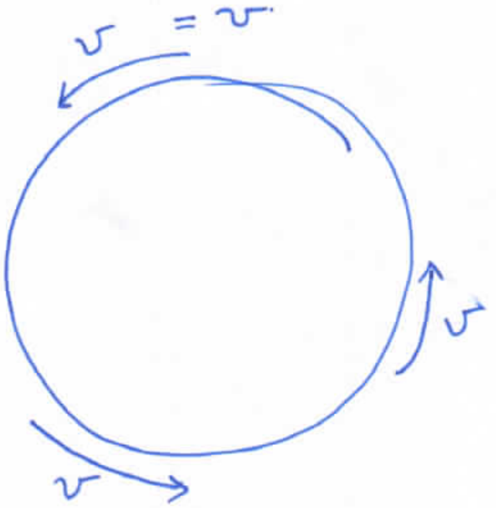
QUICK RECALL OF PREVIOUS CLASS

CIRCULAR MOTION

UNIFORM CIRCULAR MOTION

Speed of the body is constant



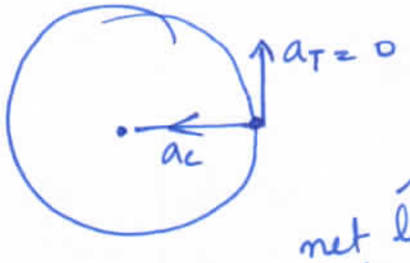
1) Speed = const

2) $\because$ speed of const

tangential acceleration $\rightarrow a_T = 0$ (a_T direction at any point is tangential)

3) centripetal acceleration $\rightarrow a_c = \frac{v^2}{r}$ (a_c direction at any point is towards the center of circle)

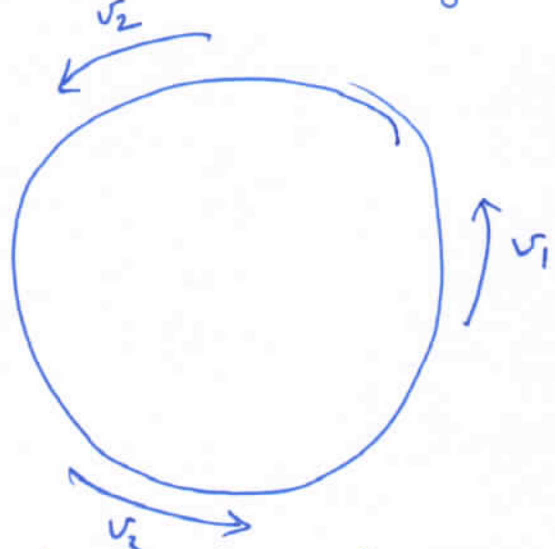
magnitude = const ($\because v$ is const)



$a = a_c$ ← net linear acceleration.

NON UNIFORM CIRCULAR MOTION

Speed is changing

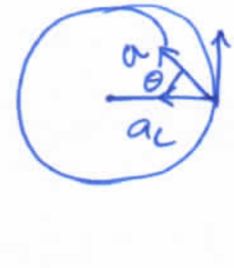


1) speed is changing.

2) $a_T \neq 0$

3) $a_c = \frac{v_p^2}{r}$

($\because$ speed is changing. magnitude of a_c is changing at each and every point)



$a = \sqrt{a_T^2 + a_c^2}$

$\vec{\theta}$ = angular displacement

$\vec{\omega}$ = angular velocity (angular frequency)

$= \frac{d\vec{\theta}}{dt}$ = rate of change of angular displacement

$\vec{\alpha}$ = angular acceleration

$= \frac{d\vec{\omega}}{dt}$ = rate of change of angular velocity

direction ?

Given by Right Hand thumb Rule.

Roll your fingers in direction of circular motion

The direction of these quantities is direction of your thumb.

$$\vec{\theta} = \frac{x}{r} \qquad \vec{\omega} = \frac{v}{r} \qquad \vec{\alpha} = \frac{a_T}{r}$$

UNIFORM

$a_T = 0$

$\vec{\alpha} = 0$

$\vec{\omega} = \text{const} = \frac{v}{r}$

$\theta = \omega t$

NON UNIFORM.

$a_T \neq 0$.

$\vec{\alpha} \neq 0$.

α is const.

$a_T \neq 0 = \text{const}$.

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 - \omega_i^2 = 2\alpha\theta$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

α is not const.

