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Covariance matrix adaptation evolution strategy based design of centralized PID controller

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ABSTRACT

In this paper, design of centralized PID controller using Covariance Matrix Adaptation Evolution Strategy (CMAES) is presented. Binary distillation column plant described by Wood and Berry (WB) having two inputs and two outputs and by Ogunnike and Ray (OR) having three inputs and three outputs are considered for the design of multivariable PID controller. Optimal centralized PID controller is designed by minimizing IAE for servo response with unit step change. Simulations are carried out using SIMULINK-MATLAB software. The statistical performances of the designed controllers such as best, mean, standard deviations of IAE and average functional evaluations for 20 independent trials. For the purpose of comparison, recent version of real coded Genetic Algorithm (RGA) with simulated binary crossover (SBX) and conventional BLT method are used. In order to validate the performance of optimal PID controller for robustness against load disturbance rejection, load regulation experiment with step load disturbance is conducted. Also, to determine the performance of optimal PID controller for robustness against model uncertainty, servo and load response with +20% variations in gains and dead times is conducted. Simulation results reveal that for both OR and WB systems, CMAES designed centralized PID controller is better than other methods and also it is more robust against model uncertainty and load disturbance.

1. Introduction

Proportional-Integral-Derivative (PID) control offers the simplest and yet most efficient solution to many real-world control problems. Three-term functionality of PID controller covers treatment of both transient and steady state responses. The popularity of PID control has grown tremendously, since the invention of PID control in 1910 and the Ziegler-Nichol's straight forward tuning method in 1942. More than 90% of industrial controllers are still implemented based around PID control algorithms, as no other controllers match the simplicity, clear functionality, applicability and ease of use offered by the PID controllers (Ang, Chang, & Li, 2005). Ziegler-Nichols and Cohen-Coon are the most commonly used conventional methods for tuning PID controllers and neural network, fuzzy based approach, neuro-fuzzy approach and evolutionary computation techniques are the recent methods (Astrom & Hagglund, 1995).

Compared to the SISO system, the control of multivariable systems has always been a challenge to control system designers due to its complex interactive nature. In last several decades, designing controllers for MIMO systems has attracted a lot of research interests and many multivariable control approaches have been proposed (Luyben, 1986, 1990; Monica, Yu, & Luyben, 1988; Wang, Zou, Lee, & Qiang, 1997). Many researchers have already reported the optimal design of PID controller for MIMO system using Evolutionary Algorithms (EA) such as Genetic Algorithm (Chang, 2007; Zuo, 1995), Particle Swarm Optimization (Su & Wong, 2007).

Recently, Covariance Matrix Adaptation Evolution Strategy (CMAES) with the ability of learning of correlations between parameters and the use of the correlations to accelerate the convergence of the algorithm is proposed (Kern et al., 2004). Owing to the learning process, the CMAES algorithm performs the search, independent of the coordinate system, reliably adapts topologies of arbitrary functions, and significantly improves convergence rate especially on non-separable and/or badly scaled objective functions. CMAES algorithm has been successfully applied in varieties of engineering optimization problems (Baskar, Alphones, Suganthan, Ngo, & Zheng, 2005). This algorithm outperforms all other similar classes of learning algorithms on the benchmark multimodal functions (Kern et al., 2004). In general, EAs are robust search and optimization methodologies, able to cope with ill-defined problem domains such as multimodality, discontinuity, time-variance, randomness and noise. Willjuice and Baskar (2009) have demonstrated the application of various EAs for the design of decentralized PID controller of WB system. Effect of load disturbance and parameter variation has not been studied for the designed optimal decentralized PID controller.

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This paper focuses mainly on the design of centralized PID controller using CMAES algorithm for distillation column plant of WB and OR systems and also validates performance of controller over robustness against load disturbance rejection and model uncertainty.

For the purpose of comparison, conventional BLT method (Monica et al., 1988) and recent version of RGA with Simulated Binary Crossover (SBX) and non-uniform polynomial mutation is used (Deb, 2001).

The remaining part of the paper is organized as follows. Section 2 introduces PID controller structure for SISO and MIMO systems. Section 3 describes the CMAES algorithm. Section 4 introduces the MIMO systems considered for PID controller tuning. Section 5 describes the CMAES implementation details of multivariable PID controller. Section 6 reveals the simulation results. Finally, conclusions are given in Section 7.

2. PID controller structure

A standard PID controller structure is also known as the "three-term" controller, whose transfer function is generally written in the ideal form in (1) or in the parallel form in (2).

$$G(s) = K_P\left(1 + \frac{1}{T_I s} + T_D s\right), \tag{1}$$

$$G(s) = K_P + \frac{K_I}{s} + K_D s, \tag{2}$$

where K_P is the proportional gain, T_I is the integral time constant, T_D is the derivative time constant, $K_I = K_P/T_I$ is the integral gain and $K_D = K_P T_D$ is the derivative gain.

The "three term" functionalities are highlighted below.

- The proportional term – providing an overall control action proportional to the error signal through the all pass gain factor.
- The integral term – reducing steady state errors through low frequency compensation by an integrator.
- The derivative term – improving transient response through high frequency compensation by a differentiator.

