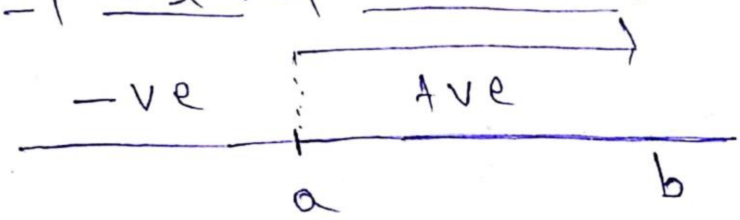
① VC dimension of a function is defined by the maximum point the target function can shatter.

Targed Function

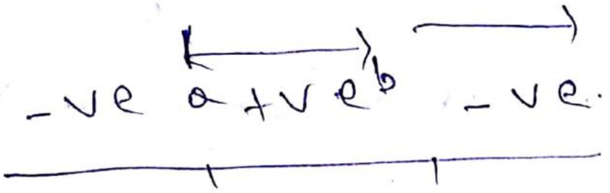
$$[a,b] = \{x \in R \mid a \le x \le b\}$$

two points canbe shattered using the target function. (-ve, +ve)

graphical representation

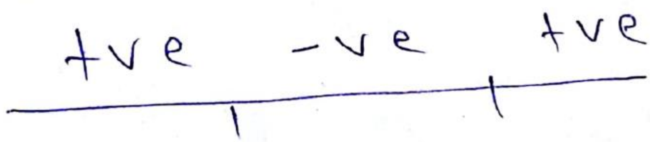



the given target function cannot Scatter 3 points in all the cases like +ve, -ve, +ve




can be scattered

but it can't scatter the following case.




So the VC dimension of the given function is 2.

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2(a)

$$F(x|\theta) = \frac{1}{\theta-1} e^{-\frac{x}{\theta-1}}$$

$$\Rightarrow L\left(F(x|\theta)\right)$$

$$= \log \prod_n \frac{1}{\theta-1} e^{-\frac{x^n}{\theta-1}}$$

$$= \sum_n \log \frac{1}{\theta-1} - \sum_n \frac{x^n}{\theta-1}$$

$$\Rightarrow \frac{\partial L(F(x|\theta))}{\partial \theta} = 0$$

$$\Rightarrow \sum_n \frac{1}{\frac{1}{\theta-1}} \frac{-1}{(\theta-1)^2} - \sum_n \frac{x^n}{(\theta-1)^2}(-1) = 0$$

$$\Rightarrow \sum_n \frac{(\theta-1)(-1)}{(\theta-1)^2} + \sum_n \frac{x^n}{(\theta-1)^2} = 0$$

