

## A TWO-STEP APPROACH TO CONCEPTUAL DESIGN OF MECHANICAL DEVICES

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**Abstract.** In this paper, a two-step approach to conceptual design of mechanical devices and machines is described. The first step is to synthesise, using a set of basic elements and their combination rules, a set of solution concepts, in terms of their topological as well as spatial configurations, which satisfy an instantaneous part of the required temporal function. In the second step, each such solution will undergo a temporal reasoning process so that it can be worked out (i) whether or not the solution can satisfy the complete function, and if it does, (ii) what interfaces are required between its elements. To support the above steps, (i) an instantaneous representation of the basic elements in terms of their topological and spatial characteristics, and (ii) a temporal representation of these elements in terms of their 'sequence diagrams', have been developed. These representations and the reasoning processes under development for carrying out each of these steps, along with implementation issues and results, are discussed with examples.

### 1. Introduction

Engineering design may be considered the activity of recognising and transforming a perceived need into the description of a technical means that can satisfy the need; this description can be used subsequently to realise the design. Research presented in this paper is focused on the development of computational systems for generating topological and spatial configurations of design concepts for mechanical transmission devices and systems from functional requirements of design problems, and is an extension of the work in Chakrabarti, (1991), part of which was reported in Chakrabarti and Bligh (1992). Design problems for mechanical transmission systems can be usually expressed using time-varying inputs and outputs. The goal is to develop representations and synthesis processes for generating design solution-concepts to satisfy time-varying functional requirements, using a knowledge base of basic components and rules of combination. The approach to synthesis has two principal steps, see Figure 1. One is to develop the necessary representations and processes using which, given a time-varying transmission design problem, abstract spatial configurations of solution concepts to solve an instantaneous part of the problem could be generated from a given

knowledge base of components and combination-rules. The second is to develop the necessary representations, knowledge base and processes, which can be used to ensure that the solutions generated above satisfy the complete time-varying functions of the problem. In this paper, approaches taken to support these steps are presented with examples.

## 2. Instantaneous Synthesis

In transmission design, devices are designed to amplify forces, transmit torques, etc. In other words, the functional requirements are to transmit and transform forces and motions. This can be expressed as a transformation among a set of input characteristics and a set of output characteristics; each of these characteristics may be required to change with time. At one instant of time, the characteristics of the input (or output) constitutes the input (or output) characteristics. The transformation at an instant between the input and the output characteristics is an instantaneous transformation; an ordered set of instantaneous transformations can be used to express the overall functional requirements of a problem.

A solution concept is an abstract description of a system of identifiable individual elements which can satisfy given functional requirements. For instance, one concept, for transmitting a force on the same plane but into a different direction and location, could be a system where an input rack takes the input force, rotates an intermediate pinion, which then moves an output rack in the required direction to provide the required output force.

Known simple devices were analysed from their input-output point of view to identify their energy transforming elements, the contribution of these elements to the overall input-output transformation of the device, and the rules for combining these elements so that the device can work the way it does. Following the power flow path from the input to the output points, these transmission elements were identified. These elements can be viewed as transformers which transform some characteristics of the inputs and outputs. For instance, in a bicycle drive, which transforms a foot-force into a rear-wheel rotation, the element connecting the paddle to the front sprocket transforms only the position of the I/O variables, moving it from the paddle-push point to the centre of the sprocket, while the part of the sprocket connecting its centre to the chain transforms both the position and the kind (from force to torque) of the I/O variables. It is noted that when two transformers are connected to each other, the energy receiving transformers take the outputs of the other transformers as their inputs. The rules of combination of these transformers therefore are: (1) a set of transformers can be connected only by connecting the inputs of one set of transformers with the outputs of the other; and (2) a connection is possible only when the inputs involved in the connection would have the same characteristics as the outputs at that connection.

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Now, given a system of connected transformers, where the transformer-transformations (which transformer does what) and the connections (which transformer's input is connected to which transformer's output) involved are known, it is possible to analyse that system to: (1) check if it is a valid system, i.e., whether the connections are valid; and (2) identify, for a valid system, its transformations, i.e., the transformations between the characteristics of its I/O variables.

The instantaneous transformation of a given system can be deduced using the information about its constituent transformers, connections, and their rules of combination. Knowledge, involved in the description above, is formally represented next, so that procedures can be written to synthesise systems that would satisfy a given instantaneous transformation.

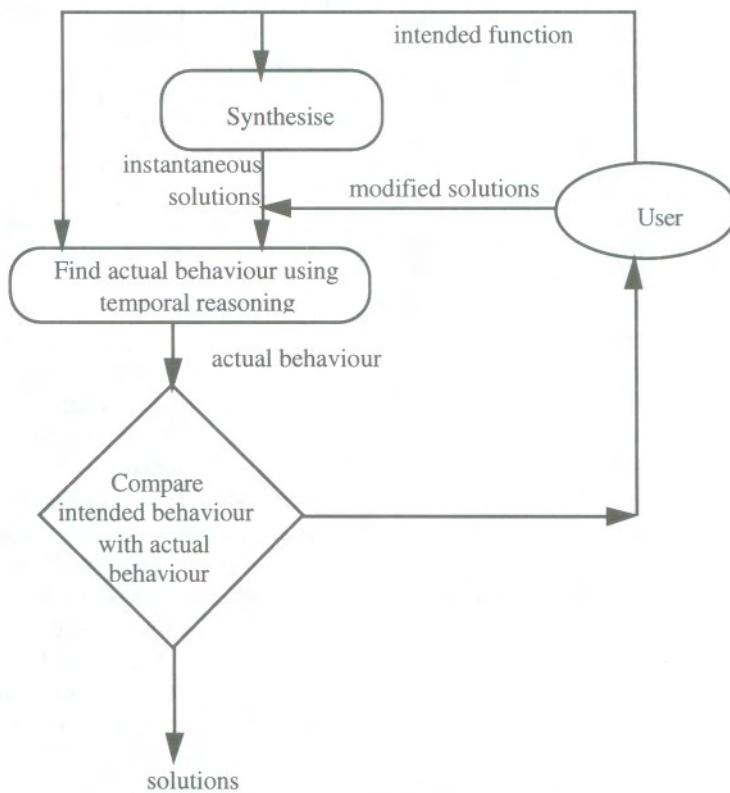


Figure 1. The conceptual design approach.

