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ORIGINAL RESEARCH PAPER

Balancing control of a motorcycle

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ABSTRACT

This article deals with balancing an autonomous motorcycle model along a straight line and curve lines. The dynamic model of the motorcycle balancing is described with an inverted physical pendulum loaded with torque. The torque is provided by the inertia of a rotor driven by a direct current motor. The lean angle of the motorcycle is measured by a smart sensor, which is the feedback signal for the linear quadratic regulator control system. The main purpose of this study is to compensate the error of the smart sensor. Controlling the necessary lean angle of the motorcycle during cornering is also addressed.

KEYWORDS

autonomous motorcycle, self-balancing, optimal control, sensor drifting, simulation

1. INTRODUCTION

Nowadays, personal transport devices e.g., Segways, hover-boards and unicycles, are becoming more and more popular. Their essential feature is that the balance is provided by a controller while driving [1–3]. The dynamic model of these devices is a moving inverted physical pendulum [4]. In addition to balancing, the driving of autonomous vehicles is also widely investigated in literature, e.g., [5–8]. This article does not address the complex problem of vehicle tracking [9], only balancing.

In this article, self-balancing of an autonomous motorcycle is investigated. Its dynamic model is also equivalent to balancing an inverted physical pendulum. The lean angle of the motorcycle is measured by a smart sensor, which is the feedback signal in the control system. The balancing of Segways, hover-boards and unicycles is performed by Proportional Integral Derivative (PID) in [1, 3, 4] or by fuzzy logic [2] controllers. The use of an optimal controller, i.e., a Linear Quadratic Regulator (LQR) [10–12], is also frequent in engineering practice.

Errors caused by various disturbances of lean angle sensors are discussed in detail in a review paper by Shipeng et al. [13]. The drift of the reference point due to heat is regularly recalibrated to a known fixed angular position in [14].

This study sets up an electromechanical model for the mathematical description of motorcycle balancing. The controller required to balance the motor is designed with the LQR method [11, 15]. Among the state variables, the system directly measures the lean angle of the motorcycle and the angular velocity of a balancing rotor. Special attention is paid to determine the theoretically possible maximum lean angle, which gives the limits of the disturbance of the self-balancing. The drift of the smart sensor can cause an unexpected malfunction, e.g., the speed of the balancing rotor increases with the magnitude of the error even in equilibrium. Special attention is focused on the correction of the reference point of the smart sensor. When the motorcycle is cornering, the system automatically calculates the necessary lean angle for the controller to compensate the centrifugal effect.

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2. DESIGNING A REGULATOR FOR MOTORCYCLE BALANCING

2.1. Electromechanical model of the motorcycle

Figure 1 shows a ~17-percent scale model of a motorcycle, which is 0.3 m long, 0.18 m high and its weight is $m = 0.421$ kg.

Balancing a motorcycle can also be thought of the problem of inverted physical pendulum torque (see Fig. 2). The Direct Current (DC) motor driving the rotor shown in Fig. 1 provides the balancing torque M_m, which is shown in Fig. 2.

The dynamic model of an inverted pendulum can be given by an impulse-momentum equation for rigid body in plane motion for fixed axis z at point A [16]:

$$-J_a\ddot{\varphi} = -mgL_s\sin\varphi + M_m, \tag{1}$$

where J_a and m denote the inertia and the mass of the motorcycle, respectively; L_s is the distance between the center of gravity and the ground; and φ is the lean angle. It is noted that in engineering practice $\sin(\varphi) \cong \varphi$ if $\varphi<30°$ because the error is smaller than 5%. Therefore Eq. (1) can be linearized as:

$$\ddot{\varphi} - \frac{mgL_s\varphi}{J_a} = -\frac{M_m}{J_a}. \tag{2}$$

Electromechanical equations of a rotor driven by a DC motor [17, 18] are written as:

$$L\frac{di}{dt} + Ri + k_e\omega_r = u(t), \tag{3}$$

$$J_r\frac{d\omega_r}{dt} = k_m i, \tag{4}$$

where L is the inductance; R is the resistance; J_r is the inertia of the rotor; i is the current; ω_r is the angular velocity of the rotor; $u(t)$ is the regulated terminal voltage of the motor; and k_e, k_m are the voltage and torque constants, respectively.

