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Dual Nature of Radiation and Matter

Electron Emission.

The phenomenon of emission of electrons from a metal surface is called electron emission.

It is of the following types.

(a) Thermionic Emission:- Here electrons are emitted from the metal surface with the help of thermal energy.

(b) Field emission:- Electrons are emitted from a metal surface by subjecting it to a very strong electric field.

(c) Photoelectric emission:- Electrons are emitted from a metal surface with the help of photons (light). These photo (light) generated electrons are called photoelectrons.

Work function

The minimum amount of energy required by an electron to just escape from the metal surface is known as work function of the metal. It is denoted by ϕ_0 and measured in eV.

One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 Volt.

$$1 eV = 1.602 \times 10^{-19} J.$$

The work function ϕ_0 depends on the properties of the metal and the nature of its surface.
eg:. The work function of platinum is the highest $\phi_0 = 5.65\ eV$ while it is the lowest for caesium $\phi_0 = 2.14\ eV$.

Photo electric Effect.

The phenomenon of emission of electrons from a metal surface when electromagnetic radiations of sufficiently high frequency are incident on it; it is called photo electric effect.

Alkali metal like Li, Na, K, Cs show photoelectric effect with visible light. Metals like Zn, Cd, Mg respond to UV light.

Hertz's Observations.

The phenomenon of photo electric effect was discovered by Heinrich Hertz in 1887. While demonstrating the existence of EM waves Hertz found that high voltage sparks passed across the metal electrodes of the detector loop more easily when the cathode was illuminated by UV light from an arc lamp. The UV light falling on the metal surface caused the emission of negatively charged particles, which are now known to be electrons.

Hallwach's and Lenard's. Observations.

During the years 1886 - 1902 Wilhelm Hallwachs and Philipp Lenard investigated the phenomenon of Photoelectric emission in detail.

Lenard observed that when UV radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes, current flows in the circuit. As soon as the UV radiations are stopped, the current flow also stopped. The observation indicated that when UV radiations fall on the emitter plate C electrons are ejected from it which are attracted towards the positive, collector plate A by the electric field. The electrons flow through the evacuated glass tube, resulting in the current flow. This current was called photo electric current.

Hallwachs connected a zinc plate to an electroscope. He observed that the zinc plate became (i) uncharged if initially -vely charged (ii) +vely charged if initially uncharged and (iii) more +vely charged if initially +vely charged. From these observations, he concluded that some negatively charged particles were emitted by the zinc plate when exposed to UV light.

Hallwachs and Lenard also observed that when UV light fell on the emitter plate, no electrons were emitted at all when the frequency of the incident light was smaller than a certain minimum value, called the threshold frequency.

Experimental Study Of Photo electric effect.

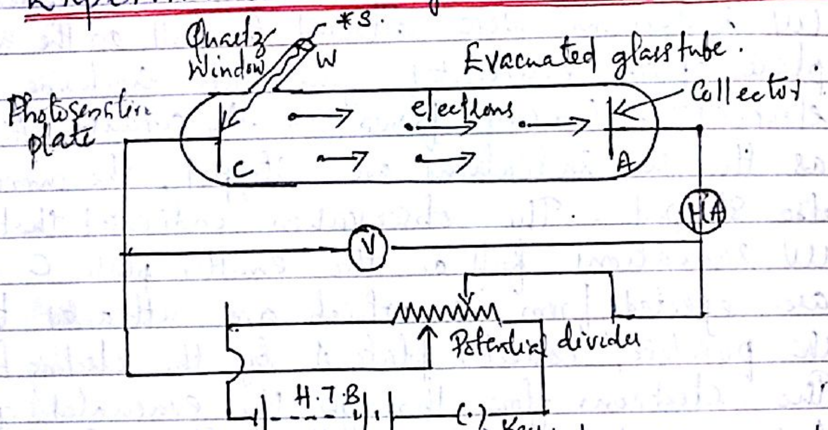


Figure shows the experimental arrangement used for the study of photo electric effect. It consists of an evacuated glass / quartz tube which encloses a photosensitive plate C and another metal pipe A. The two electrodes are connected to a high tension battery, a potential divider arrangement and a micro ammeter μA.

When monochromatic radiations of a sufficiently high frequency fall on the plate C electrons are emitted which are collected by the plate A. So a current, called photo electric current flows in the outer circuit which is measured by the microammeter μA.

1. Effect of intensity of light on photo current.

The collector A is maintained at a +ve potential with respect to emitter C so that electrons emitted from C are attracted towards collector A. Keeping the frequency of the incident radiation and the accelerating potential fixed, the intensity of light is varied and the resulting photo electric current is measured in each time. It is found that photocurrent increases linearly with intensity of incident light.

Since the photo electric current is directly proportional to the no: of photo electrons emitted per second. this implies that the no: of photo electrons emitted per second is directly proportional to the intensity of incident radiation.

2. Effect of potential on photo electric current.

If we keep the intensity and the frequency of the incident radiation fixed and increase the positive potential called accelerating potential on plate A gradually. It is found that the photoelectric current increases with the increase in accelerating potential till a stage is reached when the photoelectric current becomes maximum and does not increase further with the increase in the accelerating potential. This maximum value of the photoelectric current is called the saturation current. At this stage, all the electrons emitted by the plate ~~A~~ are collected by the plate A.

