

CIRCULAR MOTION TUTORIAL.

Pg 69-70

3, 5, 6

✓ Comprehension.

Pg 70

✓ Matrix Type

Pg 71

✓ 1, 2, 3, 5

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✓ 2, 2, 4, 5, 7

Pg 79

9, 11, 12, 14, 16, 17, 19

Pg 80-81

Comp 1 & 2

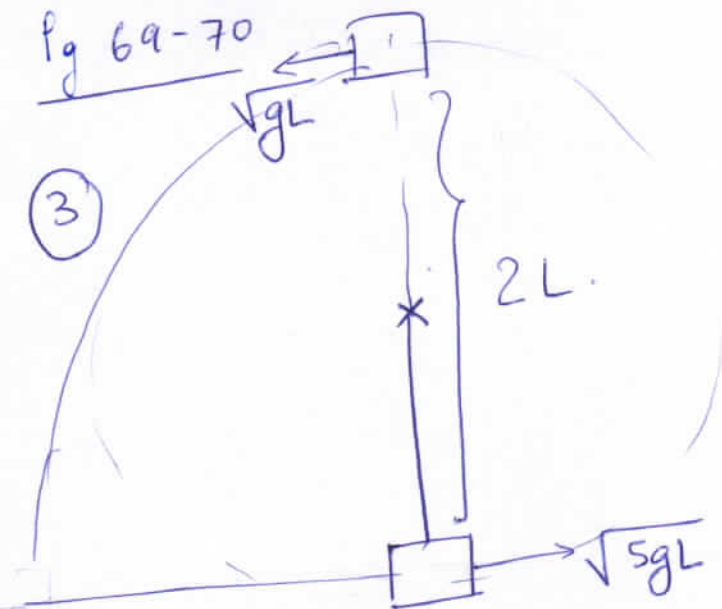
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(3)



$$H = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2H}{g}}$$

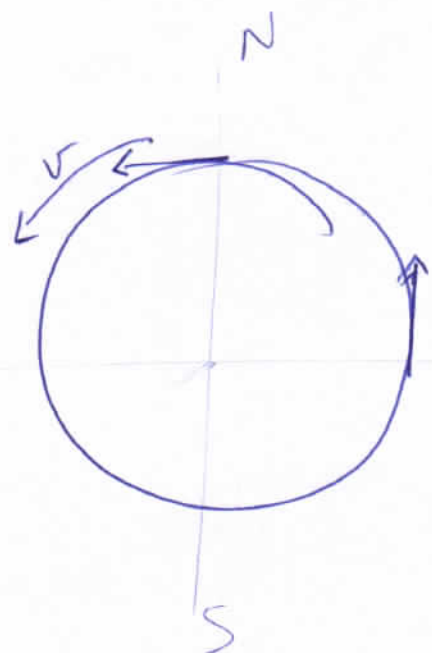
$$t = \sqrt{\frac{2 \times 2L}{g}}$$

$$R = \sqrt{gL} \times t$$

$$= \sqrt{gL} \times \sqrt{\frac{4L}{g}}$$

$$= 2L$$

(5)



F

$$= \sqrt{2} v \quad \sqrt{v^2 + v^2}$$

$$\vec{v}_2 - \vec{v}_1$$

$$\vec{v}_2 + (-\vec{v}_1)$$

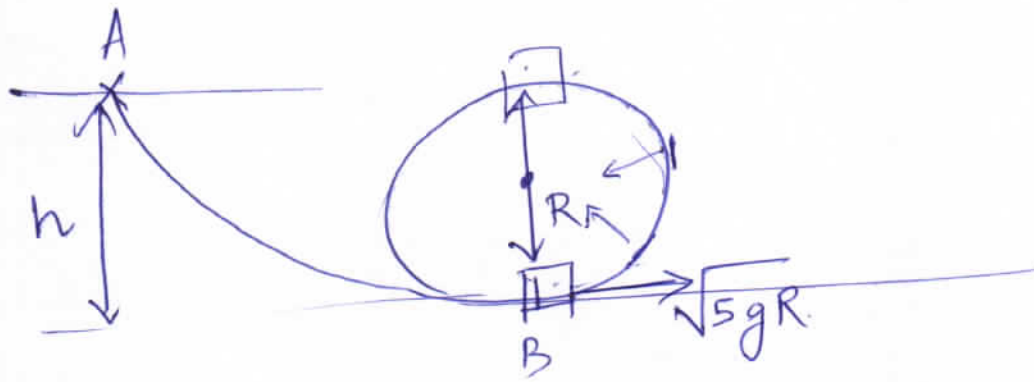
$$\sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos \theta}$$

