Multivariate Data Analysis - Presentation

## Multivariate Data Analysis Spring 2019 (37459-2019-SPRING-CITY)

### Assignment: 1

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## Question 1

### Step 1: Download the dataset from online

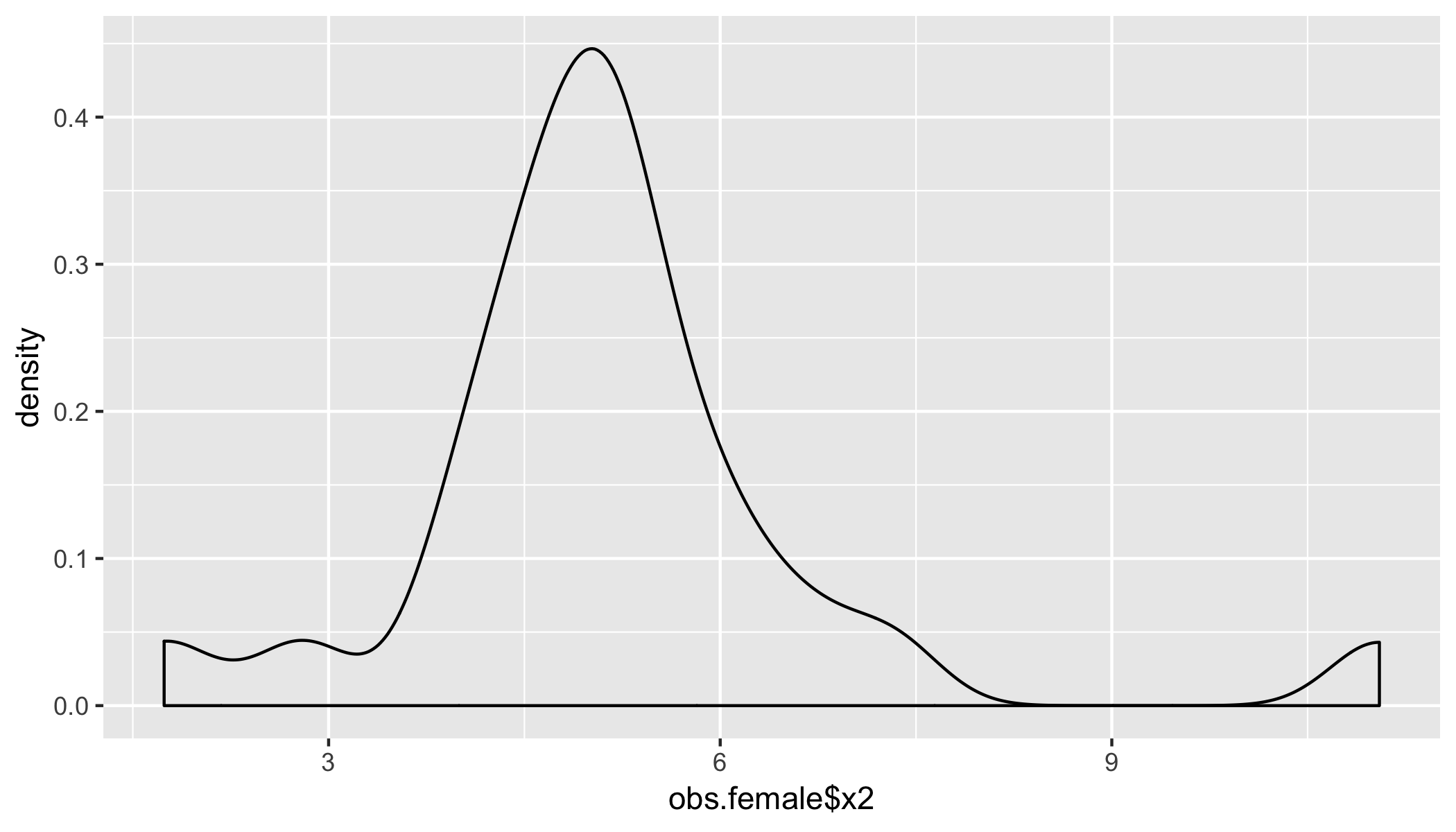
$$\_{n=1}^{1

wd <- getwd()  
res <- readLines("http://users.stat.umn.edu/~kb/classes/5401/files/data/JWData5.txt")  
# Figure out which lines we need. Each table reference will appear twice. Need to know the table you want and the next in the list  
  
start <- grep("T06\_12", res)[2]  
  
end <- grep("T06\_13", res)[2]  
  
rawtable <- res[start:(end - 2)]  
  
# Use the first line to find the dimensions of the table  
  
infovec <- strsplit(rawtable[1], " ")[[1]]  
  
infovec <- infovec[infovec != ""]  
  
length <- as.numeric(infovec[2])  
  
# Extract the rows containing the data (the last few rows)  
  
start <- length(rawtable) - length + 1  
  
rawdata <- rawtable[start:length(rawtable)]  
  
# Split the row into each data point  
  
final\_data <- strsplit(rawdata, " ")  
  
# Organise into a matrix, removing blank entries  
  
datatable <- matrix(0, length, as.numeric(infovec[3]))  
  
for (i in 1:length) {  
 row <- final\_data[[i]]  
  
 datatable[i, ] <- as.numeric(row[row != ""])  
}  
  
# Arrange in a data frame  
  
data.frame <- as.data.frame(datatable)  
colnames(data.frame) <- c("Gender", "x1", "x2", "x3", "x4")