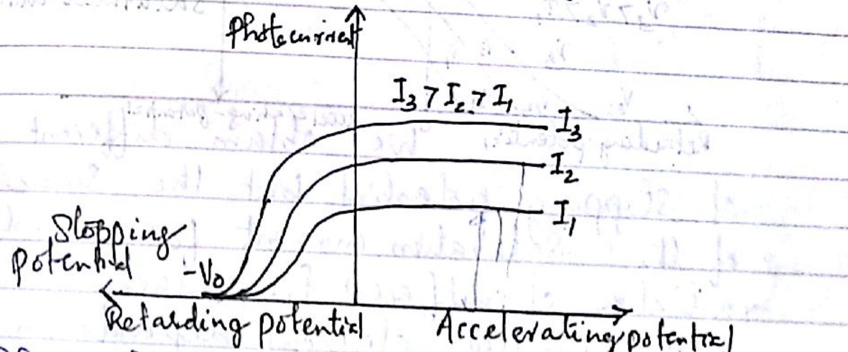
Now apply a negative potential on plate A with respect to plate C and increase its magnitude gradually. It is seen that the photocurrent decreases rapidly until it becomes zero for a certain value of negative potential on plate A. The value of the retarding potential at which the photoelectric current becomes zero is called cutoff or stopping potential for the given frequency of the incident radiation.

At the stopping potential V_0 when no photoelectrons are emitted, the work done by stopping potential on the fastest electrons must be equal to its K.E

i.e $K_{max} = eV_0$.

$\frac{1}{2} m v^2_{max} = eV_0$.

If we repeat the experiment with incident radiation of the same frequency but of higher intensity i.e ($I_3 > I_2 > I_1$), we find that the values of saturation currents have increased in proportional to the intensity of incident radiation, while the stopping potential is still the same. Thus for a given frequency of incident radiation, the stopping potential is independent of its intensity.

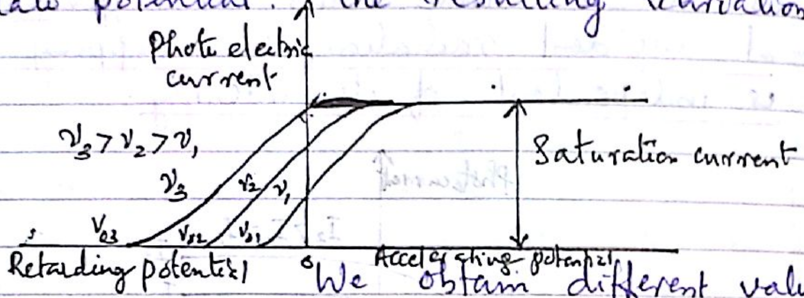


3. Effect of frequency of incident radiation on stopping potential.

P.T.O →

~~To study the effect of frequency on photoelectric effect, the intensity of incident radiation at each frequency is adjusted in such a way that the saturation current is same each time when the plate A is at a +ve potential.~~

To study the relation between the frequency ν of the incident radiation and the stopping potential V_0. We suitably adjust the same intensity of light radiation at various frequencies and study the variation of photocurrent with collector plate potential. The resulting variation is

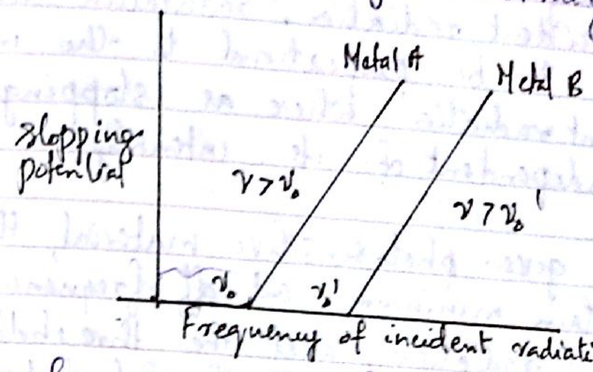


We obtain different values of stopping potential but the same value of the saturation current for incident radiation of different frequencies. The energy of the emitted electrons depends on the frequency of the incident radiations. The stopping potential is more negative for higher frequencies of incident radiation. For frequencies $\nu_3 > \nu_2 > \nu_1$, the corresponding stopping potentials vary in the order

$$V_{03} > V_{02} > V_{01}$$

This implies that greater the frequency of incident light greater is the maximum K.E of the photoelectrons i.e greater is the retarding potential to stop them completely i.e stopping potential.

The graph between the frequency of incident radiation and the corresponding stopping potential for different metals, we get a straight line graphs.



These graphs reveal the following facts.

(i) The stopping potential or the maximum K.E of the photo e's increases linearly with the frequency of the incident radiation, but is independent of its intensity.

(ii) For a frequency ν of incident radiations lower than the cut off frequency ν_0, no photoelectric emission is possible even if the intensity is large. This minimum cut-off frequency ν_0 is called the threshold frequency. It is different for different metals.

Laws of Photoelectric emission.

1. For a given photosensitive material and frequency of incident radiation, the photoelectric current is

directly proportional to the intensity of light.

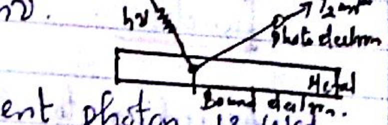
(ii) For a given photosensitive material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation where as stopping potential is independent of its intensity.

(iii) For a given photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation, called the threshold frequency below which no emission of photo electrons take place. Above the threshold frequency, the stopping potential or equivalently the maximum K.E of the emitted photo e's increases linearly with the frequency of the incident radiation; but is independent of its intensity.

(iv) The photoelectric emission is an instantaneous process. The time lag between the incident light radiation and the emission of photo e's is very small, even less than 10^{-9} sec.

Einstein's Photo electric equation.

Einstein explained photo electric effect on the basis of Plank's quantum theory according to which a light radiation travels in the form of discrete photons. The energy of each photon is $h\nu$.



The energy $h\nu$ of the incident photon is used up in two parts.

