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Introduction to Logic:-

What is logic?
↳ What? , How? , Why?

What and why of logic?

Logic is the study of methods and principles to discern good arguments from bad ones. Formulation and understanding of arguments

What is Argument?

1. A group of two or more statements.
2. So related that one statement is claimed to follow from the other
3. ~~All specs~~

ex:

- All specs-wearing animals are humans.
- Mohan is a specs-wearing animal
- Therefore Mohan is a human.

The basic constituent of an argument is proposition or statements

Premise(s):- statements which are supporting
Conclusion(s): statements which are supported by premise.

* Relationship of premise and conclusion is called as implication. i.e premises implies conclusion

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Premis

since, because, for, given that, as, owing to as much as, for the reason that, in that, may be inferred from, seeing that ic.

Conclusion

Therefore, accordingly

Good Argument (valid)

1. Premises are acceptable (True)
2. Premises provide sufficient support to the conclusion
3. Premises are relevant to the conclusion.

Argument

* In deductive argument stat 1 and 2 are true.

* In inductive argument,

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All statement → Simple

→ compound

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Simple statement represents a simple fact in reality and if the reality the way it is claimed by the statement then the statement is true otherwise it is false.

Compound statement is that statement in which simple statements occurs as components.

ex: Ramesh is a boy and Sona is a woman. = R·S
R S

constants :- A . . . Z

p, q, r, s :- variables

ex: Ramesh is a boy
L, R = True/false.

A conjuctive compound statement is true only when both of its constituent statements are true.

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Compound Statements

(1) Conjuctions

(2) Disjunctions: Either p or q ($p \vee q$) or unless q

Disjunct

A disjunctive statement is false only when all the disjunct are false.

3. Implication (conditional or hypothetical stateme

If P then Q

$P \supset Q$

antecedent

consequent.

* If A conditional statement is false only when its antecedent is true and its consequent is false.

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

* A conditional statement is true in all such cases when its actecedents are false.

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* A conditional statement is true in all such cases where its consequents is true.

Simple statement		Compound statements		conjuction	Disjun	Implica
p	$\sim p$	p	q	$p \cdot q$	$p \vee q$	$p \supset q$
T	F	T	T	T	T	T
F	T	T	F	F	T	F
		F	T	F	T	T
		F	F	F	F	T

2^n = no. of simple statement

$2^2 = 4 \quad \frac{4}{2} = \frac{2}{2} = 1$

Inclusive sense of disjunction — poor or disable will get sch-

Exclusive sense of disjunction — Tea or coffee = Rs 5

Either p or q but not both

$= (p \vee q) \cdot \sim (p \cdot q)$

conjuction

Exclusive sense of dis.

Truth table

F

T

T

F

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Q Diff b/w disjunction inclusive and exclusive

Disjunction is a compound statement which is identified on the basis of indicators such as "either ... or ..." or "unless". These are two senses of disjunction.

1. Inclusive sense of disjunction
2. Exclusive sense of disjunction

Inclusive s. of d is false only when all the disjuncts are false for eg: Either P or q

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive s. of. d is false ~~when~~ in two conditions
(a) when all the disjuncts are false (p v q)
(b) when all the disjuncts are true.

for eg: Either p or q but not both ~~p~~

p	q	(p v q) . ~(p.q)
T	T	F
T	F	T
F	T	T
F	F	F

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$\sim(S\cdot H) = \sim S \vee \sim H$

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Shivam and saransh will not both go to library

Shivam and Saransh will both not go to library

a.

S	H	$\sim S$	$\sim H$	$S\cdot H$	$\sim S \vee \sim H$	$\sim(S\cdot H)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

It is not the case that both will go to library

b) $\sim S \cdot \sim H \equiv \sim(S \vee H)$

S	H	$\sim S$	$\sim H$	$\sim S\cdot \sim H$	$\sim(S \vee H)$
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Materially equivalent expressions

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Negation	conjunction	Disjunction	Implication conditional statement
$\sim$	$\cdot$	$\vee$	$\supset$

ex: R will go library if and only if (iff) M will go to
a. library. $(R \equiv M)$

b. If R will go to lib, then M will go to lib and if M will go to lib, then R will go to lib.

