LIMITS TUTORIAL

Pg 82, 83	SOLVED EX	1, 3, 7
Pg 84, 85	SOLVED EX	10, 12, 14
Pg 86, 87	SOLVED EX	15, 16, 20
Pg 88, 89	SOLVED EX	21, 23, 24..
Pg 90, 91	SOLVED EX	2, 4, 5
Pg 95, 96		3, 6, 8, 11, 12, 13
Pg 96		16, 19, 20
Pg 97, 98, 99		1, 3, 6, 9, 11, 17, 18, 21, 22, 25, 30
Pg 100, 101		35, 38, 43, 44
Pg 102, 103		7, 8, 12, 13, 14
Pg 105.		Comp 1, 2
Pg 109		1, 2, 4.

Pg 95, 96

(3) $$\lim_{x\to 1}\left(\frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}}\right)\left(\frac{\sqrt{x+8}+\sqrt{8x+1}}{\sqrt{x+8}+\sqrt{8x+1}}\right)\left(\frac{\sqrt{5-x}+\sqrt{7x-3}}{\sqrt{5-x}+\sqrt{7x-3}}\right)$$

$$\lim_{x\to 1}\frac{(-7x+7)}{(-8x+8)}\left(\frac{\sqrt{5-x}+\sqrt{7x-3}}{\sqrt{x+8}+\sqrt{8x+1}}\right)$$

$$\lim_{x\to 1}\frac{-7\cancel{(x-1)}}{-8\cancel{(x-1)}}\left(\frac{\sqrt{5-x}+\sqrt{7x-3}}{\sqrt{x+8}+\sqrt{8x+1}}\right)$$

$$=\frac{\cancel{-}7}{\cancel{-}8}\left(\frac{\sqrt{4}+\sqrt{4}}{\sqrt{9}+\sqrt{9}}\right)=\frac{7}{\cancel{8}_2}\times\frac{\cancel{4}}{6}=\frac{7}{12}$$

(6) $$\lim_{x\to\infty}\left(a^x+b^x+c^x\right)^{1/x}\qquad 0<a<b<c$$

$$\lim_{x\to\infty}\left(c^x\left(\frac{a^x}{c^x}+\frac{b^x}{c^x}+1\right)\right)^{1/x}$$

$$\lim_{x\to\infty} c\left(1+\left(\frac{a}{c}\right)^x+\left(\frac{b}{c}\right)^x\right)^{1/x}$$

$\because 0<a<b<c$

$\frac{a}{c}<1$

$\frac{b}{c}<1$

$$=c$$

(8) $$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{\sin 5x}\left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right)$$

$$\lim_{x\to 0}\frac{x}{\sin 5x\left(\sqrt{x+4}+2\right)}=\frac{1}{5\times(2+2)}=\frac{1}{20}$$

(11) $\lim_{x\to 1} (1-x)\tan\left(\frac{\pi x}{2}\right)$

$\lim_{x\to 1} \frac{(1-x)}{\cot\left(\frac{\pi x}{2}\right)}$

$d(\cot x) = -\operatorname{cosec}^2 x.$

$= \lim_{x\to 1} \frac{-1}{-\operatorname{cosec}^2\left(\frac{\pi x}{2}\right)\times\frac{\pi}{2}}$

$= \frac{-1}{-1\times\frac{\pi}{2}} = \frac{2}{\pi}$

(12) If α, β are roots of x^2+bx+c

find $\lim_{x\to\beta} \frac{1-\cos(x^2+bx+c)}{(\beta-x)^2}$

$1-\cos\theta$

$= 2\sin^2\frac{\theta}{2}$

$x^2+bx+c = (x-\alpha)(x-\beta)$

$\lim_{x\to\beta} \frac{1-\cos\{(x-\alpha)(x-\beta)\}}{(\beta-x)^2}$

$\lim_{x\to 0} \frac{\sin kx}{x} = k$

$\lim_{x\to\beta} \frac{2\sin^2\left\{\frac{(x-\alpha)(x-\beta)}{2}\right\}}{(\beta-x)^2}$

$\lim_{x\to\beta} \frac{2\sin\left\{\frac{(x-\alpha)(x-\beta)}{2}\right\}}{(x-\beta)} \frac{\sin\left\{\frac{(x-\alpha)(x-\beta)}{2}\right\}}{(x-\beta)}$

$= \cancel{2}\frac{(x-\alpha)}{\cancel{2}}\frac{(x-\alpha)}{2} = \frac{1}{2}(\beta-\alpha)^2$

