CALCULUS.

DIFFERENTIAL

INTEGRAL.

CONSTANT

TEMPERORY

FOR A PARTICULAR PROBLEM.

$v = 20\,m/s$

is a temporary const for problem.

PERMANENT

π, G

(problem Independent)

VARIABLE.

If a quantity can be assigned different values

car moving at $v = 20\,m/s$.

t	x
1 s	20 m
2 s	40 m
3 s	60 m.

$$y = 2x - 3$$

y is a function of x.

or $y = f(x)$

dependent variable: y

independent variable: x

$$y = x^2 - 2x + 3$$

$$y = \cos x$$

$y = z^2 + 2$

y is not a function of variable x

but its a function of variable z.

$y = x^2 + z + 3$ $\quad$ $y = f(x)$

$z \longrightarrow$ constant for this problem.

Differentiation

$y = f(x)$

y depends on x

& y will change with change in x.

if x changes by Δx $\quad$ $x \longrightarrow x + \Delta x$

$\hookrightarrow$ y changes by Δy. $\quad$ $y \longrightarrow y + \Delta y$.

Difference + Division $\longrightarrow$ Differentiation.

$$\frac{\text{Change in function}}{\text{Change in independent variable value}} = \frac{(y+\Delta y) - y}{(x+\Delta x) - x}$$

if $\Delta x \longrightarrow 0$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$ ← differention of y w.r.t x.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$\frac{d}{dx}(y)$$

differential operator.

$$y = x^2 \qquad \frac{dy}{dx} = \frac{d}{dx}(y) \rightarrow \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$y + \Delta y = (x + \Delta x)^2$$

$$\Delta y = (x+\Delta x)^2 - x^2 \qquad \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 - \cancel{x^2}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} = 2x$$

i) $y = x^n$

$\frac{dy}{dx} = n x^{n-1}$

$y = x^2 \Rightarrow \frac{dy}{dx} = 2x^{2-1}$
$= 2x$

ii) $y = c$

$\frac{dy}{dx} = 0$

iii) $y = x^n$ $\qquad \frac{dy}{dx} = n x^{n-1}$

$z = c x^n$

$z = cy$

$\frac{dz}{dx} = \frac{d(cy)}{dx} = c \frac{dy}{dx}$

eg. $y = 3x^3$

$\frac{dy}{dx} = 3(3x^2)$
$= 9x^2$

iv) $y = x^n$

$z = x^n + c$

$z = y + c$

$\frac{dz}{dx} = \frac{dy}{dx}$

$y = x^4 + 5$

$\frac{dy}{dx} = 4x^3$

$$y = f_1(x) \pm f_2(x)$$

$$\frac{dy}{dx} = \frac{d f_1(x)}{dx} \pm \frac{d f_2(x)}{dx}$$

eg. $y = x^2 + 2x^4 + 5$

$$\frac{dy}{dx} = 2x + 8x^3 + 0$$

$$= 2x + 8x^3$$

$$y = f_1(x) \times f_2(x)$$

$u = f_1(x)$
$v = f_2(x)$.

$$y = u \times v$$

$$\frac{dy}{dx} = \frac{d}{dx}(u \times v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

eg. $y = \underbrace{2x^3}_{u} \underbrace{\sin x}_{v}$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}.$$

$$\frac{dy}{dx} = 2x^3 \cos x + \sin x (6x^2)$$

$y = \underbrace{x^4 \sin x}_{u} \underbrace{\cos x}_{v}$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$x^4 \sin x(-\sin x) + \cos x\left(\frac{d(x^4 \sin x)}{dx}\right)$$

$$\frac{d(\underset{u}{x^4}\,\underset{v}{\sin x})}{dx} = x^4 \cos x + \sin x(4x^3)$$

$$-x^4 \sin^2 x + \cos x\left(x^4 \cos x + \sin x(4x^3)\right)$$

$$x^4(\cos^2 x - \sin^2 x) + 4x^3 \sin x \cos x.$$

$$y = \frac{x^3}{\sin x} \qquad y = x^3 \operatorname{cosec} x$$

$$y = \frac{f_1(x)}{f_2(x)}$$

$$y = \frac{u}{v}. \qquad \Rightarrow y = u\left(\frac{1}{v}\right)$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{\sin x(3x^2) - x^3(\cos x)}{(\sin x)^2}$$

