Dynamic Programming
Day 1

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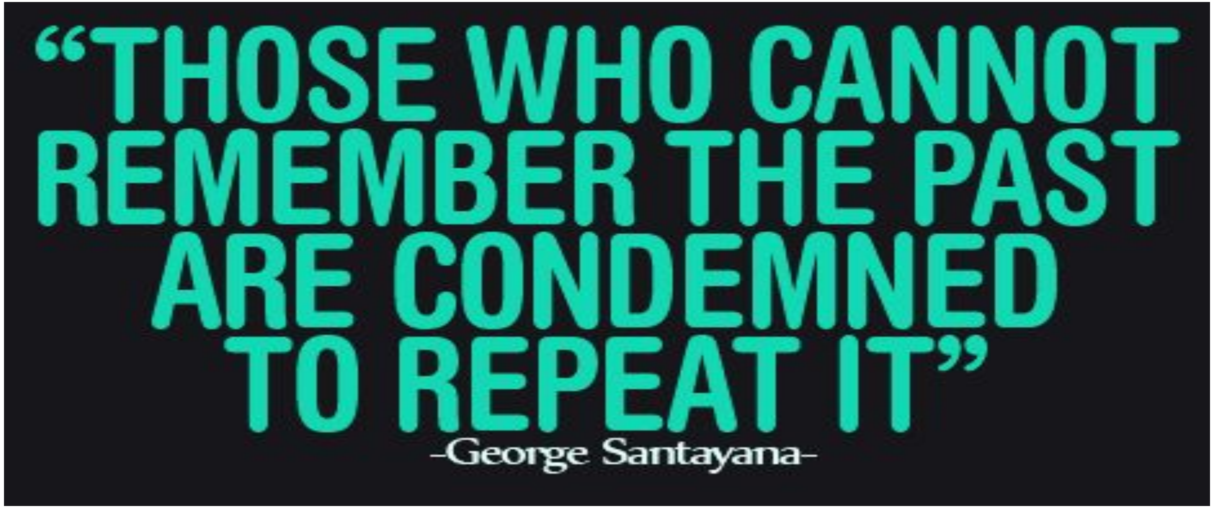
Why Dynamic Programming?