$$= \sqrt{v^2 + v^2 + 2vv \cos 90}$$

$$= \sqrt{v^2 + v^2}$$

$$= \sqrt{2} v \quad \text{South west}$$

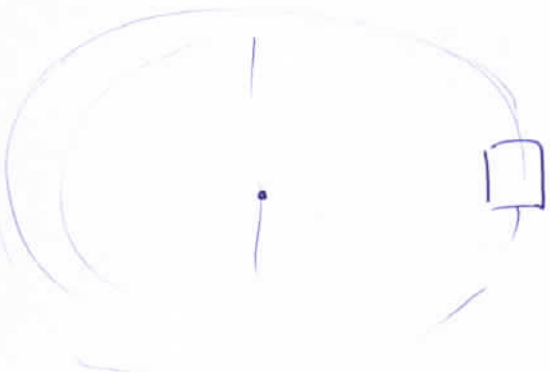
6



$$N = \frac{mv^2}{R} - mg$$

$$\begin{aligned} \text{A.} \quad mgh + \frac{1}{2}m(0)^2 &= \text{B.} \quad 0 + \frac{1}{2}m(\sqrt{5gR})^2 \\ mgh &= \frac{1}{2}m \times 5gR \\ h &= \frac{5}{2}R = \frac{5}{2} \times 5 = 12.5 \text{ m.} \end{aligned}$$

Comprehension :



$$\begin{aligned} \omega_0 &\longrightarrow 0 \\ 0 &\longrightarrow t \end{aligned}$$

$$\begin{aligned} \alpha &\propto \sqrt{\omega} \\ \alpha &= -k\sqrt{\omega} \\ \frac{d\omega}{dt} &= -k\sqrt{\omega} \\ \int_{\omega_0}^0 \frac{d\omega}{\sqrt{\omega}} &= \int_0^t -k dt \end{aligned}$$

$$\int_{\omega_0}^0 \omega^{-1/2} d\omega = -k \int_0^t 1 dt$$

$$x^n = \frac{x^{n+1}}{n+1}$$

$$\left[\frac{\omega^{1/2}}{1/2} \right]_{\omega_0}^0 = -k t \left[1 \right]_0^t$$

$$\frac{0^{1/2}}{1/2} - \frac{\omega_0^{1/2}}{1/2} = -k(t-0)$$

$$-2\omega_0^{1/2} = -k t$$

$$4\omega_0 = k^2 t^2 \Rightarrow t^2 = 4 \frac{\omega_0}{k^2}$$

$$\lambda = 4 \quad (\text{D})$$

$$d = \omega \frac{d\omega}{d\theta}$$

$$a = v \frac{dv}{ds} \quad a = \frac{dv}{dt}$$

$$\frac{\omega d\omega}{d\theta} = -k\sqrt{\omega}$$

$$\frac{\omega d\omega}{\sqrt{\omega}} = -k d\theta$$

$$\Rightarrow \int_{\omega_0}^0 \sqrt{\omega} d\omega = \int_0^\theta -k d\theta$$

$$\frac{\omega^{3/2}}{3/2} \Bigg|_{\omega_0}^0 = -k\theta \Bigg|_0^\theta$$

$$\frac{0^{3/2}}{3/2} - \frac{\omega_0^{3/2}}{3/2} = -k(\theta - 0)$$

$$\frac{2\omega_0\sqrt{\omega_0}}{3} = k\theta$$

$$\theta = \frac{2\omega_0\sqrt{\omega_0}}{3k} \quad (B)$$

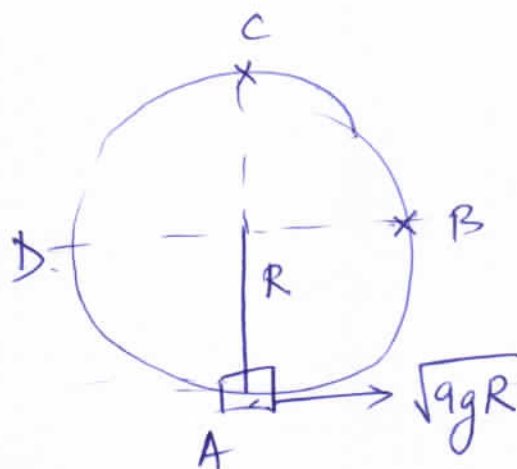
$$\begin{aligned}\omega_0^{3/2} &= \omega_0^{1+1/2} \\ &= \omega_0 \omega_0^{1/2} \\ &= \omega_0 \sqrt{\omega_0}\end{aligned}$$