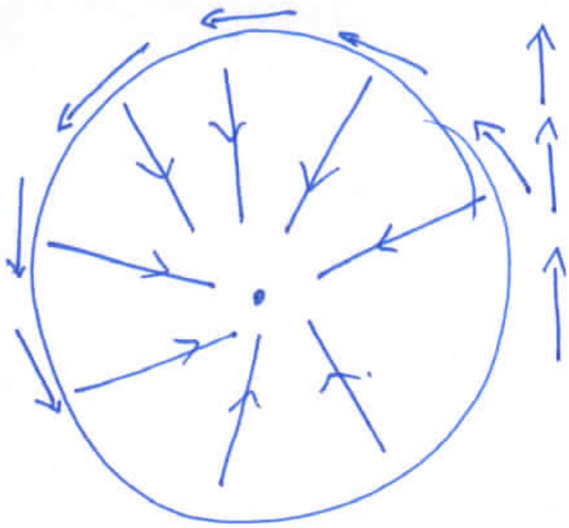
$a_T \neq 0 \neq \text{const}$.

Use calculus equations.

$\omega = \frac{d\theta}{dt}$

$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

$\alpha = \frac{\omega \, d\omega}{d\theta}$



$$a_c = \frac{v^2}{r}$$

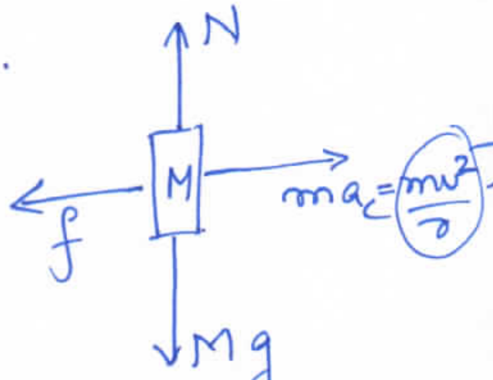
$$F_c = m a_c = \frac{mv^2}{r}$$

Force causing the circular motion is called the centripetal Force denoted by F_c

What Force acts as centripetal Force for below cases.

1) If we whirl a stone with the help of ~~the~~ a rope (Tension in the rope)

2) Car/bike moving along a circular track

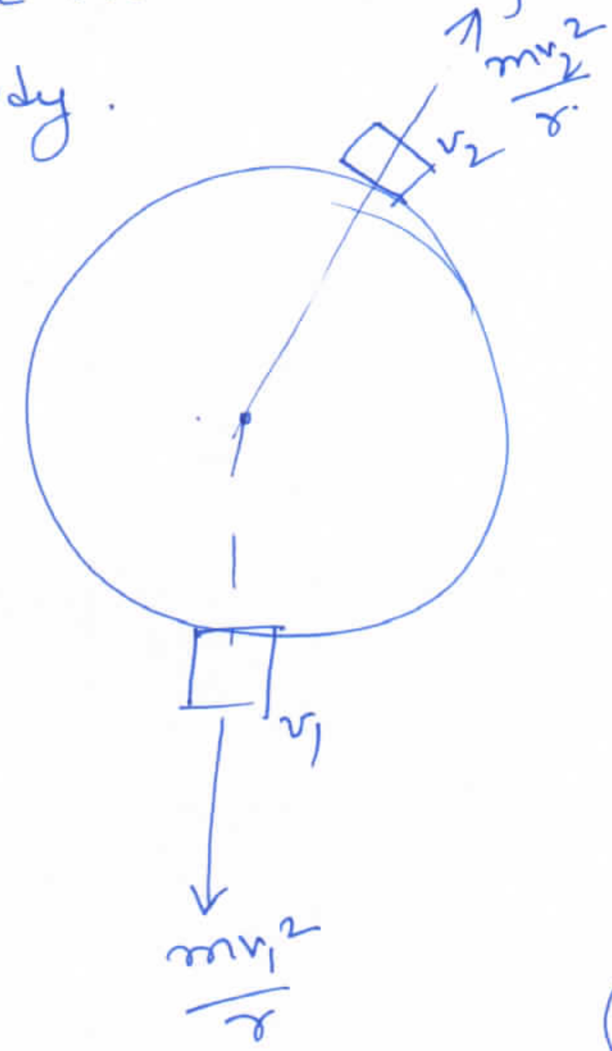


$$f = m a_c = \frac{mv^2}{r}$$

$$f = \frac{mv^2}{r}$$

A pseudo force named as centrifugal force is applied on body doing circular motion to elemenate the effect of centripetal force causing the ~~mo~~ circular motion.