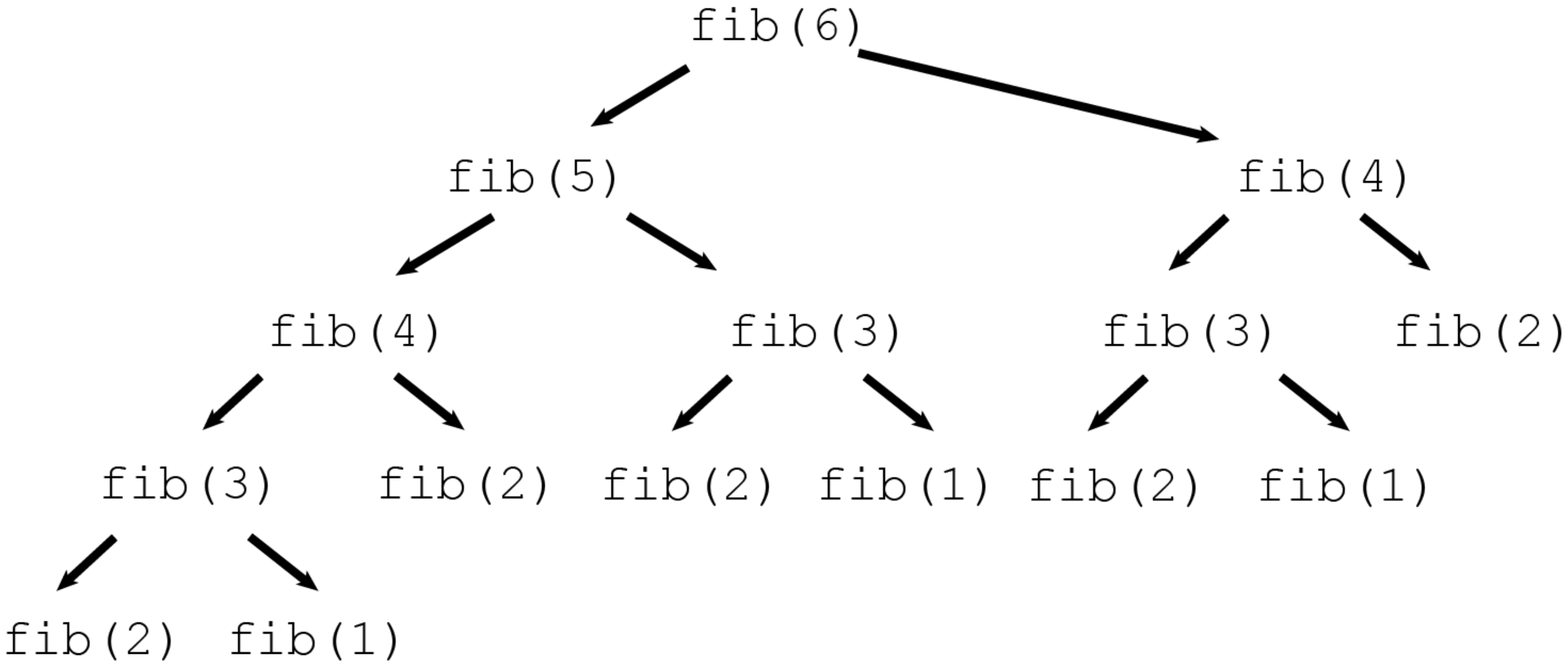


Why Dynamic Programming?

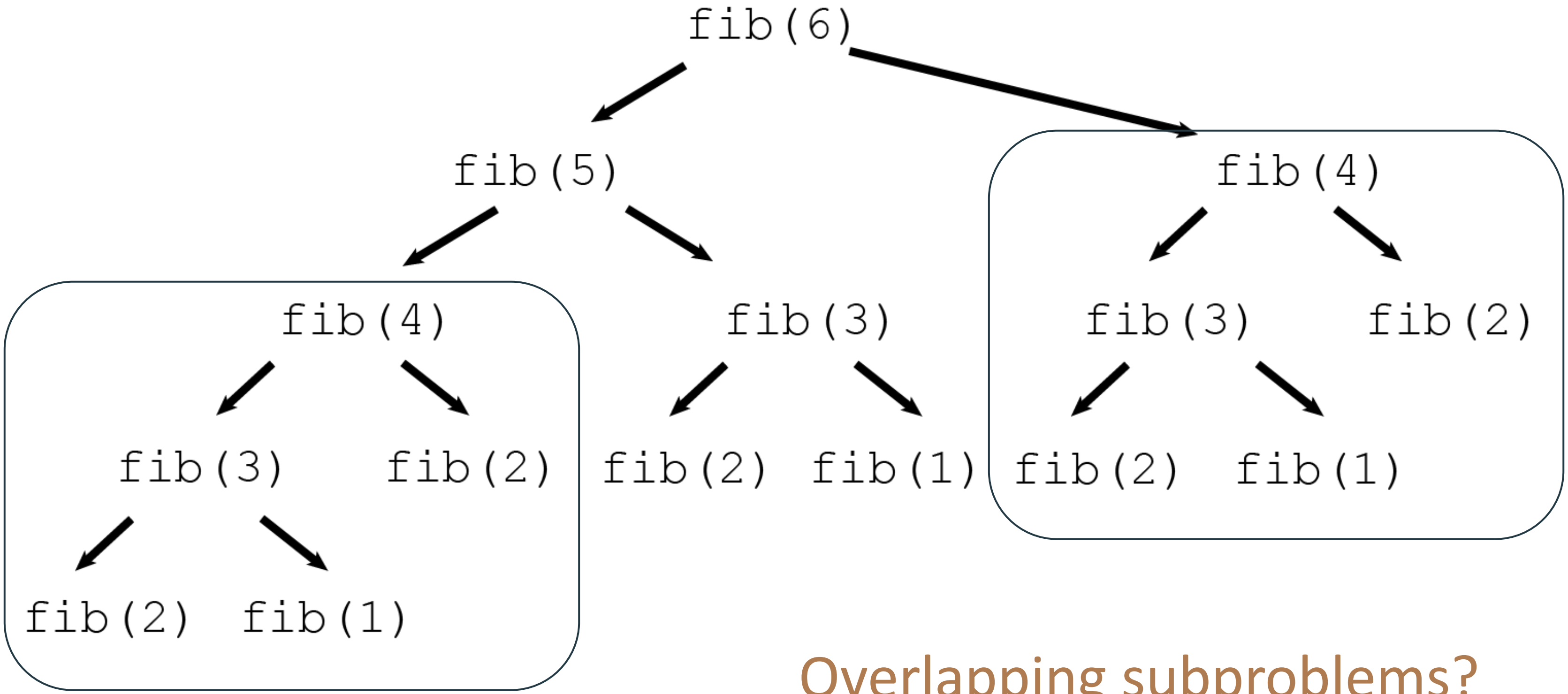
- Overlapping subproblems
- Maximize/Minimize some value
- Finding number of ways
- Covering all cases (DP vs Greedy)
- Check for possibility

Need of DP

- Let’s understand this from a problem
    - Find n^{th} fibonacci number
    - $F(n) = F(n - 1) + F(n - 2)$
    - $F(1) = F(2) = 1$



Any problem here?



Overlapping subproblems?

