SETS

Collection of objects.

Each object in the set is a member of the set.

eg. A set of even integers.

2, 4, 6 - - - - -

$\mathbb{R}^-$ $\mathbb{R}^+$

0 (non-negative real numbers)

Set of Real Numbers. R $\mathbb{R}$

Set of Rational Numbers

(Any number which can be written as $\frac{p}{q}$; $p, q \in I$ & $q \neq 0$

Set of Irrational Numbers

(which cannot be written in the form $\frac{p}{q}$; $p, q \in I$ & $q \neq 0$

$\sqrt{2}$ $3+\sqrt{2}$

Integers. I

Fractions.

~~Natural Numbers~~

-2, -1, 0, 1, 2, 3, 4

Natural Number. N

1, 2, 3, 4 - - -

Whole Numbers. W

0, 1, 2, 3, 4, 5 - - - -

Representation of Sets.

① Tabular form / Roaster form. – In this representation all elements are listed and they are seperated by commas within braces { }

eg $\{1, 2, 3, 4, 5 \ldots\ldots\}$

Set of Natural Numbers.

② Set Builder form.

In this form, a variable x which stands for each element of the set is written within braces after giving semi colon (:) or oblique line (/) the properties possessed by each element of the set

$\{x : x \text{ is a natural number}\}$

Question.

① Write the set A which contains whole numbers less than 11 in Tabular as well as set builder form

$A = \{0, 1, 2 \ldots\ldots 10\}$ $A = \{x : x \text{ is a whole number less than } 11\}$

② Write down the Set Builder form of the following sets.

$A = \{3, 6, 9, 12\}$ $B = \{1, 4, 9, 16 \ldots\ldots 100\}$

$A = \{A : 3 \leq A \leq 12, A \in I\}$ $A = \{x : x = 3n \text{ where } n \in N \text{ and } 1 \leq n \leq 4\}$

$B = \{x : x = n^2, n \in N \ \& \ 1 \leq n \leq 10\}$

TYPES OF SET

1) NULL OR EMPTY SET $\{\ \}$ ϕ

A set having no element.

2) SINGLETON SET

A set having a single element.

eg. $\{1\}$ $\{2\}$

3) PAIR SET

A set having two elements.

4) FINITE & INFINITE SET

FINITE $\downarrow$ Counting comes to an end

INFINITE $\downarrow$ uncountable.

CARDINAL NUMBER OF A FINITE SET

$\downarrow$

number of distinct element in a finite set.

eg. $A = \{2, 3, 4, 5\}$ 4 $n(A)$

5) EQUAL SETS.

Two sets A & B are equal if every element $a \in A$ is an element of set B & every element $b \in B$ is an element of set A

$A = \{2, 2, 3\}$ & $B = \{2, 3\}$

6) EQUIVALENT SETS

Two sets A & B are equivalent if number of elements of set A is equal to number of elements of set B

SUBSET

A set A is said to be a subset ~~of set~~ of B if every element $a \in A$ also belongs to B.

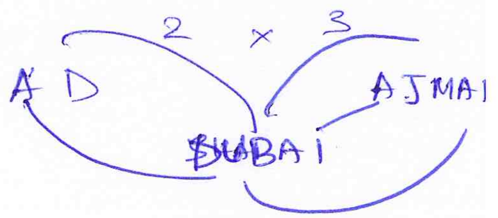
$$A \subseteq B$$

PROPER SUBSET

A set A is said to be a proper subset of B if every element $a \in A$ also belongs to B & atleast one element $b \in B$ ~~should not~~ $\notin A$.

