Multivariate Data Analysis - Assignment 2

## Multivariate Data Analysis Spring 2019 (37459-2019-SPRING-CITY)

### Assignment: 2

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## Part A

## Question 1

For a given mean vector and covariance matrix, we can simulate random samples from the multivariate normal distribution in R using the ‘mvrnorm’ function from **MASS** package.

# Question 1  
# Mean vector  
mv<-rep(0, 3)  
  
# Cov matrix  
vcmat <- 1/5630 \* matrix(c(575,-60,10,-60,300,-50,10,-50,196),nrow=3,byrow=TRUE)  
  
# Covariance matrix  
print(vcmat)

## [,1] [,2] [,3]  
## [1,] 0.102131439 -0.010657194 0.001776199  
## [2,] -0.010657194 0.053285968 -0.008880995  
## [3,] 0.001776199 -0.008880995 0.034813499

#MVN  
mnd <- mvrnorm(n=1000,mv,vcmat)

### Question 1a

Calculate the least square estimates using R function for Y2 and Y3 where:

and

Perform a linear regression to find the coefficients , and .

# Question 1a  
  
# Convert matrix to a data.table  
mnd\_df <- as.data.frame(as.table(mnd))  
setDT(mnd\_df)  
mnd\_dt <- dcast(mnd\_df, Var1~Var2, value.var = 'Freq')  
mnd\_dt[,Var1:=NULL]  
  
colnames(mnd\_dt) <- c('Y1', 'Y2', 'Y3')  
  
model\_1<-lm(Y2~Y1, data = mnd\_dt)  
  
model\_summary <- summary(model\_1)  
  
# Coefficent of Y1  
beta2\_1 <- model\_summary$coefficients[[2]]  
print(beta2\_1)

## [1] -0.1026132

model\_2<-lm(Y3~Y1+Y2, data = mnd\_dt)  
  
model\_summary <- summary(model\_2)  
  
#Coefficent of Y1  
beta3\_1 <- model\_summary$coefficients[[2]]  
print(beta3\_1)

## [1] 0.01863946

#Coefficent of Y2  
beta3\_2 <- model\_summary$coefficients[[3]]  
print(beta3\_2)

## [1] -0.1715777

### Question 1b

Estimate

# Question 1b  
sigma\_2\_square <- (summary(model\_1)$sigma)^2  
print(sigma\_2\_square)

## [1] 0.04970386

### Question 1c

Estimate

# Question 1c  
sigma\_3\_square <- (summary(model\_2)$sigma)^2  
print(sigma\_3\_square)

## [1] 0.03380183

### Question 1d

Construct the 3x3 matrix from coefficients

T <- matrix(c(1,-1\*beta2\_1,-1\*beta3\_1,0,1,-1\*beta3\_2,0,0,1),nrow = 3)  
print(T)

## [,1] [,2] [,3]  
## [1,] 1.00000000 0.0000000 0  
## [2,] 0.10261322 1.0000000 0  
## [3,] -0.01863946 0.1715777 1

print(round(T%\*%t(T)))

## [,1] [,2] [,3]  
## [1,] 1 0 0  
## [2,] 0 1 0  
## [3,] 0 0 1

### Question 1e

Compute

TT <- T%\*%vcmat%\*%t(T)  
print(TT)

## [,1] [,2] [,3]  
## [1,] 0.1021314387 -0.0001771578 -0.0019560125  
## [2,] -0.0001771578 0.0521742203 0.0002596199  
## [3,] -0.0019560125 0.0002596199 0.0333720531

### Question 1f

Calculate given:

where is a 3x3 diagonal matrix, with entries on the main diagonal as , , and has already been calculated earlier.

To make as close to possible, let us assume:

Based on that we can calculate the value of as:

vcmat\_inv <- solve(vcmat)  
round(vcmat\_inv)

## [,1] [,2] [,3]  
## [1,] 10 2 0  
## [2,] 2 20 5  
## [3,] 0 5 30

y <- vcmat\_inv[1,1]   
sigma\_1\_square <-y-(T[2,1]^2\*sigma\_2\_square+ T[3,1]\*sigma\_3\_square)  
  
sigma\_1\_square <-y-(sigma\_2\_square+ sigma\_3\_square)  
  
D = matrix(c(sigma\_1\_square, 0, 0, 0, sigma\_2\_square, 0, 0, 0, sigma\_3\_square),nrow = 3)  
print(D)

## [,1] [,2] [,3]  
## [1,] 9.916494 0.00000000 0.00000000  
## [2,] 0.000000 0.04970386 0.00000000  
## [3,] 0.000000 0.00000000 0.03380183

print(round(vcmat\_inv, 2))

## [,1] [,2] [,3]  
## [1,] 10 2 0  
## [2,] 2 20 5  
## [3,] 0 5 30

print(round(t(T)%\*%solve(D)%\*%T))

## [,1] [,2] [,3]  
## [1,] 0 2 -1  
## [2,] 2 21 5  
## [3,] -1 5 30

## Question 2

### Load the dataset from local storage

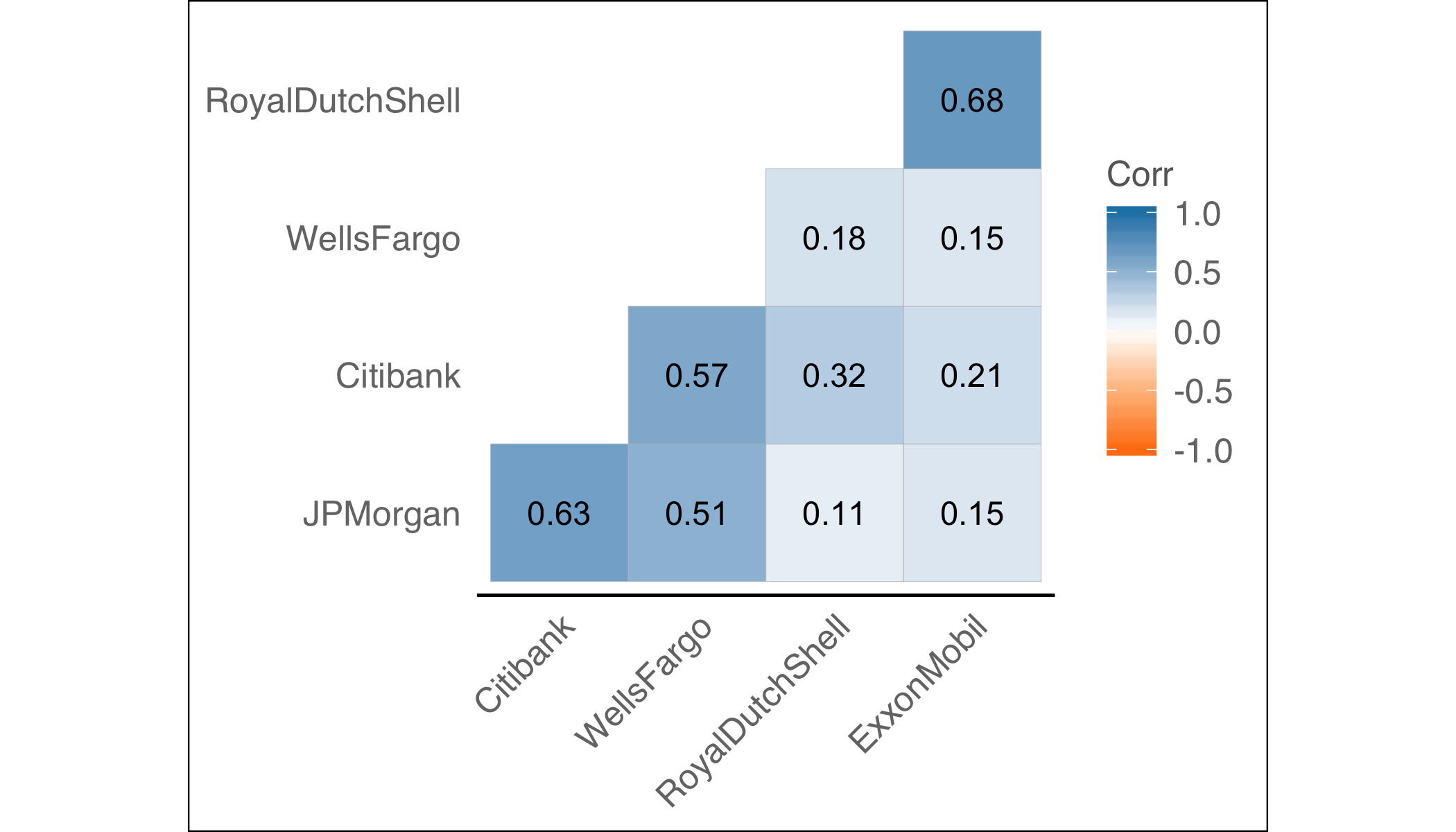
Load the dataset using ‘fread’ from **data.table** package. The stock data consists of weekly returns of five different stocks. The weekly returns of each stock are defined as (current week closing price - previous week closing price)/(previous week closing price) for 103 successive weeks.

dat <- fread('Data/stockdata.csv')  
summary(dat)

## JPMorgan Citibank WellsFargo   
## Min. :-0.045867 Min. :-0.0597924 Min. :-0.0362141   
## 1st Qu.:-0.013564 1st Qu.:-0.0132409 1st Qu.:-0.0080536   
## Median : 0.003363 Median : 0.0017339 Median : 0.0003354   
## Mean : 0.001063 Mean : 0.0006554 Mean : 0.0016261   
## 3rd Qu.: 0.016804 3rd Qu.: 0.0140293 3rd Qu.: 0.0100178   
## Max. : 0.048480 Max. : 0.0525266 Max. : 0.0406957   
## RoyalDutchShell ExxonMobil   
## Min. :-0.053948 Min. :-0.063605   
## 1st Qu.:-0.014470 1st Qu.:-0.012539   
## Median : 0.006335 Median : 0.005215   
## Mean : 0.004049 Mean : 0.004039   
## 3rd Qu.: 0.022237 3rd Qu.: 0.021622   
## Max. : 0.061994 Max. : 0.078416

The observations in the data appear to be independently distributed but the rate of return across stocks are correlated. Generally, one would expect the stocks would move together in response to general economic conditions. The correlation below shows a relationship between JPMorgan, Citibank and WellsFargo (banking stocks) and also relationship between Royal Dutch Shell and ExxonMobil (oil stocks)

ggcorrplot(cor(dat, use = "pairwise.complete.obs"), hc.order = FALSE, type = "lower",  
 ggtheme = ggthemes::theme\_gdocs,  
 colors = c("#ff7f0e", "white", "#1f83b4"),  
 lab = TRUE)+  
 theme(panel.grid.major=element\_blank())



### Question 2a

Perform factor analysis using principal component analysis method. Looking at the importance of the components, the first two principal components explains 80% of the variance in the data. The proportion of the variance explained by component 3 is less than 0.2 () ( being 5 in this case). A rule of thumb suggests retaining only those components whose variances individually are greater than . So, we will retain only the first two principal components.

# Principal component analysis method  
dat\_pc<-princomp(dat)  
summary(dat\_pc, loadings = TRUE)

## Importance of components:  
## Comp.1 Comp.2 Comp.3 Comp.4  
## Standard deviation 0.03680217 0.02635056 0.01585365 0.01188352  
## Proportion of Variance 0.52926066 0.27133298 0.09821584 0.05518400  
## Cumulative Proportion 0.52926066 0.80059364 0.89880948 0.95399348  
## Comp.5  
## Standard deviation 0.01085046  
## Proportion of Variance 0.04600652  
## Cumulative Proportion 1.00000000  
##   
## Loadings:  
## Comp.1 Comp.2 Comp.3 Comp.4 Comp.5  
## JPMorgan 0.223 0.625 0.326 0.663 0.118  
## Citibank 0.307 0.570 -0.250 -0.414 -0.589  
## WellsFargo 0.155 0.345 -0.497 0.780  
## RoyalDutchShell 0.639 -0.248 -0.642 0.309 0.148  
## ExxonMobil 0.651 -0.322 0.646 -0.216

Based on the importance of the components, we have seen that first two components explains most of the variance, hence we are going to perform factor analysis using m=2 factors.

fact\_pc<-principal(dat, nfactors=2,rotate="none")  
print(fact\_pc)

## Principal Components Analysis  
## Call: principal(r = dat, nfactors = 2, rotate = "none")  
## Standardized loadings (pattern matrix) based upon correlation matrix  
## PC1 PC2 h2 u2 com  
## JPMorgan 0.73 -0.44 0.73 0.27 1.6  
## Citibank 0.83 -0.28 0.77 0.23 1.2  
## WellsFargo 0.73 -0.37 0.67 0.33 1.5  
## RoyalDutchShell 0.60 0.69 0.85 0.15 2.0  
## ExxonMobil 0.56 0.72 0.83 0.17 1.9  
##   
## PC1 PC2  
## SS loadings 2.44 1.41  
## Proportion Var 0.49 0.28  
## Cumulative Var 0.49 0.77  
## Proportion Explained 0.63 0.37  
## Cumulative Proportion 0.63 1.00  
##   
## Mean item complexity = 1.6  
## Test of the hypothesis that 2 components are sufficient.  
##   
## The root mean square of the residuals (RMSR) is 0.1   
## with the empirical chi square 19.17 with prob < 1.2e-05   
##   
## Fit based upon off diagonal values = 0.95

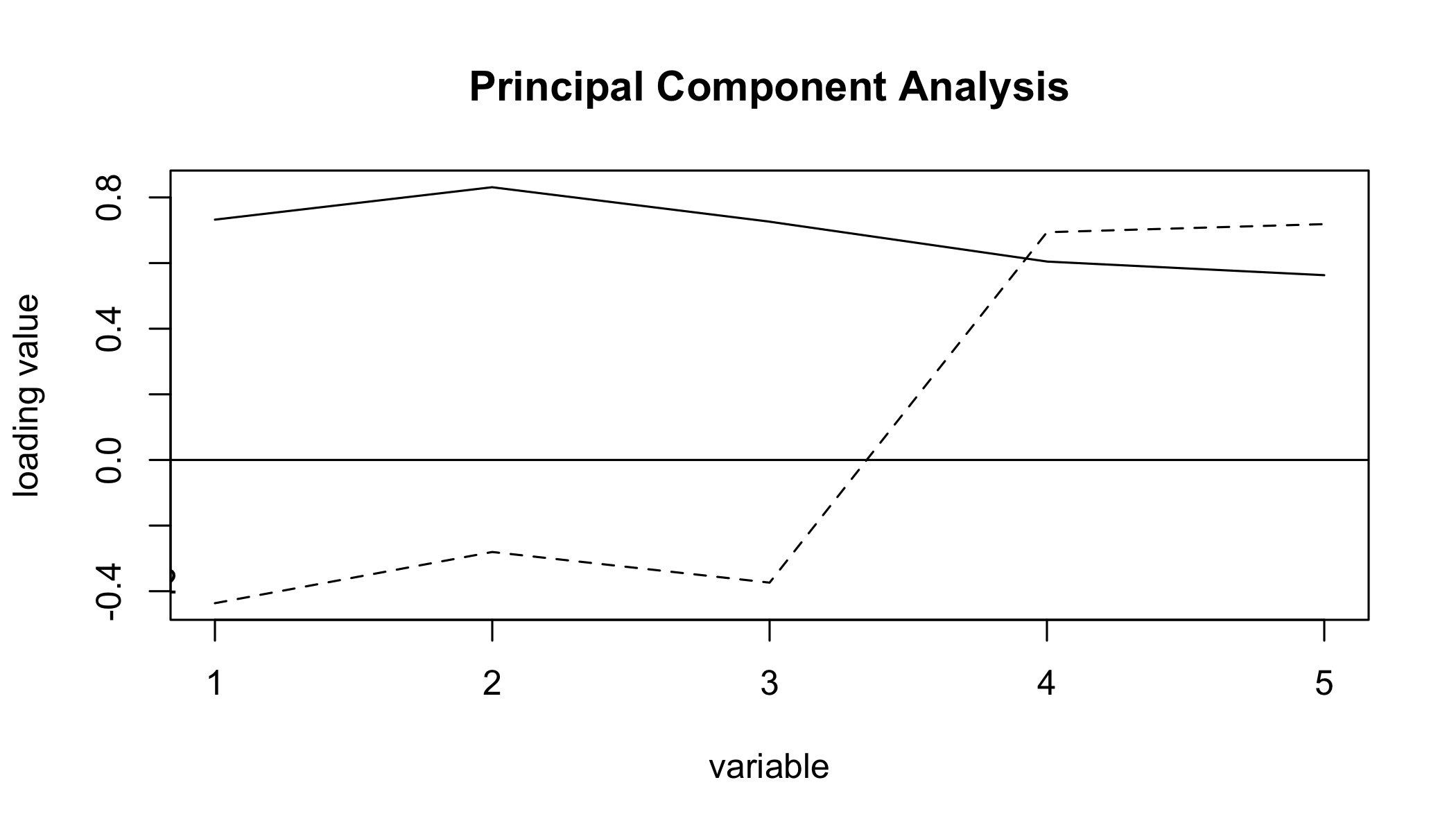
### Question 2b

Looking at the factor loadings, all of the stocks load highly on the first factor while the second factor shows contrasting loading of banking stocks (negative low loadings) to the oil stocks (positive high loadings). It is clear that the factor represents general economic conditions and can be called as *Market Factor*. The second factor seems to differentiate stocks in different industries and can be called as *Industry Factor*.

