# TRIGONOMETRIC EQ. TUTORIAL

Pg 27 — 1, 2, 3

Pg 28 — 6, 8

Pg 37 — 2, 4, 5

Pg 38 — 2, 4, 5, 7, 11, 13, 14, 15

Pg 39 — 3, 5

Pg 42 — 1, 2, 3, 4

Pg 27

① $\cos 3x = 0$

$\cos 3x = \cos \frac{\pi}{2}$

$3x = 2n\pi \pm \frac{\pi}{2}$

$x = \frac{1}{3}\left(2n\pi \pm \frac{\pi}{2}\right)$

$\cos x = 0$

$\cos x = \cos \frac{\pi}{2}$

$x = 2n\pi \pm \frac{\pi}{2}$

② $\cos x - 2\sin^2 \frac{x}{2} = 0$

$\cos x - (1 - \cos x) = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$\cos x = \cos \frac{\pi}{3}$

$x = 2n\pi \pm \frac{\pi}{3}$

③ $\sin 2x = \sqrt{2}\cos x$

$2\sin x \cos x - \sqrt{2}\cos x = 0$

$\sqrt{2}\cos x(\sqrt{2}\sin x - 1) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{\sqrt{2}}$

$\cos x = \cos \frac{\pi}{2}$ $\sin x = \sin \frac{\pi}{4}$

$x = 2n\pi \pm \frac{\pi}{2}$ $x = n\pi + (-1)^n \frac{\pi}{4}$

Pg 28 (6)

Solve $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

$$3\cos 2\theta - \sqrt{3}\sin 2\theta = 0$$

$$\sqrt{3^2+(\sqrt{3})^2}\left(\frac{3}{\sqrt{3^2+(\sqrt{3})^2}}\cos 2\theta - \frac{\sqrt{3}}{\sqrt{3^2+(\sqrt{3})^2}}\sin 2\theta\right) = 0$$

$$\sqrt{12}\left(\cos\alpha\cos 2\theta - \sin\alpha\sin 2\theta\right) = 0$$

$$\cos\alpha = \frac{3}{\sqrt{12}}$$

$$\sqrt{12}\cos(\alpha+2\theta) = 0$$

$$\cos(\alpha+2\theta) = \cos\frac{\pi}{2}.$$

$$\alpha+2\theta = 2n\pi \pm \frac{\pi}{2}$$

$$2\theta = 2n\pi \pm \frac{\pi}{2} - \alpha$$

$$\theta = n\pi \pm \frac{\pi}{4} - \frac{\alpha}{2}$$

(8) Solve $\tan\theta + \tan 2\theta + \tan\theta\tan 2\theta = 1$.

$$\tan\theta + \tan 2\theta = 1 - \tan\theta\tan 2\theta$$

$$\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = 1$$

$$\tan 3\theta = \tan\frac{\pi}{4}$$

$$3\theta = n\pi \pm \frac{\pi}{4}$$

$$\theta = \frac{n\pi}{3} \pm \frac{\pi}{12}$$

Pg 37 (2) $\sin x + \sin 3x = \sin 2x$ $\quad 0 \le x \le 2\pi$

$$2\sin 2x\cos x = \sin 2x$$

$$\sin 2x(2\cos x - 1) = 0$$

$\sin 2x = 0$ or $\cos x = \frac{1}{2}$

$2x = n\pi$

$x = \frac{n\pi}{2}$

$\cos x = \cos\frac{\pi}{3}$

$x = 2n\pi \pm \frac{\pi}{3}$

$x \Rightarrow n=0 \quad 0$
$n=1 \quad \frac{\pi}{2}$
$n=2 \quad \pi$
$n=3 \quad \frac{3\pi}{2}$
$n=4 \quad 2\pi$

$x \Rightarrow n=0 \quad \frac{\pi}{3}$
$n=1 \quad 2\pi - \frac{\pi}{3}$
$= \frac{5\pi}{3}$

④ $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$

$\cos\theta - \cos 2\theta = \sin 2\theta - \sin\theta$

$2\sin\frac{3\theta}{2}\sin\frac{\theta}{2} = 2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}$

$\sin\frac{\theta}{2}\left\{\sin\frac{3\theta}{2} - \cos\frac{3\theta}{2}\right\} = 0$

$\sin\frac{\theta}{2} = 0$

$\frac{\theta}{2} = n\pi$

$\theta = 2n\pi$

$\sin\frac{3\theta}{2} = \cos\frac{3\theta}{2}$

$\tan\frac{3\theta}{2} = 1$

$\tan\frac{3\theta}{2} = \tan\frac{\pi}{4}$

$\frac{3\theta}{2} = n\pi \pm \frac{\pi}{4}$

$\theta = \frac{2}{3}\left(n\pi \pm \frac{\pi}{4}\right)$

⑤ $\tan\theta + \tan 2\theta + \tan 3\theta = 0$

$\tan\theta + \tan 2\theta + \left(\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta}\right) = 0$

$(\tan\theta + \tan 2\theta)\left\{1 + \frac{1}{1 - \tan\theta\tan 2\theta}\right\} = 0$

$\left(\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta}\right)(2 - \tan\theta\tan 2\theta) = 0$

$$(\tan 3\theta)(\tan\theta\tan 2\theta - 2) = 0$$

$$\tan 3\theta = 0$$

$$3\theta = n\pi$$

$$\theta = \frac{n\pi}{3}.$$

$$\tan\theta\tan 2\theta = 2$$

$$\tan\theta\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = 2$$

$$\frac{2\tan^2\theta}{1-\tan^2\theta} = 2$$

$$\tan^2\theta = 1 - \tan^2\theta$$

$$2\tan^2\theta = 1$$

$$\tan^2\theta = \frac{1}{2}$$

$$\tan^2\theta = \tan^2\alpha.$$

$$\theta = n\pi + \alpha.$$

$$\alpha = \tan^{-1}\frac{1}{\sqrt{2}}$$