$$A \subset B$$

Total number of subsets for a set A containing n elements. $= \boxed{2^n}$



Total number of proper subsets $= \boxed{2^n - 1}$

$\{ \underset{2}{\underline{\ \ }}^{①} \times \underset{2}{\underline{\ \ }}^{②} \times \underset{2}{\underline{\ \ }}^{③} \ldots \underline{\ \ }^{ⓝ} \times 2 \}$ $\quad 2^n$

elements		0	1	2	3	4			
1	$\longrightarrow$	1,	1				=	2	2^1
2	$\longrightarrow$	1,	2	1			=	4	2^2
3	$\longrightarrow$	1,	3	3	1		=	8	2^3
4	$\longrightarrow$	1,	4,	6,	4,	1	=	16	2^4

$\{2, 3\}$

$n \longrightarrow$ $\quad 2^n$

POWER SET

A set of all subsets of a given set (A) is called power set of A.

eg. $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$
$\{\ \}$

$n(P(A)) = 2^n$

Ques. If set $A = \{1, 3, 5\}$
Find cardinality of $P(A)$ & $P(P(A))$

$2^3 = 8$ $\qquad$ $2^8 = 256$

OPERATIONS ON SETS

If A & B are 2 sets.

① UNION OF SETS : $A \cup B = \{x : x \in A \text{ or } x \in B\}$

eg $A = \{1, 2\}$ $\qquad$ $B = \{2, 4, 5\}$

$A \cup B = \{1, 2, 4, 5\}$

2) INTERSECTION OF SETS : $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A \cap B = \{2\}$

(3) COMPLIMENT OF A SET

A

A' or A^c $= U - A$

↑ Universal set.

$U = \{x : x \in N\}$

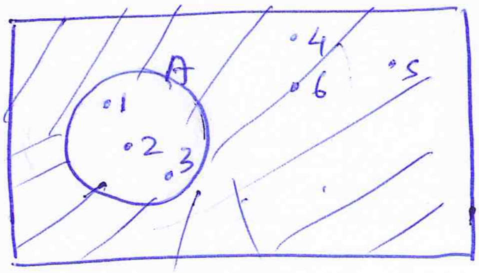
$A = \{1, 2, 3\}$

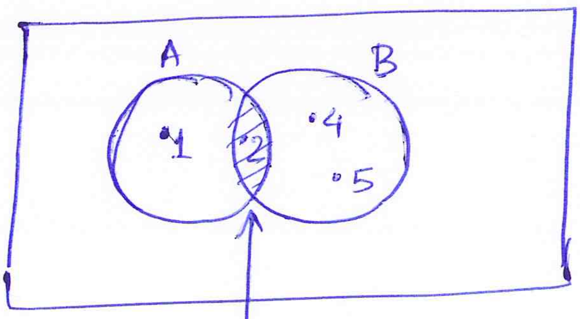
$A^c = \{4, 5, 6, 7 \ldots\}$

Seeing the operations diagramatically

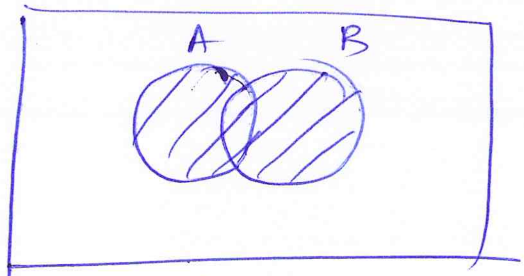
Venn Diagram :

Universal Set is represented by a rectangular region & sets are represented by circles or any closed geometrical shape. Elements of a set are denoted by points within a set.





$A \cap B$



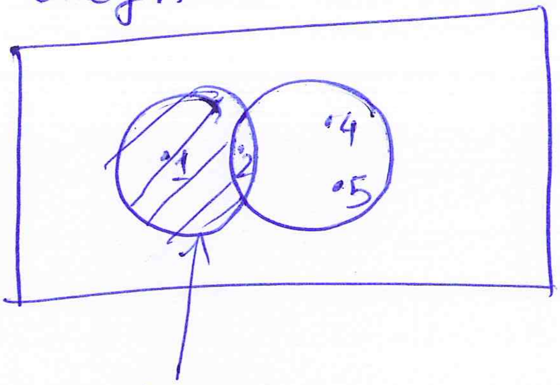
$A \cup B$

DIFFERENCE OF SETS

$$A - B = \{x : x \in A \ \& \ x \notin B\}$$

only A

$$= A - A \cap B$$



A − B

B − A
only B

$$x^2(x^3 - 6x^2 + 11x - 6) = 0$$

$$x^2(x-1)(x^2 - 5x + 6)$$

$$x^3 - x^2 - 5x^2 + 5x + 6x - 6$$

Q1) If $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\} = \{0, 1, 2, 3\}$

