ROTATIONAL MOTION

Circular Motion – Point mass moves in a circular path.

Rotational motion – System of particles is moving in a circular path about a fixed point

System of particles – Group of particles at fixed distances from each other.

↓

body

↓

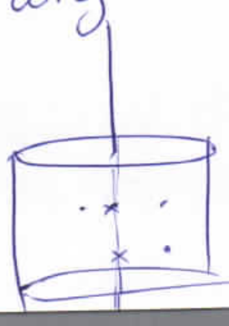
Rigid body (No matter what is the case the distance between the particles doesn't change by normal forces)

Rotational Motion

When a system of particles is moving in such a way that all its particles have same angular velocity about a common axis

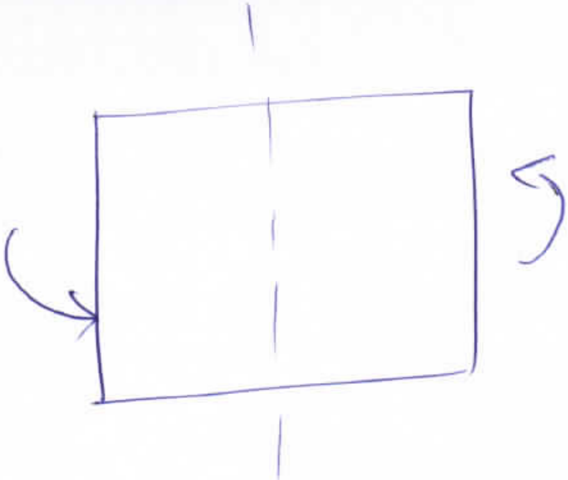
↓

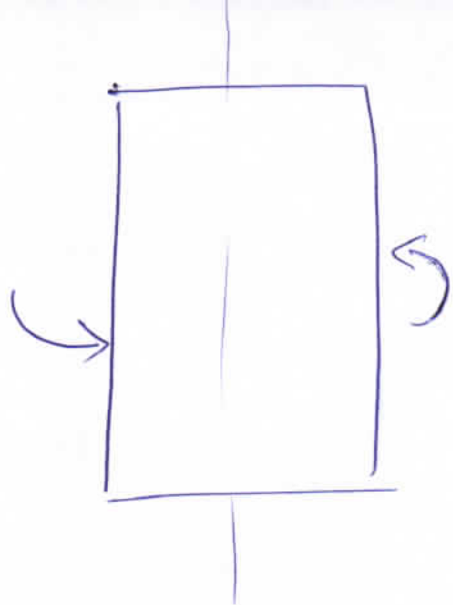
Axis of rotation.

