153 PROBLEMS with SOLUTIONS*

$153 = 1^3 + 5^3 + 3^3$

$153 = 1! + 2! + 3! + 4! + 5!$

*Answers/Solutions are provided to only TathaGat students and Total Gadha CBT-club members

http://www.totalgadha.com

PROBLEMS

1. The number of persons who booked ticket for the New Year's concert is a perfect square. If 100 more persons booked ticket then the number of spectators would be a perfect square plus 1. If still 100 more persons booked ticket then the number of spectators would be again a perfect square. How many persons booked ticket for the concert?

2. If the last digits of the products $1\cdot 2$, $2\cdot 3$, $3\cdot 4$, ..., $n(n+1)$ are added, the result is 2010. How many products are used?

3. What is the remainder obtained when 2^{32} is divided by 641?

4. How many integers may be the measure, in degrees, of the angles of a regular polygon?

5. Several sets of prime numbers, such as {7, 83, 421, 659}, use each of the nine nonzero digits exactly once. What is the smallest possible sum such a set of primes could have?

6. How many ordered triplets (a, b, c) of non – zero real numbers have the property that each number is the product of the other two?

7. Rajat decided to tell the truth on Mondays, Thursdays and Saturdays, but lie on every other day. One day he says, "I will tell the truth tomorrow." What day of the week he made this statement?

8. For what smallest positive integral n, factorial of n is divisible by 414?

9. 2010 inhabitants of TG Land are divided into two groups: the Truth tellers – who always tell the truth and the Liars – who always tell a lie. Each person is exactly one of the following – a cricketer, a guitarist or a swimmer. Each inhabitant was asked the three questions: 1) Are you a cricketer? 2) Are you a guitarist? 3) Are you a swimmer? 1221 persons answered "yes" to the first question. 729 persons answered "yes" to second question and 660 persons answered "yes" to third question. How many "Liars" are present on the TG Land?

10. Isosceles triangle ABC has the property that, if D is a point on AC such that BD bisects angle ABC, then triangle ABC and BCD are similar. If BC has length of one unit, then what is the length of AB?

11. Vertices A, B and C of a parallelogram ABCD lie on a circle and D lies inside the circle such that line BD intersects the circle at point P. Given that $\angle APC = 75°$ and $\angle PAD = 19°$, what is the measure of $\angle PCD$?

12. Read the following 5 statements carefully:

(i) Statement (ii) is true.
(ii) At most one of the given five statements is true.
(iii) All of the given statements are true.
(iv) .
(v) .

The last two statements are printed in invisible ink. Which of the statements are true?

13. While adding all the page numbers of a book, I found the sum to be 1000. But then I realized that two page numbers (not necessarily consecutive) have not been counted. How many different pairs of two page numbers can be there?

14. How many positive integers are equal to 12 times the sum of their digits?

15. An 8 cm by 12 cm rectangle is folded along its long side so that two diagonally opposite corners coincide. What is the length of crease formed?

16. A point P inside an equilateral triangle, ABC is located at a distance of 3, 4 and 5 units respectively from A, B and C. What is the area of the triangle ABC?

17. Ten boxes each contain 9 balls. The balls in one box each weigh 0.9 kg; the rest all weigh 1 kg. In how many least number of weighing you can determine the box with the light balls?

18. A given circle has n chords. Each chord crosses every other chord but no three chords meet at the same point. How many regions are in the circle?

19. Find all prime numbers *p* for which *5p + 1* is a perfect square.

20. A programmer carelessly increased the tens digit by 1 for each multi-digit Fermat number in a lengthy list produced by a computer program. Fermat numbers are integers of the form: $N = 2^{2^n} + 1$ for integer n > 1. How many numbers on this new list are primes?

21. What is the greatest common divisor of the 2010 digit and 2005 digit numbers below?

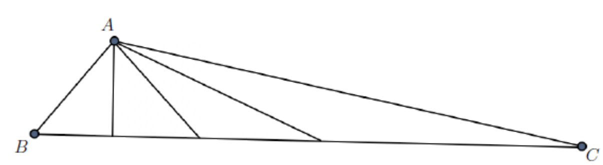
$$\underbrace{33333...333}_{2010\ 3's} \quad \underbrace{7777...77}_{2005\ 7's}$$

22. Two players play a game on the board below as follows. Each person takes turns moving the letter **A** either downward at least one rectangle or to the left at least one rectangle (so each turn consists of moving either downward or to the left but not both). The first person to place the letter **A** on the rectangle marked with the letter **B** wins. How should the first player begin this game if we want to assure that he wins? Answer with the number given on the rectangle that he should move the letter **A** to.