$A = \{x : x^2 - 5x + 6 = 0\} = \{2, 3\}$

$B = \{x : x^2 - 3x + 2 = 0\} = \{1, 2\}$

a) $A \cap B = \{2\}$

b) $(A \cap B)' = \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$

Q2 If $U = \{x : x \in N \ \& \ x \leq 9\} = \{1, 2, 3, \ldots 9\}$

$A = \{x : x = 2n, n \in N \ \& \ n \leq 4\} = \{2, 4, 6, 8\}$

$B = \{2, 3, 5, 7\}$

a) $(A \cup B)'$ $\quad A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$ $\quad (A \cup B)' = U - A \cup B = \{1, 9\}$

b) $A' \cap B'$ $\quad A' = U - A = \{1, 3, 5, 7, 9\}$

$B' = U - B = \{1, 4, 6, 8, 9\}$ $\Rightarrow A' \cap B' = \{1, 9\}$

LAWS OF ALGEBRA OF SETS

Suppose A, B & C are three non-empty sets.

1) Idempotent law

a) $A \cup A = A$ b) $A \cap A = A$

2) Identity law

a) $A \cup \phi = A$ b) $A \cap U = A$

3) Commutative law

a) $A \cup B = B \cup A$ b) $A \cap B = B \cap A$

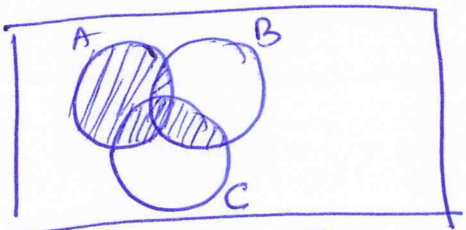
4) Associative law.

a) $(A \cup B) \cup C = A \cup (B \cup C)$ b) $(A \cap B) \cap C = A \cap (B \cap C)$

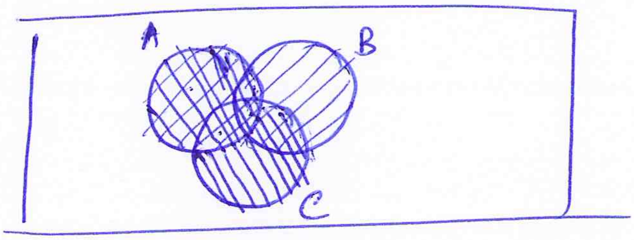
5) Distributive law

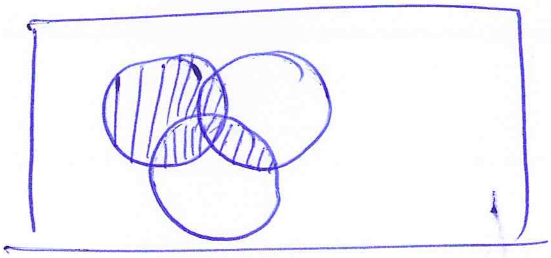
a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