$$c f(x)$$

$$y = \sin(x^3) \qquad \frac{dy}{dx} \qquad f(cx^2)$$

$$y = f(u) \qquad y = \sin u$$

$$u = f(x) \qquad u = x^3$$

$$y = f(u)$$
$$u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 3x^2$$

$$= \cos(x^3)\,(3x^2)$$

$$\frac{d}{dx}\sin x = \cos x.$$

$$\frac{d}{dx}\sin u = \cos u$$

$$\cos(x^3) \times 3x^2$$

$$y = (x+2)^4$$

$$\frac{dy}{dx} = 4(x+2)^3 \times (1)$$

$$= 4(x+2)^3$$

$$x^4 \qquad 3x^3 \times 1$$

$$y = (x^3+2x)^5$$

$$\frac{dy}{dx} = 5(x^3+2x)^4 \times (3x^2+2)$$

$$y = \sin\left((2x^2+4)^3\right)$$

$$\frac{dy}{dx} = \cos\left((2x^2+4)^3\right)\left(3(2x^2+4)^2\right)(4x)$$

Find dy/dx

i) $y = \sin\left(\dfrac{x^4}{x+2}\right)$

$$\frac{d\,e^x}{dx} = e^x$$

ii) $y = \cos\left(x^3(x+2)^2\right)$

iii) $y = e^{(x^2+4)^2}$

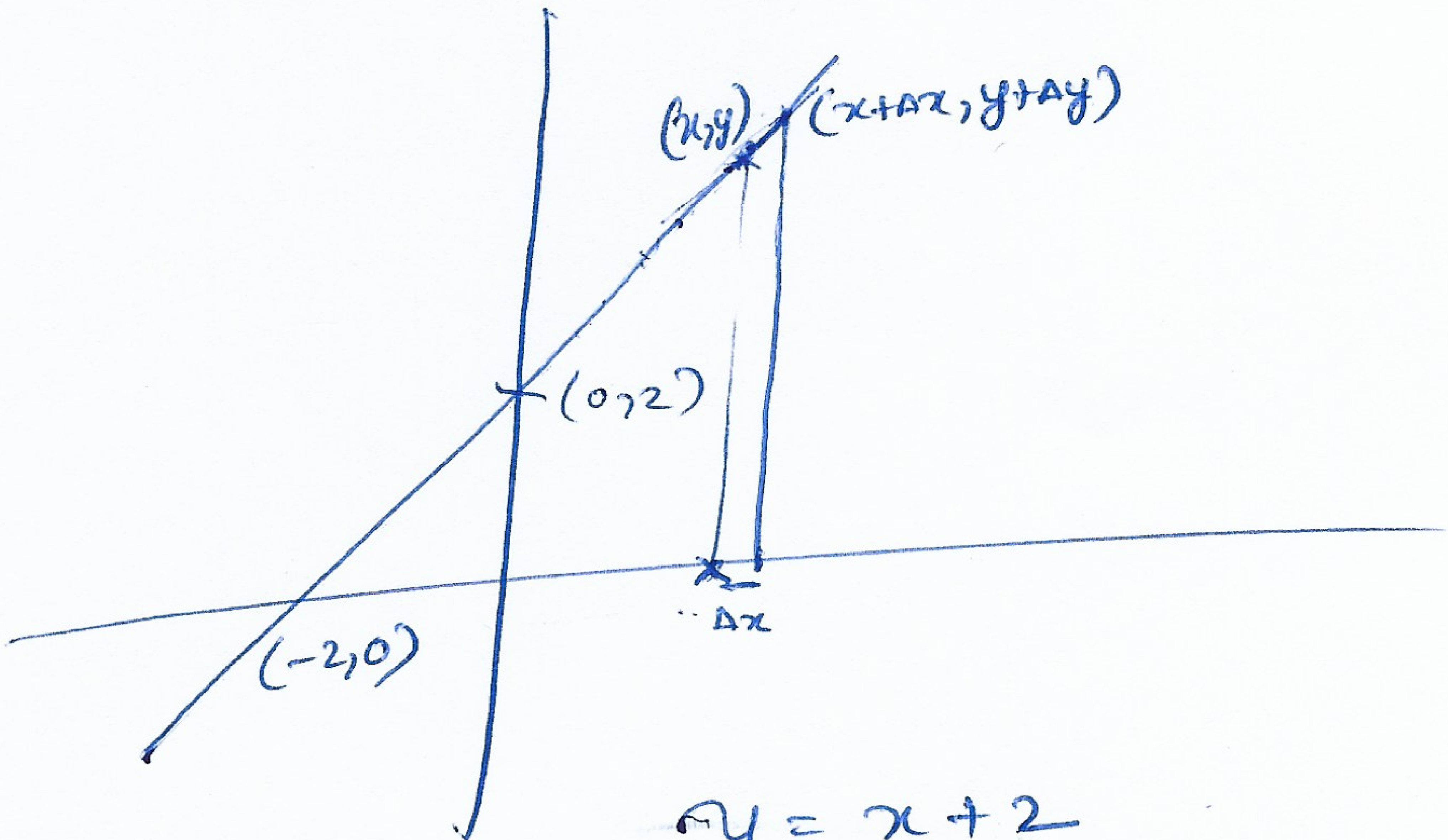
i) $$\frac{dy}{dx} = \cos\left(\frac{x^4}{x+2}\right)\left\{\frac{(x+2)4x^3 - x^4(1)}{(x+2)^2}\right\}$$

