

# SAMPLE QUESTION PAPER

CLASS-XII (2016-17)

MATHEMATICS (041)

Time allowed: 3 hours

Maximum Marks: 100

## General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

## SECTION-A

Questions from 1 to 4 are of 1 mark each.

- 1. What is the principal value of  $\tan^{-1}\left(\tan \frac{2\pi}{3}\right)$ ?
- 2. A and B are square matrices of order 3 each,  $|A| = 2$  and  $|B| = 3$ . Find  $|3AB|$
- 3. What is the distance of the point (p, q, r) from the x-axis?
- 4. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 3x^2 - 5$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $g(x) = \frac{x}{x^2+1}$ . Find  $g \circ f$

## SECTION-B

Questions from 5 to 12 are of 2 marks each.

- 5. How many equivalence relations on the set  $\{1,2,3\}$  containing (1,2) and (2,1) are there in all? Justify your answer.
- 6. Let  $l_i, m_i, n_i$ ;  $i = 1, 2, 3$  be the direction cosines of three mutually perpendicular vectors in space. Show that  $AA' = I_3$ , where  $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$ .
- 7. If  $e^y (x+1) = 1$ , show that  $\frac{dy}{dx} = -e^y$
- 8. Find the sum of the order and the degree of the following differential equations:

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1+x) = 0$$

9. Find the Cartesian and Vector equations of the line which passes through the point  $(-2, 4, -5)$  and parallel to the line given by  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$
10. Solve the following Linear Programming Problem graphically:  
 Maximize  $Z = 3x + 4y$   
 subject to  
 $x + y \leq 4, x \geq 0$  and  $y \geq 0$
11. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.
12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

### SECTION-C

Questions from 13 to 23 are of 4 marks each.

13. If  $A + B + C = \pi$ , then find the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

OR

Using properties of determinant, prove that

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

14. It is given that for the function  $f(x) = x^3 - 6x^2 + ax + b$  Rolle's theorem holds in  $[1, 3]$  with  $c = 2 + \frac{1}{\sqrt{3}}$ . Find the values of 'a' and 'b'
15. Determine for what values of x, the function  $f(x) = x^3 + \frac{1}{x^3}$  ( $x \neq 0$ ) is strictly increasing or strictly decreasing

OR

Find the point on the curve  $y = x^3 - 11x + 5$  at which the tangent is  $y = x - 11$

16. Evaluate  $\int_0^2 (x^2 + 3) dx$  as limit of sums.
17. Find the area of the region bounded by the y-axis,  $y = \cos x$  and  $y = \sin x$ ,  $0 \leq x \leq \frac{\pi}{2}$
18. Can  $y = ax + \frac{b}{a}$  be a solution of the following differential equation?  

$$y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \dots\dots\dots (*)$$
  
 If no, find the solution of the D.E. (\*).

OR

Check whether the following differential equation is homogeneous or not

$$x^2 \frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution  $y=vx$ .

19. If the vectors  $\vec{p} = a\hat{i} + \hat{j} + \hat{k}$ ,  $\vec{q} = \hat{i} + b\hat{j} + \hat{k}$  and  $\vec{r} = \hat{i} + \hat{j} + c\hat{k}$  are coplanar, then for a, b, c  $\neq 1$  show that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

20. A plane meets the coordinate axes in A, B and C such that the centroid of  $\Delta ABC$  is the point  $(\alpha, \beta, \gamma)$ . Show that the equation of the plane is  $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$
21. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.
22. The random variable X has a probability distribution P(X) of the following form, where k is some number:
- $$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$
- (i) Find the value of k (ii) Find  $P(X < 2)$  (iii) Find  $P(X \leq 2)$  (iv) Find  $P(X \geq 2)$
23. A bag contains  $(2n + 1)$  coins. It is known that 'n' of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is  $\frac{31}{42}$ , find the value of 'n'.

#### SECTION-D

Questions from 24 to 29 are of 6 marks each

24. Using properties of integral, evaluate  $\int_0^\pi \frac{x}{1+\sin x} dx$

OR

Find:  $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx$

25. Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\tan^{-1} \left( \frac{x+1}{x-1} \right) + \tan^{-1} \left( \frac{x-1}{x} \right) = -\tan^{-1} 7$$

OR

Determine whether the operation \* define below on  $\mathbb{Q}$  is binary operation or not.

$$a * b = ab + 1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in  $\mathbb{Q}$ .

26. Find the value of  $x$ ,  $y$  and  $z$ , if  $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$  satisfies  $A' = A^{-1}$

OR

Verify:  $A(\text{adj } A) = (\text{adj } A)A = |A| I$  for matrix  $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

27. Find  $\frac{dy}{dx}$ , if  $y = e^{\sin^2 x} \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$
28. Find the shortest distance between the line  $x - y + 1 = 0$  and the curve  $y^2 = x$
29. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:
- $$\vec{r} = (8 + 3\lambda) \hat{i} - (9 + 16\lambda) \hat{j} + (10 + 7\lambda) \hat{k}$$
- $$\vec{r} = 15 \hat{i} + 29 \hat{j} + 5 \hat{k} + \mu (3 \hat{i} + 8 \hat{j} - 5 \hat{k})$$