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Discrete Structures

well defined collection

- Cantor's def
- Intuitive principle : If $p(x)$ is a property

Then

$$S = \{x \mid p(x)\}$$ → Logical $p(x)$ is true

↓

proportion or logical statement

↓

either true or false

$p(x)$ = x is a natural no between 1 and 10

$$S = \{x \mid p(x)\}$$
$$= \{2, 3, 4, 5, 6, 7, 8, 9\}$$

let $A = \{x_1, x_2, \ldots, x_n\}$

we define a function

$$\chi_{A(x)} = \begin{cases} 0, & x \notin A \\ 1, & x \in A \end{cases}$$

Here, $\chi_A(x)$ is called a characteristic function.

$A_1 \subseteq A$

$A_1 = \{x_1, x_2\}$

~~$\chi_{A_1}(x) =$~~

$$\chi_{A_1}(x) = \{1, 1, 0, 0, \ldots\}$$

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$X_A(x) = \{1, 1, 1, 1, 1 \ldots\}$

We can write sets or its subsets in Binary form.

$X = \{x_1, x_2, \ldots x_n\}$

$|X| = n$

$P(X)$ = Power of X.

= The set of all subsets of X

eg $X = \{1, 2, 3\}$

$P(X) = \{\phi, \{1, 2, 3\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$

Total no. of elements in PS $= 2^n$.

$X_A(x) = \{\cdot, \cdot, \cdot, \ldots, \cdot\}$

2 chosen

Total $= 2 \times 2 \times 2 \times \ldots n$ times $= 2^n$

$X = \{$ 82 students $\}$

$A \subseteq X$ $\qquad$ $A = \{\underbrace{0.9, 0.8, 0.95, \ldots}_{\text{fuzzy logic}}\}$

intelligent students

ambiguity

$$\mu_A = \begin{cases} 0 & , x \notin A \quad X_A(x) \\ (0, 1) & , \text{otherwise} \\ 1 & , x \in A \end{cases}$$

$\longrightarrow$ Membership function (Artificial Intelligence)

Notation

eg)

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Relations :

Ordered pair :

Let $A \neq \phi$, $B \neq \phi$

let $a \in A$ & $b \in B$.

Then (a, b) is ordered pair.

Cartesian product :

Let $A \neq \phi$, $B \neq \phi$

Then $A \times B = \{(a, b) \mid a \in A, b \in B\}$

is called the cartesian product of A and B.

Let no. of elements in A = m

" " in B = n

Then, no. of elements in

$A \times B = m \times n$

No. of subsets of $A \times B$ = 2^{mn}

Function :

Let $X \neq \phi$ & $Y \neq \phi$ then

a function f from X to Y is a rule according to which each element of X is associated with a unique element of Y.

[illegible]tion : A function f from X to Y is denoted as

$f : X \longrightarrow Y$

$X = \{1, 2, 3\}$, $Y = \{3, 5, 7\}$

$f : X \longrightarrow Y$ such that

$f(x) = 2x + 1$

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Domain = X

Codomain = Y

Range set / Image set = $\{3, 5, 7\}$

- **Binary Relation :-** A binary relation / operation on a set A is a function such that $f : X \times X$

$$f : X \times X \longrightarrow Y$$
$$f(x, y) = t, \quad x, y, t \in X$$

$$* : X \times X \longrightarrow X$$
$$x * y = t \in X$$
$$\text{i.e.} \quad \boxed{x * y \in X}$$

eg) ① operation addition is binary operation on set of natural no.

② " multiplication " " " .

③ subtraction " is not " "

- **Unary Operation :-**

$$c : P(X) \longrightarrow P(X)$$
$$A \mapsto A^c$$

- **Relation :** A relation between two sets A and B (or A to B) is a subset of $A \times B$ which consists those ordered pair (a, b) such that a is related b according to some rule.

$A = \{1, 3, 5, 7, 9\}$ $\quad |A| = 5$

$B = \{2, 4, 6, 8, 11\}$ $\quad |B| = 5$

$A \times B = 25$

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$R \subseteq A \times B$

Def: (a,b) iff $a \leq b$

$$R = \{ (1,2), (1,4), (1,6), (1,8), (1,11), (3,4), (3,6), (3,8), (3,11), (5,6), (5,8), (5,11), (7,8), (7,11), (9,11) \}$$

Reflexive Relation:

Let $A \neq \emptyset$ then a relation R is said to be reflexive if $(a,a) \in R \; \forall \, a \in A$ or $aRa \; \forall \, a \in A$.

Cartesian product of A with itself $(A \times A)$

Symmetric Relation: $aRb \Rightarrow bRa$

$$(a,b) \Leftrightarrow (b,a)$$

Transitive: aRb and bRc then aRc.

if $(a,b) \in R$ and $(b,c) \in R$
then, $(a,c) \in R$

Let $A = \{a_1, a_2, a_3, \ldots, a_n\}$

and $A \times A = \{ (a_1,a_1), (a_1,a_2), \ldots (a_1,a_n)$
$(a_2,a_1), (a_2,a_2), \ldots (a_2,a_n)$
$\ldots$
$(a_n,a_1), (a_n,a_2), \ldots (a_n,a_n) \}$

$$M = A \times A = \begin{bmatrix} (a_1,a_1) & (a_1,a_2) & \cdots & (a_1,a_n) \\ (a_2,a_1) & (a_2,a_2) & \cdots & (a_2,a_n) \\ \cdots & & & \\ \cdots & & & \\ (a_n,a_1), & (a_n,a_2) & \cdots & (a_n,a_n) \end{bmatrix}$$

No of entries $= n \times n$

Total no. of diagonal elements $= n$

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Let R be a Reflexive relation then the set R will have include diagonal elements of matrix M