direction of the centrifugal force is away from the center of ~~cir~~ circular motion and in the line joining the center & the body.



$$f = \frac{mv^2}{r}$$

$$\mu N = \frac{m v_{MAX}^2}{r}$$

$$V_{MAX} = \sqrt{\frac{r \mu N}{m}}$$

$$= \sqrt{\frac{r \mu \cancel{m} g}{\cancel{m}}}$$

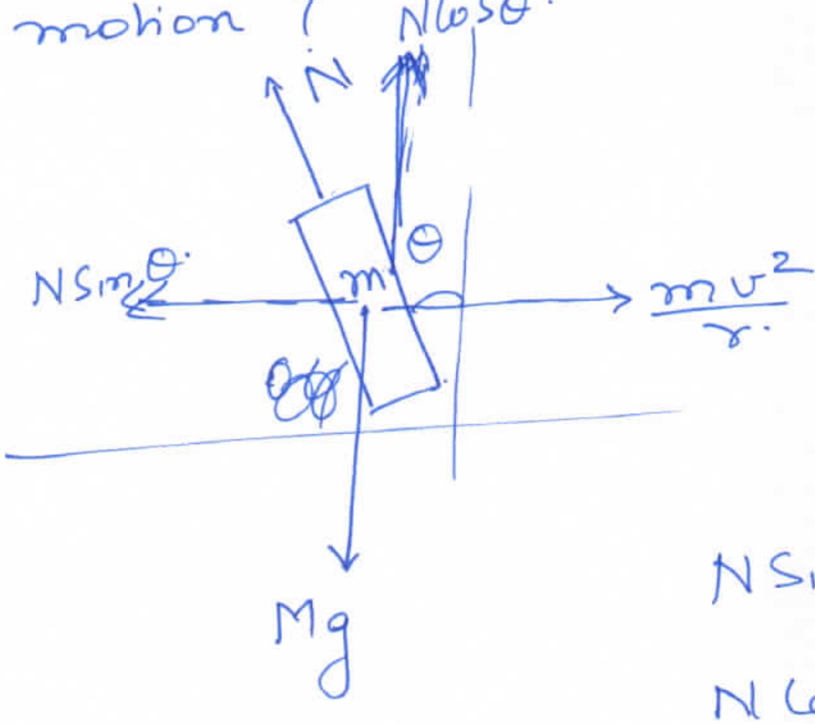
maximum velocity at which body can do circular motion of radius r $\Rightarrow$ $V_{MAX} = \sqrt{\mu r g}$

if $V \leq V_{MAX}$ then body can complete circular motion.

to increase V_{MAX}. $\Rightarrow$ solution is to increase the radius of circular motion.

Suppose friction effect is negligible

Is it possible to ~~comp~~ do circular motion?



$$N \sin\theta = \frac{mv^2}{r}$$

$$N \cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg}$$

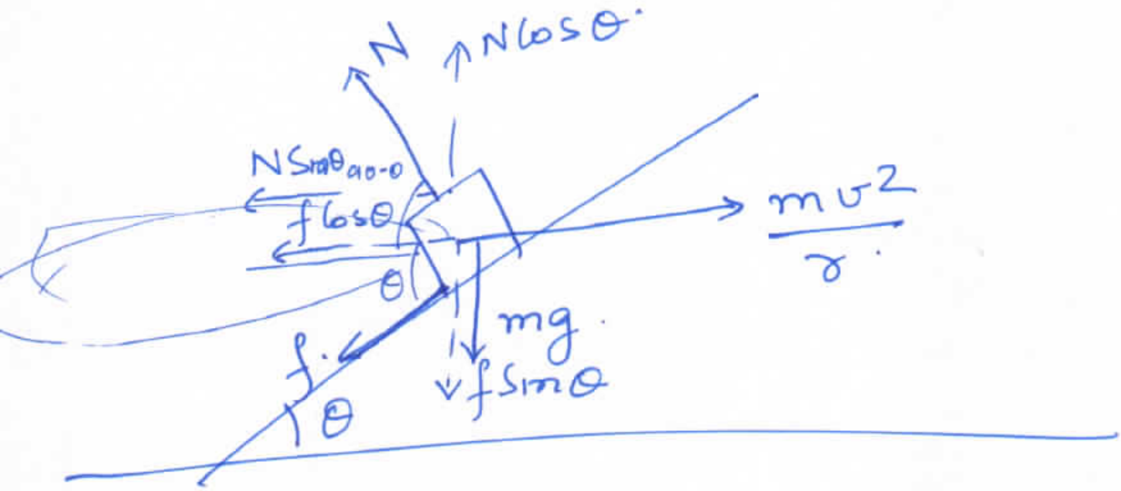
$$v = \sqrt{rg\tan\theta}$$

$v \uparrow$ $\quad \theta \uparrow$ $\quad$ or $\quad r \uparrow$.