$$\langle \omega \rangle = \frac{\text{total angular displacement}}{\text{total time}}$$

$$= \frac{\frac{2\omega_0\sqrt{\omega_0}}{3k}}{\frac{2\sqrt{\omega_0}}{k}} = \frac{\omega_0}{3} \quad (A)$$

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Matrix



$$P \cdot F = 0$$

T.E B

= T.E A.

$$\frac{1}{2} m v_B^2 + mgR = \frac{1}{2} m (\sqrt{9gR})^2 + 0$$

$$\frac{1}{2} m v_B^2 + mgR = \frac{9}{2} mgR$$

$$\frac{1}{2} m v_B^2 = \frac{7mgR}{2}$$

$$v_B = \sqrt{7gR}$$

A \rightarrow R.

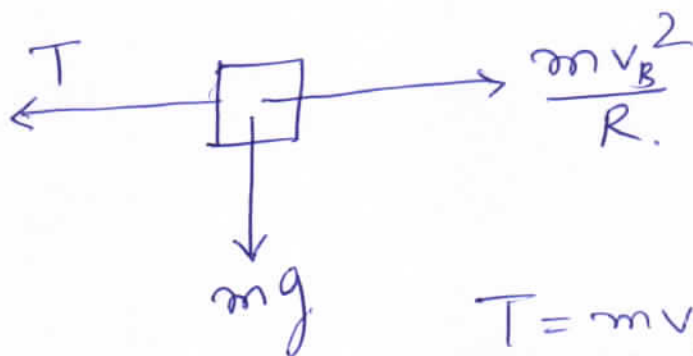
T.E C.

T.E A.

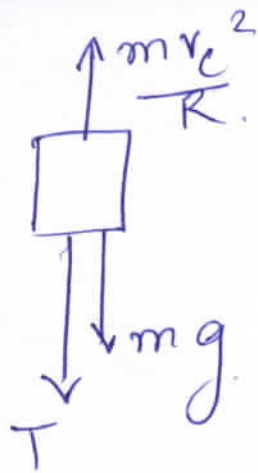
$$\frac{1}{2} m v_C^2 + mg(2R) = \frac{9}{2} mgR$$

$$\frac{1}{2} m v_C^2 = \frac{5mgR}{2}$$

$$v_C = \sqrt{5gR}$$

B \rightarrow Q.C \rightarrow P.

$$T = \frac{m v_B^2}{R} = \frac{m (\sqrt{7gR})^2}{R} = 7mg$$



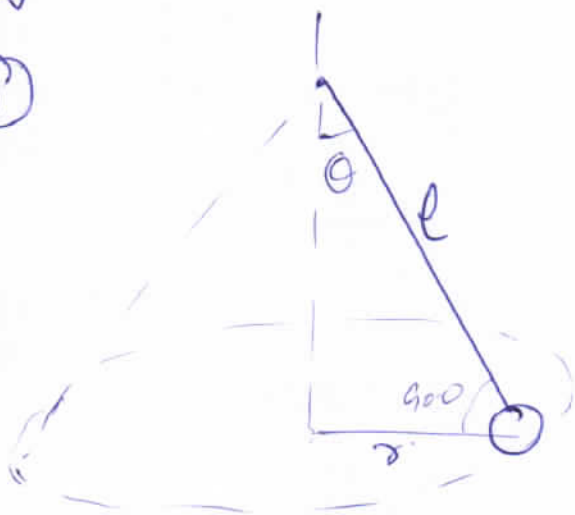
$$T + mg = \frac{mv_c^2}{R}$$

$$\begin{aligned} T &= \frac{mv_c^2}{R} - mg \\ &= \frac{m(\sqrt{5gr})^2}{R} - mg \\ &= 4mg. \end{aligned}$$

