Multivariate Data Analysis - Presentation

## Multivariate Data Analysis Spring 2019 (37459-2019-SPRING-CITY)

### Assignment: 1

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## Question 1

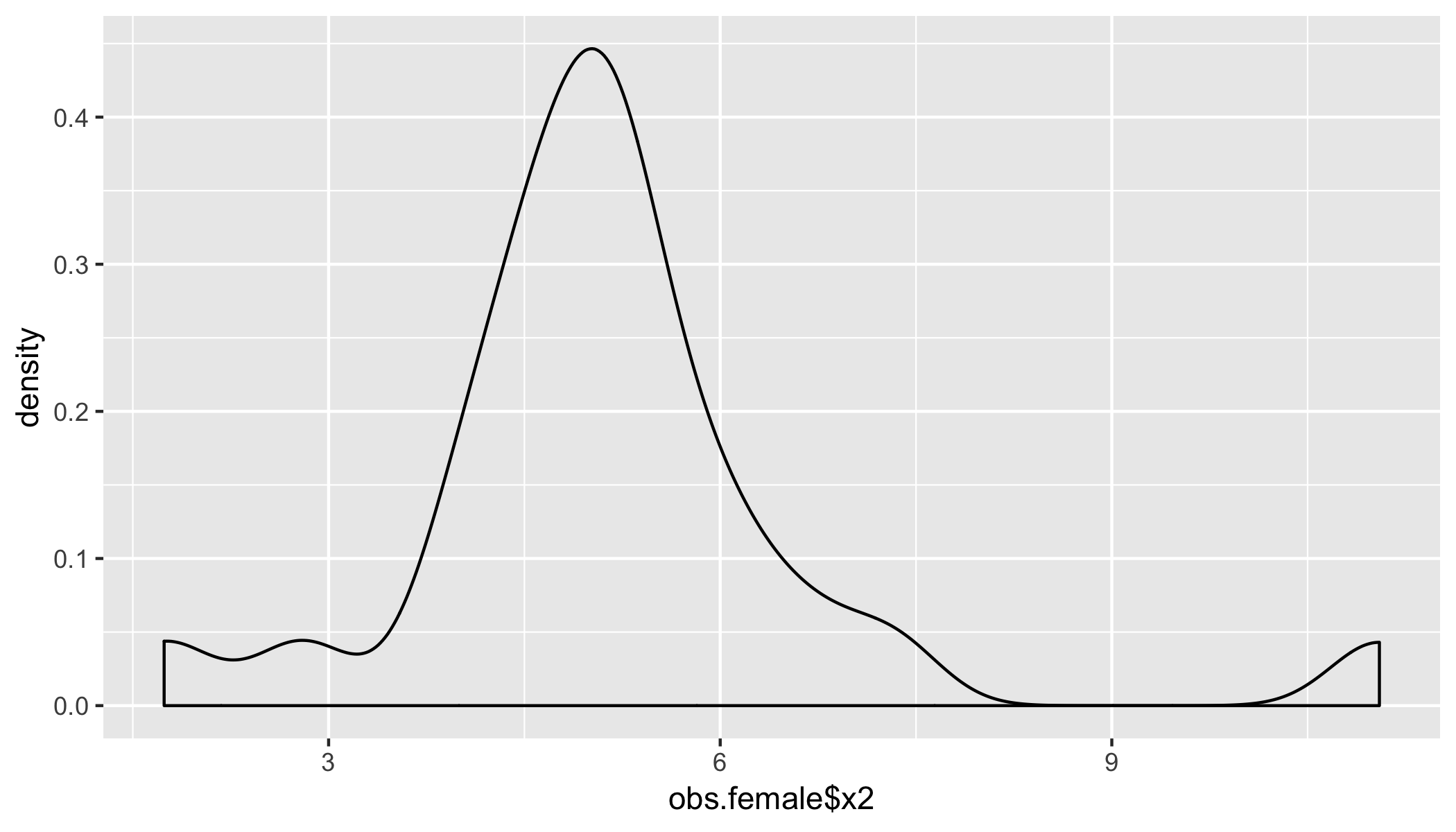
### Step 1: Download the dataset from online

wd <- getwd()  
res <- readLines("http://users.stat.umn.edu/~kb/classes/5401/files/data/JWData5.txt")  
# Figure out which lines we need. Each table reference will appear twice. Need to know the table you want and the next in the list  
  
start <- grep("T06\_12", res)[2]  
  
end <- grep("T06\_13", res)[2]  
  
rawtable <- res[start:(end - 2)]  
  
# Use the first line to find the dimensions of the table  
  
infovec <- strsplit(rawtable[1], " ")[[1]]  
  
infovec <- infovec[infovec != ""]  
  
length <- as.numeric(infovec[2])  
  
# Extract the rows containing the data (the last few rows)  
  
start <- length(rawtable) - length + 1  
  
rawdata <- rawtable[start:length(rawtable)]  
  
# Split the row into each data point  
  
final\_data <- strsplit(rawdata, " ")  
  
# Organise into a matrix, removing blank entries  
  
datatable <- matrix(0, length, as.numeric(infovec[3]))  
  
for (i in 1:length) {  
 row <- final\_data[[i]]  
  
 datatable[i, ] <- as.numeric(row[row != ""])  
}  
  
# Arrange in a data frame  
  
data.frame <- as.data.frame(datatable)  
colnames(data.frame) <- c("Gender", "x1", "x2", "x3", "x4")