$\uparrow$
easier

Case where friction is not neglibible & object is bending while doing circular motion.

BANKED ROAD



$$N\cos\theta = mg + f\sin\theta$$

$$\frac{mv^2}{r} = N\sin\theta + f\cos\theta$$

$f_{MAX} = \mu N$

$$N\cos\theta = mg + \mu N\sin\theta$$

① $N(\cos\theta - \mu\sin\theta) = mg$

$$\frac{mv_{MAX}^2}{r} = N\sin\theta + \mu N\cos\theta$$

② $\frac{mv_{MAX}^2}{r} = N(\sin\theta + \mu\cos\theta)$

$$\frac{②}{①} \Rightarrow \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{\frac{\cancel{m}v_{MAX}^2}{r}}{\cancel{m}g}$$

$$\Rightarrow \frac{\tan\theta + \mu}{1 - \mu\tan\theta} \times rg = v_{MAX}^2$$

$$v_{MAX} = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

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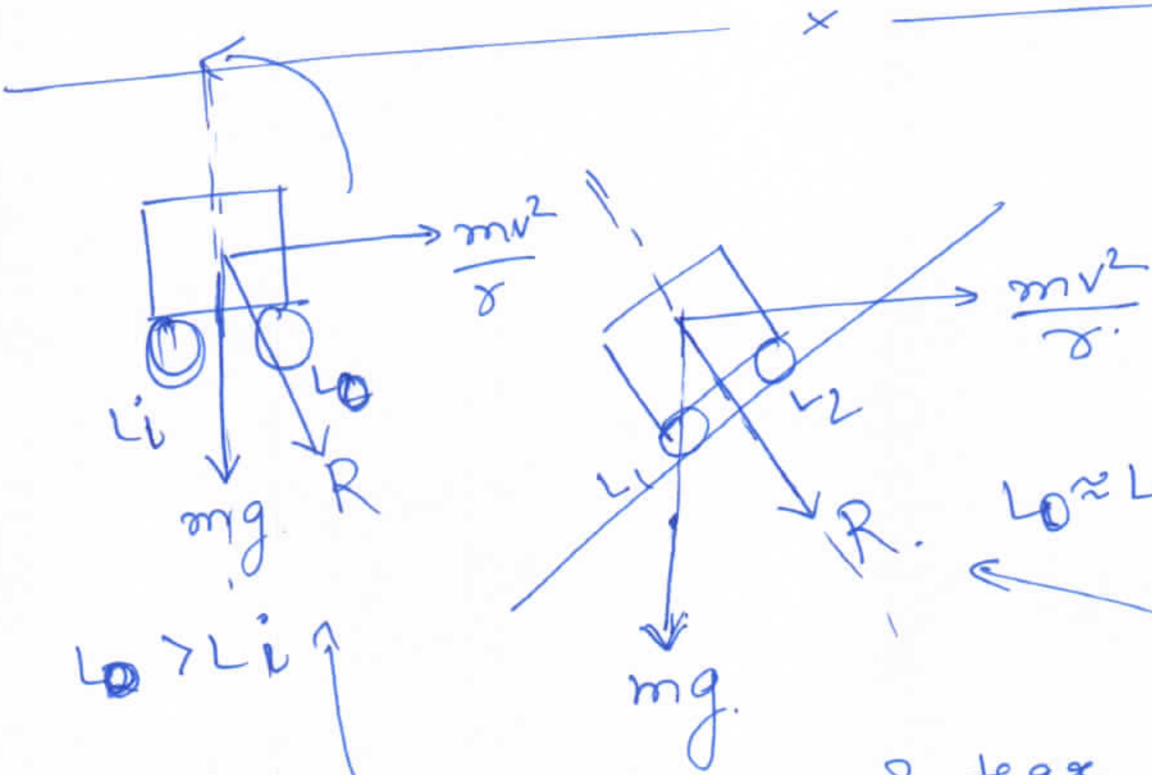
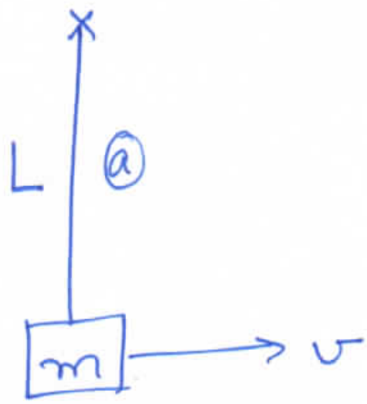