In Eq. (4), the rotor is driven by the electrical torque of the motor, but friction and air resistance are neglected:

$$M_m = k_m i. \tag{5}$$

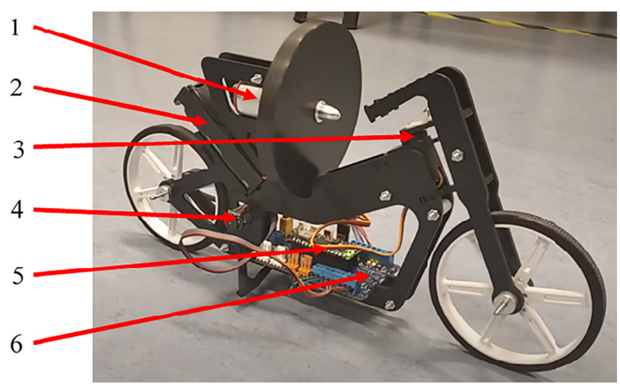

Fig. 1. Motorcycle model built by components from the Arduino Engineering Kit Rev2, 1 – DC motor with balancing rotor; 2 – Battery; 3 – Servo motor; 4 - DC motor; 5 – Arduino Nano 33 IoT development board; 6 – Arduino Motor Carrier

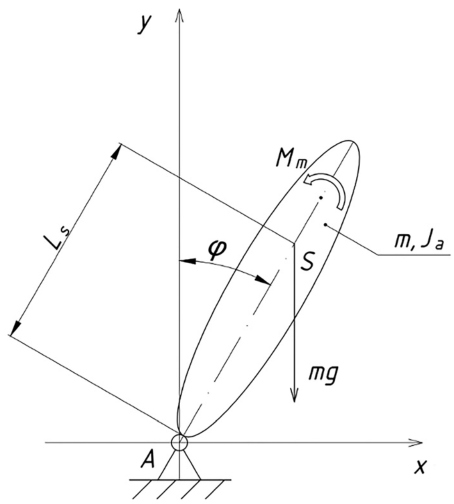

Fig. 2. Inverted pendulum loaded with external torque

2.2. Designing the regulator based on state space equations

The mathematical model Eq. (1)–(5) can also be written in the form of state variables by introducing the following new notations: $x_1 = \varphi$, $x_2 = \dot{\varphi}$, $x_3 = i$, $x_4 = \omega_r$. For the sake of simplicity furthermore let us denote the following: $c_1 = mgL_s/J_a$, $c_2 = k_m/J_a$, $c_3 = R/L$, $c_4 = k_e/L$, $c_5 = k_m/J_r$, $c_6 = 1/L$. After these, the state space equation is expressed with matrices:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u\ (t), \tag{6}$$

where $\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_1 & 0 & -c_2 & 0 \\ 0 & 0 & -c_3 & -c_4 \\ 0 & 0 & c_5 & 0 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ c_6 \\ 0 \end{bmatrix}$.

In order to have a stable self-balancing system the input voltage of the DC motor driving the rotor is defined by negative feedback of the state variables:

$$u(t) = -\mathbf{k}^T\mathbf{x}, \tag{7}$$

where $\mathbf{k}^T$ is the row vector of the gains. The gain parameters of the feedback are determined by designing a linear quadratic controller, which is an optimal controller. The method is based on constrained minimization of the following quadratic cost function [15, 17]:

$$J(\mathbf{x},\ u(t)) = \frac{1}{2}\int_0^T \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + ru(t)^2\right)dt, \tag{8}$$

where $\mathbf{Q} = \mathbf{Q}^T$ and $\mathbf{Q} \geq \mathbf{0}$, $r>0$ are penalty parameters. These are defined by the user, depending on the degree to which the various state variables and the actuator intervention are to be minimized [12, 15, 19, 20]. The constraint is the reordered Eq. (6) to zero. Minimizing the functional Eq. (8) results in practically zero values both for the vector $\mathbf{x}$ and the voltage $u(t)$.