## 2.1. REPRESENTATION OF DESIGN PROBLEMS

An instantaneous multiple input-output design problem can be viewed as a transformation between the characteristics of a set of input vectors and a set of output vectors. A vector would have a kind, an orientation in space, a sense of

its orientation, a magnitude and a position in space associated with it. The possible ways a vector can be spatially located is potentially infinite. To keep this synthesis problem sufficiently tractable, only the orthogonal orientations (along i, j and k axes only) of these vectors, and concept configurations, are considered. Consequently, the design problem-representation constructs are:

<b>Input1</b>	<b>Output1</b>
kind: (force/torque/lin. vel./ang. vel.)	kind: (force/torque/lin. vel./ang. vel.)
orientation: ( $i / j / k$ )	orientation: ( $i / j / k$ )
sense: (+ / -)	sense: (+ / -)
magnitude: (some number)	magnitude: (some number)
position: ( $x_1i + y_1j + z_1k$ )	position: ( $x_2i + y_2j + z_2k$ )
<b>Input2</b>	<b>Output2</b>
kind: (force/torque/lin. vel./ang. vel.)	kind: (force/torque/lin. vel./ang. vel.)
orientation: ( $i / j / k$ )	orientation: ( $i / j / k$ )
sense: (+ / -)	sense: (+ / -)
magnitude: (some number)	magnitude: (some number)
position: ( $x_1i + y_1j + z_1k$ )	position: ( $x_2i + y_2j + z_2k$ )
...	...
...	...

where  $i$ ,  $j$  and  $k$  are unit vectors in a rectangular space coordinate system, and lin.vel. and ang.vel. stand respectively for linear and angular velocity.

## 2.2 REPRESENTATION OF SOLUTION CONCEPTS

The solution elements are vector transformers which transform a set of input vectors into a set of output vectors. The I/O points of the vectors associated with a transformer specify the positions of these vectors in space, and the spatial separation between these I/O points becomes the position transformation of the transformer. A vector transformer is represented by a 3-tuple of vectors: a set of input vectors  $I$ , a set of output vectors  $O$ , and a set of length vectors  $L$  (created to explicitly reason about the position changes involved in a solution), see Figure 2. An I-vector or an O-vector has a kind, orientation, sense, magnitude and position, while an L-vector has an orientation (given by the line joining the positions of the I- and O-vectors), sense (directed from the input point towards the output point), magnitude (spatial separation between the input and the output points) and a position. These characteristics are variously coupled, depending on the characteristics of the specific transformer involved. Within the confines of orthogonality restrictions, the possible spatial relations between these vectors are shown in Figure 3. This is based on the fact that the spatial relationship between two vectors (i.e., input and output) can be expressed using a combination of two parameters: (i) whether or not they are parallel, and (ii) whether they intersect or not. Using appropriate values of these vector characteristics, known transformers can be classified in terms of a set of kind transformations, orientation transformations, sense transformations etc., where a transformation

is a compatible set of values of the considered characteristic of the three vectors of the transformer. So, a shaft would be a type PI (parallel & intersecting) element (Figure 3) having torque as the input and output kind, having orientation transformations including (i i i) where all the vectors are oriented along the i vector, and having sense transformations including (+ - +) which in conjunction with the orientation transformation mean that its I-, L- and O-vectors are directed in positive i, negative i, and positive i respectively. The position transformation of a single transformer is given by the characteristics of its length vector(s). The magnitude transformation is defined as the relationship between the magnitude of the O-vector and that of the I-vector and is governed by physical principles/constraints constituting the behaviour of the transformer, which include the constant power flow criterion. In the case of a shaft, for instance, this relation is 1, i.e., the input and output torques are of the same magnitude.

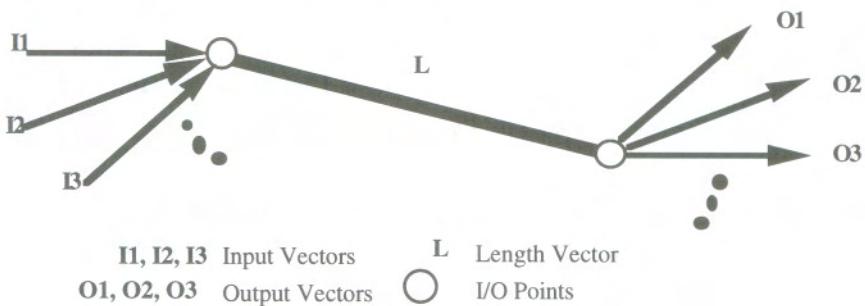


Figure 2. Representation of a general transformer.

### 2.3. PROCEDURES AND AN EXAMPLE

The problem considered here is to generate solutions to an instantaneous requirement of a transmission design problem expressed as a transformation between an input of specified characteristics and an output having specified characteristics. The approach taken is to solve one part of it at a time, while moving from the general to the specific requirements. Kind synthesis (i.e., synthesising solutions which will provide only the I/O kinds of the design problem) is first considered. Each solution produced above is then evaluated for orientation, thereby configuring some of its valid orientations. These orientations are then checked for the sense requirements, and valid sense configurations are computed. The magnitude and position requirements are used as constraints to evaluate these solutions for validity at appropriately informed phases.