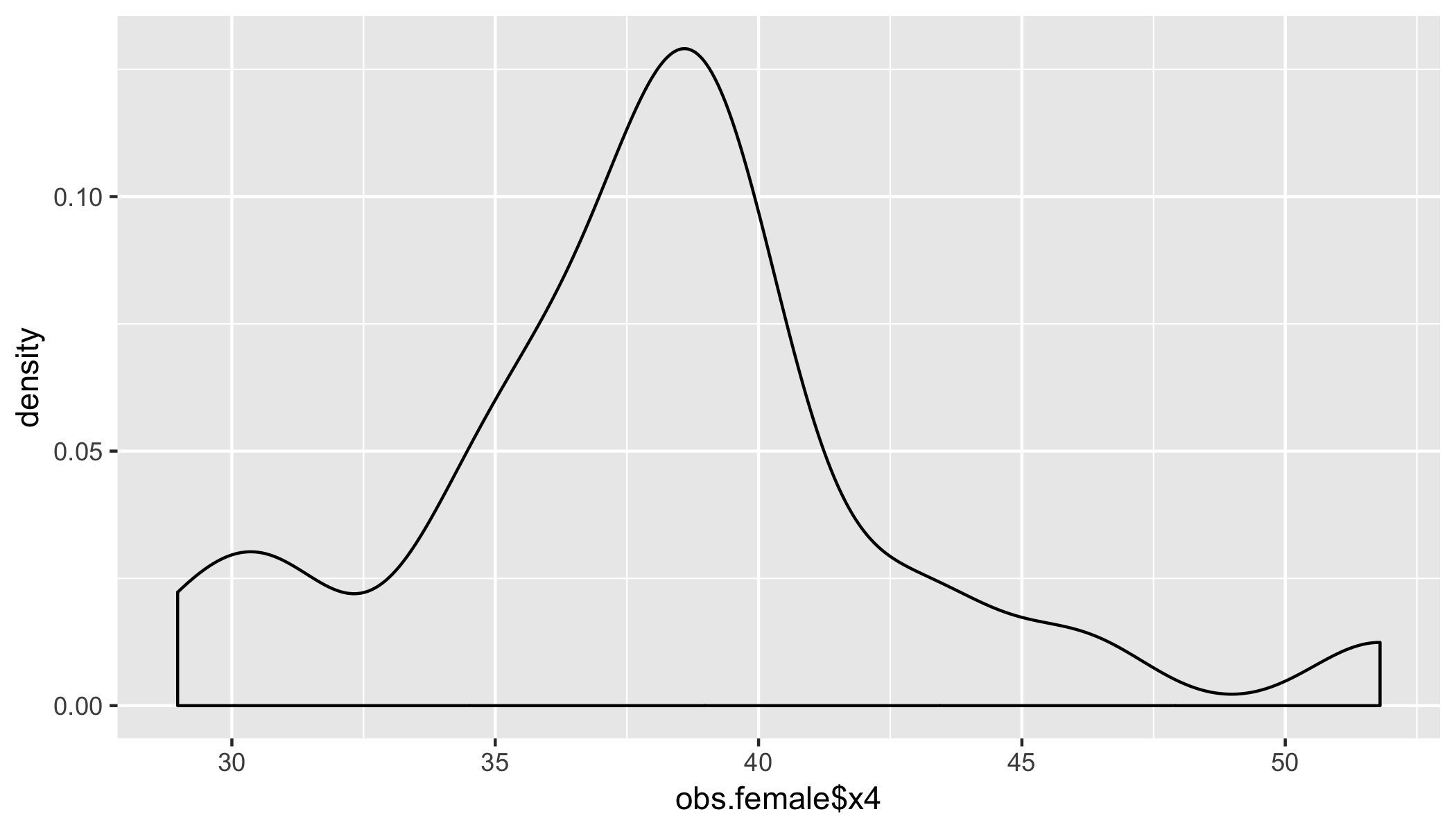
### Question 1a

We are using the data.table package from R to read the file in to memory

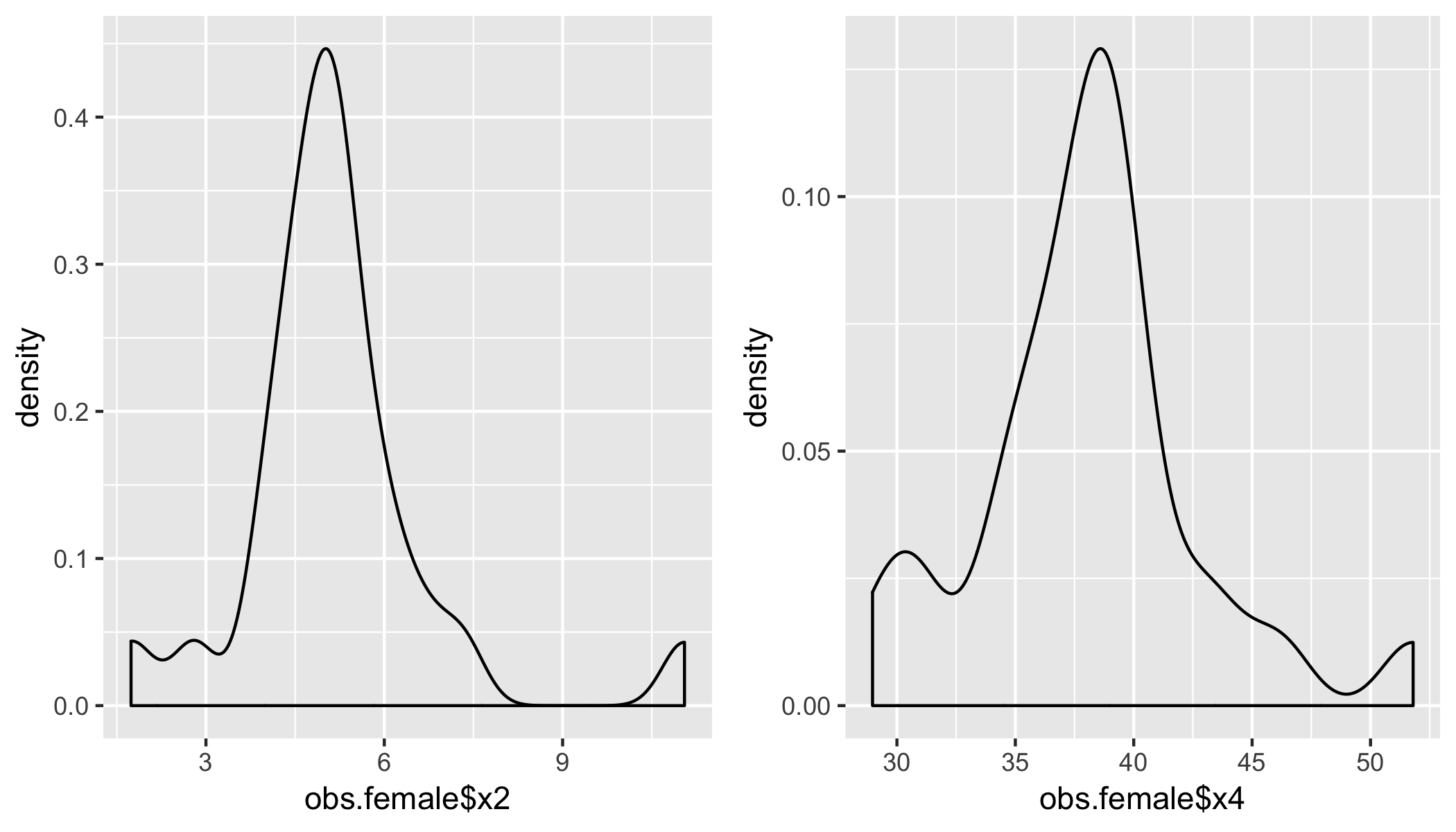
male <- 1  
female <- 2  
  
setDT(data.frame)  
obs.female <- data.frame[Gender==2]  
  
  
  
# Question 1a  
p1<-ggplot(obs.female, aes(x=obs.female$x2)) + geom\_density()  
p2<-ggplot(obs.female, aes(x=obs.female$x4)) + geom\_density()  
p1



p2



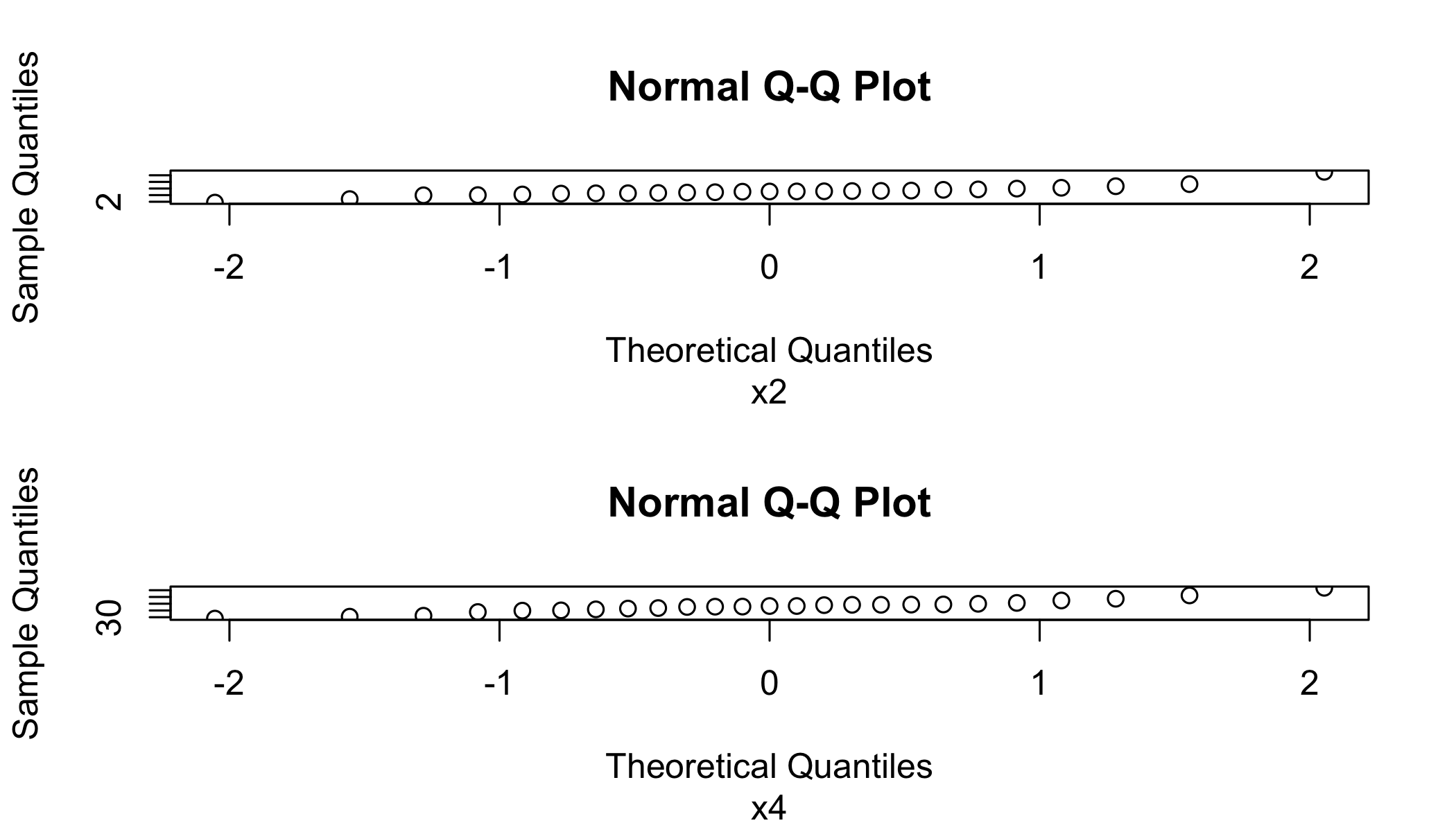
grid.arrange(p1, p2, ncol=2)



obs.female[,list(x1)]

## x1  
## 1: 0.29  
## 2: 0.28  
## 3: 0.31  
## 4: 0.30  
## 5: 0.28  
## 6: 0.11  
## 7: 0.25  
## 8: 0.26  
## 9: 0.39  
## 10: 0.37  
## 11: 0.31  
## 12: 0.35  
## 13: 0.29  
## 14: 0.33  
## 15: 0.18  
## 16: 0.28  
## 17: 0.44  
## 18: 0.22  
## 19: 0.34  
## 20: 0.30  
## 21: 0.31  
## 22: 0.27  
## 23: 0.66  
## 24: 0.37  
## 25: 0.35  
## x1

par(mfrow = c(2, 1))  
setDF(obs.female)  
qqnorm(obs.female[,3], sub = colnames(obs.female)[3])  
qqnorm(obs.female[,5], sub = colnames(obs.female)[5])



?qqnorm

### Question 1b

# Question 1b  
  
  
mvtest<-roystonTest(wine[,10:11],qqplot=TRUE)

## Warning: 'roystonTest' is deprecated.  
## Use 'mvn' instead.  
## See help("Deprecated")

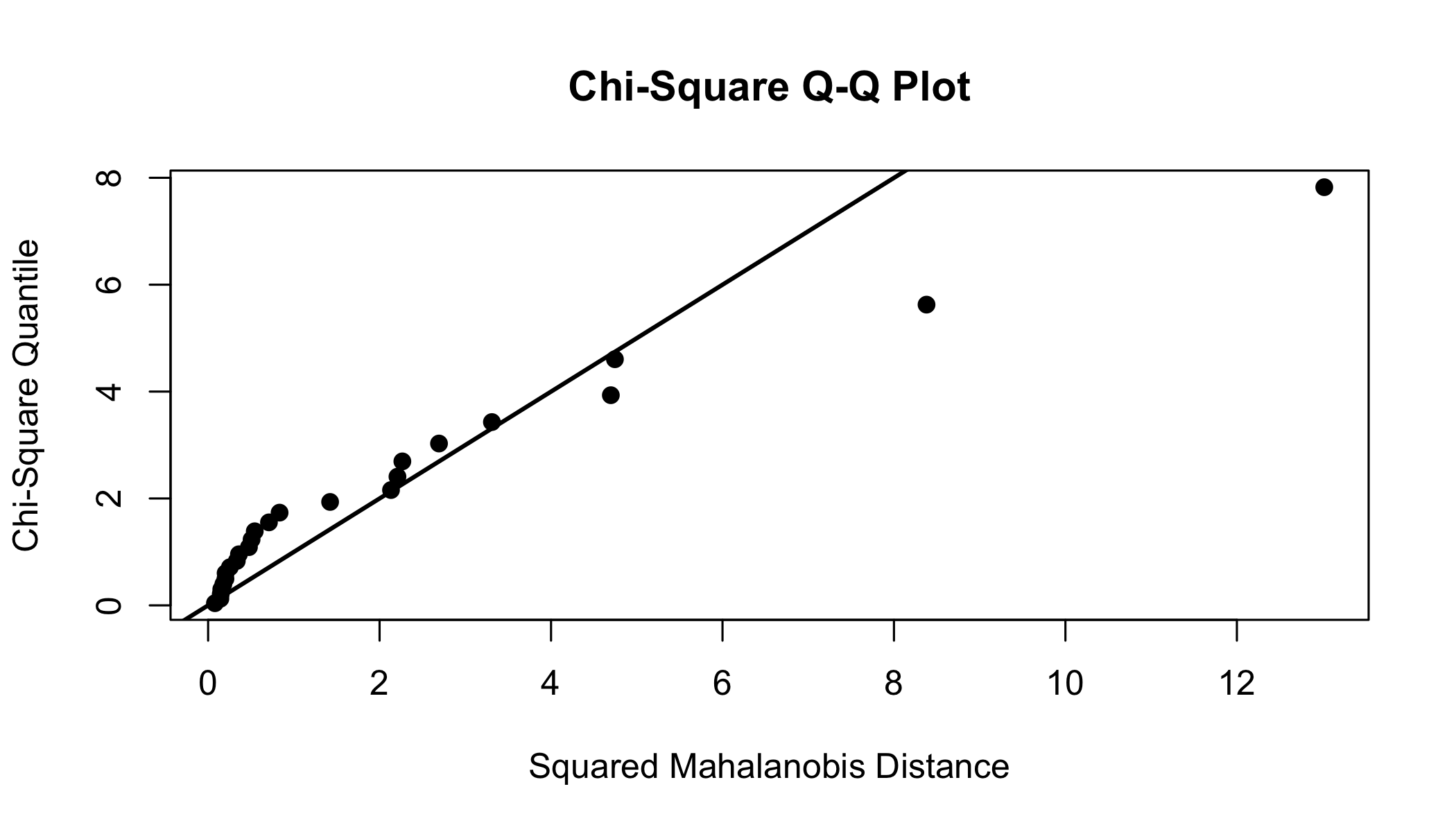
?mvn()