(a) a part of the energy of the photon is used to liberating the electron from the metal surface which is equal to the work function ϕ_0 of the metal.

(b) the remaining energy of the photon is used in imparting K.E to the ejected electron.

ie $h\nu = \phi_0 + K_{max}$

$$\therefore \boxed{K_{max} = \frac{1}{2}mv^2_{max} = h\nu - \phi_0} \quad \text{—} ①$$

If the incident photon is of threshold frequency ν_0

$\therefore \phi_0 = h\nu_0$.

$$\therefore \boxed{K_{max} = h\nu - h\nu_0 = h(\nu - \nu_0)} \quad \text{—} ②$$

Equ's ① and ② are called Einstein's photoelectric equations

Explanation of laws of Photo electric effect on the basis of Einstein's Photo electric equation.

1. Explanation of effect of intensity:-

The increase of intensity means the increase in the number of photons striking the metal surface per unit time. As each ~~electron~~ photon ejects only one electron. So the number of ejected ~~photons~~ photo electrons increases with the increase in intensity of incident radiation.

2. Explanation of threshold frequency:-

If $\nu < \nu_0$ ie the frequency of incident radiation is less than the threshold frequency

$K_{max} = h(\nu - \nu_0)$ is negative.

This has no physical meaning. So photo electric emission does not occur below the threshold frequency.

3. Explanation of Kinetic energy:-

If $\nu > \nu_0$ ie the frequency of incident radiation is more than the threshold frequency

$$K_{max} = h(\nu - \nu_0)$$

or $K_{max} \propto \nu$ or $eV_0 \propto \nu$ $(K_{max} = eV_0)$

ie above the threshold frequency the maximum K.E of the electrons increases linearly with the frequency ν of the incident radiation. or stopping potential is directly proportional to the frequency of the radiation

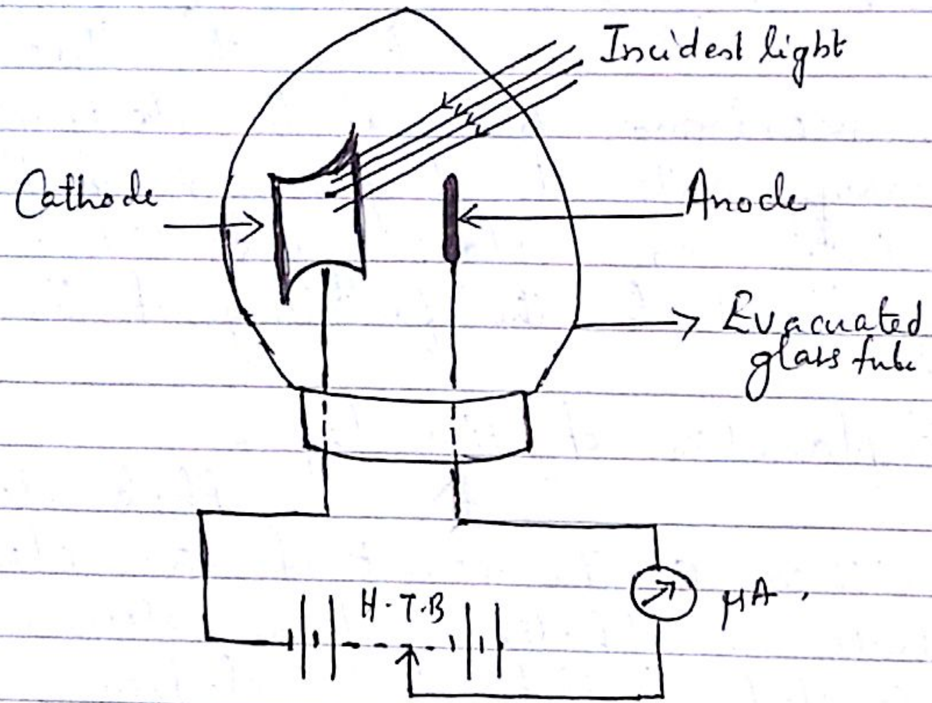
4. Explanation of time lag:-

Photo electric emission is the result of an elastic collision between a photon and an electron. In photo electric effect each electron absorb a photon. This process is instantaneous.

Photo cell.

A photo cell or a photo electric cell is an arrangement which converts light energy into electric energy. It works on the principle of photo electric emission.

Construction:- It consists of an evacuated glass tube which encloses two electrodes The cathode or emitter is a parabolic metal plate coated with a layer of photo sensitive material. The anode or a collector a thin rod of Pt or Ni. The two electrodes are connected externally to a high tension battery and a micro ammeter.



Working:- When light of frequency greater than the threshold frequency falls on the cathode, photo electrons are emitted which are attracted by the collector. The circuit gets completed and a current starts flowing in the circuit. As the number of photo electrons emitted is proportional to the intensity of incident light, the photoelectric current indicated by the microammeter gives a measure of the intensity of light.

Uses of Photo-cells.

1. In burglar's alarms 2. In counting devices
3. In automatic street light system.
4. In cinematography to the reproduction of sound.

Determination of Plank's constant and work function.

According to Einstein's photoelectric equation

$$K_{max} = h\nu - \phi_0.$$

If V_0 is the stopping potential then

$$K_{max} = eV_0.$$

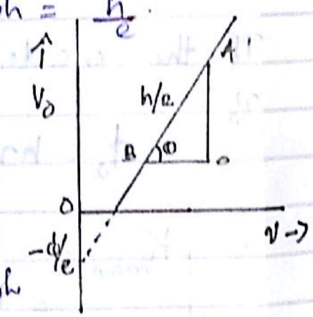
$$\therefore \quad eV_0 = h\nu - \phi_0.$$

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e} \quad \text{—} (1)$$

which is equal to the eq^n^ of a straight line $y = mx + c$. — (2).

where m is the slope of the graph.