ii) $$\frac{dy}{dx} = -\sin\left(x^3(x+2)^2\right)\left\{x^3\left(2(x+2)\right) + (x+2)^2 3x^2\right\}$$

iii) $$\frac{dy}{dx} = e^{(x^2+4)^2}\left\{2(x^2+4)(2x)\right\}$$

$$\frac{d\tan(2x)}{dx} = \sec^2(2x)\{2\}$$

$$\frac{d\tan x}{dx} = \sec^2 x.$$



$$\lim_{\Delta x \to 0} \frac{(y+\Delta y) - y}{(x+\Delta x) - x}$$

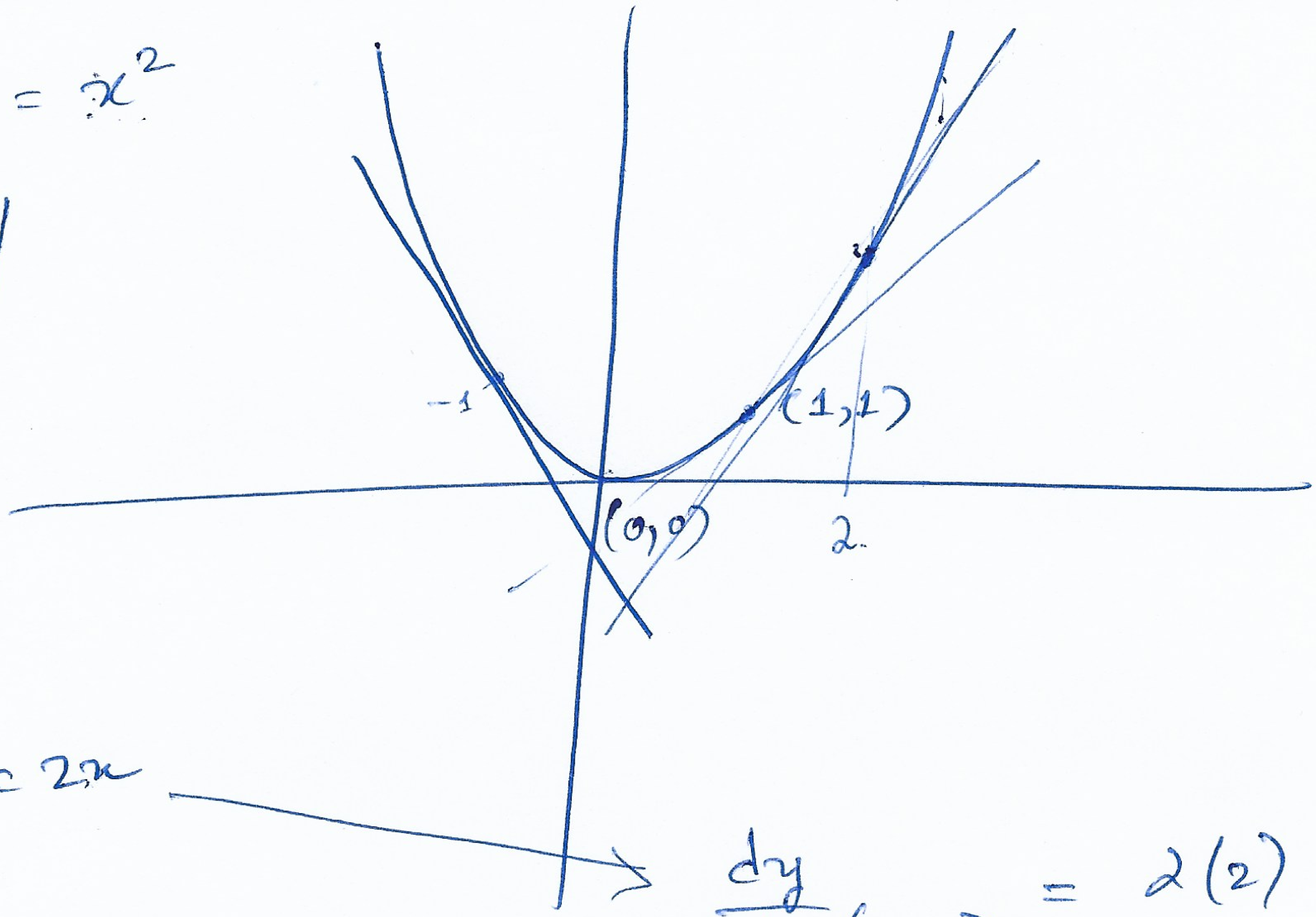
$y = x+2$

$\dfrac{dy}{dx}(x=x_0)$

gives you the slope of tangent drawn to the curve $y=f(x)$ at $x=x_0$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$y = x^2$



