QUADRATIC EQUATIONS TUTORIAL

Pg 90

3, 4, 5, 6, 7, 10

Pg 91 21, 22

Pg 92

4, 6, 14, 10

Pg 93

16, 17

Pg 95

1, 5

Pg 98

Comprehension 3

Pg 90

(3) $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$

$$\frac{(5-2\sqrt{6})(5+2\sqrt{6})}{(5+2\sqrt{6})} = \frac{1}{5+2\sqrt{6}}$$

$$(5+2\sqrt{6})^{x^2-3} + \left(\frac{1}{5+2\sqrt{6}}\right)^{x^2-3} = 10$$

$$(5+2\sqrt{6})^{x^2-3} = y$$

$$y + \frac{1}{y} = 10$$

$$y^2 - 10y + 1 = 0$$

$$y = \frac{10 \pm \sqrt{100-4}}{2} = 5 \pm 2\sqrt{6}$$

$5+2\sqrt{6}$, $(5-2\sqrt{6})$

$(5+2\sqrt{6})^{-1}$

$$(5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})^1$$

$x^2-3 = 1 \Rightarrow x = \pm 2$

$$(5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})^{-1}$$

$x^2-3=-1$ $\quad x^2 = 2$ $\quad x = \pm\sqrt{2}$

(4) If ~~a root of~~

$x^2 + px + 12 = 0$

$x^2 + px + q = 0$

$\alpha\beta = 12 \quad \beta = 3$

$\alpha + \beta = -p$

$\cancel{4+3} \quad p = -7$

$x^2 - 7x + 12$

$x^2 - 7x + q = 0$

$D = 0$

$49 - 4q = 0 \Rightarrow q = \frac{49}{4}$

(5) $\alpha + \beta = \frac{-b}{a}$

$= -2\frac{(m+n)}{a}$

$= -(m+n)$

Square $= (m+n)^2$ = root,

$x = -\frac{b}{2a} \pm \frac{\sqrt{D}}{2a}$.

$-\frac{b}{2a} + \frac{\sqrt{D}}{2a}, \frac{-b}{2a} - \frac{\sqrt{D}}{2a}$

$\frac{\sqrt{D}}{a}$

$\underline{\alpha - \beta} = \frac{\sqrt{b^2 - 4ac}}{a}$

$= \frac{b^2 - 4ac}{a^2}$

$= \frac{4(m+n)^2 - 4 \times 2(m^2+n^2)}{4}$

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$(m+n)^2 - 2(m^2+n^2)$

$= -(m-n)^2$

eq. $\Rightarrow x^2 - 4mnx - (m^2-n^2)^2$

The equation $= (x-(m+n)^2)(x+(m-n)^2)$

$= x^2 - 4mnx - (m^2-n^2)^2$

(6) Given $ax^2+bx+c=0$

if α, β are roots

$\beta = \alpha^2$

Now $\alpha+\beta = \frac{-b}{a}$ $\qquad \alpha\beta = \frac{c}{a}$

$\alpha+\alpha^2 = \frac{-b}{a}$ $\qquad \alpha^3 = \frac{c}{a}$

$a\alpha(1+\alpha) = -b$ $\qquad \alpha^3 a = c$

$b = -a\alpha(1+\alpha)$

Take LHS $= b^3+a^2c+ac^2$

$= [a\alpha(1+\alpha)]^3 + a^2(\alpha^3 a) + a(\alpha^3 a)^2$

$b^3 = -a^3\alpha^3(\alpha^3+1+3\alpha(1)(\alpha+1))$

$b^3 = -a^{\cancel{3}2}\frac{c}{\cancel{a}}\left(\frac{c}{a}+1+3\left(-\frac{b}{a}\right)\right)$