Remaining element in M = $n^2 - n$

$R = \{ \underbrace{(a_1, a_1) \ldots (a_n a_n}_{n \text{ times}}, \underbrace{\overset{\downarrow}{2} \; \overset{\downarrow}{2} \; \overset{\downarrow}{2} \ldots}_{} \}$

$2^{n^2-n} = 2^{n(n-1)}$

→ In the reflexive relation apart from diagonal elements of matrix some more elements out of n^2-n elements will appear. Now, from remaining n^2-n elements every element have two choices either it can be included or may not be included. Thus, for remaining n^2-n elements we have $\underbrace{2 \cdot 2 \cdot 2 \cdot \ldots 2}_{n^2-n \text{ times}}$ choices $= 2^{n^2-n}$ choices

Consequently, the no of reflexive relations on a set A of n elements $= 2^{n^2-n} = 2^{n(n-1)}$.

Q) No. of symmetric relation on a set of n element.

Ans) $M = A \times A = $

$$\begin{bmatrix} (a_1, a_1), & (a_1, a_2) & \ldots & (a_1, a_n) \\ (a_2, a_1), & (a_2, a_2) & \ldots & (a_2, a_n) \\ - & - & - & - \\ - & - & - & - \\ (a_n, a_1), & (a_n, a_2) & \ldots & (a_n, a_n) \end{bmatrix}$$

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Sol) Let $A = \{a_1, a_2, a_3, \ldots a_n\}$
we can write $A \times A$ in the matrix form

$$M = \begin{bmatrix} (a_1,a_1) & (a_1,a_2) & \cdots & (a_1,a_n) \\ (a_2,a_1) & (a_2,a_2) & \cdots & (a_2,a_n) \\ \vdots & & & \\ (a_n,a_1) & (a_n,a_2) & \cdots & (a_n,a_n) \end{bmatrix}_{n \times n}$$

principle diagon

Any relation R is a subset of $A \times A$ ie $R \subseteq A \times A$
ie elements of R, are some entries from matrix M.

We know that R is symmetric if $(a,b) \in R \Rightarrow (b,a) \in R$.
We have $\frac{n^2-n}{2}$ elements above and below the principle diagram.

$$R = \{2, 2, 2 \ldots 2^{\frac{n^2-n}{2}}\} = \frac{n^2-n}{2}$$

Total choices of elements below the diagonal $= 2^{\frac{n^2-n}{2}}$
For diagonal elements:
$2 \times 2 \times 2 \times \ldots n \text{ times} = 2^n$.

$$\boxed{\text{Total choice} = 2^{\frac{n^2-n}{2}} \cdot 2^n}$$

The elements below the principle diagonal & above the principle diagonal will be present in the symmetric relation in the similar manner. Therefore, we check the presence of elements below the principal diagonal in the symmetric relation R.

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→ No. of ways in which we can check the belongingness of an element in R = 2 (either it can be in R or it cannot be in R).

→ Total no. of ways in which elements below the principal diagonal can be present in R = $2 \cdot 2 \cdot 2 \cdot 2 \cdots 2 = 2^{\frac{n^2-n}{2}}$

→ Further, n diagonal elements of matrix M can be present in R $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdots n$ times $= 2^n$

∴ Total no. of elements in a ~~transitive~~ symmetric relation are $2^n \cdot 2^{\frac{n^2-n}{2}}$

Q) No. of elements in antisymmetric relation on a set of n elements $= 2^n \cdot 3^{\frac{n^2-n}{2}}$

→ Partition of a set :

Let $X \neq \phi$

and suppose $S_1, S_2, S_3 \ldots S_n$ be n-subsets of X. Then the set $P = \{S_1, S_2, S_3 \ldots S_n\}$ is called partition of X.

if ① $S_1 \cup S_2 \cup S_3 \cup \ldots S_n = X$

② $S_i \cap S_j = \phi$ when $i \neq j$.

Rule : partition cannot contain ϕ.

Ex) ① $\phi = \{1, 2, 3\}$

$S_1 = \{2\}$

$S_2 = \{1, 3\}$

$P = \{S_1, S_2\}$

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Exa

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$S_1 = \{1\}$ $S_2 = \{2\}$ $S_3 = \{3\}$

$P = \{1, 2, 3\}$

→ Union of $S_1, S_2, S_3 = X$

→ Intersection of any two $= \phi$.

Equivalence relation: If it satisfies three properties, then it is said to be equivalence relation

① Reflexive

② Symmetric

③ Transitive

Equivalence class: Let A be any set and R be a relation defined on A

Let $a \in A$, then equivalence class of a is defined as

$$[a] = \{x \in A \mid aRx\}$$

Example :- Let $A = \{1, 2, 3, 4\}$

let $R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$

Clearly, R is equivalence relation.

Equivalence class $P_1 = \{1, 2, 3\}$ $P_2 = \{4\}$

$[1] = \{1, 2, 3\}$

$[2] = \{1, 2, 3\}$

$[3] = \{1, 2, 3\}$

$[4] = \{4\}$

There are two equivalent classes.

Partition This set is classes.

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No. of equivalence relations = No. of partition of that set. ③

→ Fundamental Theorem of Equivalence Relation: There is one-to one correspondence between the number of equivalence relations defined on set of n-elements and no. of parition of the set.

→ Partial Order Relations:

Let $A \neq \phi$. Then R is said to be a partially ordered relation.

① R is reflexive
② R is anti-symmetric
③ R is transitive.

Ex) Let $X \neq \phi$ and $P(X)$ be power set of X. we define a relation '$\leq$' on $P(X)$ such that $A \leq B$ iff $A \subset B$.