(13) $\lim_{x\to 0} \frac{\ln(1+x)}{3^x - 1} = \lim_{x\to 0} \frac{\frac{\ln(1+x)}{x}}{\frac{3^x-1}{x}}$

$$= \frac{\lim_{x\to 0} \frac{\ln(1+x)}{x}}{\lim_{x\to 0} \frac{3^x-1}{x}} = \frac{1}{\ln 3}$$

(16) $\lim_{x\to 1} (1+\sin \pi x)^{\cot \pi x} = e^{\lim_{x\to 1} \cot \pi x (1+\sin\pi x - 1)}$

1^{∞} $f(x)^{g(x)} = e^{\lim_{x\to a} g(x)\{f(x)-1\}}$

$$= e^{\lim_{x\to 1} \cos \pi x}$$

$$= e^{-1} = \frac{1}{e}$$

(19) $\lim_{x\to 0} \left(\frac{\sin 2x + a \sin x}{x^3}\right) = b$ $\quad a, b \in R.$

L.H
$= \lim_{x\to 0} \left(\frac{2\cos 2x + a\cos x}{3x^2}\right)$

$a + 2 = 0$

$a = -2.$

L.H
$= \lim_{x\to 0} \left(\frac{-4\sin 2x + 2\sin x}{6x}\right)$

L.H
$= \lim_{x\to 0} \left(\frac{-8\cos 2x + 2\cos x}{6}\right) = \frac{-8+2}{6} = -1 = b$

20) If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+1}{x-1} - ax\right)$ be finite find a & the limit.

$$\lim_{x\to\infty}\left(\frac{x^2+1-ax^2+ax}{x-1}\right)$$

$$\lim_{x\to\infty}\left(\frac{x^2(1-a)+ax+1}{x-1}\right) \qquad 1-a=0 \quad a=1$$

$$\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right) = \underline{1}$$

Pg 105

Comp 1 $\quad 1^\infty \quad \lim\limits_{x\to a}\{f(x)\}^{g(x)} = e^{\lim\limits_{x\to a} g(x)\{f(x)-1\}}$

1) $\lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}} = e^{\alpha}$

$\alpha = \lim\limits_{x\to a} \tan\dfrac{\pi x}{2a}\left(2-\dfrac{a}{x}-1\right) = \lim\limits_{x\to a}\dfrac{x-a}{x\cot\left(\frac{\pi x}{2a}\right)} = \lim\limits_{x\to a}\dfrac{1}{x}\times\lim\limits_{x\to a}\dfrac{x-a}{\cot\frac{\pi x}{2a}}$

by L·H $= \dfrac{1}{a}\lim\limits_{x\to a}\dfrac{1}{-\operatorname{cosec}^2\left(\frac{\pi x}{2a}\right)\times\frac{\pi}{2a}} = \dfrac{-2}{\pi}$

$\alpha = -\dfrac{2}{\pi} \quad$ Ans $= e^{\alpha} = e^{-2/\pi}$

2) $\lim\limits_{x\to\infty}\left(\dfrac{x^2-2x+2}{x^2-4x+1}\right)^x = e^{\alpha} \qquad \alpha = \lim\limits_{x\to\infty} x\left\{\dfrac{x^2-2x+2}{x^2-4x+1}-1\right\}$

$\alpha = \lim\limits_{x\to\infty} x\left(\dfrac{2x+1}{x^2-4x+1}\right) \Rightarrow \lim\limits_{x\to\infty}\left(\dfrac{2x^2+x}{x^2-4x+1}\right) \Rightarrow \lim\limits_{x\to\infty}\left(\dfrac{2+\frac{x}{x^2}}{1-\frac{4}{x}+\frac{1}{x^2}}\right) = 2$

$\alpha = 2 \quad$ Ans $= e^{\alpha} = e^2$

3) $\lim\limits_{x\to 0}\left\{\tan\left(\dfrac{\pi}{4}+x\right)\right\}^{\frac{1}{x}} = e^{\alpha}$

$\alpha = \lim\limits_{x\to 0}\dfrac{1}{x}\left\{\tan\left(\dfrac{\pi}{4}+x\right)-1\right\}$

$$\alpha = \lim_{x \to 0} \frac{\left\{ \tan\left(\frac{\pi}{4} + x\right) - 1 \right\}}{x}$$

L.H

$$\alpha = \lim_{x \to 0} \frac{\sec^2\left(\frac{\pi}{4} + x\right)(0 + 1)}{1} = 2$$

Ans $e^{\alpha} = e^{2}$