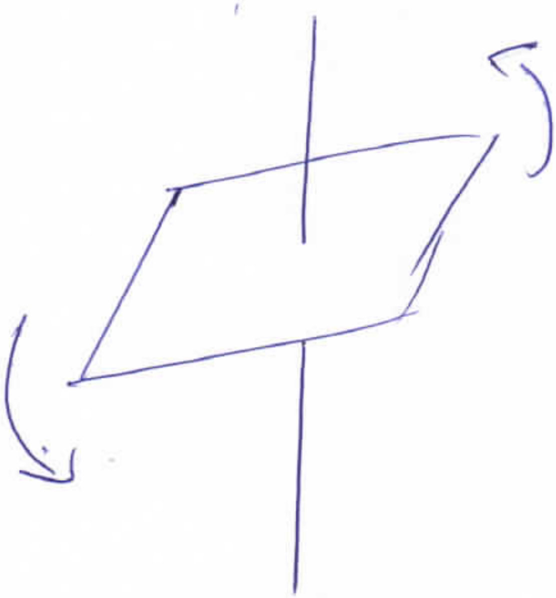


locus

line joining the centers of each particle circular motion

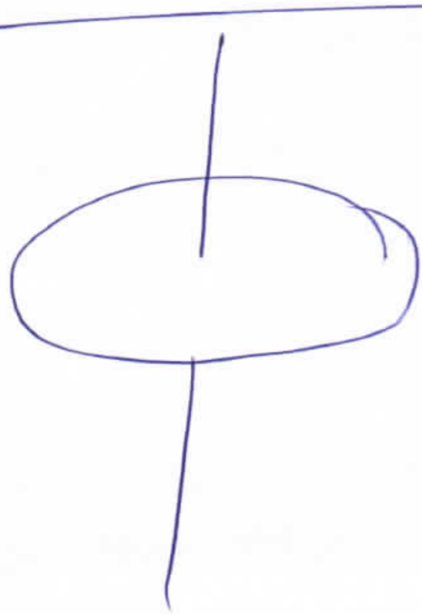




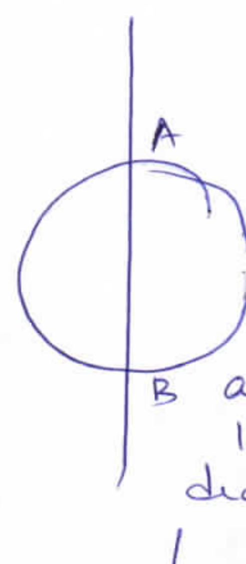


Axis of Rotation is always normal to the plane of rotation

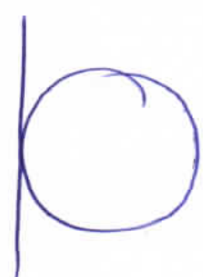
<u>Nomenclature of Axis</u>



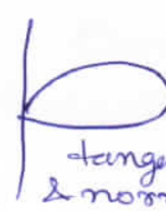
passing through the center and normal to the plane



along its diameter



tangential to the surface



tangential & normal

CENTRE OF MASS.

Its the specific location for a system of particles.
(a point)

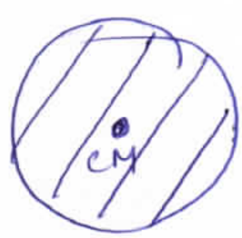
We assume all the mass of the system of particles to be at that point.

Centre of Mass of Different bodies:

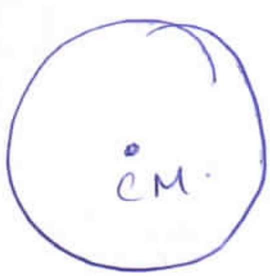
If mass of body is symmetrical then the geometrical centre is the centre of mass.

Ring

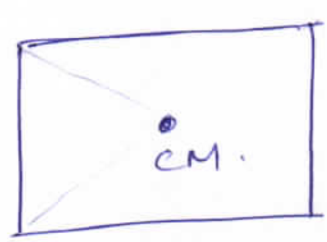
Disc.



Rod

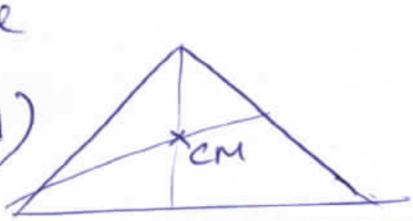


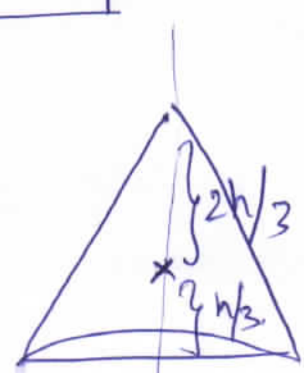
sphere.



