Assignment 3 - Constrained optimization

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Due March 1, 2019 at 11:59 p.m.

For full credit, submit your assignment as a single zip file containing the pdf of your responses and all the code used to answer the questions. Make sure to include all your `.py` files in the zip file that you submit. A reminder that it is OK to discuss but the code and responses should be worked on individually.

1. Consider the following optimization problem.

$$\min_{x_1,x_2} (x_1-2)^4 + (x_2-2)^4$$
$$\text{subject to } x_1 + x_2 = 2$$

   (a) Solve the above optimization problem with a quasi-Newton update and a line search algorithm of your choice using:

      i. the quadratic penalty method (using as large a ρ as you can)
      ii. the method of Lagrange multipliers

   and report the solution using both methods.

   (b) Plot (1) the contours of the objective function, (2) the zero-contour for the constraint (the line along which the constraint is satisfied), and (3) path of the (x_1, x_2) iterates, all on the same set of axes. Start from an initial guess of $(x_1, x_2) = (-1, 1)$.

2. Consider the following optimization problem.

$$\min_{x_1,x_2} (1-x_1)^2 + 100(x_2 - x_1^2)^2$$
$$\text{subject to } x_1 + x_2 = 1$$

   (a) Solve the above optimization problem with a quasi-Newton update and a line search algorithm of your choice using:

      i. the quadratic penalty method (using as large a ρ as you can)
      ii. the method of Lagrange multipliers

   and report the solution using both methods.

   (b) Plot (1) the contours of the objective function, (2) the zero-contour for the constraint (the line along which the constraint is satisfied), and (3) path of the (x_1, x_2) iterates, all on the same set of axes. Start from an initial guess of $(x_1, x_2) = (-1, 1)$.

3. List the pros and cons of the method of Lagrange multipliers and the quadratic penalty method.