

(a) After 0th iteration:

$$V(S_0) = 0, \text{ ~~So~~ } V(S_1) = 0, V(S_2) = 0, V(S_3) = 0$$

After 1st iteration

$$V(S_0) = 1, V(S_1) = 2, V(S_2) = 3, V(S_3) = 0$$

After 2nd iteration:

$$V(S_0) = 1 + (0.9 \times (0.5 \times 2) + 0.5 \times 3)$$

$$= 3.25$$

$$V(S_1) = 2 + 0.9 (\max(0.5 \times 10 + 0.5 \times 2, 1 \times 3))$$

$$= 2 + 0.9 (\max(6, 3)) = 2 + 0.9 \times 6$$

$$= 7.4$$

$$V(S_2) = 3 + 0.9 \times 1 \times 1$$

$$= 3.9$$

$$V(S_3) = 10 + 0.9 \times 1 \times 10$$

$$= 19$$

After 3rd iteration

$$V(S_0) = 3.25 + 0.9 \times (0.5 \times 7.4 + 0.5 \times 3.9)$$

$$= 8.335$$

$$V(S_1) = 7.4 + 0.9 (\max(0.5 \times 19 + 0.5 \times 7.4, 1 \times 3.9))$$

$$= 19.28$$

$$V(S_2) = 3.9 + 0.9 \times 1 \times 3.25$$

$$= 6.825$$

$$V(S_3) = 19 + 0.9 \times 1 \times 19$$

$$= 36.1$$

(b) Optimal policy = Q value at any possible actions of any possible state

The optimal policy is from $S_1 \rightarrow S_3$

$$V(S_2) = 10 + 0.9 (V(S_3))$$

$$0.1 V^*(S_2) = 10$$

$$V^*(S_2) = 100$$

$$V^*(S_1) = 2 + 0.9 (0.5 V^*(S_1) + 0.5 V^*(S_3))$$

$$V^*(S_1) = 2 + 0.9 (0.5 V^*(S_1) + 50)$$

$$V^*(S_1) = \frac{47}{0.55} = 85.45$$

(c) (i) False, MDP isn't always cyclic

(ii) False, MDP isn't cyclic always so the value doesn't keep increasing and has no bounds.

(iii) True, values would be immediate rewards & there is no updates.

(iv) True

(v) True

Q2 Each value = $2^{10} - 255 \rightarrow 2^8$
8 bits for each value.

\therefore each has R, G, B

thus 1 pixel = $3 \times 8 = 24 = 3$ bytes

whole image = $N^2 \times 24 = 24N^2$ bits = $3N^2$ bytes

if there are k clusters.

bits req'd $\Rightarrow 2^b = k$

$$b = \log_2 k$$

$$\therefore \text{total bits reqd} \Rightarrow 3N^2 \times \log_2 k \\ = (3 \log_2 k) N^2$$

$$\text{compression ratio} = \frac{\text{un compressed}}{\text{compressed}} \\ = \frac{24N^2}{3N^2 \log_2 k} \\ = \frac{8}{\log_2 k}$$

2) It is prop to $\frac{1}{\log_2 k}$

⇒ higher the k higher the ratio.