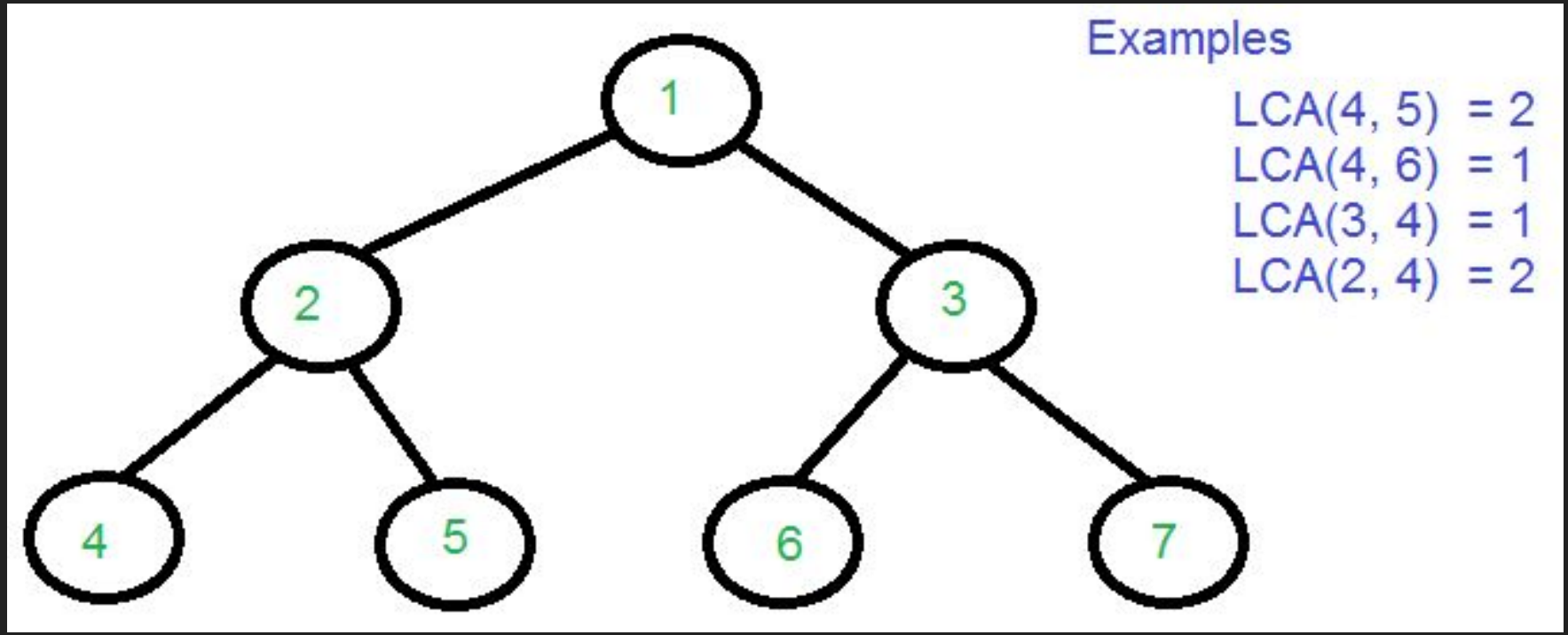
What is LCA



Subproblem:
Find the kth parent of a node in a tree.

Binary Lifting !!!

Let’s express k in binary k = 11

$11 = 2^3 + 2^1 + 2^0$

Let’s say we are at node u.

V1 = 8^{th} parent of u

V2 = 2^{nd} parent of v1

V3 = 1^{st} parent of v2

V3 is our answer.

So, let’s say we store the $(2^k)^{th}$ parent for each node in the tree. How much memory are we using?

NLogN

If we have N nodes, then we only have to store k parents such that $2^k \leq N$. So, $k \leq \log N$ for each node. Hence, NlogN

Now, after this preprocessing we can get the $(2^k)^{th}$ parent of each node in O(1).

So, in order to find kth parent of any node, we need to do at most LogK operations.

Pre-processing for binary lifting

```
//Assume you have the just immediate parent already.
//If you don't have that, you can run a dfs and get that.
for (int i = 1; i < limit; i++)
{
    for (int j = 0; j < n; j++)
    {
        //2^ith parent of node j
        int x = parent[j][i - 1];  // x = 2^(i-1)th parent
        parent[j][i] = parent[x][i - 1]; // 2^ith parent = 2^(i-1)th parent of x
    }
}
```

Finding k^{th} parent of a node

```
int kth_parent(int node, int **parent, int k) {
    if (k == 0)
        return node;
    int log_val = log2(k);
    int max_power = pow(2, log_val);
    return kth_parent(parent[node][log_val], parent, k - max_power);
}
```

Practice Problem

https://cses.fi/problemset/task/1750

Finding the LCA

```
int find_lca(int a, int b, int *level, int limit)
{
    if (level[a] > level[b])
        swap(a, b);
    int d = level[b] - level[a];
    b = kth_parent(b, parent, d);

    //now a and b are at same level
    if (a == b) // return a if both are at same node
        return a;


    for (int i = limit - 1; i >= 0; i--)
    {
        //try to move the nodes a and b upwards till they both are not equal
        if (parent[a][i] != -1 && (parent[a][i] != parent[b][i]))
        {
            a = parent[a][i];
            b = parent[b][i];
        }
    }
    //now a and b's just next parent is their LCA
    return parent[a][0];
}
```

Distance Queries

Given 2 nodes A and B, find the distance between them.

1) Unweighted Tree
2) Weighted Tree

Problems

Counting Paths:
https://cses.fi/problemset/task/1136\

Video Editorial:
https://www.youtube.com/watch?v=8Nq3THy2Kw0