For optimum performance, K_P, K_I (or T_I) and K_D (or T_D) are tuned by minimizing the performance measures such as IAE, ISE and ITAE.

2.1. PID controller for MIMO system

Consider a multivariable PID control structure as in Fig. 1,

$$\begin{aligned}
&\text{where, desired output vector: } Y_d = [y_{d1}, y_{d2}, \ldots, y_{dn}]^T;\\
&\text{Actual output vector: } Y = [y_1, y_2, \ldots, y_n]^T;\\
&\text{Error vector: } E = Y_d - Y = [y_{d1} - y_1, y_{d2} - y_2, \ldots, y_{dn} - y_n]^T\\
&\qquad = [e_1, e_2, \ldots, e_n]^T;\\
&\text{Control input vector: } U = [u_1, u_2, \ldots, u_n]^T;
\end{aligned}$$

$n \times n$ Multivariable process:

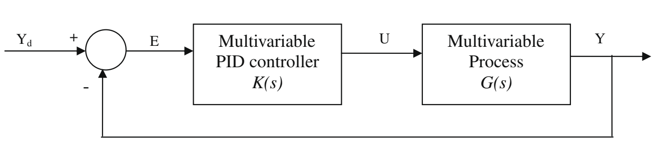

Fig. 1. A multivariable PID control system.

$$G(s) = \begin{bmatrix} g_{11}(s) & \cdots & g_{1n}(s) \\ \vdots & \ddots & \vdots \\ g_{n1}(s) & \cdots & g_{nn}(s) \end{bmatrix}; \tag{3}$$

$n \times n$ Centralized PID controller:

$$K(s) = \begin{bmatrix} k_{11}(s) & \cdots & k_{1n}(s) \\ \vdots & \ddots & \vdots \\ k_{n1}(s) & \cdots & k_{nn}(s) \end{bmatrix}. \tag{4}$$

The form of $k_{ij}(s)$ is either in (1) or (2). In this work, "parallel form" of PID controller in (2) is used and can be rewritten as

$$k_{ij}(s) = k_{P_{ij}} + \frac{k_{I_{ij}}}{s} + k_{D_{ij}} s. \tag{5}$$

For convenience, let $\theta_{ij} = [k_{P_{ij}}, k_{I_{ij}}, k_{D_{ij}}]$, represents the gains vector of ith row and jth column sub PID controller in $K(s)$.

2.2. Performance index

In the design of PID controller, the performance criterion or objective function is first defined based on the desired specifications such as time domain specifications, frequency domain specifications and time-integral performance. The commonly used time-integral performance indexes are integral of the square error (ISE), integral of the absolute value of the error (IAE) and integral of the time-weighted absolute error (ITAE). Minimization of IAE as given in (6) is considered as the objective in this paper.

$$\mathrm{IAE} = \int_0^\infty (|e_1(t)| + |e_2(t)| + \cdots + |e_n(t)|)\,dt. \tag{6}$$

3. Covariance matrix adaptation evolution strategy (CMAES)

CMAES is a class of continuous EA; it generates new population members by sampling from a probability distribution that is constructed during the optimization process. CMAES is derived from the concept of self-adaptation in evolution strategies, which adapts the covariance matrix of a multivariate normal search distribution. One of the key concepts of this algorithm involves the learning of correlations between parameters and the use of the correlations to accelerate the convergence of the algorithm. Owing to the learning process, the CMAES algorithm performs the search independent of the coordinate system, reliably adapts topologies of arbitrary functions, and significantly improves convergence rate especially on non-separable and/or badly scaled objective functions. This algorithm outperforms all other similar classes of learning algorithms on the benchmark multimodal functions (Hansen, 2006). The adaptation mechanism of CMAES consists of two parts:

(i) The adaptation of the covariance matrix **C** and
(ii) The adaptation of the global step size.

The covariance matrix **C** is adapted by the evolution path and vector difference between the μ best individuals in the current and previous generation. The detailed CMAES algorithm is explained in following steps:

Step 1: Generate an initial random solution.
Step 2: The offspring at $g+1$ generation x_k^{g+1} are sampled from a Gaussian distribution using covariance matrix and global step size at generation g.

$$x_k^{(g+1)} = z_k, \quad z_k = N(\langle x\rangle_\mu^{(g)}, \sigma^{(g)^2}\mathbf{C}^{(g)}) \quad k = 1, \ldots, \lambda, \tag{7}$$

where $\langle x\rangle_{\mu}^{(g)} = \sum_{i=1}^{\mu} x_i^{(g)}$ with μ being the selected best individuals from the population.

The parameters c_c, c_{cov}, c_σ and d required for further computations are by default given in terms of number of decision variables (n) and μ as follows:

$$c_c = \frac{4}{n+4}, \quad c_\sigma = \frac{10}{n+20}, \quad d = \max\left(1, \frac{3\mu}{n+10}\right) + c_\sigma,$$
$$c_{\mathrm{cov}} = \frac{1}{\mu}\frac{2}{\left(n+\sqrt{2}\right)^2} + \left(1 - \frac{1}{\mu}\right)\min\left(1, \frac{2\mu - 1}{(n+2)^2 + \mu}\right). \tag{8}$$

The parameters c_σ and c_{cov} control independently the adaptation time scales for the global step size and the covariance matrix.

