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1.a)

(a)

Model-3 :- $p(x) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu) \right]$.

$$l(x) = \prod_{t=1}^{N} \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu) \right].$$

Here, Consider $\Sigma = \alpha_i I$

$$L(x|\mu_1,\alpha_1) = \sum_{t=1}^{N_1} -\frac{d}{2}\log 2\pi - \frac{1}{2}\log|\alpha_1 I| - \frac{1}{2}(x^t-\mu_1)^T(\alpha_1 I)^{-1}(x^t-\mu_1)$$

$$\Rightarrow \sum_{t=1}^{N_1} -\frac{d}{2}\log 2\pi - \frac{1}{2} d\alpha_1 - \frac{1}{2}\left(\frac{1}{\alpha_1}\right).(x^t-\mu_1)^T(x^t-\mu_1).$$

$$\frac{\partial L}{\partial \mu_1} = 0 \Rightarrow \sum_{t=1}^{N_1} (x^t-\mu_1)^T\left(\frac{1}{\alpha_1}\right) = 0.$$

$$\Rightarrow \mu_1 = \frac{\sum_{t=1}^{N_1} x^t}{N_1}.$$

Illy, $\mu_2 = \frac{\sum_{t=1}^{N_2} x^t}{N_2}$.

Combining, $\boxed{\mu_i = \frac{\sum_{t=1}^{N} r_i^t x^t}{\sum_{t=1}^{N} r_i^t}}$

$$\frac{\partial L}{\partial \alpha_1} = 0 \Rightarrow \frac{dN_1}{\alpha_1} - \sum_{t=1}^{N_1} \frac{(x-\mu_1)^2}{\alpha_1^2} = 0.$$

$$\Rightarrow d\alpha_1 N_1 - \sum_{t=1}^{N_1} (x-\mu_1)^2 = 0$$

$$\Rightarrow \alpha_1 = \frac{\sum_{t=1}^{N_1} (x^t-\mu_1)^2}{N_1 d}.$$

Illy, $\boxed{\alpha = \frac{\sum_{t=1}^{N} r_i^t (x^t-\mu_i)^2}{d \cdot \sum_{t=1}^{N} r_i^t}}$

Model-2 :-

We pool the data based on the proportions of $P(c_i)$.

So, $S = P(c_1) \cdot S_1 + P(c_2) S_2$.

1.b) and 1.c)

The error rate for Model 1 = 0.14; for Model 2 = 0.12 and for Model 3 = 0.3. From Model 1 to Model 2 to Model 3, we are decreasing the complexity of the model by decreasing the number of parameters to be computed for the model. We observe that as we simplify the models, the error first decreases and then increases. This would imply that Model 2 matches the actual problem complexity better than Model 1 and Model 3.