$A \cup (B \cap C)$





$(A \cup B) \cap (A \cup C)$

6) De Morgan's law

a) $(A \cup B)' = A' \cap B'$

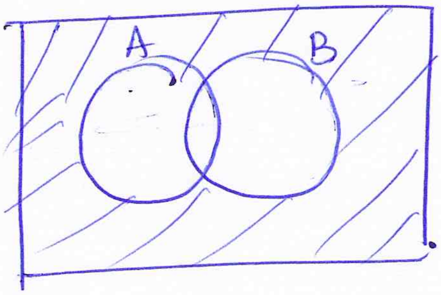
b) $(A \cap B)' = A' \cup B'$

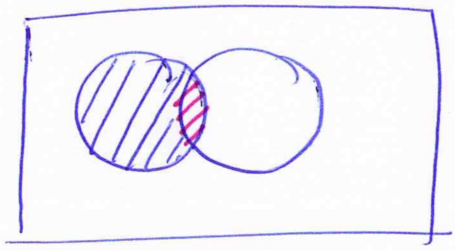
c) $A - (B \cup C) = (A - B) \cap (A - C)$

d) $A - (B \cap C) = (A - B) \cup (A - C)$

Q If A & B are two non empty sets.

find a) $A \cap (A \cup B)' = \phi$ b) $(A \cap B) \cup (A - B) = A$





$(A \cap B) \cup (\text{only } A)$ Co

$A \cap A' \cap B'$

$\phi \cap B' = \phi$

$(\text{only } A) \cup (A \cap B)$

D.L $(\text{Only } A \cup A) \cap (\text{Only } A \cup B)$

$(A) \cap (A \cup B)$

$\therefore A \subseteq A \cup B$ A

Subset $\cap$ Superset
= Subset.

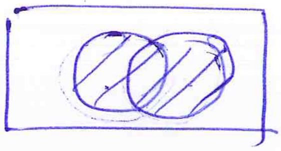
Q) Two finite sets have m & n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m & n are. $2^m - 2^n = 112$

A) 4, 7 B) 7, 4 C) 4, 4 d) 7, 7

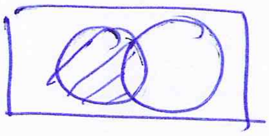
Results on number of elements in sets.

eg. $n(A)$ = number of elements in set A.

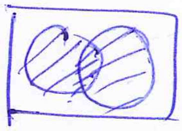
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$



$n(A - B) = n(A) - n(A \cap B)$



$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$



$n((A \cup B) \cup C) = n(A \cup B) + n(C) - n((A \cup B) \cap C)$

$= n(A) + n(B) - n(A \cap B) + n(C) - \{ n(A \cap C) + n(B \cap C) - n(A \cap B \cap C) \}$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$(A \cup B) \cap C$

$= C \cap (A \cup B)$

$= (C \cap A) \cup (C \cap B)$

$n((A \cup B) \cap C) = n((C \cap A) \cup (C \cap B)) = n(C \cap A) + n(C \cap B) - n(A \cap B \cap C)$

$n(A' \cup B') = n((A \cap B)') = n(U - (A \cap B))$

$= n(U) - n(A \cap B)$

$n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

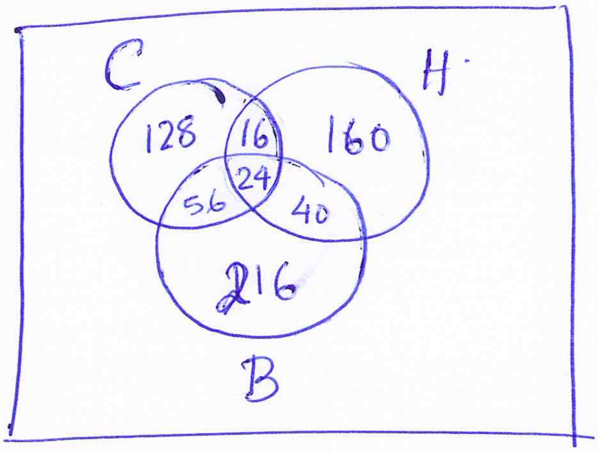
Q Out of 800 boys in a school, 224 played Cricket 240 played Hockey, 336 Basketball. Of the total 64 played both Basketball & Hockey 80 played both Cricket & Basketball, 40 played Cricket & Hockey; 24 played all the 3 games. The number of boys who did not play any games.