1	2	3	4	5	6	7	8	9	**A**
									10
									11
B									12

23. In ΔABC (not drawn to scale), the altitude from *A*, the angle bisector of $\angle BAC$, and the median from *A* to the midpoint of *BC* divide $\angle BAC$ into four equal angles. What is the measure in degrees of angle $\angle BAC$?

24. Let a_1, a_2,..a_{2011} represents the arbitrary arrangement of the numbers 1, 2,..2011. Then what is the remainder when $(a_1 - 1)(a_2 - 2)..(a_{2011} - 2011)$ is divided by 2?

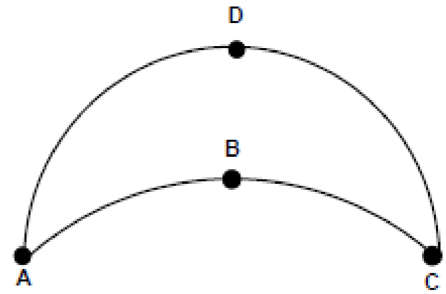


25. One side of a triangle has length 75. Of the other two sides, the length of one is double the length of the other. What is the maximum possible area for this triangle?

26. The polynomial $P(x) = a_0 + a_1x + a_2x^2 + \ldots + 10x^9$ has the property that $P\left(\frac{1}{k}\right) = \frac{1}{k}$ for k = 1, 2, 3, .., 9. Find $P\left(\frac{1}{10}\right)$.

27. How many ordered triplets (a, b, c) of positive odd integers satisfy a + b + c = 23?

28. The figure ABCD on the right is bounded by a semicircle ADC and a quarter-circle ABC. Given that shortest distance between A and C = 18 units. What is the area of region bounded by this figure?

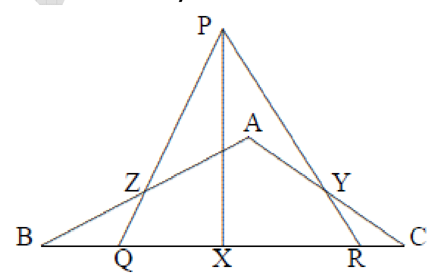


29. A palindrome is a number which reads same forward and backward, e.g. 121 is a three digit palindrome number. What is the sum of all three digit palindromes which are multiple of 13?

30. Find the sum of all the digits in the decimal representations of all the positive integers less than 1000.

31. Consider the numbers 3, 8, 13… 103, 108. What is the smallest value of *n* such that every collection of *n* of these numbers will always contain a pair which sums to 121?

32. In the diagram shown, X is the midpoint of BC, Y is the midpoint of AC and Z is the midpoint of AB. Also $\angle ABC + \angle PQC = \angle ACB + \angle PRB = 90°$. Find $\angle PXR$.



33. Let a, b, c, d be four real numbers such that

$$a + b + c + d = 8,$$
$$ab + ac + ad + bc + bd + cd = 12.$$

Find the greatest possible value of d.

34. The ordered pair of four-digit numbers (2025; 3136) has the property that each number in the pair is a perfect square and each digit of the second number is 1 more than the corresponding digit of the first number. Find all ordered pairs of five-digit numbers with the same property.

35. Exactly one of the statements in this problem is true. The first statement in this problem is false. In fact, both the first and second statements in this problem are false. How many true statements are there in this problem?

36. Given that *a* and *b* are digits from 1 to 9, what is the number of fractions of the form *a*/*b*, expressed in lowest terms, which are less than 1?

37. For a positive integer n let f(n) be the value of $\dfrac{4n + \sqrt{4n^2 - 1}}{\sqrt{2n+1} + \sqrt{2n-1}}$. Calculate

$$f(1) + f(2) + \ldots + f(40)$$

38. If N be the number of consecutive zeros at the end of the decimal representation of the expression 1! × 2! × 3! × 4! ×...........× 99! × 100! Find the remainder when N is divided by 1000?

39. What are the dimensions of the greatest *n x n* square chessboard for which it is possible to arrange 121 coins on its cells so that the numbers of coins on any two adjacent cells (i.e. that share a side) differ by 1?