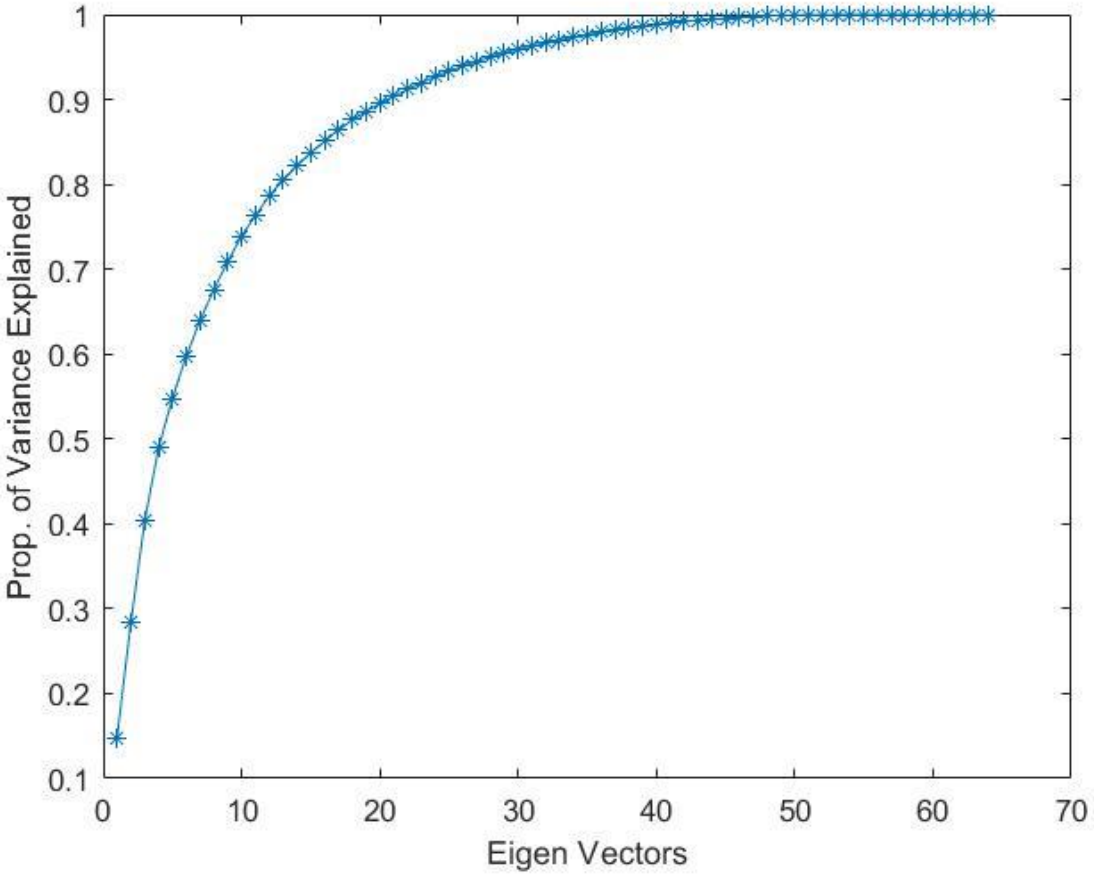
```
Printing Statistics for Model 1 :-

pc1 =0.300000
pc2 =0.700000
m1 =
1.055362   2.518051   3.296693   -1.892653   -1.391843   4.063513   -4.354007   -5.870519
m2 =
3.805217   5.374021   5.733303   1.159601   1.177740   6.799984   -2.028626   -2.504397
s1 =
0.972929   0.713549   0.456986   0.893755   0.309622   0.197526   0.736243   1.662850
0.713549   3.065819   2.598160   0.387774   1.299431   0.144218   0.916845   4.938755
0.456986   2.598160   6.661194   0.908431   1.639701   0.814769   0.077851   5.416803
0.893755   0.387774   0.908431   5.075387   0.096322   1.061116   2.397836   4.594624
0.309622   1.299431   1.639701   0.096322   2.397334   -0.019128   0.217550   2.537772
0.197526   0.144218   0.814769   1.061116   -0.019128   1.041249   -0.053108   1.860371
0.736243   0.916845   0.077851   2.397836   0.217550   -0.053108   6.515404   3.860949
1.662850   4.938755   5.416803   4.594624   2.537772   1.860371   3.860949   17.093118
s2 =
1.348581   0.865764   0.155911   0.584693   0.947281   0.254299   0.370517   0.943514
0.865764   2.816079   -0.181888   0.215242   0.714060   0.679660   -0.238329   2.419394
0.155911   -0.181888   6.673399   1.771554   1.031843   0.652603   1.679524   3.899243
0.584693   0.215242   1.771554   3.543344   0.357020   1.366504   2.120719   3.089929
0.947281   0.714060   1.031843   0.357020   2.751684   0.122527   1.434168   2.465131
0.254299   0.679660   0.652603   1.366504   0.122527   1.810718   0.344532   1.511289
0.370517   -0.238329   1.679524   2.120719   1.434168   0.344532   7.113410   2.624578
0.943514   2.419394   3.899243   3.089929   2.465131   1.511289   2.624578   13.915060
error on test set =0.140000
```

```
Printing Statistics for Model 2 :-

pc1 =0.300000
pc2 =0.700000
m1 =
1.055362   2.518051   3.296693   -1.892653   -1.391843   4.063513   -4.354007   -5.870519
m2 =
3.805217   5.374021   5.733303   1.159601   1.177740   6.799984   -2.028626   -2.504397
s1 =
1.235885   0.820100   0.246233   0.677412   0.755983   0.237267   0.480234   1.159315
0.820100   2.891001   0.652126   0.267002   0.889671   0.519027   0.108223   3.175202
0.246233   0.652126   6.669738   1.512617   1.214200   0.701253   1.199022   4.354511
0.677412   0.267002   1.512617   4.002957   0.278811   1.274888   2.203854   3.541338
0.755983   0.889671   1.214200   0.278811   2.645379   0.080031   1.069183   2.486923
0.237267   0.519027   0.701253   1.274888   0.080031   1.579877   0.225240   1.616014
0.480234   0.108223   1.199022   2.203854   1.069183   0.225240   6.934008   2.995489
1.159315   3.175202   4.354511   3.541338   2.486923   1.616014   2.995489   14.868478
s2 =
1.235885   0.820100   0.246233   0.677412   0.755983   0.237267   0.480234   1.159315
0.820100   2.891001   0.652126   0.267002   0.889671   0.519027   0.108223   3.175202
0.246233   0.652126   6.669738   1.512617   1.214200   0.701253   1.199022   4.354511
0.677412   0.267002   1.512617   4.002957   0.278811   1.274888   2.203854   3.541338
0.755983   0.889671   1.214200   0.278811   2.645379   0.080031   1.069183   2.486923
0.237267   0.519027   0.701253   1.274888   0.080031   1.579877   0.225240   1.616014
0.480234   0.108223   1.199022   2.203854   1.069183   0.225240   6.934008   2.995489
1.159315   3.175202   4.354511   3.541338   2.486923   1.616014   2.995489   14.868478
error on test set =0.120000

Printing Statistics for Model 3 :-

pc1 =0.300000
pc2 =0.700000
m1 =
1.055362   2.518051   3.296693   -1.892653   -1.391843   4.063513   -4.354007   -5.870519
m2 =
3.805217   5.374021   5.733303   1.159601   1.177740   6.799984   -2.028626   -2.504397
alpha1 =5.174377
alpha2 =4.925155
error on test set =0.230000
```