Note that if $\mu \gg n$, d is large and the change in σ is negligible compared to that of $\mathbf{C}$. The initial values are $\mathbf{P}_\sigma^{(0)} = \mathbf{P}_c^{(0)} = \mathbf{0}$ and $\mathbf{C}^{(0)} = \mathbf{I}$.

Step 3: The evolution path $\mathbf{P}_c^{(g+1)}$ is computed as follows:

$$\mathbf{P}_c^{(g+1)} = (1 - c_c)\cdot \mathbf{P}_c^{(g)} + \sqrt{c_c(2 - c_c)} \cdot \frac{\sqrt{\mu}}{\sigma^{(g)}}\left(\langle x\rangle_{\mu}^{(g+1)} - \langle x\rangle_{\mu}^{g}\right),$$
$$\mathbf{C}^{(g+1)} = (1 - c_{\mathrm{cov}})\cdot \mathbf{C}^{(g)} + c_{\mathrm{cov}} \cdot \left(\begin{array}{l} \frac{1}{\mu}\mathbf{P}_c^{(g+1)}(\mathbf{P}_c^{(g+1)})^{\mathrm{T}} \\ +\left(1 - \frac{1}{\mu}\right)\frac{1}{\mu}\sum_{i=1}^{\mu}\frac{1}{\sigma^{(g)2}}\left(x_i^{(g+1)} - \langle x\rangle_{\mu}^{(g)}\right)\left(x_i^{(g+1)} - \langle x\rangle_{\mu}^{(g)}\right)^{\mathrm{T}} \end{array}\right). \tag{9}$$

The strategy parameter $c_{\mathrm{cov}} \in [0, 1]$ determines the rate of change of the covariance matrix $\mathbf{C}$.

Step 4: Adaptation of global step size $\sigma^{(g+1)}$ is based on a conjugate evolution path $\mathbf{P}_\sigma^{(g+1)}$

$$\mathbf{P}_\sigma^{(g+1)} = (1 - c_\sigma)\cdot \mathbf{P}_\sigma^{(g)} + \sqrt{c_\sigma(2 - c_\sigma)} \cdot \mathbf{B}^{(g)}(\mathbf{D}^{(g)})^{-1}(\mathbf{B}^{(g)})^{-1} \times \frac{\sqrt{\mu}}{\sigma^{(g)}}\left(\langle x\rangle_{\mu}^{(g+2)} - \langle x\rangle_{\mu}^{g}\right) \tag{10}$$

the matrices $\mathbf{B}^{(g)}$ and $\mathbf{D}^{(g)}$ are obtained through a principal component analysis:

$$\mathbf{C}^{(g)} = \mathbf{B}^{(g)}(\mathbf{D}^{(g)})^2(\mathbf{B}^{(g)})^{\mathrm{T}}$$

where the columns of $\mathbf{B}^{(g)}$ are the normalized eigen vectors of $\mathbf{C}^{(g)}$ and $\mathbf{D}^{(g)}$ is the diagonal matrix of the square roots of the given eigen values of $\mathbf{C}^{(g)}$. The global step size $\sigma^{(g+1)}$ is determined by

$$\sigma^{(g+1)} = \sigma^{(g)}\exp\left(\frac{c_\sigma}{d}\left(\frac{\|\mathbf{P}_\sigma^{(g+1)}\|}{E(\|N(0,\mathbf{I})\|)}\right) - 1\right). \tag{11}$$

Step 5: Repeat steps 2–4 until a maximum number of function evaluations or tolerance on design variables and objective function is reached.

The key points of CMAES algorithm is summarized below:

- *Estimation principle:* The CMAES algorithm estimates the distribution parameters from a set of selected steps. The steps avoid premature convergence and support explorative search behavior.
- *Step size control:* Methods to estimate or adapt the covariance matrix do not achieve good overall step lengths. In the CMAES algorithm, the adaptation of the covariance matrix is complemented with step size control. The adjustment of the step size is based on a different adaptation principle. Cumulative path length control often adapts nearly optimal step sizes which are usually leading to considerable larger step lengths. This improves convergence speed and global search capabilities at the same time.
- *Population size, adaptation, and change rates:* Choosing the population size λ is always a compromise. Small λ leads to faster convergence and large λ helps to avoid local optima. To achieve a fast learning scheme for a covariance matrix:
  (i) The population size λ must be comparatively small and
  (ii) An adaptation procedure must be established, where parameters are updated rather than estimated from scratch in every generation.

Based on the above key points, the CMAES algorithm can improve the performance on ill-conditioned and/or non-separable problems by orders of magnitude, leaving the performance on simple problems unchanged (Hansen, 2006b).