$x = 1.$

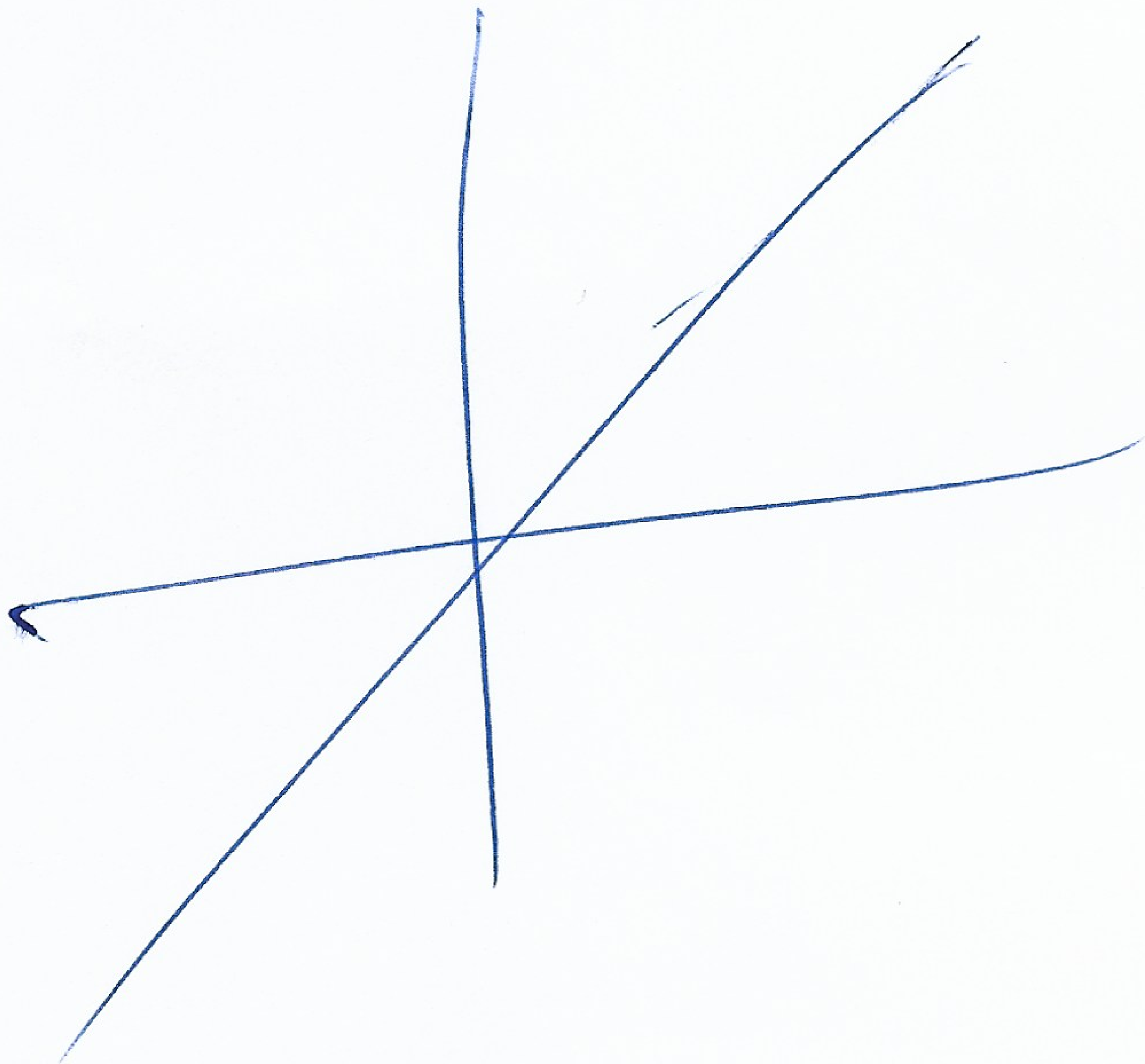
$\frac{dy}{dx} = 2x \longrightarrow \frac{dy}{dx}(x=2) = 2(2) = 4$