The steps of the controller design are summarized in accordance with [15]:

Step 1: Controllability test:

$$\mathrm{rank}\{\mathbf{C}(\mathbf{A},\mathbf{b})\} = n, \tag{9}$$

where $\mathbf{C}(\mathbf{A},\ \mathbf{b})_{4\times4} = [\mathbf{b}\ \boldsymbol{A}\mathbf{b}\ \boldsymbol{A}^2\mathbf{b}\ \boldsymbol{A}^3\mathbf{b}]$, n is the number of the state variables, which should be equal to 4.

Step 2: Construction matrix $\mathbf{Q}$:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{10}$$

The pivot entries of $\mathbf{Q}$ are set to 1 if the corresponding state variables are measured or determined numerically. Since the current is not measured in the control circuit, the third diagonal is zero.

Step 3: Numerical solution of Control Algebraic Riccati Equation (CARE) [15, 17]:

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{b}r^{-1}\mathbf{b}^T\mathbf{P} + \mathbf{Q} = 0, \tag{11}$$

where $\mathbf{P}$ is a positive definite matrix, which provides the row vector of the gains,

$$\mathbf{k}^T = r^{-1}\mathbf{b}^T\mathbf{P}. \tag{12}$$

Eqs (9) and (11) can be solved by MATLAB or Scilab program systems, when $r = 100$, the vector of gains: $\boldsymbol{k}^T = [-324.3 - 34.2\ 0.0 - 0.106\ ]$.

2.3. Static stability analysis of the control system

The balance of the motorcycle is limited on the one hand by the maximum torque of the motor driving the rotor and on the other hand by the values of the penalty parameters $\mathbf{Q}$ and r in the LQR controller. Observing Fig. 2, the torque at point A due to the gravitational force at the maximum lean angle can be balanced by the maximum torque of the DC motor applied to the rotor:

$$\varphi_{\max} = \frac{M_m^{\max}}{mgL_s} = \frac{k_m i_{\max}}{mgL_s} = 0.0468\ \mathrm{rad} = 2.679°, \tag{13}$$

where $M_m^{\max}$ is the stall torque of the DC motor, $L_s = 76.2$ mm, $i_{\max} = 2.6$ A, $k_m = 5.6596\cdot10^{-3}\frac{\mathrm{Nm}}{\mathrm{A}}$, $g = 9.81\cdot\frac{\mathrm{m}}{\mathrm{s}^2}$. Under dynamic conditions, this limit cannot be reached, i.e., $\varphi<\varphi_{\max}$. When disturbance causes a larger lean angle than $\varphi_{\max}$ the controller cannot balance the motorcycle, due to the limited torque of the DC motor, and it falls.

2.4. Resetting the reference point of the angle sensor

The reference point of the sensor measuring the lean angle drifts due to warming up. The measured angle φ_m changes slowly with time with an increasing error $\varepsilon(t)$. The following relation can be written for the measured φ_m and real φ values of the lean angle:

$$\varphi_m(t) = \varphi(t) + \varepsilon(t). \tag{14}$$

During the test, it occurred that the rotor had nonzero angular velocity even when the motorcycle was in a stable vertical position, due to drifting of the reference point. The optimal controller LQR tends to reduce the state vector $\mathbf{x}$ and the input voltage $u(t)$ depending on the feedback state variables, i.e.,

$$\lim_{t\to\infty}\mathbf{x}(t) = 0, \quad \lim_{t\to\infty}u(t) = 0. \tag{15}$$

If the motorcycle is permanently close to equilibrium, then the actual φ and $\dot{\varphi}$ can be considered to be zero, and assuming that $\varepsilon(t)$ changes slowly over time, i.e., $\dot{\varepsilon}(t) \ll 1$, therefore it is negligible. Finally based on Eqs (7), (14) and (15) the error $\varepsilon(t)$ can be obtained:

$$\varepsilon(t) = -\frac{k_4\omega_r}{k_1}. \tag{16}$$

As it is known, the error of the angle, the reference point of the angle sensor can always be reset when the motorcycle is close to equilibrium position, i.e., φ and $\dot{\varphi}$ are considered to be zero.

