MISCELLANEOUS TOPICS

Law of Large Numbers (LLN)

→ It's a theorem that describes the result of performing the same experiment a large number of times.

→ According to the law, the average of the results obtained from a large number of trials should be close to the expected value, and tend to become closer to the expected value as more trials are performed.

→ The LLN is important because it guarantees stable long-term results for the averages of some random events.

Note :- It is important to remember that the law of Large Numbers only applies when a large number of observations is considered. There is no principle that a small number of observations will coincide with the expected value or that a streak of one value will immediately be "balanced" by the others.

Example A single roll of a fair, six-sided die produces one of the numbers 1, 2, 3, 4, 5 or 6 with equal probability. Therefore, the expected value of the average of the roll is

$$\frac{1+2+3+4+5+6}{6} = 3.5$$

A/c to LLN, if a large number of six-sided dice are rolled the average of their values (or, simple mean) is likely to be close to 3.5, with the precision increasing as more dice are rolled.

~~Example~~ It follows from the LLN that the empirical probability of success in a series of Bernouilli trials will converge to the theoretical probability. For a Bernouilli random variable, the expected value is the theoretical probability of success.

and the average of n such variables (assuming they are independent and identically distributed (i.i.d)) is precisely the relative frequency.

Example
a fair coin is a Bernouilli trial. When a fair coin is flipped once, the theoretical probability that the outcome will be head is equal to $1/2$. Therefore, according to the LLN, the proportion of heads in a "large" number of coin flips "should be" roughly $1/2$. In particular, the proportion of heads after n flips will almost surely converge to $1/2$ as $n \to \infty$.

Example Another example of the LLN is the Monte Carlo methods. These are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The larger the number of repititions, the better the approximation tends to be.

Limitations

- The average of the results obtained from a large number of trials may fail to converge in some cases.
For instance, the average of the results from the Cauchy distribution or some Pareto distribution ($\alpha < 1$) will not converge as x becomes larger; the reason is heavy tails. The Cauchy distribution and the Pareto distribution represent two cases:
- the Cauchy distribution doesnot have an expectation, &
- the expectation of the Pareto distribution ($\alpha < 1$) is infinite.

Another example is where the random numbers equal the tangent of an angle uniformly distributed between $-90°$ and $+90°$. The median is zero, but the expected value does not exist, and indeed the average of n such variables has the same distribution as one such variable. It does not converge in probability towards zero (or any other value) as $n \to \infty$.

Gambler's fallacy (Monte Carlo fallacy / fallacy of the maturity of chances

- The erroneous belief that if a particular event occurs more frequently than normal during the past, it is less likely to happen in the future (or vice versa), when it has otherwise been established that the probability of such events does not depend on what has happened in the past.
- Such events, having the quality of historical independence, are referred to as statistically independent.
- The fallacy is commonly associated with gambling, wherein it may be believed for example that the next dice roll is more than usually likely to be six because there have recently been less than usual number of sixes.