Assignement 3

Anuran Pal—16ms152

January 2020

Theory

Evolution with Mutation and Selection

The replicator equation involving selection as well as mutation is given as:

$$\dot{x} = x[f_0 - \phi] - u_0 x + u_1(1 - x) \tag{1}$$

where, the x denotes the frequency of type 0 in the population.
u_0 denotes the mutation rate from type 0 to type 1 and vice versa.
f_i denotes the fitness of type i.
ϕ denotes the mean fitness of the population given by $\Sigma_i x_i f_i$.
We consider the population size to be fixed. Thus we can say x_0=1-x_1.
The frequency reaches an equilibrium population when $\dot{x}$=0. Or we can say that the mutation rate balances the selection rate.
Putting in the value of $\dot{x}$=0 and solving for equilibrium frequency x^* we get a quadratic equation whose solution is:

$$x^*_{\pm} = \frac{(u_0 + u_1 + f_0 - f_1) \pm \sqrt{(f_0 - f_1 - u_0 - u_1)^2 - 4(f_1 - f_0)u_1}}{2(f_1 - f_0)} \tag{2}$$

We will consider only the x^*_+ solution since frequency cannot be negative.
Answer 1]
I) **Putting in the values,**
$f_0 = 1.001, f_1 = 1, u_0 = 0.01, u_1 = 0.0$**,**
we get the value of x*=0.
II and III) As the expression does not have the size of the population in it, thus answer II and III should be the same.
Putting in the values,
$f_0 = 1.1, f_1 = 1, u_0 = 0.01, u_1 = 0.0$**, we get the value of x*=0.9**.

Evolution with Selection

Let us consider the total populaton strength to be **N** and the number of type 1 individual to be **i**.
Then, let $\mathbf{x}_i$ denote the probability that type 1 reaches fixation from a population strength of i. Let the probability of increase in one individual of type 1 be given as α_i and probability of decrease be given as β_i. Then the probability of fixation from a size of i of type 1 is given as,

$$x_i = \frac{1 + \Sigma_{j=1}^{i-1} \Pi_{k=1}^{j} \frac{\beta_k}{\alpha_k}}{1 + \Sigma_{j=1}^{N-1} \Pi_{k=1}^{j} \frac{\beta_k}{\alpha_k}} \tag{3}$$

Consider type 1 to have fitness **r** while type 0 have fitness 1.
Then putting the value of r in the expression for fixation probability we get:

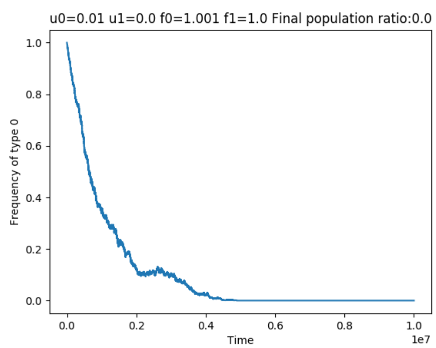
$$x_i = \frac{1 - \frac{1}{r^i}}{1 - \frac{1}{r^N}} \tag{4}$$

Answer 2]
I]The given values are r=1.01, N=100, i=1 for type1.
Putting the values in equation 4, x_1 = 0.0157.
II]The given values are r=0.99, N=100, i=1 for type1.
Putting the values in equation 4, x_1 = 0.00583.

Simulation

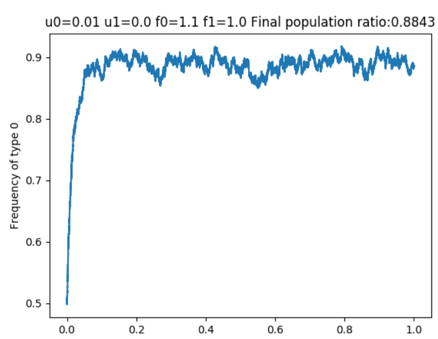
Answer1]

I]



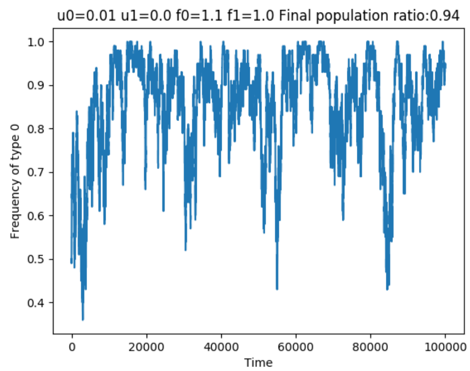
(a) Population size=10000, f0=1.001, f1=1.0, u0=0.01, u1=0.0, i=10000

II]



(a) Population size=10000, f0=1.1, f1=1.0, u0=0.01, u1=0.0, i=5000

III]



(a) Population size=100, f0=1.1, f1=1.0, u0=0.01, u1=0.0, i=50

Answer2]

I]Considering f1=1.01, the **calculated fixation probability is 0.0157, while the simulated frequency is 0.016**.
II]Considering f1=0.99, the **calculated fixation probability is 0.00583, while the simulated frequency is 0.0074**.