

Homework 5, part 2

15.1-3)

(15.1-3) As given in the book, there are two ways for ^{saving} rod-cutting problem.

1. Bottom-up-cut
2. Top-down cut.

Let's use Bottom-up cut for this case where we need to add an extra fixed cost for cutting the rod. Which should be subtracted from the prices that we get each made to small rods that are cut!

Let's consider fixed cost to be c .

Given: to subtract c from price of the pieces.

$$(i.e.) P_{i1} + P_{i2} + \dots + P_{ik}$$

$k \rightarrow$ no. of pieces, $i \rightarrow$ inches of the rod.

This is nothing but u_n (revenue) of rod

length n .

$$u_n = P_{i1} + P_{i2} + \dots + P_{ik}$$

So subtracting the fixed cost c from price would be:

$$M_n = (P_{i_1} - c) + (P_{i_2} - c) + \dots + (P_{i_k} - c)$$

$$= (P_{i_1} + P_{i_2} + P_{i_3} + \dots + P_k) - k(c)$$

no. of pieces. k times the fixed cost as k is

this can be ~~be~~ called by

$$\boxed{M_n - k(c)} \text{ when } P_{i_1} + P_{i_2} + \dots + P_k \text{ is } M_n.$$

Let's use Bottom-up approach.

MODIFIED CUT ROD (p, n, c) \rightarrow fixed cost

1. let $r[0 \dots n]$ be a new array

2. $r[0] = 0$

3. for $j = 1$ to n

4. $q = p[j]$

5. for $i = 1$ to $j-1$

6. $q_i = \max(q, p[i] + r[i-1] - c)$

7. $r[j] = q$

8. return $r[n]$.

In line number 6, we subtract the fixed cost from revenue, as discussed earlier it is similar to subtracting it from the pieces price

In line no. 4 & 5, we add $q - p[j]$ instead just to make sure that we don't subtract fixed costs for no cutting. Run the for loop from $i \leftarrow j-1$ instead of $i \leftarrow 1$. When $i = j$,

when we don't add that, we tend to subtract the fixed cost from nothing

$(1 + q = \max(q, p[i] - c)) \rightarrow$ which is not valid
hence above is the modified algorithm with fixed cost

15.2-1)

15) 2-1) Given: $\{5, 10, 3, 12, 5, 50, 6\}$

let make a sequence with given dimension

$\{x_0 \{5, 10\}, x_1 \{10, 3\}, x_2 \{3, 12\}, x_3 \{12, 5\}, x_4 \{5, 50\}, x_5 \{50, 6\}\}$

	x_0	x_1	x_2	x_3	x_4	x_5
x_0	0	150	27000	27000	20,250,000	4,050,000
x_1		0	360	27000	1080,000	40,500,000
x_2			0	180	1,35,000	2,700,000
x_3				0	3000	54,000
x_4					0	1500
x_5						0

$$\underline{l=2}$$

$$x_0 x_1 = 150$$

$$x_1 x_2 = 360$$

$$x_2 x_3 = 180$$

$$x_3 x_4 = 3000$$

$$x_4 x_5 = 1500$$

$$\underline{l=3}$$

$$\frac{x_0 x_1 x_2}{1 \ 2} = 150 \times 5 \times 3 \times 12 = 27,000 \checkmark$$

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$$\frac{x_0 x_1 x_2}{1 \ 2} = 360 \times 5 \times 10 \times 12 = 21,600$$

• ~~1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12~~

$$\frac{x_1 x_2 x_3}{1 \ 2} = 360 \times 12 \times 5 \times 10 = 21,600$$

$$\frac{x_1 x_2 x_3}{1 \ 2} = 180 \times 10 \times 3 \times 5 = 27,000 \checkmark$$

~~1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12~~

$$x_2 x_3 x_4 = 180 \times 5 \times 50 \times 3 = 1,350,000 \checkmark$$

$$x_2 x_3 x_4 = 3000 \times 3 \times 12 \times 50 = 51,600,000$$

~~1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12~~

$$x_3 x_4 x_5 = 3000 \times 50 \times 6 \times 12 = 10,800,000$$

$$x_3 x_4 x_5 = 1500 \times 12 \times 5 \times 6 = 540,000 \checkmark$$

$l=4$

$$\frac{x_0 \times x_1 \times x_2 \times x_3}{\begin{array}{c} \textcircled{1} \\ \textcircled{1} \end{array}} = 150 \times 180 = 27,000 \quad \checkmark$$

$$\frac{x_0 \times x_1 \times x_2 \times x_3}{\begin{array}{c} \textcircled{1} \\ \textcircled{2} \end{array}} = 27000 \times 5 \times 12 \times 5 \quad \left. \right\}$$

$$\frac{x_0 \times x_1 \times x_2 \times x_3}{\begin{array}{c} \textcircled{1} \\ \textcircled{2} \end{array}} = 27000 \times 5 \times 10 \times 5 \quad \left. \right\} \begin{array}{l} \text{clearly these} \\ \text{are high} \end{array}$$

$$x_1 \times x_2 \times x_3 \times x_4 = 360 \times 3000 = 1,080,000 \quad \checkmark$$

$$x_1 \times x_2 \times x_3 \times x_4 = 27000 \times 5 \times 150 \times 10 \quad \left. \right\} \begin{array}{l} \text{clearly these} \\ \text{are high} \end{array}$$

$$x_1 \times x_2 \times x_3 \times x_4 = 135000 \times 10 \times 3 \times 50 \quad \left. \right\} \begin{array}{l} \text{clearly these} \\ \text{are high} \end{array}$$

