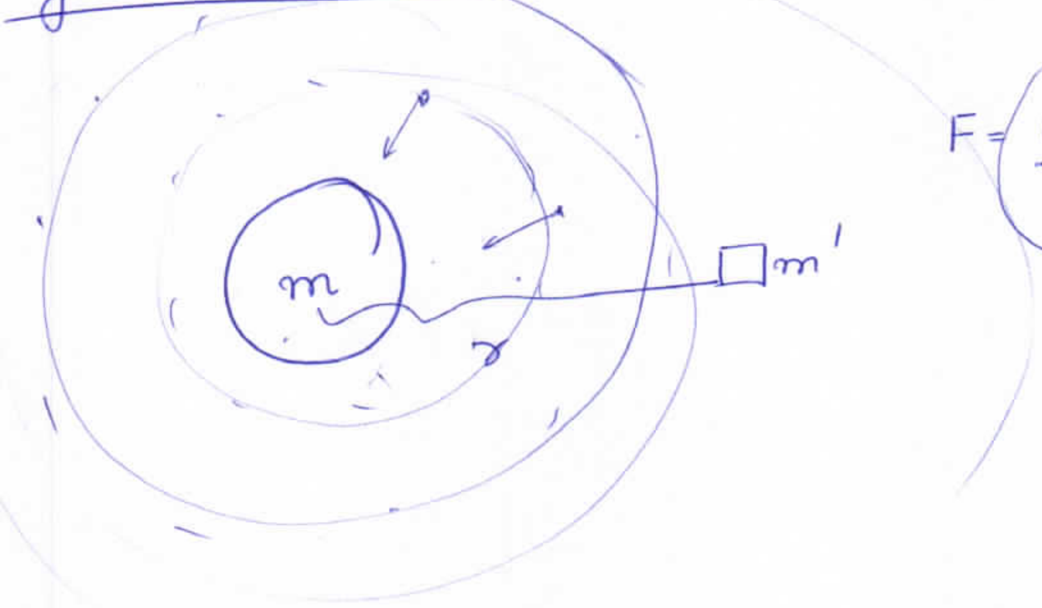
Gravitational Field.



$$F = \frac{Gm}{r^2} m'$$

Gravitational Field due a mass 'm' is the space around it where another body can experience a force due to the mass 'm'

Force at different positions in the field is different

$$\underset{\text{Intensity of Field.}}{E_g} = \frac{Gm}{r^2}$$

$E_g = g_r$ — h above surface: $\frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} \approx g\left(1-\frac{2h}{R}\right)$

— surface: $\frac{GM}{R^2} = 9.8\,m/s^2$

— h below surface: $g\left(1-\frac{h}{R}\right)$

Gravitational Potential.

$$F = \frac{Gm}{r^2} \cdot m'$$

$$W.D = \int_{\infty}^{r} F\,dr = \int_{\infty}^{r} \frac{Gmm'}{r^2}\,dr$$

$$Gmm'\int_{\infty}^{r} \frac{dr}{r^2} = -\frac{Gmm'}{r}\Big]_{\infty}^{r}$$

$$= -\frac{Gmm'}{r} - \left[-\frac{Gmm'}{\infty}\right]$$

$$W\cdot D = -\frac{Gmm'}{r}$$

$$\frac{W\cdot D}{m'} = \boxed{-\frac{Gm}{r} = V_r}$$

W·D in bringing unit mass from $\infty \longrightarrow r$

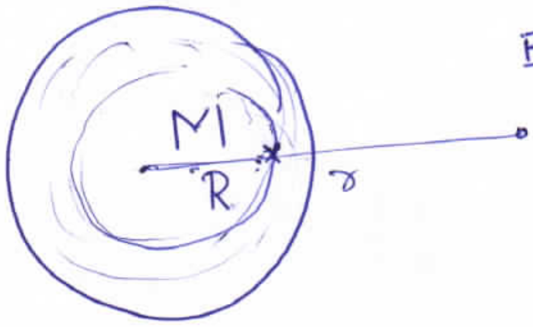
Gravitational Potential Energy:

W·D in bringing a mass m' from ∞ to a distance r from mass m

$$W\cdot D = \boxed{-\frac{Gmm'}{r} = P\cdot E}$$

Intensity of gravitational field.

Solid sphere.



$$E_g = \frac{GM}{r^2} \qquad r > R.$$

$$= \frac{GM}{R^2} \qquad r = R.$$

$$= \frac{GM'}{r^2} \qquad r < R.$$

$$\frac{M'}{M} = \frac{r^3}{R^3}$$

$$\frac{M'}{M} = \frac{V' \times d'}{V \times d'} = \frac{\cancel{\frac{4}{3}}\cancel{\pi} r^3 \cancel{d'}}{\cancel{\frac{4}{3}}\cancel{\pi} R^3 \cancel{d'}}$$

$$M' = \frac{r^3}{R^3} M$$

$$= \frac{GMr^{\cancel{3}}}{R^3 \cancel{r^2}} = \frac{GMr}{R^3} \qquad r < R$$

Hollow sphere

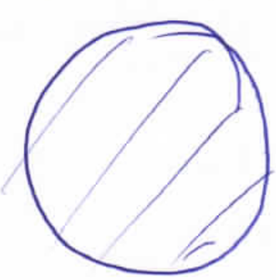
$$E_g = \frac{GM}{r^2} \qquad r > R.$$

$$= \frac{GM}{R^2} \qquad r = R$$

$$= 0 \qquad r < R$$

Gravitational Potential.