Clearly, Slope of $V_0 - \nu$ graph $= \frac{h}{e}$.



ie $m = \tan\theta = \frac{AC}{BC} = \frac{h}{e}$.

$$\therefore h = e \times \frac{AC}{BC}$$

$= e \times$ slope of $V_0 - \nu$ graph

Thus Plank's constant 'h' can be determined.

From eq^n^ (2)

C = intercept on vertical axis

From the graph.

Intercept on vertical axis $= -\frac{\phi_0}{e}$

$\therefore \phi_0 = e \times$ Magnitude of the intercept on vertical axis.

In this way, the work f^n^ ϕ_0 can be determined.

Particle Nature of light :- The Photon.

According to Plank's quantum theory of radiation an electromagnetic wave travels in the form of discrete packets of energy called quanta. One quantum of light radiation is called a photon. The main features of photons are as follows.

- A photon travels with the speed of light.
- Each photon has energy $E = h\nu$ and momentum $p = h\nu/c$ and c is the speed of light.
- All photons of light of a particular frequency ν or wavelength λ, have the same energy
  $E = h\nu$
  $= h\frac{c}{\lambda}$.
  and momentum $p = \frac{h\nu}{c} = \frac{h}{\lambda}$.
- Photons are electrically neutral and are not deflected by electric and magnetic fields.
- In a photon-particle collision, the total energy and total momentum are conserved.

Dual nature of radiation.

The phenomenon like interference diffraction and polarisation etc can be satisfactorily explained only on the basis of wave nature of light. On the other hand, the phenomenon like photo electric effect, compton effect can be explained only in terms of quantum theory of light i.e by particle nature of light. This shows that light radiation has dual nature i.e it sometimes behaves like wave and sometimes as a particle.

Wave nature of matter.

In 1924, the French physicist Louis Victor de Broglie put forward the bold hypothesis that moving particles of matter should display wave-like properties under suitable conditions. He reasoned that nature was symmetrical and that the two basic physical entities - matter and energy. If radiation shows dual nature, so should matter.

∴ The wave associated with material particles in motion are called matter or de Broglie waves and their wavelength is called de-Broglie wavelength.

de-Broglie's wave equation

Considering photon as an e.m wave of frequency ν its energy from Planck's quantum theory is given by

$$E = h\nu \quad -①$$

According to Einstein's mass energy relation

$$E = mc^2 \quad -②$$

$$\therefore \quad h\nu = mc^2$$

$$h\frac{c}{\lambda} = mc^2$$

$$\frac{h}{\lambda} = mc$$

or $\lambda = \frac{h}{mc} = \frac{h}{p}$ where λ is the wavelength of the radiation of frequency ν and momentum $p = mc$.

A particle of mass 'm' moving with velocity v then

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

This is de-Broglie's wave equation for material particles.

de-Broglie wavelength of an electron of Kinetic energy K.

Consider an electron of mass 'm' and charge e. Let v be the velocity attained by an electron when it is accelerated from rest to attain K.E K.

Then $K = \frac{1}{2}mv^2$

$mv^2 = 2K$

$\Rightarrow \frac{m^2 v^2}{m} = 2K$

$m^2v^2 = 2mK$

$(mv)^2 = 2mK$

$p^2 = 2mK$ where p is the momentum $p = mv$.

$\therefore p = \sqrt{2mK}$

If it is accelerated through a potential difference V

$\therefore K = eV$

$\therefore$ the above eqn becomes $p = \sqrt{2meV}$

Hence the de-Broglie wavelength of the electron is $\lambda = \frac{h}{p}$

$$\boxed{\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}} \quad \text{— (1)}$$

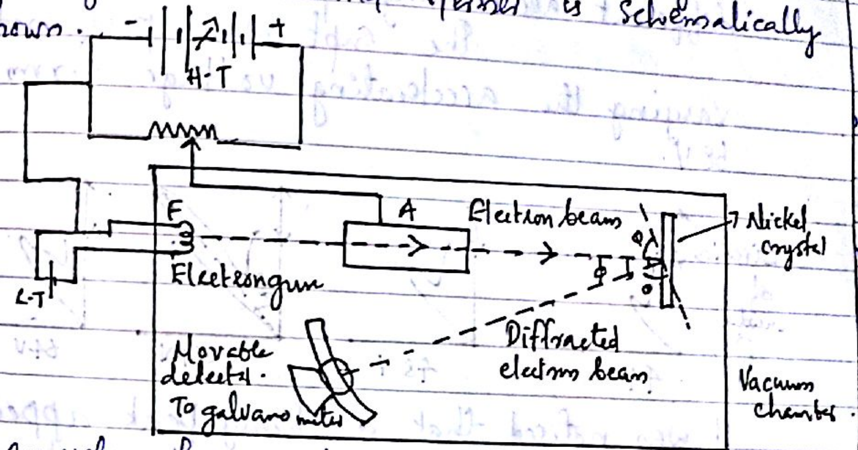
Substituting $h = 6.63 \times 10^{-34}$ Js
$m = 9.1 \times 10^{-31}$ kg
$e = 1.6 \times 10^{-19}$ C in eqn (1)

$\therefore \lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}}$ m

or $\boxed{\lambda = \frac{12.27}{\sqrt{V}} A^0 \text{ or } \frac{1.227}{\sqrt{V}} \text{nm}}$

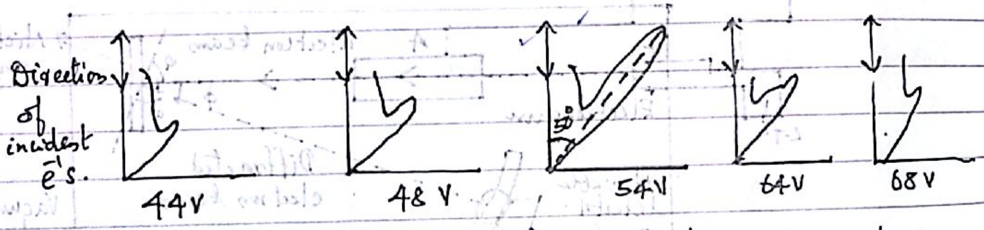
Davisson and Germer Experiment.