4. MIMO systems

Most of the industrial processes belong to the category of MIMO system, which requires MIMO control techniques to improve performance, even though they are naturally more difficult to exploit than SISO system. Binary distillation column plant described by Wood and Berry (WB) (Chang, 2007; Monica et al., 1988; Wang et al., 1997) is considered for 2×2 system. The transfer function and the load transfer function for this system as specified by (Monica et al., 1988) is given in Table 1. The transfer function of WB process has first order dynamics and significant time delays and it has a strong interaction between inputs and outputs. Ogunnaike-Ray column (OR) (Monica et al., 1988) is considered for the 3×3 system. The system and load transfer functions of this MIMO system are given in Table 1. In this paper, centralized PID controller structure is used for optimizing the IAE performance for set point regulation using CMAES and RGA.

5. CMAES implementation of multivariable PID controller

CMAES implementation of two inputs and two outputs Binary distillation column plant described by WB is described below. The centralized PID controller $K(s)$ for this system is given in (12).

$$K(s) = \begin{bmatrix} k_{11}(s) & k_{12}(s) \\ k_{21}(s) & k_{22}(s) \end{bmatrix}. \tag{12}$$

Table 1
System and load transfer functions.

System Transfer Functions

WB

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{1+16.7s} & \frac{-18.9e^{-3s}}{1+21s} \\ \frac{6.6e^{-7s}}{1+10.9s} & \frac{-19.4e^{-3s}}{1+14.4s} \end{bmatrix}$$

OR

$$G(s) = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.61e^{-3.6s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.01e^{-1.2s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix}$$

Load Transfer Functions

WB

$$G_L(1) = \frac{3.8e^{-8s}}{14.9s+1}, \quad G_L(2) = \frac{4.9e^{-3s}}{13.2s+1}$$

OR

$$G_L(1) = \frac{0.14e^{-12s}}{(19.2s+1)^2}, \quad G_L(2) = \frac{0.53e^{-10.5s}}{6.9s+1}, \quad G_L(3) = \frac{-11.54e^{-0.6s}}{7.01s+1}$$

In order to obtain the optimum performance, the parameters of $K(s)$ i.e., $[\theta_{11}, \theta_{12}, \theta_{21}\theta_{22}] = [k_{P_{11}}, k_{I_{11}}, k_{D_{11}}, k_{P_{12}}, k_{I_{12}}, k_{D_{12}}, k_{P_{21}}, k_{I_{21}}, k_{D_{21}}k_{P_{22}}, k_{I_{22}}, k_{D_{22}}]$ are optimized by optimization algorithms. The chromosome representation of $K(s)$ is given in Fig. 2.

All the elements of chromosomes of the population are randomly initialized within the search space specified by their lower and upper bounds of individual parameters for WB system as given in (Chang, 2007). The inequality conditions for the parameter ranges of centralized PID controllers for WB and OR systems are given in (13) and (14), respectively.

$$-1 \leq K_{P_{ij1}}, K_{I_{ij}}, K_{D_{ij}} \leq 1, \quad i,j = 1,2, \tag{13}$$

$$-5 \leq K_{P_{ij1}}, K_{I_{ij}}, K_{D_{ij}} \leq 5, \quad i,j = 1,2,3. \tag{14}$$

Unlike other EAs, CMAES requires only a seed vector to generate initial population using Gaussian sampling. The initial seed vector is selected randomly within its minimum and maximum limits of PID parameters. Initial coordinate wise standard deviation vector ($\vec{\sigma}$) is set at 0.25 times range of each coordinate. Offspring population is generated around initial seed vector by sampling a multivariate normal distribution with a known covariance matrix and overall standard deviation. The covariance matrix **C** determines **B** and **D**, and is adapted by means of the summation of weighted center of mass differences.

The population size (λ) is fixed at 50 for WB system and 100 for OR system. Tolerance values for objectives (*TolFun*) and coordinates (*TolX*) are assumed as 10^{-5} and 10^{-5}, respectively, for the last 20 generations. Original MATLAB code for CMAES algorithm is taken from the website (Hansen, 2006a).

6. Simulation results

In this paper, design of centralized multivariable PID controller for distillation column plant described by WB and OR, using CMAES is conducted. For simulating distillation column plant, MATLAB-SIMULINK software is employed. Simulations are carried out using Core 2 Duo Processor 2.2 GHz, 2 GB RAM PC. IAE is determined for step response of set point regulation. Simulations are carried out using a stopping criteria namely, Maximum Number of function evaluations (Fevalmax) with tolerance on design variables and objective function. Fevalmax is set at 10,000 functional evaluations for WB system and 20,000 for OR system. Tolerance is fixed as 10^{-5} for the last 20 generations. Owing to the randomness of the CMAES, many trials with independent population initializations is made to acquire a useful conclusion of the performance of the algorithm. Hence, best, mean, standard deviation of IAE measure and average functional evaluations in 20 independent trials are reported. For the purpose of comparison, recent version of RGA with SBX crossover is used for the design of centralized multivariable PID controller. Also, already reported controller parameters of BLT method is considered for comparison (Monica et al., 1988).

$k_{P_{11}}$	$k_{I_{11}}$	$k_{D_{11}}$	$k_{P_{12}}$	$k_{I_{12}}$	$k_{D_{12}}$	$k_{P_{21}}$	$k_{I_{21}}$	$k_{D_{21}}$	$k_{P_{22}}$	$k_{I_{22}}$	$k_{D_{22}}$

Fig. 2. Chromosome of centralized PID controller.

