Assignement 2

Anuran Pal—16ms152

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Theory

Let us consider the total populaton strength to be **N** and the number of type 0 individual to be **i**.
Then, let $\mathbf{x}_i$ denote the probability that type 0 reaches fixation from a population strength of i. Let the probability of increase in one individual of type 0 be given as α_i and probability of decrease be given as β_i. Then the probability of fixation from a size of i of type 0 is given as,

$$x_i = \frac{1 + \Sigma_{j=1}^{i-1} \Pi_{k=1}^{j} \frac{\beta_k}{\alpha_k}}{1 + \Sigma_{j=1}^{N-1} \Pi_{k=1}^{j} \frac{\beta_k}{\alpha_k}} \tag{1}$$

Consider type 0 to have fitness **r** while type 1 have fitness 1.
Then putting the value of r in the expression for fixation probability we get:

$$x_i = \frac{1 - \frac{1}{r^i}}{1 - \frac{1}{r^N}} \tag{2}$$

Answer 1]
Given N=500 and r=1.
Fixation probability of type 0 when i=N/2 and r=1 is (from equation 1),

$$x_{N/2} = \frac{1 + N/2 - 1}{1 + N - 1} = \frac{1}{2} = 0.5 \tag{3}$$

Thus, $\mathrm{x}_{N/2}$(given r=1) = $\frac{1}{2}$.

Answer 2]
Given N=100 and r=1.01.
Fixation probability of type 0 when i=1 and r=1.01 is (from equation 2),

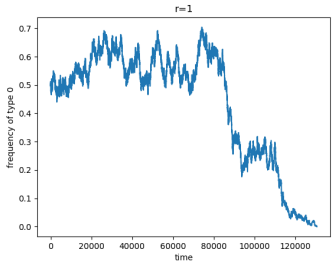
$$x_1 = \frac{1 - \frac{1}{1.01^1}}{1 - \frac{1}{1.01^N}} \approx 0.0157 \tag{4}$$

Given N=100 and r=0.99.
Fixation probability of type 0 when i=N/2 is (from equation 2),

$$x_{50} = \frac{1 - \frac{1}{0.99^{N/2}}}{1 - \frac{1}{0.99^N}} \approx 0.3769. \tag{5}$$

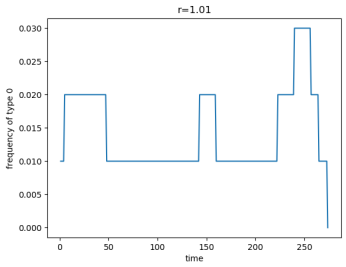
Simulation

Answer 1]
Running the simulation for population size of 500 for equal fitness and i=250 for 100 times we expect a **0.5** probability of type 0 to be fixed in the population. **The simulation gives us values ranging between 0.4 to 0.6 approximately**.
Following is a plot of frequency of type 0 and time for a particular trial run:

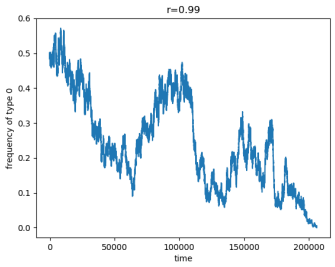


(a) Population size = 500. r = 1. i = 250

Answer 2 I]
Running the simulation for population size of 100 for r=1.01, i=1 for 1000 times we expect a **0.0157** probability of type 0 to be fixed in the population. Implies the fixation probability of type 1 will be 1-0.0157=0.9843. **The simulation gives us a probability lying between 0.01 and 0.02 approximately.**
Answer 2 II]
Running the simulation for population size of 100 for r=0.99, i=50 for 1000 times we expect a **0.3769** probability of type 0 to be fixed in the population. Implies the fixation probability lying between 1-0.3769=0.6321. **The simulation gives us a probability between 0.39 and 0.35 approximately.**
Following are two plots of frequency of type 0 vs time for r=0.99(type0=50) and r=1.01(type1=1):



(a) Population size=100, type 0 frequency=1, r=1.01



(b) Population size=500, type 0 frequency=50 r=0.99