$$\Rightarrow -\frac{n}{(\theta-1)} + \sum_n \frac{x^n}{(\theta-1)^2} = 0$$

$$\Rightarrow \sum_n \frac{x^n}{(\theta-1)^2} = \frac{n}{(\theta-1)}$$

$$\Rightarrow n(\theta-1) = \sum_n x^n$$

$$\Rightarrow \theta = \frac{\sum_n x^n}{n} + 1$$

(b) $f(x|\Theta) = (\Theta - 1) \cdot x^{\Theta - 2}$

$$L(f(x|\Theta)) = \log \prod_n (\Theta - 1) x_n^{\Theta - 2}$$

$$= \sum_n \log(\Theta - 1) + \sum_n \log x_n^{\Theta - 2}$$

$$\frac{\partial L(f(x|\Theta))}{\partial \Theta} = 0$$

$$\Rightarrow \sum_n \frac{1}{\Theta - 1} + \sum_n \log x_n = 0$$

$$\Rightarrow \frac{n}{\Theta - 1} + \sum_n \log x_n = 0$$

$$\Rightarrow -\frac{n}{\Theta - 1} = \sum_n \log x_n$$

$$\Rightarrow \Theta - 1 = \frac{-n}{\sum_n \log x_n}$$

$$\Rightarrow \Theta = 1 - \frac{n}{\sum_n \log x_n}$$

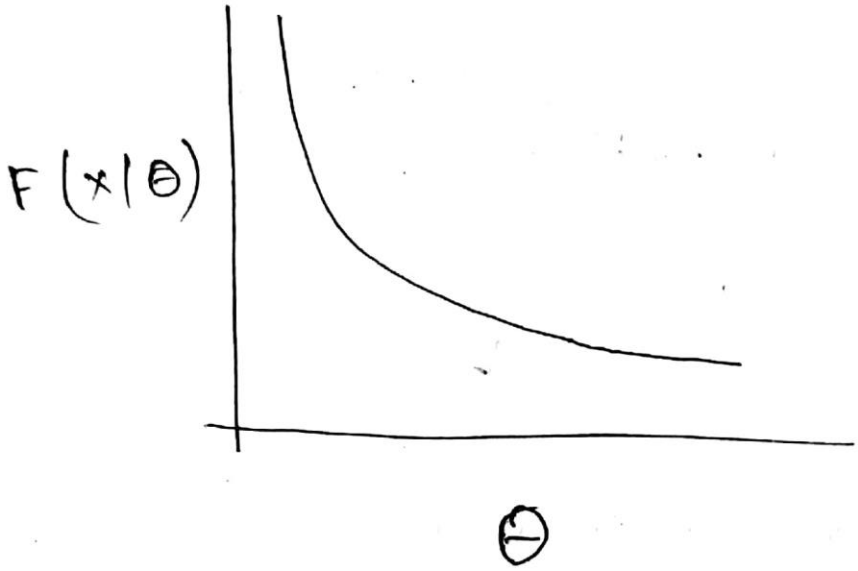
(c) $F(x|\Theta) = 1/\Theta$

For n samples

$$F(x|\Theta) = \prod_{n} 1/\Theta$$

$$= 1/\Theta^n$$

graph of $1/\Theta^n$




Since $1/\Theta^n$ is strictly decreasing MLE$(F(x|\Theta))$ can be found by the Maximum value of $x$ since $0 \leq x \leq \Theta$

~~3(a)~~

3(a) For bernouli densities

$$P(x/c) = p^{x}(1-p)^{1-x}$$

given:

$$P(x=0 \mid C_1) \equiv P_1$$

$$P(x=0 \mid C_2) \equiv P_2$$

According to bayes theorem

$$P(C/x) = \frac{P(C) \cdot P(x/c)}{P(x)}$$

According to the prediction rule

$$\begin{cases} C_1 & \text{if } P(C_1/x) > P(C_2/x) \\ C_2 & \text{otherwise} \end{cases}$$

In the calculation of $P(C_1/x)$ and $P(C_2/x)$ we can ignore $P(x)$

so

$P(C_1/x)$

$$= P(C_1) \cdot P(x/C_1)$$

$$= P(C_1) \cdot P_1^{(1-x)}(1-P_1)^{x}$$

$$P(c_2/x) = P(c_2) \cdot P_2^{(1-x)} (1-P_2)^x$$

For $x = 0$

$$\begin{cases} c_1 & P(c_1)\, P_1 > P(c_2) \cdot P_2 \\ c_2 & \text{otherwise} \end{cases}$$

For $x = 1$

$$\begin{cases} c_1 & P(c_1)(1-P_1) > P(c_2) \cdot (1-P_2) \\ c_2 & \text{otherwise.} \end{cases}$$

(b) $x$ is d-dimensional bernouli density.

$$P_{ij} \equiv P(x_j = 0 \mid c_i)$$

$$i = 1, 2, \quad j = 1, 2 \ldots D$$

$$1 - P_{ij} = P(x_j = 1 / c_i)$$

Since $x_j$ are independent bernouli densities.

$$P(x/c_i) = \prod_{j=1}^{D} (1-P_{ij})^{x_j} P_{ij}^{(1-x_j)}$$

$$P(x/c_1) = \prod_{j=1}^{D} (1-P_{1j})^{x_j} P_{1j}^{(1-x_j)}$$

$P(x/c_2)$

$$= \prod_{j=1}^{D} (1-P_{2j})^{x_j} \, P_{2j}^{(1-x_j)}$$

condition

$$\begin{cases} c_1 & \text{if } P(c_1/x) > P(c_2/x) \\ c_2 & \text{otherwise} \end{cases}$$

$$= \begin{cases} c_1 & \prod_{j=1}^{D} (1-P_{1j})^{x_j} \, P_{1j}^{(1-x_j)} \, P(C_1) \geq \prod_{j=1}^{D} (1-P_{2j})^{x_j} \, P_{2j}^{(1-x_j)} \, P(C_2) \\ c_2 & \text{otherwise} \end{cases}$$