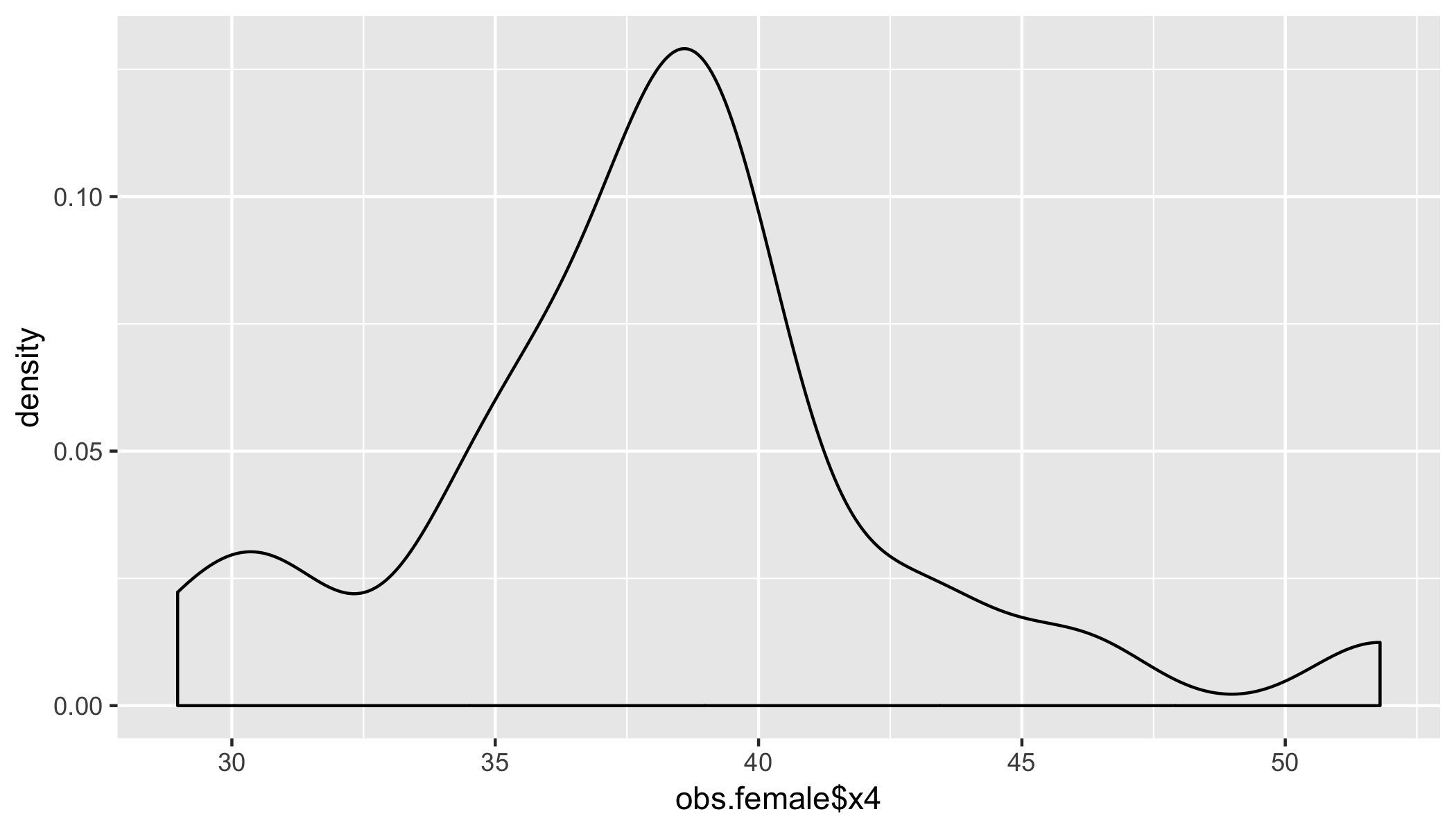
### Question 1a

We are using the data.table package from R to read the file in to memory

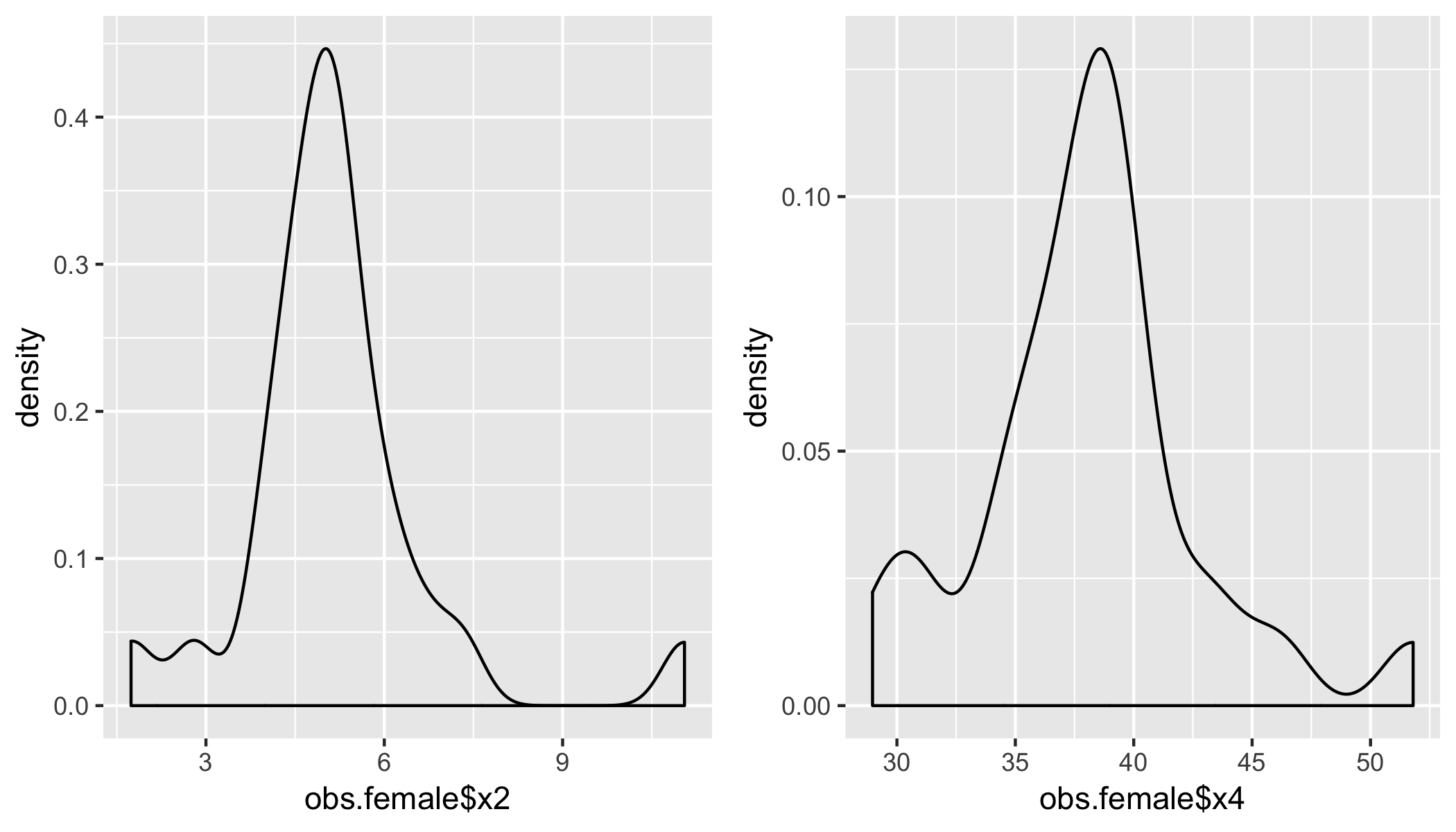
male <- 1  
female <- 2  
  
setDT(data.frame)  
obs.female <- data.frame[Gender==2]  
  
  
  
# Question 1a  
p1<-ggplot(obs.female, aes(x=obs.female$x2)) + geom\_density()  
p2<-ggplot(obs.female, aes(x=obs.female$x4)) + geom\_density()  
p1



p2



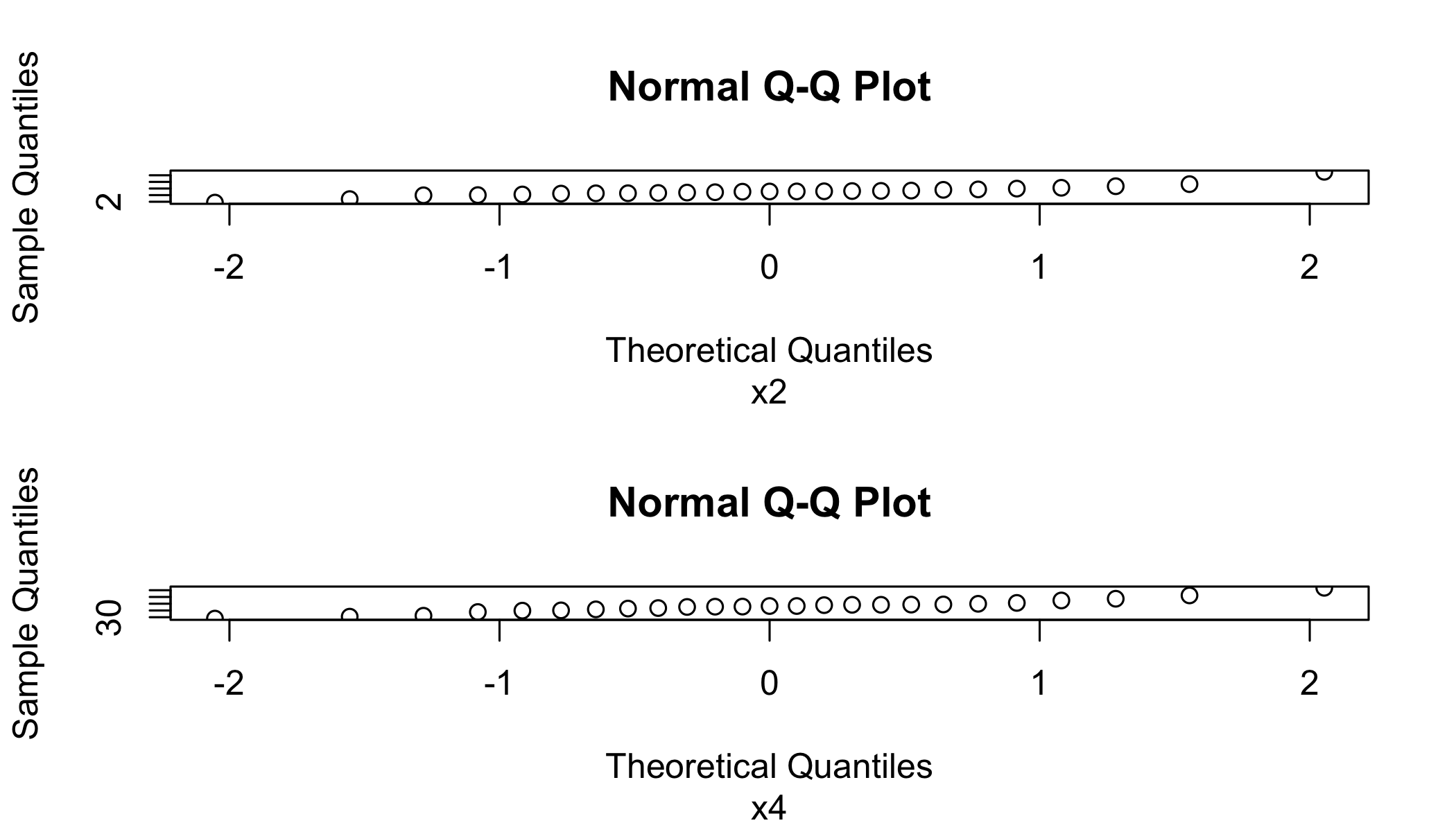
grid.arrange(p1, p2, ncol=2)



obs.female[,list(x1)]

## x1  
## 1: 0.29  
## 2: 0.28  
## 3: 0.31  
## 4: 0.30  
## 5: 0.28  
## 6: 0.11  
## 7: 0.25  
## 8: 0.26  
## 9: 0.39  
## 10: 0.37  
## 11: 0.31  
## 12: 0.35  
## 13: 0.29  
## 14: 0.33  
## 15: 0.18  
## 16: 0.28  
## 17: 0.44  
## 18: 0.22  
## 19: 0.34  
## 20: 0.30  
## 21: 0.31  
## 22: 0.27  
## 23: 0.66  
## 24: 0.37  
## 25: 0.35  
## x1

par(mfrow = c(2, 1))  
setDF(obs.female)  
qqnorm(obs.female[,3], sub = colnames(obs.female)[3])  
qqnorm(obs.female[,5], sub = colnames(obs.female)[5])



?qqnorm

### Question 1b

# Question 1b  
  
  
mvtest<-roystonTest(wine[,10:11],qqplot=TRUE)

## Warning: 'roystonTest' is deprecated.  
## Use 'mvn' instead.  
## See help("Deprecated")

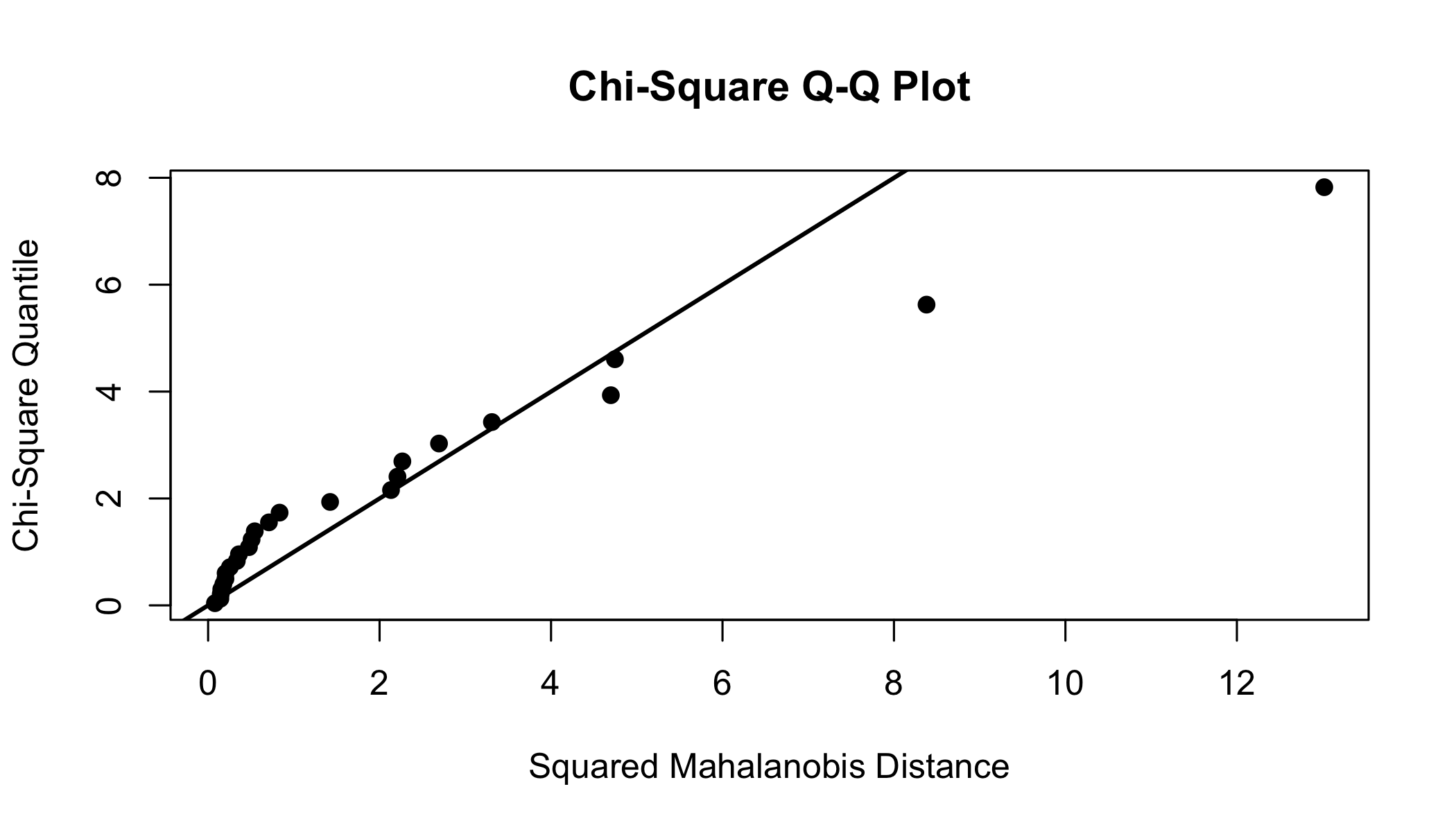
?mvn()