① Reflexive

we have, $A \subset A \quad \forall A \in P(X)$.

$A \leq A$

$\therefore$ '$\leq$' is Reflexive

② Anti-symmetric

($A R B$ and $B R A$ $\Rightarrow A = B$)

let $A \leq B$ and $B \leq A$

$\Rightarrow A \subseteq B$ and $B \subseteq A$

$\Rightarrow A = B$

$\therefore$ Antisymmetric is satisfied

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Transitive

let $A \leq B$ and $B \leq C$

$\Rightarrow A \subseteq B$ and $B \subseteq C$

$\Rightarrow A \subseteq C \Rightarrow A \leq C$

$\therefore$ 'Transitive' is satisfied

1. Give two examples of each of the following
   1. Reflexive Relation
   2. Symmetric ——
   3. Antisymmetric ——
   4. Transitive ——
   5. Equivalent ——
   6. Partial order ——

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

2. Find all partitions of the set $(1, 2, 3, 4)$.
~~A×A~~ (Incomplete)

Partially ordered set (POSET).

Let $X \neq \phi$ and '$\geq$' is a partial order on X
Then $(X, \geq)$ is called as partially ordered set.

Example: let $X \neq \phi$, then $(P(X), \leq)$ is a partially ordered set
when $A \leq B$ iff $A \subseteq B \; \forall \; A, B \in P(X)$

Divisiblity of two numbers:

Let $a, b \in N$, then we say that a/b (pronounced as 'a' divides 'b')

Denominator (b), Numerator (a)

if $\exists$ an Integer l such that $\boxed{b = a \cdot l}$

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eg : $5/30$ ✓

$5/29$ ×

Ex 2) Let N be the set of natural numbers we define a relation '$\leq$' on N such that

$a \leq b$ iff $a|b$.

① Reflexive :

$a|a \quad \forall\, a \in N$

$\Rightarrow a \leq b \quad \forall\, a \in N$

$\therefore$ '$\leq$' is reflexive

② Antisymmetric :

Let $a \leq b$ and $b \leq a$

$\therefore$ $a|b$ and $b|a$ (By definition)

$\Rightarrow a = b$

$\therefore$ '$\leq$' is antisymmetric

③ Transitive :

Let $a \leq b$ and $b \leq c$

$\Rightarrow a|b$ and $b|c$

$\Rightarrow a|c$

$\Rightarrow a \leq c$

$\therefore$ '$\leq$' is transitive.

$\therefore$ $(N, \leq)$ is POSET

m^n

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Let D_{14} = The set of divisor of 14.
ie $D_{14} = \{1, 2, 7, 14\}$
Let $a \le b$ iff $a | b$ $\forall a, b \in D_{14}$.
It is POSET

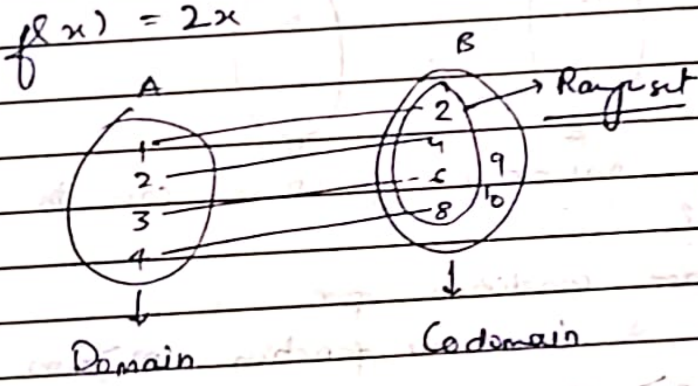
Functions

$f : A \longrightarrow B$

$A = \{1, 2, 3, 4\}$

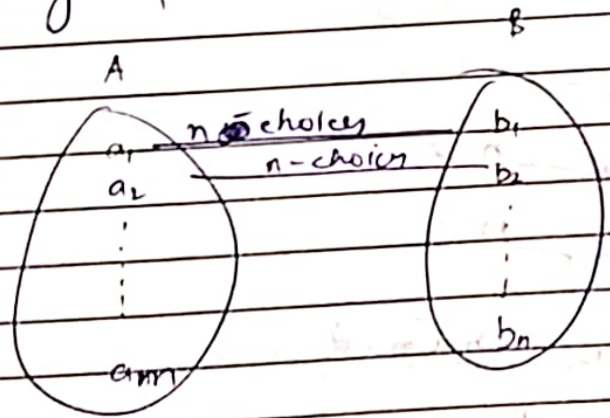
$B = \{2, 4, 6, 8, 9, 10\}$

$f(x) = 2x$



Let $|A| = m$ $\quad$ $|B| = n$

How many functions can be defined from A to B



∴ Total no. of choice $= n \times n \times \cdots m$ time
$= n^m$.

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→ **One - One function:**

A function $f: A \to B$ is said to be one - one if distinct elements of A have distinct images.

ie Let $x, y \in A$ such that $x \neq y$

Then $f(x) \neq f(y)$

or

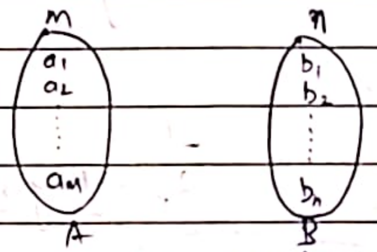
$f(x) = f(y)$

$\Rightarrow x = y$

$m > n$

$m < n$

$m = m$



Q) How many one - one functions can be defined from A to B?