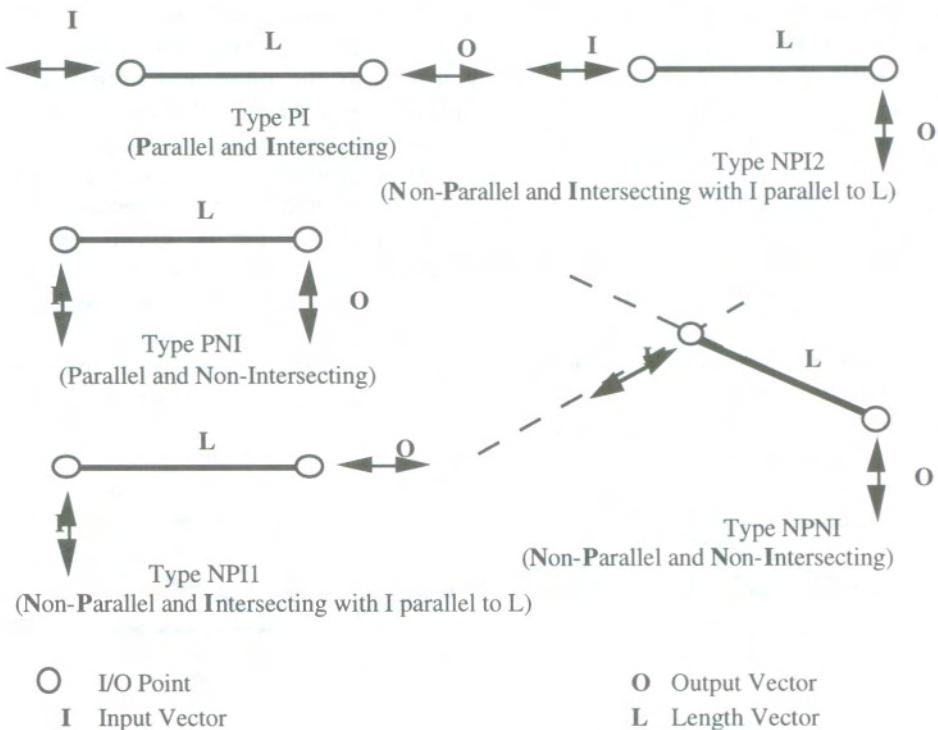


Figure 3. Various possible spatial relations between the i/o vectors of a transformer.

The kind synthesis procedure synthesises solution concepts, using transformers and their rules of combination from a known set, to a given design problem of transforming an input of a given kind to an output of a desired kind (e.g., force to torque). We view the problem as a search problem, where there are defined initial (input kinds) and goal (output kinds) states, and the problem is to move from the initial state to the goal state using valid operators (known transformers) that change the state of the problem. The resulting solutions should be causal networks of operators connecting the given input kinds to the desired output kinds. The termination of this process is ensured by specifying the maximum number of operators that can be used in producing a solution. For instance, suppose we want to devise solution concepts to the problem of using a small hand force for locking and unlocking a toilet door and providing a signal. To illustrate how the various procedures discussed in this section work, this problem will be solved through the rest of this section. There are two problems. One is (*un*)locking of the door, and the other, *indicating* to people outside that it is (*un*)locked. Suppose we have already decided that the door would be locked by inserting a slider into a slot, and the indication (whether it is locked or not) would be

given by bringing into people's view some suitable signal (such as a colour code, as often found in houses in England). Let us also suppose that it is decided that the input would be a couple, i.e., a pair of equal and opposite non-collinear forces. Using the problem representation constructs that our synthesis procedures can recognise, we can represent this problem as a transformation problem between two input forces (and associated motion) and two output linear motions (one for locking, and the other for indication). The function of the problem at an instant can be described as an instantaneous transformation between two input forces and two output linear motions. A complete specification of this problem would be:

**Input-1**

Kind: force  
Orientation:  $\mathbf{k}$   
Sense: +  
Magnitude: magnitude-1  
Position:  $(x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k})$

**Output-1**

Kind: linear motion  
Orientation:  $\mathbf{i}$   
Sense: +  
Magnitude: magnitude-3  
Position:  $(x_3\mathbf{i} + y_3\mathbf{j} + z_3\mathbf{k})$

**Input-2**

Kind: force  
Orientation:  $\mathbf{k}$   
Sense: -  
Magnitude: magnitude-2  
Position:  $(x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k})$

**Output-2**

Kind: linear motion  
Orientation:  $\mathbf{i}$   
Sense: -  
Magnitude: magnitude-4  
Position:  $(x_4\mathbf{i} + y_4\mathbf{j} + z_4\mathbf{k})$

Now starting with the kind synthesis of the problem, the output of the procedure would be a list of causal networks, each of which could be represented as a directed graph of a set of structures. For instance, for a suitable knowledge base, if  $r$ , the maximum allowable number of operator, is set as 5 for this problem, one (see Figure 4) of the solutions produced would be:

The solution:

(lever-1 input force-1  $\rightarrow$  intermediate torque)  
(lever-2 input force-2  $\rightarrow$  intermediate torque)  
(lever-3 intermediate torque  $\rightarrow$  intermediate force)  
(lever-4 intermediate torque  $\rightarrow$  output force-2)  
(tie-rod-1 intermediate force  $\rightarrow$  output force-1)

Any of these solution concepts must also satisfy that the input and the output should have some orientations in space. The problem of orientation configuration is to produce the possible configurations of the solutions. This could be viewed as a constraint propagation problem. The input and the output points of a solution (i.e., a graph) are constrained to have the required input and output orientations. The task is to find, from the knowledge base of possible orientation transformations for all the transformers, of the solution-graph, that are compatible with the specified orientation constraints. As an example, we consider the orientation requirements of the problem discussed in the kind synthesis example, and try to configure the orientations (if any) for a solution generated there. Here the problem is:

Orientation Transformation:      Orientation of input-1  $k \rightarrow$   
     Orientation of output-1  $i$ ,  
     Orientation of input-2  $k$   
     Orientation of output-2  $i$ .

The output of this procedure, for the above solution concept, would be a list of a set of ordered sequences of the I-, L- and O-vector orientations of the structures constituting the concept, which could be used for producing directed graphs or sketches of the orientations of the concept. The solution in Figure 4, for instance, can be oriented in only one way, which is represented, using the above information generated by the orientation configuration procedure, in the graph and the graphical representations shown in Figure 5a and b respectively.

The sense configuration procedure is similar to the orientation configuration procedure, and for the above problem, if a positive input-1 and a negative input-2 is to be transformed into a positive output-1 and a negative output-2, on the orientation configuration of Figure 5, the number of direction configurations (which consists of orientation and sense information) produced is 4, two of which are shown using the graph and the graphical representations in Figure 6a and b respectively. The sense configuration 2 in Figure 6 of the solution in Figure 5 can be taken as an abstract representation of the spatial arrangement of a solution which still can be seen in use in many old houses in England (see Figure 8a for a schematic diagram of this solution).