## Help on topic 'mvn' was found in the following packages:  
##   
## Package Library  
## mclust /usr/local/lib/R/3.5/site-library  
## MVN /usr/local/lib/R/3.5/site-library  
##   
##   
## Using the first match ...

setDT(obs.female)  
obs.female[,list(x2,x4)]

## x2 x4  
## 1: 5.04 33.85  
## 2: 3.95 35.82  
## 3: 4.88 36.40  
## 4: 5.97 37.87  
## 5: 4.57 38.30  
## 6: 1.74 39.19  
## 7: 4.66 39.21  
## 8: 5.28 39.94  
## 9: 7.32 42.41  
## 10: 6.22 28.97  
## 11: 4.20 37.80  
## 12: 5.10 31.10  
## 13: 4.46 38.30  
## 14: 5.60 51.80  
## 15: 2.80 37.60  
## 16: 4.01 36.78  
## 17: 6.69 46.16  
## 18: 4.55 38.95  
## 19: 5.73 40.60  
## 20: 5.12 43.69  
## 21: 4.77 30.40  
## 22: 5.16 39.46  
## 23: 11.05 39.34  
## 24: 5.23 34.86  
## 25: 5.37 35.07  
## x2 x4

par(mfrow = c(1, 1))  
mvn(obs.female[,list(x2,x4)], mvnTest='royston', multivariatePlot='qq')



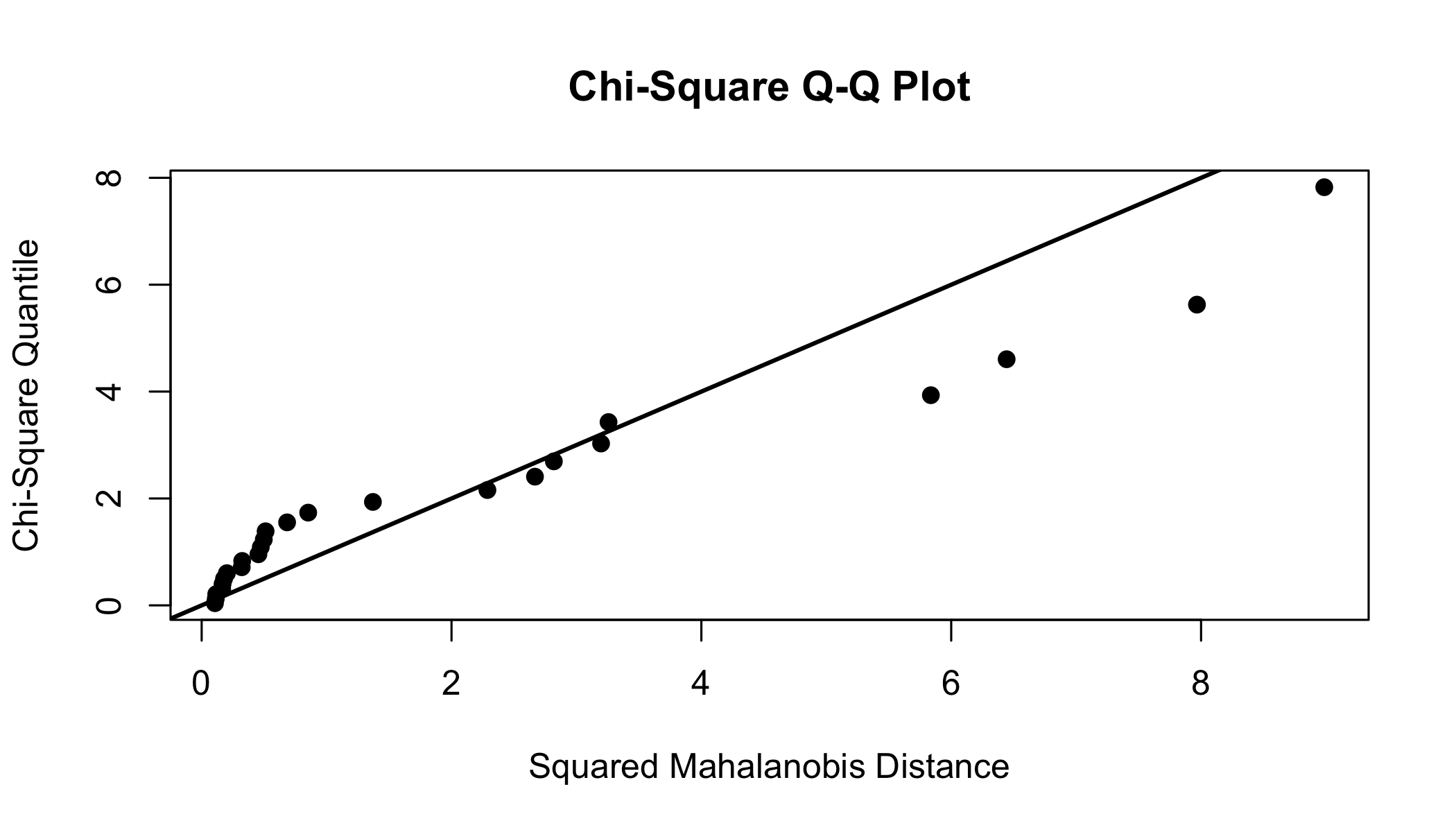
## $multivariateNormality  
## Test H p value MVN  
## 1 Royston 15.09293 0.0005281256 NO  
##   
## $univariateNormality  
## Test Variable Statistic p value Normality  
## 1 Shapiro-Wilk x2 0.8475 0.0016 NO   
## 2 Shapiro-Wilk x4 0.9441 0.1839 YES   
##   
## $Descriptives  
## n Mean Std.Dev Median Min Max 25th 75th Skew Kurtosis  
## x2 25 5.1788 1.667531 5.1 1.74 11.05 4.55 5.60 1.3623856 4.2103263  
## x4 25 38.1548 4.822948 38.3 28.97 51.80 35.82 39.46 0.5594578 0.9846515

### Question 1c

# Question 1c  
#  
#One approach to dealing with data that do not follow a multivariate normal distribution  
#is to perform a transformation. In practice, data are often transformed using a log transformation or a square root transformation to stabilise the variances of the response in  
#each of the treatment groups when they differ significantly. These two transformations  
#are in fact special cases of the Box–Cox transformation.  
#

### Question 1d

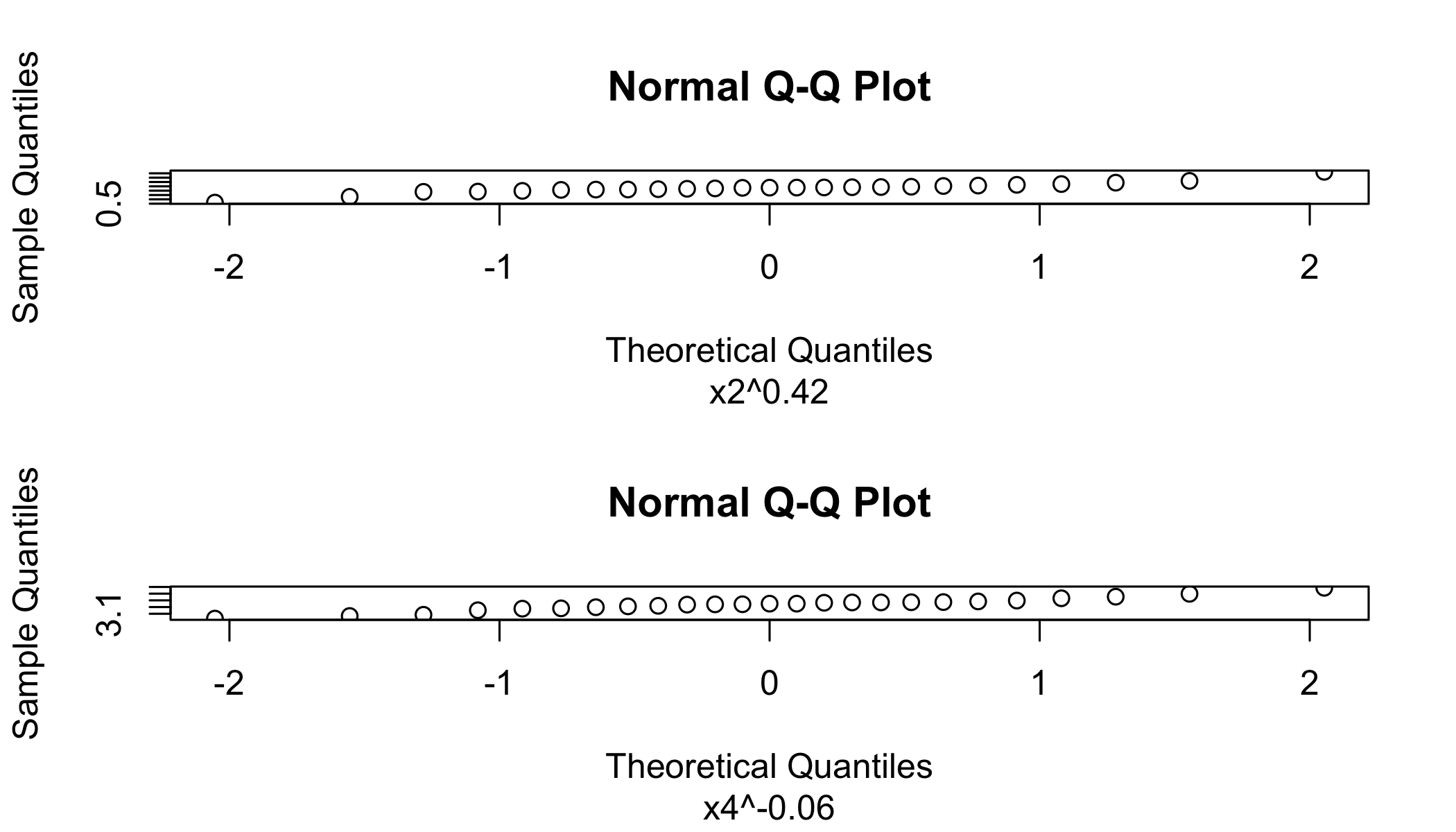
# Question 1d  
  
trans<-powerTransform(obs.female[,list(x2,x4)])  
obs.female.sub<-obs.female[,list(x2,x4)]  
obs.female.sub<-bcPower(obs.female.sub,trans$lambda)  
  
par(mfrow = c(1, 1))  
mvn(obs.female.sub, mvnTest='royston', multivariatePlot='qq')