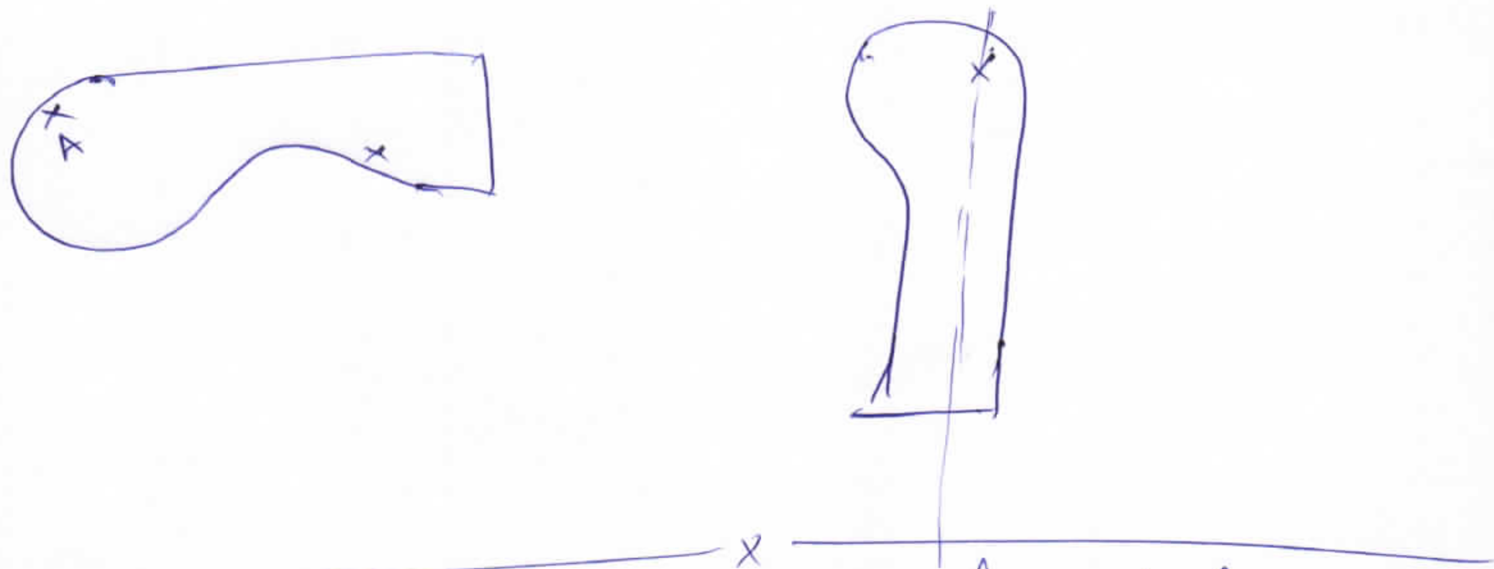
Rectangle.

triangle
(centroid)



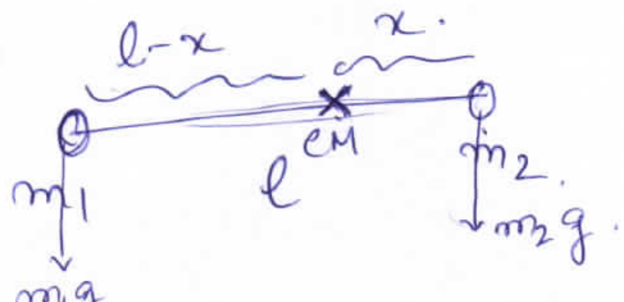


Suppose I have a asymmetrical body.



Calculating CM for a system of particles.

i) 2 particle system.



$m_2 > m_1$

$$-m_2 g x + m_1 g(l - x) = 0$$

Moment about a point = $F \times (\perp$ distance from the point$)$

$$m_2 g x = m_1 g (l - x)$$

$$m_2 x + m_1 x = m_1 l$$

$$\boxed{x = \frac{m_1 l}{m_1 + m_2}}$$

x is distance of CM from m_2

ii) Many particle system.

May or may not be in same plane.

let m_1 be at (x_1, y_1, z_1) m_2 at (x_2, y_2, z_2) . - - m_n at (x_n, y_n, z_n)