- First condition for one - one function $n \geq m$

Ans No. of one-ones f^ns $\begin{cases} {}^nP_m & \text{if } n \geq m \\ 0 & \text{otherwise} \end{cases}$

$a_1 \underline{\quad n \quad}$

$a_2 \underline{\quad n-1 \quad}$

$a_3 \underline{\quad n-2 \quad}$

$a_4 \underline{\quad n-3 \quad}$

$\vdots$

$a_m \underline{\quad n-(m-1) \quad}$

Total choices $= [(n-(m-1)][(n-(m-2)] \ldots (n-2)(n-1)n$

$\Rightarrow \dfrac{[1 \cdot 2 \cdot 3 \cdot 4 \ldots (n-m)][(n-(m-1)][n-(m-2)] \ldots (n-1)n}{1 \cdot 2 \cdot 3 \cdot 4 \ldots (n-m)}$

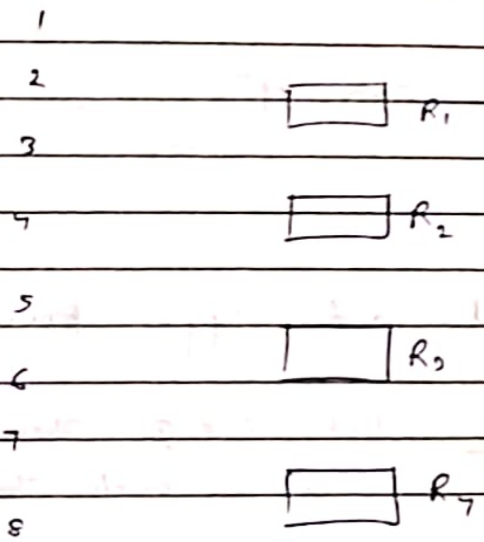
$= {}^nP_m = \dfrac{n!}{\ldots}$

No. of one-one and onto functions.

$$^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! \quad (n=m)$$

No. of onto functions that can be defined.

Ans [redacted]



No. of choices in which one room can be kept vacant $= {}^4C_1 \times 3^8$

" two rooms $= {}^4C_2 \times 2^8$

" three room $= {}^4C_3 \times 1^8$

Total choices $= 4^8$

∴ No. of ways in which 8 guests can be accomodated in 4 room provided no room is left vacant $= 4^8 - ({}^4C_1 \times 3^8 + {}^4C_2 \times 2^8 + {}^4C_3 \times 1)$

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Onto: Each element of co-domain should have a pre-image

Ans: $n^m - \sum_{r=1}^{n-1} {}^nC_r (n-r)^m$

Exercise ① Give two examples of the following :-

① Partially Ordered Set
② One - One f^n
③ Onto - f^n
④ One-one & Onto f^n

→ **Lattice :-**

Lower Bound and Upper Bound of a set

- **Lower Bound:** Let $S \neq \phi$ Then lower bound l of S is a real number such that $l \leq x \;\forall\; x \in S$

- **Upper Bound:** Let $S \neq \phi$ Then upper bound u of S is a real number such that $x \leq u \;\forall\; x \in S$

Finite Set

$S = \{1, 2, 3, 4\}$

Q) What is lower bound of S ?

Ans $1 \leq x \;\forall\; x \in S$ ✓

$0 \leq x \;\forall\; x \in S$ ✓

lower bound of $-1 \leq x \;\forall\; x \in S$ ✓

$2 \leq x \;\forall\; x \in S$ ✗

∴ A given set can have infinite number of lower bounds

↓

1 is greatest among all lower bounds. This is called Greatest Lower Bound (GLB) or Infimum of set (S)

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Greatest Lower Bound or Infimum of a Set:

Let $S \neq \phi$, Then a real no. l is said to be a greatest lower Bound of S if :-

$l \leq x \ \forall \ x \in S$

If l' is another lower bound ~~among~~ of S then $l' \leq l$.

Ex ① Let $S = \{1, 2, 4\}$

Then $glb(S) = 1$

and set S has infinite no. of lower bounds.

② $S = (0, 1)$

$= \{x \mid 0 < x < 1\}$

0 1

$glb(S) = 0$

Note: The glb of a set may or may not belong to the set itself.

Least Upper Bound or Supremum of a Set:

(lub)

Let $S \neq \phi$, then a real no u is said to be lub S if

$x \leq u \ \forall \ x \in S$

If u' is another upper bound of S then $u \leq u'$

① $S = \{1, 2, 3, 4\}$

$lub(S) = 4$

② $S = (0, 1)$

$lub = 1$

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Note: The least upper bound or greatest lower bound of a set if exists is unique.

→ Lattice

Let $(L, \leq)$ be a partially ordered set then
$\leq$ ↓ usual subset relation

① L is said to be a lattice if
$glb\{a,b\} = Inf\{a,b\} = a \wedge b$ exists in L
$\forall\ a, b \in L$

② $lub\{a,b\} = Sup\{a,b\} = a \vee b$ exists in L
$\forall\ a, b \in L$

Ex) ① Let $P(S)$ be the power set of S
Then $(P(S), '\subseteq')$ is a POSET
$P(S) = \{\underbrace{\{S\}, \{\ldots\}, \{\ldots\}, \ldots}_{\text{All subsets of } S}\}$

A B

Let $A \vee B = A \cup B = Sup\{A, B\} \in P(S)$,
$A \wedge B = A \cap B = Inf\{A, B\} \in P(S)$

$A \subseteq A \cup B$	$\Rightarrow A \leq A \cup B$
$B \subseteq A \cup B$	$\Rightarrow B \leq A \cup B$

$\therefore P(S)$ is a lattice

Ex Let L be a lattice then

① $a \vee b = b$ iff $a \leq b$

* ② $a \wedge b = b$ iff $a \geq b$

$a \le b$ $\boxed{a \vee b \le b}$
$b \le b$
$\sup\{a, b\} \le b$

$a \le a \vee b$
$b \le a \vee b$

$a \le b$
$b \le b$
$\sup\{a, b\} \le a \vee b$

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Proof Let $a \le b$

we have $a \vee b = \sup\{a, b\}$ or $\text{lub}\{a, b\}$

$\therefore a \le a \vee b$

and $b \le a \vee b$ —— ① (By def of sup)

By Given cond.