3(c) given

$P_{11} = 0.6$

$P_{12} = 0.1$

$P_{21} = 0.6$

$P_{22} = 0.9$

$(x_1, x_2) \in \{(0,0), (0,1), (1,0), (1,1)\}$

First case

$P(C_1) = 0.2 \quad P(C_2) = 0.8$

$x = (0,0)$

$P(x/C_1) = P_{11} \cdot P_{12}$

$= 0.1 \times 0.6 = 0.06$

$P(x/C_2)$

$= P_{21} \cdot P_{22}$

$= 0.6 \times 0.9 = 0.54$

$$P(C_1/x) = \frac{P(C_1) \times P(x/C_1)}{P(C_1) \times P(x/C_1) + P(C_2) \times P(x/C_2)}$$

$$= \frac{0.2 \times 0.06}{0.2 \times 0.06 + 0.8 \times 0.54}$$

$= 0.012 / 0.444 = 0.0270$

$$P(c_2/x) = \frac{P(c_2) \times P(x/c_2)}{P(c_2) \times P(x/c_2) + P(c_1)\,P(x/c_1)}$$

$$= \frac{0.8 \times 0.54}{0.8 \times 0.54 + 0.2 \times 0.06}$$

$$= \frac{0.432}{0.444} = \cancel{0.97\ldots}$$

$= 0.973$

For $x_1$ $(0,1)$

$$P(x/c_1) = P_{11}(1-P_{12})$$

$$= 0.6 \times 0.9 = 0.54$$

$$P(x/c_2) = P_{21}(1-P_{22})$$

$$= 0.6 \times 0.1 = 0.06$$

$$P(c_1/x) = \frac{0.2 \times 0.54}{0.2 \times 0.54 + 0.06 \times 0.8}$$

$$= \frac{0.108}{0.108 + 0.048}$$

$= 0.692$

$$P\left(\frac{c_2}{x}\right) = \frac{0.048}{0.108 + 0.048}$$

$$= 0.307$$

$x = 1, 1$

$$P\left(\frac{x}{c_1}\right) = (1 - P_{11})(1 - P_{12})$$

$$= 0.36$$

$$P\left(\frac{x}{c_2}\right) = (1 - P_{21})(1 - P_{22})$$

$$= 0.04$$

$$P\left(\frac{c_1}{x}\right) = \frac{0.2 \times 0.36}{0.2 \times 0.36 + 0.04 \times 0.8}$$

$$= \frac{0.072}{0.104} = 0.692$$

$$P\left(\frac{c_2}{x}\right) = \frac{0.04 \times 0.8}{0.2 \times 0.36 + 0.04 \times 0.8}$$

$$= 0.307$$

$x = (1, 0)$

$$P(x/C_1) = (1 - P_{11})\,P_{12}$$

$$= (1 - 0.6)\,0.1 = 0.04$$

$$P(x/C_2) = (1 - P_{21})\,P_{22}$$

$$= 0.4 \times 0.9 = 0.36$$

$$P(C_1/x) = \frac{0.2 \times 0.04}{0.2 \times 0.04 + 0.8 \times 0.36}$$

$$= 0.027$$

$$P(C_2/x) = \frac{0.8 \times 0.36}{0.2 \times 0.04 + 0.8 \times 0.36}$$

$$= 0.973$$

Second case

$$P(C_1) = 0.6$$

$$P(C_2) = 0.4$$

$x = 0, 0$

$$P(x/C_1) = P_{11} \times P_{12} = 0.06$$

$$P(x/C_2) = P_{21} \times P_{22} = 0.54$$

$$P(C_1/x) = \frac{0.6 \times 0.06}{0.6 \times 0.06 + 0.54 \times 0.4}$$

$$= 0.14$$

$$P(C_2/x) = \frac{0.54 \times 0.4}{0.6 \times 0.06 + 0.54 \times 0.4}$$

$$= 0.857$$

$\underline{x = 0, 1}$

$$P(x \mid c_1) = P_{11}(1 - P_{12}) = 0.6 \times 0.9$$
$$= 0.54$$

$$P(x \mid c_2) = P_{21}(1 - P_{22}) = 0.6 \times 0.1$$
$$= 0.06$$

$$P(c_1 / x) = \frac{0.6 \times 0.54}{0.4 \times 0.06 + 0.54 \times 0.6}$$
$$= 0.931$$

$$P(c_2 / x) = \frac{0.4 \times 0.06}{0.4 \times 0.06 + 0.54 \times 0.6}$$
$$= 0.069$$