let centre of mass coordinates be

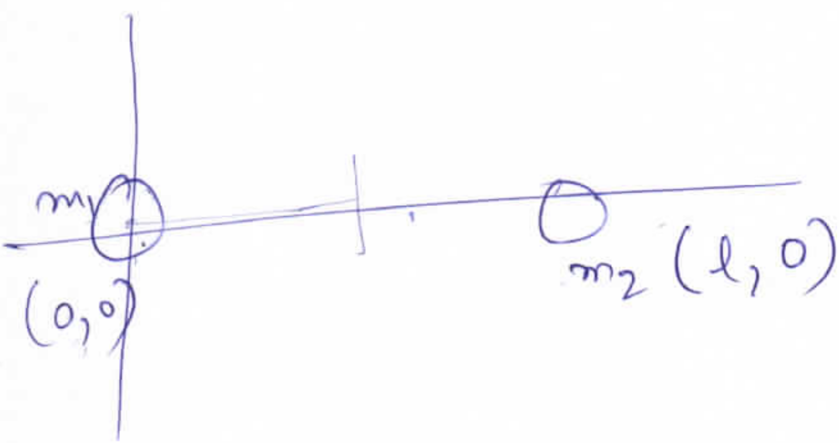
(x_{cm}, y_{cm}, z_{cm})

taking moments about the origin.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 \cdots m_n x_n}{m_1 + m_2 + m_3 + \cdots m_n}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots m_n y_n}{m_1 + m_2 + \cdots m_n}$$

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \cdots m_n z_n}{m_1 + m_2 + m_3 \cdots m_n}$$

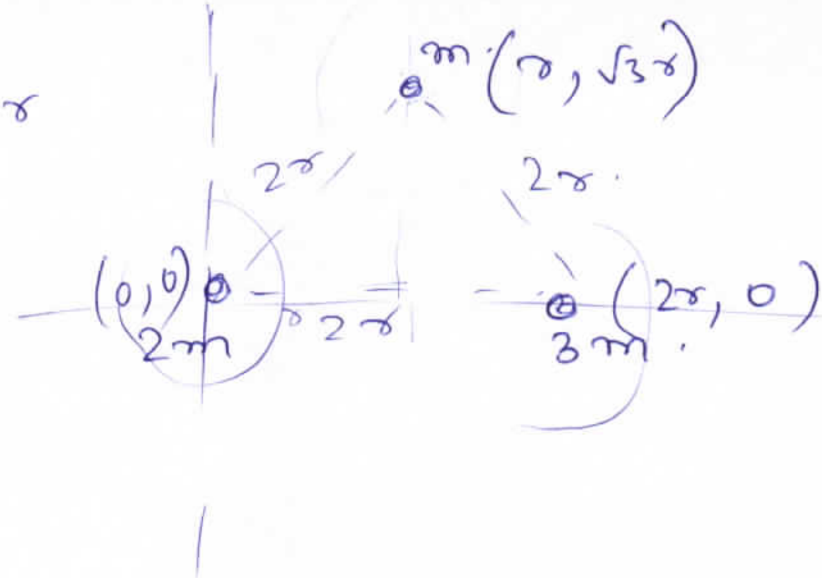


$$x_{cm} = \frac{m_1 \times 0 + m_2 \times l}{m_1 + m_2} = \frac{m_2 l}{m_1 + m_2}$$

$$y_{cm} = \frac{m_1 \times 0 + m_2 \times 0}{m_1 + m_2} = 0$$

$$\left(\frac{m_2 l}{m_1 + m_2}, 0\right)$$

Calculate CM for



$$x_{CM} = \frac{2m \times 0 + m \times r + 3m \times 2r}{2m + m + 3m}$$

$$= \frac{7\cancel{m}r}{6\cancel{m}} = \frac{7r}{6}$$

$$y_{CM} = \frac{2m \times 0 + 3m \times 0 + m \times \sqrt{3}r}{2m + m + 3m}$$

$$= \frac{\sqrt{3}r}{6}$$

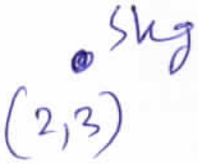
$\left(\frac{7r}{6}, \frac{\sqrt{3}r}{6}\right)$ from $2m$ mass.