Table 2
Parameter selection.

Evolutionary algorithms	Parameter
RGA	$P_c = 0.8$ $P_m = 1/n$ $\eta_c = 5$ $\eta_m = 20$
CMAES (For OR system)	$\lambda = 100$ $c_\sigma = 0.7450$ $d = 2.9814$ $c_c = 0.3077$ $c_{cov} = 0.2798$

6.1. Parameter tuning

In CMAES algorithm, cumulation for step size (c_σ), damping for step size (d), cumulation for distribution (c_c), and change rate of the covariance matrix (c_{cov}) are determined using Eq. (8). The initial parameters of CMAES for OR system are reported in Table 2. Optimal parameter combinations for RGA is experimentally determined by conducting a series of experiments with different parameter settings before conducting actual runs to collect the results. The parameters actually used in the simulations are summarized in the Table 2.

6.2. Tuning of centralized PID controller

Best centralized PID controller and corresponding IAE value for 20 trials of PID controller design using CMAES algorithm for WB and OR systems are reported in Table 3. For the purpose of comparison, optimal controller obtained by RGA and already reported optimal controller by conventional BLT method (Monica et al., 1988) are given in the Table 3.

Table 3
Performance of centralized PID controller.

System	Method	Optimal controller	IAE
WB	BLT	$\begin{bmatrix} 0.03750 + \frac{0.0452}{s} & 0 \\ 0 & -0.075 - \frac{0.0032}{s} \end{bmatrix}$	23.5568
	RGA	$\begin{bmatrix} 0.7175 + \frac{0.2368}{s} + 0.7134s & -0.3262 - \frac{0.1204}{s} - 0.1957s \\ -0.3698 + \frac{0.0120}{s} - 0.0897s & 0.0143 - \frac{0.0175}{s} - 0.2415s \end{bmatrix}$	8.2021
	CMAES	$\begin{bmatrix} 0.6414 + \frac{0.2403}{s} + 0.7765s & -0.2525 - \frac{0.1320}{s} - 0.1738s \\ -0.5307 + \frac{0.0120}{s} - 0.2237s & 0.1220 - \frac{0.0188}{s} - 0.0594s \end{bmatrix}$	7.9251
OR	BLT	$\begin{bmatrix} 1.5070 + \frac{0.0920}{s} & 0 & 0 \\ 0 & -0.2930 - \frac{0.0163}{s} & 0 \\ 0 & 0 & 2.6326 + \frac{0.3975}{s} \end{bmatrix}$	245.4433
	RGA	$\begin{bmatrix} -0.1634 - \frac{0.1248}{s} + 3.0976s & 0.1761 + \frac{0.2669}{s} + 1.1934s & 0.1165 - \frac{0.0375}{s} + 0.0411s \\ 0.9584 + \frac{0.4403}{s} - 1.0962s & -0.9027 - \frac{0.3797}{s} - 0.5727s & -0.0094 + \frac{0.0762}{s} + 0.1208s \\ -2.4719 - \frac{4.8952}{s} - 1.5357s & -4.1153 - \frac{2.7327}{s} + 4.8441s & 4.7839 + \frac{4.2005}{s} + 4.6310s \end{bmatrix}$	138.3504
	CMAES	$\begin{bmatrix} 0.9809 + \frac{0.3500}{s} + 1.5661s & -0.6980 - \frac{0.3887}{s} - 0.8211s & 0.0218 - \frac{0.0097}{s} - 0.0201s \\ 0.3516 + \frac{0.2091}{s} - 1.5712s & -0.6386 - \frac{0.4787}{s} - 1.3162s & -0.0206 - \frac{0.0125}{s} + 0.0032s \\ 0.6374 + \frac{2.5495}{s} - 0.7100s & -0.0822 - \frac{3.5826}{s} - 0.8898s & 4.9210 + \frac{5.0}{s} + 4.1608s \end{bmatrix}$	38.4445

Time response of WB system with best PID controller obtained by CMAES is shown in Figs. 3 and 4 for output response y_1 and y_2, respectively. Time responses of obtained using CMAES and RGA are nearly equal and better than BLT method. IAE value obtained by CMAES method is lesser than other methods. Time responses of OR system with best PID controller obtained by CMAES method are shown in Figs. 5–7 for output response y_1, y_2 and y_3, respectively. Time response of OR system using CMAES optimized controller is better than RGA and BLT methods in terms of very low peak overshoot for outputs y_1 and y_3, less rise time for output y_2 and y_3, quick settling time for all outputs and also gives less IAE value.

The statistical performance such as best, mean, standard deviation of IAE and Average Functional Evaluations (AFE) of twenty independent trials of CMAES and RGA for the design of centralized PID controller for WB and OR systems are given in Table 4.