$\frac{dy}{dx} = 2(1)$

$= 2$

$y = (x+2)^3$

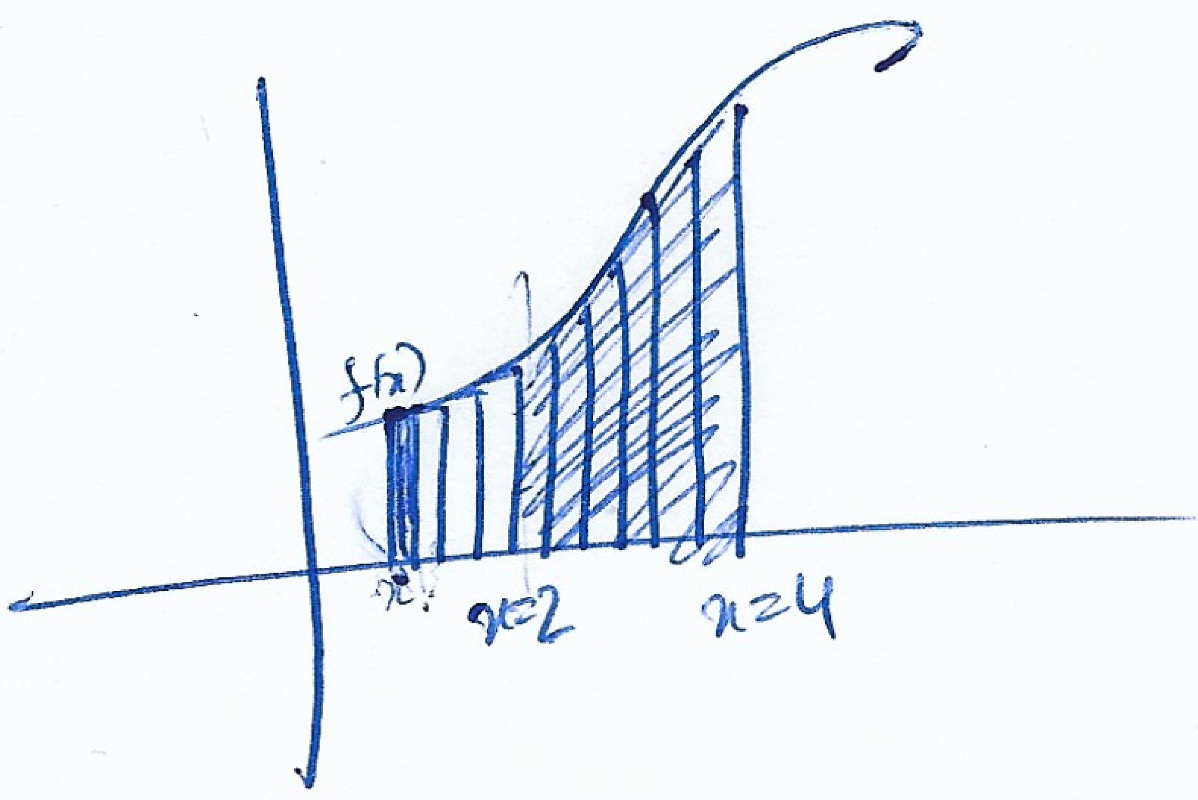
$\frac{dy}{dx} = 3(x+2)^2$



INTEGRATION

Differentiation = Difference + division.

$\frac{\text{Addition} \quad + \quad \text{Multiplication}}{\text{Integration.}}$