M	R	$M \supset R$	$R \supset M$	$(M \supset R) \cdot (R \supset M)$	$M \equiv$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Material equivalence is true only when both the components have same value. If they are diff. value material equivalence is false

a) iff $(R \supset M) \cdot (M \supset R)$

Pg 14-15 Exercise I, II, III
Pg 18-19 Exercise I, II & III

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$A \equiv B$

$\Downarrow$

equivalent to

$(A \cdot B) \vee (\sim A \cdot \sim B)$

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Arguement forms / Rules of inference (9)

Valid Arguement Form

1. **Modus Pones (MP)**

$P \supset Q$
P

$\therefore Q$

2. **Modus Tollens (MT)**

$P \supset Q$
$\sim Q$

$\therefore \sim P$

3. **Conjunction**

P
Q

$\therefore P \cdot Q$

4. **Addition (Add)**

P
Q

$\therefore P \vee Q$

5. Constructive Dilemma (CD)

$(P \supset Q) \cdot (R \supset S)$
$P \vee R$
$\therefore Q \vee S$

6. Destructive Dilemma (DD)

$(P \supset Q) \cdot (R \supset S)$ (where $\cdot$ is any conjunction like but, and, however)
$\sim Q \vee \sim S$
$\therefore \sim P \vee \sim R$

7. Simplification

$P \cdot Q$
$\therefore P$

8. Disjunctive syllogism (DS)

$P \vee Q$
$\sim P$
$\therefore Q$

9. Hypothetical synogism (HS)

~~$P \vee Q$~~	$P \supset Q$
~~$\sim P$~~	$Q \supset R$
~~$\therefore Q$~~	$\therefore P \supset R$

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Symbolisation

ex: If A wins the first game then both C and D will their first game.

$$A \supset (C \cdot D)$$

ex: Either if A will win its first game then C win its first game or D wins its first game.

$$(A \supset C) \vee D$$

ex: If A doesn't win its first game then both C and D donot win their first game

$$\{(\sim A) \supset [\sim(C \cdot D)]\}$$

$$\sim A \supset \sim(C \vee D)$$

Homework

Pg 14, 15 ex 1,2,3

Pg 18, 19 ex 1,2,3

Q.

1. $A \supset B$

2. $C \supset D$

3. $(\sim B \vee \sim D) \cdot (\sim A \vee \sim B)$

$\therefore \sim A \vee \sim \cancel{B} C$

4. $(A \supset B) \cdot (C \supset D)$ conjuction on 1 and 2
5. $\sim B \vee \sim D$ by simplication 3
5. $\sim A \vee \sim C$ by 4 and 5 DD

Q.

1. $A \supset (B \cdot \sim C)$

2. $(B \vee C) \supset D$

3. A / $\therefore D$

~~4. D~~

4. $B \cdot \sim C$ by 1 and 3 MP

5. B by 4 simplicatin

6. $B \vee C$ by addition 5

7. D by 2 and 6 MP

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Rules of replacement

1) Demorgan Theoram

$\sim(p \cdot q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \cdot \sim q$

2) Commutation (Com)

$p \cdot q \equiv q \cdot p$

$p \vee q \equiv q \vee p$

3) Association

$p \vee (q \vee r) \equiv (p \vee q) \vee r$

$p \cdot (q \cdot r) \equiv (p \cdot q) \cdot r$

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4) <u>Distribution</u>

$$p \cdot (q \vee r) \equiv (p \cdot q) \vee (p \cdot r)$$

$$p \vee (q \cdot r) \equiv (p \vee q) \cdot (p \vee r)$$

5) <u>Double negation</u>

$$p \equiv \sim p$$

ex:- It is not the case that it is not raining

≡ It is raining

6) <u>Transposition (Trans)</u>

$$p \supset q \equiv \sim q \supset \sim p$$

7) <u>Material Implication</u>

$$p \supset q \equiv \sim p \vee q$$

8) <u>Material equivalence</u>

$$(p \equiv q) \equiv [(p \supset q) \cdot (q \supset p)]$$

$$\equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$$

9) Exportation

$[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$

10) $p \equiv (p \cdot p)$

$p \equiv (p \vee p)$

* These can be used with half part of the statement also

* MI, Demorgan, simpli

N: nebors
W: writing

P: Par get inv in their chi ed.
S: School, ys or leng
L: Lean phone
R: Reading wh cmp
I: They are cintrod to absts
M: Maths will un