## $multivariateNormality  
## Test H p value MVN  
## 1 Royston 9.917541 0.00702209 NO  
##   
## $univariateNormality  
## Test Variable Statistic p value Normality  
## 1 Shapiro-Wilk x2^0.42 0.8958 0.0148 NO   
## 2 Shapiro-Wilk x4^-0.06 0.9589 0.3935 YES   
##   
## $Descriptives  
## n Mean Std.Dev Median Min Max 25th  
## x2^0.42 25 2.324406 0.63061301 2.348569 0.624426 4.176261 2.126111  
## x4^-0.06 25 3.241326 0.09893204 3.250670 3.027693 3.487365 3.197568  
## 75th Skew Kurtosis  
## x2^0.42 2.539104 0.15554247 2.5537890  
## x4^-0.06 3.274264 0.04045563 0.4334795

### Question 1e

# Question 1e  
  
par(mfrow = c(2, 1))  
setDF(obs.female.sub)  
qqnorm(obs.female.sub[,1], sub = colnames(obs.female.sub)[1])  
qqnorm(obs.female.sub[,2], sub = colnames(obs.female.sub)[2])



## Question 2

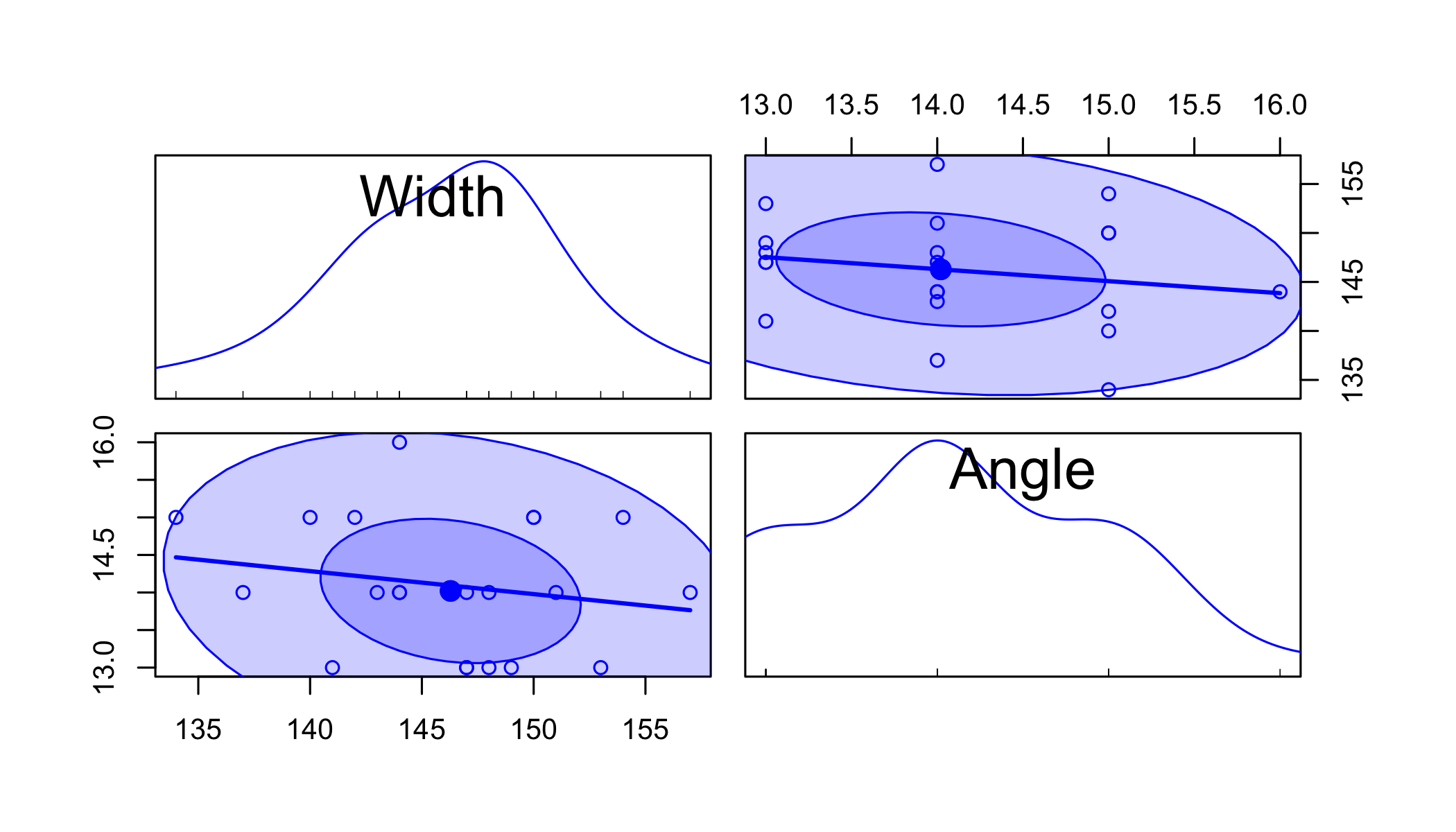
### Question 2a

# Question 2a  
  
concinna <- fread('Data/Concinna.csv')  
concinna[, Species:=NULL]  
  
setDF(concinna)  
  
for (i in 1:2) {  
 ci <- round(t.test(concinna[, i])$conf.int, 3)  
 cat(paste("The 95% CI for ", colnames(concinna)[i], " is: (", ci[1], ", ",  
 ci[2], ")\n"))  
}

## The 95% CI for Width is: ( 143.629 , 148.752 )  
## The 95% CI for Angle is: ( 13.691 , 14.5 )

scatterplotMatrix(~ Width+Angle,  
 data=concinna, smooth=FALSE, ellipse=TRUE, by.groups=TRUE,  
 diagonal="none")

## Warning in applyDefaults(diagonal, defaults = list(method =  
## "adaptiveDensity"), : unnamed diag arguments, will be ignored



### Question 2b

# Question 2b  
# u1 or Width is outside the 95% CI, so possibly not a concinna species

### Question 2c

# Question 2c  
  
Xbar <- colMeans(concinna)  
S <- cov(concinna)  
n <- nrow(concinna)  
p <- ncol(concinna)  
for (i in 1:2) {  
 lower <- round(Xbar[i] - sqrt(p \* (n - 1)/(n - p) \* qf(0.95, p, n - p)) \*  
 sqrt(S[i, i]/n), 3)  
 5  
 upper <- round(Xbar[i] + sqrt(p \* (n - 1)/(n - p) \* qf(0.95, p, n - p)) \*  
 sqrt(S[i, i]/n), 3)  
 cat(paste("The 95% CI for ", colnames(concinna)[i], " is: (", lower, ", ",  
 upper, ")\n"))  
}

## The 95% CI for Width is: ( 142.847 , 149.534 )  
## The 95% CI for Angle is: ( 13.567 , 14.624 )

### Question 2d

# Question 2d  
for (i in 1:2) {  
 ci <- round(t.test(concinna[, i], conf.level = (1 - 0.05/p))$conf.int, 3)  
 cat(paste("The 95% Bonferroni CI for ", colnames(concinna)[i], " is: (",  
 ci[1], ", ", ci[2], ")\n"))  
}

## The 95% Bonferroni CI for Width is: ( 143.215 , 149.166 )  
## The 95% Bonferroni CI for Angle is: ( 13.625 , 14.565 )

