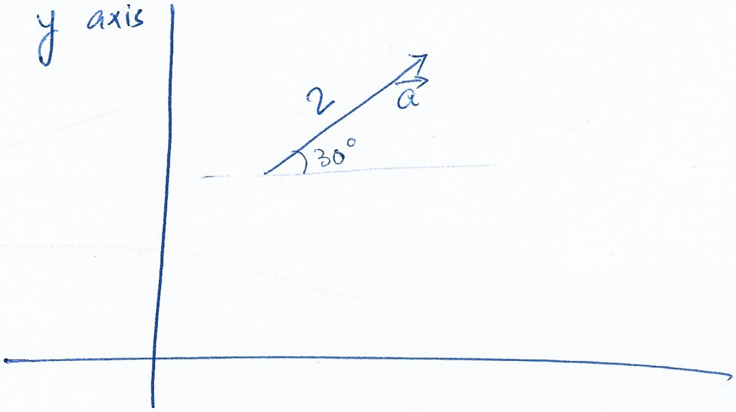
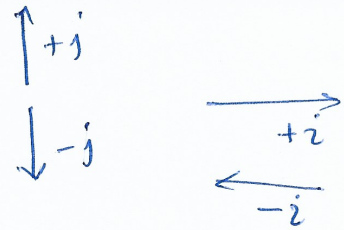
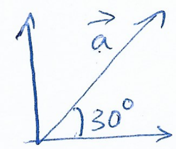




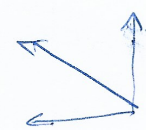
$$\vec{a} = 2\cos 30^\circ i + 2\sin 30^\circ j$$