During the conceptual design phase, there is no way of synthesising solution concepts using specified magnitude or position requirements. These requirements therefore would be used as constraints, usually in more detailed phases, to check the validity of candidate solution concepts.

### 3. Temporal Reasoning

The solutions generated by the above synthesis programs are ensured to work for the instant for which they are synthesised. In order to ensure, in a qualitative sense, that they also have the potential of satisfying the function for the other instants of time, we need a qualitative temporal reasoning tool which will allow one to work out (i) whether or not a given concept has this potential, and (ii) if it does, the kind of interfaces required between the elements of the solution to achieve this.

In order to have such a temporal reasoning tool, we need a temporal representation of the elements and their functions. This representation should be such it should make explicit (i) the motions of an element as a given input is fed into it, which helps work out how its output would behave, and, (ii) relative motions between elements, which determines how the interface between them should be.

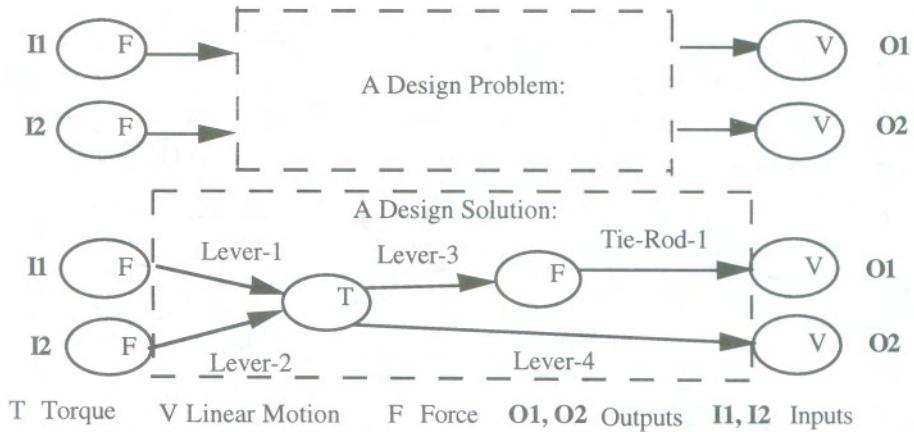
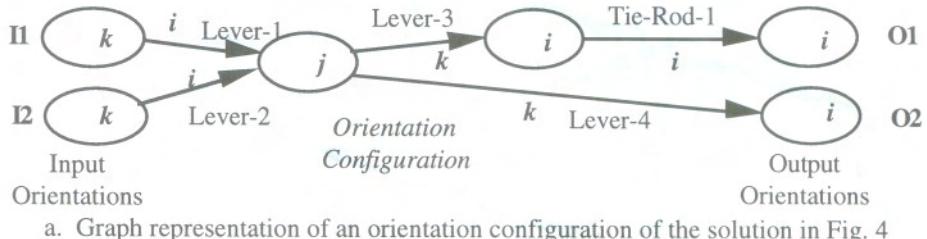


Figure 4. A kind synthesis example: the toilet-door lock problem and one of its solutions.



a. Graph representation of an orientation configuration of the solution in Fig. 4

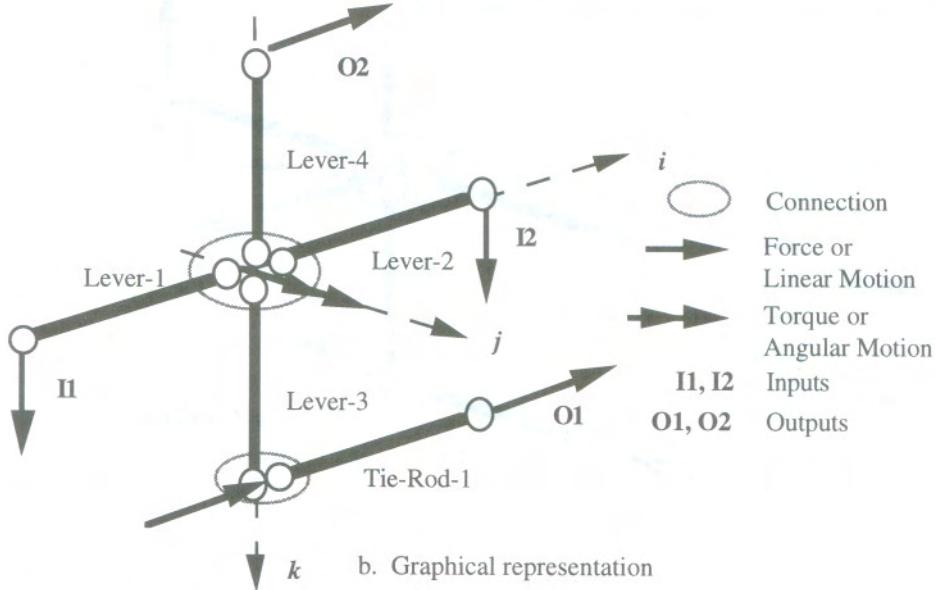


Figure 5. Representations of one orientation configuration of the solution in Figure 4.

