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3C03

Assignment.

Q1. Given a random sample $(X_1, X_2, \ldots, X_n)$
$\mu = \theta_1$ (mean) & $\sigma^2 = \theta_2$ (variance).

Likelihood funcⁿ $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(\frac{(x_i-\theta_1)^2}{2\theta_2}\right)$

To maximize take log on both sides.

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n}\left[-\frac{1}{2}\ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2}\right]$$

(i). differentiate wrt θ_1 [for θ_1]

$$\frac{d}{d\theta_1} \ln L[\theta_1, \theta_2] = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\theta = \sum_{i=1}^{n} \frac{x_i}{n} \quad [\text{mean}].$$

(ii). differentiate wrt θ_2 [for θ_2]

$$\frac{d}{d\theta_2} \ln L[\theta_1, \theta_2] = \sum_{i=1}^{n}\left[\frac{-1}{2\theta_2} + \frac{(x_i-\theta_1)^2}{2\theta_2^2}\right] = 0.$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{2} \sum_{i=1}^{n} (x_i - \theta_i)^2 \quad [\text{variance}].$$

Q2. Binomial Distribution $B(m, \theta)$. $\quad p = \theta$
PMF (Prob mass function). $\quad q = 1 - \theta$.

$$f(x, m, \theta) = {}^{m}C_{x}\, \theta^{x} (1-\theta)^{m-x}.$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}.$$

$$\ln L(\theta) = \sum_{i=1}^{n}\left[\ln {}^{m}C_{x_i} + x_i \ln\theta + m - x_i \ln(1-\theta)\right]$$

differentiate wrt θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right] - 0$$

Find θ.

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right] = 0.$$

$$\sum_{i=1}^{n} \left[\frac{(1-\theta)(x_i) - (m-x_i)\theta}{\theta(1-\theta)} \right] = 0.$$

$$\sum_{i=1}^{n} (1-\theta)\, x_i - (m-x_i)\theta = 0.$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i\, n.$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m.n}$$