = \frac{1}{C\omega}$

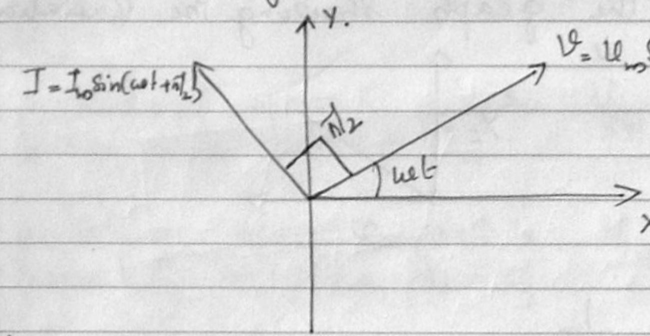
$\therefore I_m = \frac{v_m}{X_C}$ is the peak value of current.

From equ's (1) and (2) it is clear that in a capacitive a.c circuit current leads the applied voltage by $\pi/2$. or the voltage lags the current by $\pi/2$.

The graph showing the variation of V and I versus ωt is



The Phasor diagram for a capacitive circuit is



Average power consumed in a capacitive circuit.

The instantaneous power supplied to the capacitor is

$$P_c = IV$$

$$= i_m \sin(\omega t + \pi/2) \cdot v_m \sin \omega t.$$

$$= I_m v_m \cos \omega t \sin \omega t.$$

$$= \frac{I_m v_m}{2} 2\cos \omega t \sin \omega t.$$

$$= \frac{I_m v_m}{2} \sin 2\omega t.$$

So the average power over a complete cycle is

$$P_c = \left\langle \frac{I_m v_m}{2} \sin 2\omega t \right\rangle$$

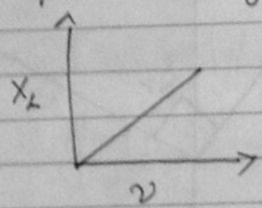
$$= \frac{I_m v_m}{2} \langle \sin 2\omega t \rangle = \frac{I_m v_m}{2} \times 0 = 0$$

Thus the average power supplied to a capacitor over one complete cycle is zero.

Variation of inductive reactance and capacitive reactance with frequency

We have $X_L = L\omega$
$= L \cdot 2\pi\nu$
$X_L \propto \nu$.

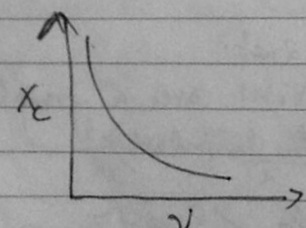
ie inductive reactance varies directly with frequency. The graph showing the variation of X_L and ν is



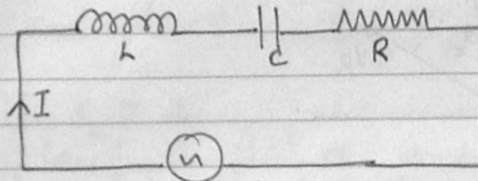
Also $X_C = \frac{1}{C\omega}$
$= \frac{1}{C \cdot 2\pi\nu}$

ie $X_C \propto \frac{1}{\nu}$

ie capacitive reactance varies inversely with frequency. The graph showing the variation of X_C and ν is



Series LCR circuit.



$V = V_m \sin\omega t$.

Figure shows a series LCR circuit connected to an a.c source of emf

$V = V_m \sin\omega t$ ——— (1)

Let I be the current in the series circuit at any instant.

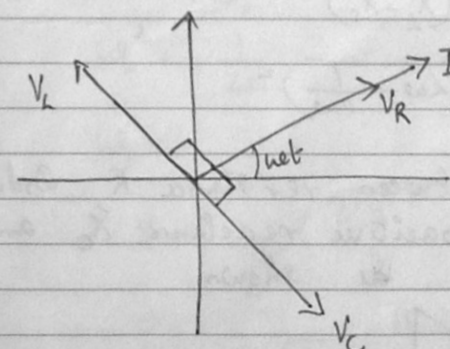
Then

1. Voltage across R will be $V_{Rm} = I_m R$ in same phase with current I_m.

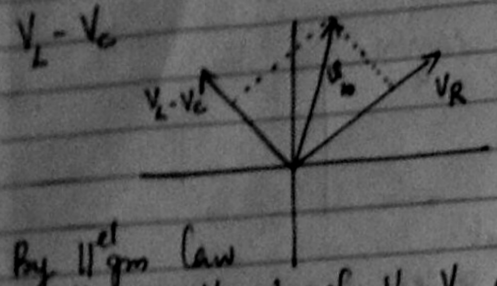
2. Voltage across L will be $V_{LM} = I_m X_L$ ahead of current I_m in phase by $\pi/2$.

3. Voltage across C will be $V_{CM} = I_m X_C$ lags of current I_m in phase by $\pi/2$.

Then it can be represented by using phasor diagram as.