$n(U) = 800$

$n(C) = 224$

$n(H) = 240$

$n(B) = 336$



$n(U) - \underline{n(C \cup H \cup B)}$

$$n(C \cup H \cup B) = n(C) + n(H) + n(B) - n(C \cap H) - n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)$$

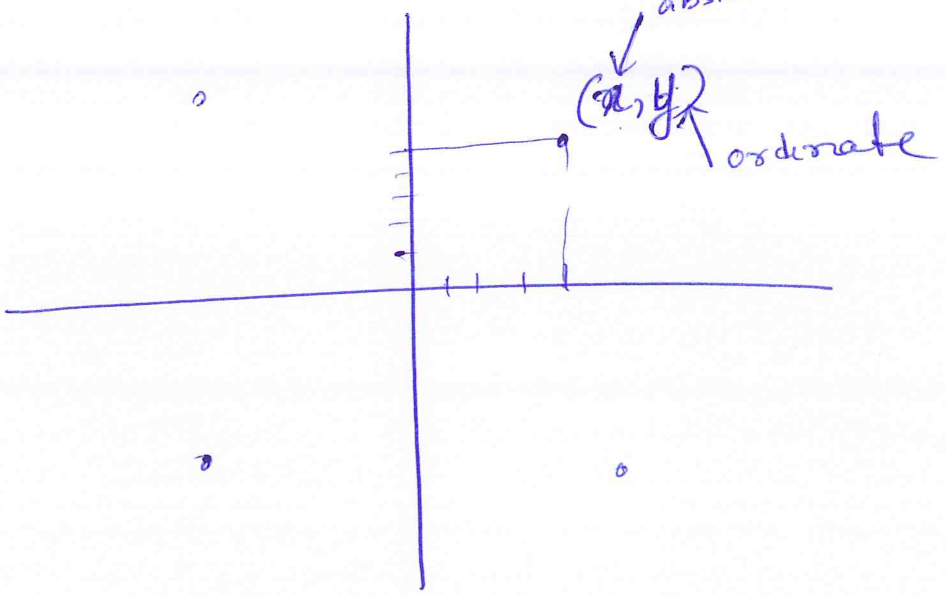
$$= 224 + 240 + 336 - 40 - 64 - 80 + 24$$

$$= 640$$

$800 - 640 = \underline{160}$

CARTESIAN PRODUCT OF TWO SETS.

(a,b)



if $A = \{1,2,3\}$ & $B = \{4,5\}$ are two sets

$A \times B = \{(a,b) : a \in A \text{ \& } b \in B\}$.

$= \{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$

$\underline{n(A \times B) = n(A) \times n(B)}$.

$B \times A = \{(b,a) : b \in B \text{ \& } a \in A\}$.

$= \{(4,1),(4,2),(4,3),(5,1),(5,2),(5,3)\}$.

$A \times B \neq B \times A$ $\quad (A \neq B)$

What if one of the sets is ϕ

$A \times \phi = \phi$

Relations of Sets.

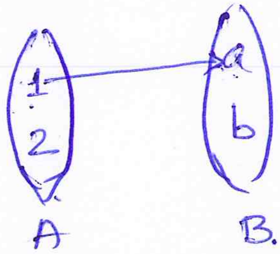
A relation from set A to set B is any subset of ~~the~~ $A \times B$

eg. $A = \{1, 2\}$ & $B = \{a, b\}$

$$A \times B = \{(1,a), (1,b), (2,a), (2,b)\}$$

$R_1 = \{(1,a)\}$

$R_1 : A \longrightarrow B.$



$a = R_1(1)$

a is called image of 1 in R_1

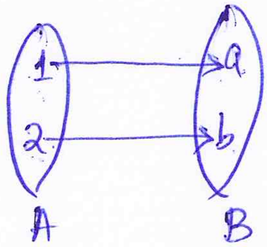
$R_2 = \{(1,b)\}$

|

|

$R_{10} = \{(1,a), (2,b)\}$

$R_{10} : A \longrightarrow B$



$a = R_{10}(1)$

$b = R_{10}(2)$

2 is the pre-ima
of b in R_{10}

$R_{16} = \{(1,a), (1,b), (2,a), (2,b)\}$

Collection of first elements for all ordered pairs of a relation. is called <u>domain of relation</u>.