For WB and OR systems, the performance of CMAES is better than RGA in terms of IAE and standard deviation in IAE values in 20 trials. All the 20 trails, both algorithms found the stable controller for WB system. Owing to the increased number of parameters to be optimized, OR system is more complex. For OR problem, only CMAES algorithm is able to find the stable controller in all trails. This is very clear from the results of mean and best IAE value for the 20 trails. RGA algorithm is able to give stable controller only once in 20 independent trials. For the remaining trials, the obtained IAE value is very large due to unstable operation. From these experiments, it is very clear RGA fails to give good performance for complex systems but CMAES algorithm is able to give consistent performances for even complex systems like OR.

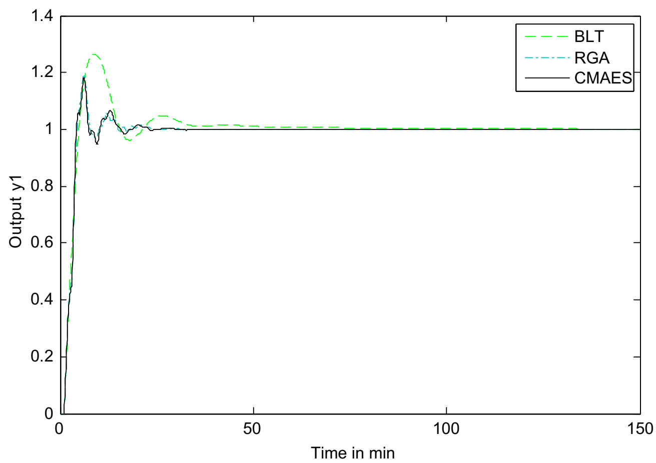

Fig. 3. Output response y_1 for WB system.

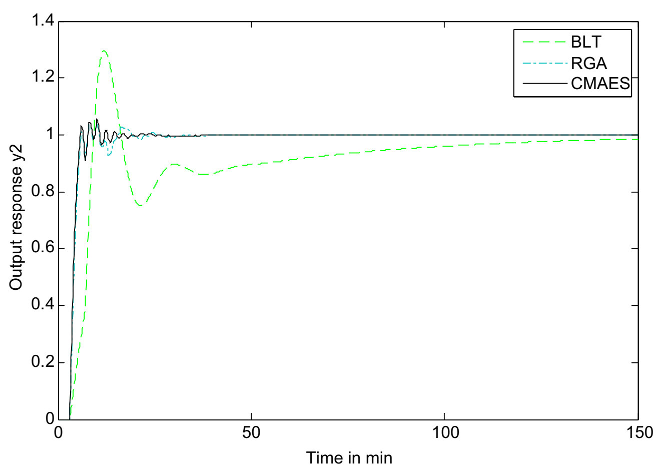

Fig. 4. Output response y_2 for WB system.

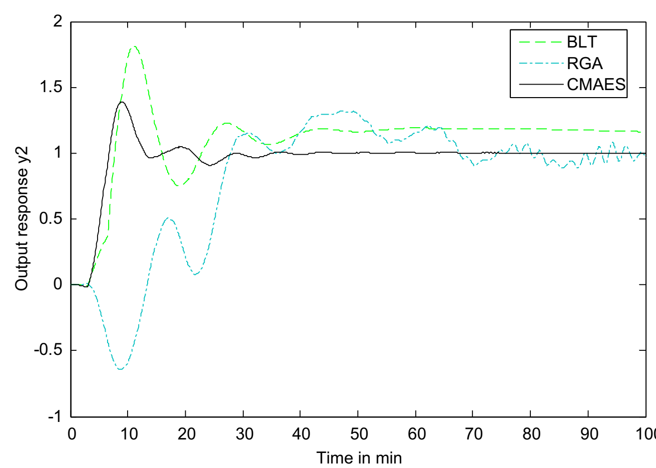

Fig. 6. Output response y_2 for OR system.

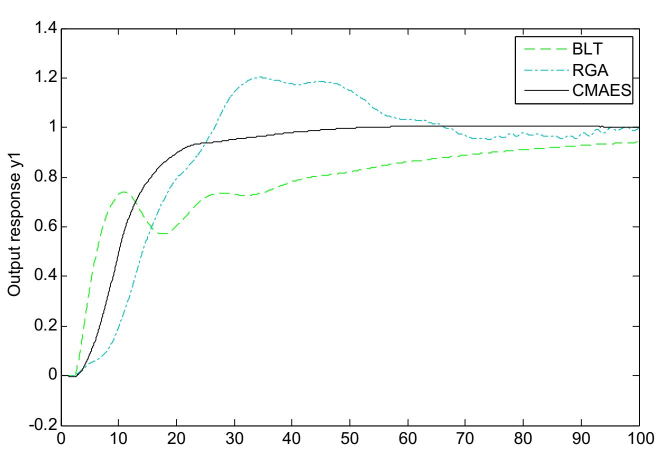

Fig. 5. Output response y_1 for OR system.