As V_L and V_c are in opposite direction - their resultant is



By ||gm law the resultant of $V_L - V_C$ and V_R will be

$$V_m^2 = V_{Rm}^2 + (V_{Lm} - V_{Cm})^2$$

$$= (I_m R)^2 + (I_m X_L - I_m X_C)^2$$

$$= I_m^2 \left[R^2 + (X_L - X_C)^2 \right]$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \quad \text{——— ②}$$

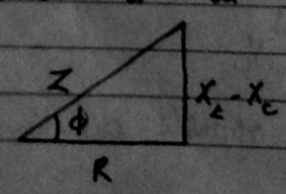
Clearly by analogy to the resistance in a circuit. $\sqrt{R^2 + (X_L - X_C)^2}$ is the effective resistance of the series LCR circuit called impedance Z.

Thus $I_m = \frac{V_m}{Z}$

where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

The relation between resistance R inductive reactance X_L, capacitive reactance X_C and impedence Z is as shown.



The Δ^{le} so obtained is called impedance triangle.

and $\tan\phi = \frac{X_L - X_C}{R}$ ——— ③.

If $X_L > X_C$

$\tan\phi$ is positive and the LCR circuit is predominantly inductive ie the current lags the voltage by $\pi/2$.

If $X_C > X_L$

$\tan\phi$ is negative and the LCR circuit is predominantly capacitive ie the current leads the voltage by $\pi/2$.

If $X_L = X_C$.

$Z = \sqrt{R^2} = R$

$\tan\phi =$

Clearly the impedance is R, the LCR circuit is purely resistive. The current and voltage are in same phase and the current in the circuit is maximum. This condition of the LCR circuit is called resonance.

ie $X_L = X_C$ If ω is varied, then at a particular frequency ω_0.

$$L\omega_0 = \frac{1}{C\omega_0}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\pi\nu = \frac{1}{\sqrt{LC}}$$

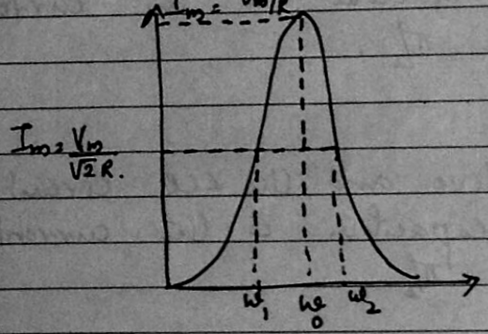
$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

This frequency ω_0 is called resonant frequency

At resonant frequency current amplitude is maximum

i.e. $I_m = \frac{V_m}{R}$

Figure shows the variation of I_m with ω.



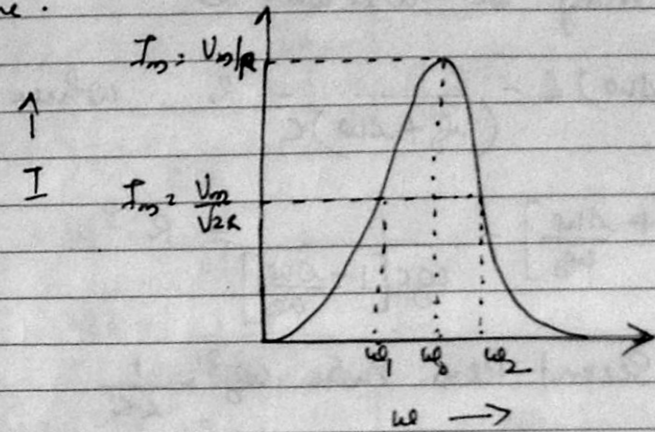
Sharpness of resonance :- Q- factor

The sharpness of resonance is measured by a factor called quality or Q- factor of the circuit.
The resonant frequency is independent of R but the sharpness of peak depends on R. The peak is higher for small values of R and flat for large R.

Q- factor of a series resonant circuit is defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value at resonant frequency.

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0}{2\Delta\omega} = \frac{\text{Resonant frequency}}{\text{Bandwidth}}.$$

where ω_1 and ω_2 are the frequencies at which the current falls to $\frac{1}{\sqrt{2}}$ times its resonant value.



$\therefore \omega_1 = \omega_0 - \Delta\omega$, $\omega_2 = \omega_0 + \Delta\omega$.

$\therefore \omega_2 - \omega_1 = (\omega_0 + \Delta\omega) - (\omega_0 - \Delta\omega)$
$= 2\Delta\omega$. is called bandwidth

Larger the value of Q- factor the smaller is the value of $2\Delta\omega$ or the bandwidth and sharper is the peak in the current.

Expression for Q- factor

At ω_0 the impedence is equal to R while at ω_1 and ω_2 its value is $\sqrt{2}R$.