If there are large bodies (instead of point masses)

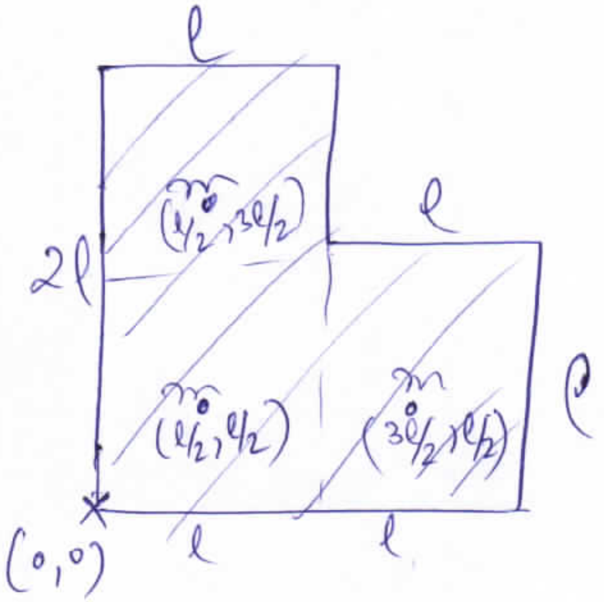
then the body should be converted

into point mass by concentrating its

mass at the CM of the body.



→ Calculate location of CM of the lamina shown.



mass = 3m.

$$x_{CM} = \frac{m \times l/2 + m \times l/2 + m \times 3l/2}{m + m + m}$$

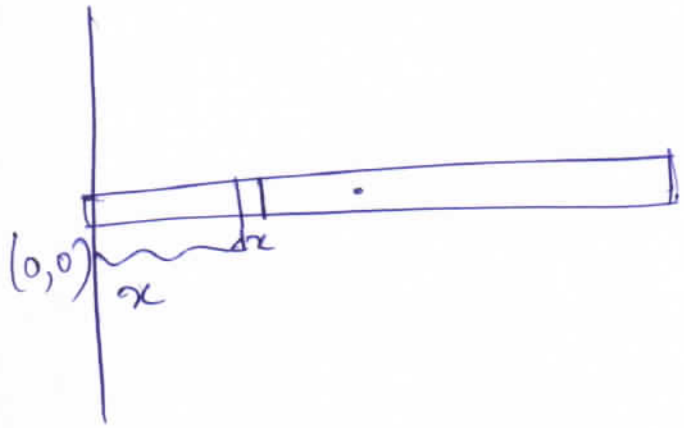
$$= 5l/6$$

$$y_{CM} = \frac{m \times l/2 + m \times \frac{3l}{2} + m \times l/2}{m + m + m}$$

$$= 5l/6$$

CENTRE OF MASS OF CONTINUOUS BODIES.

a) UNIFORM STRAIGHT ROD of mass M length L.



$$dm = M \times \frac{dx}{L}$$

$$CM = \frac{\int_0^L dm \times x}{\int_0^L dm}$$

$$= \frac{\int_0^L \frac{Mx\,dx}{L}}{\int_0^L \frac{M\,dx}{L}}$$

$$= \frac{\cancel{\frac{M}{L}} \left[\frac{x^2}{2}\right]_0^L}{\cancel{\frac{M}{L}} \left[x\right]_0^L} = \frac{\frac{L^2}{2} - 0}{L - 0}$$

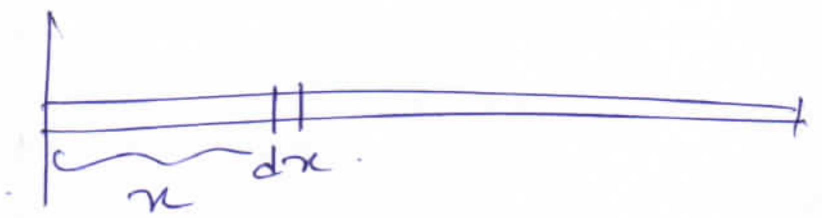
$$= L/2.$$

b) Rod Mass M, L.

Mass per unit length (λ) varies with distance from its left end.

as $\lambda = kx$ ← x is distance from left end.