So, looking at the factor variances and loadings, it can be summarized that the weekly rates of stocks are determined by general market conditions and activities in the respective industries and some of it is explained by other/residual factors.

The chart below shows the same story where all the variables load highly on the first factor and first 3 variable load negatively on second factor while the last 2 variables load positively on second factor.

factor.plot(fact\_pc)

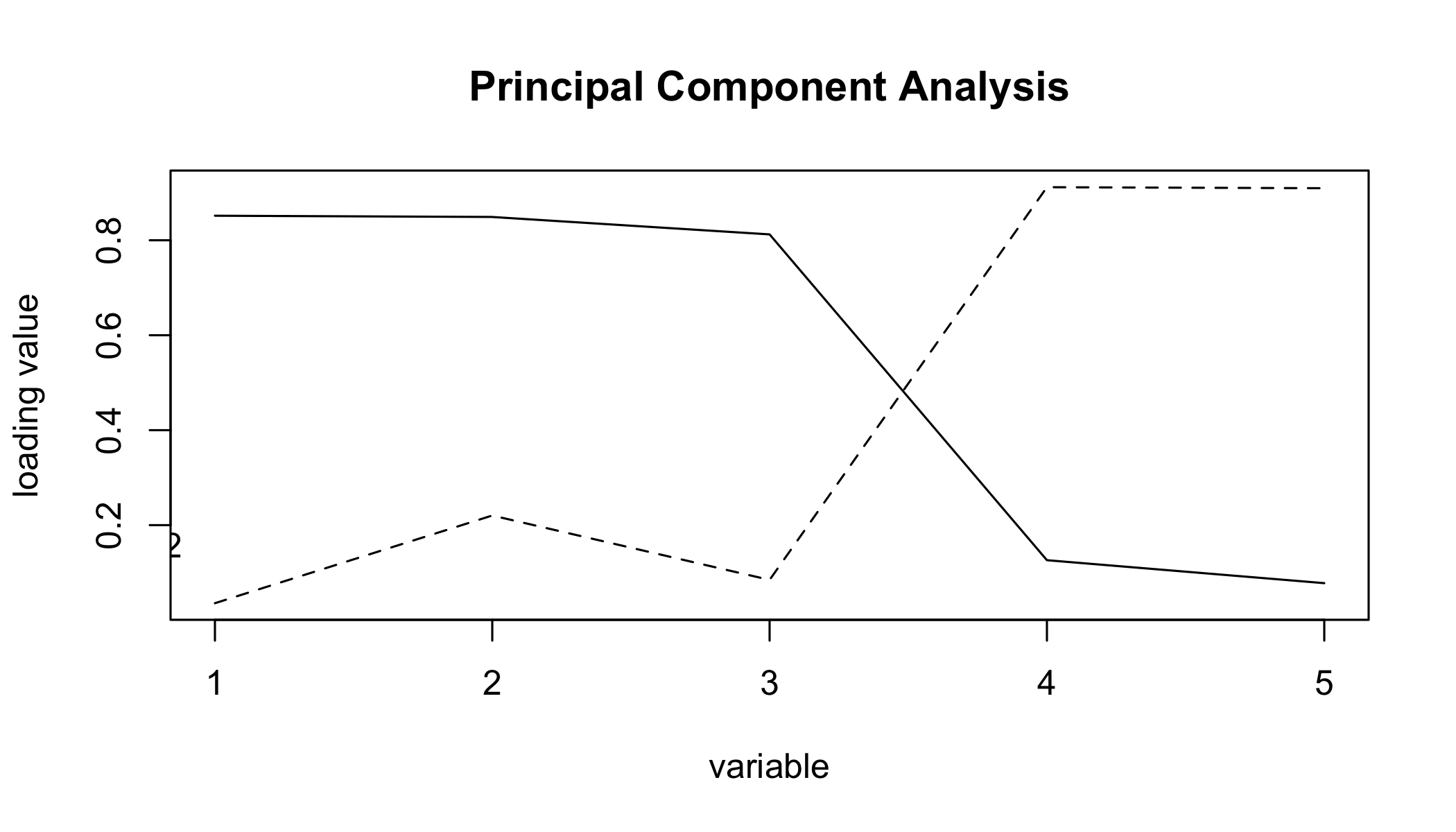


Let’s look at the factor anlysis with rotated loadings. Here, we are using the same principal components analysis method with **varimax** rotation. We will still use two factors and looking at the loadings, we now see a different pattern. The banking stocks load heavily on the the factor 1 while the oil stocks load heavily on the second factor . Factor 1 represent those economic forces that causes the bank stocks to move together while the Factor 2 represents the economic forces that affect the oil stocks. The chart confirms the above findings.

fact\_pc\_rot <-principal(dat, nfactors=2,rotate="varimax")  
print(fact\_pc\_rot)

## Principal Components Analysis  
## Call: principal(r = dat, nfactors = 2, rotate = "varimax")  
## Standardized loadings (pattern matrix) based upon correlation matrix  
## RC1 RC2 h2 u2 com  
## JPMorgan 0.85 0.04 0.73 0.27 1.0  
## Citibank 0.85 0.22 0.77 0.23 1.1  
## WellsFargo 0.81 0.08 0.67 0.33 1.0  
## RoyalDutchShell 0.13 0.91 0.85 0.15 1.0  
## ExxonMobil 0.08 0.91 0.83 0.17 1.0  
##   
## RC1 RC2  
## SS loadings 2.13 1.72  
## Proportion Var 0.43 0.34  
## Cumulative Var 0.43 0.77  
## Proportion Explained 0.55 0.45  
## Cumulative Proportion 0.55 1.00  
##   
## Mean item complexity = 1  
## Test of the hypothesis that 2 components are sufficient.  
##   
## The root mean square of the residuals (RMSR) is 0.1   
## with the empirical chi square 19.17 with prob < 1.2e-05   
##   
## Fit based upon off diagonal values = 0.95

factor.plot(fact\_pc\_rot)



### Question 2c

Table 9.8 on page 510 of Johnson and Wichern shows the unrotated and rotated factors for the same dataset obtained using the maximum likelihood method. In our previous steps we have used the principal component method for obtaining the factors. Here, we will use another method (without rotation) to calculate the factors. The unrotated factors using the maximum likelihood method shows that the oil stocks load heavily on the the factor 1 () while the banking stock load heavily on the second factor . Factor 1 represent those economic forces that causes the oil stocks to move together while the Factor 2 represents the economic forces that affect the banking stocks.

# Maximum likelihood method  
fact\_ml <- factanal(x = dat,factors = 2, rotation = 'none')  
print(fact\_ml)

##   
## Call:  
## factanal(x = dat, factors = 2, rotation = "none")  
##   
## Uniquenesses:  
## JPMorgan Citibank WellsFargo RoyalDutchShell   
## 0.417 0.275 0.542 0.005   
## ExxonMobil   
## 0.530   
##   
## Loadings:  
## Factor1 Factor2  
## JPMorgan 0.121 0.754   
## Citibank 0.328 0.786   
## WellsFargo 0.188 0.650   
## RoyalDutchShell 0.997   
## ExxonMobil 0.685   
##   
## Factor1 Factor2  
## SS loadings 1.622 1.610  
## Proportion Var 0.324 0.322  
## Cumulative Var 0.324 0.646  
##   
## Test of the hypothesis that 2 factors are sufficient.  
## The chi square statistic is 1.97 on 1 degree of freedom.  
## The p-value is 0.16

The rotated factors using the maximum likelihood method are similar to the rotated factors using the principal component method and shows that the banking stocks load heavily on the the factor 1 ( )while the oil stock load heavily on the second factor .

# Maximum likelihood method with rotation  
fact\_ml\_rot <- factanal(x = dat,factors = 2, rotation = 'varimax')  
print(fact\_ml\_rot)

##   
## Call:  
## factanal(x = dat, factors = 2, rotation = "varimax")  
##   
## Uniquenesses:  
## JPMorgan Citibank WellsFargo RoyalDutchShell   
## 0.417 0.275 0.542 0.005   
## ExxonMobil   
## 0.530   
##   
## Loadings:  
## Factor1 Factor2  
## JPMorgan 0.763   
## Citibank 0.819 0.232   
## WellsFargo 0.668 0.108   
## RoyalDutchShell 0.113 0.991   
## ExxonMobil 0.108 0.677   
##   
## Factor1 Factor2  
## SS loadings 1.725 1.507  
## Proportion Var 0.345 0.301  
## Cumulative Var 0.345 0.646  
##   
## Test of the hypothesis that 2 factors are sufficient.  
## The chi square statistic is 1.97 on 1 degree of freedom.  
## The p-value is 0.16

## Question 3

### Load the dataset from local storage

egyptskull <- fread('Data/egyptskull.csv')  
  
summary(egyptskull)

## MB BH BL NH   
## Min. :119 Min. :120.0 Min. : 81.00 Min. :44.00   
## 1st Qu.:131 1st Qu.:129.0 1st Qu.: 93.00 1st Qu.:49.00   
## Median :134 Median :133.0 Median : 96.00 Median :51.00   
## Mean :134 Mean :132.5 Mean : 96.46 Mean :50.93   
## 3rd Qu.:137 3rd Qu.:136.0 3rd Qu.:100.00 3rd Qu.:53.00   
## Max. :148 Max. :145.0 Max. :114.00 Max. :60.00   
## Epoch   
## Min. : 150   
## 1st Qu.: 200   
## Median :1850   
## Mean :1900   
## 3rd Qu.:3300   
## Max. :4000

egyptskull[, Epoch:= as.factor(Epoch)]

### Question 3a

Logistic regression is a statistical model that in its basic form uses a logistic function to model a binary dependent variable, although many more complex extensions exist. In regression analysis, logistic regression (or logit regression) is estimating the parameters of a logistic model (a form of binary regression).

In statistics, the logistic model is used to model the probability of a certain class or event existing such as pass/fail, win/lose, alive/dead or healthy/sick.

### Question 3b

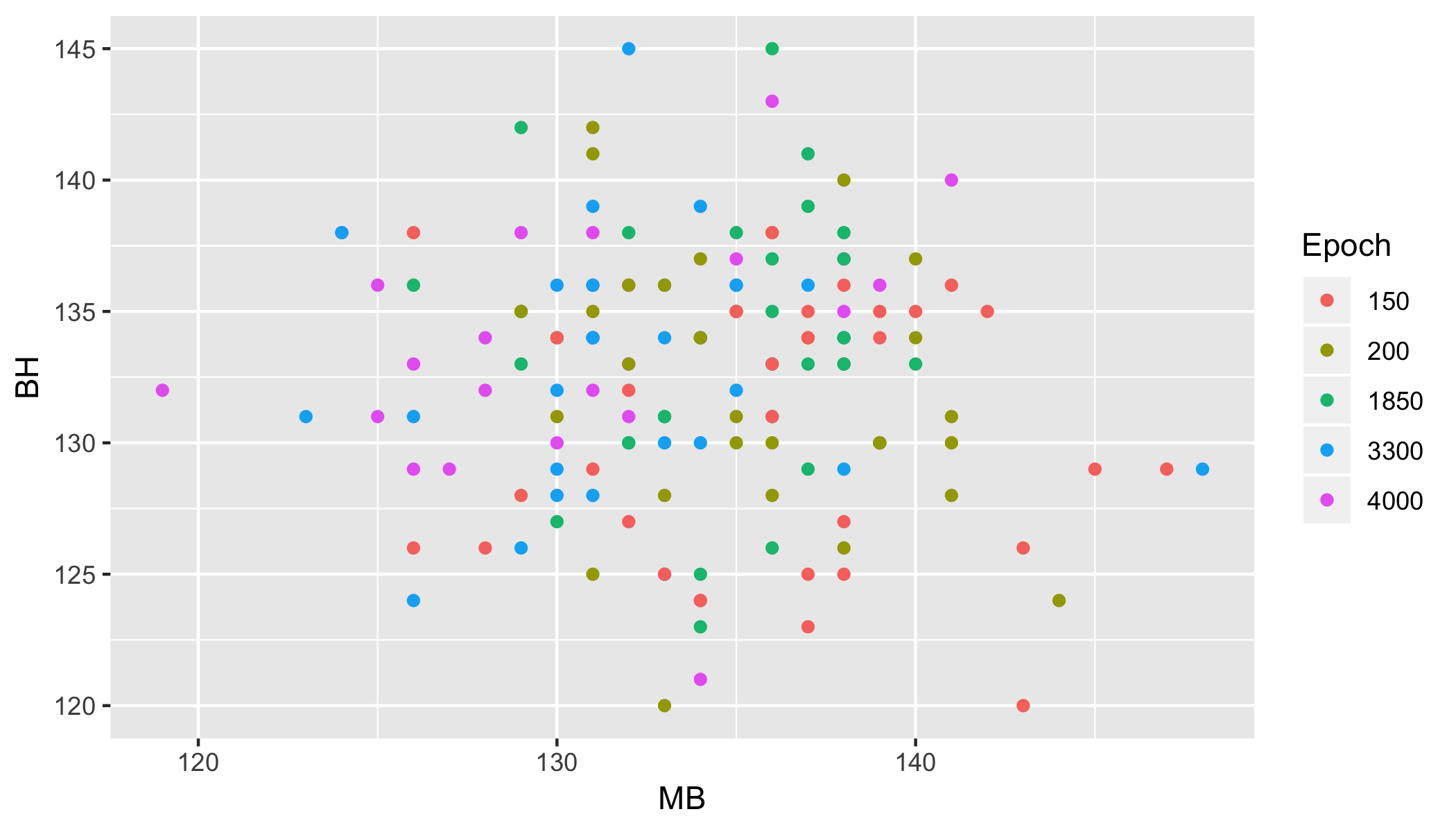
Classification trees are used to predict membership of cases or objects into classes of a categorical dependent variable from their measurements on one or more predictor variables. Classification tree analysis has traditionally been one of the main techniques used in data mining.