$\therefore Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

Becomes

$$\sqrt{2}R = \sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}$$

or $2R^2 = R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2$

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = R^2$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

which may be written as

$$(\omega_0 + \Delta\omega) L - \frac{1}{(\omega_0 + \Delta\omega) C} = R \quad \text{where } \omega_2 = \omega_0 + \Delta\omega$$

$$\omega_0 L\left[1 + \frac{\Delta\omega}{\omega_0}\right] - \frac{1}{\omega_0 C\left[1 + \frac{\Delta\omega}{\omega_0}\right]} = R$$

In the second term sub: $\omega_0^2 = \frac{1}{LC}$

or $C = \frac{1}{L\omega_0^2}$

$$\therefore \omega_0 L\left[1 + \frac{\Delta\omega}{\omega_0}\right] - \frac{1}{\omega_0 \times \frac{1}{L\omega_0^2}\left[1 + \frac{\Delta\omega}{\omega_0}\right]} = R$$

$$\omega_0 L\left[1 + \frac{\Delta\omega}{\omega_0}\right] - L\omega_0\left[1 + \frac{\Delta\omega}{\omega_0}\right]^{-1} = R$$

$$\omega_0 L\left[1 + \frac{\Delta\omega}{\omega_0}\right] - L\omega_0\left[1 - \frac{\Delta\omega}{\omega_0}\right] = R$$

$(1+x)^{-n} = 1 - nx$

$$\omega_0 L \cdot \frac{2\Delta\omega}{\omega_0} = R$$

$$L \cdot 2\Delta\omega = R$$

$$\Delta\omega = \frac{R}{2L}$$

Sharpness of resonance or Q factor $Q = \frac{\omega_0}{2\Delta\omega}$

$$Q = \frac{\omega_0}{2 \cdot \frac{R}{2L}}$$

$$\boxed{Q = \frac{\omega_0 L}{R}} \quad \text{— ①}$$

But $\omega_0 = \frac{1}{\sqrt{LC}}$

$\omega_0^2 = \frac{1}{LC}$

$\omega_0 L = \frac{1}{\omega_0 C}$

$$\therefore \boxed{Q = \frac{1}{\omega_0 CR}} \quad \text{— ②}$$

Also $\omega_0 = \frac{1}{\sqrt{LC}}$

or $\frac{1}{\omega_0} = \sqrt{LC}$

$$Q = \frac{\sqrt{LC}}{CR} = \frac{1}{R}\sqrt{\frac{LC}{C^2}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} \quad \text{— ③}$$

From equ's ① ② & ③ it is clear that if Q-factor is large i.e. R is low or L is large, the band width $2\Delta\omega$ is small. This means that the resonance is sharp or the series resonant circuit is more selective.

Power in an a.c circuit. - The Power factor.

Suppose in an a.c circuit, the voltage and current at any instant are given by

$V = V_m \sin \omega t$

This voltage is applied to a series LCR circuit, driven a current in the circuit given by.

$$I = I_m \sin(\omega t + \phi)$$

where $I_m = \frac{V_m}{Z}$ and $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

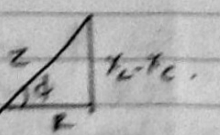
Instantaneous power

$$P = VI$$
$$= V_m \sin \omega t \cdot I_m \sin(\omega t + \phi)$$
$$= V_m I_m \sin \omega t \sin(\omega t + \phi)$$
$$= V_m I_m \sin \omega t\,[\sin \omega t \cos\phi + \cos \omega t \sin\phi]$$
$$= V_m I_m\,[\sin^2 \omega t \cos\phi + \sin \omega t \cos \omega t \sin\phi]$$
$$= V_m I_m\left[\sin^2 \omega t \cos\phi + \frac{2 \sin \omega t \cos \omega t}{2} \sin\phi\right]$$
$$= V_m I_m\left[\left(\frac{1 - \cos 2\omega t}{2}\right)\cos\phi + \frac{\sin 2\omega t}{2} \sin\phi\right]$$
$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos 2\omega t \cos\phi + \sin 2\omega t \sin\phi\right]$$
$$= \frac{V_m I_m}{2}\left[\cos\phi - [\cos 2\omega t \cos\phi - \sin 2\omega t \sin\phi]\right]$$
$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right] \quad -(1)$$

The average power over a cycle is given by the average of the two terms in RHS of equ (1). It is only the second term which is time dependent. Its average is zero.

$$\therefore P = \frac{V_m I_m}{2} \cos\phi.$$
$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi.$$
$$= V_{rms} I_{rms} \cos\phi.$$

$\cos\phi = \frac{R}{Z}$.



$$\therefore \boxed{P = V_{rms} I_{rms} \cdot \frac{R}{Z}}$$

The quantity $\cos\phi$ is called power factor.

Special Cases:

Case 1

Resistive circuit:- If the circuit contains only pure R. In this case $\phi = 0$, because V & I are in same phase.

$\cos\phi = 1$

$\therefore P = V_{rms} I_{rms}$, thus the maximum power is dissipated.

Case II.

Purely inductive circuit:- If the circuit contains only inductor. In this case $\phi = \pi/2$ because voltage leads the current by $\pi/2$.

$\therefore \cos\phi = 0$

$\therefore P = 0$ no power is dissipated even though a current is flowing in the circuit

Case III.

Purely capacitive circuit:- If the circuit contains only capacitor. In this case $\phi = \pi/2$ because current leads the voltage by $\pi/2$.

$\therefore \cos\phi = 0$

$P = 0$ no power is dissipated even though a current is flowing in the circuit.

Case IV.

Series LCR circuit:- In series LCR circuit power dissipated is $P = V_{rms} I_{rms} \cos\phi$

where $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

So ϕ may be non zero in a RC, RL and LCR circuit. So power is consumed in such circuit containing R only.