40. Let PQR be an isosceles triangle with PQ = PR, and suppose that M is a point on the side QR with QR > QM > MR. Let QS and RT be diameters of the respective circumcircles of triangles PQM and PRM. What is the ratio QS : RT?

41. "You eat more than I do," said Tweedledee to Tweedledum.
"That is not true," said Tweedledum to Tweedledee.
"You are both wrong," said Alice to them both.
"You are right," said the White Rabbit to Alice.
How many of the four statements were true?

42. The road from village *P* to village *Q* is divided into three parts. If the first section was 1.5 times as long and the second one was 2/3 as long as they are now, then the three parts would be all equal in length. What fraction of the total length of the road is the third section?

43. Four different digits are chosen, and all possible positive four-digit numbers of distinct digits are constructed out of them. The sum of the four-digit numbers is 186 648. How many different sets of such four digits can be chosen?

44. x = ±1 ±2 ±3 ±4 ±5 ±6 ±7 ±8 ±9 ±10. How many possible values can x take?

45. Points X and Y are on the sides PQ and PR of triangle PQR respectively. The segments QY and RX intersect at the point Z. Given that QY = RY, PQ = RZ and ∠QPR = 60°. Find ∠RZY.

46. Let O, A, B, C be four points in a plane such that OA = OB = 15 and OC = 7. What is the maximum area of the triangle ABC?

47. A particular month has 5 Tuesdays.
The first and the last day of the month are not Tuesday.
What day is the last day of the month?

48. Find the minimum value of $\dfrac{\left(x+\dfrac{1}{x}\right)^6-\left(x^6+\dfrac{1}{x^6}\right)-2}{\left(x+\dfrac{1}{x}\right)^3+\left(x^3+\dfrac{1}{x^3}\right)}$ for x > 0.

49. What is the sum of the series: $2^2 + 4^2 + 6^2 + 10^2 + 16^2 + ... + 754^2 + 1220^2$?

50. Determine *F(2010)* if for all real x and y, *F(x)F(y) – F(xy) = x + y*.

51. How many 4 digit number exist in which, when two digits are removed, 35 remains (e.g. 2315 and 3215 will be there in the list)?

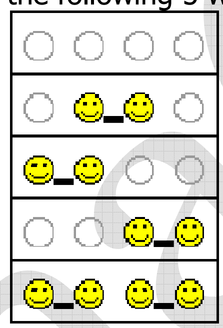
52. On a circle there are 10 points each of which is connected with each other with a straight line. How many triangles will be formed which lies completely inside the circle?

53. Let f(n) be the sum of the distinct positive prime divisors less than 50 for all positive integers n. For example: f(15) = 3 + 5 = 8 and f(61) = 0. Find the remainder when f(1) + f(2) + … + f(99) is divided by 1000.

54. Two players A and B play a game moving alternately starting with A on a 1×100 grid of unpainted hundred unit squares. A has to paint three unpainted consecutive squares blue and B has to paint four unpainted consecutive squares red in their respective turns. The player who can not paint the squares in his turn loses. Who has the winning strategy?

55. Three men - Arthur, Bernard and Charles – with their wives – Ann, Barbara and Cynthia, not necessarily in order – make some purchases. When their shopping is finished each finds that the average cost in dollars of the articles he or she has purchased is equal to the number of his or her purchases. Arthur has bought 23 more articles than Barbara, and Bernard has bought 11 more than Ann. Each husband has spent $63 more than his wife. What is the total amount spent by Charles and Cynthia?

56. In TG's birthday bash people arrive in twos and want to sit next to their partner. How many ways can a row of 10 chairs be filled with couples or be left empty?
For instance, a row of 4 chairs can be filled in the following 5 ways:



57. How many sequences of 1's and 2's sum to 15?

58. A closed bag contains 3 green hats and 2 red hats. Amar, Akbar, Anthony all close their eyes, take a hat, put it on, and close the bag. When they open their eyes, Amar looks at Akbar and Anthony, but can't deduce the color of his own hat. Akbar now tries to deduce his own hat's color but can't be certain. What color is Anthony's hat?