In computer science, Decision tree learning uses a decision tree (as a predictive model) to go from observations about an item (represented in the branches) to conclusions about the item’s target value (represented in the leaves). It is one of the predictive modeling approaches used in statistics, data mining and machine learning. Tree models where the target variable can take a discrete set of values are called classification trees; in these tree structures, leaves represent class labels and branches represent conjunctions of features that lead to those class labels. Decision trees where the target variable can take continuous values (typically real numbers) are called regression trees.

### Question 3c

Scatter Plot

ggplot(egyptskull, aes(x=MB, y=BH, group=Epoch))+  
 geom\_point(aes(color=Epoch))



### Question 3d

Split the dataset in to train and test datasets. For multionomial regression, we need to create 5 different response variables to denote the five levels of Epoch categories.

egyptskull[, Epoch\_1:=ifelse(Epoch == 4000, 1, 0)]  
egyptskull[, Epoch\_2:=ifelse(Epoch == 3300, 1, 0)]  
egyptskull[, Epoch\_3:=ifelse(Epoch == 1850, 1, 0)]  
egyptskull[, Epoch\_4:=ifelse(Epoch == 200, 1, 0)]  
egyptskull[, Epoch\_5:=ifelse(Epoch == 150, 1, 0)]  
  
egyptskull\_train <- egyptskull[,.SD[1:25], by = list(Epoch)]  
egyptskull\_test <- egyptskull[,.SD[26:30], by = list(Epoch)]  
  
egyptskull\_train[, .N, by = list(Epoch)]

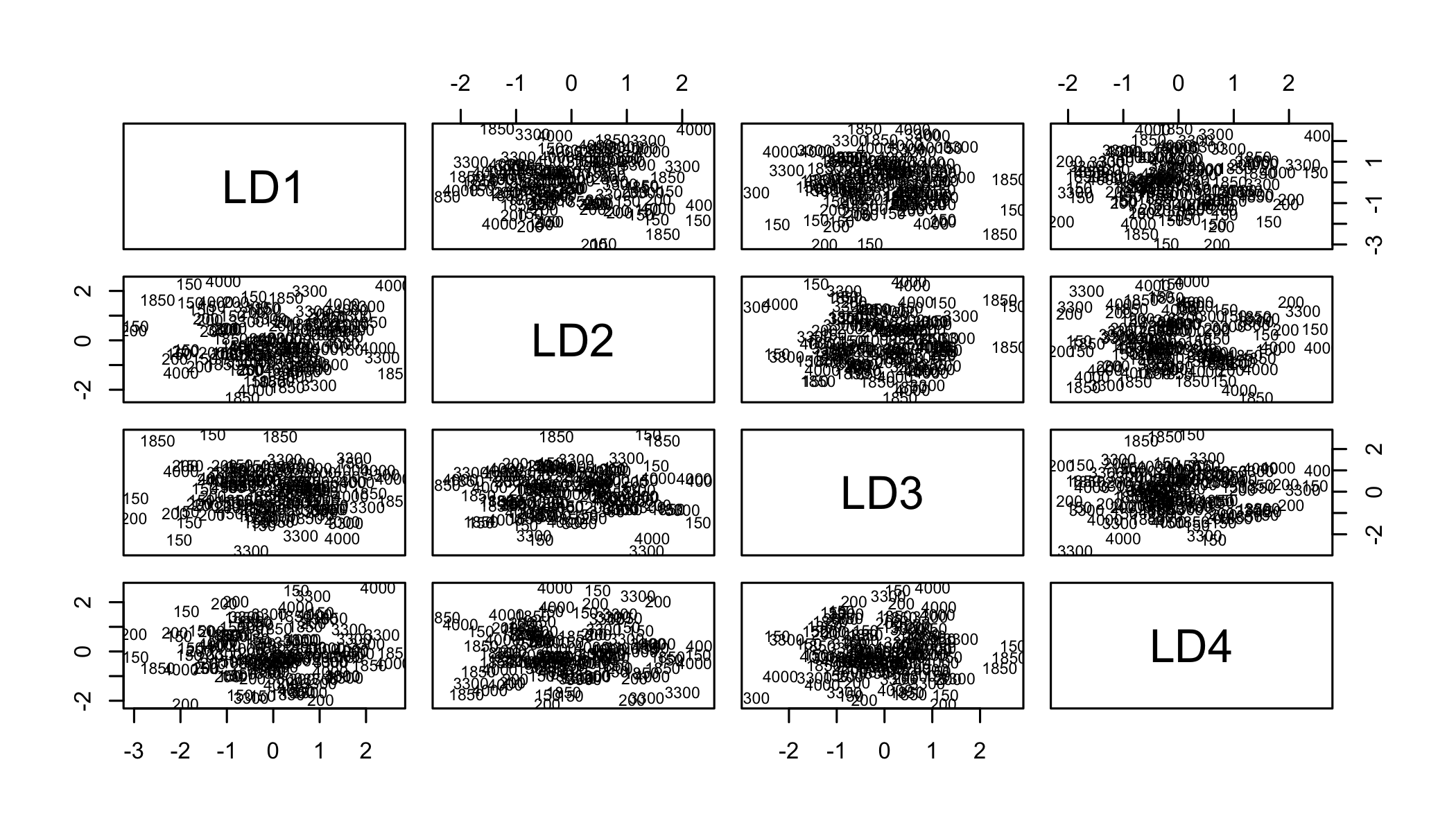
## Epoch N  
## 1: 4000 25  
## 2: 3300 25  
## 3: 1850 25  
## 4: 200 25  
## 5: 150 25

egyptskull\_test[, .N, by = list(Epoch)]

## Epoch N  
## 1: 4000 5  
## 2: 3300 5  
## 3: 1850 5  
## 4: 200 5  
## 5: 150 5

Perform LDA

####### LDA  
model\_lda <- lda(Epoch ~ MB+BH+BL+NH, data=egyptskull\_train)  
plot(model\_lda)



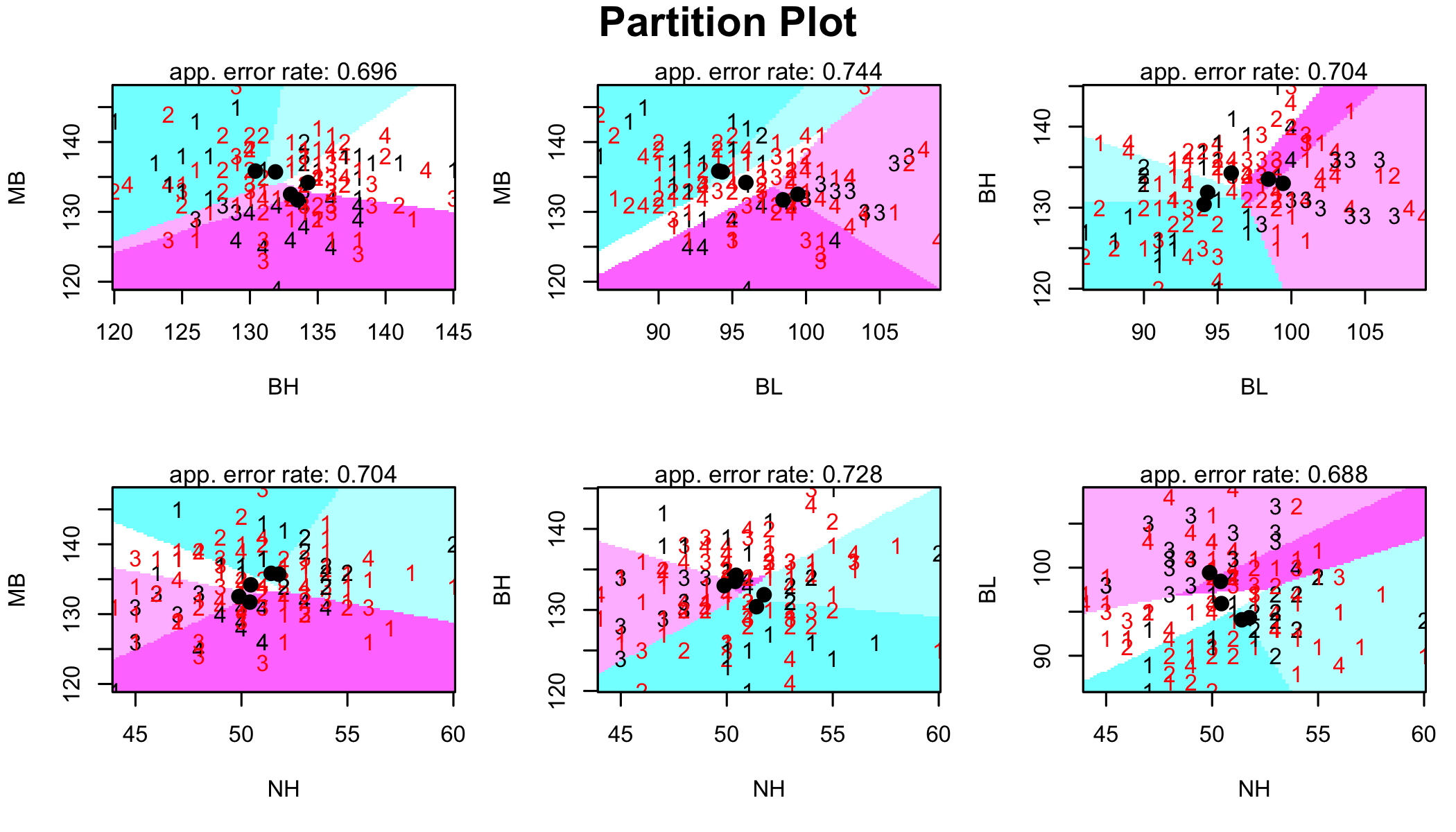
egyptskull\_test$lda\_predict<-predict(model\_lda,egyptskull\_test[,2:5])$class  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$lda\_predict)

##   
## 150 200 1850 3300 4000  
## 150 3 0 1 1 0  
## 200 1 1 2 1 0  
## 1850 1 1 2 0 1  
## 3300 0 3 0 2 0  
## 4000 0 2 0 2 1

# error rate  
mean(egyptskull\_test$lda\_predict != egyptskull\_test$Epoch)

## [1] 0.64

partimat(as.factor(Epoch) ~ MB+BH+BL+NH, data=egyptskull\_train,method="lda")



Perform QDA

######## QDA  
model\_qda<-qda(Epoch ~ MB+BH+BL+NH, data=egyptskull\_train)  
model\_qda

## Call:  
## qda(Epoch ~ MB + BH + BL + NH, data = egyptskull\_train)  
##   
## Prior probabilities of groups:  
## 150 200 1850 3300 4000   
## 0.2 0.2 0.2 0.2 0.2   
##   
## Group means:  
## MB BH BL NH  
## 150 135.84 130.40 94.08 51.40  
## 200 135.72 131.88 94.32 51.76  
## 1850 134.20 134.28 95.92 50.44  
## 3300 132.52 133.00 99.44 49.88  
## 4000 131.72 133.52 98.44 50.40

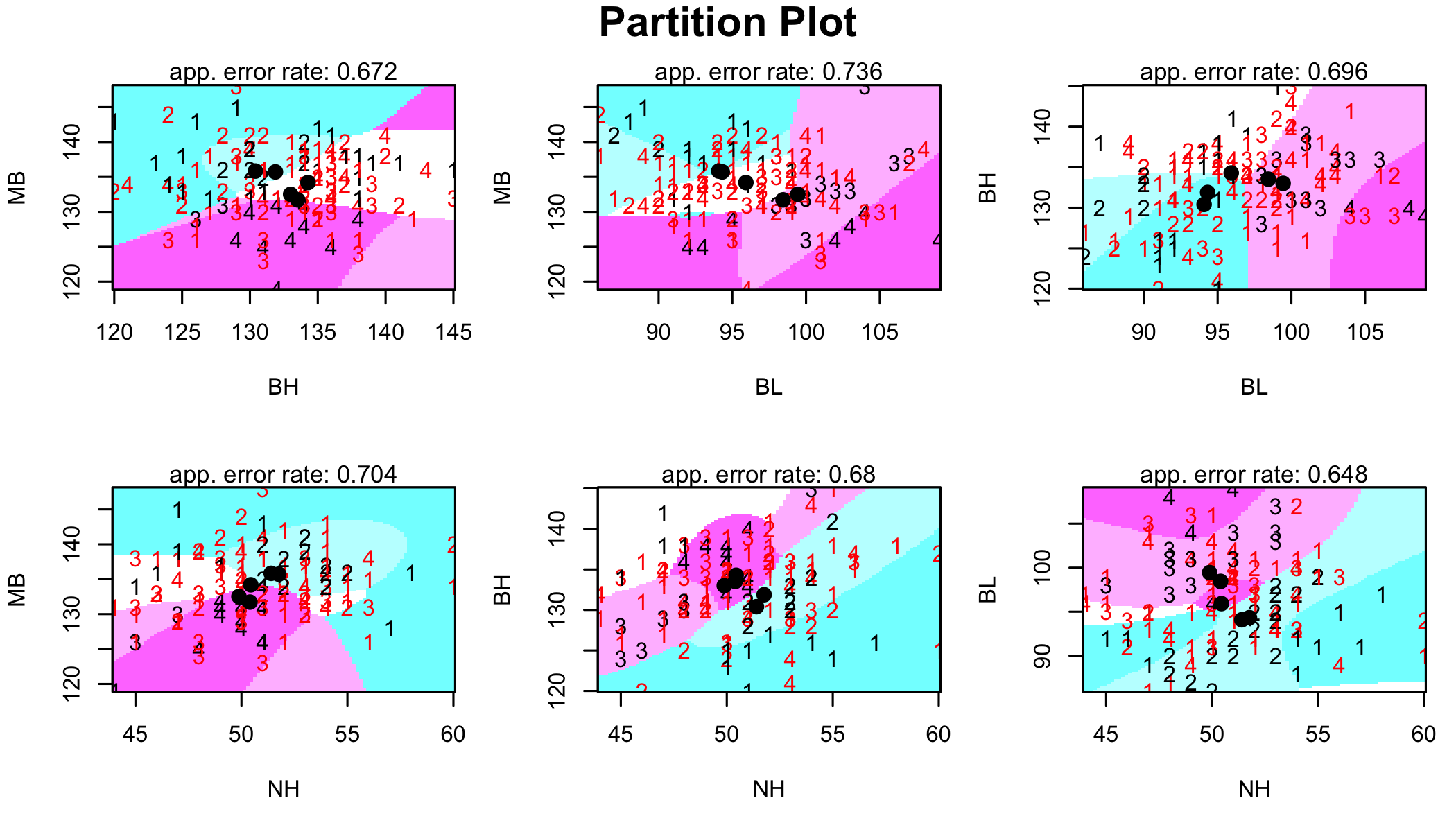
egyptskull\_test$qda\_predict<-predict(model\_qda,egyptskull\_test[,2:5])$class  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$qda\_predict)