Memoization

- Why calculate F(x) again and again when we can calculate it once and use it every time it is required?
  - Check if F(x) has been calculated
    - If No, calculate it and store it somewhere
    - If Yes, return the value without calculating again

Without DP

```
int functionEntered = 0;
int helper(int n){
    functionEntered++;
    if(n == 1 || n == 2){
        return 1;
    }
    return helper(n - 1) + helper(n - 2);
}
void solve(){
    int n;
    cin >> n;
    cout << helper(n) << nline;
    cout << functionEntered << nline;
}
```

functionEntered = 1664079
with n = 30

With DP

```cpp
int functionEntered = 0;
int dp[40];
int helper(int n){
    functionEntered++;
    if(n == 1 || n == 2){
        return 1;
    }
    if(dp[n] != -1)
        return dp[n];
    return dp[n] = helper(n - 1) + helper(n - 2);
}
void solve(){
    int n;
    cin >> n;
    for(int i = 0; i <= n; i++)
        dp[i] = -1;
    cout << helper(n) << nline;
    cout << functionEntered << nline;
}
```

functionEntered = 57
with n = 30

Let’s solve another problem!

Given a 2D grid (N X M) with numbers written in each cell, find the path from top left (0, 0) to bottom right (n - 1, m - 1) with minimum sum of values on the path

1	5	8
6	2	7
9	3	4

Naive Way

Explore all paths. Standing at (i, j) try both possibilities (i + 1, j), (i, j + 1)

Every cell has two choices

Time complexity: $O(2^{m*n})$?

Actual Time complexity: $O(C(n + m - 2, m - 1))$

Efficient Way

Overlapping subproblems

Memoization

Time complexity: O(n * m)

Space complexity: O(n * m)

```
int grid[n][m]; // input matrix

int dp[n][m]; // every value here is -1

// subproblem: f(i, j) represents minimum sum path from (i, j) to (n - 1, m - 1)
int f(int i, int j){ //
    if(i >= n || j >= m){ // moving outside the grid // not allowed
        return INF;
    }
    if(i == n - 1 && j == m - 1) // reached the destination
        return grid[n - 1][m - 1];

    if(dp[i][j] != -1) // this state has been calculated before
        return dp[i][j];

    // state never calculated before
    dp[i][j] = grid[i][j] + min(f(i, j + 1), f(i + 1, j));
    return dp[i][j];
}

void solve(){
    cout << f(0, 0) << nline;
}
```

Important Terminology

State: A subproblem that we want to solve. The subproblem may be complex or easy to solve but the final aim is to solve the final problem which may be defined by a relation between the smaller subproblems. Represented with some parameters.

Transition: Calculating the answer for a state (subproblem) by using the answers of other smaller states (subproblems). Represented as a relation b/w states.

Exercise

Fibonacci Problem:

- State
  - dp[i] or f(i) meaning i^{th} fibonacci number
- Transition
  - dp[i] = dp[i - 1] + dp[i - 2]

Exercise

Matrix Problem:

- State
  - dp[i][j] = shortest sum path from (i, j) to (n - 1, m - 1)

- Transition
  - dp[i][j] = grid[i][j] + min(dp[i + 1][j], dp[i][j + 1])

Let’s solve another problem

Given an array of integers (both positive and negative). Pick a subsequence of elements from it such that no 2 adjacent elements are picked and the sum of picked elements is maximized.

1	4	2	-10	10	5

Sum = 14

1	4	2	-10	10	5

Sum = 13

Some ways to solve the problem

1. Having 2 parameters to represent the state

State:

dp[i][0] = maximum sum in (0 to i) if we don't pick ith element

dp[i][1] = maximum sum in (0 to i) if we pick ith element

Transition:

dp[i][0] = max(dp[i - 1][1], dp[i - 1][0])

dp[i][1] = arr[i] + dp[i - 1][0]

Final Answer:

max(dp[n - 1][0], dp[n - 1][1])

Some ways to solve the problem

2. Having only 1 parameter to represent the state

State:

dp[i] = max sum in (0 to i) not caring if we picked i^{th} element or not

Transition: 2 cases

- pick i^{th} element: cannot pick the last element : arr[i] + dp[i - 2]
- leave i^{th} element: can pick the last element : dp[i - 1]

dp[i] = max(arr[i] + dp[i - 2], dp[i - 1])

Final Answer:

dp[n - 1]

```cpp
int a[n]; // input array

int dp[n]; // filled with -INF to represent uncalculated state

// f(i) = max sum till index i
int f(int index){
    if(index < 0) // reached outside the array
        return 0;
    if(dp[index] != -INF) // state already calculated
        return dp[index];

    // try both cases and store the answer
    dp[index] = max(a[index] + f(index - 2), f(index - 1));
    return dp[index];
}

void solve(){
    cout << f(n - 1) << nline;
}
```

Time and Space Complexity in DP

Time Complexity:

Estimate: Number of States * Transition time for each state

Exact: Total transition time for all states

Space Complexity:

Number of States * Space required for each state