Case V.

Power dissipated at resonance LCR circuit.

At resonance $X_C = X_L$

i.e. $X_C - X_L = 0$

$$\therefore \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

$$= 0$$

$$\therefore \cos\phi = 1$$

$$P = V_{rms} I_{rms}$$

$$\therefore P = I_{rms} R I_{rms}$$

$P = I_{rms}^2 R$ i.e. maximum power is dissipated in a circuit (through R) at resonance

Wattless Current

The current in an a.c circuit is said to be wattless if the average power consumed in the circuit is zero.

Average power

$$P = I_{rms} V_{rms} \cdot \cos\phi.$$

In the case of a purely inductive or capacitive circuit the phase difference between V and I is $\pi/2$.

$$\therefore \cos\phi = \cos\pi/2 = 0$$

$\therefore$ P = 0 no power is dissipated even though a current is flowing in the circuit. This current is called Wattless or Idle current.

LC Oscillations

When a charged capacitor is allowed to discharge through a non resistive inductor electrical oscillation of constant amplitude and frequency are produced. These oscillations are called LC oscillations.

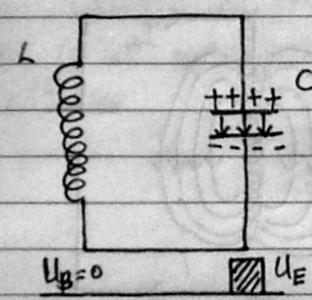


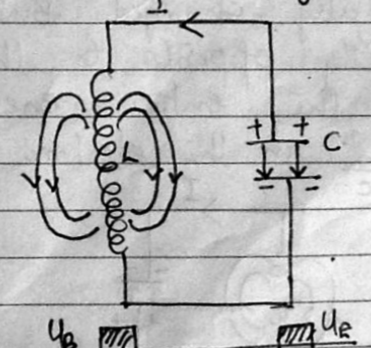
Figure shows a capacitor with initial charge q_m connected to an ideal inductor. The electrical energy stored in the charged capacitor

$$U_E = \frac{1}{2}\frac{q_m^2}{C}$$

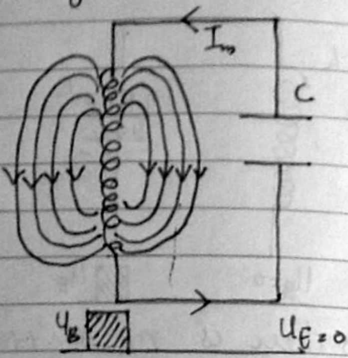
As there is no current in the circuit, the energy stored in the magnetic field of the inductor is zero.

As the circuit is closed the capacitor begins to discharge itself through the inductor causing a current I. As the current I increases it builds up magnetic field around the inductor. A part of electrical energy of the capacitor gets stored in the inductor in the form of magnetic energy

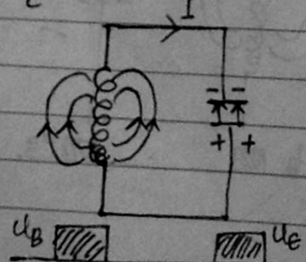
$$U_B = \frac{1}{2}LI^2.$$

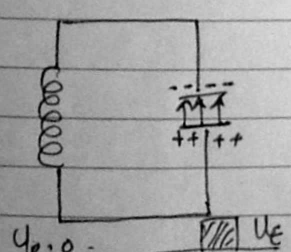


At the later instant the capacitor gets fully discharged and the potential difference across its plates becomes zero. The current reaches its maximum value I_m, the energy stored in the magnetic field is $\frac{1}{2} L I_m^2$. Thus the entire electrostatic energy of the capacitor has been converted in to the magnetic field energy of the inductor.



After the discharge of the capacitor is complete the magnetic flux linked with the inductor decreases, thus an induced current is set up. The current thus persists through with decreasing magnitude and charges the capacitor in the opposite direction. The magnetic energy of the inductor begins to change in to the electrostatic energy of the capacitor. This process continues till the capacitor is fully charged. But it is charged with a polarity opposite to that in its initial state. Thus the entire energy is again stored as $\frac{1}{2} \frac{q_m^2}{C}$ in the electric field of the capacitor.

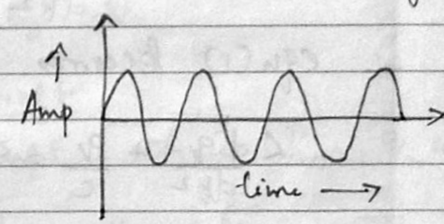




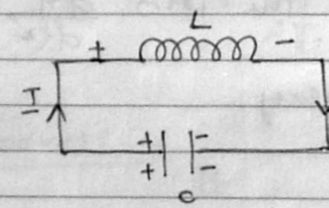
Thus the process repeats the energy of the system continuously surges back and forth between the electric field of the capacitor and the magnetic field of the inductor. This produces electrical oscillations of a definite frequency. These are called LC oscillations.