##   
## 150 200 1850 3300 4000  
## 150 2 1 0 1 1  
## 200 1 1 1 1 1  
## 1850 1 2 1 1 0  
## 3300 0 1 0 0 4  
## 4000 0 1 0 1 3

# error rate  
mean(egyptskull\_test$qda\_predict != egyptskull\_test$Epoch)

## [1] 0.72

partimat(as.factor(Epoch) ~ MB+BH+BL+NH, data=egyptskull\_train,method="qda")



Perform Multinomial Logistic

###### Multinomial Logistic   
  
model\_mnl<-vglm(formula = cbind(Epoch\_1,Epoch\_2,Epoch\_3,Epoch\_4,Epoch\_5) ~ MB+BH+BL+NH, family = multinomial, data = egyptskull\_train)  
summary(model\_mnl)

##   
## Call:  
## vglm(formula = cbind(Epoch\_1, Epoch\_2, Epoch\_3, Epoch\_4, Epoch\_5) ~   
## MB + BH + BL + NH, family = multinomial, data = egyptskull\_train)  
##   
## Pearson residuals:  
## Min 1Q Median 3Q Max  
## log(mu[,1]/mu[,5]) -3.318 -0.4637 -0.2831 -0.09212 5.238  
## log(mu[,2]/mu[,5]) -3.112 -0.4613 -0.2458 -0.07941 3.680  
## log(mu[,3]/mu[,5]) -2.784 -0.4446 -0.2796 -0.14198 5.502  
## log(mu[,4]/mu[,5]) -2.243 -0.4046 -0.2508 -0.10041 3.086  
##   
## Coefficients:   
## Estimate Std. Error z value Pr(>|z|)   
## (Intercept):1 -1.325060 13.283864 -0.100 0.92054   
## (Intercept):2 -6.528734 13.415429 -0.487 0.62650   
## (Intercept):3 -10.099703 13.091198 NA NA   
## (Intercept):4 -6.996331 12.680035 -0.552 0.58111   
## MB:1 -0.184026 0.071829 NA NA   
## MB:2 -0.135660 0.070781 NA NA   
## MB:3 -0.065723 0.067484 NA NA   
## MB:4 -0.003269 0.063397 -0.052 0.95888   
## BH:1 0.101965 0.067863 1.502 0.13297   
## BH:2 0.081905 0.068726 1.192 0.23336   
## BH:3 0.161953 0.065986 2.454 0.01411 \*   
## BH:4 0.055233 0.058324 0.947 0.34364   
## BL:1 0.185242 0.072970 2.539 0.01113 \*   
## BL:2 0.238083 0.073697 3.231 0.00124 \*\*  
## BL:3 0.052184 0.070169 0.744 0.45706   
## BL:4 -0.010218 0.068163 -0.150 0.88084   
## NH:1 -0.105379 0.103191 -1.021 0.30715   
## NH:2 -0.179985 0.107504 NA NA   
## NH:3 -0.146265 0.098590 NA NA   
## NH:4 0.022474 0.087339 0.257 0.79693   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Names of linear predictors: log(mu[,1]/mu[,5]), log(mu[,2]/mu[,5]),   
## log(mu[,3]/mu[,5]), log(mu[,4]/mu[,5])  
##   
## Residual deviance: 352.3384 on 480 degrees of freedom  
##   
## Log-likelihood: -176.1692 on 480 degrees of freedom  
##   
## Number of Fisher scoring iterations: 5   
##   
## Warning: Hauck-Donner effect detected in the following estimate(s):  
## '(Intercept):3', 'MB:1', 'MB:2', 'MB:3', 'NH:2', 'NH:3'  
##   
##   
## Reference group is level 5 of the response

predictions<-predict(model\_mnl,newdata=egyptskull\_test[,2:5],type="response")  
egyptskull\_test$pred\_mnl<-apply(predictions,1,function(i) which.max(i) )  
  
egyptskull\_test[, pred\_mnl:= c(4000, 3300, 1850, 200, 150)[pred\_mnl]]  
egyptskull\_test[, unique(Epoch)]

## [1] 4000 3300 1850 200 150   
## Levels: 150 200 1850 3300 4000

#confusion matrix  
print(table(egyptskull\_test$Epoch,egyptskull\_test$pred\_mnl))

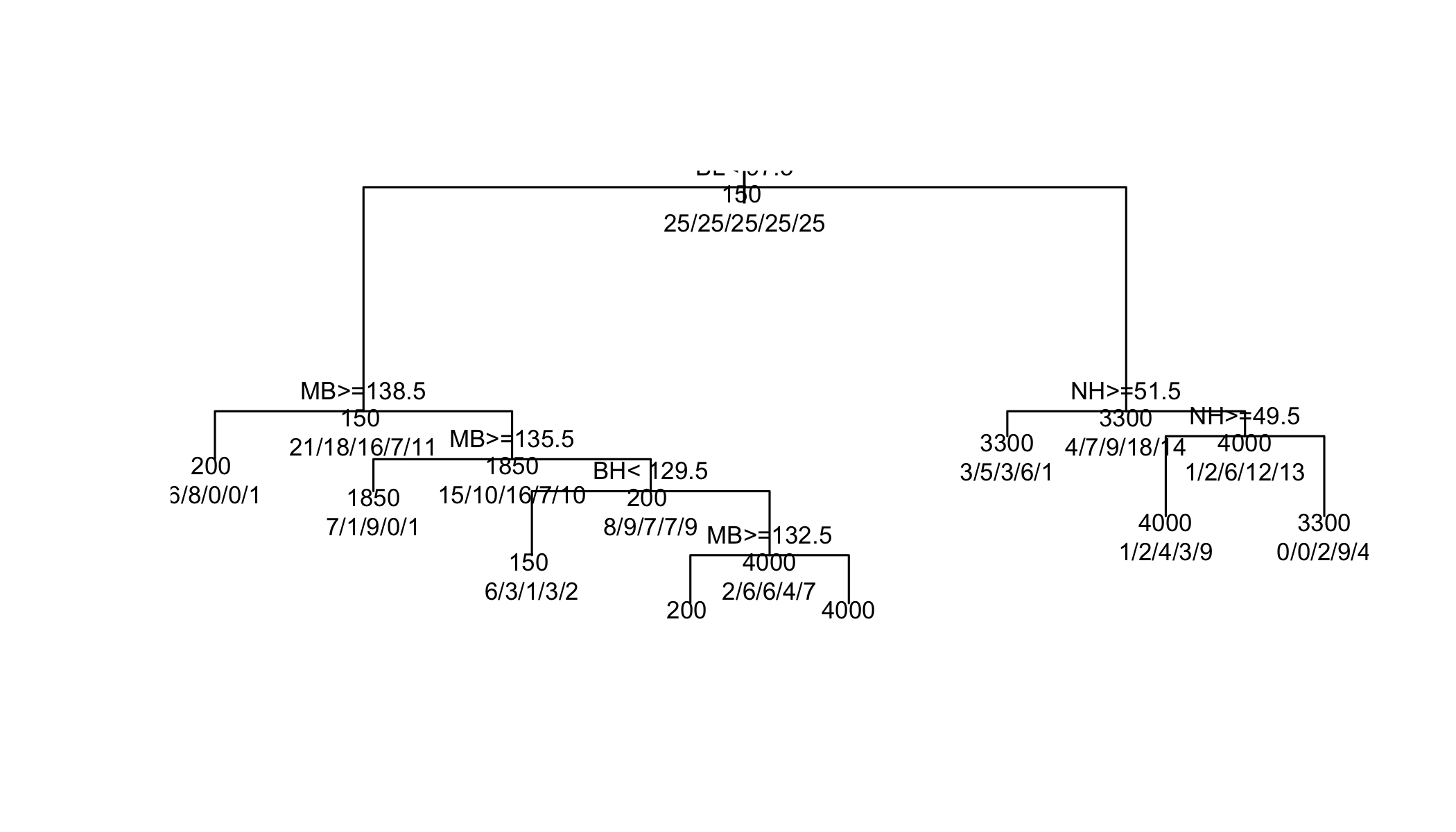
##   
## 150 200 1850 3300 4000  
## 150 3 0 1 1 0  
## 200 1 1 2 1 0  
## 1850 1 1 2 0 1  
## 3300 0 3 0 2 0  
## 4000 0 1 0 2 2

# error rate  
mean(egyptskull\_test$pred\_mnl != egyptskull\_test$Epoch)

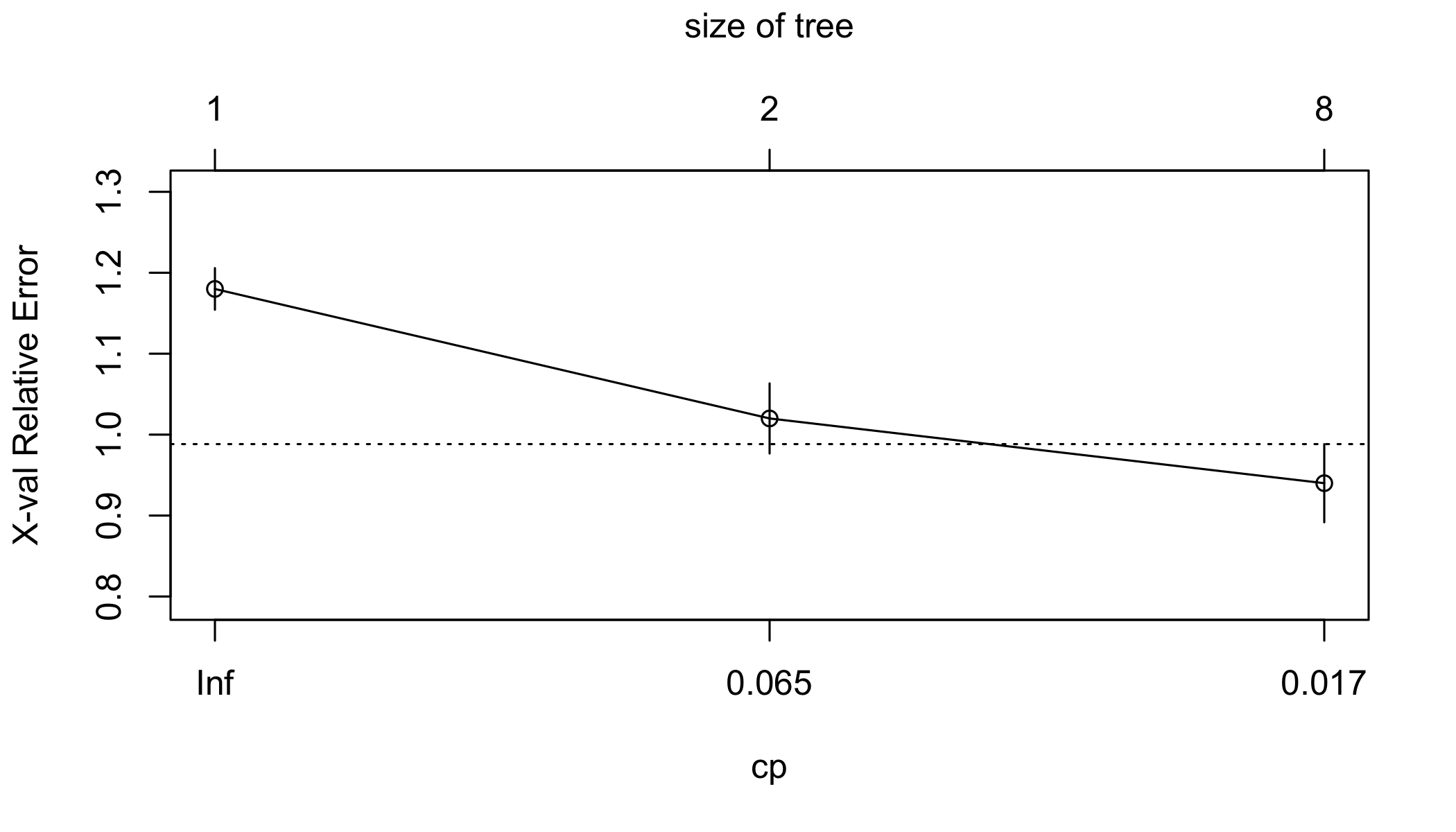
## [1] 0.6

Perform CART

######## CART  
model\_ct <- rpart(Epoch ~ MB+BH+BL+NH, data = egyptskull\_train, method="class")  
plot(model\_ct)  
text(model\_ct, use.n=TRUE, all=TRUE, cex=.7)



plotcp(model\_ct)



egyptskull\_test$pred\_ct<-predict(model\_ct,egyptskull\_test,type="vector")  
egyptskull\_test[, pred\_ct:= c(4000, 3300, 1850, 200, 150)[pred\_ct]]  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$pred\_ct)

##   
## 150 200 1850 3300 4000  
## 150 0 1 2 1 1  
## 200 3 0 2 0 0  
## 1850 0 2 1 1 1  
## 3300 4 1 0 0 0  
## 4000 1 4 0 0 0

