

Dynamic Programming

Day 6

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Solving Homework Problems

Problem 1: [Link](#)

- State
 - $dp[i] = \text{max profit from } [0 \dots i] \text{ in sorted list of projects}$
- Transition
 - $dp[i] = \max(dp[i - 1], \text{profit}[i] + dp[j])$ s.t. j is the max index for which $\text{end}[j] < \text{start}[i]$
- Base Case
 - $dp[0] = \text{profit}[0]$
- Final Subproblem
 - $dp[n - 1]$

DP with Bitmasking

- Bitmasks
- Basic operations on Bitmasks
- Limitations on “N” (You will need 2^n integers to represent all the subsets)

Problem 1:

Given a list of points on a 2D plane, rearrange these points in any way such that in the final permutation of points, the sum of distances of the adjacent elements is minimized.

Constraints: $[N \leq 15]$, $[-1e9 \leq X_i, Y_i \leq 1e9]$

Points : $\{\{0, 0\}, \{5, 6\}, \{1, 2\}\}$

Best permutation $\rightarrow \{\{0, 0\}, \{1, 2\}, \{5, 6\}\}$

Ans = $\text{Dist}(P1, P3) + \text{Dist}(P3, P2)$

Problem 1: TC: $O(n^3 2^n)$, SC: $O(n^2 2^n)$

state:

`dp[i][bitmask][last element]` = minimum sum of distances in the suffix `[i... n - 1]`
such that `bitmask` represents the elements in the first `i - 1` elements and `last element` represents the last point

transition:

check for `j`th point from `(0 to n - 1)`

can you pick the `j`th point as the `i`th element in the final array or not

`if(bitmask & (1 << j))` { whether `j`th bit is set or not

`continue;`

`}else{`

`dp[i][bitmask][last element] = min(dp[i][bitmask][last element],
(bitmask != 0 ? dist(j, last element) : 0) + dp[i + 1][bitmask | (1 << j)][j])`

`}`

base case:

`dp[n][(1 << n) - 1][anything] = 0`

final subproblem

`dp[0][0][anything]`

Problem 1: TC: $O(n^2 2^n)$, SC: $O(n \cdot 2^n)$

state:

`dp[bitmask][last element]`

`i = set_bits(bitmask)`

= minimum sum of distances in the suffix `[i... n - 1]` such that the bitmask represents the elements in the first `i - 1` elements and last element represents the last point

transition:

check for `j`th point from `(0 to n - 1)`

can you pick the `j`th point as the `i`th element in the final array or not

`if(bitmask & (1 << j))` { whether `j`th bit is set or not

`continue;`

`}else{`

`dp[bitmask][last element] = min(dp[bitmask][last element],`

`(bitmask != 0 ? dist(j, last element) : 0) + dp[bitmask | (1 << j)][j]`

`}`

base case:

`dp[(1 << n) - 1][anything] = 0`

final subproblem

`dp[0][anything]`

Problem 2: Link (Homework)

- State
 -
- Transition
 -
- Base Case
 -
- Final Subproblem
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