CENTRE FOR ADVANCEMENT OF STANDARDS IN EXAMINATIONS
(GEMS ASIAN SCHOOLS)
COMMON REHEARSAL EXAMINATIONS – JANUARY 2015
(ALL INDIA SENIOR SCHOOL CERTIFICATE EXAMINATION)
MATHEMATICS (041)

Grade: XII **Max Marks: 100**

No. of pages: 4 **Time: 3 hours.**

General Instructions:

i. *All questions are compulsory.*
ii. *The question paper consist of 26 questions divided into three sections A,B and C. Section A comprises of 6 questions of one mark each , section B comprises of 13 questions of 4 marks each and section C comprises of 07 questions of six marks each.*
iii. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
iv. *Use of calculators is not permitted. You may ask for logarithmic tables, if required.*
v. *This question paper consists of 4 printed pages.*

SECTION – A

Question numbers 1 to 6 carry 1 mark each.

1) Let f: R $\rightarrow$ R be defined by f(x) = $(2-x^7)^{\frac{1}{7}}$. Find a function g: R $\rightarrow$R such that

   gof = fog = I_R

2) Find the value(s) of $cos^{-1}\left[cos\frac{-3\pi}{5}\right]$.

3) If A is a square matrix of order 3 and $\left|A^T\right| = 5$, find the value of $\left|2\ adjA\right|$.

4) Without expanding evaluate : $\begin{vmatrix} 2x-4 & 2x+5 & 2x-7 \\ 4 & -5 & 7 \\ y-3 & y-3 & y-3 \end{vmatrix}$.

5) If 2A = $\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, find k so that A. (adjA) = kI .

6) If $\vec{a}$ and $\vec{b}$ are any two unit vectors , then prove that $\left|\vec{a}-\vec{b}\right| = 2\sin\frac{\theta}{2}$.

SECTION – B

Question numbers 7 – 19 carry 4 marks each.

7) Examine which of the following is a binary operation. i) a ∗ b =, a^b, a, b ∈ N
ii) a ∗ b = b^a, a, b ∈ Q For binary operation check the commutative and associative property.

(OR)

Check whether the relation R in the set R of reals, defined as
R = {(a, b): a≤ b^2} is an equivalence relation or not. Verify all three conditions

8) Solve for x : $\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$.

9) Using properties of determinants, prove that $\begin{vmatrix} b^2+c^2 & c^2 & b^2 \\ c^2 & c^2+a^2 & a^2 \\ b^2 & a^2 & b^2+a^2 \end{vmatrix}$ = $4a^2b^2c^2$

10)If $\sqrt{1-x^6}+\sqrt{1-y^6} = a\left(x^3-y^3\right)$, prove that $\frac{dy}{dx} = \frac{x^2\sqrt{1-y^6}}{y^2\sqrt{1-x^6}}$.

11)If $y\sqrt{1+x^2} = \log\left(\sqrt{1+x^2}-x\right)$, show that $\left(1+x^2\right)y_2 + 3xy_1 + y = 0.$

12)Find the intervals in which f(x) = sin x - $\sqrt{3}\cos x$, x ε [0, 2π] is strictly increasing or decreasing.

13) Evaluate $\int_0^{\pi} \frac{x\,dx}{4-cos^2x}$

OR

Evaluate: $\int_2^5 (x^2+3)dx$ as limit of a sum

14)Evaluate : $\int \frac{e^{\tan^{-1}x}.\left(1+x+x^2\right)}{1+x^2}dx$

15) Evaluate : $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

OR

$\int \frac{5x+3}{\sqrt{x^2+4x+10}}$ dx

16) Find the general solution for the differential equation :

$x\frac{dy}{dx} + y - x + xy\cot x = 0,\ x \neq 0$

17) Solve: x $\frac{dy}{dx} = y(\log y - \log x + 1)$.

18) If $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2\left(\vec{b} \times \vec{c}\right)$.

OR

Form the differential equation representing the family of parabolas having center at the origin and axis as the x-axis.

19) Two dice are thrown simultaneously. Let X denote the number of ones. Find the probability distribution of X. Also find the mean and variance of X using the probability distribution table.

<u>SECTION – C</u>

Question numbers 20-26 carry 6 marks each:

20) Two schools A and B wants to award their students who won gold, silver and bronze medals in CBSE Athletic meet. The total amount awarded for 1 gold, 1 silver and 1 bronze is Rs 2200.The school A awarded Rs 4200 and school B, Rs 4900. School A won 2 gold, 1 silver and 3 bronze whereas school B won 3 gold, 2 silver and 1 bronze. Using matrices, find the award money for each prize. **What is the importance of sports in education?**

21) Make a sketch of the region given below and find the area using integration

$\{(x, y): 0 \leq y \leq x^2 + 3, 0 \leq y \leq 2x + 3, 0 \leq x \leq 3\}$.

22)Find the equation of the plane through the line of intersection of the planes

x- 2y+ z=1 and 2x + y+ z= 8 and parallel to the line $\frac{x-2}{1}=\frac{y+3}{2}=\frac{z-4}{1}$. Also find the distance from the point (1, 2, 3) to the plane formed.

23) Find the Cartesian and vector equation of the planes passing through the intersection of the planes $\vec{r}\cdot(2\vec{i}+6\vec{j})+12=0$ and $\vec{r}\cdot(3\vec{i}-\vec{j}+4\vec{k})=0$, which are at unit distance from the origin.

OR

Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are co planar. Also find the equation of the plane containing the lines.

24) A 15-year old child rides his motor cycle at 50 km/hour, the cost of petrol is Rs 5 per Km. If he rides at a speed of 70 km/hr. He has only Rs 400 to spend on petrol and wishes to travel maximum distance within one hour. Form a LPP and solve graphically. **Should a child below 18-years be allowed to drive a motor cycle? Write your opinion.**

25) Answering a question on a multiple choice test with four choices for each question, a student knows, guesses or copies the answer. Let ½ be the probability that he knows the answer and ¼ be the probability that he guesses the answer. Assume that a student who copies the answer will be correct with probability ¾. What is the probability that the student knows the answer given that he answered it correctly?

26) An open box with a square base is to be made out of a given quantity of sheet of area $81m^2$. Find the maximum volume of the box.

OR

If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

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