$D(R_1) = \{1\}$ $D(R_{10}) = \{1, 2\}$

$D(R_{16}) = \{1, 1, 2, 2\}$

Collection of second element for all ordered pairs of a relation is called range of relation.

$R(R_1) = \{a\}$

$R(R_{10}) = \{a, b\}$

$R(R_{16}) = \{a, b, a, b\}$

Function

$(A \longrightarrow B)$

$f : A \longrightarrow B$

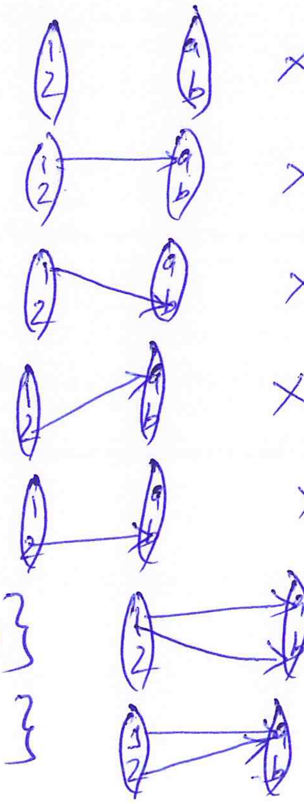
i) No element in domain is repeated

&

ii) $D = A$

↑

Domain



$R_0 = \phi$ ×

$R_1 = \{(1, a)\}$ × One-One

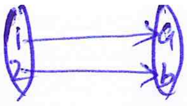
$R_2 = \{(1, b)\}$ × One-One

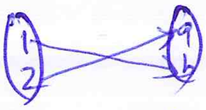
$R_3 = \{(2, a)\}$ × One-One.

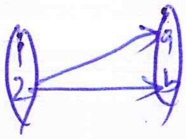
$R_4 = \{2, b\}$ × One-One.

$R_5 = \{(1, a), (1, b)\}$ × One-Many.

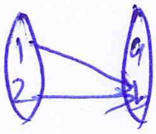
$R_6 = \{(1, a), (2, a)\}$ ✓ Many-One

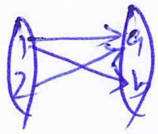
$R_7 = \{(1,a), (2,b)\}$  ✓ One - One

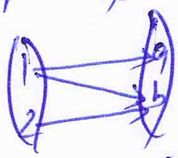
$R_8 = \{(2,a), (1,b)\}$  ✓ One - One

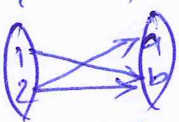
$R_9 = \{(2,a), (2,b)\}$  × One - Many

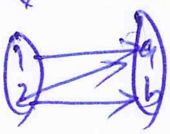
~~$R_{10} = \{(1,b), (2,a)\}$~~

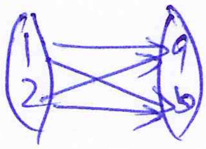
$R_{11} = \{(1,b), (2,b)\}$  ✓ Many - One.