$b^3 = -a^2c\left(\frac{a+c-3b}{\cancel{a}}\right)$

$b^3 = -a^2c - ac^2 + 3abc.$

$b^3 + a^2c + ac^2 = 3abc.$

(7) $\alpha+\beta = -b/a \qquad \alpha\beta = \dfrac{c}{a}.$

$$\frac{1}{(a\alpha+b)^3} + \frac{1}{(a\beta+b)^3}$$

$$a\alpha+b = -a\beta$$

$$\frac{1}{(-a\beta)^3} + \frac{1}{(-a\alpha)^3}$$

$$= -\frac{1}{a^3}\left(\frac{1}{\beta^3}+\frac{1}{\alpha^3}\right)$$

$$= -\frac{1}{a^3}\left(\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}\right)$$

$$= -\frac{1}{\cancel{a^3}}\left(\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{c^3/\cancel{a^3}}\right)$$

$$= \cancel{+}\ \frac{(+b/a)\left((-b/a)^2-3\frac{c}{a}\right)}{c^3}$$

$$\frac{b}{a}\left(\frac{b^2-3ac}{a^2}\right)\times\frac{1}{c^3}$$

$$\Rightarrow \frac{b^3-3abc}{a^3c^3}$$

(10) $2x^2 + 3x + 5k = 0$
$x^2 + 2x + 3k = 0$ } A common root.

let α be the common root.

$$\frac{\alpha^2}{9k - 10k} = \frac{\alpha}{5k - 6k} = \frac{1}{4-3}$$

$$\frac{\alpha}{-k} = 1$$

$$\frac{\alpha^2}{\alpha} = \frac{9k - 10k}{5k - 6k}$$

$$\alpha = 1$$

$$1 + 2 + 3k = 0$$
$$k = -1$$

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(21)

$$\frac{3x^2 - 2x - 5}{x^2 - 2x + 5} > 2$$

$$\frac{3x^2 - 2x - 5}{x^2 - 2x + 5} - 2 > 0$$

$$\frac{x^2 + 2x - 15}{x^2 - 2x + 5} > 0$$

$$\frac{(x^2 + 2x - 15)(x^2 - 2x + 5)}{(\quad)^2} > 0$$

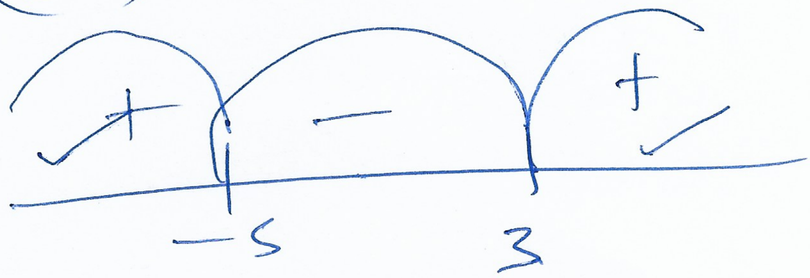
$(x+5)(x-3)(\ +ve\ ) \geq 0$

$x = 2 \pm \sqrt{4-20}$

$x^2 - 2x + 5 = (x-1)^2 + 4$

$x^2 - 2x + 1 + 4$

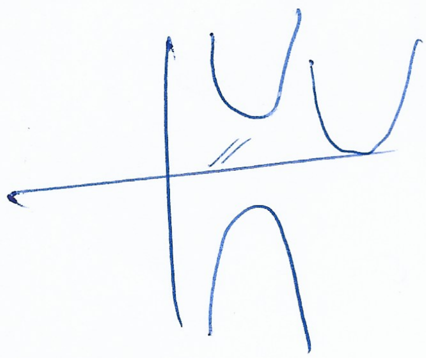
$(x-1)^2 + 4$



$x \in (-\infty, -5) \cup (3, \infty)$

(22)

$$\frac{x^2 + ax - 2}{x^2 - x + 1} < 2$$



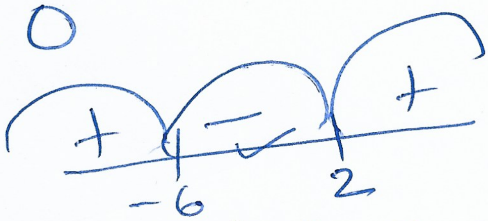
$$\frac{x^2 + ax - 2}{x^2 - x + 1} - 2 < 0$$

$$\frac{x^2 + ax - 2 - 2x^2 + 2x - 2}{\left(\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}\right)} < 0$$