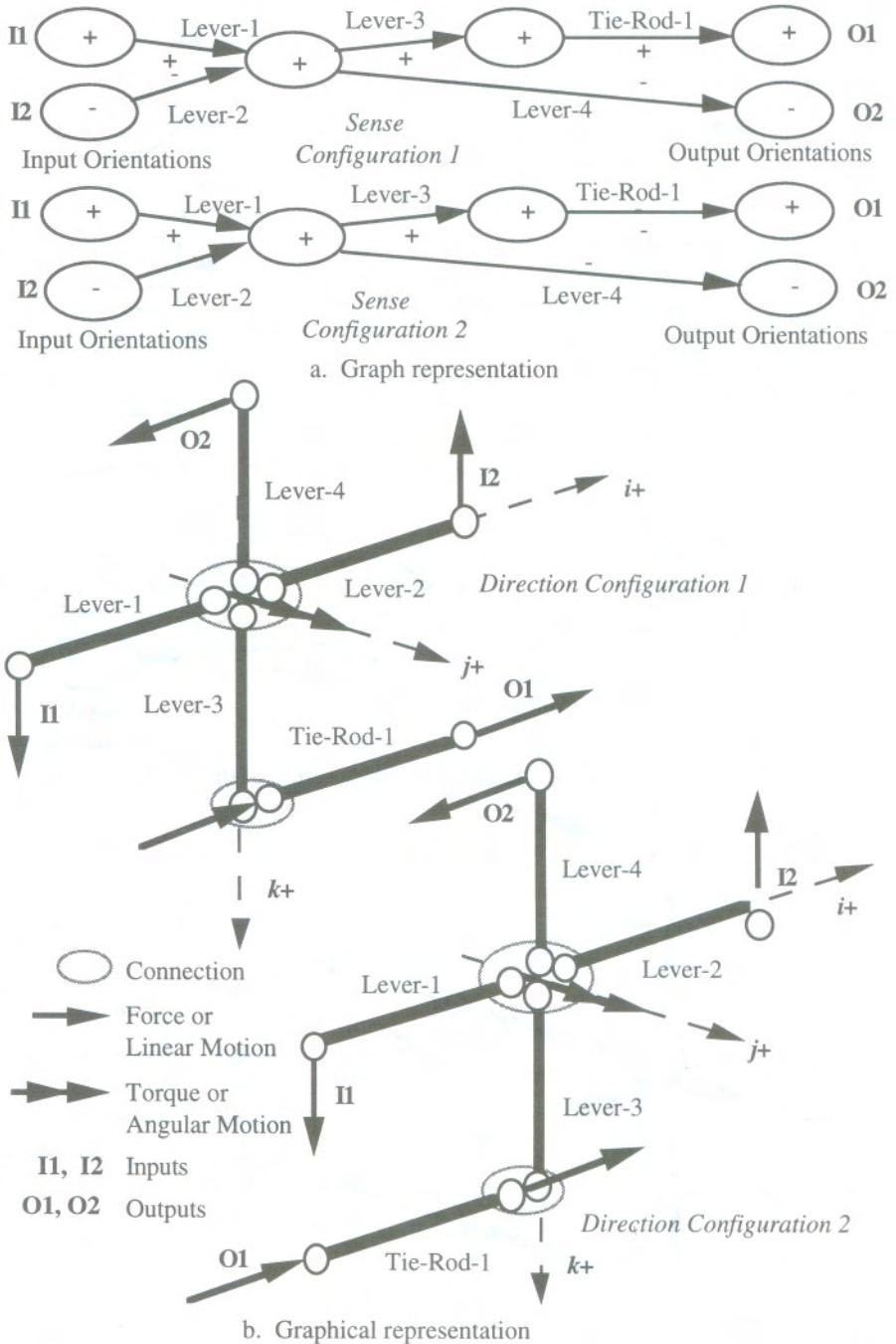
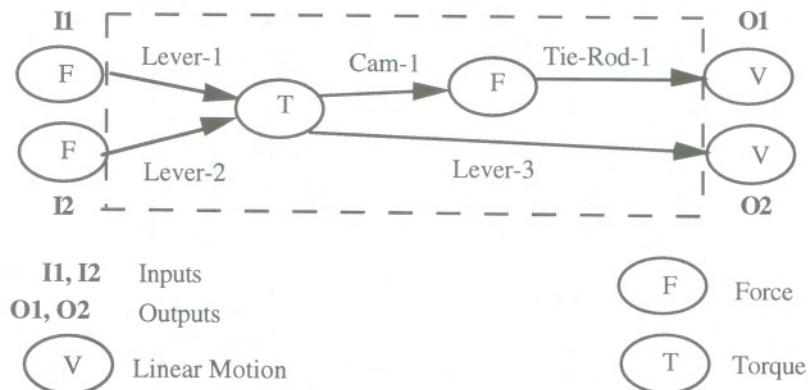
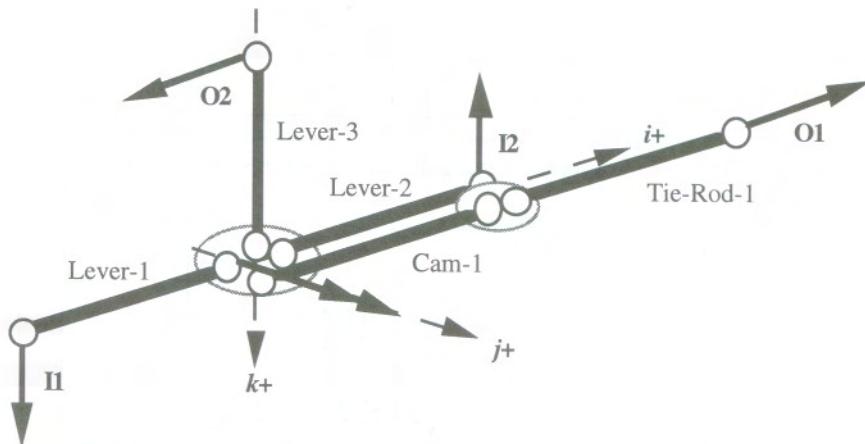


Figure 6. Representations of two direction configurations of the orientation configuration in Figure 5.