$a < b$

$b \le b$

$\Rightarrow \sup\{a, b\} < b$

$\Rightarrow a \vee b \le b$ —— ②

from ① & ②

$\boxed{a \vee b = b}$

$a \ge b$
$a \le b$
$\Rightarrow \boxed{a = b}$

Ex) $(N, \le)$ ↓ usual sense

Let $a, b \in N$

$a \vee b = \max\{a, b\}$

$a \wedge b = \min\{a, b\}$

'$\le$' in usual sense.

Is $(N, \le)$ a Lattice ?

$(N, \le)$

① Reflexive

$a \le a \quad \forall a \in N$

② Antisymmetric

if $a \le b$ and $b \le a$

Then $a = b$

③ Transitive

Let $a <= b$ and $b <= c$

then $a <= c$

$\Rightarrow \therefore (N, \le)$ is POSET

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Let $a, b \in N$

$a \vee b = \max\{a, b\} \in N$

$\max\{2, 3\} = 3 \in N$

$a \wedge b = \min\{a, b\} \in N$

$\therefore (N, \leq)$ is a lattice.

Q) $(N, /)$ is a Lattice

divisibility

Sol) $a \leq b$ iff a/b $\forall$ $a, b \in N$

① Reflexive

Let we have

$a/a \quad \forall \quad a \in N$

$\therefore a \leq a \quad \forall \quad a \in N$

$\therefore$ '$\leq$' is Reflexive

② Antisymmetric

We have

$a \leq b$ and $b \leq a$

$\Rightarrow a/b$ and b/a

which is possible iff $a = b$

$\therefore$ '$\leq$' is antisymmetric

③ Transitive

Let $a \leq b$ and $b \leq c$ $\forall$ $a, b, c \in N$

$\therefore a/b$ and b/c

$\Rightarrow a/c$

$\Rightarrow a \leq c \quad \therefore$ '$\leq$' is transitive

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$\therefore$ $(N, \leq)$ is POSET

$2 \leq 5 \Rightarrow 2/5$ ✗

$Sup\{a,b\} = a \vee b \Rightarrow 5 \leq 5 \Rightarrow 5/5$ ✓

$Inf\{a,b\} = a \wedge b \Rightarrow 2 \leq 2 \Rightarrow 2/2$ ✓

$5 \leq 2 \Rightarrow 5/2$ ✗

$Sup\{a,b\} = lcm\{a,b\} \in N$

$Inf\{a,b\} = gcd\{a,b\} \in N$

$Sup\{2,5\} = 2 \vee 5 = lcm\{2,5\} = 10 \in N$

$2 \leq 10 \Rightarrow 2/10$ ✓

$5 \leq 10 \Rightarrow 5/10$ ✓

$Inf\{2,5\} = gcd\{2,5\} = 1 \in N$

$1 \leq 2 \Rightarrow 1/2$ ✓

$1 \leq 5 \Rightarrow 1/5$ ✓

$\therefore$ $(N, /)$ is a lattice

→ **Properties of Lattice :-**

Let $(L, \leq, \vee, \wedge)$ be a lattice then following properties are satisfied.

① **Idempotent Property:**

$a \vee a = a \quad | \quad a \wedge a = a \quad \forall \; a \in L$

② **Absorption Law:**

$a \vee (a \wedge b) = a \quad | \quad a \wedge (a \vee b) = a$

↓

b is absorbed

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① **Associative :**

$a \vee (a \vee c) = (a \vee b) \vee c \quad | \quad a \wedge (b \wedge c) = (a \wedge b) \wedge c$

② **Commutative :**

$a \vee b = b \vee a$ and $a \wedge b = b \wedge a \quad \forall \ a, b \in L.$

Q) Idempotent Law follows from Absorption Law.

Sol) From the absorption law, we have

$a \vee (a \wedge b) = a$ —① $\forall \ a, b \in L$

If $a, b \in L$ then $a \vee b \in L$

$\therefore$ In Eq ① we can replace $\boxed{b}$ by $a \vee b$

$a \vee (a \wedge (a \vee b)) = a$

$a \vee a = a \quad \forall \ a \in L$ ($\because a \wedge (a \vee b) = a$ using absorption law)

Similarly $a \wedge a = a \quad \forall \ a \in L$

$\therefore$ Idemp Law follows from absorption Law

* Q) Let $(L_1, \leq, \wedge, \vee)$ and $(L_2, \leq, \wedge, \vee)$ be two lattices then define $L_1 \times L_2$ is also a Lattice

① $L_1 \times L_2$ is: POSET

② $Sup\{(a_1, b_1), (a_2, b_2)\} = \{a_1 \vee a_2, b_1 \vee b_2\}$

③ $Inf\{(a_1, b_1), (a_2, b_2)\} = \{a_1 \wedge a_2, b_1 \wedge b_2\}$

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→ Another definition of Lattice :

A non empty set L with two binary operations '∨' and '∧' is said to be a lattice if it satisfies the following properties :

① Commutative Property :

$$a \vee b = b \vee a$$
$$a \wedge b = b \wedge a \quad \forall \; a, b \in L$$

② Associative Property :