$-x^2 + (a+2)x - 4 < 0$

$a \in (-6, 2)$

$x^2 - (a+2)x + 4 > 0$



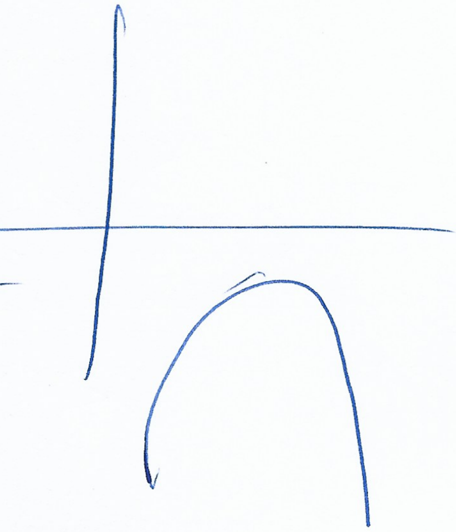
$(a+2)^2 - 4 \times 4 < 0$

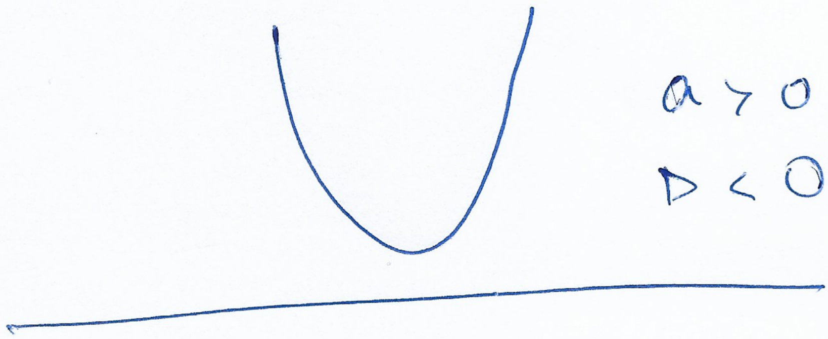
$a^2 + 4a - 12 < 0$

$(a+6)(a-2) < 0$

$$x^2 - (a+2)x + 4 < 0$$

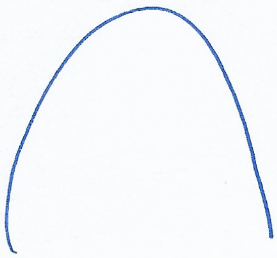
$$\cancel{-x^2 + (a+2)x - 4 > 0}$$





$a > 0$

$D < 0$



$a < 0$

$D < 0$

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4) $y = px + a\sqrt{1+p^2}$

$\Downarrow$ quadratic in p

$$\left(a\sqrt{1+p^2}\right)^2 = (y-px)^2$$

$$a^2(1+p^2) = y^2 + p^2x^2 - 2yxp$$

$$p^2(a^2-x^2) + 2xyp + a^2 - y^2 = 0$$

$\Downarrow$ $D = 0$

$$(2xy)^2 - 4(a^2-x^2)(a^2-y^2) = 0$$

$$\cancel{4x^2y^2} - 4a^4 + 4a^2y^2 + 4a^2x^2 - \cancel{4x^2y^2} = 0$$

$$\cancel{4a^2}(x^2+y^2) = \cancel{4}a^{4\,2}$$

$$x^2+y^2 = a^2$$

6) $$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$$ $\quad \alpha, -\alpha$

$$(2x+p+q)r = x^2 + (p+q)x + pq$$

$$x^2 + (p+q-2r)x + pq - pr - rq = 0$$

$$\alpha - \alpha = -\frac{(p+q-2r)}{1} \Rightarrow p+q = 2r$$

$$\alpha(-\alpha) = -\alpha^2 = \frac{pq-pr-rq}{1} = pq - (p+q)\frac{(p+q)}{2}$$

$$pq - \frac{(p+q)^2}{2} = -\frac{(p^2+q^2)}{2}$$

(14)

$$\left.\begin{array}{l} x^2+ax+bc=0 \\ x^2+bx+ca=0 \end{array}\right\} \begin{array}{c} \alpha, \beta_1 \\ \longrightarrow \\ \alpha, \beta_2 \end{array}$$ common root.