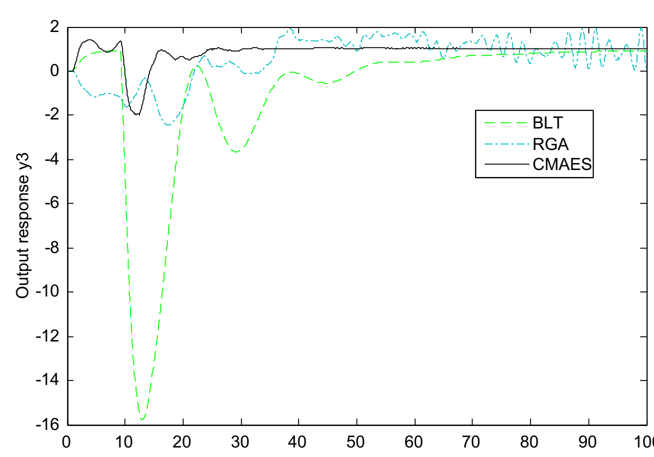

Fig. 7. Output response y_3 for OR system.

Table 4
Statistical performance of CMAES and RGA-centralized PID controller.

System	Method	Best value	Mean value	Standard deviation	Average functional evaluations
WB	RGA	8.2021	29.2679	36.5185	9801
	CMAES	7.9251	7.9251	0	10,016
OR	RGA	138.3504	6.04e+08	1.4579e+09	19,415
	CMAES	38.4445	41.4622	3.1033	20,002

Figs. 8 and 9 show the convergence characteristics of CMAES and RGA for WB and OR systems, respectively. Higher values of IAE during initial generations/iterations indicate unstable PID controller settings. For convenience, IAE for the unstable response is limited to 1000. Owing to the self learning behaviour of CMAES algorithm, convergence characteristic shows large variations during initial search. From the figures it is clear that convergence of CMAES algorithms is faster than RGA and also converges to less IAE value. Even after 20,000 function evaluations, RGA is not converged to a stable value. In order to find requirement of additional functional evaluations in RGA, two additional experiments were conducted.

At first, RGA algorithm is applied for designing centralized PID controller for OR system with 1,00,000 function evaluations with 100 population size for 10 runs. Even after this only two runs produced stable controller values with the minimum IAE value of 88.350. Secondly, for the same problem, RGA with 1,00,000 with 200 populations for 10 runs, produced three stable controller values, with the minimum IAE value of 52.559. From these experiments it is clear that, CMAES algorithm is more suitable for designing centralized PID controller for higher order system due to self-adaptive learning feature of CMAES algorithms.

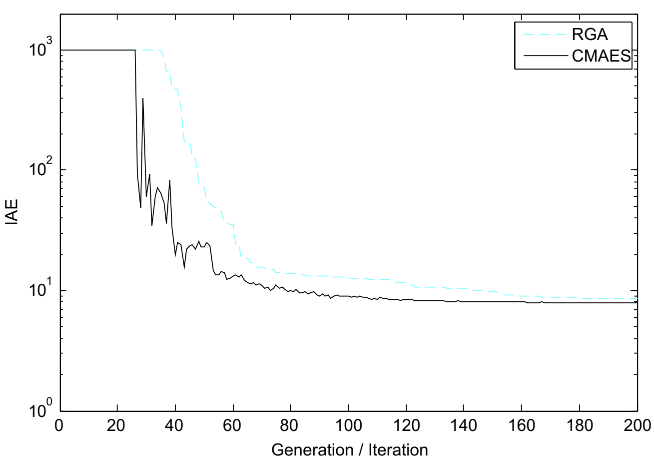

Fig. 8. Convergence characteristics for WB system.

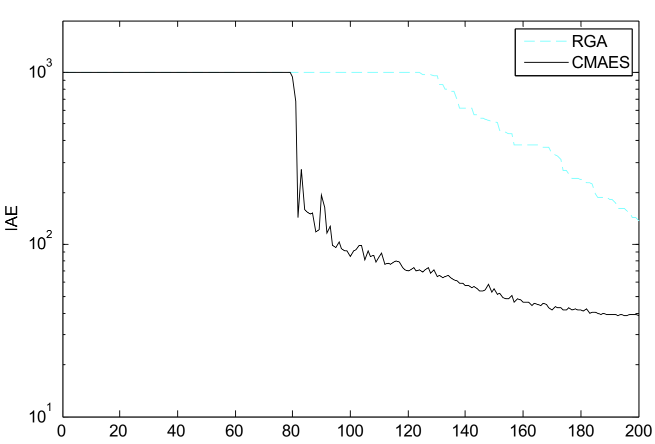

Fig. 9. Convergence characteristics for OR system.

6.3. Robustness against load disturbance

In order to validate the performance of multivariable PID controller designed using CMAES algorithm for the load disturbance rejection, an experiment with unit step load change is carried out. IAE value for this experiment for WB and OR systems are reported in Table 5. For WB system, centralized controller designed using CMAES and RGA are more robust against load disturbance as compared to BLT method. But for OR system, controller designed using CMAES is more robust than RGA and BLT methods. Fig. 10 shows the load regulation of output y_3 for OR system with optimal controller obtained using CMAES, RGA and BLT methods. This figure clearly indicates the better load disturbance rejection capability of CMAES designed controller compared to others.