$\underline{x = 1, 0}$

$$P(x / c_1) = (1 - P_{11}) P_{12}$$
$$= 0.4 \times 0.1$$
$$= \cancel{0.36}\ 0.04$$

$$P(x / c_2) = (1 - P_{21}) P_{22}$$
$$= 0.4 \times 0.9$$
$$= 0.36$$

$$P(c_1|x) = \frac{0.04 \times 0.6}{0.04 \times 0.6 + 0.36 \times 0.4}$$

$$= \cancel{0.166}\ 0.143$$

$$P(c_2|x) = \frac{0.36 \times 0.4}{0.04 \times 0.6 + 0.36 \times 0.4}$$

$$= 0.857$$

$x = (1,1)$

$$P(x|c_1) = (1 - P_{11})(1 - P_{12})$$

$$= 0.36$$

$$P(x/c_2) = (1 - P_{21})(1 - P_{22})$$

$$= 0.04$$

$$P(c_1/x) = \frac{0.36 \times 0.6}{\cancel{0.232}\ 0.36 \times 0.6 + 0.4 \times 0.04} = 0.93$$

$$P(c_2/x) = \frac{0.04 \times 0.4}{0.36 \times 0.6 + 0.4 \times 0.04}$$

$$= 0.068$$

Third

$P(C_1) = 0.8 \quad P(C_2) = 0.2$

$x = 0, 0$

$$P(x/c_1) = P_{11} \cdot P_{12} = 0.06$$

$$P(x/C_2) = P_{21} \cdot P_{22}$$

$$= 0.54$$

$$P(C_1/x) = \frac{0.6 \times 0.8}{0.06 \times 0.8 + 0.54 \times 0.2}$$

$$= 0.307$$

$$P(C_2/x) = \frac{0.54 \times 0.2}{0.06 \times 0.8 + 0.54 \times 0.2}$$

$$= 0.692$$

2) $x = (0, 1)$

$$P(x/C_1) = P_{11}(1 - P_{12}) = 0.54$$

$$P(x/C_2) = P_{21}(1 - P_{22}) = 0.06$$

$$P(C_1/x) = \frac{0.8 \times 0.54}{0.8 \times 0.54 + 0.2 \times 0.06}$$

$$= 0.97$$

$$P(C_2/x) = \frac{0.2 \times 0.06}{0.8 \times 0.54 + 0.2 \times 0.06}$$

$$= 0.027$$

3 $x = (1, 0)$

$$p(x/c_1) = (1-P_{11})\,P_{12}$$
$$= 0.04$$

$$p(x/c_2) = (1-P_{21})\,P_{22}$$
$$= 0.36$$

$$p(c_1/x) = \frac{0.8 \times 0.04}{0.8 \times 0.04 + 0.2 \times 0.36}$$
$$= 0.307$$

$$p(c_2/x) = \frac{0.2 \times 0.36}{0.8 \times 0.04 + 0.2 \times 0.36}$$
$$= 0.692$$

4 $x = 1, 1$

$$p(x/c_1) = (1-P_{11})(1-P_{12})$$
$$= 0.36$$
$$p(x/c_2) = (1-P_{21})(1-P_{22})$$
$$= 0.04$$
$$p(c_1/x) = \frac{0.8 \times 0.36}{0.8 \times 0.36 + 0.2 \times 0.04}$$
$$= 0.973$$

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$$P(C_2/x) = \frac{0.2 \times 0.04}{0.8 \times 0.26 + 0.2 \times 0.04}$$

$$= 0.027$$

4

```
[p1,p2,pc1,pc2]=Bayes_Learning(TD,TV);
```

| sigma | PC1 | PC2 | error |
|---|---|---|---|
| -5.0000 | 0.0067 | 0.9933 | 23.5955 |
| -4.0000 | 0.0180 | 0.9820 | 20.2247 |
| -3.0000 | 0.0474 | 0.9526 | 22.4719 |
| -2.0000 | 0.1192 | 0.8808 | 21.3483 |
| -1.0000 | 0.2689 | 0.7311 | 23.5955 |
| 0 | 0.5000 | 0.5000 | 28.0899 |
| 1.0000 | 0.7311 | 0.2689 | 28.0899 |
| 2.0000 | 0.8808 | 0.1192 | 32.5843 |
| 3.0000 | 0.9526 | 0.0474 | 32.5843 |
| 4.0000 | 0.9820 | 0.0180 | 32.5843 |
| 5.0000 | 0.9933 | 0.0067 | 31.4607 |

ERORRATE

```
>> Bayes_Testing(TestData,p1,p2,pc1,pc2)

ans =

    'Error rate in percentage: 14.606742'
```