59. Find the sum of all remainders when $n^5 - 5n^3 + 4n$ is divided by 120 for all positive integers $n \geq 2010$.

60. The equation $x^2 + ax + (b + 2) = 0$ has real roots. What is the minimum value of $a^2 + b^2$?

61. There are 12 balls of equal size and shape, but one is either lighter or heavier than the other 11. For how many minimum number of times weighing required with ordinary beam balance to determine the faulty ball?

62. Sanjeev: I am thinking of a two digit number. Bet you can't guess it.
Kamal: Bet I can.
Sanjeev: Well, I'll only tell you the remainders of my number with anything from 1 to 10. How many questions do you think that you will have to ask?
Kamal: Hmmm! That depends on how lucky I am. But I'm not going to take chances. I am sure that I can guess your number with exactly _______ questions.
How many questions does Kamal tell Sanjeev he will ask?

63. What is the smallest possible difference between a square number and a prime number, if prime is greater than 3 and the square number is greater than prime?

64. 101 digits are chosen randomly and two numbers a, b are formed using all the digits exactly once. What is the probability that $a^4 = b$?

65. Let ABCD be a quadrilateral. The circumcircle of the triangle ABC intersects the sides CD and DA in the points P and Q respectively, while the circumcircle of CDA intersects the sides AB and BC in the points R and S. The straight lines BP and BQ intersect the straight line RS in the same points M and N respectively. If $\angle BQP = 90°$, find $\angle PMR$.

66. Kamal and Rajeev are playing the following game. They take turns writing down the digits of a six-digit number from left to right; Kamal writes the first digit, which must be nonzero, and repetition of digits is not permitted. Kamal wins the game if resulting six-digit number is divisible by 2, 3 or 5, and Rajeev wins otherwise.
Who has a winning strategy?

67. What is the least number of links you can cut in a chain of 21 links to be able to give someone all possible number of links up to 21?

68. Every blip is a blop. Half of all blops are blips, and half of all bleeps are blops. There are 30 bleeps and 20 blips. No bleep is a blip.
How many blops are neither blips nor bleeps?

69. Several weights are given, each of which is not heavier than 1 kg. It is known that they cannot be divided into two groups such that the weight of each group is greater than 1 kg. Find the maximum possible total weight of these weights.

70. Find the largest prime number p such that p^3 divided 2009! + 2010! + 2011!

71. How many integers less than 500 can be written as the sum of 2 positive integer cubes?

72. Three boys Ali, Bashar and Chirag are sitting around a round table in that order. Ali has a ball in his hand. Starting from Ali the boy having the ball passes it to either of the two boys. After 6 passes the ball goes back to Ali. How many different ways can the ball be passed?

73. There are 21 girls standing in a line. You have only nine chairs. In how many ways you can offer these chairs to nine select girls (one for each girl) such that number of standing girls between any two selected girls is odd?

74. In a group of people, there are 19 who like apples, 13 who like bananas, 17 who like cherries, and 4 who like dates. (A person can like more than 1 kind of fruit.) Each person who likes bananas also likes exactly one of apples and cherries. Each person who likes cherries also likes exactly one of bananas and dates. Find the minimum possible number of people in the group.

75. Let M and P be the points on sides AC and BC of ΔABC respectively such that AM : MC = 3 : 1 and BP : PC = 1 : 2. If Q is the intersection point of AP and BM and area of ΔBPQ is 1 square unit, find the area of ΔABC.

76. How many pairs of non-negative integers (x, y) satisfy $(xy - 7)^2 = x^2 + y^2$?

77. What is the 50th digit after decimal for:

$$\sqrt{\frac{2009\times2010\times2011\times2012+1}{4}}\,?$$

78. Year is 2051 and there is a strange game being played by 2051 inhabitants of TG Land. All 2051 inhabitants are standing in a circle. Now TG appears and randomly selects a person who shouts loudly IN, then person standing next clockwise say OUT, as must be the rule of the game, and get out of the circle. Again next person says IN and remain in his position and next says OUT and go out of circle. This process continues for a long time and in the end there is only one person remaining in the original circle. What is the position of the last survivor in the original circle, if first person selected by TG is numbered as 1 and numbers increases clockwise?

79. DaGny bought a rare earring set for $700, sold it for $800, bought it back for $900 and sold it again for $1000. How much profit did she make?

80. ABCD is a convex quadrilateral that is not parallelogram. P and Q are the midpoints of diagonals AC and BD respectively. PQ extended meets AB and CD at M and N respectively. Find the ratio of area(ΔANB) : area(ΔCMD).