## Help on topic 'mvn' was found in the following packages:  
##   
## Package Library  
## mclust /usr/local/lib/R/3.5/site-library  
## MVN /usr/local/lib/R/3.5/site-library  
##   
##   
## Using the first match ...

setDT(obs.female)  
obs.female[,list(x2,x4)]

## x2 x4  
## 1: 5.04 33.85  
## 2: 3.95 35.82  
## 3: 4.88 36.40  
## 4: 5.97 37.87  
## 5: 4.57 38.30  
## 6: 1.74 39.19  
## 7: 4.66 39.21  
## 8: 5.28 39.94  
## 9: 7.32 42.41  
## 10: 6.22 28.97  
## 11: 4.20 37.80  
## 12: 5.10 31.10  
## 13: 4.46 38.30  
## 14: 5.60 51.80  
## 15: 2.80 37.60  
## 16: 4.01 36.78  
## 17: 6.69 46.16  
## 18: 4.55 38.95  
## 19: 5.73 40.60  
## 20: 5.12 43.69  
## 21: 4.77 30.40  
## 22: 5.16 39.46  
## 23: 11.05 39.34  
## 24: 5.23 34.86  
## 25: 5.37 35.07  
## x2 x4

par(mfrow = c(1, 1))  
mvn(obs.female[,list(x2,x4)], mvnTest='royston', multivariatePlot='qq')



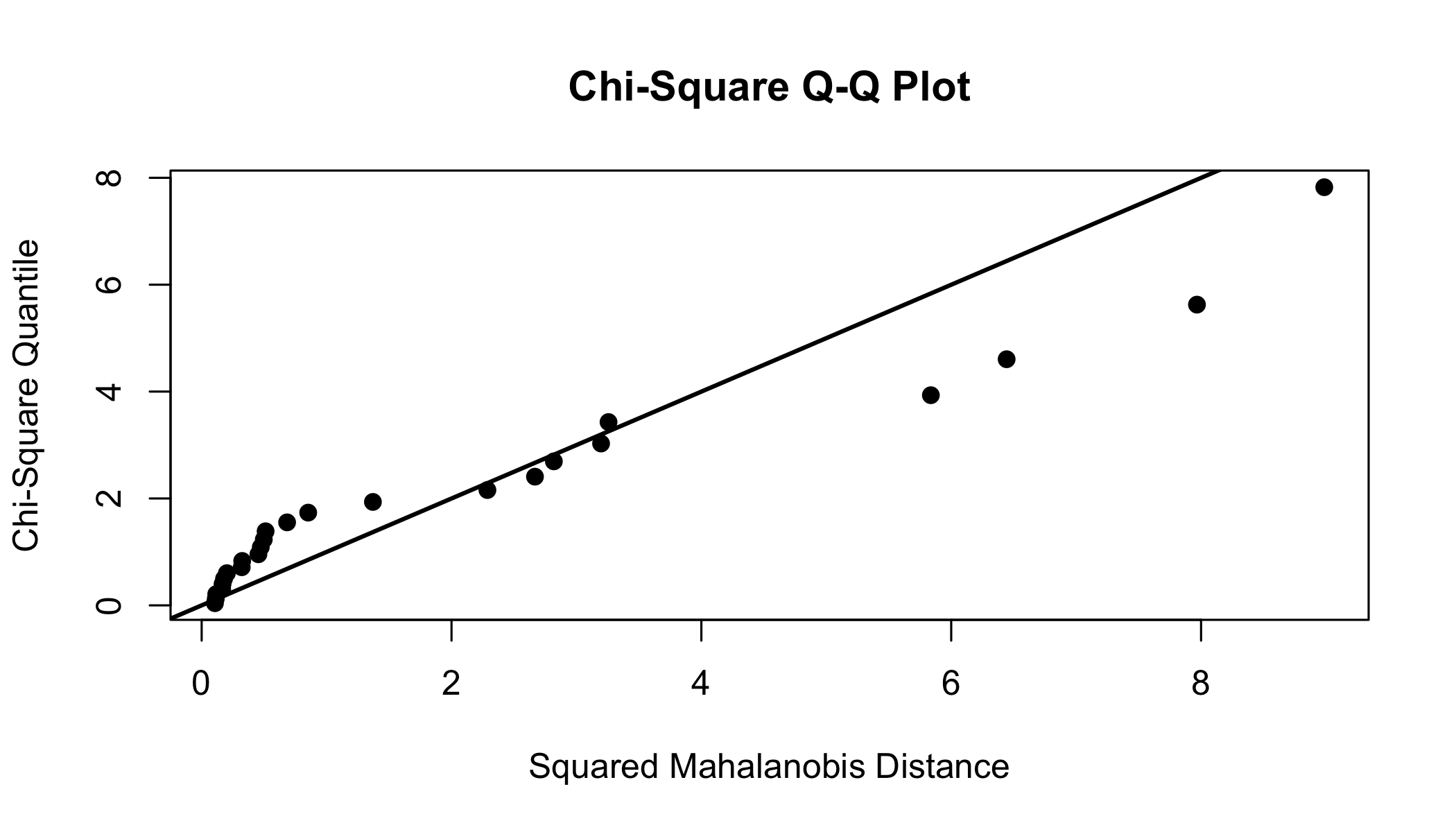
## $multivariateNormality  
## Test H p value MVN  
## 1 Royston 15.09293 0.0005281256 NO  
##   
## $univariateNormality  
## Test Variable Statistic p value Normality  
## 1 Shapiro-Wilk x2 0.8475 0.0016 NO   
## 2 Shapiro-Wilk x4 0.9441 0.1839 YES   
##   
## $Descriptives  
## n Mean Std.Dev Median Min Max 25th 75th Skew Kurtosis  
## x2 25 5.1788 1.667531 5.1 1.74 11.05 4.55 5.60 1.3623856 4.2103263  
## x4 25 38.1548 4.822948 38.3 28.97 51.80 35.82 39.46 0.5594578 0.9846515

### Question 1c

# Question 1c  
#  
#One approach to dealing with data that do not follow a multivariate normal distribution  
#is to perform a transformation. In practice, data are often transformed using a log transformation or a square root transformation to stabilise the variances of the response in  
#each of the treatment groups when they differ significantly. These two transformations  
#are in fact special cases of the Box–Cox transformation.  
#

### Question 1d

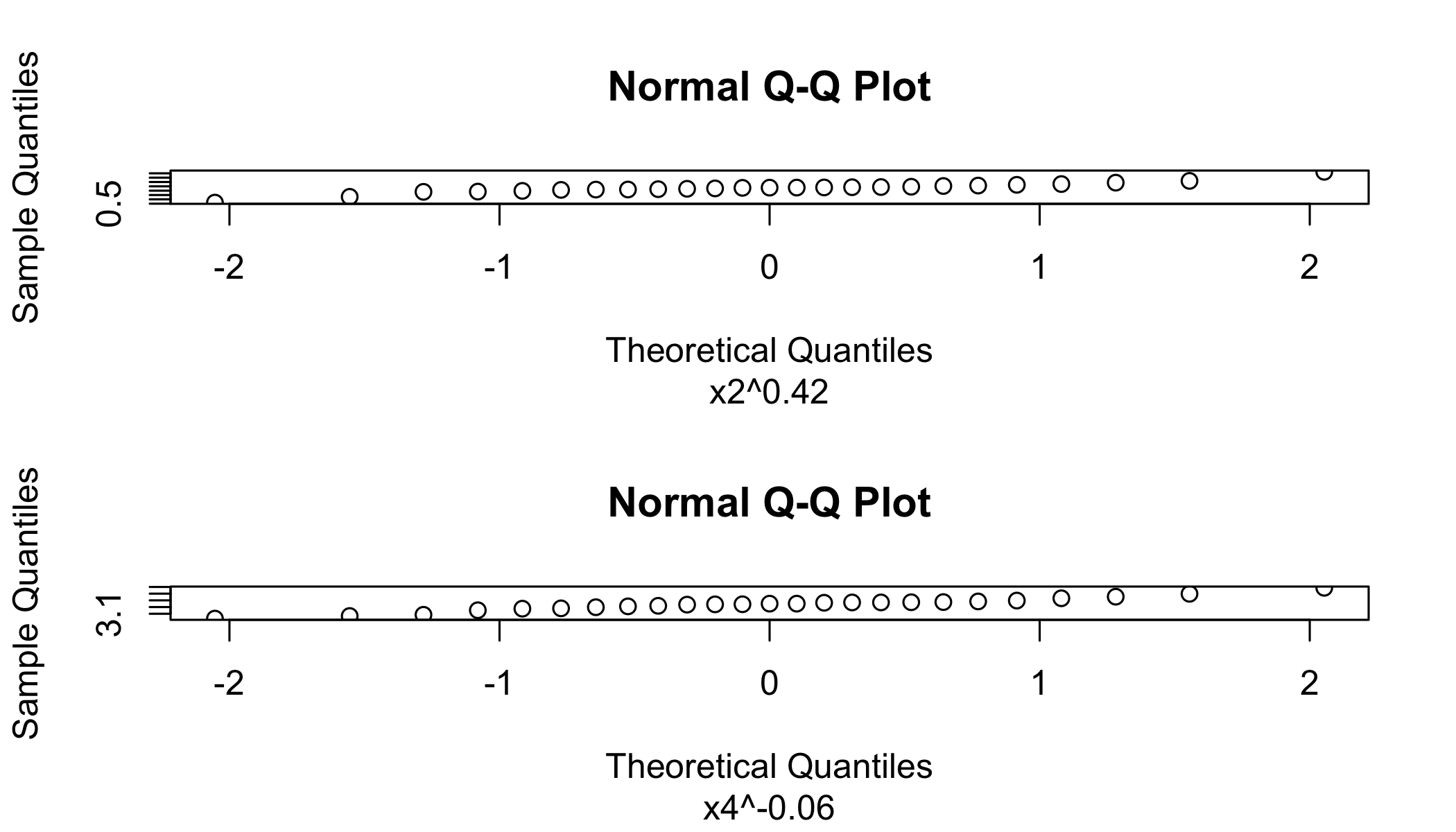
# Question 1d  
  
trans<-powerTransform(obs.female[,list(x2,x4)])  
obs.female.sub<-obs.female[,list(x2,x4)]  
obs.female.sub<-bcPower(obs.female.sub,trans$lambda)  
  
par(mfrow = c(1, 1))  
mvn(obs.female.sub, mvnTest='royston', multivariatePlot='qq')