2.a)

```
error ratio for k = 1 is 0.053872
error ratio for k = 3 is 0.040404
error ratio for k = 5 is 0.043771
error ratio for k = 7 is 0.053872
```

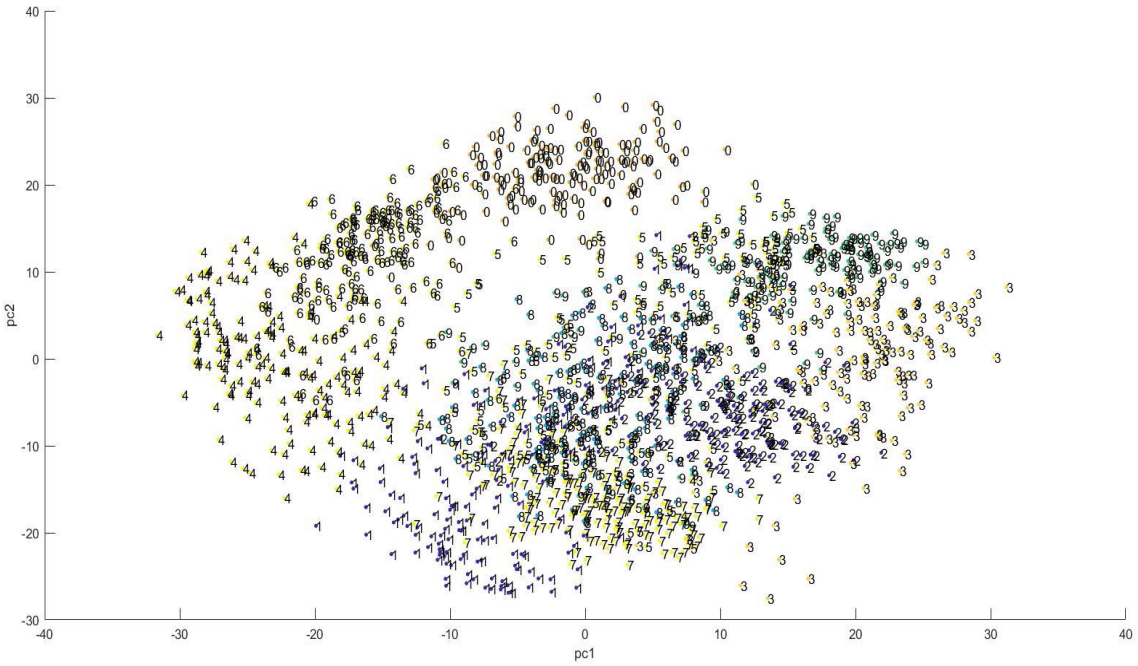
2.b)