81. How many positive integers N are there such that $3 \times N$ is a three digit number and $4 \times N$ is a four digit number?

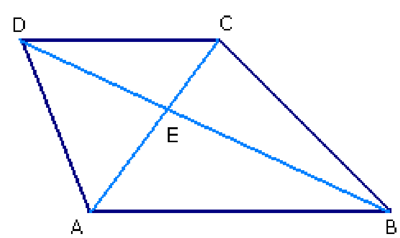
82. Lara is deciding whether to visit Kullu or Cherapunji for the holidays. She makes her decision by rolling a regular 6-sided die. If she gets a 1 or 2, she goes to Kullu. If she rolls a 3, 4, or 5, she goes to Cherapunji. If she rolls a 6, she rolls again. What is the probability that she goes to Cherapunji?

83. The numbers 201, 204, 209, 216, 225, ... are of the form $a_n = 200 + n^2$ where n = 1, 2, 3, 4, 5, ... For each n, let D_n be the greatest common divisor of a_n and a_{n+1}. What is the maximum value of D_n?

84. Messrs Baker, Cooper, Parson and Smith are a baker, a cooper, a parson and a smith. However, no one has the same name as his vocation. The cooper is not the namesake of Mr. Smith's vocation; the baker is neither Mr. Parson nor is he the namesake of Mr. Baker's vocation.

What is Mr. Baker's vocation?

85. In trapezium ABCD, AB||CD. If area(ΔABE) = $\log_a 11$, area(ΔCDE) = $\log_{11} a$, and area(ΔABC) = 11, find the area of ABCD.



86. Let P(x) be a polynomial such that:

$P(x) = x^{19} - 2011x^{18} + 2011x^{17} - ... - 2011x^2 + 2011x$. Calculate P(2010).

87. How many 9-digit numbers (in decimal system) divisible by 11 are there in which every digit occurs except zero?

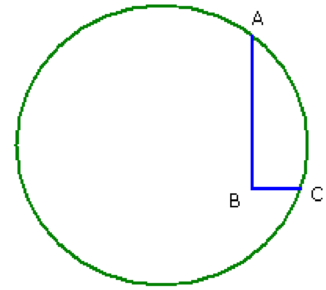
88. There are four unit spheres inside a larger sphere, such that each of them touches the large sphere and the other three unit spheres. What is the radius of large sphere?

89. For the real numbers a, b and c, it is known that

$$\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=1\text{, and}$$

$$a + b + c = 1.$$

Find the value of the expression, M = $\frac{1}{1+a+ab}+\frac{1}{1+b+bc}+\frac{1}{1+c+ca}$.



90. In the circle shown, radius = $\sqrt{50}$, AB = 6, BC = 2, $\angle$ABC = 90°. Find the distance from B to the centre of the circle.

91. Today is Friday. What day will it be after 4^{2010} days?

92. Solve the congruence cryptarithm *LIFE* $\equiv$ *SIZE* (mod *ELS*) in base 6 with E, L and S nonzero, all alphabets representing different numerals and Z > L > S.

Write the 6 letter-word denoting the digits 012345 as answer.

93. Find the sum $1+\frac{3}{2}+\frac{5}{4}+\frac{7}{8}+\ldots$ up to infinity.

94. Two boats start at same instant from opposite ends of the river traveling across the water perpendicular to shores. Each travels at a constant but different speed. They pass at a point 720 meters from the nearest shore. Both boats remain at their slips for 15 minutes before starting back. On the return trip, they pass 400 meters from the other shore. Find the width of the river.

95. Larry, Curly, and Moe had an unusual combination of ages. The sum of any two of the three ages was the reverse of the third age (*e.g.*, 16 + 52 = 68, the reverse of 86). All were under 100 years old. What was the sum of the ages?

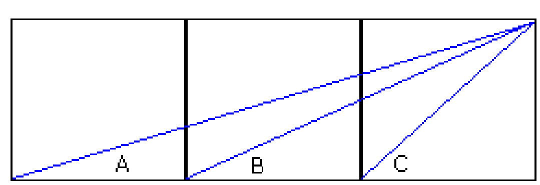
96. Find the sum of all four-digit numbers N whose sum of digits is equal to 2010 – N.