## $multivariateNormality  
## Test H p value MVN  
## 1 Royston 9.917541 0.00702209 NO  
##   
## $univariateNormality  
## Test Variable Statistic p value Normality  
## 1 Shapiro-Wilk x2^0.42 0.8958 0.0148 NO   
## 2 Shapiro-Wilk x4^-0.06 0.9589 0.3935 YES   
##   
## $Descriptives  
## n Mean Std.Dev Median Min Max 25th  
## x2^0.42 25 2.324406 0.63061301 2.348569 0.624426 4.176261 2.126111  
## x4^-0.06 25 3.241326 0.09893204 3.250670 3.027693 3.487365 3.197568  
## 75th Skew Kurtosis  
## x2^0.42 2.539104 0.15554247 2.5537890  
## x4^-0.06 3.274264 0.04045563 0.4334795

### Question 1e

# Question 1e  
  
par(mfrow = c(2, 1))  
setDF(obs.female.sub)  
qqnorm(obs.female.sub[,1], sub = colnames(obs.female.sub)[1])  
qqnorm(obs.female.sub[,2], sub = colnames(obs.female.sub)[2])



## Question 2

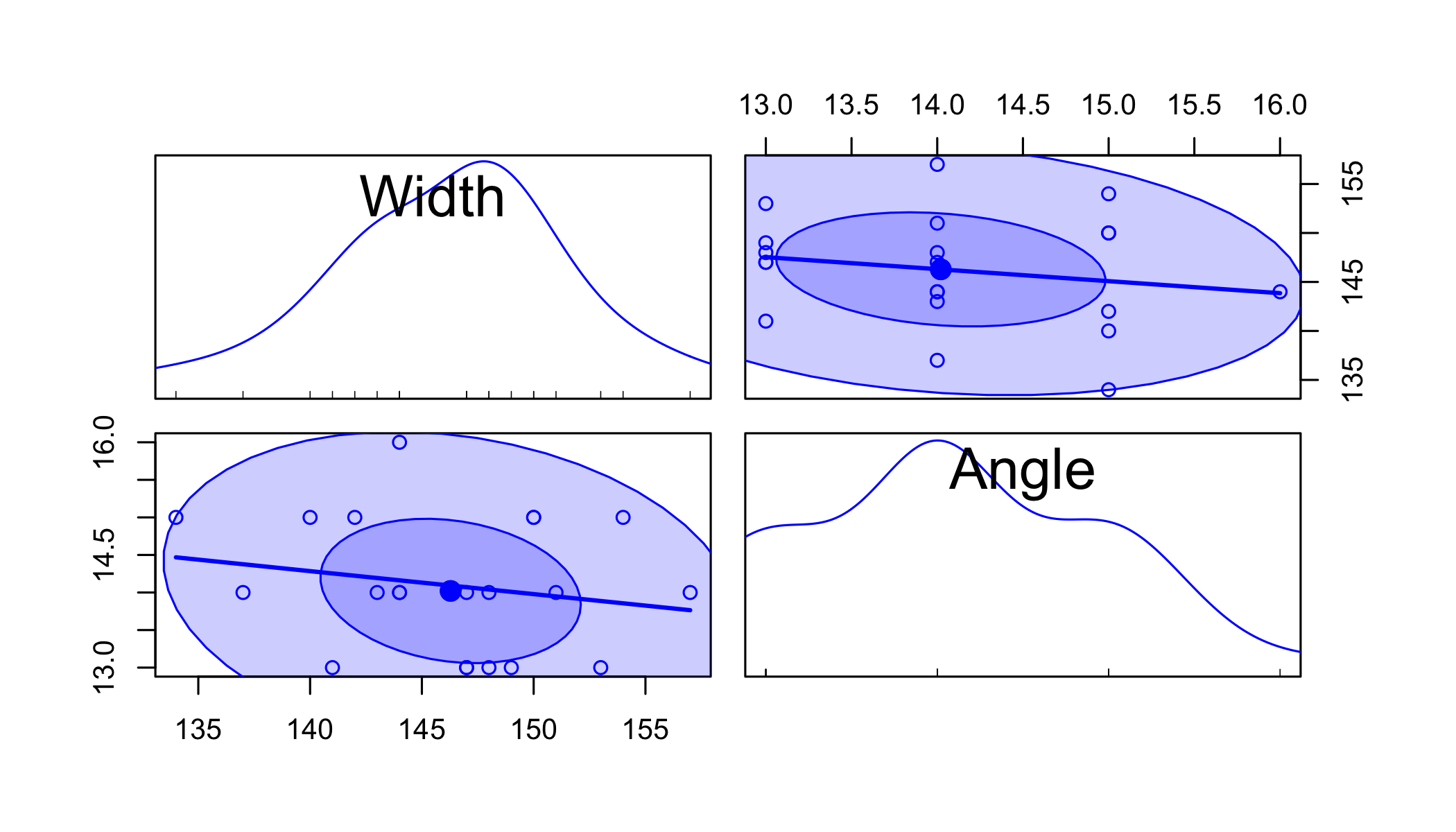
### Question 2a

# Question 2a  
  
concinna <- fread('Data/Concinna.csv')  
concinna[, Species:=NULL]  
  
setDF(concinna)  
  
for (i in 1:2) {  
 ci <- round(t.test(concinna[, i])$conf.int, 3)  
 cat(paste("The 95% CI for ", colnames(concinna)[i], " is: (", ci[1], ", ",  
 ci[2], ")\n"))  
}

## The 95% CI for Width is: ( 143.629 , 148.752 )  
## The 95% CI for Angle is: ( 13.691 , 14.5 )

scatterplotMatrix(~ Width+Angle,  
 data=concinna, smooth=FALSE, ellipse=TRUE, by.groups=TRUE,  
 diagonal="none")

## Warning in applyDefaults(diagonal, defaults = list(method =  
## "adaptiveDensity"), : unnamed diag arguments, will be ignored



### Question 2b

# Question 2b  
# u1 or Width is outside the 95% CI, so possibly not a concinna species

### Question 2c

# Question 2c  
  
Xbar <- colMeans(concinna)  
S <- cov(concinna)  
n <- nrow(concinna)  
p <- ncol(concinna)  
for (i in 1:2) {  
 lower <- round(Xbar[i] - sqrt(p \* (n - 1)/(n - p) \* qf(0.95, p, n - p)) \*  
 sqrt(S[i, i]/n), 3)  
 5  
 upper <- round(Xbar[i] + sqrt(p \* (n - 1)/(n - p) \* qf(0.95, p, n - p)) \*  
 sqrt(S[i, i]/n), 3)  
 cat(paste("The 95% CI for ", colnames(concinna)[i], " is: (", lower, ", ",  
 upper, ")\n"))  
}

## The 95% CI for Width is: ( 142.847 , 149.534 )  
## The 95% CI for Angle is: ( 13.567 , 14.624 )

### Question 2d

# Question 2d  
for (i in 1:2) {  
 ci <- round(t.test(concinna[, i], conf.level = (1 - 0.05/p))$conf.int, 3)  
 cat(paste("The 95% Bonferroni CI for ", colnames(concinna)[i], " is: (",  
 ci[1], ", ", ci[2], ")\n"))  
}

## The 95% Bonferroni CI for Width is: ( 143.215 , 149.166 )  
## The 95% Bonferroni CI for Angle is: ( 13.625 , 14.565 )