The wave nature of electrons was first experimentally verified by C. J Davisson and L. H Germer. The experimental arrangement used by Davisson and Germer is schematically shown.



It consists of an electron gun which comprises of a tungsten filament F, coated with barium oxide and heated by a low voltage power supply. Electrons emitted by the filament are accelerated to a desired velocity by applying suitable voltage from a high voltage power supply. They are made to pass through a cylinder with fine holes along its axis, producing a fine collimated beam. The beam is made to fall on the surface of a nickel crystal. The electrons are scattered in all directions by the atoms of the crystal. The electrons scattered in different directions by the atoms of Ni crystal are received by a movable detector which is just an electron collector. Thus we measure scattered electron intensity as a f^n of the scattering angle ϕ, the angle between the incident and the scattered electron beam. The expt. is repeated for different accelerating potentials V.

The expt. was performed by varying the accelerating voltage from 44V to 68V.



It was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelerating voltage of 54V at scattering angle $\phi = 50°$

From fig. the glancing angle θ is given by

$$\theta + \phi + \theta = 180°$$
$$2\theta + \phi = 180°$$
$$2\theta = 180 - \phi = 180 - 50$$
$$2\theta = 130°$$
$$\therefore \theta = \frac{130}{2} = 65°$$

The interatomic separation for Ni crystal is

$$d = 0.914\ A°$$

For first order (n=1) diffraction the Bragg's law is $\boxed{n\lambda = 2d\sin\theta}$

$$2d\sin\theta = \lambda$$
$$\therefore \lambda = 2 \times 0.914 \times \sin 65°$$
$$= 1.65\ A°.$$

From de-Broglie hypothesis the wavelength associated with an electron beam accelerated through 54V must be

$$\lambda = \frac{h}{p} = \frac{12.27\ A°}{\sqrt{V}}$$
$$= \frac{12.27}{\sqrt{54}} = 1.66\ A°.$$

The experimentally measured wavelength is close to that estimated from de-Broglie hypothesis. This proves the existence of de-Broglie waves.

Exercises. (Numericals)

1. Find the (a) maximum frequency and (b) minimum wavelength of X-rays produced by 30KV electrons.

Ans: We have $h\nu_{max} = eV$

$$\nu_{max} = \frac{eV}{h}$$
$$= \frac{1.6 \times 10^{-19} \times 30 \times 10^{3}}{6.6 \times 10^{-34}}$$
$$= 7.24 \times 10^{18}\ Hz$$

(b) $\lambda_{max} = \frac{c}{\nu_{max}} = \frac{3\times10^{8}}{7.24\times10^{18}}$

$= 0.0414\times10^{-9}m$

$= 0.0414 nm.$

2. The work function of Caesium metal is 2.14 eV. When light of frequency 6×10^{24} Hz is incident on the metal surface, photo emission of electrons occurs. What is the
(a) maximum K.E of the emitted $\bar{e}$s
(b) stopping potential &
(c) maximum speed of the emitted photo-$\bar{e}$'s?

Ans: (a) $\phi_0 = 2.14 eV$

$\nu = 6\times10^{24} Hz.$

$K_{max} = h\nu - \phi_0$

$= \frac{6.6\times10^{-34}\times6\times10^{24}}{1.6\times10^{-19}} - 2.14$

$= 2.48 - 2.14$

$0.34 eV.$

(b) Stopping potential.

$eV_0 = K_{max}$

$eV_0 = \frac{K_{max}}{e} = \frac{0.34eV}{e}$

$= 0.34 V.$

(c) Maximum speed $= V_{max} = \sqrt{\frac{2\times K.E}{m}} = \sqrt{\frac{2\times0.34\times1.6\times10^{-19}}{9.1\times10^{-31}}}$

$= 1195604\times10^{6}$

$= 345.8\times10^{3} m/s$

3. The photo electric cut-off voltage in a certain experiment is 1.5 V. What is the maximum K.E of photoelectrons emitted?

Ans. $V_0 = 1.5 V$

$K_{max} = eV_0$

$= 1.6\times10^{-19}\times1.5$

$= 2.4\times10^{-19} J.$

4. Monochromatic light of wavelength 632.8 nm is produced by a He-Ne laser. The power emitted is 9.42 mW.
(i) Find the energy and momentum of each photon in the light beam
(ii) How many photons per second, on the average arrive at a target irradiated by this beam?
(iii) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon.

Ans. $\lambda = 632.8 nm$

$= 632.8\times10^{-9} m.$

$P = 9.42\times10^{-3} W.$

(i) Energy of each photon $E = \frac{hc}{\lambda}$

$= \frac{6.63\times10^{-34}\times3\times10^{8}}{632.8\times10^{-9}} = 3.14\times10^{-19} J$

Momentum $P = \frac{h}{\lambda}$

$= \frac{6.63 \times 10^{-34}}{632.8 \times 10^{-9}}$

$= 1.05 \times 10^{-27}$ kgm/s.