2.5. Cornering maneuver

The theory discussed in the previous subsections dealt with balancing in a stationary position or during motion in a straight line. When the motorcycle is cornering the centrifugal force tilts the structure out of the vertical position. Therefore, it should be leant to a certain angle in order to achieve equilibrium between the torques of the gravity and the centrifugal forces:

$$mgL_s\sin\varphi = \frac{mv_s^2}{r_s}L_s\cos\varphi. \tag{17}$$

Hence the stable lean angle φ_s in the course of cornering is:

$$\varphi_s \cong \frac{v_s^2}{r_s\cdot g}, \tag{18}$$

where r_s is the radius of cornering circle measured from its center to the gravity center of the motorcycle, and v_s is the velocity of the motorcycle.

Taking into account the calculated lean angle for the LQR controller, the function of Eq. (8) is modified as follows:

$$J(\mathbf{x},u(t)) = \frac{1}{2}\int_0^T\left(\begin{bmatrix}(\varphi-\varphi_s) & \dot{\varphi} & i & \omega_r\end{bmatrix}\mathbf{Q}\begin{bmatrix}(\varphi-\varphi_s) \\ \dot{\varphi} \\ i \\ \omega_r\end{bmatrix} + ru(t)^2\right)dt, \tag{19}$$

where $u(t) = -(k_1(\varphi_m - \varphi_s) + k_2\dot{\varphi}_m + k_4\omega_r)$.

This theoretical consideration makes it suitable for this particular motorcycle to run on a complex track, performing complicated maneuvers.

Experience has also confirmed that the theoretically possible maximum angular deviation, i.e., the disturbance is very small (approx. 2.5°). The system is not able to compensate larger dynamic disturbances; therefore it is only advisable to perform cornering, acceleration and deceleration gradually near the stable position.

Based on the theoretical context described in Section 2, a program in C++ programming language for the balance control, forward and reverse motions and cornering of a motorcycle, has been developed and uploaded onto an Arduino Nano 33 IoT development board. The tests proved the efficient operation of the motorcycle within the interference limits described here.

3. SIMULATION OF THE SENSOR RESETTING

Based on the theory derived in Subsection 2.2 and 2.3, a simulation program is developed using the Scilab software. The following model parameters have been determined based on the data of datasheets and experiments: $J_a = 3.5 \cdot 10^{-3}$ kgm^2, $J_r = 9.5 \cdot 10^{-5}$ kgm^2, $R = 2.3077$ Ω, $L = 3.4 \cdot 10^{-4}$ H, $k_e \cong k_m$, $U_{\max} = 6$ V.

Assuming initial disturbance $\dot{\varphi}(0) = 0.43$ rad/s, the following three different operating conditions have been analyzed:

- Case A: Ideally operating lean angle sensor is assumed with constant 0 reference point;