97. DaGny has 11 different colors of fingernail polish. Find the number of ways she can paint the five fingernails on her left hand by using at least three colors such that no two consecutive finger nails have same color. Also she is to apply only one color at one fingernail which is quite unusual for her.

98. A box contains 300 matches. Kamal and Sandeep take turns removing no more than half the matches in the box. The player who cannot move loses. What should be Kamal's first move to ensure his win if he is starting the game?

99. Let N be an integer such that $2N^2$ has exactly 28 distinct positive divisors and $3N^2$ has exactly 24 distinct positive divisors. How many distinct positive divisors does $6N^2$ have?

100. Three unit squares are joined as shown. Find the measure of $\angle$A + $\angle$B + $\angle$C.



101. What is the 625^{th} term of the series where each term is made up of even digits only?

2, 4, 6, 8, 20, 22, 24, 26, 28, 40, 42, ...

102. The houses in a street are spaced so that each house of one lane is directly opposite to a house of other lane. The houses are numbered 1, 2, 3, … and so on up one side, continuing the order back down the other side. Number 39 is opposite to 66. How many houses are there?

103. In a polygon, internal angles have the measures of 90° and 270° only. If there are 18 angles of measure 270°, then what is the number of angles with measure of 90°?

104. How many pair of positive integers (a, b) are there such that their LCM is 2012?

105. What is the sum of all natural numbers which are less than 2012 and co-prime to it?

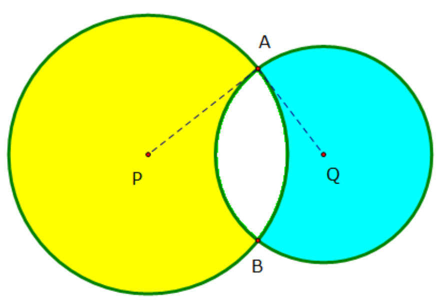
106. How many positive integers N satisfy: (i) $N < 1000$ and (ii) $N^2 - N$ is divisible by 1000?

107. 25 is a square number and can be written as average of two different square numbers i.e. 1 and 49. How many other square numbers from 1 to
625 inclusive can be written as average of two different square numbers?

108. I can break a block of 7 kg in smaller blocks of integral weights in four ways i.e. {1, 2, 4}, {1, 2, 2, 2}, {1, 1, 1, 4}, {1, 1, 1, 1, 1, 1, 1} such that I can measure each weight from 1 kg to 7 kg in exactly one way in either case.
For example, using 1st case this is the only possible combination of weights to measure 1 kg to 7 kg: 1 = 1, 2 = 2, 3 = 1 + 2, 4 = 4, 5 = 1 + 4, 6 = 2 + 4, and 7 = 1 + 2 + 4. So find the number of ways a block of 14 kg can be broken under similar conditions e.g. {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1} is a valid case but {1, 2, 3, 4, 4} is not.

109. I am twice as old as you were when I was as old as you are. What is the ratio of ages of mine and yours?

110. Circles with centers P and Q have radii 20 and 15 cm respectively and intersect at two points A, B such that ∠PAQ = 90°. What is the difference in the area of two shaded regions?



111. What is the largest integer that is a divisor of

$$(n + 1)(n + 3)(n + 5)(n + 7)(n + 9)$$

for all positive even integers n?

112. For how many ordered pairs of positive integers (x, y), xy/(x+y) = 9?

113. Amu, Bebe, Chanda and Dori played with a deck of 52 cards. In one game, Dori was dealing out the cards one by one to the players, starting with Amu, followed by Bebe, Chanda and Dori in this order, when suddenly some of the cards she had not dealt out yet slipped out of her hands and fell on the floor. The girls noticed that the number of cards on the floor was 2/3 of the number of cards Amu had already got, and the

number of cards that Chanda had got was 2/3 of those in the remaining part of the deck in Dori's hand that she had not dealt out yet. How many cards had Dori dealt out altogether?

114. In a city, 2/3 of the men and 3/5 of the women are married. (Everyone has one spouse and the spouses live in the same city.) What fraction of the inhabitants of the city is married?

115. The sum of all interior angles of eight polygons is 3240°. What is the total number of sides of polygons?

116. Consider a triangle ABC with BC = 3. Choose a point D on BC such that BD = 2. Find the value of

$$AB^2 + 2AC^2 - 3AD^2.$$

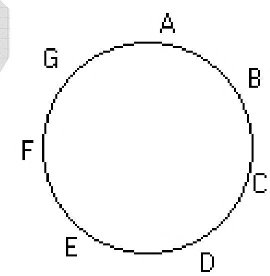
117. Determine the number of divisors of 2012^8 that are less than 2012^4.