(ii) Number of photons arriving per second

$N = \frac{P}{E}$

$= \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}}$

$= 3 \times 10^{16}$ photons per second.

(iii) Momentum of a Hydrogen atom

= Momentum of photon

$mV = p$

$V = \frac{p}{m} = \frac{1.05 \times 10^{-27}}{1.67 \times 10^{-27}}$

$= 0.63$ m/s.

5. The energy flux of sunlight reaching the surface of the earth is $1.388 \times 10^3 W/m^2$. How many photons per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Ans: $N = \frac{P}{E}$.

No. of photons per m^2 = $\frac{\text{Power per } m^2}{\text{Energy}}$.

$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{550 \times 10^{-9}}$

$= 3.62 \times 10^{-19}$ J

∴ No of photons per $m^2 = \frac{1.388 \times 10^3}{3.62 \times 10^{-19}}$

$= 3.8 \times 10^{21}$

6. In an experiment on photo electric effect the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} Vs. Calculate the Planck's constant.

$\frac{\Delta V}{\Delta \nu} = 4.12 \times 10^{-15}$ Vs.

Plank's constant $h = \frac{\Delta V}{\Delta \nu} \times e$

$= 4.12 \times 10^{-15} \times 1.6 \times 10^{-19}$

$= 6.592 \times 10^{-34}$ Js

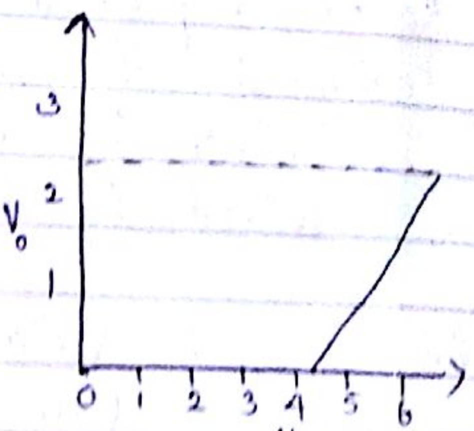
7. What is the de-Broglie wavelength associated with an electron, accelerated through a potential difference of 100 volts?

$V = 100 V$

$\lambda = \frac{12.27}{\sqrt{V}} A^\circ = \frac{12.27}{\sqrt{100}} = \frac{12.27}{10} = 1.227 A^\circ$.

Previous years CBSE questions.

1. For the photo electric effect in Na, figure shows the plot of cut-off voltage with frequency Hz of incident radiation. Calculate the
(i) threshold frequency
(ii) Work function of Na.



Ans: (i) from the graph threshold frequency

$$\nu_0 = 4.5 \times 10^{14} \text{ Hz}$$

(ii) Work function $\phi_0 = h\nu_0$

$$= 6.6 \times 10^{-34} \times 4.5 \times 10^{14}$$

$$= 2.97 \times 10^{-19} \text{ J.}$$

$$= \frac{2.97 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$= \underline{1.86 \text{ eV}}.$$

2. An electron, an α-particle and a proton have same K E, which one of the particles has the shortest de-Broglie wavelength?

Ans:

$$m_e < m_p < m_\alpha$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}.$$

For given K $\quad \lambda \propto \frac{1}{\sqrt{m}}$

As $m_e < m_p < m_\alpha$

$\therefore \lambda_e > \lambda_p > \lambda_\alpha$ from the relation $\lambda \propto \frac{1}{\sqrt{m}}$

$\therefore$ α-particle has shortest de-Broglie wavelength

3. An α-particle and a proton are accelerated through the same potential difference. Calculate the ratio of linear momenta acquired by the two.

Ans. $p = \sqrt{2meV}$ for $\bar{e}$

for a particle of charge q

$p = \sqrt{2mqV}$

For α-particle $\rightarrow$ $P_\alpha = \sqrt{2m_\alpha q_\alpha V}$

For a proton $\rightarrow$ $P_p = \sqrt{2m_p q_p V}$

$$\frac{P_\alpha}{P_p} = \frac{\sqrt{2m_\alpha q_\alpha V}}{\sqrt{2m_p q_p V}} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}}$$

$m_\alpha = 4m_p$

$q_\alpha = 2q_p$

$$= \sqrt{\frac{4m_p \times 2q_p}{m_p \times q_p}}$$

$$= \sqrt{\frac{8}{1}} = \frac{2\sqrt{2}}{1}$$

$$= \underline{2\sqrt{2} : 1}$$

4. An α-particle and a proton are accelerated through the same potential difference V. Find the ratio of de-Broglie wavelength associated with them.

Ans: $\lambda = \frac{h}{\sqrt{2meV}}$

For a particle of charge q $\lambda = \frac{h}{\sqrt{2mqV}}$

In case of α-particle $\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha q_\alpha V}}$

In case of proton $\lambda_p = \frac{h}{\sqrt{2m_p q_p V}}$

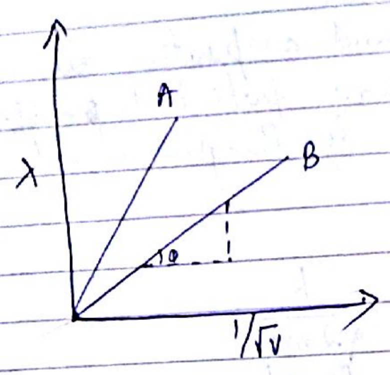
$$\therefore \frac{\lambda_\alpha}{\lambda_p} = \frac{h}{\sqrt{2m_\alpha q_\alpha V}} \times \frac{\sqrt{2m_p q_p V}}{h}$$

$$= \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}} = \sqrt{\frac{m_p q_p}{4m_p \times 2q_p}}$$

$$= \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}}$$

$$= \underline{1 : 2\sqrt{2}}$$

5. Two lines A and B shown in the graph plot the de-Broglie wavelength λ as a function of $\frac{1}{\sqrt{V}}$ for two particles have same charge. Which of the two represent the particle of heavier mass?