### Question 2e

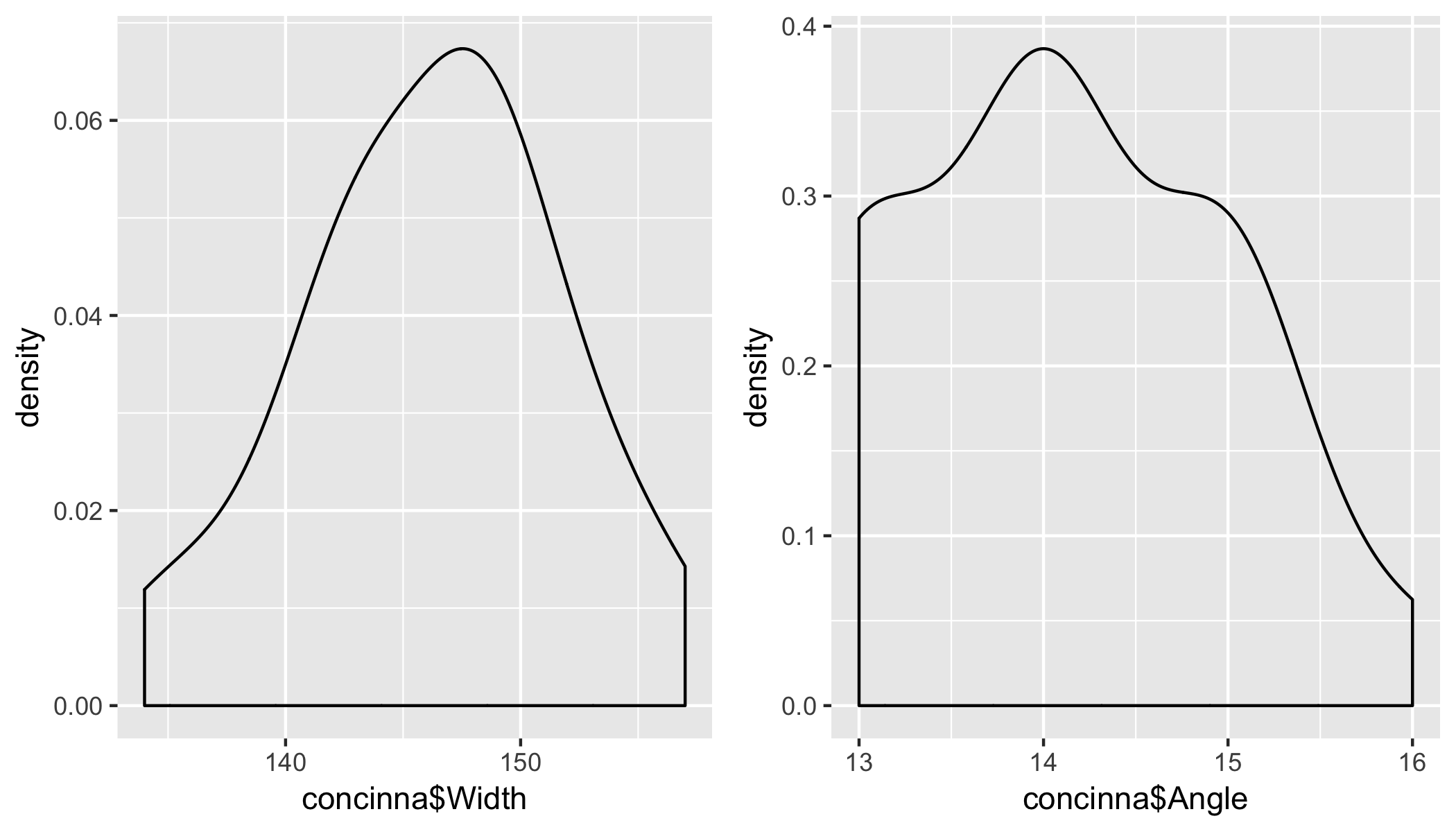
# Question 2e  
  
###  
### Write commentary

### Question 2f

# Question 2f  
  
par(mfrow = c(2, 1))  
mvn(concinna, mvnTest='royston', multivariatePlot='qq')

## $multivariateNormality  
## Test H p value MVN  
## 1 Royston 6.941658 0.03106203 NO  
##   
## $univariateNormality  
## Test Variable Statistic p value Normality  
## 1 Shapiro-Wilk Width 0.9881 0.9936 YES   
## 2 Shapiro-Wilk Angle 0.8669 0.0084 NO   
##   
## $Descriptives  
## n Mean Std.Dev Median Min Max 25th 75th Skew  
## Width 21 146.19048 5.6268912 147 134 157 143 150 -0.2111542  
## Angle 21 14.09524 0.8890873 14 13 16 13 15 0.2347644  
## Kurtosis  
## Width -0.5081691  
## Angle -1.0324544

p1<-ggplot(concinna, aes(x=concinna$Width)) + geom\_density()  
p2<-ggplot(concinna, aes(x=concinna$Angle)) + geom\_density()  
p1  
p2  
grid.arrange(p1, p2, ncol=2)



### Question 3

### Question 3a

# Question 3a  
  
##### Cov matrix  
  
vcmat <- 1/5630 \* matrix(c(575,-60,10,-60,300,-50,10,-50,196),nrow=3,byrow=TRUE)  
  
# inverse  
vcmat.inv <-solve(vcmat)

### Question 3b

# Question 3b

### Question 3c

# Question 3c  
#MVN  
  
  
mv<-rep(0, 3)  
  
mnd <- mvrnorm(n=1000,mv,vcmat)

### Question 3d

# Question 3d  
  
#Pair 1  
x <- seq(from=mv[1]-4\*vcmat[1,1],to=mv[1]+4\*vcmat[1,1],by=vcmat[1,1]/25)  
y <- seq(from=mv[2]-4\*vcmat[2,2],to=mv[2]+4\*vcmat[2,2],by=vcmat[2,2]/25)  
length(x)

## [1] 201

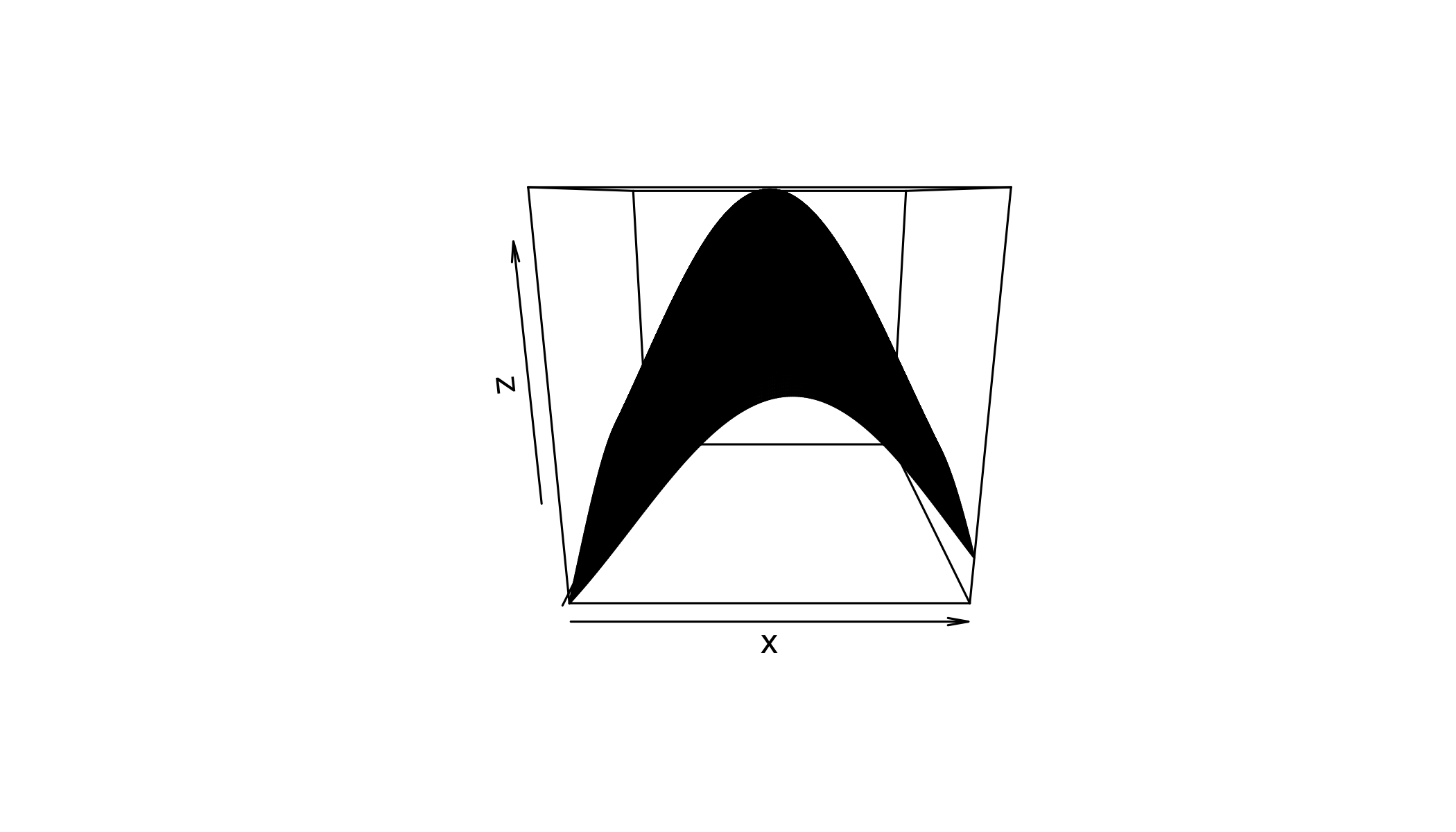
vcmat[1:2,1:2]

## [,1] [,2]  
## [1,] 0.10213144 -0.01065719  
## [2,] -0.01065719 0.05328597

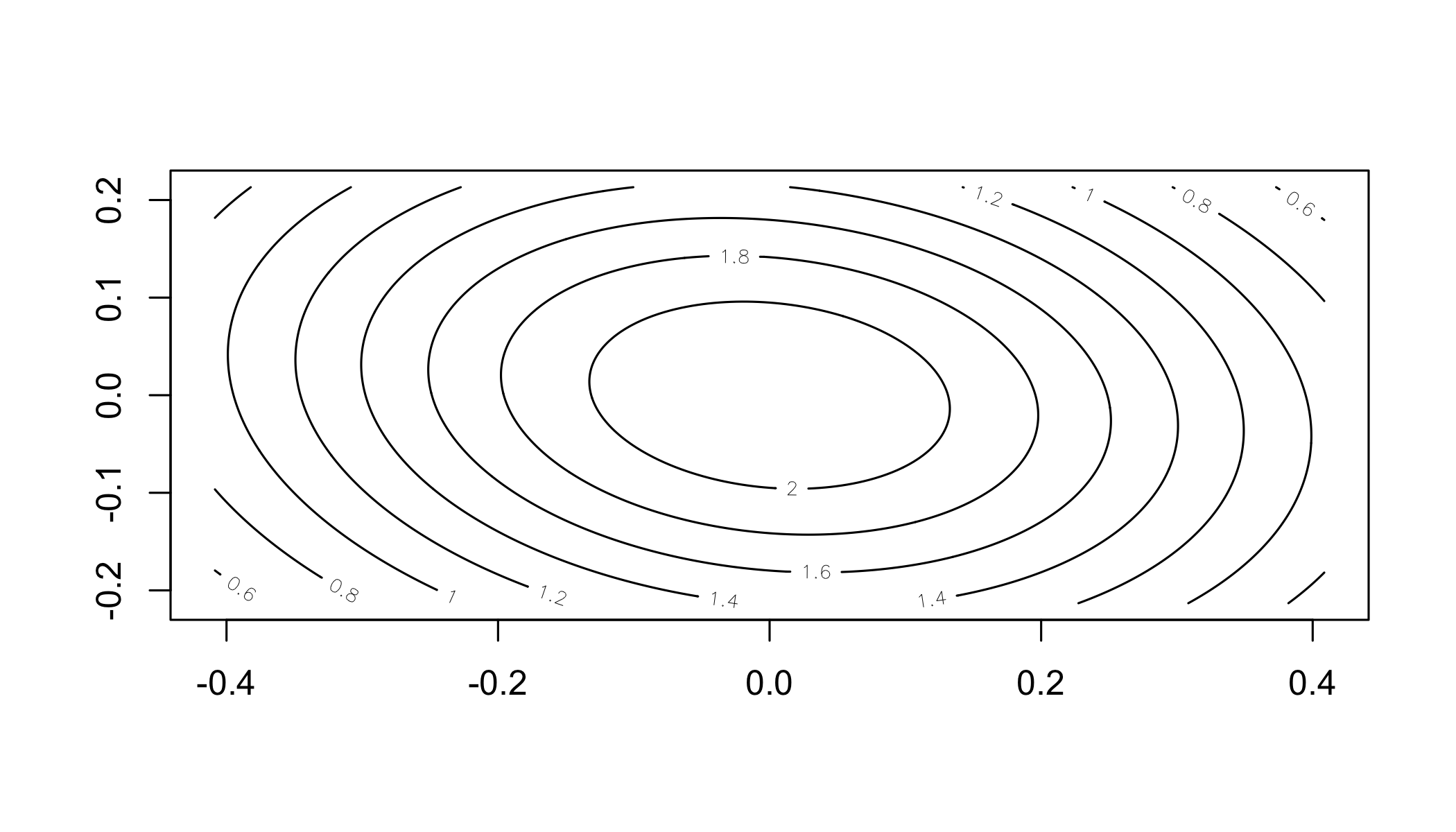
mv[1:2]