# error rate  
mean(egyptskull\_test$pred\_ct != egyptskull\_test$Epoch)

## [1] 0.96

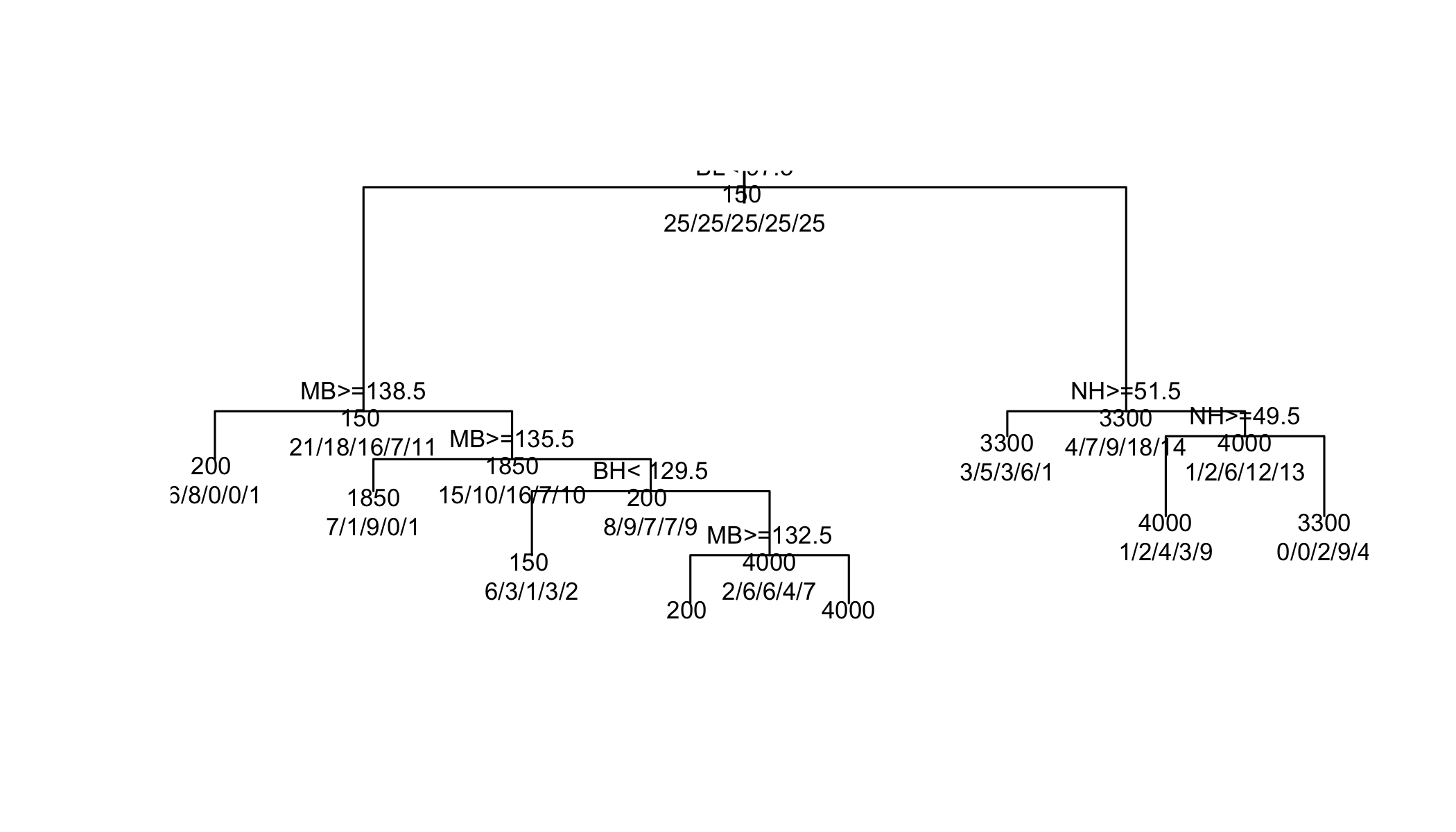
model\_ct$cptable

## CP nsplit rel error xerror xstd  
## 1 0.14 0 1.00 1.18 0.02570603  
## 2 0.03 1 0.86 1.02 0.04332205  
## 3 0.01 7 0.68 0.94 0.04828250

model\_ct\_fit<- prune(model\_ct, cp=model\_ct$cptable[which.min(model\_ct$cptable[,"xerror"]),"CP"])  
  
summary(model\_ct\_fit)

## Call:  
## rpart(formula = Epoch ~ MB + BH + BL + NH, data = egyptskull\_train,   
## method = "class")  
## n= 125   
##   
## CP nsplit rel error xerror xstd  
## 1 0.14 0 1.00 1.18 0.02570603  
## 2 0.03 1 0.86 1.02 0.04332205  
## 3 0.01 7 0.68 0.94 0.04828250  
##   
## Variable importance  
## MB BL NH BH   
## 38 24 23 15   
##   
## Node number 1: 125 observations, complexity param=0.14  
## predicted class=150 expected loss=0.8 P(node) =1  
## class counts: 25 25 25 25 25  
## probabilities: 0.200 0.200 0.200 0.200 0.200   
## left son=2 (73 obs) right son=3 (52 obs)  
## Primary splits:  
## BL < 97.5 to the left, improve=4.122761, (0 missing)  
## MB < 135.5 to the right, improve=2.933333, (0 missing)  
## BH < 127.5 to the left, improve=2.449634, (0 missing)  
## NH < 51.5 to the right, improve=1.866667, (0 missing)  
## Surrogate splits:  
## BH < 138.5 to the left, agree=0.640, adj=0.135, (0 split)  
## MB < 124.5 to the right, agree=0.592, adj=0.019, (0 split)  
##   
## Node number 2: 73 observations, complexity param=0.03  
## predicted class=150 expected loss=0.7123288 P(node) =0.584  
## class counts: 21 18 16 7 11  
## probabilities: 0.288 0.247 0.219 0.096 0.151   
## left son=4 (15 obs) right son=5 (58 obs)  
## Primary splits:  
## MB < 138.5 to the right, improve=3.0044720, (0 missing)  
## BH < 129.5 to the left, improve=2.4397140, (0 missing)  
## BL < 90.5 to the left, improve=1.2458510, (0 missing)  
## NH < 53.5 to the right, improve=0.8067706, (0 missing)  
##   
## Node number 3: 52 observations, complexity param=0.03  
## predicted class=3300 expected loss=0.6538462 P(node) =0.416  
## class counts: 4 7 9 18 14  
## probabilities: 0.077 0.135 0.173 0.346 0.269   
## left son=6 (18 obs) right son=7 (34 obs)  
## Primary splits:  
## NH < 51.5 to the right, improve=2.0485170, (0 missing)  
## BL < 98.5 to the left, improve=1.7383390, (0 missing)  
## BH < 129.5 to the left, improve=0.8667263, (0 missing)  
## MB < 134.5 to the right, improve=0.7785146, (0 missing)  
## Surrogate splits:  
## BH < 135.5 to the right, agree=0.731, adj=0.222, (0 split)  
##   
## Node number 4: 15 observations  
## predicted class=200 expected loss=0.4666667 P(node) =0.12  
## class counts: 6 8 0 0 1  
## probabilities: 0.400 0.533 0.000 0.000 0.067   
##   
## Node number 5: 58 observations, complexity param=0.03  
## predicted class=1850 expected loss=0.7241379 P(node) =0.464  
## class counts: 15 10 16 7 10  
## probabilities: 0.259 0.172 0.276 0.121 0.172   
## left son=10 (18 obs) right son=11 (40 obs)  
## Primary splits:  
## MB < 135.5 to the right, improve=2.8471260, (0 missing)  
## BH < 129.5 to the left, improve=2.3860150, (0 missing)  
## NH < 54.5 to the right, improve=1.1392830, (0 missing)  
## BL < 90.5 to the right, improve=0.7244507, (0 missing)  
## Surrogate splits:  
## BH < 136.5 to the right, agree=0.724, adj=0.111, (0 split)  
##   
## Node number 6: 18 observations  
## predicted class=3300 expected loss=0.6666667 P(node) =0.144  
## class counts: 3 5 3 6 1  
## probabilities: 0.167 0.278 0.167 0.333 0.056   
##   
## Node number 7: 34 observations, complexity param=0.03  
## predicted class=4000 expected loss=0.6176471 P(node) =0.272  
## class counts: 1 2 6 12 13  
## probabilities: 0.029 0.059 0.176 0.353 0.382   
## left son=14 (19 obs) right son=15 (15 obs)  
## Primary splits:  
## NH < 49.5 to the right, improve=2.1636740, (0 missing)  
## BL < 101.5 to the left, improve=1.0882350, (0 missing)  
## BH < 129.5 to the left, improve=0.8051665, (0 missing)  
## MB < 136.5 to the right, improve=0.5690045, (0 missing)  
## Surrogate splits:  
## BH < 130.5 to the right, agree=0.618, adj=0.133, (0 split)  
## BL < 103.5 to the left, agree=0.618, adj=0.133, (0 split)  
## MB < 130.5 to the right, agree=0.588, adj=0.067, (0 split)  
##   
## Node number 10: 18 observations  
## predicted class=1850 expected loss=0.5 P(node) =0.144  
## class counts: 7 1 9 0 1  
## probabilities: 0.389 0.056 0.500 0.000 0.056   
##   
## Node number 11: 40 observations, complexity param=0.03  
## predicted class=200 expected loss=0.775 P(node) =0.32  
## class counts: 8 9 7 7 9  
## probabilities: 0.200 0.225 0.175 0.175 0.225   
## left son=22 (15 obs) right son=23 (25 obs)  
## Primary splits:  
## BH < 129.5 to the left, improve=1.473333, (0 missing)  
## BL < 92.5 to the left, improve=1.250384, (0 missing)  
## MB < 128.5 to the right, improve=0.861039, (0 missing)  
## NH < 53.5 to the right, improve=0.587500, (0 missing)  
## Surrogate splits:  
## NH < 46.5 to the left, agree=0.70, adj=0.200, (0 split)  
## BL < 91.5 to the left, agree=0.65, adj=0.067, (0 split)  
##   
## Node number 14: 19 observations  
## predicted class=4000 expected loss=0.5263158 P(node) =0.152  
## class counts: 1 2 4 3 9  
## probabilities: 0.053 0.105 0.211 0.158 0.474   
##   
## Node number 15: 15 observations  
## predicted class=3300 expected loss=0.4 P(node) =0.12  
## class counts: 0 0 2 9 4  
## probabilities: 0.000 0.000 0.133 0.600 0.267   
##   
## Node number 22: 15 observations  
## predicted class=150 expected loss=0.6 P(node) =0.12  
## class counts: 6 3 1 3 2  
## probabilities: 0.400 0.200 0.067 0.200 0.133   
##   
## Node number 23: 25 observations, complexity param=0.03  
## predicted class=4000 expected loss=0.72 P(node) =0.2  
## class counts: 2 6 6 4 7  
## probabilities: 0.080 0.240 0.240 0.160 0.280   
## left son=46 (8 obs) right son=47 (17 obs)  
## Primary splits:  
## MB < 132.5 to the right, improve=1.8600000, (0 missing)  
## BL < 95.5 to the left, improve=0.9473016, (0 missing)  
## NH < 49.5 to the right, improve=0.8600000, (0 missing)  
## BH < 133.5 to the right, improve=0.3917460, (0 missing)  
## Surrogate splits:  
## BL < 96.5 to the right, agree=0.76, adj=0.250, (0 split)  
## NH < 53.5 to the right, agree=0.72, adj=0.125, (0 split)  
##   
## Node number 46: 8 observations  
## predicted class=200 expected loss=0.625 P(node) =0.064  
## class counts: 1 3 1 3 0  
## probabilities: 0.125 0.375 0.125 0.375 0.000   
##   
## Node number 47: 17 observations  
## predicted class=4000 expected loss=0.5882353 P(node) =0.136  
## class counts: 1 3 5 1 7  
## probabilities: 0.059 0.176 0.294 0.059 0.412

plot(model\_ct\_fit)  
text(model\_ct\_fit, use.n=TRUE, all=TRUE, cex=.7)



egyptskull\_test$pred\_ct\_fit<-predict(model\_ct\_fit,egyptskull\_test,type="vector")  
egyptskull\_test[, pred\_ct\_fit:= c(4000, 3300, 1850, 200, 150)[pred\_ct\_fit]]  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$pred\_ct\_fit)

##   
## 150 200 1850 3300 4000  
## 150 0 1 2 1 1  
## 200 3 0 2 0 0  
## 1850 0 2 1 1 1  
## 3300 4 1 0 0 0  
## 4000 1 4 0 0 0

# error rate  
mean(egyptskull\_test$pred\_ct\_fit != egyptskull\_test$Epoch)

## [1] 0.96

Build a neural network

##### Nnet  
model\_nnet<-nnet(Epoch ~ MB+BH+BL+NH, data = egyptskull\_train,size=5,decay=0.1)

## # weights: 55  
## initial value 296.976158   
## iter 10 value 201.182897  
## iter 20 value 195.424673  
## iter 30 value 184.350099  
## iter 40 value 175.251644  
## iter 50 value 174.245517  
## iter 60 value 173.916475  
## iter 70 value 173.279070  
## iter 80 value 171.057928  
## iter 90 value 167.997382  
## iter 100 value 165.494561  
## final value 165.494561   
## stopped after 100 iterations

egyptskull\_test$pred\_nnet<-predict(model\_nnet,egyptskull\_test,type="class")  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$pred\_nnet)

##   
## 150 200 3300 4000  
## 150 2 2 1 0  
## 200 1 2 1 1  
## 1850 4 0 1 0  
## 3300 2 1 2 0  
## 4000 2 0 2 1

# error rate  
mean(egyptskull\_test$pred\_nnet != egyptskull\_test$Epoch)

## [1] 0.72

### Question 3e

# LDA  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$lda\_predict)

##   
## 150 200 1850 3300 4000  
## 150 3 0 1 1 0  
## 200 1 1 2 1 0  
## 1850 1 1 2 0 1  
## 3300 0 3 0 2 0  
## 4000 0 2 0 2 1

# error rate  
mean(egyptskull\_test$lda\_predict != egyptskull\_test$Epoch)

## [1] 0.64

# QDA  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$qda\_predict)

##   
## 150 200 1850 3300 4000  
## 150 2 1 0 1 1  
## 200 1 1 1 1 1  
## 1850 1 2 1 1 0  
## 3300 0 1 0 0 4  
## 4000 0 1 0 1 3

# error rate  
mean(egyptskull\_test$qda\_predict != egyptskull\_test$Epoch)

## [1] 0.72

# Multinomial  
#confusion matrix  
print(table(egyptskull\_test$Epoch,egyptskull\_test$pred\_mnl))

##   
## 150 200 1850 3300 4000  
## 150 3 0 1 1 0  
## 200 1 1 2 1 0  
## 1850 1 1 2 0 1  
## 3300 0 3 0 2 0  
## 4000 0 1 0 2 2

# error rate  
mean(egyptskull\_test$pred\_mnl != egyptskull\_test$Epoch)

## [1] 0.6

# CART  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$pred\_ct\_fit)

##   
## 150 200 1850 3300 4000  
## 150 0 1 2 1 1  
## 200 3 0 2 0 0  
## 1850 0 2 1 1 1  
## 3300 4 1 0 0 0  
## 4000 1 4 0 0 0

# error rate  
mean(egyptskull\_test$pred\_ct\_fit != egyptskull\_test$Epoch)

## [1] 0.96

# Nnet  
#confusion matrix  
table(egyptskull\_test$Epoch,egyptskull\_test$pred\_nnet)

##   
## 150 200 3300 4000  
## 150 2 2 1 0  
## 200 1 2 1 1  
## 1850 4 0 1 0  
## 3300 2 1 2 0  
## 4000 2 0 2 1

# error rate  
mean(egyptskull\_test$pred\_nnet != egyptskull\_test$Epoch)

## [1] 0.72

### Question 3f

####### Predict  
egyptskull\_val <- data.table(rbind(c(128, 143, 103, 50)   
 , c(129, 126, 91, 50)  
 , c(130, 127, 99, 45)  
 , c(130, 131, 98, 53)  
 , c(134, 124, 91, 55)  
 , c(130, 130, 104, 49)  
 , c(134, 139, 101, 49)  
 , c(136, 133, 91, 49)  
 ))  
  
names(egyptskull\_val) <- names(egyptskull)[1:4]  
  
# Use multinomial  
  
predictions<-predict(model\_mnl,newdata=egyptskull\_val,type="response")  
  
egyptskull\_val$pred\_mnl<-apply(predictions,1,function(i) which.max(i) )  
  
egyptskull\_val[, pred\_mnl:= c(4000, 3300, 1850, 200, 150)[pred\_mnl]]

## Part B

## Question 1

### Load the dataset from web

#url <- 'https://web.stanford.edu/~hastie/Papers/LARS/diabetes.data'  
#diabetes\_orig <- fread(url, sep = '\t')  
  