Ans. Slope of the graph $= \tan\theta = \frac{\lambda}{1/\sqrt{V}}$

$= \lambda\sqrt{V}$

We have $\lambda = \frac{h}{\sqrt{2mqV}}$

For same charge $\lambda \propto \frac{1}{\sqrt{mV}}$

or $\lambda\sqrt{V} \propto \frac{1}{\sqrt{m}}$. ie slope $\propto \frac{1}{\sqrt{m}}$

Slope of A > slope of B

∴ mass of A < mass of B

B represents heavier mass.

6. X-rays of wavelength λ, falls on a photosensitive surface emitting electrons. Assuming that the work function of the surface can be neglected. Prove that de-Broglie wavelength of electrons will be $\sqrt{\frac{h\lambda}{2mc}}$.

Ans: According to Einstein's photo $\bar{e}$ equation

$h\nu = \phi_0 + \frac{1}{2}mv^2$

As given work function is neglected

$\therefore h\nu = \frac{1}{2}mv^2$.

$\frac{hc}{\lambda} = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$

$p^2 = \frac{hc \times 2m}{\lambda}$

$p = \sqrt{\frac{2mhc}{\lambda}}$

de-Broglie wavelength $\lambda_d = \frac{h}{p} = \frac{h}{\sqrt{\frac{2mhc}{\lambda}}} = h\sqrt{\frac{\lambda}{2mhc}}$

$= \sqrt{\frac{h^2\lambda}{2mhc}}$

$\lambda_d = \sqrt{\frac{h\lambda}{2mc}}$

7. Why de-Broglie waves associated with a moving foot ball not possible?

The wavelength λ of a photon and the de-Broglie wavelength of an $\bar{e}$ have the same value. Show that energy of the photon is $\frac{2\lambda mc}{h}$ times the K.E of the electron?

Ans:

de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$.

Due to the mass of a foot ball and the velocity of its motion de-Broglie wave length λ associated with a moving foot ball is very small.

Hence its wave nature is not visible.

Energy of the photon $= h\nu$

$= \frac{hc}{\lambda}$

K.E of electrons $= \frac{1}{2}mv^2$

$= \frac{m^2v^2}{2m} = \frac{p^2}{2m}$

$\frac{\text{Energy of photon}}{\text{K.E of electrons}} = \frac{\frac{hc}{\lambda}}{p^2/2m} = \frac{hc}{\lambda} \times \frac{2m}{p^2}$

$= \frac{hc}{\lambda} \times \frac{2m}{h^2/\lambda^2}$

where $p = \frac{h}{\lambda}$

$= \frac{hc}{\lambda} \times \frac{2m\lambda^2}{h^2}$

$= \frac{2m\lambda c}{h}$

ie Energy of photon $= \frac{2m\lambda c}{h} \times$ K.E of electrons.

8. The threshold frequency of a metal is ν_0. When a light of frequency $2\nu_0$ is incident on the metal plate the maximum velocity of electrons emitted is V_1. When the frequency of incident radiation is increased to $5\nu_0$ the maximum velocity of electrons emitted is V_2. Find the ratio of V_1 to V_2.

Ans. According to Einstein's photo electric equation

$h\nu = \phi_0 + \frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = h\nu - \phi_0$

$= h\nu - h\nu_0$

$= h(\nu - \nu_0)$

Case I $\quad \frac{1}{2}mv_1^2 = h(2\nu_0 - \nu_0)$

$= h\nu_0 \quad -①$

Case II $\quad \frac{1}{2}mv_2^2 = h(5\nu_0 - \nu_0)$

$= h4\nu_0 \quad -②$

$\therefore \frac{①}{②} \Rightarrow \frac{V_1^2}{V_2^2} = \frac{1}{4}$

or $\frac{V_1}{V_2} = \frac{1}{2} \qquad V_1 : V_2 = 1:2$

9. Two metals A and B have work functions 4eV and 10eV respectively. Which metal has higher threshold ~~frequency~~ wavelength?

Ans.

$\phi_0 = h\nu_0$

$\phi_0 = \frac{h.c}{\lambda_0}$

or $\phi_0 \propto \frac{1}{\lambda_0}$ or $\lambda_0 \propto \frac{1}{\phi_0}$

λ_0 is higher for A because ϕ_0 is low for metal A.

10. If the frequency of the incident light on a metal surface is doubled will the K.E of the photons be doubled? Give reason.

Ans: I Case $h\nu = \phi_0 + K_1$ —— (1)

II Case $h(2\nu) = \phi_0 + K_2$ —— (2)

$$\frac{(2)}{(1)} \quad \frac{h(2\nu)}{h\nu} = \frac{\phi_0 + K_2}{\phi_0 + K_1}$$

$$2 = \frac{\phi_0 + K_2}{\phi_0 + K_1}$$

$$2(\phi_0 + K_1) = \phi_0 + K_2$$

$$K_2 = 2\phi_0 + 2K_1 - \phi_0$$

$$= 2K_1 + \phi_0$$

$\therefore$ K_2 is more than doubled when the frequency of incident radiation is doubled.

11. Radiation of frequency 10^{15} Hz is incident on three photosensitive surfaces A, B and C. Following observation are recorded.

Surface A :- No photo emission occurs.