$$\vec{a} = 2i + 3j$$

$$\vec{a} = xi + yj$$

$$|\vec{a}| = \sqrt{x^2+y^2}$$



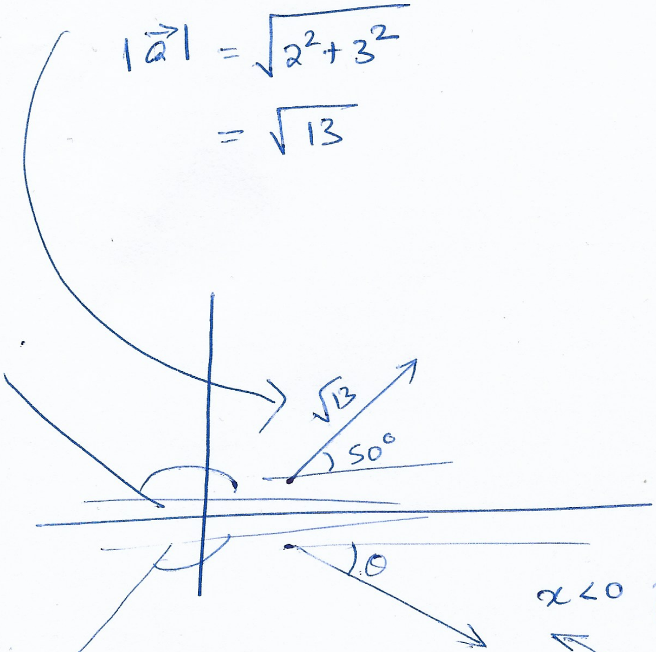
$$|\vec{a}| = \sqrt{2^2+3^2}$$

$$= \sqrt{13}$$

$$\tan\theta = \left|\frac{y}{x}\right|$$

$$\tan\theta = \left|\frac{3}{2}\right|$$

θ is angle made with +ve x axis



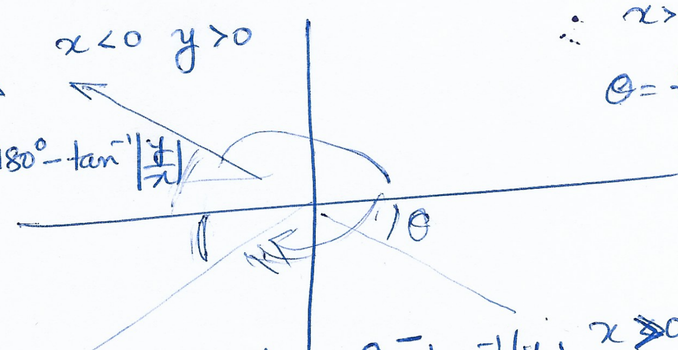
$$\vec{a} = 2i - 3j$$

$\therefore$ $x>0$ $y>0$

$$\theta = \tan^{-1}\left|\frac{y}{x}\right|$$

$x<0$ $y>0$

$$\theta = 180^\circ - \tan^{-1}\left|\frac{y}{x}\right|$$

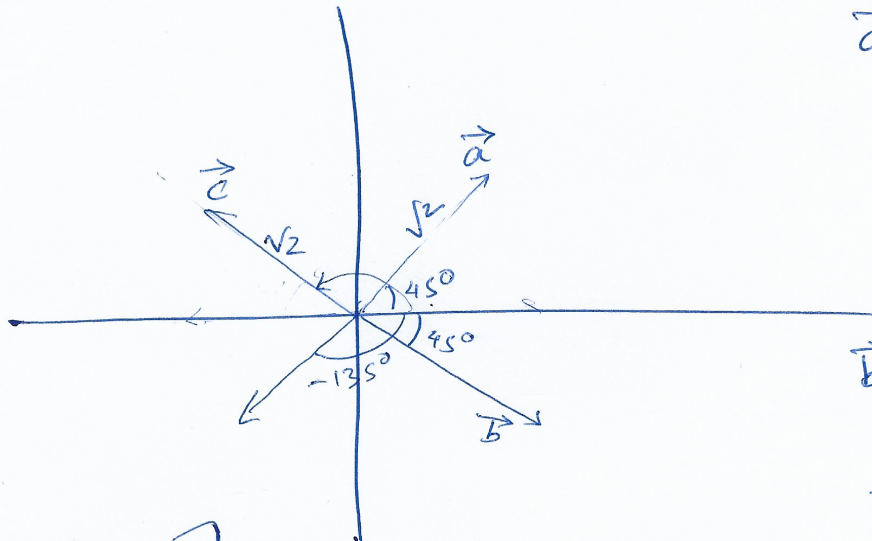


$$\theta = -\tan^{-1}\left|\frac{y}{x}\right|$$

$x>0$ $y<0$

$$-180^\circ + \tan^{-1}\left|\frac{y}{x}\right|$$

	magnitude	
$\vec{a} = i + j$	$\sqrt{1^2+1^2} = \sqrt{2}$	$\theta = \tan^{-1}\left\lvert\frac{y}{x}\right\rvert = 45^\circ$
$\vec{b} = i - j$	$\sqrt{1^2+(-1)^2} = \sqrt{2}$	$\theta = -\tan^{-1}\left\lvert\frac{y}{x}\right\rvert = -45^\circ$
$\vec{c} = -i + j$	$\sqrt{1^2+1^2} = \sqrt{2}$	$\theta = 180 - \tan^{-1}\left\lvert\frac{y}{x}\right\rvert = 135^\circ$
$\vec{d} = -i - j$	$\sqrt{1^2+1^2} = \sqrt{2}$	$\theta = -180 + \tan^{-1}\left\lvert\frac{y}{x}\right\rvert = -135^\circ$



$\vec{a} = \sqrt{2}\cos 45\, i + \sqrt{2}\sin 45\, j$
$= i + j$

$\vec{b} = \sqrt{2}\cos(-45)\, i + \sqrt{2}\sin(-45)\, j$
$= i - j$

$$\left.\begin{aligned} \cos(-\theta) &= \cos\theta \\ \sin(-\theta) &= -\sin\theta \\ \tan(-\theta) &= -\tan\theta \end{aligned}\right\} \forall\, \theta$$

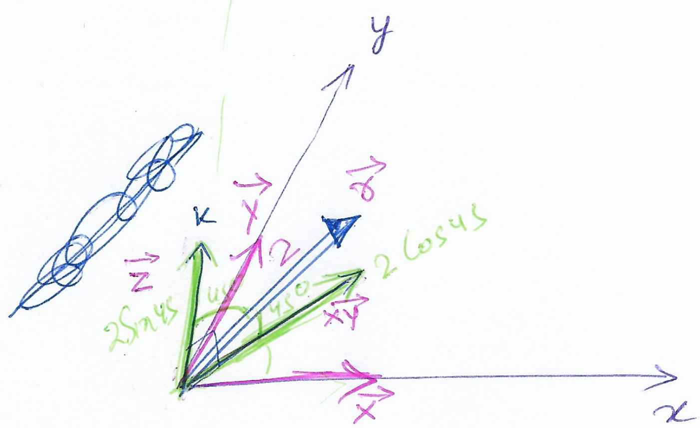
$-180^\circ < \theta \leq 180$

$\tan(-135) = -\tan 135$

$\vec{c} = \sqrt{2}\cos 135\, i + \sqrt{2}\sin 135\, j$
$= -i + j$

$\vec{d} = \sqrt{2}\cos(-135)\, i + \sqrt{2}\sin(-135)\, j$
$= \sqrt{2}\cos(135)\, i - \sqrt{2}\sin 135\, j$
$= -i - j$

$\vec{a} = i + j + k$



$$\vec{r} = \vec{z} + \vec{xy}$$

$$\vec{xy} = \vec{x} + \vec{y}$$

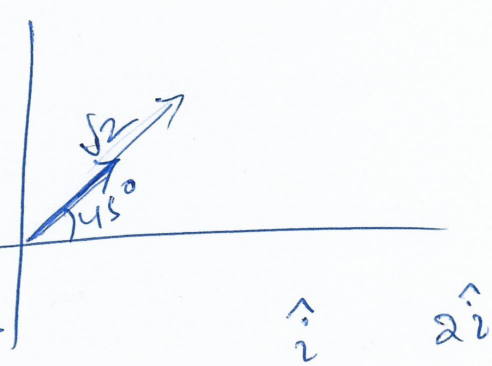
$$\vec{r} = \vec{z} + \vec{x} + \vec{y}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