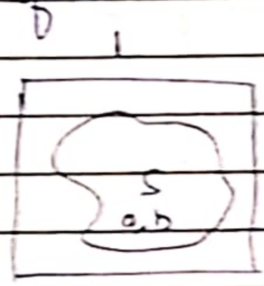
$$(a \vee b) \vee c = a \vee (b \vee c)$$
and $$(a \wedge b) \wedge c = a \wedge (b \wedge c) \quad \forall \; a, b, c \in L$$

③ Absorption Law :-

$$a \vee (a \wedge b) = a \quad , \quad a \wedge (a \vee b) = a$$
$$\forall \; a, b \in L$$

→ Sub lattice : Let L be a lattice, Then a subset S of L is said to be a sub lattice of L.

if $a \vee b \in S \;\; \forall \; a, b \in S$
and $a \wedge b \in S \;\; \forall \; a, b \in S$



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Example) Let $X = \{a, b, c\}$,

$P(X)$ = power set = $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}\}$

$(P(X), \cup, \cap)$ is a Lattice.

Take $\vee \longrightarrow \cup$

$\wedge \longrightarrow \cap$

① Let $A, B \in P(X)$ then

$A \cup B \in P(X)$

$A \cap B \in P(X)$

$\therefore$ $\cap, \cup$ are Binary Operations on $P(X)$

② Commutative :

$A \cup B = B \cup A$, $A \cap B = B \cap A$

$\forall A, B \in P(X)$

③ Associativity :

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

④ Absorption :

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

$\therefore$ Absorption is satisfied

$\therefore$ $(P(X), \cup, \cap)$ is a Lattice.

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Now, Take $S = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$

$\cup$	ϕ	X	$\{a\}$	$\{b\}$	$\{a,b\}$
ϕ	ϕ	X	$\{a\}$	$\{b\}$	$\{a,b\}$
X	X	X	X	X	X
$\{a\}$	$\{a\}$	X	$\{a,b\}$	$\{b\}$	$\{a,b\}$
$\{b\}$	$\{b\}$	X	$\{a,b\}$	$\{b\}$	$\{a,b\}$
$\{a,b\}$	$\{a,b\}$	X	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$

$\therefore$ $A \cup B \in S$ for $A, B \in S$

and $A \cap B \in S$ $\forall$ $A, B \in S$

$\therefore$ S is a sublattice of $P(X)$

Now, Take $S_1 = \{\phi, X, \{a\}, \{b\}, \{c\}\}$

when we take $\to \{a,b\} \notin S_1$

(Hence it union is not a sub lattice

Q) $\to$ Lattice Homomorphism :-

function

~~A function f~~

Let L_1 and L_2 be two lattices then a function $f: L_1 \to L_2$ is said to be a homomorphism if two properties are satisfied

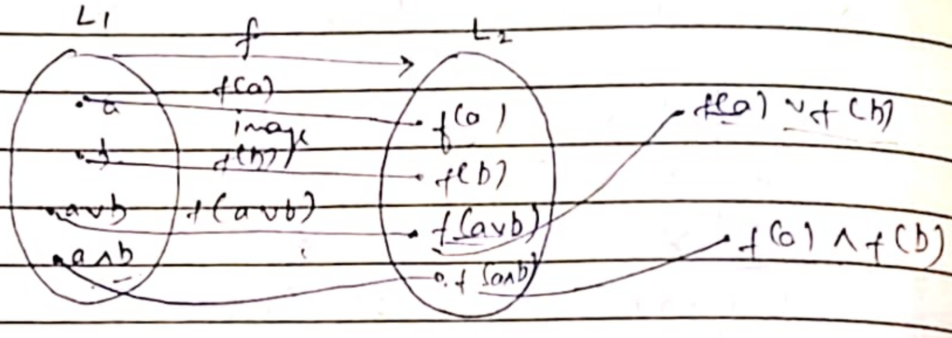
(1) $f(a \vee b) = f(a) \vee f(b)$

(2) $f(a \wedge b) = f(a) \wedge f(b)$

$\forall a, b \in L_1$

set	$\to$ sub set
lattice	$\to$ sub lattice
Boolean Algebra	$\to$ sub algebra
Groups	$\to$ sub groups
Rings	$\to$ Sub Rings
field	$\to$ sub field
Vector Space	$\to$ subspace

Also, the lattices L_1 & L_2 are said to be Homomorphic



L1 & L2 are homomorphic
f is a lattice homomorphism

→ Lattice Isomorphism : A Homomorphism $f : L_1 \to L_2$ is said to be an isomorphism if f is one-one and onto.
function

Structure 1   Structure - 2

Q)1 Give two examples of lattice Homomorphism / Isomorphism

Q)2 Give two examples of lattices which are not
ⓐ Homomorphic

Q)3 Give ~~two~~ one example of two lattices which are homomorphic but not isomorphic

→ Bounded Lattice : A lattice L is said to be bounded if $\exists\ l \in L$ such that $l \leq x\ \forall\ x \in L$ and $u \in L$ such that $x \leq u\ \forall\ x \in L$.

eg) $(P(S), \cup, \cap)$

we have $\emptyset \in P(S)$
s.t
$\emptyset \leq A \quad \forall\ A \in P(S)$
$(\because \emptyset \subseteq A)$

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we have $S \in P(S)$
s.t
$A \subseteq S \quad \forall A \in P(S)$

→ Boolean Algebra:

Let $B \neq \phi$ and '+', '*' be two binary operations on B. Let ' ' ' is a unary operation on B. Also, B has at least two elements '0' and '1'. Then, B is said to be a Boolean Algebra if it satisfies following axioms.