### Question 2e

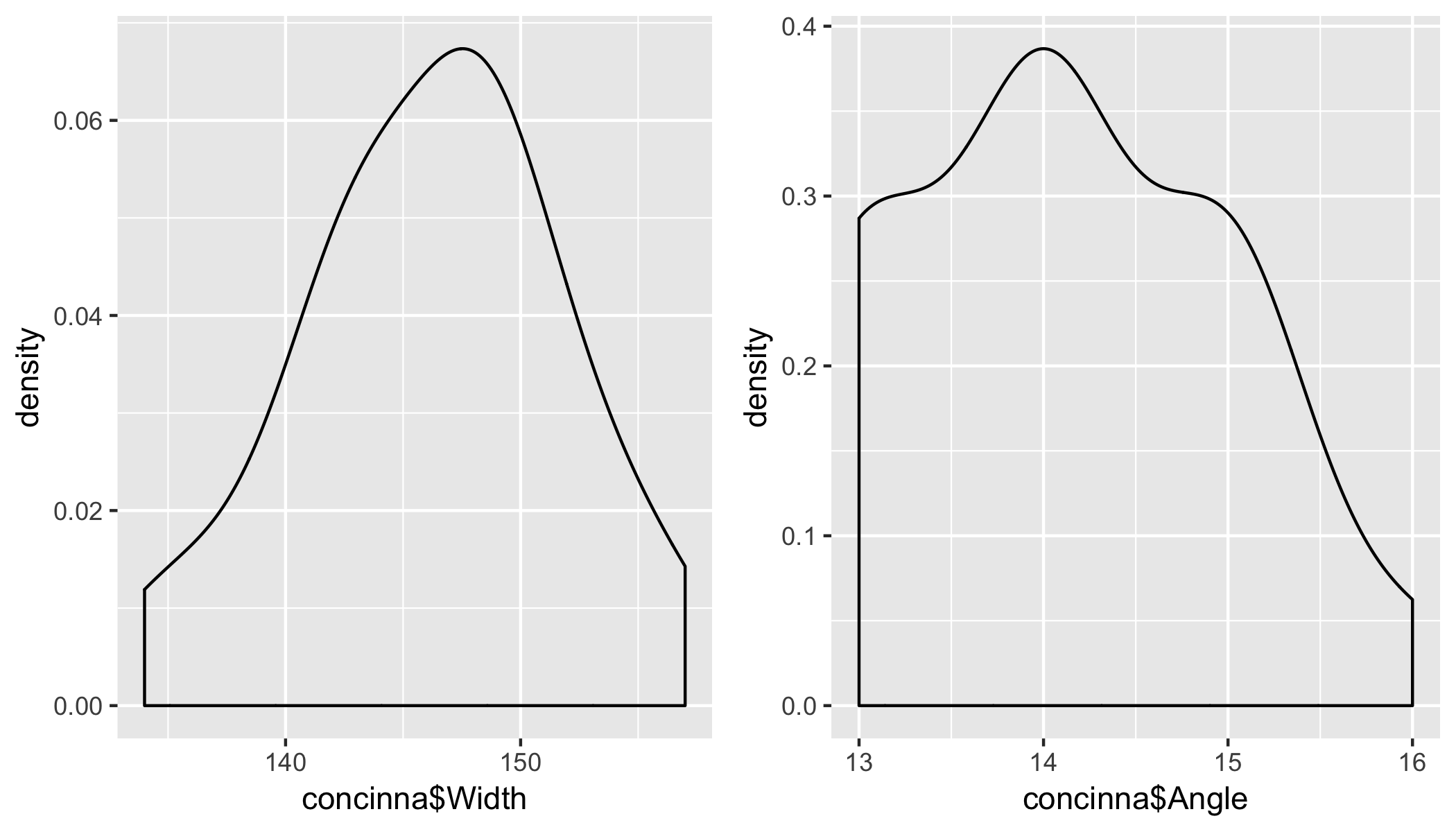
# Question 2e  
  
###  
### Write commentary

### Question 2f

# Question 2f  
  
par(mfrow = c(2, 1))  
mvn(concinna, mvnTest='royston', multivariatePlot='qq')

## $multivariateNormality  
## Test H p value MVN  
## 1 Royston 6.941658 0.03106203 NO  
##   
## $univariateNormality  
## Test Variable Statistic p value Normality  
## 1 Shapiro-Wilk Width 0.9881 0.9936 YES   
## 2 Shapiro-Wilk Angle 0.8669 0.0084 NO   
##   
## $Descriptives  
## n Mean Std.Dev Median Min Max 25th 75th Skew  
## Width 21 146.19048 5.6268912 147 134 157 143 150 -0.2111542  
## Angle 21 14.09524 0.8890873 14 13 16 13 15 0.2347644  
## Kurtosis  
## Width -0.5081691  
## Angle -1.0324544

p1<-ggplot(concinna, aes(x=concinna$Width)) + geom\_density()  
p2<-ggplot(concinna, aes(x=concinna$Angle)) + geom\_density()  
p1  
p2  
grid.arrange(p1, p2, ncol=2)



### Question 3

### Question 3a

# Question 3a  
  
##### Cov matrix  
  
vcmat <- 1/5630 \* matrix(c(575,-60,10,-60,300,-50,10,-50,196),nrow=3,byrow=TRUE)  
  
# inverse  
vcmat.inv <-solve(vcmat)

### Question 3b

# Question 3b

### Question 3c

# Question 3c  
#MVN  
  
  
mv<-rep(0, 3)  
  
mnd <- mvrnorm(n=1000,mv,vcmat)

### Question 3d

# Question 3d  
  
#Pair 1  
x <- seq(from=mv[1]-4\*vcmat[1,1],to=mv[1]+4\*vcmat[1,1],by=vcmat[1,1]/25)  
y <- seq(from=mv[2]-4\*vcmat[2,2],to=mv[2]+4\*vcmat[2,2],by=vcmat[2,2]/25)  
length(x)

## [1] 201

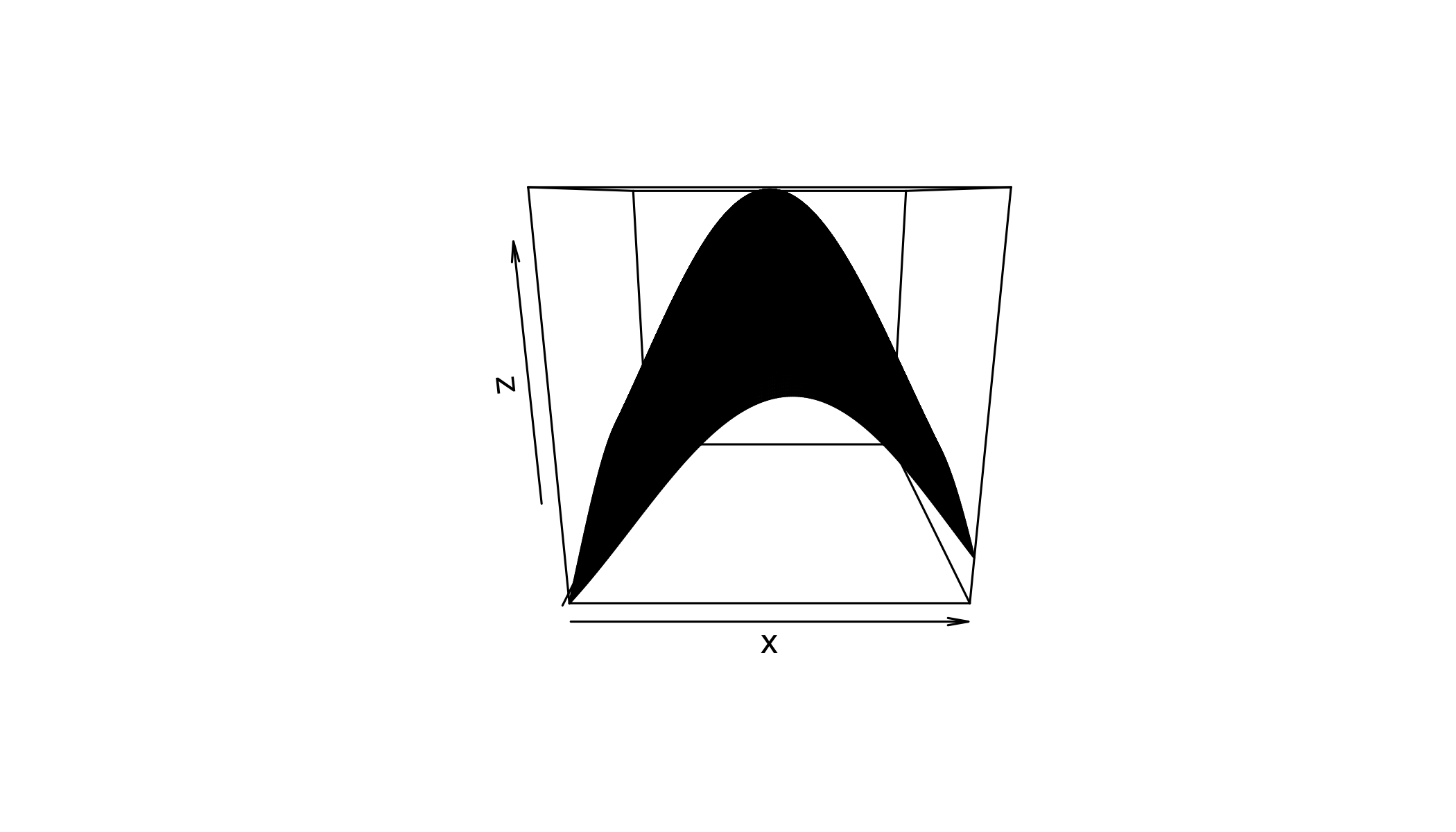
vcmat[1:2,1:2]

## [,1] [,2]  
## [1,] 0.10213144 -0.01065719  
## [2,] -0.01065719 0.05328597

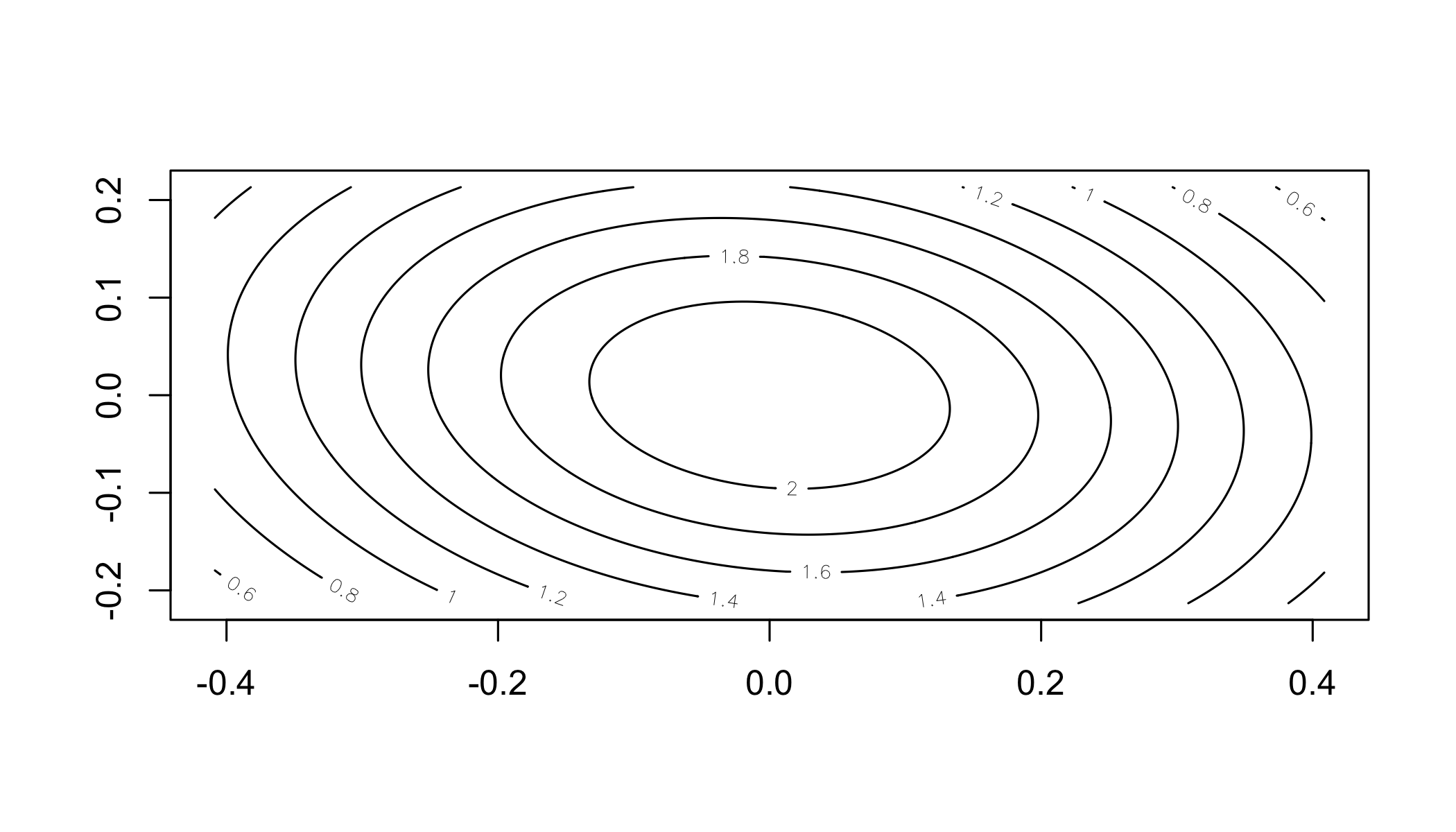
mv[1:2]