K = 21

```
error ratio for k = 1 is 0.043771
error ratio for k = 3 is 0.040404
error ratio for k = 5 is 0.043771
error ratio for k = 7 is 0.040404
```

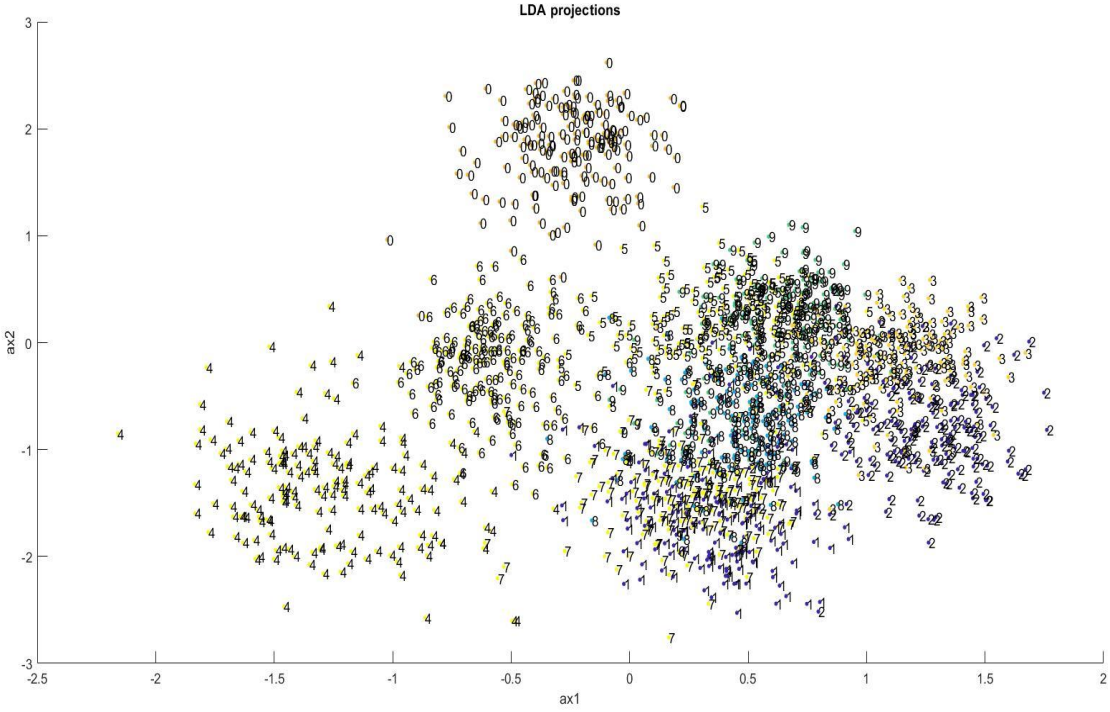
2.c)



