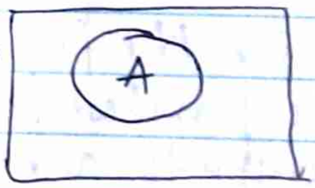
Probability and Counting | statistics 110 | Lecture 1

words, sentences, clarity, honesty

Applications :-

(I) History: Mosteller-Wallace Federalist papers (History of US)
(II) Govt: IQSS (Harvard Institute for Quantitative Social Science)
(III) Finance → Stats 123
(IV) Gambling → the historical roots of the subject are exactly in games of chances - gambling
Fermat-Pascal (1650's)
(V) Life - statistics is the logic of uncertainty (Maths is the logic of certainty)
Everyone has a lot of uncertainties and ~~probility~~ probability and statistics are how we quantify and update our beliefs and deal with uncertainty.

A sample space is the set of all possible outcomes of an experiment.
An event is a subset of the sample space. S



Naive definition of Probability

(Only use this when we have strong justification for doing so)

$$P(A) = \frac{\#\text{favourable outcomes to A}}{\#\text{ possible outcomes}}$$

event

Assumptions - ~~All~~

(I) All outcomes are equally likely
(II) finite sample space

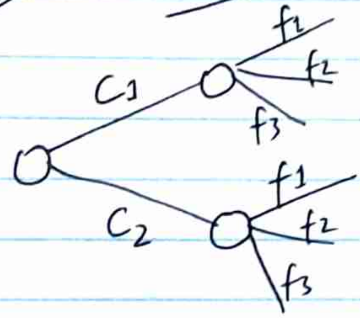
Reasonable assumption in some problems where we have some kind of symmetry.

Counting

Multiplication Rule: If we have an experiment with n_1 possible outcomes, and for each outcome of 1st experiment there are n_2 outcomes for 2nd expt, . . . , for each outcome of $(r-1)$th expt. there are n_r outcomes for rth experiment, then overall there are $n_1 n_2 \dots n_{r-1} n_r$ possible outcomes.

~~Proof~~ Induction

Example icecream — 3 flavours (f_1, f_2, f_3), 2 type of cones (c_1, c_2)



6 possibilities $= 2 \times 3$ (choose cone first, flavours next)
$= 3 \times 2$ (choose flavours first, cone next)

Example prob. of full house of pocker, 5 card hand
Completely shuffled and all 5 cards are equally likely.

Individual cards are ranked, from highest to lowest

A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, 2

$$\frac{{}^{13}C_1 \times {}^{4}C_3 \times {}^{12}C_1 \times {}^{4}C_2}{{}^{52}C_5}$$

Can choose any one of 13 cards

Choose 3 out 4.

eg. 3 7's, 2 10's

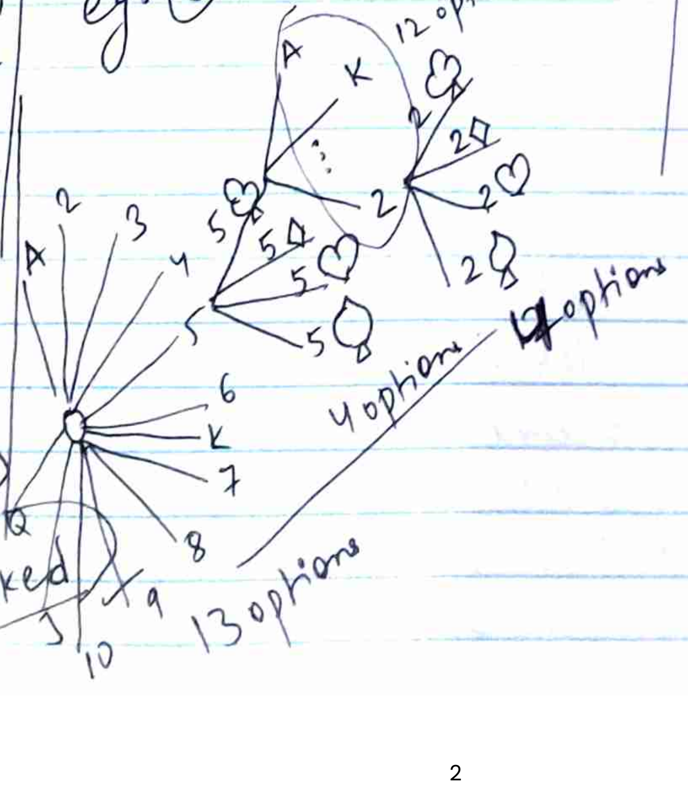
12 options (excluding 5)

full house | full boat | boat | full hand

3 cards of 1 rank, 2 cards of another rank

eg. 3♣ 3♡ 3♠ 6♣ 6♡

Suits
clubs ♣
diamonds ◇
hearts ♡
spades ♠
→ Suits are not ranked



Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad 0 \text{ if } k > n$$

# of subsets of size k, of group of n people where order does not matter

$$\frac{n\cdot(n-1)\cdot(n-2)\cdots(n-k+1)}{k!}$$

We could have chosen them in any order, so we have to divide it by k! because we overcounted by that factor

$$= \frac{n!}{(n-k)!\,k!}$$

Sampling table: Choose k objects out of n. How many ways ~~of doing that is~~ are there to do it?

we have some population of items or people or anything and we are drawing a sample

	Order matters	order doesn't matter
Sample with replacement	n^k	${}^{n+k-1}C_k$
without replacement	${}^nP_k$	${}^nC_k$ or $\binom{n}{k}$

pick k times from a set of n objects, where order does not matter, with replacement.

i.e, we pick 1 object, place it back, then pick another or same object, place it back, we do this k times.