B_1: Commutative law:

$a+b = b+a, \quad a*b = b*a \quad \forall a,b \in B$

B_2: Identity law:

$a+0 = a, \quad a*1 = a \quad \forall a \in B$

B_3: Distributive law:

$a*(b+c) = a*b + a*c, \quad a+(b*c) = (a+b)*(a+c)$
$\forall a, b, c \in B$

B_4: Complement law:

$a+a' = 1, \quad a*a' = 0 \quad \forall a \in B$

0 stands for identity element wrt + operation.
1 stands for identity element wrt * operation.

+ → does not mean usual plus
* → does not mean usual multiplication
0 → " " zero.
1 → " " one

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$S \neq \phi$, $P(S)$

ϕ, S $\to$ universal set

$A \cup B \in P(S)$ $\forall$ $A, B \in P(S)$

$A \cap B \in P(S)$ $\forall$ $A, B \in P(S)$

$\cup, \cap$ are binary operation.

$A \in P(S)$

$\to A' \in P(S)$

C.O.C B_1 $+ \longrightarrow \cup$

$* \longrightarrow \cap$

$' \longrightarrow '$

$0 \longrightarrow \phi$

$1 \longrightarrow S$

B_1 $A + B = A \cup B = B \cup A = B + A$

$\Rightarrow A + B = B + A$ $\forall$ $A, B \in P(S)$

B_2: $A \in P(S)$

$A + \phi = A \cup \phi = A$

$A * S = A \cap (S) = A$

Similarly B_3 & B_4

$\therefore$ $P(S)$ is Boolean Algebra

Notation: $B(+, *, ', 0, 1)$

eg $P(S)(\cup, \cap, ', \phi, S)$

Eg 2) $D_{14} = \{1, 2, 7, 14\}$

$+ \longrightarrow$ LCM

$* \longrightarrow$ GCD

$a' = \frac{14}{a}$

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lcm	1	2	7	14
1	1	2	7	14
2	2	2	14	14
7	7	14	7	14
14	14	14	14	14

all the elements of D_{14}

gcd	1	2	7	14
1	1	1	1	1
2	1	2	1	2
7	1	1	7	14
14	1	2	7	14

all the elements of D_{14}

'	1	2	7	14
$a' = 14/a$	14	7	2	1

B_1 : $lcm(a, b) = lcm(b, a)$ $\forall$ $a, b \in D_{14}$

B_2 $gcd(a, b) = gcd(b, a)$ $\forall$ $a, b \in D_{14}$

B_3 : $a \in D_{14}$

$a + (1) = l(a, 1) = a$

$a * (14)$ $gcd(a, 14) = a$

$0 \longrightarrow 1$

$1 \longrightarrow 14$

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$D = \{1, 3\}$

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B. Complement

$a + (14) = lcm(a, 14) = 14$

$a * (1) = gcd(a, 1) = 1$

Q) D_{35} is a Boolean Algebra.

Q) Let D_n be the set of all divisors of n find the least value of n s.t. D_n is not a Boolean Algebra. $(n = 3)$

Q) $B = \{0, 1\}$ is the smallest Boolean Algebra

→ Properties of Boolean Algebra

Let B be a Boolean algebra, then elements of B satisfies the following properties:

① Idempotent Law:

$a + a = a$ and $a * a = a \quad \forall a \in B$

② Boundness law:

$a + 1 = 1$ and $a * 0 = 0 \quad \forall a \in B$

③ Absorption Law:

$a + (a * b) = a$ and $a * (a + b) = a$

$\forall a, b \in B$

④ Associative law:

$a + (b + c) = (a + b) + c$

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and $a * (b * c) = (a * b) * c$
$\forall a, b, c \in B$

Ex 1) Let $B(+, *, ', 0, 1)$ be a Boolean Algebra. Then
$0' = 1$ and $1' = 0$

Then, Let $B(+, *, ', 0, 1)$ be a Boolean algebra,
then

(i) $(a+b)' = a' * b'$ $\forall a, b \in B$
(ii) $(a * b)' = a' + b'$ $\forall a, b \in B$

Identity Complement

To prove

Proof. ① $(a+b)' = a' * b'$

C I D C

We know

Commutative

$a + a' = 1$

$a * a' = 0$

Distributive

$(a+b) * (a' * b') = 0$

Proof :

$\Rightarrow a * (a' * b') + b * (a' * b')$ ∵ (Distributive law)

$\Rightarrow (a * a) * b' + b * (a' * b')$ ∵ (Associative law)

$\Rightarrow (a * a') * b' + b * (b' * a')$ ∵ (Commutative law)

$\Rightarrow \underbrace{(a * a')} * b' + \underbrace{(b * b')} * a'$ ∵ (Associative law)

$\Rightarrow 0 * b' + 0 * a'$ ∵ (Complement law)

$\Rightarrow 0 + 0$ ∵ (Boundary law)

$\Rightarrow 0$ ∵ (B)

$(a+b) * (a' * b') = 0$ —— ①

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$(a+b) + (a' \cdot b') = 1$

$\Rightarrow (a+b+a') \cdot (a+b+b')$ ($\because$ Distributive law)

$\Rightarrow (1+b) \cdot (1+a)$ ($\because$ complement law)

$\Rightarrow 1 \cdot 1$ ($\because$ Boundedness law)

$(a+b) + (a' \cdot b') = 1$ —— (2)

" Proved "

Using (1) & (2) we have

$(a+b)' = a' \cdot b'$

By principle of duality

$(a \cdot b)' = a' + b'$

$\therefore$ De Morgan's law hold in a Boolean Algebra

Ex) $1' = 0$ and $0' = 1$

Sol) We have

$a + a' = 1$

ie $1 = a + a'$

Taking complement on b/s

$(1)' = (a + a')'$

$(1)' = a' \cdot (a')'$

$(1)' = a' \cdot a$

$\Rightarrow \boxed{(1)' = 0}$

"Proved"