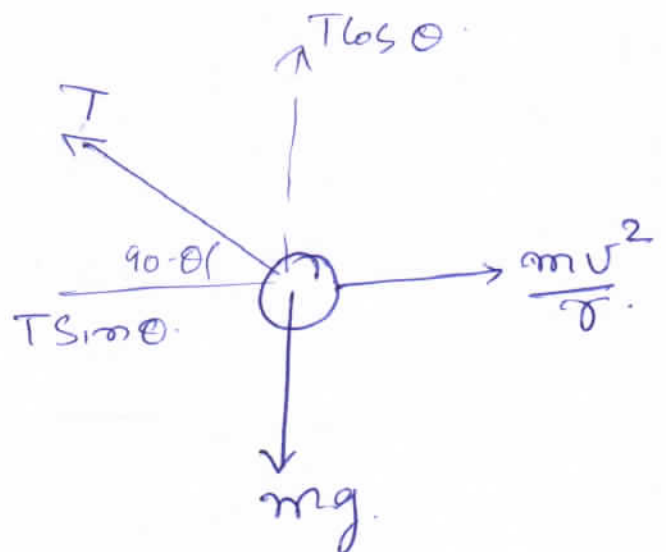
$$D \longrightarrow T$$

Pg 72-75

(P)



$$\begin{aligned} \tan \theta \sin \theta &= \frac{r}{\ell} \\ &= \frac{0.5}{1} = \frac{1}{2} \\ \theta &= 30^\circ \end{aligned}$$



$$\begin{aligned} T \cos \theta &= mg \\ T \sin \theta &= \frac{mv^2}{r} \\ \tan \theta &= \frac{v^2}{rg} \end{aligned}$$

$$\tan 30^\circ = \frac{v^2}{rg}$$

$$\frac{1}{\sqrt{3}} = \frac{v^2}{0.5 \times 10} \Rightarrow v^2 = \frac{5}{\sqrt{3}}$$

$$T = \frac{2\pi r}{v} = \frac{2 \times 3.14 \times 0.5}{\sqrt{\frac{5}{\sqrt{3}}}} = 1.8 \text{ seconds.}$$