118. How many numbers in the following sequence are prime numbers?

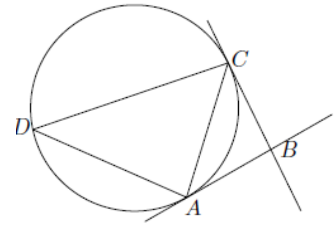
$$\{1, 101, 10101, 1010101, 101010101, \ldots\ldots\}$$

119. Find all triples of natural numbers (a, b, c) such that a, b and c are in geometric progression, and a + b + c = 111.

120. What is the smallest integer n for which $\sqrt{n} - \sqrt{n-1} < 0.01$?



121. Seven people, A, B, C, D, E, F and G can sit down for a meal at a round table as shown. Each person has two neighbours at the table: for example, A's neighbours are B and G. There are other ways in which the people can be seated round the table. Last month they dined together on a number of occasions, and no two of the people were neighbours more than once. How many meals could they have had together during the month?



122. Points A, D and C lie on the circumference of a circle. The tangents to the circle at points A and C meet at the point B. If ∠DAC = 83° and ∠DCA = 54°. Find ∠ABC.

123. How many 4-digit numbers uses exactly three different digits?

124. The ratio of two six digit numbers abcabc and ababab is 55 : 54. Find the value of a + b + c.

125. Find the infinite sum:

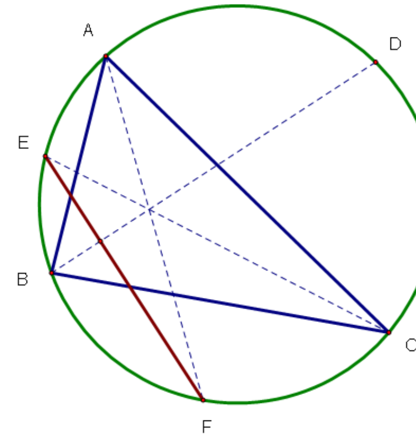
$$\frac{1}{4} + \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \frac{5}{64} + \frac{8}{128} + \frac{13}{256} + \frac{21}{512} + \ldots$$

126. What is the probability of tossing a coin 6 times such that no two consecutive throws result in a head?

127. In how many ways 3 letters can be selected from 3 identical A's, 3 identical B's and 3 identical C's?

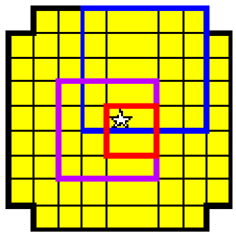
128. For how many pairs of positive integers (x, y) both $x^2 + 4y$ and $y^2 + 4x$ are perfect squares?

129. Jar X contains six liters of a 46% milk solution; Jar Y contains three liters of a 43% milk solution and Jar Z contains one liter of p% milk solution. q/r liters of solution from Jar Z is transferred to Jar X and remaining solution from Jar Z is transferred to Jar Y such that resulting two solutions both contain 50% milk solution. Also q and r are positive integers co-prime to each other. Find the value of p + q + r.



130. ABC is an isosceles right triangle inscribed in a circle such that $\angle B = 90°$. BD, CE and AF are angle bisectors of triangle ABC as shown. What is the measure of smaller angle of intersection of BD and EF?

131. In the diagram, three squares are shown, all containing the star. Altogether, how many squares containing the star can be found in the diagram?



132. Find all integers x, y, z (such that $x \le y \le z$) greater than 1 for which $xy - 1$ is divisible by z, $yz - 1$ is divisible by x, and $zx - 1$ is divisible by y.

133. In a rectangle ABCD, AB = 13 and BC = 8. PQ lies inside the rectangle such that BP = 11, DQ = 6 and AB||PQ, BP||DQ. Find the length of PQ.

134. What are the last two digits of the sum obtained by adding all the possible remainders of numbers of the form 2^n, n being a non-negative integer, when divided by 100?