## [1] 0 0

z <- matrix(0,201,201)  
for(i in 1:201){  
 for(j in 1:201){  
 z[i,j]<-dmnorm(c(x[i],y[j]), mean = mv[1:2], vcmat[1:2,1:2], log = FALSE)  
 }  
}  
?dmnorm  
persp(x,y,z, axes = TRUE,box = TRUE)



contour(x,y,z, axes = TRUE)



?dmnorm()

### Question 3e

# Question 3e  
options(scipen = 999)  
B<-eigen(vcmat)  
B$values

## [1] 0.10453986 0.05453633 0.03115472

B$vectors

## [,1] [,2] [,3]  
## [1,] 0.97591370 -0.2153318 0.03499535  
## [2,] -0.21190556 -0.8975461 0.38666136  
## [3,] 0.05185053 0.3847638 0.92155755

#   
# P<-B$vectors  
# D<-B$values\*diag(3)  
# Pt<-t(P)  
#   
# round(P%\*%D%\*%Pt)  
#   
# round(P%\*%P1)  
#   
# round(P1%\*%P)  
#   
# A

### Question 3f

# Question 3f  
  
# ??  
cov(mnd)

## [,1] [,2] [,3]  
## [1,] 0.105859771 -0.008897214 0.003580874  
## [2,] -0.008897214 0.051560365 -0.009203564  
## [3,] 0.003580874 -0.009203564 0.036048772

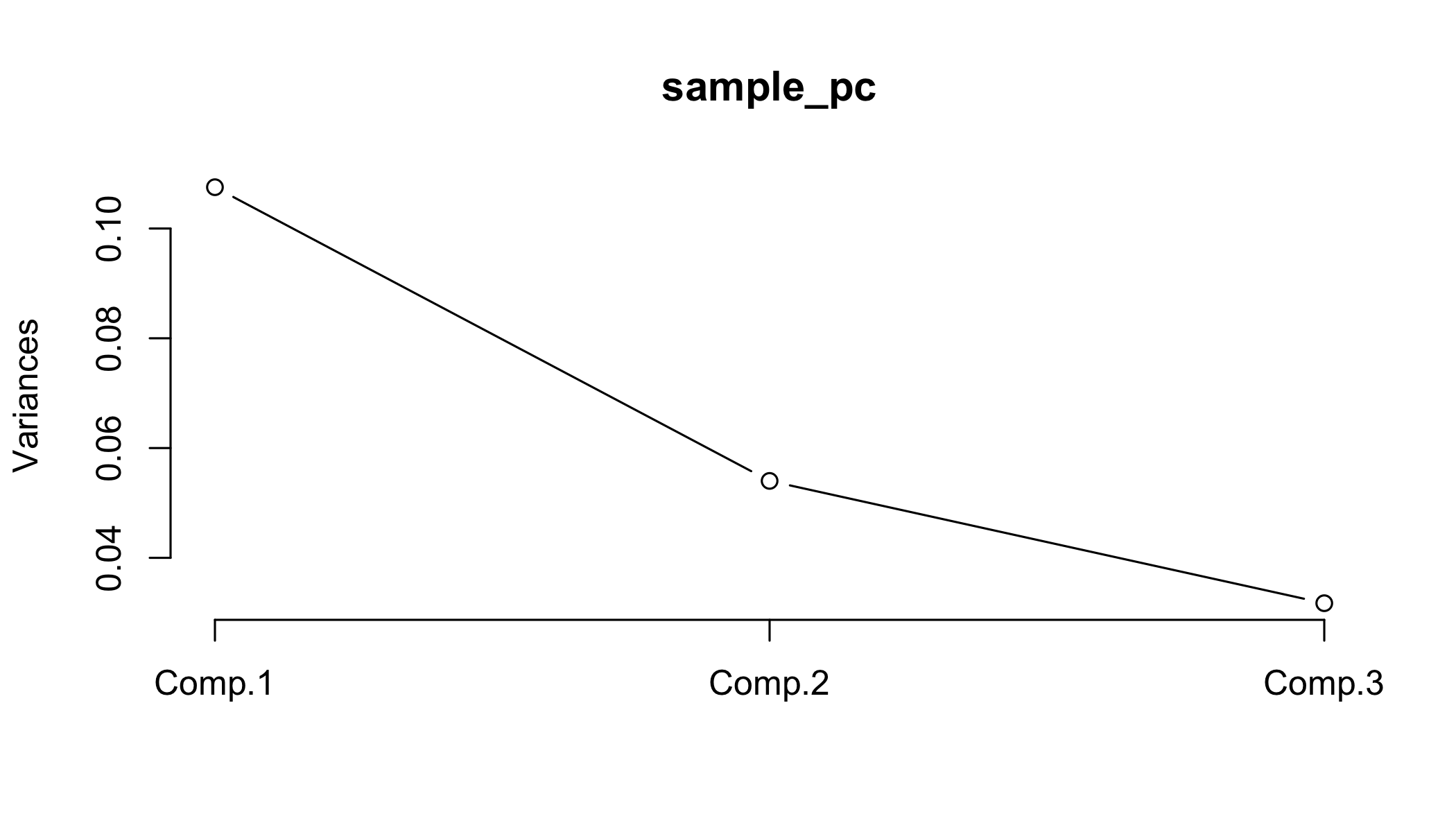
# ??  
vcmat

## [,1] [,2] [,3]  
## [1,] 0.102131439 -0.010657194 0.001776199  
## [2,] -0.010657194 0.053285968 -0.008880995  
## [3,] 0.001776199 -0.008880995 0.034813499

sample\_pc<-princomp(mnd)  
summary(sample\_pc, loadings = TRUE)

## Importance of components:  
## Comp.1 Comp.2 Comp.3  
## Standard deviation 0.3279124 0.2324122 0.1781389  
## Proportion of Variance 0.5563383 0.2794739 0.1641879  
## Cumulative Proportion 0.5563383 0.8358121 1.0000000  
##   
## Loadings:  
## Comp.1 Comp.2 Comp.3  
## [1,] 0.983 0.182   
## [2,] -0.168 0.890 0.424  
## [3,] -0.418 0.906

par(mfrow = c(1, 1))  
screeplot(sample\_pc, npcs = 3, type = "lines")