$$T \cos \theta = mg$$

$$T \times \frac{\sqrt{3}}{2} = 0.5 \times 10$$

$$T = \frac{2 \times 5}{\sqrt{3}} = \frac{10}{\sqrt{3}} \text{ N}$$

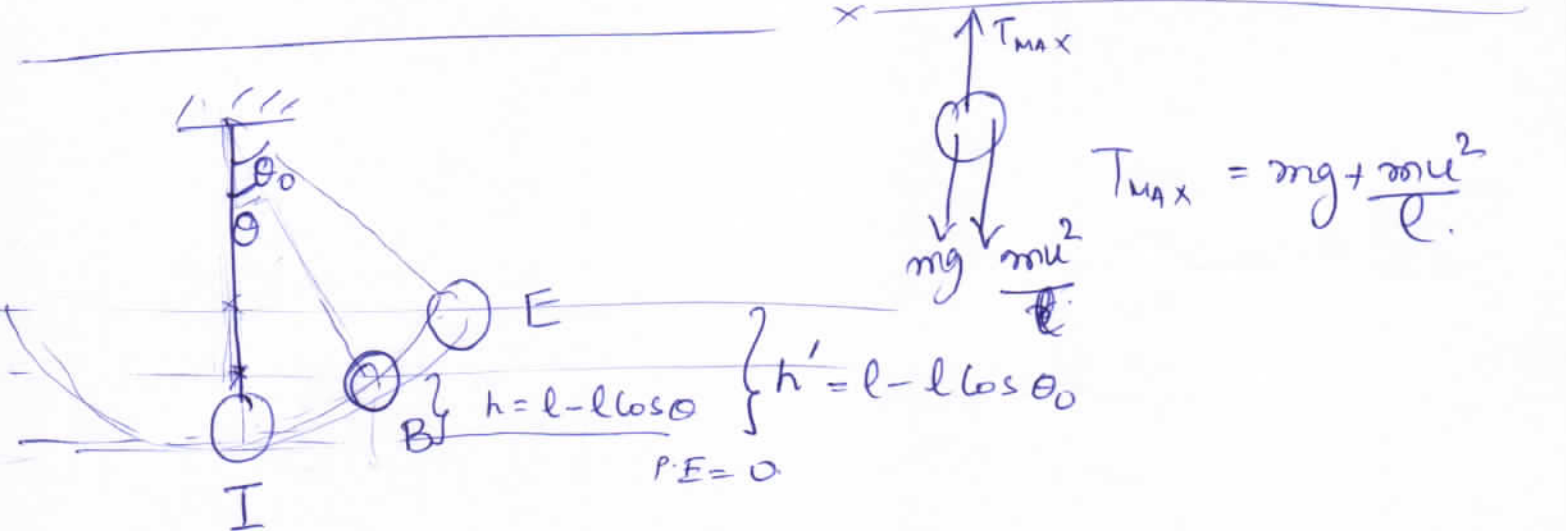


Diagram illustrating a pendulum bob of mass m and length l swinging from an initial position I to a final position E . The initial position I is at an angle θ_0 from the vertical, and the final position E is at an angle θ . The vertical height difference between I and E is $h = l - l \cos \theta$. The horizontal distance from the vertical line to E is $h' = l - l \cos \theta_0$. The potential energy at E is $P.E = 0$.

Free-body diagram of the bob at position E shows forces T_{\max} (up), mg (down), and $\frac{mu^2}{l}$ (down). The equation $T_{\max} = mg + \frac{mu^2}{l}$ is written next to it.

$$\frac{1}{2}mv^2 + \underbrace{mgl(1 - \cos \theta)}_B = \frac{1}{2}mu^2 + 0 = \frac{1}{2}m(0)^2 + \underbrace{mgl(1 - \cos \theta)}_E$$

$$\frac{mu^2}{l} = \frac{mv^2}{l} + \frac{2mgl(1-\cos\theta)}{l}$$

$$T_{\max} = \frac{mu^2}{l} + mg$$

$$= \left\{ \frac{mv^2}{l} + 2mg(1-\cos\theta) \right\} + mg$$

$$= \frac{mv^2}{l} + 3mg - 2mg\cos\theta$$

$$= \frac{mv^2}{l} + mg(3-2\cos\theta)$$

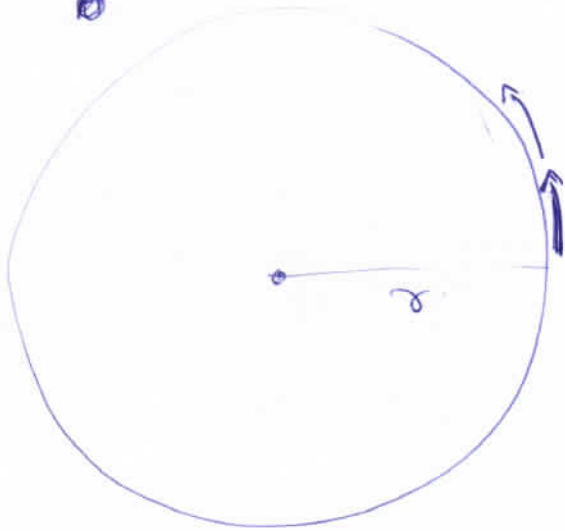
$$\frac{1}{2}mv^2 + mgl(1-\cos\theta) = mgl(1-\cos\theta_0)$$

$$\frac{v^2}{2} + gl - gl\cos\theta = gl - gl\cos\theta_0$$

$$\cos\theta_0 = \cos\theta - \frac{v^2}{2gl}$$

$$\theta_0 = \cos^{-1}\left(\cos\theta - \frac{v^2}{2gl}\right)$$

(22)



$$v = \beta \sqrt{s} \quad \frac{dv}{ds} = \frac{\beta}{2\sqrt{s}}$$

$$a_c = \frac{v^2}{r} = \frac{\beta^2 s}{r}$$

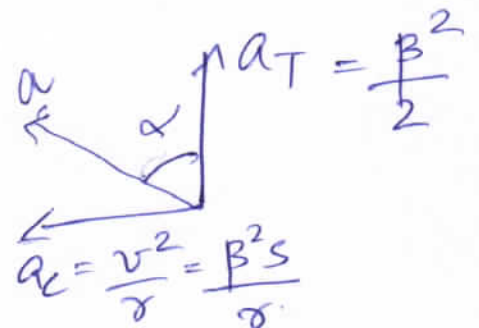
$$a_T = v \frac{dv}{ds} = \left(\beta \sqrt{s} \right) \frac{\beta}{2\sqrt{s}} = \frac{\beta^2}{2}$$