$\int f(x)\,dx$

VARIABLE FUNCTION

INDEFINITE INTEGRAL

$\int_{x=2}^{x=4} f(x)\,dx$

definite Integral $\Rightarrow$ constant value as result

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1} + c}$$

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} x^{n+1} = (n+1) x^n$$

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$$

$$\frac{\cancel{(n+1)} x^n}{\cancel{(n+1)}}$$

eg $\int x^3 dx = \frac{x^4}{4} + c$

$$\int 4x^4 dx = 4 \int x^4 dx$$

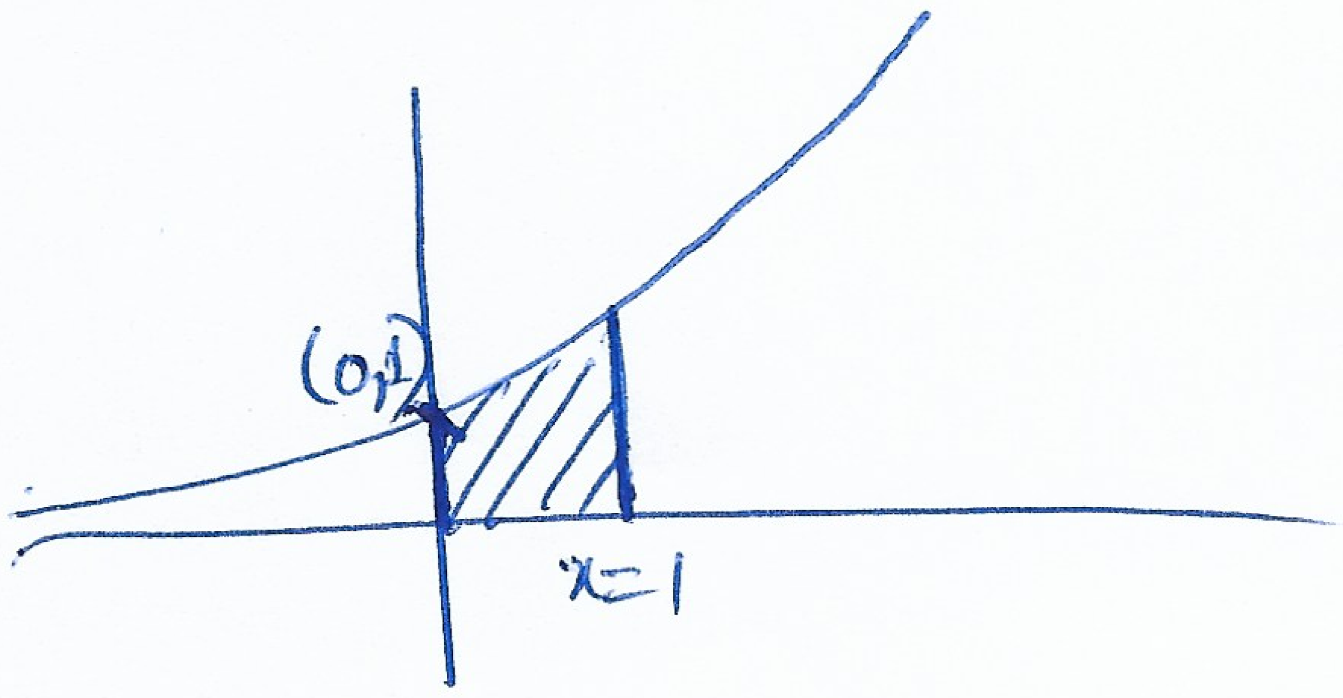
$$= 4 \times \frac{x^5}{5} + c$$

$$\boxed{\int \frac{1}{x} dx = \ln x + c}$$

$\log_e(x) = \ln(x)$

$e = 2.713$

$$\int e^x dx = e^x + c$$



$$\int_0^1 e^x dx = e^x \Big]_0^1 = e^1 - e^0$$

$$= e - 1$$

$$\boxed{\begin{aligned} \int f(x) dx &= G(x) + c \\ \int_a^b f(x) dx &= G(b) - G(a) \end{aligned}}$$

eg. If $\underset{\text{Force}}{F(x)} = (2\underset{\text{displacement}}{x} - 2)\ N$

Find Work done.

$$W \cdot D = \int F\,dx$$

$$\int c\,dx = cx$$