If there is no loss of energy, the amplitude of the oscillations remains constant. Such oscillations are called undamped oscillations. The frequency of oscillation is given by

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$



Mathematical treatment of LC Oscillations



Consider an LC circuit containing a capacitor of capacitance C initially charged to q_m, be connected to an inductor L. As the circuit is closed, the charge on the capacitor begins to decrease giving rise to a current

$$I = -\frac{dq}{dt}$$

Induced emf across the inductor $= -L\frac{dI}{dt}$

P.d across the capacitor $= \frac{q}{C}$

According to Kirchoff's law (loop rule)

$$-L\frac{dI}{dt} + \frac{q}{c} = 0 \quad \text{— ①}$$

But $I = -\frac{dq}{dt}$

$$\frac{dI}{dt} = -\frac{d}{dt}\left(\frac{dq}{dt}\right)$$
$$= -\frac{d^2q}{dt^2}$$

∴ equ (1) becomes

$$L\frac{d^2q}{dt^2} + \frac{q}{c} = 0$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \quad \text{— ②}$$

This equation has the form $\frac{d^2x}{dt^2} + \omega_0^2 x = 0$ for a SH oscillator.
with natural frequency

$$\omega_0^2 = \frac{1}{LC}$$

or $\omega_0 = \frac{1}{\sqrt{LC}}$

with solution $q = q_m \cos\omega_0 t$

$$\frac{dq}{dt} = \frac{d}{dt}(q_m \cos\omega_0 t) = q_m - \sin\omega_0 t \cdot \omega_0.$$
$$= -q_m\omega_0 \sin\omega_0 t$$

$$\frac{d^2q}{dt^2} = \frac{d}{dt}(-q_m\omega_0 \sin\omega_0 t) = -q_m\omega_0 \cdot \cos\omega_0 t \cdot \omega_0$$
$$= -q_m\omega_0^2 \cos\omega_0 t.$$

∴ equ (2) becomes

$$-q_m\omega_0^2 \cos\omega_0 t + \omega_0^2 \cdot q_m \cos\omega_0 t = 0.$$

∴ $q = q_m \cos\omega_0 t$ is the solution
and $I = -\frac{dq}{dt}$

$$= -\frac{d}{dt}(q_m \cos\omega_0 t)$$
$$= -q_m - \sin\omega_0 t \cdot \omega_0.$$
$$= q_m\omega_0 \sin\omega_0 t$$
$$= I_m \sin\omega_0 t \quad \text{where } I_m = q_m\omega_0$$

Thus the frequency of oscillation

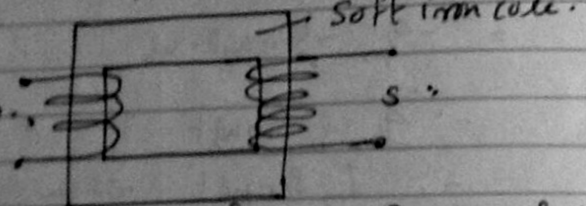
$$\nu_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.$$

Transformers. It is device for converting an alternating current at low voltage into that at high voltage and vice versa.

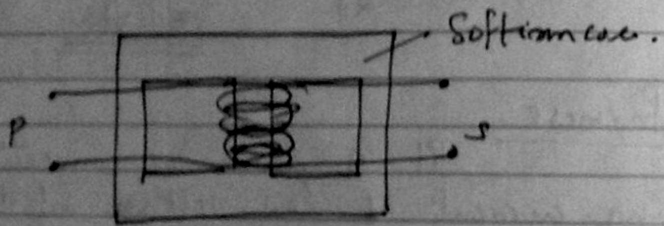
Principle:- It works on the principle of mutual induction.

Construction:- A transformer essentially consists of two coils of insulated copper wire having different number of turns and wound on the same soft iron core. The coil P to which electrical energy is supplied is called the primary and the coils from which energy is drawn or output is obtained is called the secondary. To prevent energy losses due to eddy currents, a laminated core is used. Two types of arrangements are generally used for winding of primary and secondary coils in a transformer.

1. Core type transformer. In the core type transformers the primary and secondary coils are wound on separate limbs of the core



2. Shell type transformer:- In the shell type transformers, the primary and secondary coils are wound one over another on the same limb of the iron core.



Working:- As the alternating current flows through the primary, it generates an alternating magnetic flux in the core which also passes through the secondary. This changing flux sets up an induced emf in the secondary. If there is no flux leakage of magnetic flux, then flux linked with each turn of the primary will be equal to that linked with each turn of the secondary.

Theory

According Law of EM induction

$$\mathcal{E} = -N \frac{d\phi}{dt}.$$

Induced emf in the primary coil

$$\mathcal{E}_p = -N_p \frac{d\phi}{dt} \quad \text{——— (1)}$$

Induced emf in the secondary coil

$$\mathcal{E}_s = -N_s \frac{d\phi}{dt} \quad \text{——— (2)}.$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{N_p}{N_s}$$

or $\boxed{\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p}}$ is called the turns ratio or transformer ratio.

If $\mathcal{E}_s = \left(\frac{N_s}{N_p}\right) \mathcal{E}_p$.