6.4. Robustness against parameter variation

In order to determine the robustness of optimal multivariable PID controller designed using CMAES against parameter variation, servo and load response are conducted with +20% increase in gains and dead times of WB and OR systems. The obtained IAE values for both responses for WB and OR systems are reported in Table 6. For WB system, robustness against parameter variation of CMAES and RGA designed controllers are better than those designed by BLT method. On the other hand, for OR system, controller designed by CMAES is more robust against parameter variation than those designed by RGA and BLT methods. For the demonstration purpose of robust performance of CMAES designed centralized PID controller against parameter variation, servo response y_1 for OR system and load response y_2 for OR system with

Table 5
Performance of load disturbance rejection.

System	Method	IAE
WB	BLT	96.4342
	RGA	38.4828
	CMAES	37.1756
OR	BLT	83.9726
	RGA	123.0422
	CMAES	13.5221

Table 6
Robustness against +20% parameter variations.

System	Method	IAE-servo response	IAE-load response
WB	BLT	25.3159	96.9645
	RGA	11.4859	38.4808
	CMAES	12.9005	42.7898
OR	BLT	288.0416	91.7775
	RGA	6.521e+03	4.967e+03
	CMAES	158.0199	24.9614

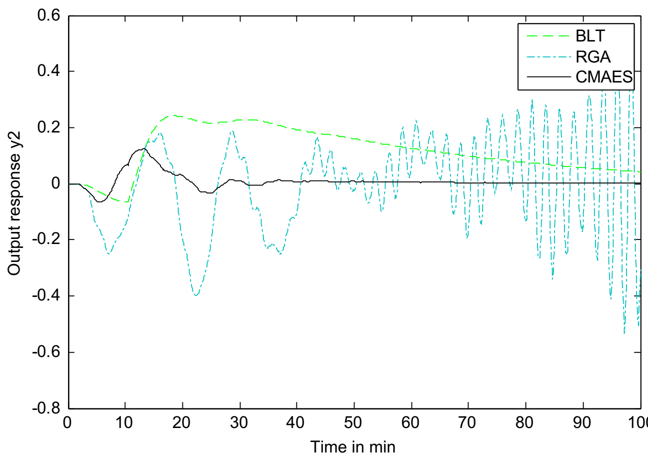

Fig. 10. Load regulation response y_2 for OR system.

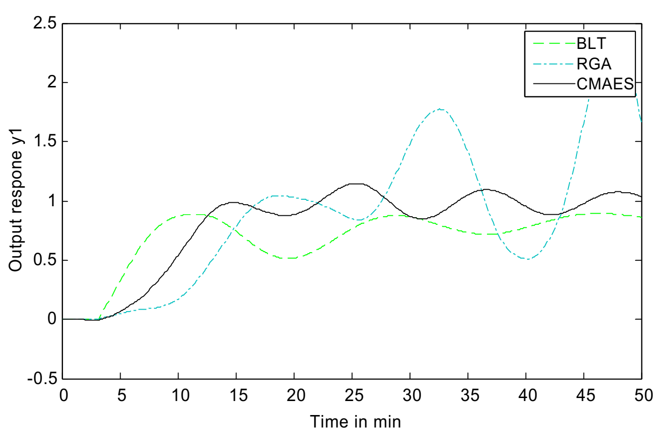

Fig. 11. Servo response y_1 for OR system with +20% increase in gain and dead time.

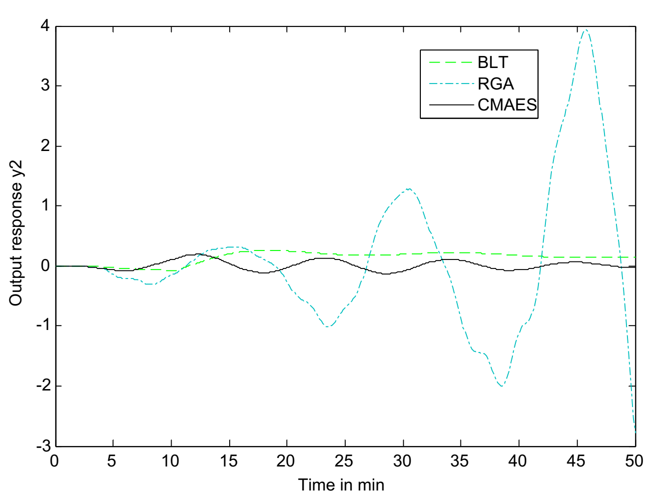

Fig. 12. Load response y_2 for OR system with +20% increase in gain and dead time.

+20% increase in gain and dead time are shown in Figs. 11 and 12, respectively.

7. Conclusions

The CMAES algorithm is applied to design the centralized PID controller for multivariable system by minimizing the IAE value for servo response. WB and OR systems have been employed to illustrate the applicability of the CMAES algorithm. The performance of CMAES algorithm is compared with RGA, and already reported BLT method. The best, mean, standard deviation of IAE value obtained from 20 independent trials are considered for checking the performances of CMAES and RGA. Simulation results reveal that the CMAES algorithm performs better than other algorithms in terms of best and mean IAE value and consistency. For both OR and WB systems, CMAES designed centralized PID controller is more robust against model uncertainty and load disturbance.

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