a. A new design solution to the problem in fig. 4



b. One direction configuration to the above solution

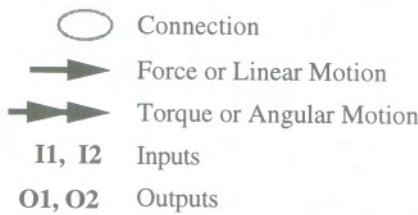
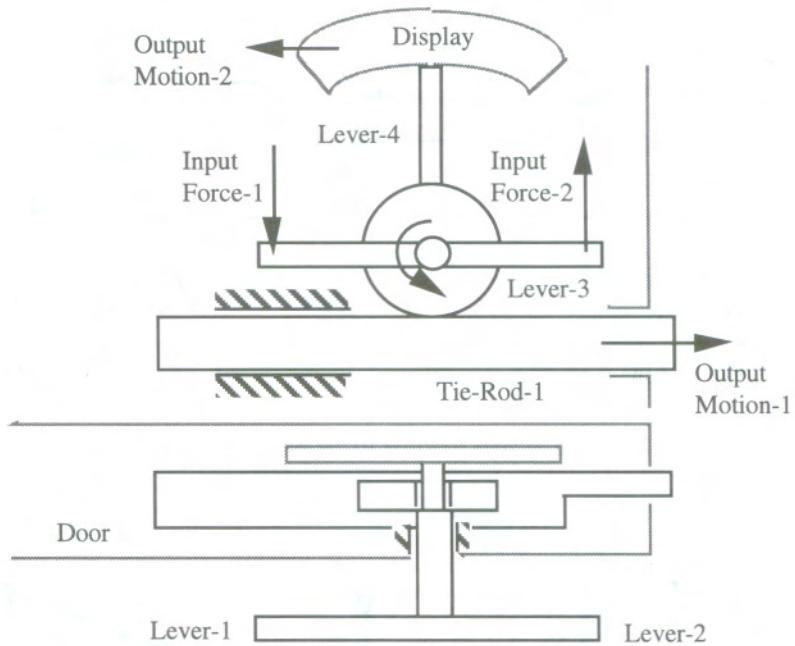
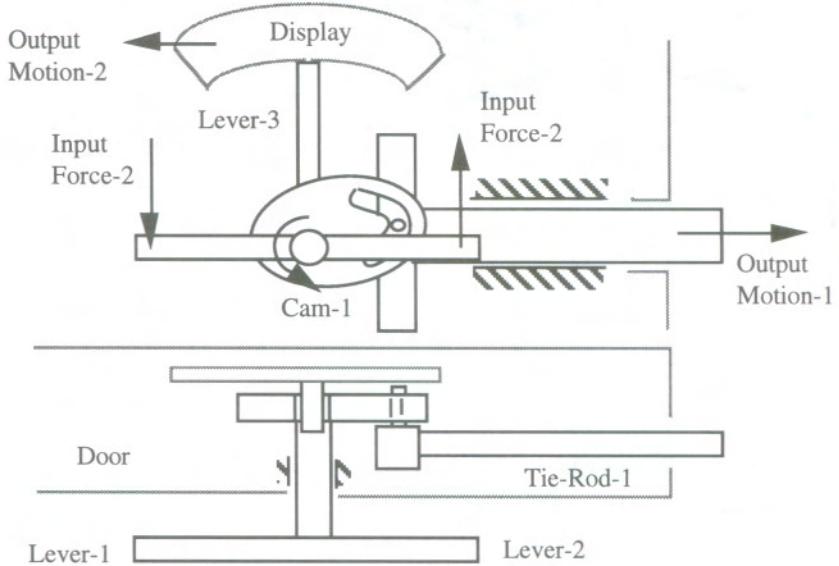


Figure 7. Another solution to the toilet door lock problem in Figure 4.



a. A schematic of the solution having direction configuration 2 in fig. 6



b. A Schematic of the Solution Shown in Fig. 7

*Figure 8. Schematic representations of an existing solution and a new solution generated by the synthesis procedures, to the toilet door-lock problem.*

*Sequence diagram* is a representation which has been developed for these purposes. This is inspired by the configuration space approach (Lozano-Perez, 1983) where various configurations that an object can take is represented in a multi-dimensional space. The difference is that in sequence diagram, the various configurations that an element can take, for a given sequence of inputs, is placed on a sequence axis. For instance, consider the motion of a lever (a type NPNI element as in Figure 2) for a continuous input. As a result of continuous rotation, the input point of the lever always remains stationary (i.e., on a straight line parallel to the sequence axis), while its other points move in concentric circles (thereby remaining on helical curves of various diameter in the sequence diagram). The sequence diagram for a lever in continuous , therefore, is a helical surface, as shown in Figure 9. Similarly, for a tie rod which can move only along its axis (a type PI element, see Figure 2), its sequence diagram for a continuous unidirectional motion is given by a plane surface, see Figure 10.

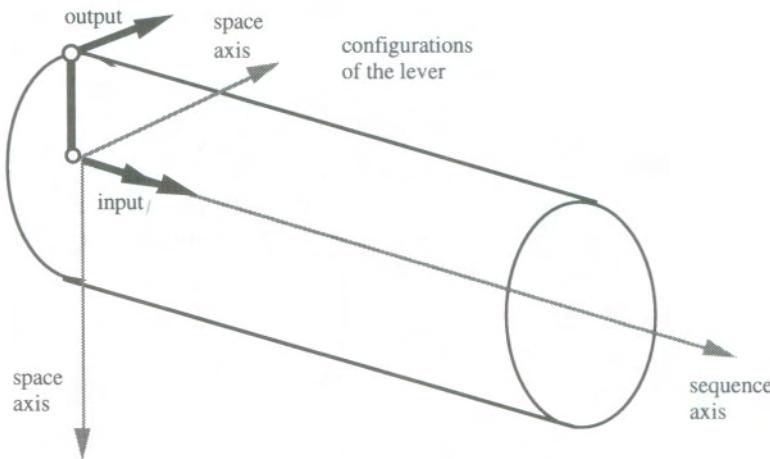


Figure 9. Sequence diagram for a type NPNI lever with continuous rotation as input.

For a given element and an input motion in a given direction, there are three basic sequence diagram constructs: one each for whether the motion is positive, negative or zero. For instance, for a type PI tie-rod, where a translation is the input, the basic sequence diagram constructs for a given direction are shown in Figure 11.