Find its CM



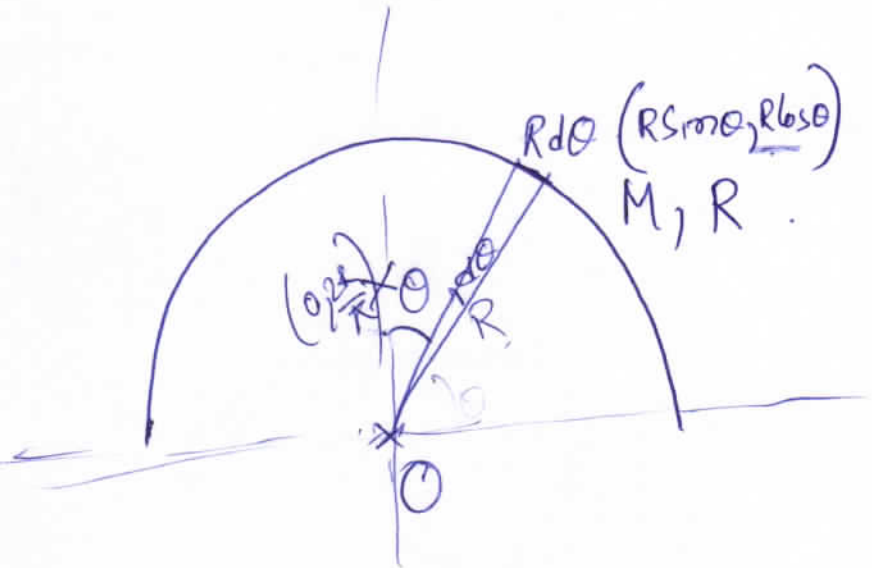
$$dm = kx \times dx.$$

$$CM = \frac{\int_0^L dm\, x}{\int_0^L dm}$$

$$= \frac{\int_0^L kx \times x\, dx}{\int_0^L kx\, dx} = \frac{\left. \frac{kx^3}{3} \right]_0^L}{\left. \frac{kx^2}{2} \right]_0^L}$$

$$= 2L/3$$

c) Centre of mass of a uniform semicircular wire of Mass M & Radius R.



$$dm = \frac{M}{\pi R} \times R\,d\theta$$

$$y_{cm} = \frac{\int dm\, y}{\int dm}$$

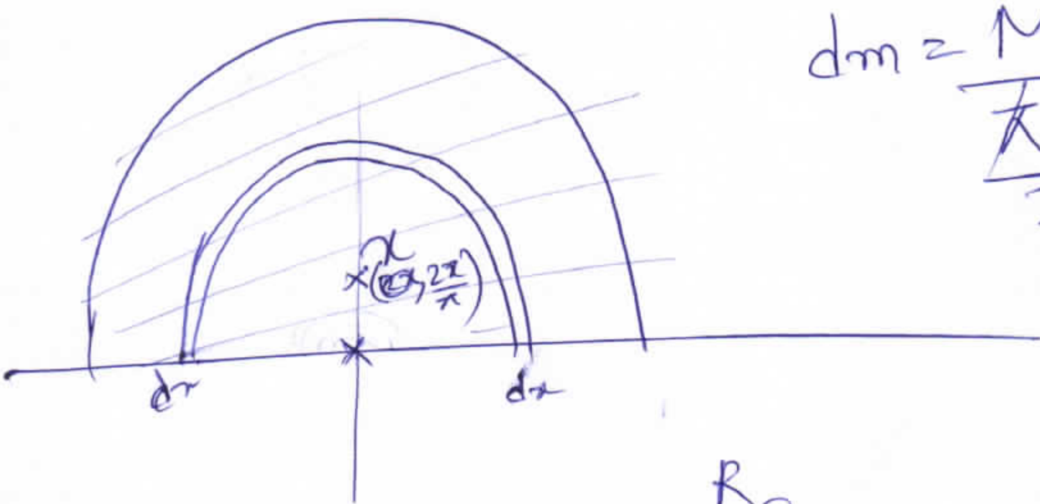
$$= \frac{\int_{-\pi/2}^{\pi/2} \frac{\cancel{M}}{\pi \cancel{R}} \cancel{R}\,d\theta \times R\cos\theta}{\int_{-\pi/2}^{\pi/2} \frac{\cancel{M}}{\cancel{\pi} R} \times \cancel{R}\,d\theta}$$