$$v^2 = 2as$$

$$v = \sqrt{2as} = \sqrt{2a} \sqrt{s}$$

$$\Rightarrow \beta = \sqrt{2a}$$

$$\frac{\beta^2}{2} = a_T$$

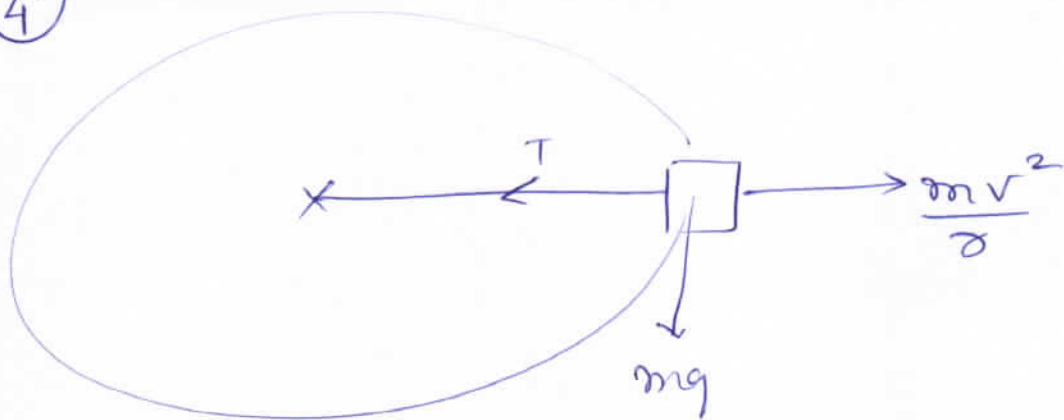


$$\tan \alpha = \frac{a_c}{a_T} = \frac{\beta^2 s / r}{\beta^2 / 2} = \frac{2s}{r}$$

$$\alpha = \tan^{-1} \frac{2s}{r}$$

$$a = \sqrt{a_T^2 + a_c^2} = \sqrt{\frac{\beta^4}{4} + \frac{\beta^4 s^2}{r^2}} = \beta^2 \sqrt{\frac{r^2 + 4s^2}{4r^2}}$$

4)



$$T = \frac{mv^2}{r}$$

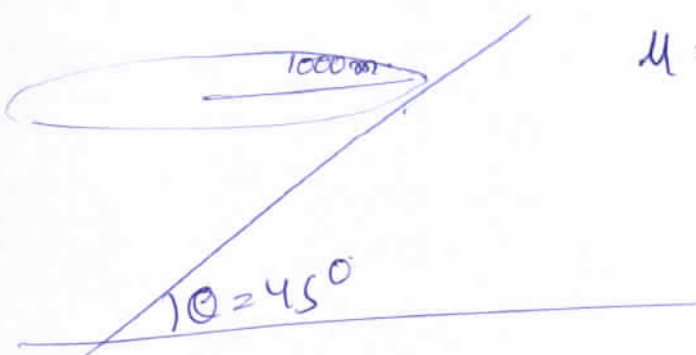
$$v = \sqrt{\frac{Tr}{m}}$$

$$v_{\text{MAX}} = \sqrt{\frac{T_{\text{MAX}} r}{m}}$$

$$= \sqrt{\frac{25 \times 1.96}{0.25}}$$

$$= \sqrt{196} = 14 \text{ m/s}$$

5)



$$\mu = 0.5$$

$$v_{\text{MAX}} = \sqrt{\frac{rg(\mu + \tan \theta)}{1 - \mu \tan \theta}}$$

$$= \sqrt{\frac{1000 \times 10 (0.5 + 1)}{1 - 0.5}}$$

$$= \sqrt{10^4 \times 3}$$

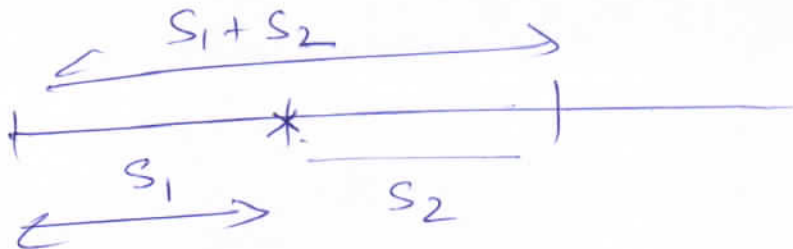
$$= \sqrt{3} \times 100$$

$$= 1.732 \times 100 = 173.2 \text{ m/s}$$

A)