Once the constructs for the set of elements comprising a given solution is available, the given input to the solution can be used to develop the complete sequence diagram of the input element, and the output of this element can be used, as an input, to construct the sequence diagrams for the elements connected to it. Continuing this for all the elements of a solution would complete the temporal reasoning process; the sequence curves for the output points of the output elements of the solution would be the temporal outputs

that the solution would generate if each element preserved its individual behaviour. As an example, consider the design of a device for transforming a continuous rotation into an oscillatory translation. Let us assume that the instantaneous synthesis procedure has generated, amongst other solution, the solution shown in Figure 12. Here, a lever takes an instantaneous rotation and transforms it into a linear motion before transmitting it to a tie-rod element of type PNI; this tie-rod is connected to a axially constrained tie-rod (of type PI) which produces an instantaneous translation at an output point. Taking continuous rotation as the temporal input, the sequence diagram for the input element (the lever) can be expressed using only its basic diagram construct for increasing motion, and is a helical surface. The output point of the lever is the helical curve of the largest diameter, and should always be in contact with the type PNI tie-rod. In other words, the sequence diagram surface of this tie-rod should contain this helical curve. Thus, using this contact constraint, it can be worked out (i) whether or not constant contact is a possibility here, and if so, (ii) what the sequence diagram of the tie-rod should be. In this case, all the sequence constructs of the type PNI tie-rod would be in use, and should produce the 'wiggly fence' containing the helical curve (see Figure 13a) as its sequence diagram. The sequence diagram of the third element (i.e., type PI tie-rod) can be similarly found from the curve of the output point of the type NPI tie-rod, and would look like the wiggly plane in Figure 13b. The curve of its output point is the overall temporal output of this solution, and is given in Figure 13c. The interface requirements between two interfacing elements can be worked out by comparing the sequence diagrams of the elements in the direction of motion in a given direction.

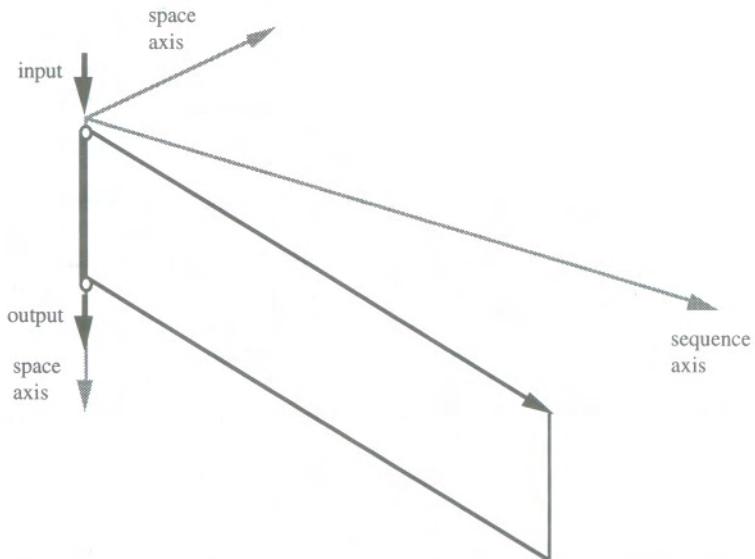


Figure 10. Sequence diagram for a type PI tie-rod in continuous translation.

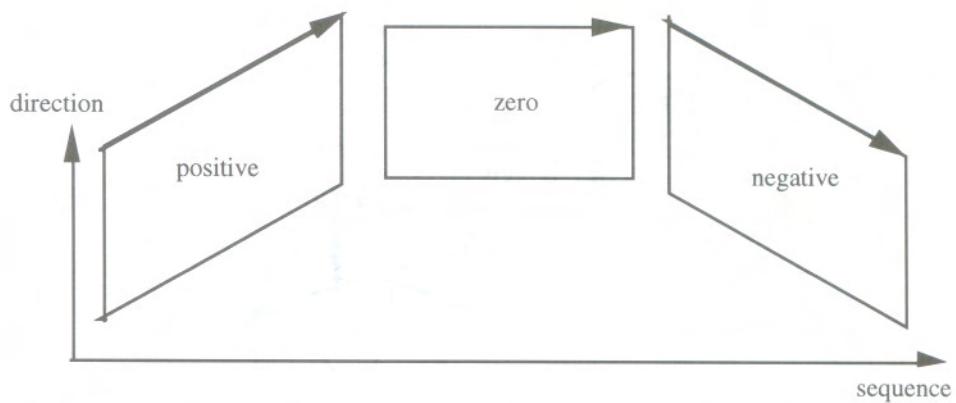


Figure 11. Basic sequence diagram constructs for a type PI tie-rod element for motion in the given direction.

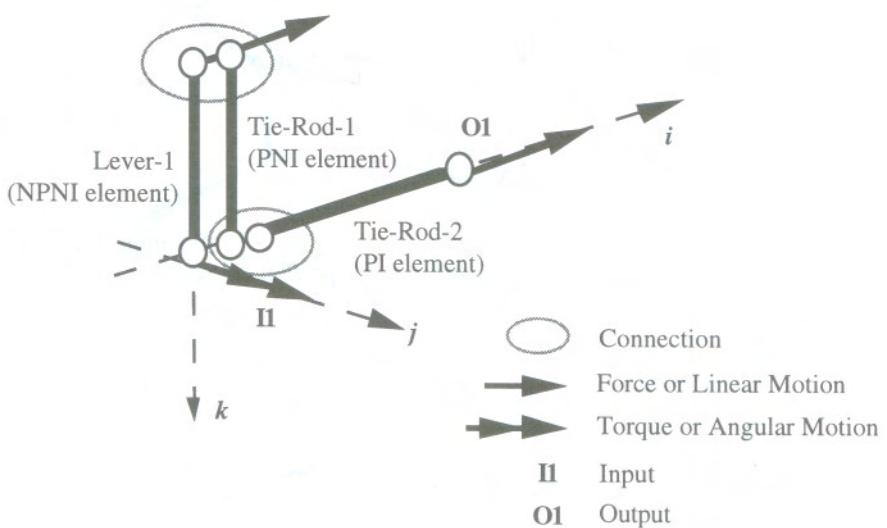
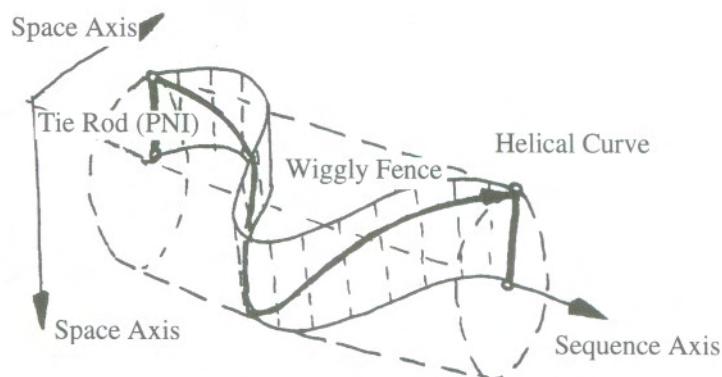
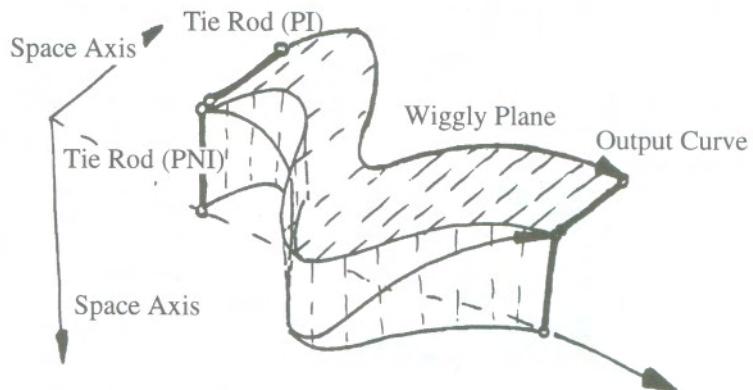