Surface B :- Photoemission occurs but photo electrons have zero K.E

Surface C :- Photo emission occurs but photo electrons have some K.E

Based on Einstein's photo electric equation explain the three observations.

Ans: For A :- Threshold frequency is more than 10^{15} Hz. Hence no photo emission is possible.

For B :- Threshold frequency is equal to the frequency of given radiation. Thus photo emission takes place but K.E is zero.

For C :- Photo emission occurs and photo electrons have some kinetic energy this is because the threshold frequency is less than 10^{15} Hz.

12. Graph shows the variation of stopping potential with frequency (ν) of the incident radiation for two metals M_1 and M_2. (i) What are values of work function for M_1 and M_2.

(ii) The values of the stopping potential for M_1 & M_2 for a frequency $\nu_3 (> \nu_{02})$ of the incident radiations are V_1 and V_2 respectively. Show that slope of the lines equals to $\frac{V_1 - V_2}{\nu_{02} - \nu_{01}}$

P.T.O →

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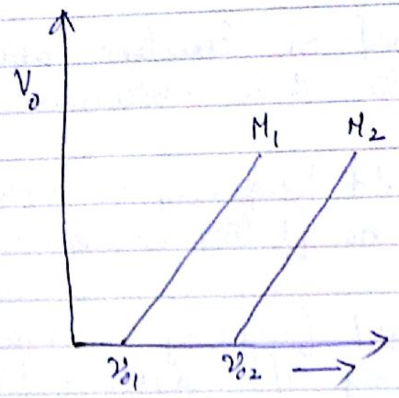

Ans. (i) Work function of M_1 $\phi_1 = h\nu_{01}$
Work function of M_2 $\phi_2 = h\nu_{02}$.

(ii) We have $h\nu = K + \phi_0$
$= eV + \phi_0$
$= eV + h\nu$

For M_1 ; $h\nu_3 = eV_1 + h\nu_{01}$

$eV_1 = h\nu_3 - h\nu_{01}$
$= h(\nu_3 - \nu_{01})$

or $V_1 = \frac{h(\nu_3 - \nu_{01})}{e}$ — (1)

Similarly $V_2 = \frac{h(\nu_3 - \nu_{02})}{e}$ — (2)

$V_1 - V_2 = \frac{h}{e}\left[\nu_3 - \nu_{01} - \nu_3 + \nu_{02}\right]$
$= \frac{h}{e}\left[\nu_{02} - \nu_{01}\right]$

$\frac{h}{e} = \frac{V_1 - V_2}{\nu_{02} - \nu_{01}}$ This is the slope of the graph.

13. The following table gives the values of work function for a few photosensitive material

Sl No	Metal	ϕ_0 (eV)
1	Na	1.92
2	K	2.15
3	Mo	4.17

If each of these metals is exposed to radiation of wavelength 300nm, which of them will not emit photo electrons and why ?

Ans. $\phi = h\nu$ $\lambda = 300nm$
$= h\frac{c}{\lambda}$ $= 300 \times 10^{-9} m$

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9} m}$.

$= 6.6 \times 10^{-19} J$

$= \frac{6.6 \times 10^{-19}}{1.6 \times 10^{-19}}$

$= 4.16 eV$

Mo will not emit photo electrons because the energy of incident photon is less than the work function of Mo

14. Radiations of frequencies ν_1 and ν_2 are made to fall on a photo sensitive surface. The stopping potentials required for stopping the most energetic emitted photo electrons in the two cases are V_1 and V_2 respectively. Prove that the Plank's constant

$$h = \frac{e(V_2 - V_1)}{\nu_2 - \nu_1} \text{ and threshold frequency } \nu_0 = \frac{\nu_1 V_2 - \nu_2 V_1}{V_2 - V_1}$$

Ans:. $h\nu = K + \phi_0$

$h\nu = eV + h\nu_0$

or

For frequency ν_1 =

$$h\nu_1 = eV_1 + h\nu_0$$

$$eV_1 = h(\nu_1 - \nu_0)$$

$$\text{or } V_1 = \frac{h}{e}[\nu_1 - \nu_0] \quad \text{—(1)}$$

Similarly $V_2 = \frac{h}{e}[\nu_2 - \nu_0]$ —(2)

$$V_2 - V_1 = \frac{h}{e}[\nu_2 - \nu_0 - \nu_1 + \nu_0]$$

$$V_2 - V_1 = \frac{h}{e}[\nu_2 - \nu_1]$$

$$\text{or } h = \frac{e[V_2 - V_1]}{\nu_2 - \nu_1}$$

Also

$$h\nu_1 = eV_1 + h\nu_0$$

$$\text{or } h\nu_0 = h\nu_1 - eV_1$$

$$= \frac{e[V_2 - V_1]}{\nu_2 - \nu_1} \times \nu_1 - eV_1$$

$$= \frac{e(V_2 - V_1)\nu_1 - eV_1(\nu_2 - \nu_1)}{\nu_2 - \nu_1}$$

$$= \frac{eV_2\nu_1 - \cancel{eV_1\nu_1} - eV_1\nu_2 + \cancel{eV_1\nu_1}}{\nu_2 - \nu_1}$$

$$h\nu_0 = \frac{e[V_2\nu_1 - V_1\nu_2]}{\nu_2 - \nu_1}$$

Sub: h

$$\frac{\cancel{e}[V_2 - V_1)}{\cancel{\nu_2 - \nu_1}}\nu_0 = \frac{\cancel{e}[V_2\nu_1 - V_1\nu_2]}{\cancel{\nu_2 - \nu_1}}$$

$$\text{or } \nu_0 = \frac{V_2\nu_1 - V_1\nu_2}{V_2 - V_1}$$