$$cm = \left(0, \frac{2R}{\pi}\right)$$

$$y_{cM} = \frac{R \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta}{\int_{-\pi/2}^{\pi/2} d\theta} \qquad \frac{R\sin\theta\Big]_{-\pi/2}^{\pi/2}}{\theta\Big]_{-\pi/2}^{\pi/2}}$$

$$= \frac{2R}{\pi}$$

d) Centre of mass of a uniform semicircular plate of Mass M & Radius R.



$$dm = \frac{M}{\frac{\cancel{\pi} R^2}{2}} \times \cancel{\pi} x \times dx = \frac{2M}{R^2}\, x\, dx.$$

$$y_{cm} = \frac{\int_0^R dm \times \frac{2x}{\pi}}{\int_0^R dm}$$

$$y_{cm} = \frac{\int_0^R \frac{2\cancel{M}}{R^2} \times x\, dx \times \frac{2x}{\pi}}{\int_0^R \frac{2\cancel{M}}{R^2}\, x\, dx}$$

$$y_{cm} \cdot \frac{2}{\pi} \int_0^R x^2 dx \Big/ \int_0^R x\, dx = \frac{4R}{3\pi}$$

H.W.

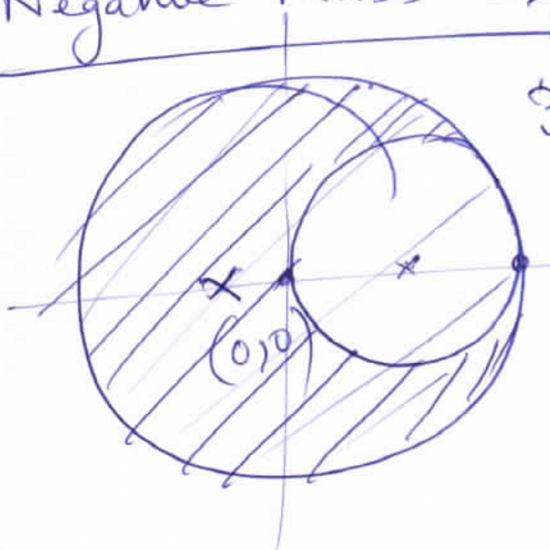
Find CM of hollow Hemisphere (M, R)

↓

Using above result

Find CM of solid Hemisphere (M, R)

Negative Mass Effect

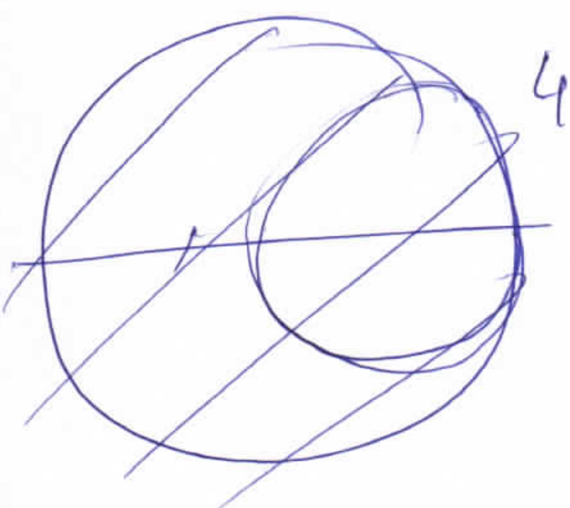


3M, R.

$$\pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \frac{3\pi R^2}{4}$$

$$\frac{3\pi R^2}{4} \propto 3M$$

$$\pi R^2 \propto 4M$$



4M, R.

-M, R/2.

4M (0,0) -M (R/2, 0)

$$x_{cm} = \frac{4M \times 0 + (-M)\frac{R}{2}}{4M + (-M)} = \frac{-MR/2}{3M} = \frac{-R}{6}$$