## [1] 0 0

z <- matrix(0,201,201)  
for(i in 1:201){  
 for(j in 1:201){  
 z[i,j]<-dmnorm(c(x[i],y[j]), mean = mv[1:2], vcmat[1:2,1:2], log = FALSE)  
 }  
}  
?dmnorm  
persp(x,y,z, axes = TRUE,box = TRUE)



contour(x,y,z, axes = TRUE)



?dmnorm()

### Question 3e

# Question 3e  
options(scipen = 999)  
B<-eigen(vcmat)  
B$values

## [1] 0.10453986 0.05453633 0.03115472

B$vectors

## [,1] [,2] [,3]  
## [1,] 0.97591370 -0.2153318 0.03499535  
## [2,] -0.21190556 -0.8975461 0.38666136  
## [3,] 0.05185053 0.3847638 0.92155755

#   
# P<-B$vectors  
# D<-B$values\*diag(3)  
# Pt<-t(P)  
#   
# round(P%\*%D%\*%Pt)  
#   
# round(P%\*%P1)  
#   
# round(P1%\*%P)  
#   
# A

### Question 3f

# Question 3f  
  
# ??  
cov(mnd)

## [,1] [,2] [,3]  
## [1,] 0.0998254399 -0.004362749 0.0009796658  
## [2,] -0.0043627489 0.053763775 -0.0081996086  
## [3,] 0.0009796658 -0.008199609 0.0363164738

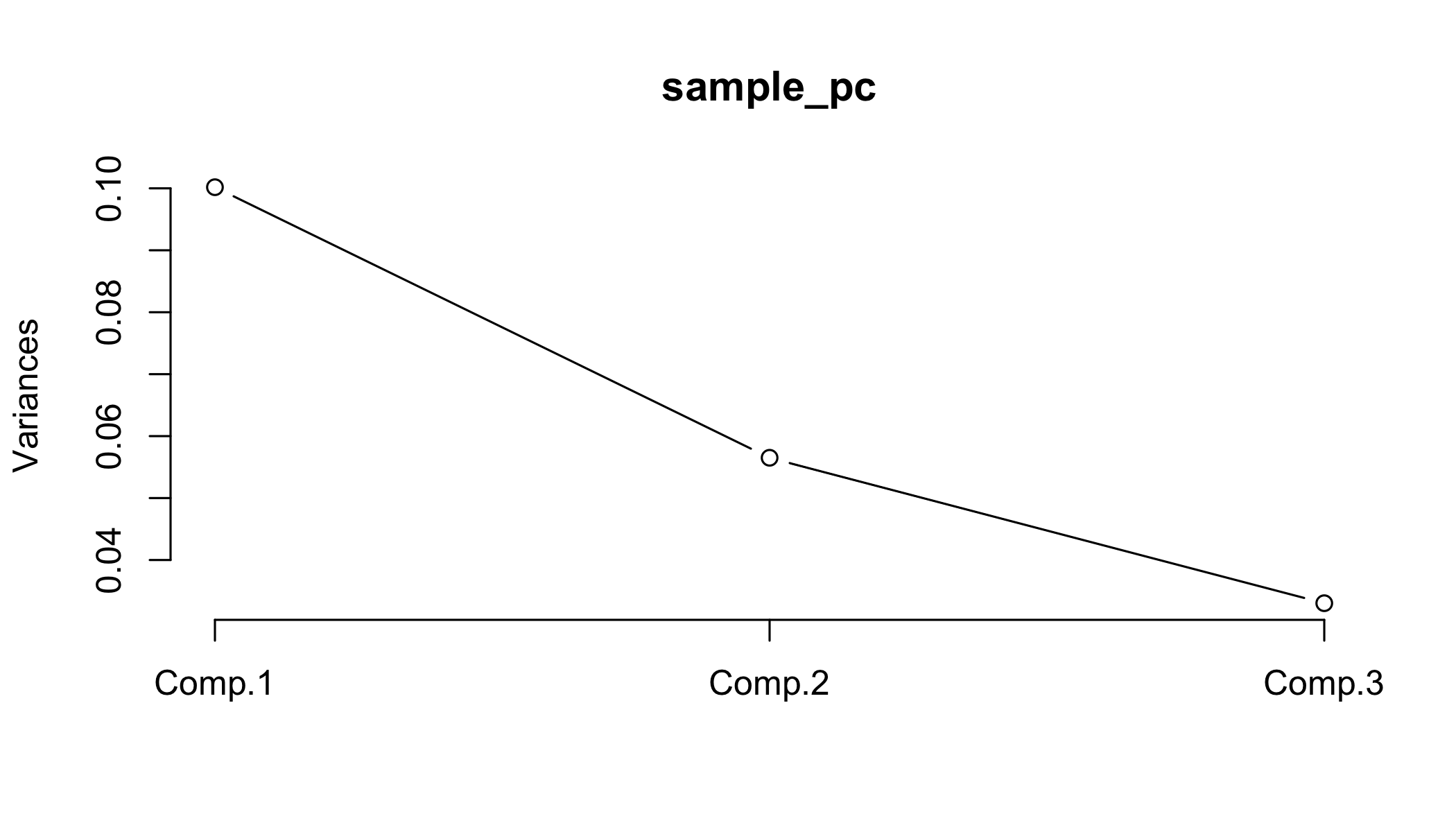
# ??  
vcmat

## [,1] [,2] [,3]  
## [1,] 0.102131439 -0.010657194 0.001776199  
## [2,] -0.010657194 0.053285968 -0.008880995  
## [3,] 0.001776199 -0.008880995 0.034813499

sample\_pc<-princomp(mnd)  
summary(sample\_pc, loadings = TRUE)

## Importance of components:  
## Comp.1 Comp.2 Comp.3  
## Standard deviation 0.3165173 0.2377081 0.1817345  
## Proportion of Variance 0.5280700 0.2978410 0.1740890  
## Cumulative Proportion 0.5280700 0.8259110 1.0000000  
##   
## Loadings:  
## Comp.1 Comp.2 Comp.3  
## [1,] 0.995 0.102   
## [2,] 0.924 0.370  
## [3,] -0.369 0.929

par(mfrow = c(1, 1))  
screeplot(sample\_pc, npcs = 3, type = "lines")