135. On planet LOGIKA, there live two kinds of inhabitants; black and white ones and they answer every question posed to them in a Yes or No. Black inhabitants of northern hemisphere always lie while white inhabitants of northern hemisphere always tell the truth. Also white inhabitants of southern hemisphere always lie while black inhabitants of southern hemisphere always tell the truth. On a dark night, there is an electricity failure and you meet an inhabitant without knowing your location on the planet. What single yes/no question can you ask the inhabitant to determine color of the inhabitant?

136. N is product of first 50 prime numbers. A is a factor of N and B is a factor of A. How many ordered pairs (A, B) exist?

137. For how many positive integers, N > 2, (N - 2)! + (N + 2)! is a perfect square?

138. Let P = $\frac{1}{1\times2}+\frac{1}{3\times4}+\frac{1}{5\times6}+...+\frac{1}{2013\times2014}$ and

Q = $\frac{1}{1008\times2014}+\frac{1}{1009\times2013}+\frac{1}{1010\times2012}+....+\frac{1}{2014\times1008}$.

Find P/Q.

139. C and D are two points on a semicircle with AB as diameter such that AC – BC = 7 and AD – BD = 13. AD and BC intersect at P. Find the difference in area of triangles ACP and BDP.

140. TG *fashions* hold its annual sale on the eve of Pi-day (14th March) and offered a discount of 90% on all its apparels. But this month it is offering the usual 80% discount. How much percent more I need to pay now than that on the annual sale's eve for purchase of similar clothing?

141. In how many ways 1,000,000 can be expressed as sum of a square number and a prime number?

142. How many ordered triples of three positive integers (a, b, c) exist such that $a^3 + b^3 + c^3 = 2011$?

143. In a quadrilateral ABCD, sides AD and BC are parallel but not equal and sides AB = DC = x. The area of the quadrilateral is 676 cm^2. A circle with centre O and radius 13 cm is inscribed in the quadrilateral such that it is tangent to each of the four sides of the quadrilateral. Determine the length of x.

144. Kiran, Shashi and Rajni are Kiran's spouse, Shashi's sibling and Rajni's sister-in-law in no particular order. Also Kiran's spouse and Shashi's sibling are of same sex. Who among the three is a married male?

145. A and B start running from two opposite ends of a 1000m racing track. A and B travel with a speed of 8m/s and 5m/s respectively. How many times they meet, while running, in first 1000s after start?

146. According to death-will of Mr. Ranjan, all of this money was to be divided among his children in the following manner: `N to the first born plus 1/17 of what remains, `2N to the second born plus 1/17 of what then remains, `3N to the third born plus 1/17 of what then remains, and so on. When the distribution of the money was complete, each child received the same amount and no money was left over. Determine the number of children.

147. One number is removed from the set of integers from 1 to n. The average of the remaining numbers is 40.75. Which integer was removed?

148. What is the 2037^{th} positive integer that can be expressed as the sum of two or more consecutive positive integers? (The first three are 3 = 1+2, 5 = 2+3, and 6 = 1+2+3.)

149. Determine the number of ordered triplets (A, B, C) of sets which have the property that
(i) $A \cup B \cup C = \{1, 2, 3, \ldots, 1000\}$, and
(ii) $A \cap B \cap C = \varnothing$.

150. In a parallelogram ABCD, let M be the midpoint of the side AB and N the midpoint of BC. Let P be the intersection point of the lines MC and ND. Find the ratio of area of ▲s APB: BPC: CPD: DPA.

151. Kali-Jot is a game played by two players each of them having some number of marbles with her. One of the two players has to determine whether the number of marbles with other player is even or odd. A particular game of Kali-Jot has seven players and starts with players P_1 and P_2 on field and the other players P_3, P_4, P_5, P_6, P_7 waiting in a queue for their turn in order. After each game is played, the loser goes to the end of the queue; the winner adds 1 point to her score and stays on the field; and the player at the head of the queue comes on to contest the next point. Game continues until someone has scored 11 points. At that moment, it was found out that a total of 43 points have been scored by all seven players together. Who is the winner?

152. For positive real numbers; A, B, C, D such that A + B + C + D = 8, find the minimum possible value of $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$.

153. In two equilateral triangles ABC and BMN, $\angle ABM = 120^\circ$. AN & CM intersects at O. Find $\angle MON$.

TathaGat Tutorials Private Limited

115, IInd Floor, New Delhi House

Barakhamba Road, New Delhi-110001

011-43580693, 011-43580694