$$\frac{\alpha^2}{a^2c-b^2c} = \frac{\alpha}{bc-ca} = \frac{1}{b-a}$$

$$\alpha = c$$

$$\alpha^2 = \frac{c(a^2-b^2)}{(b-a)} \Rightarrow \alpha^2 = -c(a+b)$$

$$c^2 = -c(a+b) \Rightarrow a+b = -c$$

$$\cancel{\alpha + \beta_1 = -a}$$

$$\cancel{\beta_1 = -a-c}$$

$$\cancel{\alpha + \beta_2 = -b}$$

$$\cancel{\beta_2 = -b-c}$$

$$\alpha\beta_1 = bc$$

$$\alpha\beta_2 = ac$$

$$\left.\begin{array}{l} \beta_1 = b \\ \beta_2 = a. \end{array}\right\}$$

$$x^2 - (a+b)x + ab = 0$$

$$x^2 + cx + ab = 0$$

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(16) $$\frac{mx^2+3x+4}{x^2+2x+2} < 5$$

$$\frac{mx^2+3x+4-5x^2-10x-10}{x^2+2x+2} < 0$$

$$\frac{(m-5)x^2-7x-6}{x^2+2x+2} < 0$$

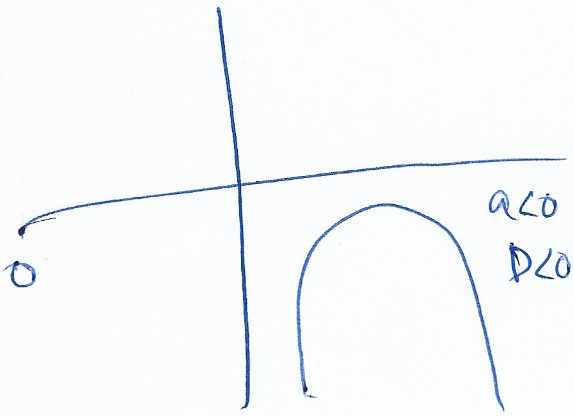
$$(m-5)x^2-7x-6 < 0$$

$$b^2-4ac < 0$$

$$49-4(m-5)(-6) < 0$$

$$49+24(m-5) < 0$$

$$24m-71 < 0$$



$m < \frac{71}{24}$ $\quad D<0$

&

$m < 5$ $\quad a<0$

$m < 71/24$.

(17) $$\frac{x^2}{x^2+x+1} \; ; \; x \in R$$

Let $\dfrac{x^2}{x^2+x+1} = y$

$$x^2 = yx^2 + xy + y$$

$$0 = x^2(y-1) + xy + y$$

As $x \in R$

$$D \geq 0$$

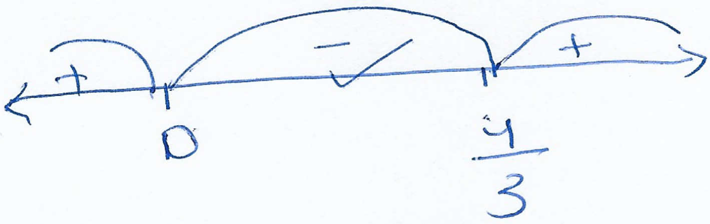
$$(y)^2 - 4(y-1)y \geq 0$$

$$y^2 - 4y^2 + 4y \geq 0$$

$$-3y^2 + 4y \geq 0$$

$$3y^2 - 4y \leq 0$$

$$y(3y-4) \leq 0$$



$y \in \left[0, \frac{4}{3}\right]$

~~As $y \geq 0$~~

max value is $\frac{4}{3}$ (B)