### Question 3g

# Question 3g  
  
# Comp1 and Comp2 explains 82%

### Question 3h

# Question 3h  
mnd

## [,1] [,2] [,3]  
## [1,] -0.1403275053 -0.038265363381 -0.2203961729  
## [2,] 0.5705769080 0.025171579277 -0.3140261607  
## [3,] 0.1052650055 -0.095400883610 0.1568703542  
## [4,] 0.0363753499 -0.247110607289 0.1079459548  
## [5,] 0.2027718785 0.041177422030 0.0094761725  
## [6,] -0.4661457348 -0.082811300363 0.1172550732  
## [7,] -0.3727841481 -0.061554676072 0.0309128044  
## [8,] 0.3290536575 -0.239769270656 0.1143222712  
## [9,] 0.5731604540 -0.040922543867 -0.0922221174  
## [10,] -0.1380999981 0.309280322057 -0.0628146607  
## [11,] -0.3360452732 0.134941693919 0.1171147051  
## [12,] 0.2027984818 0.092392276076 -0.3002917099  
## [13,] 0.1056859109 0.124788755203 -0.0465547078  
## [14,] 0.5974482889 -0.060354063991 0.0719932883  
## [15,] -0.4796274245 -0.272356789191 0.1344438127  
## [16,] -0.2799918853 -0.214659899115 -0.2772743276  
## [17,] -0.4349810667 0.223032967587 -0.0089042907  
## [18,] 0.4279867559 0.013849674745 0.0665667345  
## [19,] -0.2243457061 -0.198018019301 0.0739297005  
## [20,] -0.0635745268 -0.124761112842 0.0995159997  
## [21,] -0.0085123650 0.201464212515 -0.2783572127  
## [22,] 0.1828375861 -0.235084764794 -0.1210989776  
## [23,] 0.2069040603 -0.106752631209 0.0756050800  
## [24,] 0.2126539150 0.011232057981 0.1253065616  
## [25,] 0.3633277476 0.115177679953 -0.0593460331  
## [26,] -0.4763319849 -0.172222362068 0.1036212613  
## [27,] 0.3549981118 0.084518350018 0.0925931488  
## [28,] 0.1036767056 -0.064387609886 0.3738454068  
## [29,] 0.4458164153 -0.047639065779 -0.1565102339  
## [30,] -0.0574863645 0.452190360977 -0.0222736813  
## [31,] -0.4902769139 0.320150232759 -0.1454223970  
## [32,] 0.1921602060 -0.085870136650 0.2563746807  
## [33,] -0.3997752666 0.001886410840 -0.1808264338  
## [34,] 0.1796325966 0.398513873300 -0.2192828744  
## [35,] -0.2821660927 0.147246390545 -0.4302725230  
## [36,] -0.0118294834 0.229729579351 0.0681040132  
## [37,] 0.6196437188 0.065375132756 -0.1192708001  
## [38,] 0.0075095335 -0.311263199362 0.1330060563  
## [39,] -0.2521840075 -0.128077066420 -0.5286860837  
## [40,] 0.4155581696 0.369501906102 0.0267741248  
## [41,] -0.1055451424 0.046164555216 0.1090276658  
## [42,] -0.0059099413 0.118295466129 0.1418252159  
## [43,] 0.3781608038 0.159593775260 -0.0614972589  
## [44,] 0.6441433499 -0.092046625053 0.0957925895  
## [45,] 0.0861223607 -0.436153108628 -0.0672702199  
## [46,] 0.4660733712 -0.035918304156 -0.3117802130  
## [47,] 0.2500481951 0.121632698884 0.2725723953  
## [48,] 0.0463038152 -0.377928927140 0.1531290139  
## [49,] 0.0508022877 -0.215547411749 0.0235664307  
## [50,] -0.7112348142 -0.211272203545 0.1187050409  
## [51,] -0.3999723360 0.215483038127 0.1717743852  
## [52,] 0.0076066243 -0.324944901911 0.2399917306  
## [53,] -0.0390600179 0.269174810411 0.1584273564  
## [54,] -0.7178361988 -0.205382636110 -0.2170284164  
## [55,] 0.0389449477 0.153109437872 0.1415663905  
## [56,] -0.5708914280 0.431572034708 -0.2130193529  
## [57,] -0.2380151185 -0.282949660102 0.2432403294  
## [58,] -0.0401121556 0.214765386872 0.1916682419  
## [59,] -0.3421754113 0.280297367542 -0.0565967455  
## [60,] 0.2871111455 -0.148443144973 -0.1394370206  
## [61,] -0.0968861636 -0.371769838614 -0.0405652875  
## [62,] -0.2534262557 0.205329341731 -0.0591540852  
## [63,] 0.4828472147 0.011485948832 0.2295751118  
## [64,] 0.3999383159 -0.121711018132 0.1663374170  
## [65,] -0.0051476732 -0.171044055589 0.2519747382  
## [66,] -0.1929049288 0.162034867937 0.0141572164  
## [67,] 0.3479008574 0.022439162160 -0.3605893783  
## [68,] -0.0118925054 -0.163781773143 0.1899472544  
## [69,] 0.0819191641 -0.110866005527 0.1967802633  
## [70,] 0.0777821027 -0.060518718521 0.3578110276  
## [71,] 0.3688311944 0.080251822073 -0.0906949923  
## [72,] 0.1900529239 -0.484017082664 0.0055468204  
## [73,] -0.3962102513 -0.223156453213 -0.0835694135  
## [74,] -0.3226524731 0.272005435942 0.1426618020  
## [75,] -0.2646709731 0.067690981018 -0.0795604563  
## [76,] -0.2366999084 0.026947278063 -0.0528721870  
## [77,] -0.0008537671 -0.329400017685 0.0866307566  
## [78,] -0.5336851252 0.219429808683 -0.0368365517  
## [79,] -0.4254403612 0.171284614235 0.3116185559  
## [80,] -0.1300488277 0.204971953269 0.0109257797  
## [81,] 0.6220582640 -0.068714054202 0.1957653968  
## [82,] -0.3447927071 -0.040517560913 0.5416508925  
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## [917,] 0.0767045562 -0.270702363030 -0.3374946443  
## [918,] -0.3789667369 0.078509959700 -0.1824830547  
## [919,] -0.5293512349 -0.308618356955 -0.1446536878  
## [920,] 0.1081330448 0.329266618290 -0.0824666687  
## [921,] -0.0241802491 0.196289415681 -0.2317775677  
## [922,] -0.0369192574 -0.076778209225 -0.0959797819  
## [923,] -0.4533720687 -0.261591327781 0.2234290180  
## [924,] -0.0710321168 0.081683642543 -0.0989301854  
## [925,] 0.3358183939 0.019851906004 0.2090311261  
## [926,] 0.0320303819 0.143475629628 0.0076276336  
## [927,] 0.3547293671 0.201484817288 -0.2249852424  
## [928,] -0.0512544008 0.045165246033 0.0131183298  
## [929,] 0.4985091187 -0.257768730395 -0.1709974556  
## [930,] -0.4119436460 0.184204463564 -0.0157044297  
## [931,] 0.3176583202 0.314720904450 -0.1612416531  
## [932,] 0.2199423146 -0.194754520417 -0.4091705058  
## [933,] -0.0114573478 -0.117412405238 -0.3415751334  
## [934,] 0.6256264722 -0.122533964897 0.0339230878  
## [935,] -0.1492183184 0.084955121060 0.1058607832  
## [936,] 0.5661484106 0.348449089164 -0.0275934730  
## [937,] 0.4269430757 0.126360799864 -0.1503951682  
## [938,] 0.2041101318 -0.027318098731 -0.0197261766  
## [939,] 0.0698495065 -0.278668530040 0.0710822783  
## [940,] -0.4903164249 0.114596055215 0.1669755665  
## [941,] 0.3996083786 0.021117192183 0.0920349737  
## [942,] 0.2418467191 0.549936312797 -0.0541341866  
## [943,] 0.0867847538 -0.167297788245 0.1601286239  
## [944,] 0.1096615977 -0.245106593823 0.1327160107  
## [945,] 0.4953960256 -0.029186059432 -0.0354338731  
## [946,] -0.3889936003 0.188287556390 0.1553465600  
## [947,] -0.3587127382 -0.031272295917 0.0210941743  
## [948,] 0.0645062018 0.199268339493 0.4114259264  
## [949,] 0.0521851833 0.121187062540 0.0183336362  
## [950,] 0.0326557768 0.064970301850 0.0076730252  
## [951,] -0.1070623453 0.021463034576 -0.0765195845  
## [952,] 0.7524663573 0.061777467565 0.0002298994  
## [953,] 0.5908611943 0.060103882743 0.2321529016  
## [954,] 0.5458541695 0.210629969938 -0.1867025652  
## [955,] 0.1246842269 -0.271289384335 0.2188410069  
## [956,] 0.3995224985 0.122955637386 -0.0020864974  
## [957,] 0.1944265605 -0.198258577805 0.2222890171  
## [958,] 0.0209168382 -0.309929086150 -0.1004523122  
## [959,] 0.3088685858 -0.365793737575 0.3388572041  
## [960,] 0.8271973519 -0.128727875888 0.3145365464  
## [961,] -0.0149465120 0.367329276497 0.3191757651  
## [962,] -0.2233914613 -0.205654836549 0.0332173548  
## [963,] -0.2902723007 -0.129055614056 -0.0301548262  
## [964,] -0.0068406704 0.264077291871 0.0192412108  
## [965,] 0.1541482070 0.131802512225 -0.2366723327  
## [966,] -0.3778709516 -0.039941174972 -0.0512821564  
## [967,] -0.2638471730 -0.098072943827 0.1055363905  
## [968,] -0.8350202186 0.188730873731 0.1845777925  
## [969,] 0.0308982971 -0.337983326444 -0.0061785986  
## [970,] -0.2547007431 0.070348124333 -0.1875827337  
## [971,] 0.7138782604 -0.077788941329 -0.0648939244  
## [972,] 0.1115541341 -0.193619324871 -0.0953050972  
## [973,] -0.3388588251 0.190433429497 -0.2320753088  
## [974,] 0.3215021751 0.234823093374 0.0353157954  
## [975,] -0.0752673137 0.261826932413 -0.2218392123  
## [976,] 0.5608481665 -0.257330326907 -0.0264661077  
## [977,] -0.1650004162 0.209284978538 0.0342128238  
## [978,] -0.1950909930 -0.018222819446 0.2697423904  
## [979,] 0.1869899810 -0.273008203318 0.2369492717  
## [980,] 0.0609458172 -0.048892865256 0.3237348072  
## [981,] 0.0568304812 -0.084761921213 -0.0304818809  
## [982,] 0.2485684799 -0.243343551167 0.2007542117  
## [983,] 0.4365864550 0.148449189716 0.0964976108  
## [984,] -0.5749645265 0.152292645948 -0.0860169780  
## [985,] 0.5652976932 -0.171123782342 -0.0009353783  
## [986,] 0.2288811929 0.099089410114 -0.0552962144  
## [987,] -0.4163304407 -0.072746188475 -0.3849083154  
## [988,] 0.3533646056 -0.298297442504 0.2985249178  
## [989,] 0.2256182173 0.009533272500 0.0226062639  
## [990,] -0.2224565704 -0.416026874731 -0.0174081090  
## [991,] 0.2404153796 0.219526961206 0.0872296609  
## [992,] 0.8072952191 0.179723110466 0.2396838371  
## [993,] 0.4052945865 0.139499831795 -0.3168038323  
## [994,] -0.4995730919 -0.051354309688 0.3904240710  
## [995,] 0.5489764181 -0.330387926794 -0.0169888246  
## [996,] -0.6864078639 -0.188380793894 0.0427814562  
## [997,] -0.0822991148 0.060042642826 0.0392552122  
## [998,] 0.1284531909 0.194446138354 -0.1209278678  
## [999,] -0.0234721747 -0.159153312973 -0.0698827099  
## [1000,] -0.4214554040 -0.355965168985 0.3765017123

mnd\_df <- as.data.frame(as.table(mnd))  
setDT(mnd\_df)  
mnd\_dt <- dcast(mnd\_df, Var1~Var2, value.var = 'Freq')  
mnd\_dt[,Var1:=NULL]  
dim(mnd\_dt)

## [1] 1000 3

summary(mnd\_dt)

## A B C   
## Min. :-1.083204 Min. :-0.6251425 Min. :-0.657656   
## 1st Qu.:-0.210222 1st Qu.:-0.1535174 1st Qu.:-0.139250   
## Median : 0.011451 Median :-0.0034875 Median :-0.003815   
## Mean : 0.009491 Mean : 0.0003379 Mean :-0.003520   
## 3rd Qu.: 0.222775 3rd Qu.: 0.1579936 3rd Qu.: 0.130231   
## Max. : 0.934796 Max. : 0.8761535 Max. : 0.569680

colnames(mnd\_dt) <- c('Y1', 'Y2', 'Y3')  
  
model<-lm(Y2~Y1, data = mnd\_dt)  
  
model\_summary <- summary(model)  
#Coefficent of Y1  
model\_summary$coefficients[[2]]

## [1] -0.08404717

### Question 3i

# Question 3i  
  
model<-lm(Y3~Y1+Y2, data = mnd\_dt)  
  
model\_summary <- summary(model)  
#Coefficent of Y1  
model\_summary$coefficients[[2]]

## [1] 0.01910112

#Coefficent of Y2  
model\_summary$coefficients[[3]]

## [1] -0.1752047

### Question 3j

# Question 3j  
#Var(Y2|Y1)

### Question 3k

# Question 3k  
#Var(Y3|Y1, Y2)  
  
#For (h)-(k), you should find two different ways (one way is via exact formulas, and the other  
#way is via simulation in R) to compute the same answer. Then one way can be a check on the  
#other.