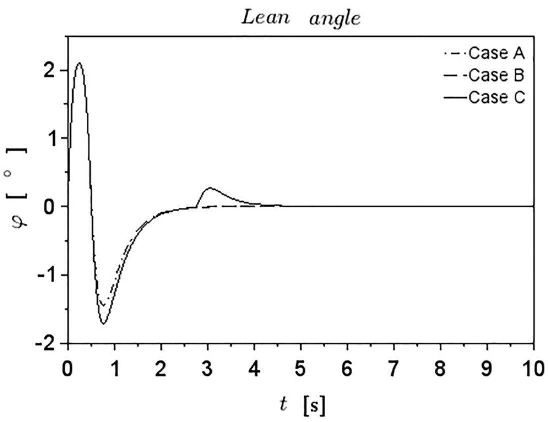

Fig. 3. Change of the lean angle

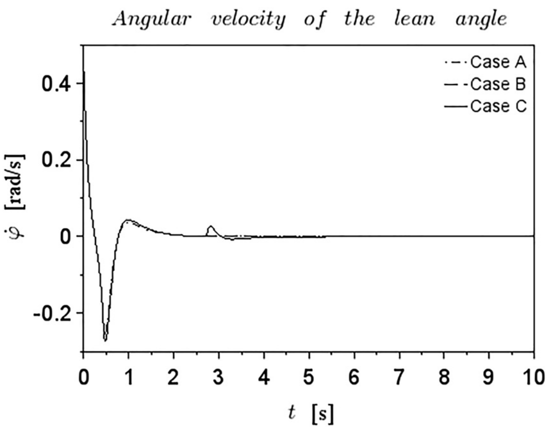

Fig. 4. Change of the time derivative of the lean angle

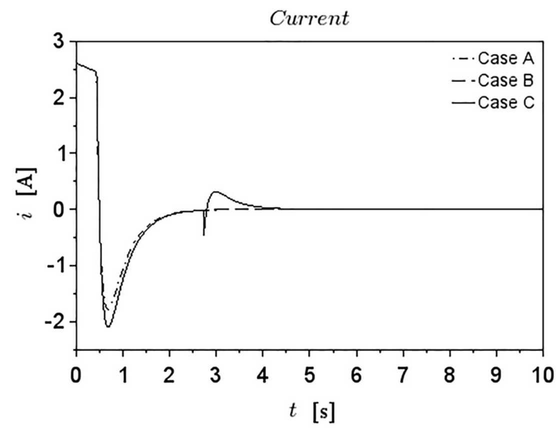

Fig. 5. Change of the current

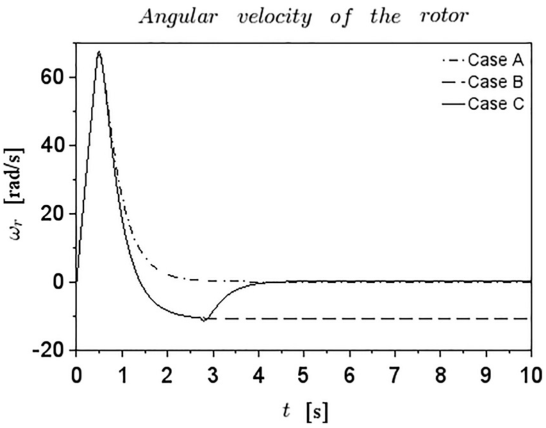

Fig. 6. Change of the angular velocity of the rotor

- Case B: The reference point of the lean angle sensor drifts with a constant error of 1.95°;
- Case C: With the same drifted reference point as in Case B, the lean angle sensor is reset automatically in the vicinity of the stand-still position of the motorcycle.

As for the lean angle, its time derivative and the current, their changes over time show good agreement in Cases A and B (see Figs 3–6). However, the angular velocities of the rotor are different due to assumed constant error of the sensor. In Case C, when φ and $\dot{\varphi}$ are considered to zero, the system automatically resets the reference point of the sensor, which results in decreasing angular velocity of the rotor and causes transients in the rest of the state variables.

4. CONCLUSIONS

This paper dealt with the balancing control of a motorcycle model, but problems of tracking a given path are not addressed. The limit of the lean angle with respect to the equilibrium position and also the necessary lean angle in the course of cornering has been determined and the appropriate tracking controller has been designed.

The self-balancing of the motorcycle is performed by an LQR optimal controller. From a practical point of view, it is important that the proposed controller is able to correct the drifted reference point of the angle sensor, and at the same time to slow down the balancing rotor. Without compensating the error of the sensor the motorcycle could be balanced efficiently only in one direction, depending on the direction of the angular velocity of the rotor.

Further development of the system is being considered, which would allow the feedback of unmeasured state variables using an observer. Modifying the structure of the system of the electromechanical equations with an integrator for the angular velocity of the rotor could be another approach to compensate for the effect of the lean angle error. In addition to balancing of the motorcycle, non-holonomic trajectory optimization based on Isidori's nonlinear control will be necessary to track a given path.

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