min value is 0

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(i) $$x^2 + \frac{x^2}{(x+1)^2} = 3$$

$$x^2(x+1)^2 + x^2 = 3(x+1)^2$$

$$x^2(x^2+2x+1) + x^2 = 3(x^2+2x+1)$$

$$x^4 + 2x^3 + x^2 + x^2 - 3x^2 - 6x - 3 = 0$$

$$x^4 + 2x^3 - x^2 - 6x - 3 = 0$$

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \cdots a_n = 0$$

$$\alpha_1 + \alpha_2 = -\frac{a_1}{a_0}$$

$$a_0x^2 + a_1x + a_2 = 0$$

$$a_0x^3 + a_1x^2 + a_2x + a_3 = 0$$

$\alpha_1, \alpha_2, \alpha_3$

$\alpha_1, \alpha_2, \alpha_3 \cdots \alpha_n$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \cdots \alpha_n = -\frac{a_1}{a_0}$$

$$\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_1\alpha_4 + \cdots \alpha_2\alpha_3 + \cdots \alpha_{n-1}\alpha_n = \frac{a_2}{a_0}$$

$$\alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_3\alpha_4 + \alpha_1\alpha_4\alpha_5 + \cdots \alpha_2\alpha_3\alpha_4 + \cdots \alpha_{n-2}\alpha_{n-1}\alpha_n = -\frac{a_3}{a_0}$$

$$\alpha_1 + \alpha_2 + \alpha_3 = -\frac{a_1}{a_0}$$

$$\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 = +\frac{a_2}{a_0}$$

$$\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0}$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = -2$$

$$\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_1\alpha_4 + \alpha_2\alpha_3 + \alpha_2\alpha_4 + \alpha_3\alpha_4 = -1$$

$$\alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_2\alpha_4 + \alpha_1\alpha_3\alpha_4 + \alpha_2\alpha_3\alpha_4 = +6.$$

$$\alpha_1\alpha_2\alpha_3\alpha_4 = -3$$

$$(x^2 + a_1 x + b_1)(x^2 + a_2 x + b_2) = 0$$

$$b_1 b_2 = -3$$

1	-3
-1	3.

$$(x^2 + a_1 x \cancel{+1})(x^2 + a_2 x + 3) = 0$$

$$x^4 + (a_1 + a_2)x^3 + (a_1 a_2 + 2)x^2 + (3a_1 \cancel{+} a_2)x - 3 = 0$$

$$\left.\begin{array}{l} a_1 + a_2 = 2. \\ 3a_1 \cancel{+} a_2 = -6 \end{array}\right] \longrightarrow \begin{array}{l} a_1 = +2. \\ a_2 = \text{[illegible]} \end{array}$$

$$4a_1 = -4$$

$$a_1 = -1$$

$$a_2 = 3.$$

$$\underbrace{(x^2 - x - 1)}_{\text{real roots}}\underbrace{(x^2 + 3x + 3)}_{\text{Imaginary roots!}} = 0$$

(5) $(x^2+2)^2 + 8x^2 = 6x(x^2+2)$

$x^4+4x^2+4 + 8x^2 \pm 6x^3 + 12x.$

$x^4 - 6x^3 + 12x^2 - 12x + 4 = 0$

$(x^2+ax+b)(x^2+cx+d) = 0$

$bd = 4$

$x^4 + (a+c)x^3 + (b+d+ac)x^2 + (ad+bc)x + bd = 0$

b	d
2	2
1	4
-1	-4
-2	-2

$a+c = -6$

$b+d+ac = 12 \longrightarrow ac = 8$

$ad+bc = -12$

$a+c = -6$

a	c	b	d
-4	-2	2	2

$(x^2-4x+2)(x^2-2x+2) = 0$

$x = \dfrac{4 \pm \sqrt{16-8}}{2}$

$= 2 \pm \sqrt{2}$

$x = \dfrac{2 \pm \sqrt{4-8}}{2}$

$x = 1 \pm i$

$\sqrt{-1} = i$