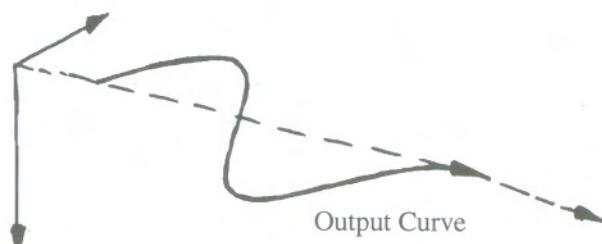
Figure 12. An instantaneous solution for transforming a rotation into a translation.



a. Sequence diagram of type PNI tie-rod containing output curve of a lever



b. Interface between sequence diagrams of type PNI tie-rod and type PI tie-rod



c. Output curve of the overall solution

*Figure 13.* Temporal reasoning sequences for the solution in Figure 12 for a continuous rotational input.

#### 4. Implementation, Performance And Related Work

The synthesis program is implemented on a Harlequin LispWorks<sup>TM</sup> environment on a SUN SPARCstation<sup>TM</sup>. Using these programs, it is possible to synthesise not only the solutions known to exist for a given problem, but also novel ones (for instance, the design in Fig. 7a and 7b a schematic of which is shown in Figure 8b). It has the potential to be useful in assisting designer in conceptual design. The procedures are exhaustive within the scope of the orthogonality restrictions, and thus could be used to find "satisficing" (Simon, 1969) solutions, if required. These programs have combinatorial problems. However, these are based on rudimentary procedures, in the sense that they are neither optimised, nor do they use any heuristics or other strong criteria for evaluation whereby the goodness of a complete solution could be estimated by examining an incomplete solution.

The temporal representation and procedures are presently being implemented on a LISP-based KnowledgeWorks<sup>TM</sup> environment.

Related works include synthesis programs developed by Hoover and Rinderle (1989), and Finger and Rinderle (1990), for geared transmission systems, by Ulrich and Seering (1989) for transducer designs (all of these are heavily based on Bond Graphs; see Paynter, 1961, for information on Bond Graphs), and, the classical "Mechanisms" approach (Reuleaux, 1876; Hoeltzel & Chieng, 1990). The present approach is inspired by Bond Graphs insofar as the concept of flow and effort is concerned, and is different from the above in at least three respects. One, this approach can be applicable to a wider domain than any of the above, a feature frequently required in mechanical designs. Two, the above procedures mainly address synthesis of topological descriptions of solutions. Three, the representation and reasoning in this approach is mainly "decompositional", whereas that in others except the Mechanisms approach is "transformational" (classifying as in Maher, 1990). The present approach is different from the Mechanisms approach in that here structures are combined first to form the essential concept, and only then are the connections chosen to enable these structures to function in the stipulated ways, whereas in the Mechanisms approach a set of connections are topologically combined to provide the freedom in motions required by the problem.

#### 5. Summary And Conclusions

This paper addresses the authors' research into developing a computational system for conceptual design of mechanical devices and machines. The knowledge representation, synthesis approach, implementation and performance issues of an implemented knowledge based system for synthesis of topological and spatial configurations of conceptual solutions to multiple input/output problems, using a knowledge base of basic components and combination rules, are discussed. Also discussed is a temporal representation

of these elements in terms of sequence diagrams, which should enable computers to support temporal reasoning about the instantaneous solutions. Work in progress is focused on implementing this representation and temporal reasoning procedures.

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### References

- Chakrabarti, A.: 1991, *Designing by Functions*, PhD Thesis, Department of Engineering, Cambridge University, UK.
- Chakrabarti A. and Bligh T. P.: 1992, A knowledge-based system for synthesis of single input single output systems in mechanical conceptual design, *Proceedings 4th Europe-USA Joint Conference on AI and Expert Systems Applications (EXPERSYS'92)*, Houston, USA, pp. 401-405.
- Finger, S. and Rinderle, J. R.: 1990, A transformational approach for mechanical design using a bond graph grammar, *EDRC Report No. 24-23-90*: Carnegie Mellon University, USA.
- Hoeltzel, D. A. and Chieng, W-H.: 1990, Knowledge-based approaches for the creative synthesis of mechanisms, *Computer-Aided Design*, **22**(1), 57-67.
- Hoover, S. P. and Rinderle, J. R.: 1989, A synthesis strategy for mechanical devices, *Research in Engineering Design*, **1**, 87-103.
- Lozano-Perez, T.: 1983, Spatial planning: A configuration space approach, *IEEE Computer*, **C-32**(2), 108-120.
- Maher, M. L.: 1990, Process models for design synthesis, *AI Magazine*, **11**(4).
- Paynter, H. M.: 1961, *Analysis and Design Of Engineering Systems*, MIT Press, MA.
- Reuleaux, F.: 1876, *The Kinematics of Machinery: Outline of a Theory of Machines*, Reprinted by Dover Publications, 1963.
- Simon, H. A.: 1969, *The Sciences of the Artificial*, MIT Press.
- Ulrich, K. T., and Seering, W. P.: 1989, Synthesis of schematic descriptions in mechanical design, *Research in Engineering Design* **1**(1), 3-18.