Solid Sphere.

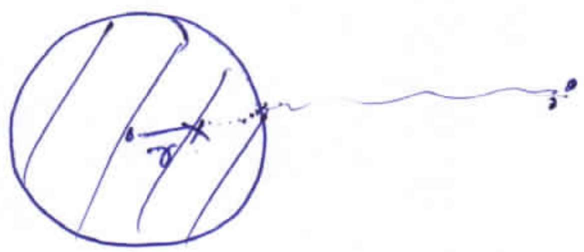


M, R

$$V_r = -\frac{GM}{r} \qquad r > R.$$

$$= -\frac{GM}{R} \qquad r = R$$

$$= -\frac{GM}{2R^3}\left(3R^2 - r^2\right) \qquad r < R.$$



$$W.D = \int_{\infty}^{r} F\,dr$$

$$= \int_{\infty}^{R} F\,dr + \int_{R}^{r} F\,dr$$

$$= \int_{\infty}^{R} \frac{GMm}{r^2}\,dr + \int_{R}^{r} \frac{GM'm}{r^2}\,dr$$

$$= \int_{\infty}^{R} \frac{GMm}{r^2}\,dr + \int_{R}^{r} \frac{G\frac{r^3}{R^3}Mm}{r^2}\,dr$$

$$= \int_{\infty}^{R} \frac{GMm}{r^2}\,dr + \int_{R}^{r} \frac{GMm}{R^3}\,r\,dr$$

$$= -\frac{GMm}{r}\Big]_{\infty}^{R} + \frac{GMm}{R^3}\,\frac{r^2}{2}\Big]_{R}^{r}$$

$$= -\left\{\frac{GMm}{R} - \underbrace{\cancel{\frac{GMm}{\infty}}}_{0}\right\} + \frac{GMm}{R^3}\left(\frac{r^2}{2} - \frac{R^2}{2}\right)$$

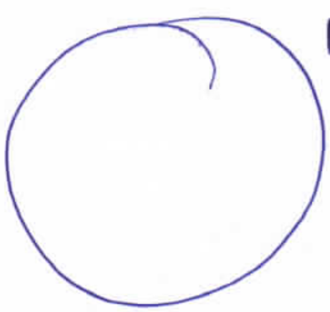
$$= -\frac{GMm}{R} + \frac{GMm\,r^2}{2R^3} - \frac{GMm}{2R}$$

$$= -\frac{3}{2}\frac{GMm}{R} + \frac{GMm}{2R^3}r^2$$

$$= -\frac{GMm}{2R^3}\left(3R^2 - r^2\right)$$

$$V_r = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$$

Hollow Sphere.



M, R.

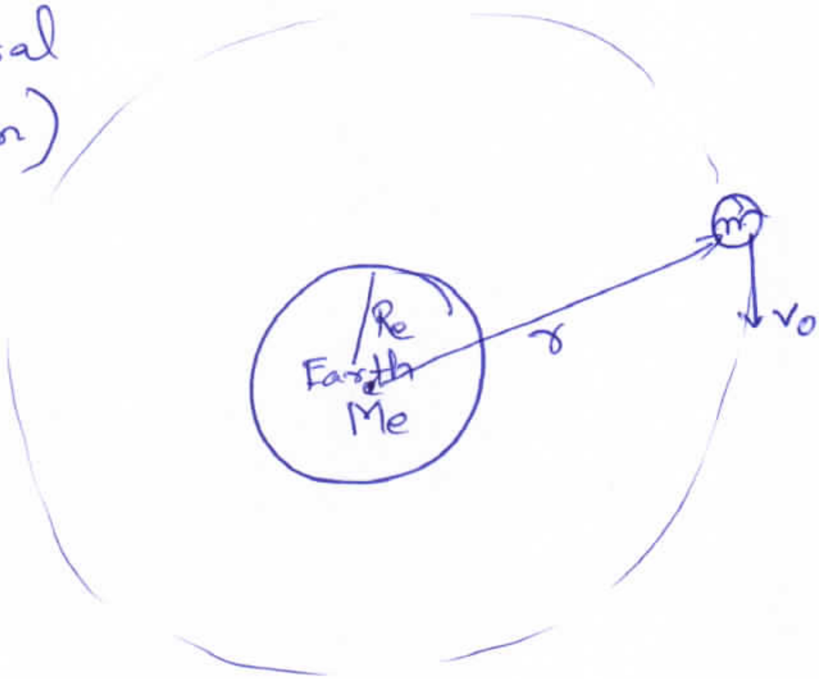
$$V_r = -\frac{GM}{r} \qquad r > R$$

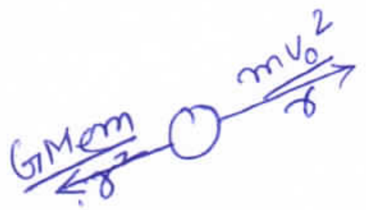
$$= -\frac{GM}{R}. \qquad r = R.$$

$$= -\frac{GM}{R}. \qquad r < R$$

Satellites

- Natural (Moon)
- Man-Made





$$\frac{GM_em}{r^{\cancel{2}}} = \frac{mv_0^2}{\cancel{r}}$$

$$v_0^2 = \frac{GM_e}{r}$$

$$v_0 = \sqrt{\frac{GM_e}{r}}$$

$r = R_e + h$ ← height above surface.