#fwrite(diabetes\_orig, 'Data/diabetes.csv')  
  
#   
# data(diabetes)  
# Xmatrix <- diabetes$x  
# yVector <- diabetes$y  
#   
  
diabetes\_orig <- fread('Data/diabetes.csv')  
dim(diabetes\_orig)

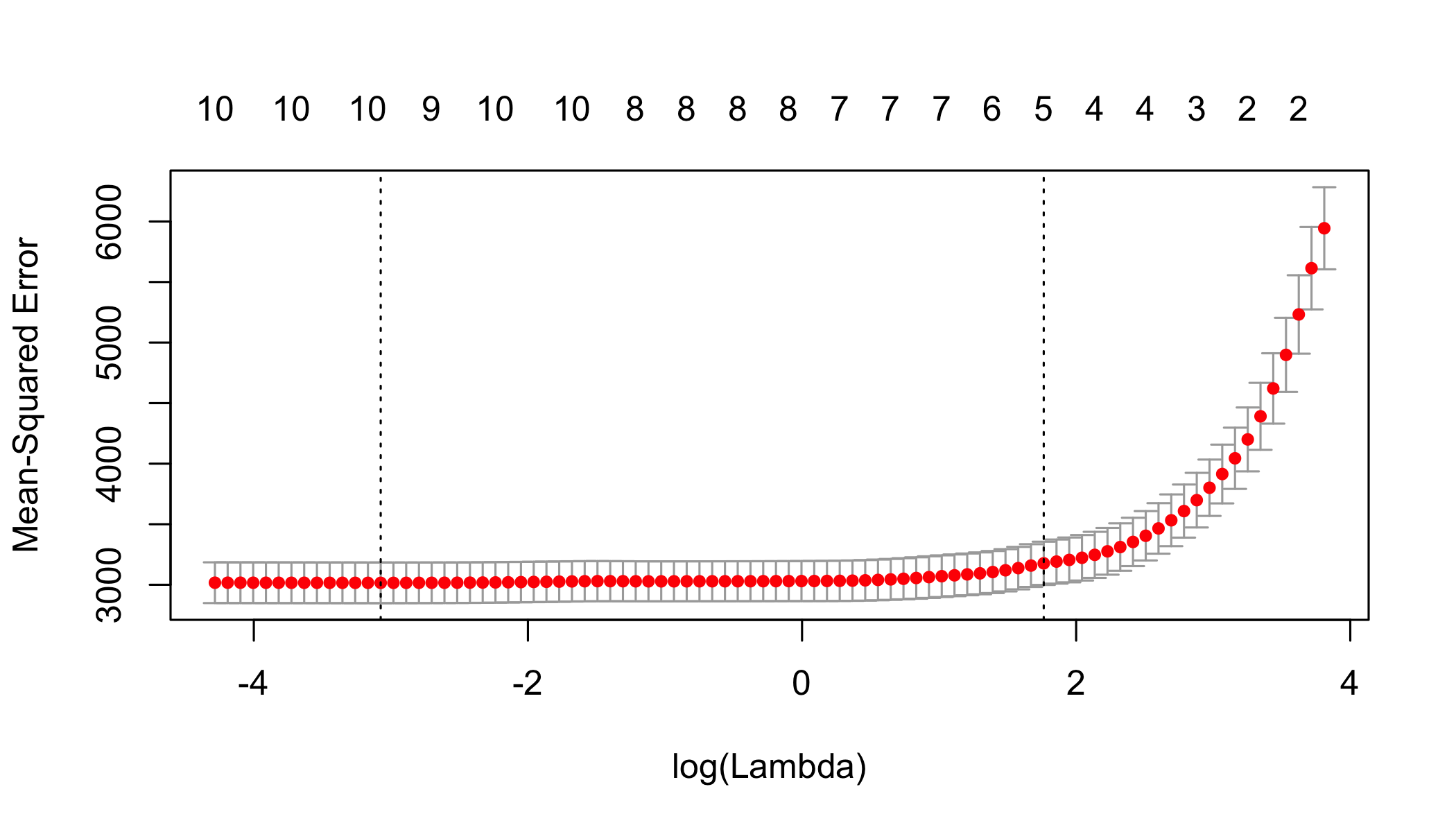
## [1] 442 11

### Question 1a

Xmatrix <- as.matrix(diabetes\_orig[,1:10])  
yVector <- diabetes\_orig$Y  
  
dim(Xmatrix)

## [1] 442 10

cvfit <- cv.glmnet(Xmatrix , yVector)  
  
plot(cvfit)



log(cvfit$lambda.min)

## [1] -3.074284

coef(cvfit, s = "lambda.min")

## 11 x 1 sparse Matrix of class "dgCMatrix"  
## 1  
## (Intercept) -307.89654852  
## AGE -0.02781562  
## SEX -22.65280922  
## BMI 5.61655326  
## BP 1.10905205  
## S1 -0.83430367  
## S2 0.52252757  
## S3 0.04724676  
## S4 5.35846043  
## S5 62.35298634  
## S6 0.27880335

log(cvfit$lambda.1se)

## [1] 1.76347

coef(cvfit, s = "lambda.1se")

## 11 x 1 sparse Matrix of class "dgCMatrix"  
## 1  
## (Intercept) -218.9741434  
## AGE .   
## SEX -1.2954159  
## BMI 5.4609190  
## BP 0.6852067  
## S1 .   
## S2 .   
## S3 -0.4571081  
## S4 .   
## S5 40.2635323  
## S6 .

LASSOfit <- glmnet(Xmatrix , yVector)  
summary(LASSOfit)

## Length Class Mode   
## a0 88 -none- numeric  
## beta 880 dgCMatrix S4   
## df 88 -none- numeric  
## dim 2 -none- numeric  
## lambda 88 -none- numeric  
## dev.ratio 88 -none- numeric  
## nulldev 1 -none- numeric  
## npasses 1 -none- numeric  
## jerr 1 -none- numeric  
## offset 1 -none- logical  
## call 3 -none- call   
## nobs 1 -none- numeric

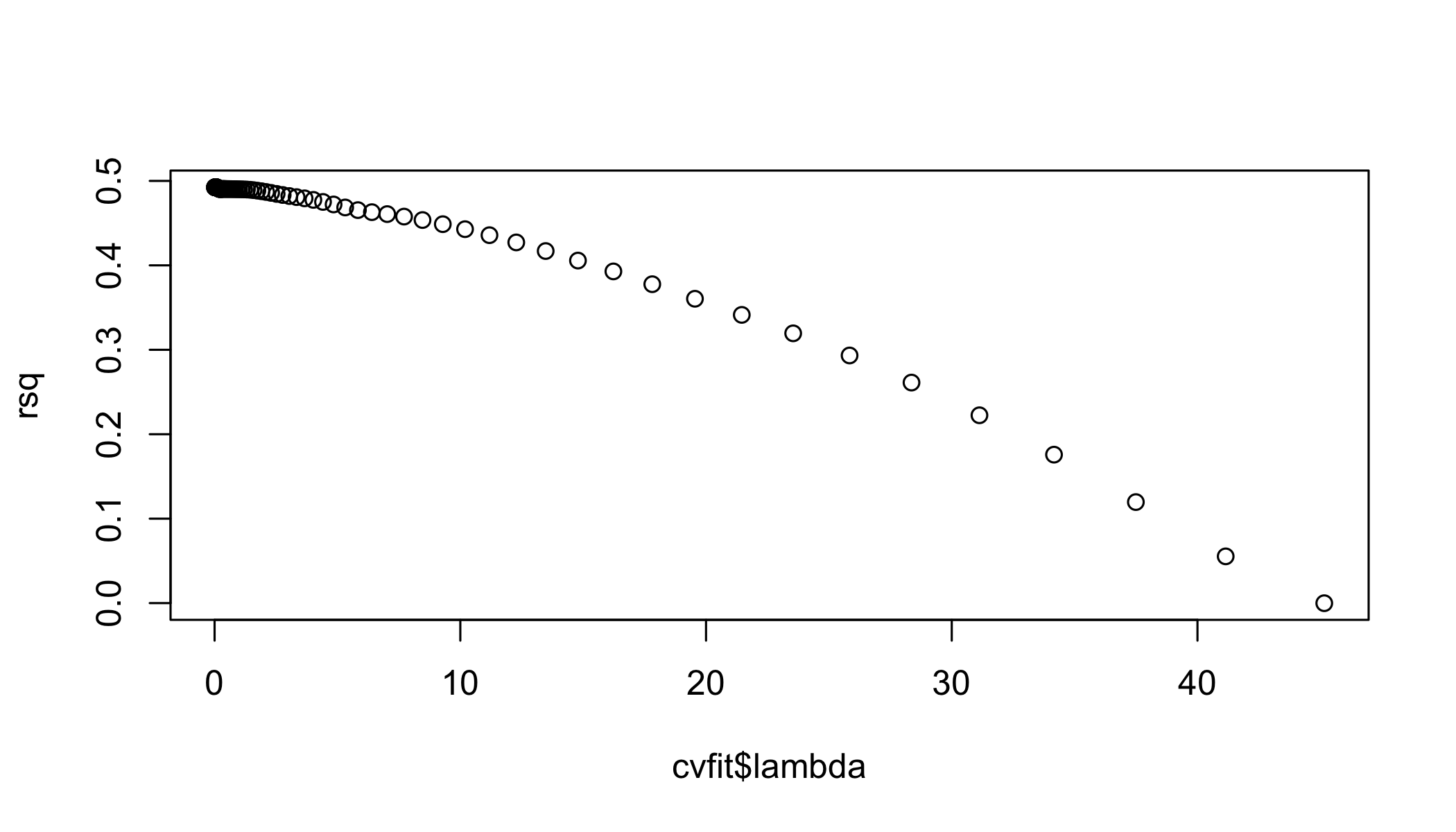
LASSOfit$lambda

## [1] 45.16003002 41.14813742 37.49265031 34.16190659 31.12705697  
## [6] 28.36181502 25.84222954 23.54647710 21.45467297 19.54869896  
## [11] 17.81204642 16.22967331 14.78787387 13.47415991 12.27715268  
## [16] 11.18648427 10.19270784 9.28721577 8.46216512 7.71040969  
## [21] 7.02543815 6.40131759 5.83264218 5.31448632 4.84236200  
## [26] 4.41217991 4.02021401 3.66306928 3.33765230 3.04114447  
## [31] 2.77097757 2.52481156 2.30051426 2.09614292 1.90992736  
## [36] 1.74025468 1.58565525 1.44479000 1.31643884 1.19949004  
## [41] 1.09293065 0.99583771 0.90737023 0.82676196 0.75331471  
## [46] 0.68639230 0.62541510 0.56985495 0.51923061 0.47310359  
## [51] 0.43107437 0.39277891 0.35788552 0.32609195 0.29712284  
## [56] 0.27072727 0.24667660 0.22476253 0.20479525 0.18660180  
## [61] 0.17002461 0.15492010 0.14115742 0.12861739 0.11719137  
## [66] 0.10678041 0.09729434 0.08865098 0.08077547 0.07359960  
## [71] 0.06706121 0.06110368 0.05567540 0.05072935 0.04622269  
## [76] 0.04211640 0.03837489 0.03496577 0.03185951 0.02902920  
## [81] 0.02645032 0.02410055 0.02195952 0.02000870 0.01823118  
## [86] 0.01661157 0.01513585 0.01379122

betaHat <- as.numeric(LASSOfit$beta)  
  
LASSOfit <- glmnet(Xmatrix , yVector , lambda=1.2)  
summary(LASSOfit)

## Length Class Mode   
## a0 1 -none- numeric  
## beta 10 dgCMatrix S4   
## df 1 -none- numeric  
## dim 2 -none- numeric  
## lambda 1 -none- numeric  
## dev.ratio 1 -none- numeric  
## nulldev 1 -none- numeric  
## npasses 1 -none- numeric  
## jerr 1 -none- numeric  
## offset 1 -none- logical  
## call 4 -none- call   
## nobs 1 -none- numeric

betaHat <- as.numeric(LASSOfit$beta)  
  
rsq <- 1 - cvfit$cvm/var(yVector)  
  
plot(cvfit$lambda,rsq)



lambda\_rsq <- data.table(cbind(lambda=cvfit$lambda,rsq))  
  
lambda\_rsq[rsq==max(rsq)]

## lambda rsq  
## 1: 0.04622269 0.4925471

LASSOfit <- glmnet(Xmatrix , yVector , lambda=1)  
betaHat <- as.numeric(LASSOfit$beta)

r-squared 49.93 better fit similar co-oefficents automated way of finding co-efficents

## Question 2

### Load the dataset

Seeds dataset contains data about four varities wheat capturing the hedonic characteristics of each variety of wheat.

seeds <- fread('Data/seeds\_dataset.txt', sep = "\t")  
col\_names\_seeds <- c("area", "perimeter", "compactness", "length\_of\_kernel", "width\_of\_kernel", "asymmetry\_coefficient", "length\_of\_kernel\_groove", "wheat\_type")  
names(seeds) <- col\_names\_seeds

### Question 2a

Estimate of covariance matrix using sample covariance method:

seeds\_vcmat <- cov(seeds[,1:7])

### Question 2b

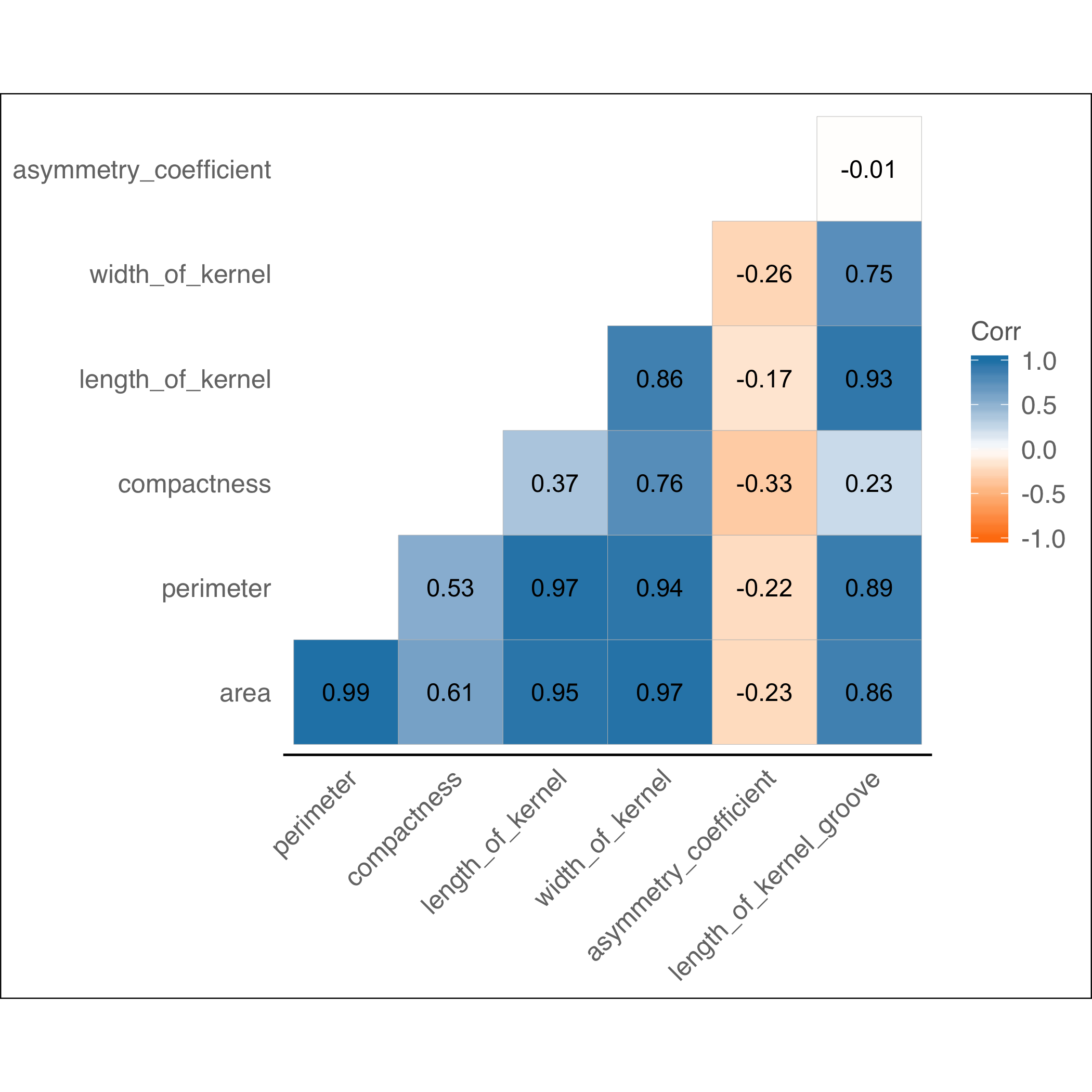
Another method is maximum likelihood estimate:

Not very different

### Question 2c

### Multicollinearity Analysis

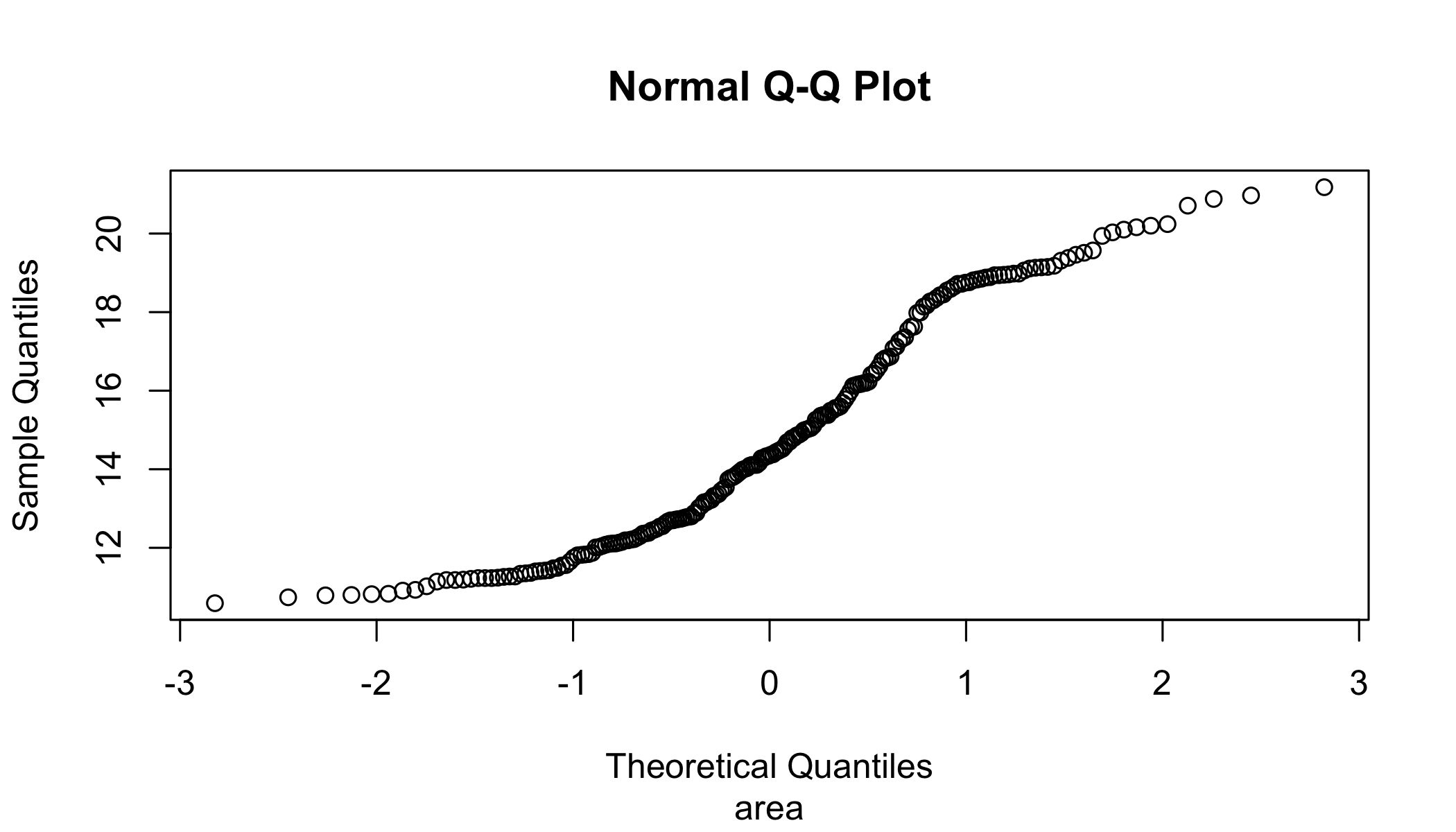
#Corr Plot  
cor\_mat <- seeds[,1:7]  
corr <- cor(cor\_mat, use = "pairwise.complete.obs")  
  
ggcorrplot(corr, hc.order = FALSE, type = "lower",  
 ggtheme = ggthemes::theme\_gdocs,  
 colors = c("#ff7f0e", "white", "#1f83b4"),  
 lab = TRUE)+  
 theme(panel.grid.major=element\_blank())

 High linear relationship among variables

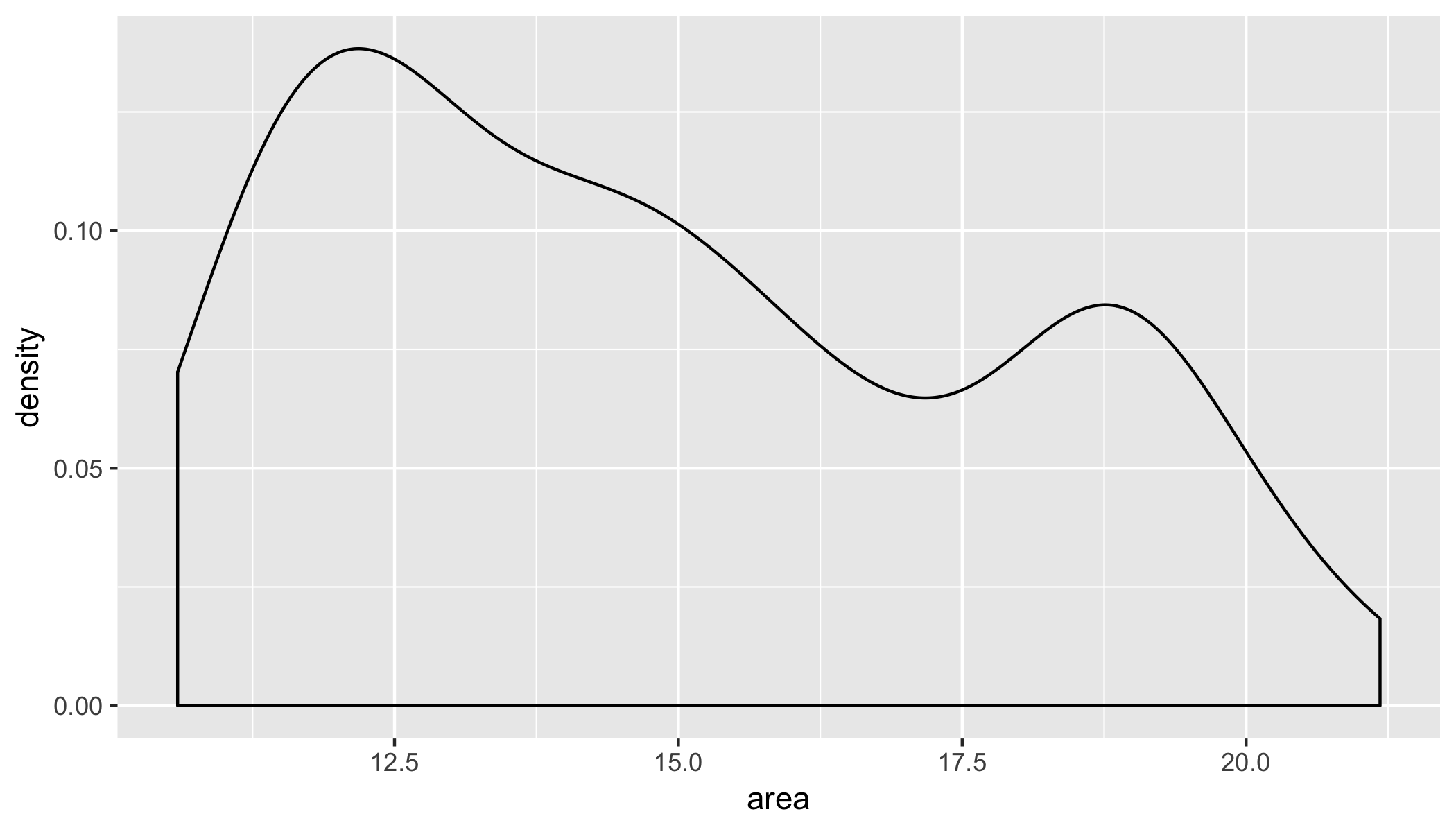
### Univariate Analysis

Shapiro Wilk test rejects the null hypothesis of sample seeds$area being univariate normal.

# Normal Q-Q plot for area  
qqnorm(seeds$area, sub = colnames(seeds)[1])



ggplot(seeds, aes(x=area)) + geom\_density()

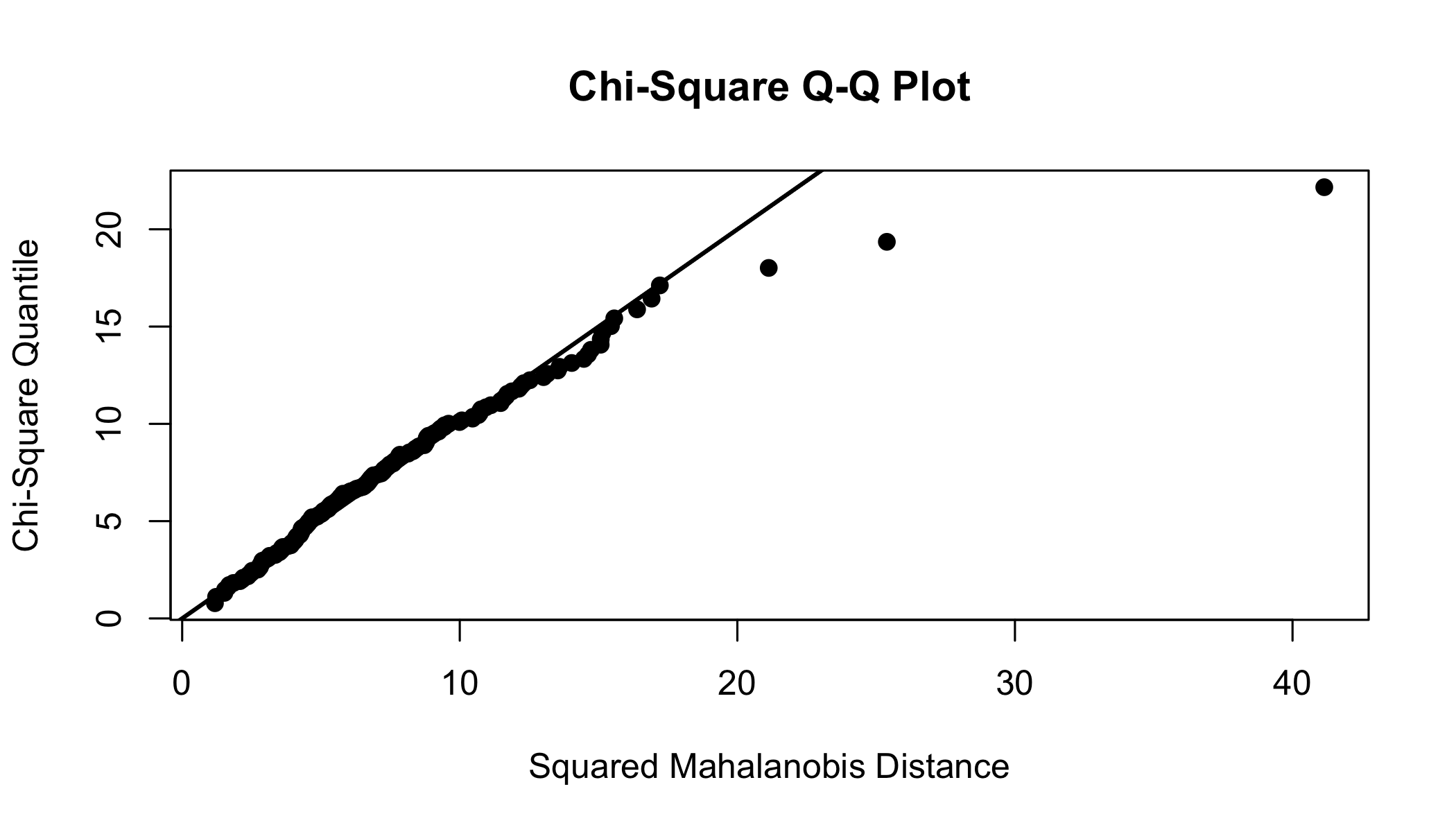


# Shapiro Wilk test for variable x2  
shapiro.test(seeds$area)

##   
## Shapiro-Wilk normality test  
##   
## data: seeds$area  
## W = 0.93259, p-value = 2.948e-08

### Multivariate Normality test

mvtest <- mvn(seeds[,1:7], mvnTest='royston', multivariatePlot='qq')