$N_s > N_p$ thus $\mathcal{E}_s > \mathcal{E}_p$ ie the output voltage is greater than the input voltage. This type of arrangement is called a step-up transformer

If $N_p > N_s$ thus $\mathcal{E}_s < \mathcal{E}_p$ ie the output voltage is less than the input voltage. This type of arrangement is called a step-down transformer.

If the transformer is assumed to be 100% efficient (ideal transformer) no energy losses.

Thus Input power = Output power

$$\mathcal{E}_p I_p = \mathcal{E}_s I_s$$

$$\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{I_p}{I_s}$$

Thus $\boxed{\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}}$

Uses of transformers

1. Small transformers are used in radio receivers loud speakers, telephones etc.

2. In stabilised power supplies.

3. In the transmission of electrical energy from the generating stations to the consumers.

Energy losses in transformers

1. Flux leakage:- The magnetic flux produced by the primary may not fully pass through the secondary. Some of the flux may leak onto air. This loss can be minimised by winding the primary and secondary over one another.
2. Copper loss (Resistance of the windings):- The wire used for the windings has some resistance and so energy is lost due to heat produced in the wire. This loss can be reduced by using thick copper wires of low resistance.
3. Eddy current loss:- The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect can be reduced by laminated iron core.
4. Hysteresis loss:- The alternating current carries the iron core through cycles of magnetisation and demagnetisation. Work is done in each of these cycles and is lost as heat. This is hysteresis loss and can be minimised by using core material having narrow hysteresis loop.

Use of transformers in long distance transmission of electric power

The large scale transmission and distribution of electrical energy over long distances is done with the use of transformers. The voltage output of the generator is stepped-up. It is then transmitted over long distances to an area sub station near the consumers. There the voltage is stepped down. It is further stepped down at distributing substations and utility poles before a power supply of 220 V reaches our homes.

Use of a series resonant circuit in tuning of a radio receiver

The tuning of a radio or TV is an example of LCR resonant circuit. Signals are transmitted by different stations at different frequencies. These frequencies are picked up by the antenna so the circuit can be driven at many frequencies. But to hear one particular radio station, we tune the radio. In tuning we vary the capacitance of a capacitor in the tuning circuit such that resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received. So the current is maximum thus the signal from the desired station can be tuned in

Thus $\boxed{\frac{E_{es}}{E_{ep}} = \frac{N_s}{N_p} = \frac{I_p}{I_s}}$

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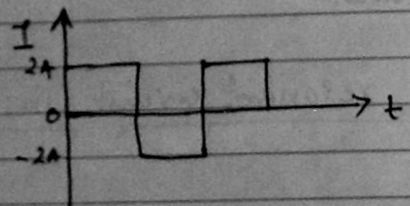
Numericals.

1. The electric mains in a house are marked 220 V, 50 Hz. Write down the equation for instantaneous voltage.

Solu.
$V_{rms} = 220\ V$
$\nu = 50\ Hz$.

Instantaneous voltage $V = V_m \sin \omega t$
$V = \sqrt{2}\, V_{rms} \sin 2\pi\nu t$
$= 1.414 \times 220 \sin[2 \times \pi \times 50\, t)$
$= \underline{311 \sin 314t}$

2. Calculate the rms value of the alternating current shown in figure



Solu
$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2}{3}}$$
$$= \sqrt{\frac{2^2 + (-2)^2 + 2^2}{3}} = \sqrt{\frac{12}{3}} = \underline{2A}$$

3. A 100 Hz a c is flowing in a 14 mH coil. Find its reactance.

Solu
$\nu = 100\ Hz$
$L = 14 \times 10^{-3}\ H$
$X_L = L\omega = L.2\pi\nu = 14 \times 10^{-3} \times 2 \times \frac{22}{7} \times 100 = 8.8\ \Omega$

4. A coil has an inductance 1 H.
(i) What is the frequency will it have a reactance of 3142 Ω ?
(ii) What should be the capacity of a capacitor which has the same reactance at that frequency?

Soln.
$L = 1H$.
$X_L = 3142\ \Omega$

(i) $X_L = L\omega$
$X_L = L.2\pi\nu$
$$\nu = \frac{X_L}{2\pi L} = \frac{3142}{2 \times 3.14 \times 1} = 500\ Hz.$$

(ii) $X_C = X_L = 3142$
$$X_C = \frac{1}{\cancel{2\pi f}} = \frac{1}{C\omega}$$
$$3142 = \frac{1}{C.2\pi\nu}$$
$$C = \frac{1}{3142 \times 2 \times 3.14 \times 500} = 0.11 \times 10^{-6}\ F$$
$= \underline{0.11\ \mu F}$

5. A pure inductor of 25 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Solu.
$L = 25 \times 10^{-3}\ H$
Emf. $V_{rms} = 220V$
$\nu = 50\ Hz$.

$X_L = L\omega = L.2\pi\nu = 25 \times 10^{-3} \times 3.14 \times 2 \times 50 = 7.85\ \Omega$
$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{X_L} = \frac{220}{7.85} = 28.03A$$

~~A coil has an inductance of ... H?~~

6. A 1.50 μF capacitor is connected to a 220 V 50 Hz source. Find the capacitive reactance and the current in the circuit. If the frequency is doubled what happens to the capacitive reactance and the current?

bottommost peak

Solu.