$R_{12} = \{(1,a), (1,b), (2,a)\}$  × Many - Many

$R_{13} = \{(1,a), (1,b), (2,b)\}$  × Many - Many

$R_{14} = \{(1,b), (2,a), (2,b)\}$  × Many - Many

$R_{15} = \{(1,a), (2,a), (2,b)\}$  × Many - Many

$R_{16} = \{(1,a), (2,a), (1,b), (2,b)\}$  × Many - Many

One - One Relation ——— Function (Domain Set A)

One - Many Relation

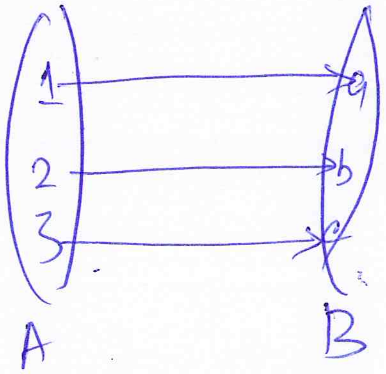
Many - One Relation ——— Function,

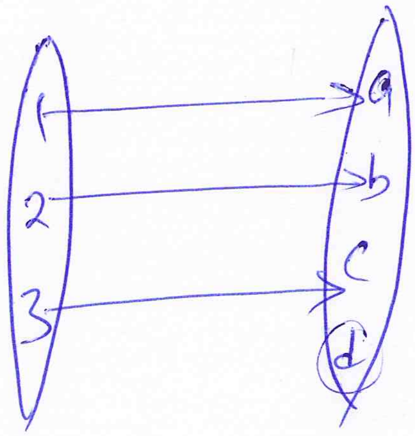
Many - Many Relation.

Domain = Set A.

One-One function : (Injective Function)

A function $f : A \longrightarrow B$ is called 1 - 1 if all elements in Range are distinct





$D = \{1, 2, 3\}$

$R = \{a, b, c\}$

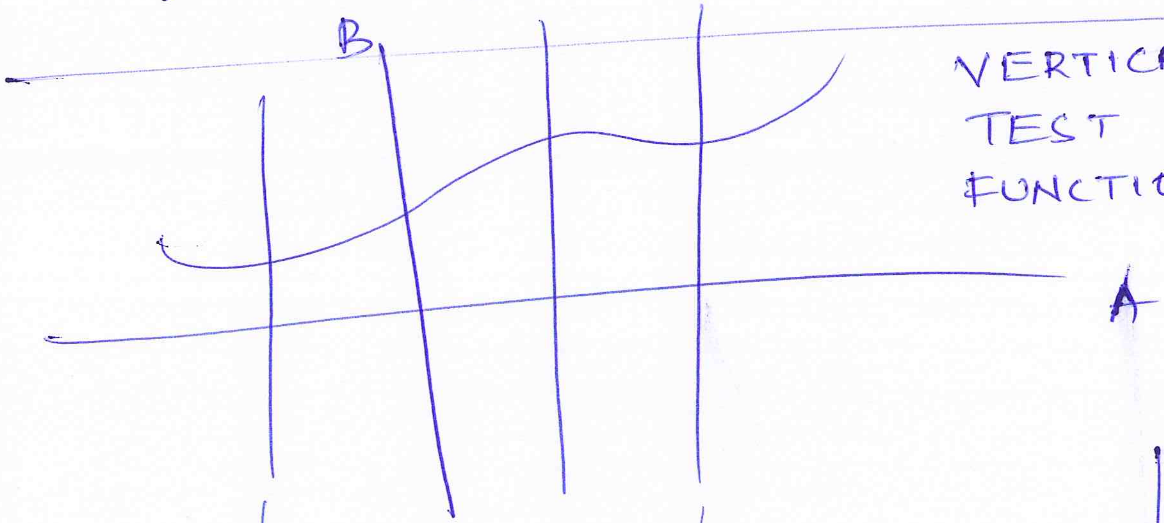
Set B is called Co-domain

If $R =$ Co-domain then it is called an ONTO function.
Surjective function.

If $R \neq$ Co-domain then it is called INTO function.

BIJECTIVE FUNCTION :

A function $f : A \longrightarrow B$ is called bijective if it is both 1 − 1 and onto.



VERTICAL LINE TEST TO CHECK FUNCTIONS