Unit vector is a vector with magnitude=1

$\vec{a} = i + j$



$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{i + j}{\sqrt{2}} = \frac{i}{\sqrt{2}} + \frac{j}{\sqrt{2}}$$

PRODUCT OF VECTORS:

SCALAR PRODUCT — DOT PRODUCT

VECTOR PRODUCT — CROSS PRODUCT

$\vec{a} \cdot \vec{b}$ = scalar

$= |\vec{a}||\vec{b}| \cos\theta$

$\vec{a} \times \vec{b} = \vec{c}$

$= |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$

$\hat{n}$ is a unit vector $\perp$ to $\vec{a}$ & $\vec{b}$

i) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

ii) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

iii) $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

iv) $\hat{i} \cdot \hat{i} = 1$

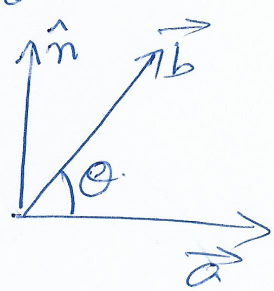
v) $\hat{j} \cdot \hat{j} = 1$

vi) $\hat{k} \cdot \hat{k} = 1$

vii) $i \cdot j = 0 \qquad j \cdot k = 0$

$i \cdot k = 0$

$\vec{a} \times \vec{b}$



viii) $(a_1 i + a_2 j + a_3 k) \cdot (b_1 i + b_2 j + b_3 k)$

$= a_1 b_1 (i \cdot i) + a_1 b_2 (i \cdot j) + a_1 b_3 (i \cdot k) +$

$(a_2 b_1)(j \cdot i) + (a_2 b_2)(j \cdot j) - - - -$

$+ (a_3 b_3)(k \cdot k)$

$= a_1 b_1 + a_2 b_2 + a_3 b_3$.

Q) Prove that $\vec{a} = 2i - 3j + k$ is perpendicular to $\vec{b} = i + j + k$

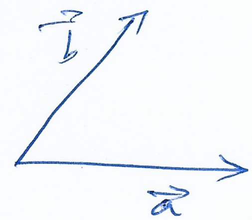
$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$2-3+1 = |\vec{a}||\vec{b}|\cos\theta$$

$$\cos\theta = 0$$

$$\theta = \underline{\underline{90^\circ}}$$

Properties of cross product:



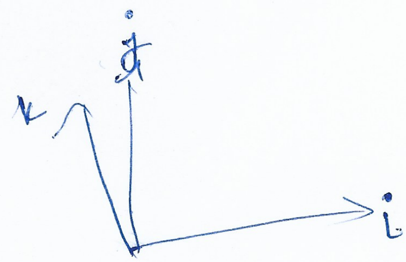
i) $\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$

ii) $\hat{i}\times\hat{i} = 0 \qquad |\hat{i}||\hat{i}|\underbrace{\sin\theta}_{0}$

$\hat{j}\times\hat{j} = 0$

$\hat{k}\times\hat{k} = 0$

$$\left.\begin{array}{l} i\times j = k \\ j\times i = -k \\ j\times k = i \\ k\times j = -i \\ k\times i = j \\ i\times k = -j \end{array}\right\}$$



$$(a_1 i + a_2 j + a_3 k)\times(b_1 i + b_2 j + b_3 k)$$

$$\underbrace{a_1b_1(i\times i)}_{0} + (a_1b_2)(k) + a_1b_3(-j) + a_2b_1(-k)$$

$$+ \underbrace{a_2b_2(j\times j)}_{0} + a_2b_3(i) + a_3b_1(j) + a_3b_2(-i) + \underbrace{a_3b_3(k\times k)}_{0}$$

$$= \underline{(a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)k}$$

$$\begin{array}{ccc} + & - & + \\ \end{array}$$
$$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= +i(a_2b_3 - b_2a_3) - j(a_1b_3 - b_1a_3) + k(a_1b_2 - b_1a_2)$$

Q Find a unit vector $\perp$ to $\vec{A} = 2i + 3j + k$ & $\vec{B} = i - j + k$ both.

$$\hat{C} = \frac{\vec{c}}{|\vec{c}|} = \frac{4i - j - 5k}{\sqrt{42}}$$

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{C} = \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 1 & -1 & 1 \end{vmatrix} = +i(3+1) - j(2-1) + k(-2-3)$$
$$= 4i - j - 5k. \qquad |\vec{c}| = \sqrt{4^2 + 1^2 + 5^2}$$

Q) Show $\vec{A} = i - j + 2k$ is $\parallel$ to vector $\vec{B} = 3i - 3j + 6k$

$$\vec{A} \times \vec{B} = \underline{|\vec{A}|}\,\underline{|\vec{B}|}\,\underbrace{\sin\theta}_{0}\,\hat{n}$$

$$\underline{\vec{A} = k(\vec{B})}.$$

$$\frac{i - j + 2k}{\sqrt{6}} \qquad \frac{i - j + 2k}{\sqrt{6}}$$

✓ ✓