### Question 3g

# Question 3g  
  
# Comp1 and Comp2 explains 82%

### Question 3h

# Question 3h  
mnd

## [,1] [,2] [,3]  
## [1,] 0.20053230662 -0.3619202879 -0.09267523951  
## [2,] 0.17858236035 -0.3742071813 0.15082839373  
## [3,] 0.22437752095 -0.3074386316 -0.29715505784  
## [4,] 0.13274928138 0.0532906607 -0.01044047794  
## [5,] -0.60484226768 0.0566828314 -0.14078878108  
## [6,] 0.78684101028 -0.5518964278 0.13659032510  
## [7,] 0.29860147999 -0.1066982109 0.09910939966  
## [8,] -0.01340746636 -0.5885935105 0.37728974046  
## [9,] -0.30664823175 -0.4535329783 0.03165500301  
## [10,] -0.25510112888 0.0528042683 0.11306877513  
## [11,] 0.02245946565 0.1464433344 0.06621671276  
## [12,] -0.19940262940 -0.0704220538 0.30107331585  
## [13,] 0.15596741570 0.0939522264 -0.07391348720  
## [14,] -0.40119911193 0.1134674874 -0.09090335196  
## [15,] 0.42350902734 0.2481666021 -0.34766018235  
## [16,] -0.26773171482 -0.2064535773 -0.22593404808  
## [17,] 0.24311150865 -0.2265659572 0.20237907695  
## [18,] 0.36222722084 0.3967774137 0.11430997623  
## [19,] -0.07535968683 0.0613963584 -0.20028587185  
## [20,] -0.72879677078 -0.2286827783 0.27408916401  
## [21,] 0.05058443929 -0.2558687904 -0.02970057906  
## [22,] -0.46863538926 0.1877791924 -0.08239495038  
## [23,] -0.01476911512 0.1236047205 -0.16926349576  
## [24,] 0.12189148444 -0.0626636129 0.22138108910  
## [25,] -0.02919412372 0.1253988623 0.12617853711  
## [26,] -0.18965507696 0.0734304473 0.23848657470  
## [27,] 0.00647146015 -0.1331110607 -0.37669104741  
## [28,] 0.41541459665 -0.4115860260 -0.03110440837  
## [29,] 0.09721280521 -0.0440432330 -0.29265300946  
## [30,] 0.20191897569 0.0101403801 -0.03939578485  
## [31,] 0.31603018310 -0.1620213343 0.20943607285  
## [32,] 0.11048098015 0.1291193057 0.23380186885  
## [33,] -0.43867924818 -0.1018121818 -0.15292591695  
## [34,] -0.12602406145 -0.1750373702 -0.12470531642  
## [35,] -0.35193334855 -0.5714499908 -0.04093445659  
## [36,] 0.38500186268 -0.3630303621 0.20895780472  
## [37,] -0.07822127791 0.0581247630 0.14242273132  
## [38,] 0.35097704061 -0.2510119755 0.23335851071  
## [39,] -0.78615742777 -0.1121221130 0.24454551747  
## [40,] -0.23885588604 -0.2603314376 0.24311090334  
## [41,] 0.39424012405 0.0825077416 0.19241451763  
## [42,] 0.01230948281 -0.1778845872 -0.24436330427  
## [43,] 0.16301049705 0.1682975274 -0.20369605733  
## [44,] -0.23181186206 0.2006712022 0.04742017214  
## [45,] 0.22129115869 -0.0260468330 0.15475061540  
## [46,] 0.09637012732 -0.1012353246 0.15705662772  
## [47,] -0.56150566178 0.2529175918 -0.31387296361  
## [48,] -0.29175310478 0.2308198222 0.04184043159  
## [49,] 0.46356713617 0.1853609270 -0.21993134255  
## [50,] 0.07939448033 -0.0949627101 0.05470973075  
## [51,] 0.00907878093 0.3944426833 -0.38045582414  
## [52,] -0.11866230225 0.1607449875 0.18984423038  
## [53,] -0.08775122316 0.2086088494 -0.12755126402  
## [54,] -0.05606766108 0.3039532946 -0.13645400968  
## [55,] 0.05608158667 -0.2698033042 0.00944042284  
## [56,] 0.13302532366 0.0266794389 -0.25713316759  
## [57,] -0.16014419141 -0.2005762359 -0.00198721626  
## [58,] -0.02112941253 -0.3591101877 0.01675119372  
## [59,] 0.26769082621 -0.0349634355 0.27904082031  
## [60,] 0.33485440161 0.2603683601 0.36131702359  
## [61,] 0.02664146674 -0.1141702914 0.02444166042  
## [62,] -0.65480783544 -0.0920986741 0.16528453425  
## [63,] -0.05420294835 -0.1744464239 0.22064324445  
## [64,] -0.07936155282 0.2867483094 0.23311012507  
## [65,] -0.16708874418 -0.0020683499 0.03201529255  
## [66,] -0.09538275073 0.1574168462 0.10879236183  
## [67,] -0.98431887283 0.2460272083 -0.03427220507  
## [68,] 0.18051189855 0.4097080768 -0.02327240317  
## [69,] 0.27187529895 0.2033587359 -0.01750665722  
## [70,] -0.16699343807 -0.2151959910 0.11873847172  
## [71,] 0.20264011684 0.0377517317 -0.06727698284  
## [72,] 0.04818840118 0.5176290089 -0.36171066060  
## [73,] 0.07764948472 -0.1679187784 0.12615653796  
## [74,] -0.22212991087 -0.4193053223 -0.02328167627  
## [75,] 0.27075702965 -0.1736874014 -0.00002866483  
## [76,] 0.10506750765 -0.1775210165 0.19440556424  
## [77,] -0.37895526176 0.0307443503 -0.22208560239  
## [78,] 0.11942994677 0.0240279795 0.08387449107  
## [79,] 0.25573258063 -0.1447389752 0.13945961410  
## [80,] 0.56189684281 0.0661207830 -0.33968144007  
## [81,] -0.42544957898 -0.0916678914 0.28382995263  
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## [919,] 0.22310832544 -0.0060977529 -0.19582330466  
## [920,] 0.10315021126 0.4122810886 0.32847230474  
## [921,] -0.15509467063 0.0218310708 0.00325191378  
## [922,] -0.17691902072 -0.3221824638 -0.19992710517  
## [923,] 0.08482211332 0.0378410751 -0.13543455954  
## [924,] -0.11970570927 0.1041679925 -0.13159187576  
## [925,] -0.57468697057 0.2753292019 -0.19661604100  
## [926,] 0.29948918477 0.0775538805 0.19767476066  
## [927,] -0.50498981659 -0.0181945609 0.19520076255  
## [928,] 0.02391636214 0.1699323945 -0.14615504753  
## [929,] 0.06445417922 0.2001805461 -0.30260386812  
## [930,] -0.68689584049 0.2278960998 -0.05615637876  
## [931,] 0.10178716534 0.1222221160 -0.23986760536  
## [932,] -0.21850055174 0.4453911785 -0.11312464245  
## [933,] 0.14735657337 0.2048056749 0.03727903293  
## [934,] -0.32771984567 -0.0448886278 -0.00593865759  
## [935,] 0.05793515928 0.0500164496 0.19696625399  
## [936,] 0.09780255568 0.1374111274 -0.11922292125  
## [937,] 0.62406898615 -0.0812918402 0.32618883327  
## [938,] -0.30451239976 -0.5206112199 0.23292018362  
## [939,] -0.26776228771 0.2446898772 0.03015544522  
## [940,] 0.16153168467 -0.1729243228 -0.19184340589  
## [941,] -0.43580770283 -0.1566250332 0.02020783229  
## [942,] -0.33208175592 0.0637326225 -0.02125849175  
## [943,] 0.12792302636 0.1270714975 0.39508600178  
## [944,] -0.12700857201 0.0830242311 -0.12382557174  
## [945,] 0.47419363190 0.1754541774 -0.28482742617  
## [946,] 0.09314055601 -0.0446092779 0.13147403779  
## [947,] 0.16346670065 0.0354399218 0.00026374232  
## [948,] -0.00561864817 -0.2206318309 0.00660624196  
## [949,] 0.04224242287 -0.0041630841 0.16297379288  
## [950,] 0.42532631390 -0.0488138765 0.05673064348  
## [951,] 0.10938153621 -0.0703325709 -0.09475141833  
## [952,] -0.01066808272 0.0426006905 0.23870480372  
## [953,] 0.06045138750 0.1227926148 -0.19516100517  
## [954,] -0.35273648093 -0.1617318938 0.28011148085  
## [955,] -0.43074110257 0.2421957231 -0.04738298037  
## [956,] -0.23122040698 -0.1102955182 -0.18863069474  
## [957,] -0.31240846750 0.3516133392 -0.22232147596  
## [958,] -0.08362974848 -0.2217952356 0.24187097460  
## [959,] 0.06740750367 -0.3180858040 -0.15717839966  
## [960,] 0.04813552974 0.2110168068 -0.03300481971  
## [961,] 0.05984547763 -0.1019422218 0.01161792776  
## [962,] 0.12292832484 0.0399460624 0.14652007409  
## [963,] 0.05431650130 -0.0297296461 0.08805062588  
## [964,] -0.19217622469 -0.1788894989 -0.32012144637  
## [965,] -0.41423859152 0.0990372244 0.16270575059  
## [966,] -0.70774678524 -0.1185075429 -0.08086853981  
## [967,] -0.12047577668 -0.1134909432 -0.04194983749  
## [968,] 0.01786102402 0.2064787250 -0.00953706038  
## [969,] -0.33344411840 0.0707921012 0.12298427185  
## [970,] -0.24943114734 0.0898026186 -0.48488735754  
## [971,] 0.46412133455 -0.0182158541 0.11080690236  
## [972,] -0.09557457952 0.2544260029 -0.06839137466  
## [973,] -0.21846899858 -0.1204402806 0.16045001443  
## [974,] -0.23112147088 -0.0998496578 0.19795708583  
## [975,] 0.45967500114 -0.2491319365 -0.12848970958  
## [976,] 0.32682757526 0.1530399910 -0.16604884340  
## [977,] -0.48490690018 -0.1469488857 -0.04955686175  
## [978,] -0.15314146713 -0.1158466038 -0.40345566932  
## [979,] 0.02648069582 0.0348236413 0.11138187847  
## [980,] 0.61480883643 -0.1130138802 0.01430467551  
## [981,] 0.30519143763 0.2191644152 -0.23991679061  
## [982,] 0.12404105885 0.0654367838 0.16141663896  
## [983,] -0.17698698993 0.0105135447 -0.32903834008  
## [984,] 0.62416520867 -0.0777159669 0.00481461491  
## [985,] 0.29302068683 0.0337640129 -0.08609203721  
## [986,] 0.34720843595 0.3176290838 -0.38649088461  
## [987,] 0.06056862746 0.1765436123 -0.18221408785  
## [988,] 0.27852902315 -0.1801642307 -0.05779798966  
## [989,] 0.00517969394 0.0747250184 0.24512910602  
## [990,] -0.35179191780 -0.2255423993 -0.10471248887  
## [991,] 0.14125684816 -0.0363141480 0.01566401778  
## [992,] -0.07191172482 0.3746213108 -0.04802169432  
## [993,] -0.13960002256 -0.1639279492 0.01798661346  
## [994,] 0.05784916238 -0.1937739416 -0.29847002888  
## [995,] -0.16607736789 0.1174327702 0.06239405794  
## [996,] -0.46184787942 0.3733468857 -0.19245300159  
## [997,] 0.39017350296 0.1758183316 0.07552109582  
## [998,] 0.06452382748 -0.1466081964 0.01380094952  
## [999,] -0.20783849313 0.0912411437 -0.27446967752  
## [1000,] 0.57482211427 -0.3006909801 -0.06672513558

mnd\_df <- as.data.frame(as.table(mnd))  
setDT(mnd\_df)  
mnd\_dt <- dcast(mnd\_df, Var1~Var2, value.var = 'Freq')  
mnd\_dt[,Var1:=NULL]  
dim(mnd\_dt)

## [1] 1000 3

summary(mnd\_dt)

## A B C   
## Min. :-0.984319 Min. :-0.779867 Min. :-0.502669   
## 1st Qu.:-0.219845 1st Qu.:-0.143964 1st Qu.:-0.126885   
## Median :-0.001908 Median : 0.009827 Median : 0.002212   
## Mean :-0.008618 Mean : 0.002850 Mean : 0.004578   
## 3rd Qu.: 0.203035 3rd Qu.: 0.165697 3rd Qu.: 0.135938   
## Max. : 1.079733 Max. : 0.689362 Max. : 0.584664

colnames(mnd\_dt) <- c('Y1', 'Y2', 'Y3')  
  
model<-lm(Y2~Y1, data = mnd\_dt)  
  
model\_summary <- summary(model)  
#Coefficent of Y1  
model\_summary$coefficients[[2]]

## [1] -0.04370378

### Question 3i

# Question 3i  
  
model<-lm(Y3~Y1+Y2, data = mnd\_dt)  
  
model\_summary <- summary(model)  
#Coefficent of Y1  
model\_summary$coefficients[[2]]

## [1] 0.003159653

#Coefficent of Y2  
model\_summary$coefficients[[3]]

## [1] -0.1522554

### Question 3j

# Question 3j  
#Var(Y2|Y1)

### Question 3k

# Question 3k  
#Var(Y3|Y1, Y2)  
  
#For (h)-(k), you should find two different ways (one way is via exact formulas, and the other  
#way is via simulation in R) to compute the same answer. Then one way can be a check on the  
#other.