We have

$a \cdot a' = 0$

ie $\Rightarrow 0 = a \cdot a'$

Taking complement on b/s

$(0)' = (a \cdot a')'$

$(0)' = a' + (a')'$

$(0)' = a' + a$

$\boxed{(0)' = 1}$ "Proved"

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→ **Boolean Homomorphism :**

Let B_1 and B_2 be two Boolean Algebra. Then a mapping $f : B_1 \to B_2$ is said to be Boolean Homomorphism if

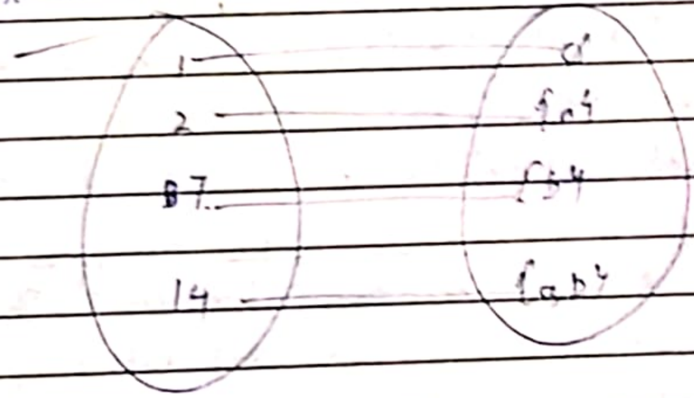
① $f(a+b) = f(a) + f(b)$ $\forall\ a, b \in B_1$
② $f(a*b) = f(a) * f(b)$ $\forall\ a, b \in B_1$
③ $f(a') = [f(a)]'$ $\forall\ a \in B_1$

If the Boolean homomorphism f is 1-1 and onto, then it is called a Boolean isomorphism.

ex 1) $B_1 = \{1, 2, 7, 14\}$ (Divisors of 14) — lcm, gcd

and $B_2 = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ (Powerset — $S = \{a, b\}$, $P(S) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$)

Define $f : B_1 \to B_2$ — one-one function



$f(1) = \phi$
$f(2) = \{a\}$
$f(7) = \{b\}$
$f(14) = \{a, b\}$

Now

$f(1') = f(14) = \{a, b\}$
$f(1) = \phi$
$(f(1))' = (\phi)' = \{a, b\}$

$f(1+2) = f(2)$ — $lcm(1, 2)$
$= \{a\}$

$f(1) + f(2) = \phi + \{a\}$
$= \phi \cup \{a\}$
$= \{a\}$

$\therefore f(1+2) = f(1) + f(2)$

$\therefore f$ is Boolean isomorphic

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Q) Give an example of Boolean algebra which are not homeomorphic and explain the reason

Q) Let B be a Boolean algebra Then show that following conditions are equivalent

① $a + b = b$ (i) ⇒ (ii)
② $a * b = a$ (ii) ⇒ (iii)
③ $a' + b = 1$ (iii) ⇒ (iv)
* ④ $a * b' = 0$ (iv) ⇒ (i)

Assignment - 1

Q) A Boolean algebra cannot have 3 distinct elements

① A Boolean " " "

② Let B be a Boolean algebra, Then prove the following:-

ⓐ $(a*b)' = a' + b'$

ⓑ $(a+b') * (a'+c) * (b+c) = a*c + a'*b + b*c$

ⓒ $(a+b)(b+c)(c+a) = ab + bc + ca$

* ⓓ $(a+b')(b+c')(c+a') = (a'+b)(b'+c)(c'+a)$

③ Let L_1 and L_2 be two sub-Lattices of L, Then show that $L_1 \cap L_2$ is also a sub-lattice of L. Is $L_1 \cup L_2$ is also a sub-lattice? Justify.

$L_1 \cap L_2$ → sub-lattice
$L_1 \cup L_2$ → not a sub lattice

④ Let B_1 and B_2 be two sub-algebras of a Boolean algebra B, Then show that $B_1 \cap B_2$ also a sub-algebra What about $B_1 \cup B_2$?

$B_1 \cap B_2$ - subalgebra
$B_1 \cup B_2$ - not a sub algebra

⑤ Explain the application of Boolean operations in simplification of switching circuits

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Saathi

① Let $(G, *)$ be a ~~group~~ non-empty set and $a * b = ab/2$ $\forall$ $a, b \in G$ then show that G is a group wrt operator '$*$'

② Give an example of two non-isomorphic Boolean algebra

③ What is Hasse diagram? Give two examples

④ Give an example of a relation which is not reflexive

Sub algebra: Let B be a Boolean algebra and $S \subseteq B$. Then S is said to be a sub-algebra of B if

① $a * b \in S$ $\forall$ $a, b \in S$

② $a + b \in S$ $\forall$ $a, b \in S$

③ $a \in S \Rightarrow a' \in S$ $\forall$ $a \in S$

$\subseteq B$ $\subseteq B$

S_1, S_2

$S_1 \cap S_2$

Let a, b be two elements of $S_1 \cap S_2$

Since ~~are~~ $a, b \in S_1$ and $a, b \in S_2$

Since S_1 & S_2 are sub-algebras of B

$a * b \in S_1$, $a * b \in S_2$

$\Rightarrow$ $a * b \in S_1 \cap S_2$

IIIly $a + b \in S_1 \cap S_2$

$\Rightarrow$ $a \in S_1$ & $a \in S_2$

$\Rightarrow$ $a' \in S_1$ & $a' \in S_2$

$\Rightarrow$ $a' \in S_1 \cap S_2$

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