## Question 4

### Question 4a

# Question 4a  
  
data(diabetes)  
x <- diabetes$x  
x\_df <- as.data.frame(as.table(x))  
setDT(x\_df)  
x\_dt <- dcast(x\_df, Var1~Var2, value.var = 'Freq')  
x\_dt[,Var1:=NULL]  
dim(x\_dt)

## [1] 442 10

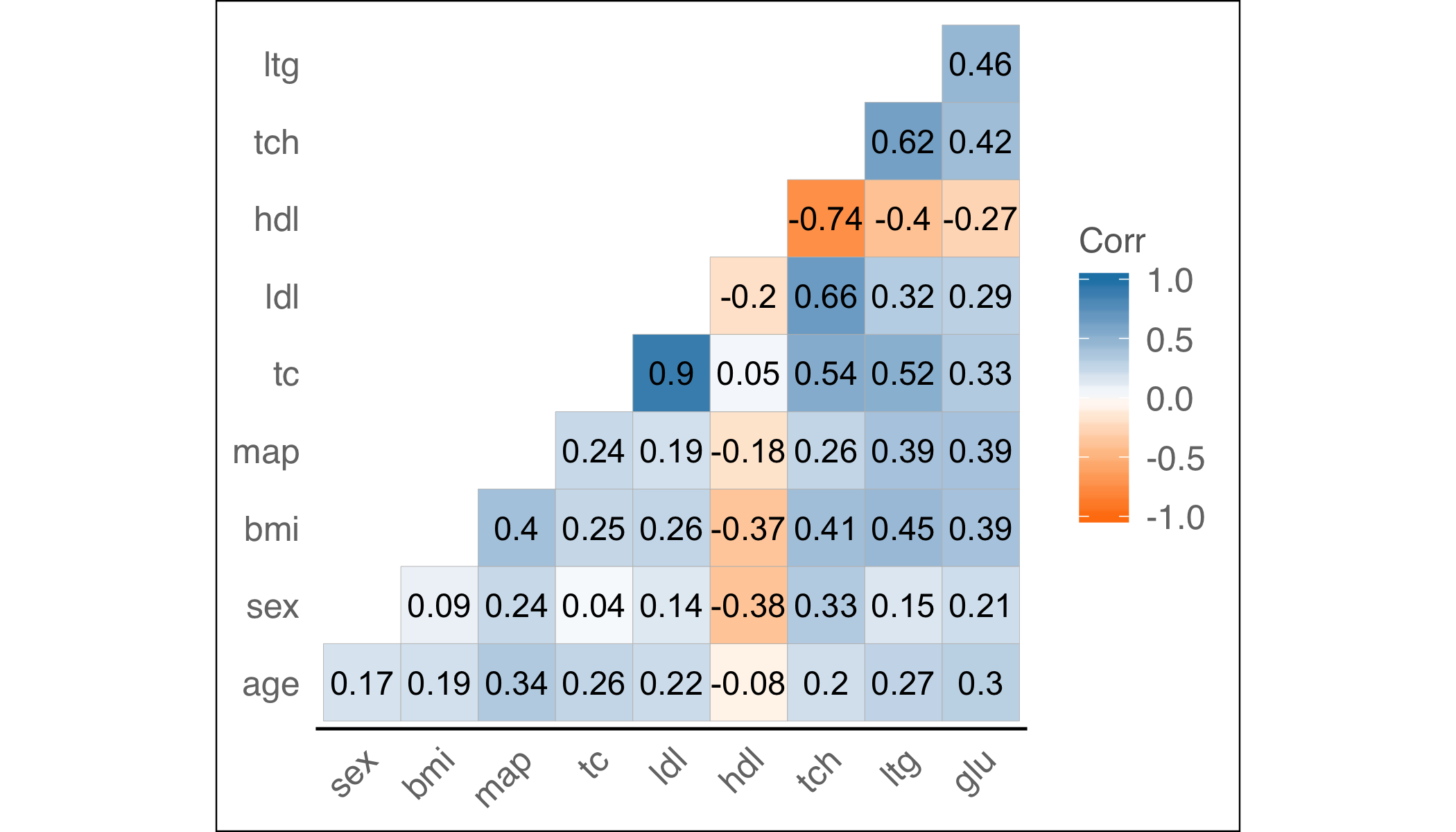
summary(x\_dt)

## age sex bmi   
## Min. :-0.107226 Min. :-0.04464 Min. :-0.090275   
## 1st Qu.:-0.037299 1st Qu.:-0.04464 1st Qu.:-0.034229   
## Median : 0.005383 Median :-0.04464 Median :-0.007284   
## Mean : 0.000000 Mean : 0.00000 Mean : 0.000000   
## 3rd Qu.: 0.038076 3rd Qu.: 0.05068 3rd Qu.: 0.031248   
## Max. : 0.110727 Max. : 0.05068 Max. : 0.170555   
## map tc ldl   
## Min. :-0.112400 Min. :-0.126781 Min. :-0.115613   
## 1st Qu.:-0.036656 1st Qu.:-0.034248 1st Qu.:-0.030358   
## Median :-0.005671 Median :-0.004321 Median :-0.003819   
## Mean : 0.000000 Mean : 0.000000 Mean : 0.000000   
## 3rd Qu.: 0.035644 3rd Qu.: 0.028358 3rd Qu.: 0.029844   
## Max. : 0.132044 Max. : 0.153914 Max. : 0.198788   
## hdl tch ltg   
## Min. :-0.102307 Min. :-0.076395 Min. :-0.126097   
## 1st Qu.:-0.035117 1st Qu.:-0.039493 1st Qu.:-0.033249   
## Median :-0.006584 Median :-0.002592 Median :-0.001948   
## Mean : 0.000000 Mean : 0.000000 Mean : 0.000000   
## 3rd Qu.: 0.029312 3rd Qu.: 0.034309 3rd Qu.: 0.032433   
## Max. : 0.181179 Max. : 0.185234 Max. : 0.133599   
## glu   
## Min. :-0.137767   
## 1st Qu.:-0.033179   
## Median :-0.001078   
## Mean : 0.000000   
## 3rd Qu.: 0.027917   
## Max. : 0.135612

#Corr Plot  
ncol(x\_dt)

## [1] 10

corr <- cor(x\_dt, use = "pairwise.complete.obs")  
  
ggcorrplot(corr, hc.order = FALSE, type = "lower",  
 ggtheme = ggthemes::theme\_gdocs,  
 colors = c("#ff7f0e", "white", "#1f83b4"),  
 lab = TRUE)+  
 theme(panel.grid.major=element\_blank())



### Question 4b

# Question 4b  
x\_dt[,y:=diabetes$y]  
  
model<-lm(y~., data = x\_dt)  
  
model\_summary <- summary(model)  
model\_summary$r.squared

## [1] 0.5177494

anova <- anova(model)  
sse <- anova$`Sum Sq`[[11]]  
vif(model)

## age sex bmi map tc ldl hdl   
## 1.217307 1.278073 1.509446 1.459429 59.203786 39.194379 15.402352   
## tch ltg glu   
## 8.890986 10.076222 1.484623

### Question 4c

# Question 4c  
model<-lm(y~age+bmi+map+hdl+ltg, data = x\_dt)  
model\_summary <- summary(model)  
model\_summary$r.squared

## [1] 0.4920391

#The idea is to reduce r square  
  
anova(model)

## Analysis of Variance Table  
##   
## Response: y  
## Df Sum Sq Mean Sq F value Pr(>F)   
## age 1 92527 92527 30.301 0.000000063252423 \*\*\*  
## bmi 1 825987 825987 270.496 < 0.00000000000000022 \*\*\*  
## map 1 119988 119988 39.294 0.000000000877373 \*\*\*  
## hdl 1 89056 89056 29.164 0.000000109511614 \*\*\*  
## ltg 1 162081 162081 53.079 0.000000000001513 \*\*\*  
## Residuals 436 1331370 3054   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

vif(model)

## age bmi map hdl ltg   
## 1.159192 1.421292 1.358792 1.260133 1.494444

### Question 4g

# Question 4g  
  
setDF(x\_dt)  
  
for (i in 1:10) {  
 ci <- round(t.test(x\_dt[, i])$conf.int, 3)  
 cat(paste("The 95% CI for ", colnames(x\_dt)[i], " is: (", ci[1], ", ",  
 ci[2], ")\n"))  
}

## The 95% CI for age is: ( -0.004 , 0.004 )  
## The 95% CI for sex is: ( -0.004 , 0.004 )  
## The 95% CI for bmi is: ( -0.004 , 0.004 )  
## The 95% CI for map is: ( -0.004 , 0.004 )  
## The 95% CI for tc is: ( -0.004 , 0.004 )  
## The 95% CI for ldl is: ( -0.004 , 0.004 )  
## The 95% CI for hdl is: ( -0.004 , 0.004 )  
## The 95% CI for tch is: ( -0.004 , 0.004 )  
## The 95% CI for ltg is: ( -0.004 , 0.004 )  
## The 95% CI for glu is: ( -0.004 , 0.004 )

### Question 4h

# Question 4h  
  
setDF(x\_dt)  
  
Xbar <- colMeans(x\_dt[1:10])  
S <- cov(x\_dt[1:10])  
n <- nrow(x\_dt[1:10])  
p <- ncol(x\_dt[1:10])  
for (i in 1:10) {  
 lower <- round(Xbar[i] - sqrt(p \* (n - 1)/(n - p) \* qf(0.95, p, n - p)) \*  
 sqrt(S[i, i]/n), 3)  
 5  
 upper <- round(Xbar[i] + sqrt(p \* (n - 1)/(n - p) \* qf(0.95, p, n - p)) \*  
 sqrt(S[i, i]/n), 3)  
 cat(paste("The 95% CI for ", colnames(x\_dt)[i], " is: (", lower, ", ",  
 upper, ")\n"))  
}

## The 95% CI for age is: ( -0.01 , 0.01 )  
## The 95% CI for sex is: ( -0.01 , 0.01 )  
## The 95% CI for bmi is: ( -0.01 , 0.01 )  
## The 95% CI for map is: ( -0.01 , 0.01 )  
## The 95% CI for tc is: ( -0.01 , 0.01 )  
## The 95% CI for ldl is: ( -0.01 , 0.01 )  
## The 95% CI for hdl is: ( -0.01 , 0.01 )  
## The 95% CI for tch is: ( -0.01 , 0.01 )  
## The 95% CI for ltg is: ( -0.01 , 0.01 )  
## The 95% CI for glu is: ( -0.01 , 0.01 )