$$x_2 \times x_3 \times x_4 \times x_5 = 180 \times 1500 = 270,000 \quad \checkmark$$

$$x_2 \times x_3 \times x_4 \times x_5 = ? \text{ will be high clearly from above.}$$

$$x_2 \times x_3 \times x_4 \times x_5 =$$

$l=5$

$$\frac{x_0 \times x_1 \times x_2 \times x_3 \times x_4}{\begin{array}{c} \textcircled{1} \\ \textcircled{1} \\ \textcircled{2} \end{array}} = 150 \times 180 \times 5 \times 5 \times 50 = 33,750,000$$

$$\frac{x_0 \times x_1 \times x_2 \times x_3 \times x_4}{\begin{array}{c} \textcircled{1} \\ \textcircled{1} \\ \textcircled{2} \end{array}} = 360 \times 3000 \times 5 \times 10 \times 50 = \text{high}$$

$$\frac{x_0 \times x_1 \times x_2 \times x_3 \times x_4}{\begin{array}{c} \textcircled{1} \\ \textcircled{1} \\ \textcircled{2} \end{array}} = 27000 \times 3000 \times 5 \times 10 \times 50 = 81,000,000$$

$$\frac{x_0 x_1 x_2 x_3 x_4}{\textcircled{2}} = 150 \times 135000 = 20,250,000$$

$$x_1 x_2 x_3 x_4 x_5 = 360 \times 3000 \times 50 \times 6 \times 10 = 324,000,000$$

$$x_1 x_2 x_3 x_4 x_5 = 180 \times 3000 \times 10 \times 3 \times 6 = 97,200,000$$

$$x_1 x_2 x_3 x_4 x_5 = 27000 \times 1500 = 40,500,000$$

$$x_1 x_2 x_3 x_4 x_5 = \frac{5490000 \times 360}{135000} = 194,400,000$$

$l=6$

$$\frac{x_0 x_1 x_2 x_3 x_4 x_5}{\textcircled{1} \textcircled{1} \textcircled{1}} = 150 \times 180 \times 1500 = 40,500,000$$

$$\frac{x_0 x_1 x_2 x_3 x_4 x_5}{\textcircled{1} \textcircled{1}} = 27000 \times 5,40,000 = \text{high}$$

$$\frac{x_0 x_1 x_2 x_3 x_4 x_5}{\textcircled{1} \textcircled{1}} = 27000 \times 1500 = 40,500,000$$

$$\frac{x_0 x_1 x_2 x_3 x_4 x_5}{\textcircled{1} \textcircled{1}} = 150 \times 2,70,000 = 40,500,000$$

$$\frac{x_0 x_1 x_2 x_3 x_4 x_5}{\textcircled{1} \textcircled{2}} = 20,250,000 \times 50 \times 6 \times 5$$

$$x_0 x_1 x_2 x_3 x_4 x_5 = 5 \times 10 \times 6 \times 40,500,000 = \text{high}$$

From above

We have 3 sequence which gives minimum value for matrix multiplication.

* * *

$$(i) (A_1 A_2) (A_3 A_4) (A_5 A_6)$$

$$(A_1 A_2 A_3 A_4) (A_5 A_6)$$

$$(A_1 A_2) (A_3 A_4 A_5 A_6)$$

15.4-1)

5-4.1)	1	0	0	1	0	1	0	1	0	1
	0	0	0	1	1	1	1	1	1	1
	1	0	1	1	1	2	2	2	2	2
	0	0	1	2	2	2	3	3	3	3
	1	0	1	2	2	3	3	4	4	4
	1	0	1	2	2	3	3	4	4	5
	0	0	1	2	3	3	4	4	5	5
	1	0	1	2	3	4	4	5	5	6
	1	0	1	2	3	4	4	5	5	6
	0	0	1	2	3	4	5	5	6	6

Longest Common Subsequence
is 6.

The LCS is.

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15-2)

15-2) longest Palindrome Sequence .

Algorithm:

Palindrome (~~A~~ A)

① r=0, l=0
② currentPal = "", longestPal = ""
③ for (c = 1; c < A.length() - 1; c++)
 l = c - 1, r = c + 1
 while (l ≥ 0 && r < A.length(),
 if (A.charAt(l) != A.charAt(r))
 break
 currentPal = A.substring(l, r + 1)
 longestPal = currentPal.length() > longestPal.length() ?
 currentPal : longestPal
 l--, r--
⑪
⑫ return longestPal .

A → input
right index → r
left index → l
c → center index

Code in Java:

```
public class Palindrome {  
    public static String getfinalLongPalindrome(final String input) {  
        int r = 0, l = 0;  
        String currentPalindrome = "", finalLongPalindrome = "";  
        for (int c = 1; c < input.length() - 1; c++) {  
            l = c - 1; r = c + 1;  
            while (l >= 0 && r < input.length()) {  
                if (input.charAt(l) != input.charAt(r)) {  
                    break;  
                }  
                currentPalindrome = input.substring(l, r + 1);  
                finalLongPalindrome = currentPalindrome.length() > finalLongPalindrome.length() ?  
                    currentPalindrome : finalLongPalindrome;  
                l--; r++;  
            }  
        }  
    }  
}
```

```
        }
    }
    return finalLongPalindrome;
}

public static void main(String args[]) {
    String str = "cabacar";
    String finalLongPali = getfinalLongPalindrome(str);
    System.out.println("String: " + str);
    System.out.println("Longest Palindrome: " + finalLongPali);
}
```

Output:

```
String: cabacar
Longest Palindrome: cabac
```

Running time would be $O(n)$ since there is only one for loop.