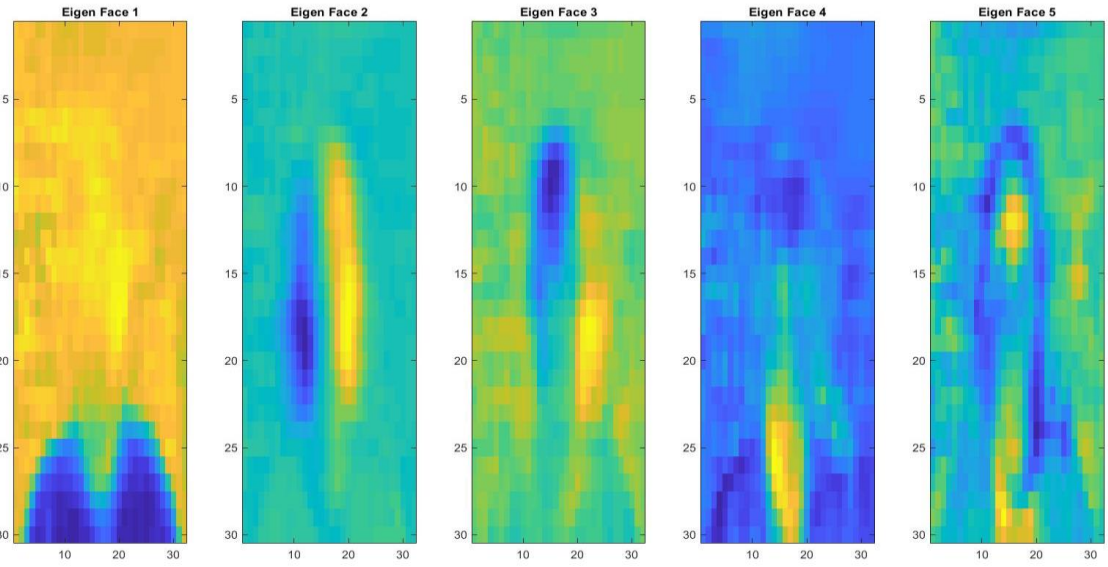
2.d)

```
error ratio for L = 2  and  k = 1  is 0.464646
error ratio for L = 2  and  k = 3  is 0.424242
error ratio for L = 2  and  k = 5  is 0.407407
error ratio for L = 4  and  k = 1  is 0.191919
error ratio for L = 4  and  k = 3  is 0.181818
error ratio for L = 4  and  k = 5  is 0.158249
error ratio for L = 9  and  k = 1  is 0.097643
error ratio for L = 9  and  k = 3  is 0.094276
error ratio for L = 9  and  k = 5  is 0.090909
```

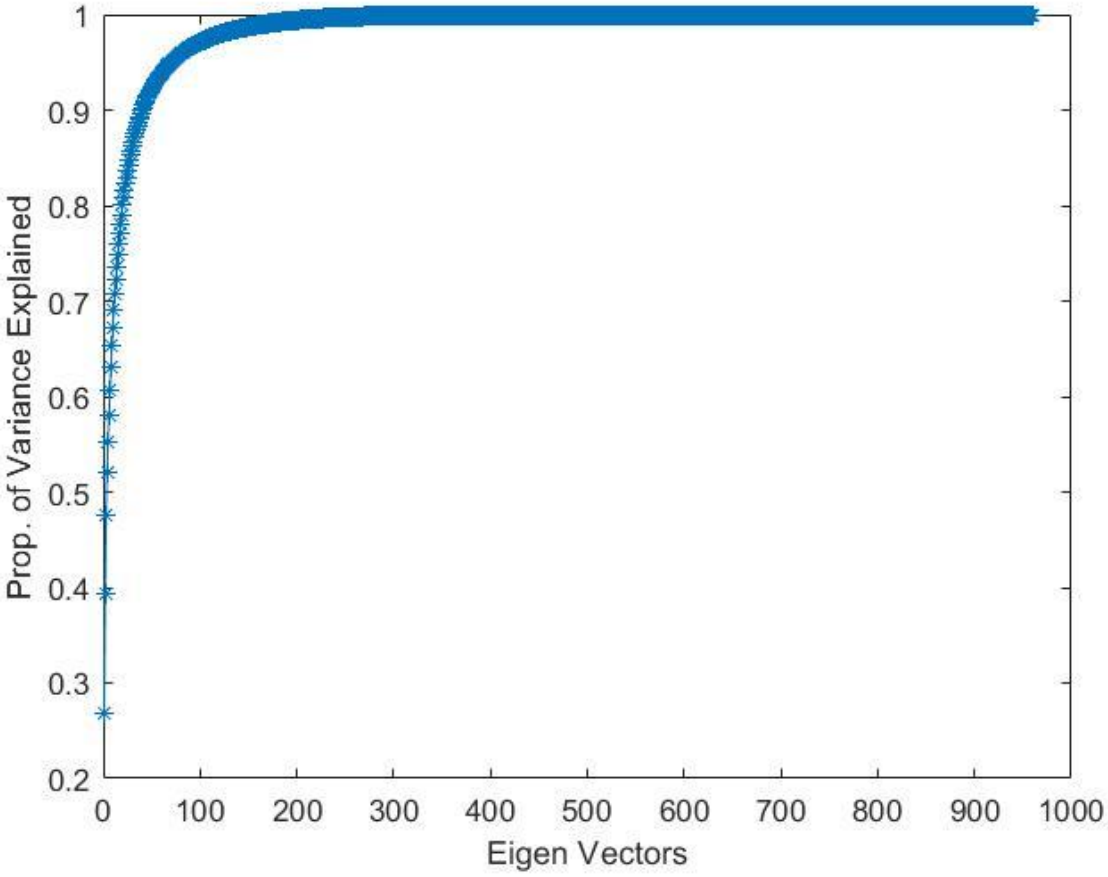
2.e)