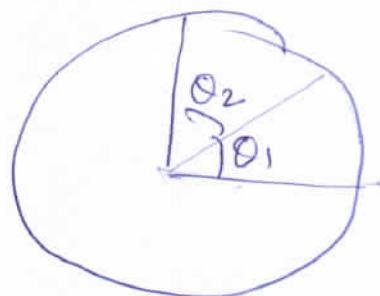
(9)

$\alpha = \text{const}$
 ω is changing



$$t : 0 \rightarrow 2 \quad \theta_1$$

$$t : 0 \rightarrow 4 \quad \theta_1 + \theta_2$$



$$\omega_0 = 0$$

$$\theta_1 = \omega_0 t + \frac{1}{2} \alpha (2)^2$$

$$\theta_1 + \theta_2 = \omega_0 t + \frac{1}{2} \alpha (4)^2$$

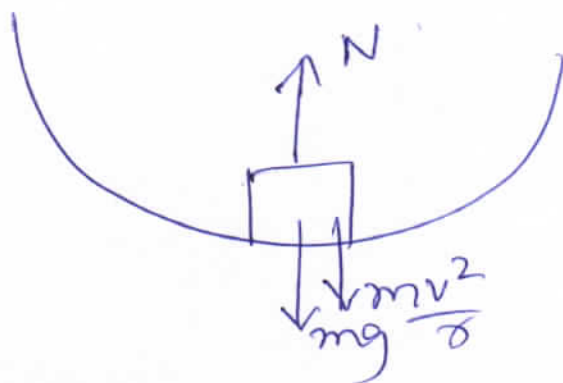
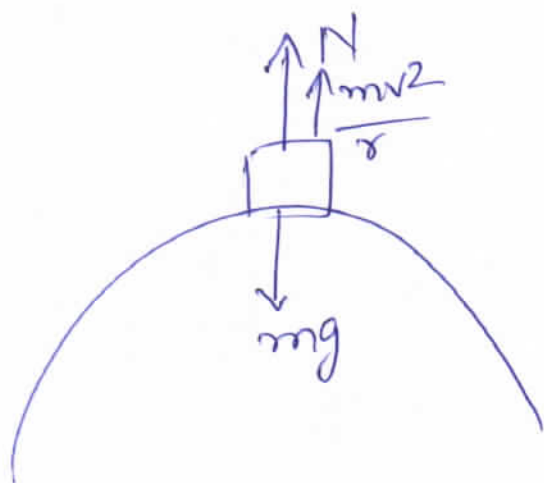
$$\Rightarrow \theta_1 = 2\alpha$$

$$\Rightarrow \theta_1 + \theta_2 = 8\alpha$$

$$\theta_2 = 6\alpha$$

$$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = 3$$

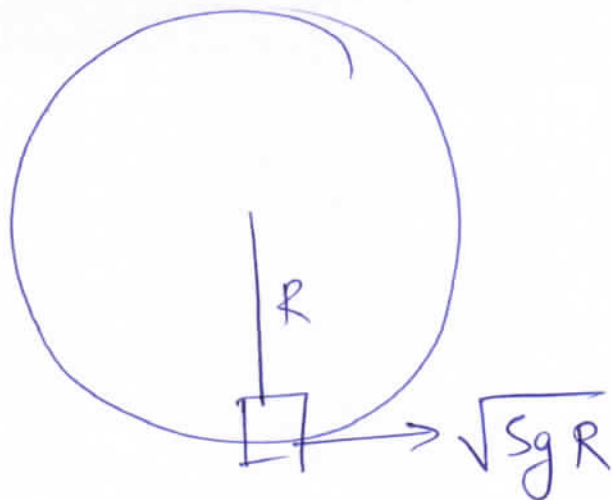
$$N = mg - \frac{mv^2}{r}$$



$$N = mg + \frac{mv^2}{r}$$

(A)