$C = 1.50\,\mu F = 1.50 \times 10^{-6} F$

$V_{rms} = 220 V$

$\nu = 50 Hz.$

$X_c = \frac{1}{C\omega}$

$= \frac{1}{C.2\pi\nu} = \frac{1}{1.50\times10^{-6} \times 2 \times 3.14 \times 50}$

$= \underline{212\,\Omega}$

$I_{rms} = \frac{V_{rms}}{X_c} = \frac{220}{212} = \underline{\underline{1.04 A}}$

Also $I_{0} = I_{rms}.\sqrt{2}$

$= 1.04 \times 1.414$

$= \underline{1.47\ A}$

$X_c = \frac{1}{C 2\pi\nu}$

$X_c \propto \frac{1}{\nu}$ if frequency is doubled the capacitive reactance is halfed.

7. When an inductor L and resistor R in series are connected to a 12 V, 50 Hz supply a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by $\pi/3$ radian. Calculate the value of R.

Solu. $V_{rms} = 12 V$

$\nu = 50 Hz.$

$I_{rms} = 0.5 A$

$\phi = \pi/3.$

$Z = \frac{V_{rms}}{I_{rms}}$

$= \frac{12}{0.5} = 24\,\Omega$

As $\cos\phi = \frac{R}{Z}$

$R = Z\cos\phi$

$= 24 \times \cos\pi/3 = 24 \times \frac{1}{2} = \underline{12\,\Omega}$

8. A bulb of resistance 10 Ω, connected to an inductor of inductance L is in series with an a.c source marked 100 V, 50 Hz. If the phase angle between the voltage and current is $\pi/4$ radian. Calculate the value of L.

Solu: $R = 10\,\Omega$

$\nu = 50 Hz.$

$V_{rms} = 100 V.$

$\phi = \frac{\pi}{4}.$

$\tan\phi = \frac{X_L}{R} = \frac{2\pi\nu L}{R}$

$L = \frac{R\tan\phi}{2\pi\nu} = \frac{10 \times \tan\pi/4}{2 \times \frac{22}{7} \times 50} = \underline{0.0318} H.$

9. A coil when connected across a 10V d.c supply draws a current of 2A. When it is connected across a 10V - 50Hz a.c supply, the same coil draws a current of 1A. Explain why it draws lesser current in the second case. Hence determine the self inductance of the coil. Take $\bar{\pi} = 3$.

Soln. The coil draws lesser current in the second case because of the reactance offered by the inductor.

In case of d.c

$V = 10V \quad I = 2A$

$R = \frac{V}{I} = \frac{10}{2} = \underline{5\Omega}$

In case of a.c

$V_{rms} = 10V \quad I_{rms} = 1A$

$Z = \frac{V_{rms}}{I_{rms}} = \frac{10}{1} = \underline{\underline{10\Omega}}$

$X_L = \sqrt{Z^2 - R^2}$

$= \sqrt{10^2 - 5^2} = \sqrt{75} = \underline{\underline{25\sqrt{3}\ \Omega}}$

$X_L = 2\pi\nu L.$

$L = \frac{X_L}{2\pi\nu} = \frac{25\sqrt{3}}{2 \times 3 \times \cancel{50}_{2}} = \underline{\underline{0.0288\,H}}$

10. A student connects a long air core coil of manganin wire to a 100V d.c source and records a current of 1.5A. When the same coil is connected across 100V, 50Hz a.c source the current reduces to 1A.

(i) Give reason for this observation.

(ii) Calculate the value of the reactance of the coil.

Soln. For d.c circuit

$R = \frac{V}{I} = \frac{100}{1.5} = \underline{\underline{66.7\,\Omega}}$

For a.c circuit

$Z = \frac{V_{rms}}{I_{rms}} = \frac{100}{1} = \underline{100\,\Omega}$

Resistance is greater for a.c than d.c so current decreases in a.c circuit.

(ii) $Z = \sqrt{R^2 + X_L^2}$

$X_L^2 = Z^2 - R^2$

$X_L = \sqrt{(100)^2 - \left(\frac{1000}{15}\right)^2} = 100\sqrt{1 - \left(\frac{10}{15}\right)^2}$

$= 100\sqrt{\frac{15^2 - 10^2}{15^2}}$

$= 100\sqrt{\frac{225 - 100}{225}}$

$= 100\sqrt{\frac{125}{225}} = \frac{100\sqrt{5}}{3}$

$= \frac{100 \times 2.23}{3}$

$= \underline{\underline{74.53\,\Omega}}$

11. When 200 Volts d.c are applied across a coil a current of 2A flows through it. When 200V a.c of 50 cps are applied to the same coil, only 1A flows. Calculate the resistance, impedance and inductance of the coil.