LDA projections
ax1
ax2

3.a)


Eigen Face 1
Eigen Face 2
Eigen Face 3
Eigen Face 4
Eigen Face 5

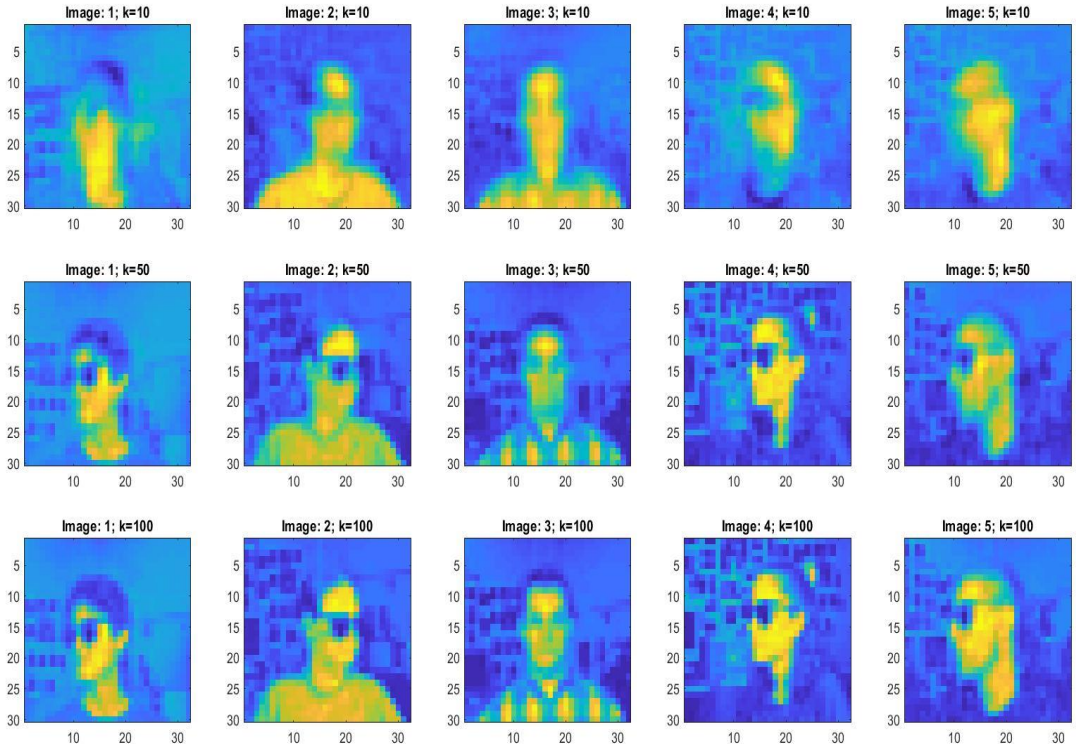
3.b)



K = 41

```
error ratio for k = 1 is 0.104839
error ratio for k = 3 is 0.241935
error ratio for k = 5 is 0.395161
error ratio for k = 7 is 0.395161
```

3.c)



The images match the original image very closely as we increase the value of k. So the reconstruction error reduces as we increase the number of components. This is because more variance inherent in the data is explained if we increase the value of k.