$L_O > L_i$ more wear & tear of outer tyres

$L_O \approx L_i$ almost equal wear & tear.

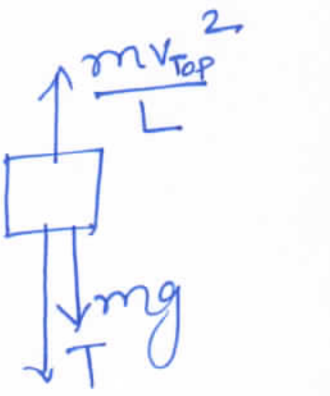
vertical plane.



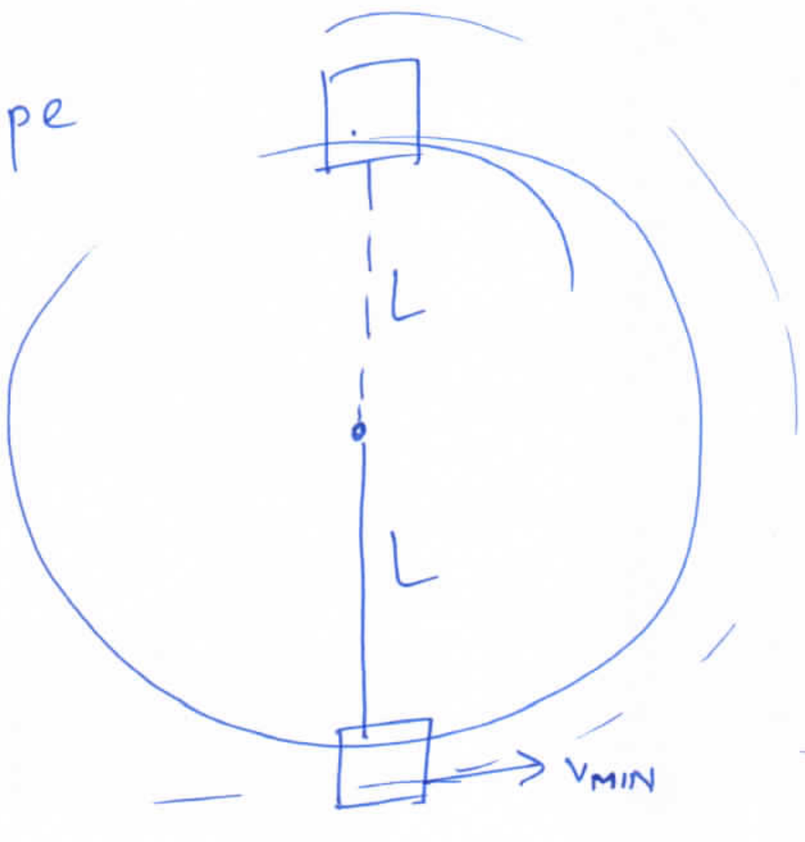
What should be minimum value of v for the body of m to complete vertical circles if

i) @ is a rope

ii) @ is a right rod.



i) rope



$$T = \frac{m v_{TOP}^2}{L} - mg$$

$$T \geq 0$$

$$v_{TOP} \geq \sqrt{gL}$$

$$0 + \frac{1}{2} m v_{MIN}^2 = mg(2L) + \frac{1}{2} m (\sqrt{gL})^2$$

$$\frac{1}{2} \cancel{m} v_{MIN}^2 = 2\cancel{m}gL + \frac{1}{2}\cancel{m}gL$$

$$v_{MIN}^2 = 5gL$$

$$v_{MIN} = \sqrt{5gL}$$

(b)

$$\frac{1}{2} m v_{MIN}^2 = mg(2L) + \frac{1}{2} m (0)^2$$

$$v_{MIN} = \sqrt{4gL}$$

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