Soln. For d.c circuit $\quad V = 200V$

$I = 2A$

$R = \frac{V}{I} = \frac{200}{2} = \underline{\underline{100\,\Omega}}$

(ii) for ac circuit

$V_{rms} = 200\,V$

$I_{rms} = 1\,A$

$\nu = 50\,Hz$

Impedence $Z = \frac{V_{rms}}{I_{rms}} = \frac{200}{1} = \underline{\underline{200\,\Omega}}$

(iii) $X_L = \sqrt{Z^2 - R^2}$

$= \sqrt{(200)^2 - (100)^2}$

$= \sqrt{30000}$

$= 100\sqrt{3}$

$\omega L = 100\sqrt{3}$

$L = \frac{100\sqrt{3}}{\omega} = \frac{100\sqrt{3}}{2\pi\nu} = \frac{100\sqrt{3}}{2 \times 3.14 \times 50}$

$= \underline{0.55\,H}$

12 An alternating current of $1.5\,mA$ rms and angular frequency $\omega = 100$ rad/s flows through a $10\,k\Omega$ resistor and $0.50\,\mu F$ capacitor in series. Calculate the value of rms voltage across the capacitor and the impedence of the circuit.

Here $\omega = 100$ rad/s

$I_{rms} = 1.5 \times 10^{-3}\,A$

$R = 10\,k\Omega$

$= 10^4\,\Omega$

$C = 0.50\,\mu F$

$= 0.50 \times 10^{-6}\,F$

$X_C = \frac{1}{C\omega} = \frac{1}{0.5 \times 10^{-6} \times 10^2} = \frac{10}{5 \times 10^{-4}} = 2 \times 10^4$

Impedence $Z = \sqrt{R^2 + X_C^2} = \sqrt{(10^4)^2 + (2 \times 10^4)^2}$

$= 10^4\sqrt{1+4}$

$= \underline{10^4 \times 2.23\,\Omega}$

$V_{rms} = I_{rms} \cdot X_C$

$= 1.5 \times 10^{-3} \times 10^4 \times 2$

$= \underline{\underline{30\,V}}$

13. A resistor of $200\,\Omega$ and a capacitor of $15\,\mu F$ are connected in series to a $220\,V$, $50\,Hz$ a.c source.

(a) Calculate the current in the circuit.

(b) Calculate the voltage across the resistor and the capacitor.

(c) Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Soln. $R = 200\,\Omega$

$C = 15\,\mu F = 15 \times 10^{-6}\,F$

$V_{rms} = 220\,V$

$\nu = 50\,Hz$.

(a) $I = \frac{V_{rms}}{Z}$.

$Z = \sqrt{R^2 + (X_C)^2}$

But $X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}}$

$= 212.3\,\Omega$

$= \sqrt{(200)^2 + (212.3)^2}$

$= \underline{\underline{291.5\,\Omega}}$

$\therefore I = \frac{220}{291.5} = \underline{0.755\,A}$

(b) $V_{rms} = I_{rms} \times R = 0.755 \times 200$

$= \underline{151\,V}$

$V_{rms} = I_{rms} \times X_C = 0.755 \times 212.3$

$= \underline{160.3\,V}$

(iii) Sum of the voltages

$= 151 + 160.3$

$= \underline{311.3\ V}$

which is more than 220 V.

These two voltages are 90° out of phase.

∴ resultant voltage

$V = \sqrt{V_R^2 + V_C^2}$

$= \sqrt{(151)^2 + (160.3)^2}$

$= \underline{220\ V}$.

14. A 0.3H inductor, 60 μF capacitor and a 60 Ω resistor are connected in series with 120 V, 60 Hz supply. Calculate

(i) impedence of the circuit

(ii) current flowing in the circuit.

Soln. $L = 0.3H$ $\quad \nu = 60Hz$

$C = 60 \times 10^{-6} F$

$R = 50 \Omega$

$X_C = \frac{1}{C\omega} = \frac{1}{60 \times 10^{-6} \times 2 \times 3.14 \times 60}$

$= \underline{44.23\ \Omega}$

$X_L = L\omega = L \cdot 2\pi\nu = 0.3 \times 2 \times 3.14 \times 60$

$= 113.04 \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(50)^2 + (113.04 - 44.23)^2} = \sqrt{50^2 + (68.81)^2}$

$= \underline{85\ \Omega}$

(ii) $I_{eff} = \frac{V_{rms}}{Z}$

$= \frac{120}{85} = \underline{1.41\ A}$.

15. A 100 mH inductor, a 20 μF capacitor and a 10 Ω resistor are connected in series to a 100 V, 50 Hz a.c source. Calculate

(i) impedence of the circuit at resonance

(ii) current at resonance

(iii) resonant frequency.

Soln. $L = 100 \times 10^{-3} H$

$C = 20 \times 10^{-6} F$

$R = 10 \Omega$

$V_{rms} = 100\ V$

$\nu = 50 Hz$.

(i) At impedence $Z = R$

$= \underline{10 \Omega}$

(ii) $I_{rms} = \frac{V_{rms}}{R} = \frac{100}{10} = \underline{10\ A}$

(iii) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100 \times 10^{-3} \times 20 \times 10^{-6}}}$

$= \frac{1}{\sqrt{2 \times 10^{-6}}} = \frac{1}{\sqrt{2}} \times 10^3$

$= \frac{1000}{1.414}$

$\therefore \nu_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14 \times \frac{1000}{1.414}} = \underline{112.6\ Hz}$