$$\int \{f_1(x) \pm f_2(x)\}dx = \int f_1(x)dx \pm \int f_2(x)dx$$

$$= \int (2x-2)\,dx$$

$$\int 1\,dx$$

$$\int x^0\,dx = \frac{x^{0+1}}{0+1} = \frac{x^1}{1} = x$$

$$= 2\int x\,dx - 2\int 1\,dx$$

$$= 2\left(\frac{x^{1+1}}{1+1}\right) - 2\left(\frac{x^{0+1}}{0+1}\right) + c$$

$$= \frac{2x^2}{2} - 2x + c$$

$$= x^2 - 2x + c$$

Find work done b/w $x = 2m$ & $x = 5m$.

$$\int_{x=2}^{x=5} F\,dx = \left. x^2 - 2x \right]_2^5$$

$$= \{5^2 - 2(5)\} - \{2^2 - 2(2)\}$$

$$= 15\ J$$

$$v(t) = t^2 + 2t$$

$$x(t = 2s)$$

$$\frac{dx}{dt} = v(t)$$

$$\int dx = \int v(t)\,dt$$

$$x = \int (t^2 + 2t)\,dt$$

$$x(t) = \frac{t^3}{3} + \frac{2t^2}{2} + c \to 0$$

$$x(2) = \frac{2^3}{3} + \frac{\cancel{2}(2^2)}{\cancel{2}}$$

$$= \frac{8}{3} + 4 = \frac{20}{3} = 6.66\ m$$

S x v a

1

$$v = f(x)$$

$$\frac{dx}{dt} = v$$

$$a = v\frac{dv}{dx}$$

$$\frac{dv}{dt} = a = \frac{d^2x}{dt^2}$$

$v = 10\,m/s$ find displacement when $t = 2\,s$.

$$\frac{dx}{dt} = 10 \qquad \int dx = \int 10\,dt$$

$$x = 10t$$