$$v_0 = \sqrt{\frac{GM_e}{R_e+h}} = \sqrt{\left(\frac{GM_e}{R_e^2}\right)\cdot\frac{R_e^2}{R_e+h}} = \sqrt{g\left(\frac{R_e^2}{R_e+h}\right)}$$

If $h << R_e$ → $v_0 \approx \sqrt{gR_e}$

$$\text{Time period} = \frac{2\pi r}{v_0} = \frac{2\pi r}{\sqrt{\frac{GM_e}{r}}} = \frac{2\pi r^{3/2}}{\sqrt{GM_e}}.$$

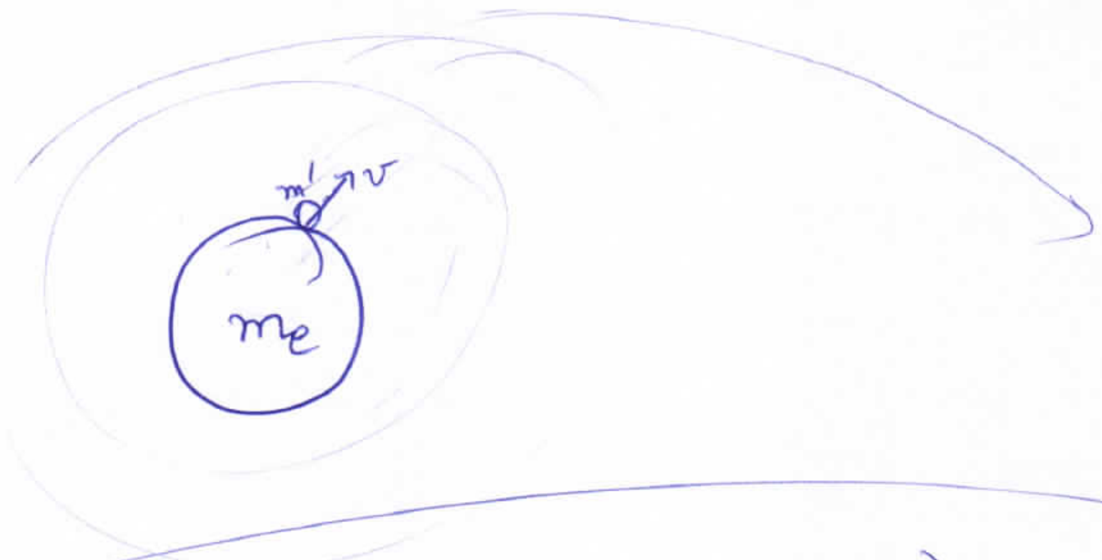
$$T = \frac{2\pi}{\sqrt{GM_e}} r^{3/2} \implies T^2 = \left(\frac{4\pi^2}{GM_e}\right) r^3$$

$\underline{T^2 \propto r^3}$ Kepler's law

Escape Velocity

↓

Min. velocity provided to a ~~body~~ mass so that it escapes the gravitational pull of the ~~field~~ body.



$$\frac{1}{2} m' v_e^2 - \frac{G m_e m'}{R_e} = \frac{1}{2} m (0)^2 - \frac{G m_e m'}{\infty}$$

0

$$\frac{1}{2} \cancel{m'} v_e^2 = \frac{G m_e \cancel{m'}}{R_e}$$

$$v_e^2 = 2 \frac{G M_e}{R_e}$$

$$v_e = \sqrt{\frac{2 G M_e}{R_e}} = \sqrt{2 g R_e} = 11.2 \text{ km/s}$$

Total energy of a satellite.

$$K.E + P.E$$

$$= \frac{1}{2} m v_0^2 - \frac{G M_e m}{\cancel{R_e}\, r}$$

$$= \frac{1}{2} m \left(\sqrt{\frac{G M_e}{r}}\right)^2 - \frac{G M_e m}{\cancel{R_e}\, r}$$

$$= \underbrace{\frac{G M_e m}{2r}}_{K.E} - \underbrace{\frac{G M_e m}{r}}_{P.E} = \underbrace{-\frac{G M_e m}{2r}}_{T.E}$$

$$\boxed{T.E = \frac{1}{2} P.E = -K.E}$$

TYPES OF SATELLITES

GEOSTATIONARY

Time period = 24 hrs

satellite should be orbiting along the equatorial plane.

POLAR.

orbit is passing through north pole & south pole

Time period = 2 hrs.

useful in taking photographs of the earth.