## Question 4

### Question 4a

# Question 4a  
  
data(diabetes)  
x <- diabetes$x  
x\_df <- as.data.frame(as.table(x))  
setDT(x\_df)  
x\_dt <- dcast(x\_df, Var1~Var2, value.var = 'Freq')  
x\_dt[,Var1:=NULL]  
dim(x\_dt)

## [1] 442 10

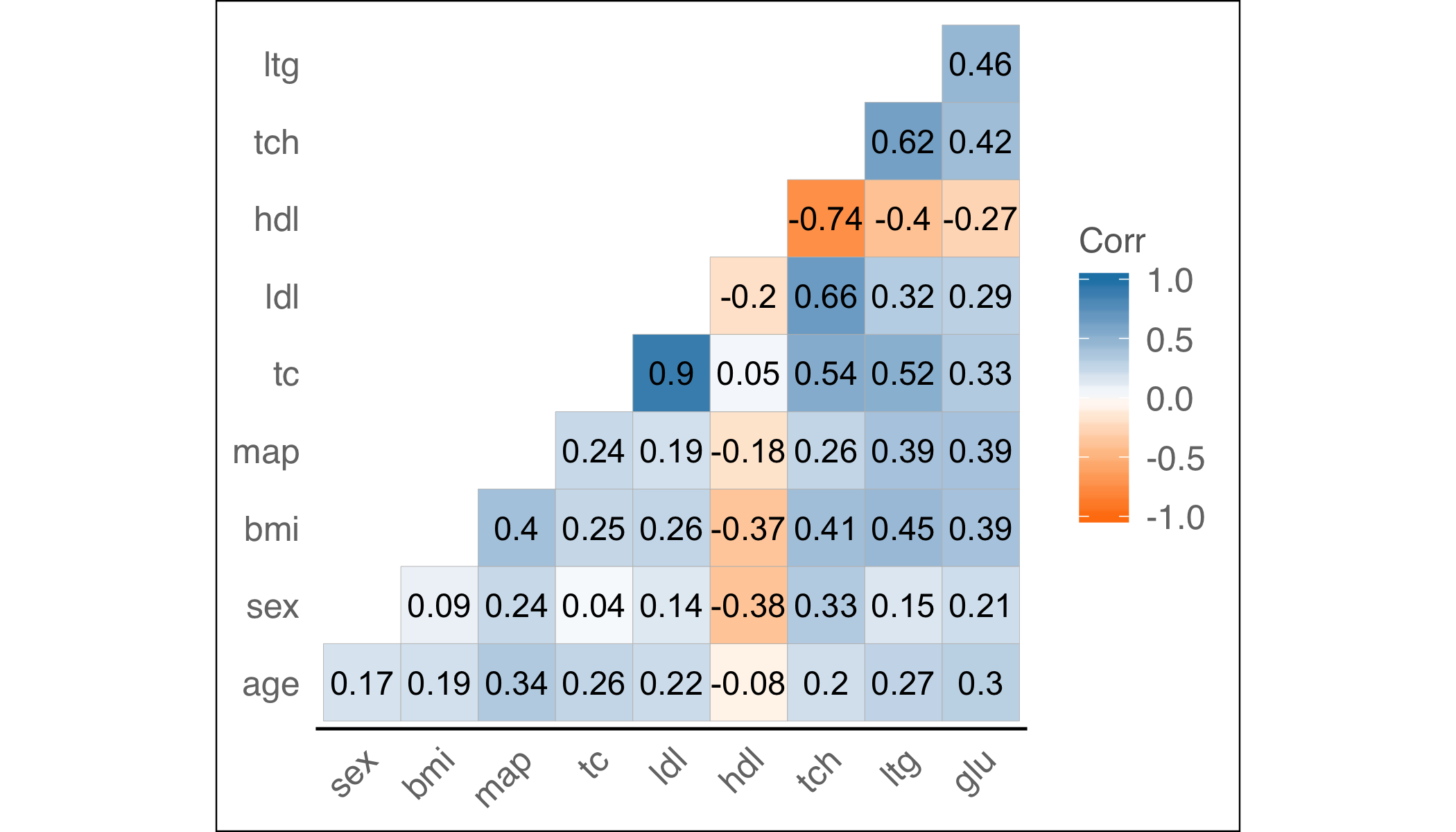
summary(x\_dt)

## age sex bmi   
## Min. :-0.107226 Min. :-0.04464 Min. :-0.090275   
## 1st Qu.:-0.037299 1st Qu.:-0.04464 1st Qu.:-0.034229   
## Median : 0.005383 Median :-0.04464 Median :-0.007284   
## Mean : 0.000000 Mean : 0.00000 Mean : 0.000000   
## 3rd Qu.: 0.038076 3rd Qu.: 0.05068 3rd Qu.: 0.031248   
## Max. : 0.110727 Max. : 0.05068 Max. : 0.170555   
## map tc ldl   
## Min. :-0.112400 Min. :-0.126781 Min. :-0.115613   
## 1st Qu.:-0.036656 1st Qu.:-0.034248 1st Qu.:-0.030358   
## Median :-0.005671 Median :-0.004321 Median :-0.003819   
## Mean : 0.000000 Mean : 0.000000 Mean : 0.000000   
## 3rd Qu.: 0.035644 3rd Qu.: 0.028358 3rd Qu.: 0.029844   
## Max. : 0.132044 Max. : 0.153914 Max. : 0.198788   
## hdl tch ltg   
## Min. :-0.102307 Min. :-0.076395 Min. :-0.126097   
## 1st Qu.:-0.035117 1st Qu.:-0.039493 1st Qu.:-0.033249   
## Median :-0.006584 Median :-0.002592 Median :-0.001948   
## Mean : 0.000000 Mean : 0.000000 Mean : 0.000000   
## 3rd Qu.: 0.029312 3rd Qu.: 0.034309 3rd Qu.: 0.032433   
## Max. : 0.181179 Max. : 0.185234 Max. : 0.133599   
## glu   
## Min. :-0.137767   
## 1st Qu.:-0.033179   
## Median :-0.001078   
## Mean : 0.000000   
## 3rd Qu.: 0.027917   
## Max. : 0.135612

#Corr Plot  
ncol(x\_dt)

## [1] 10

corr <- cor(x\_dt, use = "pairwise.complete.obs")  
  
ggcorrplot(corr, hc.order = FALSE, type = "lower",  
 ggtheme = ggthemes::theme\_gdocs,  
 colors = c("#ff7f0e", "white", "#1f83b4"),  
 lab = TRUE)+  
 theme(panel.grid.major=element\_blank())



### Question 4b

# Question 4b  
x\_dt[,y:=diabetes$y]  
  
model<-lm(y~., data = x\_dt)  
  
model\_summary <- summary(model)  
model\_summary$r.squared

## [1] 0.5177494

anova <- anova(model)  
sse <- anova$`Sum Sq`[[11]]  
vif(model)

## age sex bmi map tc ldl hdl   
## 1.217307 1.278073 1.509446 1.459429 59.203786 39.194379 15.402352   
## tch ltg glu   
## 8.890986 10.076222 1.484623

### Question 4c

# Question 4c  
model<-lm(y~age+bmi+map+hdl+ltg, data = x\_dt)  
model\_summary <- summary(model)  
model\_summary$r.squared

## [1] 0.4920391

#The idea is to reduce r square  
  
anova(model)

## Analysis of Variance Table  
##   
## Response: y  
## Df Sum Sq Mean Sq F value Pr(>F)   
## age 1 92527 92527 30.301 0.000000063252423 \*\*\*  
## bmi 1 825987 825987 270.496 < 0.00000000000000022 \*\*\*  
## map 1 119988 119988 39.294 0.000000000877373 \*\*\*  
## hdl 1 89056 89056 29.164 0.000000109511614 \*\*\*  
## ltg 1 162081 162081 53.079 0.000000000001513 \*\*\*  
## Residuals 436 1331370 3054   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

vif(model)

## age bmi map hdl ltg   
## 1.159192 1.421292 1.358792 1.260133 1.494444

### Question 4g

# Question 4g  
  
setDF(x\_dt)  
  
for (i in 1:10) {  
 ci <- round(t.test(x\_dt[, i])$conf.int, 3)  
 cat(paste("The 95% CI for ", colnames(x\_dt)[i], " is: (", ci[1], ", ",  
 ci[2], ")\n"))  
}

## The 95% CI for age is: ( -0.004 , 0.004 )  
## The 95% CI for sex is: ( -0.004 , 0.004 )  
## The 95% CI for bmi is: ( -0.004 , 0.004 )  
## The 95% CI for map is: ( -0.004 , 0.004 )  
## The 95% CI for tc is: ( -0.004 , 0.004 )  
## The 95% CI for ldl is: ( -0.004 , 0.004 )  
## The 95% CI for hdl is: ( -0.004 , 0.004 )  
## The 95% CI for tch is: ( -0.004 , 0.004 )  
## The 95% CI for ltg is: ( -0.004 , 0.004 )  
## The 95% CI for glu is: ( -0.004 , 0.004 )

### Question 4h

# Question 4h  
  
setDF(x\_dt)  
  
Xbar <- colMeans(x\_dt[1:10])  
S <- cov(x\_dt[1:10])  
n <- nrow(x\_dt[1:10])  
p <- ncol(x\_dt[1:10])  
for (i in 1:10) {  
 lower <- round(Xbar[i] - sqrt(p \* (n - 1)/(n - p) \* qf(0.95, p, n - p)) \*  
 sqrt(S[i, i]/n), 3)  
 5  
 upper <- round(Xbar[i] + sqrt(p \* (n - 1)/(n - p) \* qf(0.95, p, n - p)) \*  
 sqrt(S[i, i]/n), 3)  
 cat(paste("The 95% CI for ", colnames(x\_dt)[i], " is: (", lower, ", ",  
 upper, ")\n"))  
}

## The 95% CI for age is: ( -0.01 , 0.01 )  
## The 95% CI for sex is: ( -0.01 , 0.01 )  
## The 95% CI for bmi is: ( -0.01 , 0.01 )  
## The 95% CI for map is: ( -0.01 , 0.01 )  
## The 95% CI for tc is: ( -0.01 , 0.01 )  
## The 95% CI for ldl is: ( -0.01 , 0.01 )  
## The 95% CI for hdl is: ( -0.01 , 0.01 )  
## The 95% CI for tch is: ( -0.01 , 0.01 )  
## The 95% CI for ltg is: ( -0.01 , 0.01 )  
## The 95% CI for glu is: ( -0.01 , 0.01 )