17



$$\begin{aligned}
 V_{\min} &= \sqrt{SgR} \\
 &= \sqrt{5 \times 10 \times 6.4} \\
 &= \sqrt{320} \\
 &= 17.7 \text{ m/s.}
 \end{aligned}$$

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$$a_c = \frac{v^2}{r} = v \cdot \frac{v}{r} = v \cdot \omega \quad \text{C} \rightarrow \text{A}$$

$$= \frac{\omega^2 r^2}{r} = \omega^2 r$$

$$\omega = \frac{v}{r}$$

$$\omega = 2\pi f$$

$$v = \omega r$$

$$= (2\pi f)^2 r$$

$$= 4\pi^2 f^2 r$$

$$= 4\pi^2 \left(\frac{1}{T}\right)^2 r$$

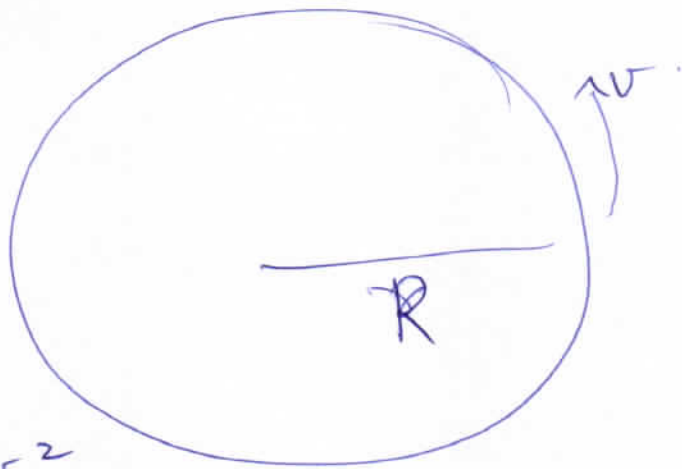
$$= \frac{4\pi^2 r}{T^2}$$

$$T = \frac{1}{f}$$

B

D

ABCD



$$K.E = \frac{1}{2}mv^2$$

$$K = as^2$$

$$\frac{1}{2}mv^2 = as^2$$

$$v^2 = \frac{2a}{m}s^2$$

$$v = \sqrt{\frac{2a}{m}}s$$

(27)

$$F_c = \frac{mv^2}{R}$$

$$= \frac{\cancel{m} \times 2a s^2}{\cancel{m} R}$$

$$= \frac{2as^2}{R} \quad (C)$$

(28)

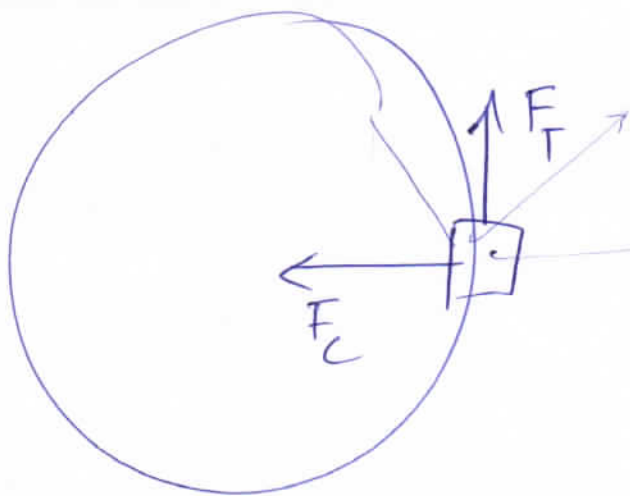
(C)

$$\frac{dv}{ds} = \sqrt{\frac{2a}{m}}$$

$$(29) F_T = ma_T$$

$$= m v \frac{dv}{ds} = m \times \sqrt{\frac{2a}{m}} s \times \sqrt{\frac{2a}{m}}$$

$$= \cancel{m} \times \frac{2a}{\cancel{m}} \times s = 2as \quad (D)$$



$$F = \sqrt{F_c^2 + F_T^2}$$

$$= \sqrt{\left(\frac{2as^2}{R}\right)^2 + (2as)^2}$$

$$= 2as \sqrt{\left(\frac{s}{R}\right)^2 + 1}$$

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