3) $P \cdot W$

4) $R \vee M$

1) $(P \vee S) \supset ((L \supset R) \cdot (I \supset M$

2) $(P \cdot \ldots) \supset \ldots$

1) $(P \vee S) \supset ((L \supset R) \cdot (I \supset M))$

2) $(P \vee N) \supset (L \vee I)$

3) $P \cdot W$ / $\therefore R \vee M$

4) P

5) $P \vee N$

6) $L \vee I$

7) $P \vee S$

8) $(L \supset R) \cdot (I \supset M)$

$R \vee M$

Q

1. If she goes on a picnic then she wears a sport clock

2. If she wears a sports clock, then she doesn't attend both the ~~blanket~~ and the dance banquet

3. If she doesn't attend the banquet but she doesn't still have a ticket

then she still has a ticket,

4. She does attend the (dance)

∴ Therefore she doesn't go to P4

1. $P \supset S$

2. $S \supset \sim(B \cdot D)$

3. $(\sim B \supset T) \cdot \sim T$; 4. D / $\therefore \sim P$

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1. $\sim B \supset T$ simp

2. $\sim T$ comut & simp

$\sim(\sim B) = B$

$B \cdot D$

$\sim S$

$\sim P$

Proving invalidity

ex: 1. $(A \supset B) \cdot (C \supset D) \rightarrow$ True

2. $A \vee C \rightarrow$ True

3. $(B \vee D) \supset (E \cdot F) \rightarrow$ True

4. $B \supset (F \supset G)$

5. $G \supset (A \supset H) \rightarrow$ True

$\therefore H$

Premises :- True

conclusion :- False

$\therefore$ Argument invalid

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H → False
A → False
C → ~~False~~ True
D → True
E → True
F → True
G → True
B → T/F

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Rule of indirect proof

1. P_1
2. P_2
3. P_3 $/\therefore A$

4. $\sim A$

P

$\sim P$

Q. Use indirect proof method to construct the proof of validity

1. $(H \supset I) \cdot (J \supset K)$
2. $(I \vee K) \supset L$
3. $\sim L$ $/\therefore \sim (H \vee J)$

5. $H \vee J$

6. $\cancel{\sim L \supset \sim (I \vee K)}$ ~~Transp~~ (2)

7. $\cancel{\sim (I \vee K)}$

6. $I \vee K$ (1), (5) CD

7. L

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8. $L \cdot V \sim (H \supset I)$

9. $\sim (H \supset I)$ DS 8, 3

* The rule of indirect proof is called as reduction and absurdum.

In using this rule we include the negation of conclusion as a premise in the argument and we derive a contradiction. This is based on the rationality we get from the notion of validity which suggest that in a valid argument it is not possible to have a false conclusion from the true premises. If the denial of conclusion leads to an absurdity or contradiction in the argument then the argument is valid.

CP	IP
1. The Rule of CP can be applied only when ~~the conditional is~~ conclusion is the conditional statement	The Rule of IP is ~~valid~~ can be applied on any statement
2. In using CP we include the antecedant of the conclusion as the premise	In using IP we include the negation of conclusion as the premise
3.	

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In the CP method we derieve the consecuent of the conclusion	In the IP method we derieve a contradiction and with the help of the contradiction we derieve the conclusion

Proofs of Tautologies

By CP Method

1. $P \supset (Q \supset P)$
2. P / ∴ $Q \supset P$ — CP
3. Q / ∴ P — CP
4. $Q \supset P$ — 1, 2 MP
5. P — 3, 4 MP

If the argument is valid the corresponding conditional statement of that argument is tautology only.

ex: $[(p \supset q) \cdot p] \supset p$ Proof it is tautology.

1. $\sim [(p \supset q) \cdot p] \supset p$

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$[(p \supset q) \cdot p] \supset p$

1.	$\sim [\sim (p \supset q) \cdot p] \supset p$	
2.	$\sim [\sim (p \supset q) \cdot p] \vee q$	1 MI
3.	$[(p \supset q) \cdot p] \cdot \sim q$	2 Dem
4.	$(p \supset q) \cdot p$	3 simp
5.	$p \supset q$	4 simp
6.	$p \cdot (p \supset q)$	4 com
7.	p	6 Simplication
8.	q	5, 7 MP
9.	$\sim q [(p \supset q) \cdot p]$	3 comm
10.	$\sim p$	9 simpli

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