mvtest$multivariateNormality

## Test H p value MVN  
## 1 Royston 64.13045 6.038104e-14 NO

mvtest$univariateNormality

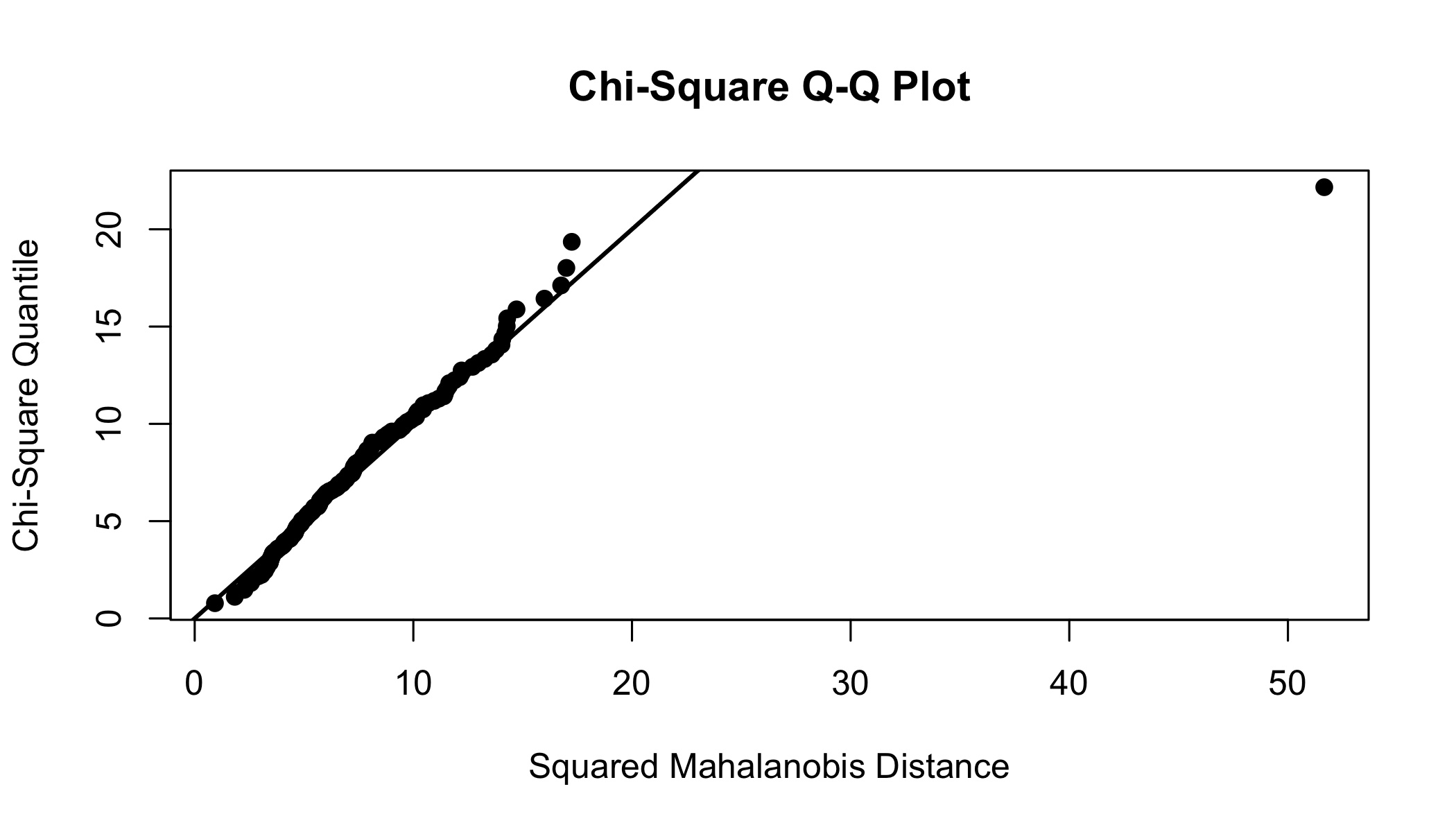
## Test Variable Statistic p value Normality  
## 1 Shapiro-Wilk area 0.9326 <0.001 NO   
## 2 Shapiro-Wilk perimeter 0.9362 <0.001 NO   
## 3 Shapiro-Wilk compactness 0.9730 5e-04 NO   
## 4 Shapiro-Wilk length\_of\_kernel 0.9438 <0.001 NO   
## 5 Shapiro-Wilk width\_of\_kernel 0.9606 <0.001 NO   
## 6 Shapiro-Wilk asymmetry\_coefficient 0.9836 0.0154 NO   
## 7 Shapiro-Wilk length\_of\_kernel\_groove 0.9249 <0.001 NO

mvtest$Descriptives

## n Mean Std.Dev Median Min Max  
## area 210 14.8475238 2.90969943 14.35500 10.5900 21.1800  
## perimeter 210 14.5592857 1.30595873 14.32000 12.4100 17.2500  
## compactness 210 0.8709986 0.02362942 0.87345 0.8081 0.9183  
## length\_of\_kernel 210 5.6285333 0.44306348 5.52350 4.8990 6.6750  
## width\_of\_kernel 210 3.2586048 0.37771444 3.23700 2.6300 4.0330  
## asymmetry\_coefficient 210 3.7002010 1.50355713 3.59900 0.7651 8.4560  
## length\_of\_kernel\_groove 210 5.4080714 0.49148050 5.22300 4.5190 6.5500  
## 25th 75th Skew Kurtosis  
## area 12.27000 17.305000 0.3941946 -1.1052328  
## perimeter 13.45000 15.715000 0.3810678 -1.1269312  
## compactness 0.85690 0.887775 -0.5302931 -0.1923636  
## length\_of\_kernel 5.26225 5.979750 0.5179985 -0.8164443  
## width\_of\_kernel 2.94400 3.561750 0.1324647 -1.1182220  
## asymmetry\_coefficient 2.56150 4.768750 0.3959475 -0.1210795  
## length\_of\_kernel\_groove 5.04500 5.877000 0.5538958 -0.8697756

### Transform to near normal

trans<-powerTransform(seeds[,1:7])  
seeds\_trans <- seeds[,1:7]  
seeds\_trans<-bcPower(seeds\_trans,trans$lambda)  
mvtest\_trans <- mvn(seeds\_trans, mvnTest='royston', multivariatePlot='qq')



mvtest\_trans$multivariateNormality

## Test H p value MVN  
## 1 Royston 57.67264 1.674206e-12 NO

mvtest\_trans$univariateNormality

## Test Variable Statistic p value Normality  
## 1 Shapiro-Wilk area^0.02 0.9450 <0.001 NO   
## 2 Shapiro-Wilk perimeter^0.05 0.9428 <0.001 NO   
## 3 Shapiro-Wilk compactness^0.26 0.9697 2e-04 NO   
## 4 Shapiro-Wilk length\_of\_kernel^-0.24 0.9540 <0.001 NO   
## 5 Shapiro-Wilk width\_of\_kernel^-0.31 0.9612 <0.001 NO   
## 6 Shapiro-Wilk asymmetry\_coefficient^0.54 0.9941 0.5774 YES   
## 7 Shapiro-Wilk length\_of\_kernel\_groove^0.1 0.9354 <0.001 NO

mvtest\_trans$Descriptives

## n Mean Std.Dev Median  
## area^0.02 210 2.7671146 0.20630365 2.7506817  
## perimeter^0.05 210 2.8574139 0.10111683 2.8428006  
## compactness^0.26 210 -0.1359227 0.02632398 -0.1329469  
## length\_of\_kernel^-0.24 210 1.4141363 0.05121080 1.4040953  
## width\_of\_kernel^-0.31 210 0.9840812 0.08095530 0.9855554  
## asymmetry\_coefficient^0.54 210 1.8217361 0.84214289 1.8470037  
## length\_of\_kernel\_groove^0.1 210 1.8396677 0.10631577 1.8025705  
## Min Max 25th 75th  
## area^0.02 2.4276846 3.16712622 2.583743 2.9503021  
## perimeter^0.05 2.6803006 3.05582763 2.771514 2.9489351  
## compactness^0.26 -0.2072624 -0.08429163 -0.151368 -0.1172097  
## length\_of\_kernel^-0.24 1.3230648 1.52736424 1.371650 1.4564393  
## width\_of\_kernel^-0.31 0.8362288 1.13358251 0.918514 1.0511577  
## asymmetry\_coefficient^0.54 -0.2492668 4.01729291 1.226098 2.4545992  
## length\_of\_kernel\_groove^0.1 1.6321132 2.07427491 1.761514 1.9433641  
## Skew Kurtosis  
## area^0.02 0.18426120 -1.2312979  
## perimeter^0.05 0.28789644 -1.1876224  
## compactness^0.26 -0.57712885 -0.1248961  
## length\_of\_kernel^-0.24 0.38694406 -0.9553874  
## width\_of\_kernel^-0.31 -0.06443810 -1.1457415  
## asymmetry\_coefficient^0.54 -0.07441285 -0.4131909  
## length\_of\_kernel\_groove^0.1 0.45214139 -0.9401847

### Multinomial logistic

seeds[, wheat\_type\_1:=ifelse(wheat\_type == 1, 1, 0)]  
seeds[, wheat\_type\_2:=ifelse(wheat\_type == 2, 1, 0)]  
seeds[, wheat\_type\_3:=ifelse(wheat\_type == 3, 1, 0)]  
  
seeds\_train <- seeds[,.SD[1:55], by = list(wheat\_type)]  
seeds\_test <- seeds[,.SD[56:70], by = list(wheat\_type)]  
  
seeds\_train[, .N, by = list(wheat\_type)]

## wheat\_type N  
## 1: 1 55  
## 2: 2 55  
## 3: 3 55

seeds\_test[, .N, by = list(wheat\_type)]

## wheat\_type N  
## 1: 1 15  
## 2: 2 15  
## 3: 3 15

model\_mnl<-vglm(formula = cbind(wheat\_type\_1,wheat\_type\_2,wheat\_type\_3) ~ area+perimeter+compactness+length\_of\_kernel+width\_of\_kernel+asymmetry\_coefficient+length\_of\_kernel\_groove, family = multinomial, data = seeds\_train)

## Warning in checkwz(wz, M = M, trace = trace, wzepsilon =  
## control$wzepsilon): 2 diagonal elements of the working weights variable  
## 'wz' have been replaced by 1.819e-12

## Warning in checkwz(wz, M = M, trace = trace, wzepsilon =  
## control$wzepsilon): 24 diagonal elements of the working weights variable  
## 'wz' have been replaced by 1.819e-12

## Warning in slot(family, "linkinv")(eta, extra = extra): fitted  
## probabilities numerically 0 or 1 occurred

## Warning in tfun(mu = mu, y = y, w = w, res = FALSE, eta = eta, extra =  
## extra): fitted values close to 0 or 1

## Warning in slot(family, "linkinv")(eta, extra = extra): fitted  
## probabilities numerically 0 or 1 occurred

## Warning in tfun(mu = mu, y = y, w = w, res = FALSE, eta = eta, extra =  
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## extra): fitted values close to 0 or 1

## Warning in checkwz(wz, M = M, trace = trace, wzepsilon =  
## control$wzepsilon): 26 diagonal elements of the working weights variable  
## 'wz' have been replaced by 1.819e-12

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## control$wzepsilon): 26 diagonal elements of the working weights variable  
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## Warning in checkwz(wz, M = M, trace = trace, wzepsilon =  
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## 'wz' have been replaced by 1.819e-12

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## Warning in vglm.fitter(x = x, y = y, w = w, offset = offset, Xm2 = Xm2, :  
## iterations terminated because half-step sizes are very small

## Warning in vglm.fitter(x = x, y = y, w = w, offset = offset, Xm2 =  
## Xm2, : some quantities such as z, residuals, SEs may be inaccurate due to  
## convergence at a half-step

summary(model\_mnl)

## Warning in temp1@family@linkinv(eta = temp1@predictors, extra =  
## temp1@extra): fitted probabilities numerically 0 or 1 occurred

## Warning in temp1@family@linkinv(eta = temp1@predictors, extra =  
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##   
## Call:  
## vglm(formula = cbind(wheat\_type\_1, wheat\_type\_2, wheat\_type\_3) ~   
## area + perimeter + compactness + length\_of\_kernel + width\_of\_kernel +   
## asymmetry\_coefficient + length\_of\_kernel\_groove, family = multinomial,   
## data = seeds\_train)  
##   
## Pearson residuals:  
## Min 1Q Median 3Q Max  
## log(mu[,1]/mu[,3]) -0.2507 -0.0004539 -1.467e-05 6.344e-05 0.2725  
## log(mu[,2]/mu[,3]) -0.1274 -0.0008345 -5.503e-05 3.206e-05 0.1407  
##   
## Coefficients:   
## Estimate Std. Error z value Pr(>|z|)  
## (Intercept):1 -615.7521 2260.9680 -0.272 0.785  
## (Intercept):2 353.9362 3002.1203 0.118 0.906  
## area:1 -18.2152 80.2874 -0.227 0.821  
## area:2 15.5862 105.5402 0.148 0.883  
## perimeter:1 53.9929 163.4872 NA NA  
## perimeter:2 2.9887 225.8194 0.013 0.989  
## compactness:1 351.1986 1373.9877 NA NA  
## compactness:2 -282.5456 1718.4672 NA NA  
## length\_of\_kernel:1 3.4585 72.2886 NA NA  
## length\_of\_kernel:2 -57.9704 76.0597 NA NA  
## width\_of\_kernel:1 1.1885 116.8446 0.010 0.992  
## width\_of\_kernel:2 -7.3878 154.7941 NA NA  
## asymmetry\_coefficient:1 -3.1679 2.9930 -1.058 0.290  
## asymmetry\_coefficient:2 -0.2879 3.7118 -0.078 0.938  
## length\_of\_kernel\_groove:1 -39.6800 49.8832 -0.795 0.426  
## length\_of\_kernel\_groove:2 -5.1394 43.7949 NA NA  
##   
## Names of linear predictors: log(mu[,1]/mu[,3]), log(mu[,2]/mu[,3])  
##   
## Residual deviance: 0.903 on 314 degrees of freedom  
##   
## Log-likelihood: -0.4515 on 314 degrees of freedom  
##   
## Number of Fisher scoring iterations: 17   
##   
## Warning: Hauck-Donner effect detected in the following estimate(s):  
## 'perimeter:1', 'compactness:1', 'compactness:2', 'length\_of\_kernel:1', 'length\_of\_kernel:2', 'width\_of\_kernel:2', 'length\_of\_kernel\_groove:2'  
##   
##   
## Reference group is level 3 of the response

predictions<-predict(model\_mnl,newdata=seeds\_test[,2:8],type="response")  
seeds\_test$pred\_mnl<-apply(predictions,1,function(i) which.max(i) )  
  
#confusion matrix  
print(table(seeds\_test$wheat\_type,seeds\_test$pred\_mnl))

##   
## 1 2 3  
## 1 14 0 1  
## 2 0 14 1  
## 3 4 0 11

# error rate  
mean(seeds\_test$pred\_mnl != seeds\_test$wheat\_type)

## [1] 0.1333333

seeds\_trans <- cbind(seeds\_trans, seeds$wheat\_type)  
names(seeds\_trans) <- col\_names\_seeds  
  
seeds\_trans[, wheat\_type\_1:=ifelse(wheat\_type == 1, 1, 0)]  
seeds\_trans[, wheat\_type\_2:=ifelse(wheat\_type == 2, 1, 0)]  
seeds\_trans[, wheat\_type\_3:=ifelse(wheat\_type == 3, 1, 0)]  
  
seeds\_trans\_train <- seeds\_trans[,.SD[1:55], by = list(wheat\_type)]  
seeds\_trans\_test <- seeds\_trans[,.SD[56:70], by = list(wheat\_type)]  
  
seeds\_trans\_train[, .N, by = list(wheat\_type)]

## wheat\_type N  
## 1: 1 55  
## 2: 2 55  
## 3: 3 55

seeds\_trans\_test[, .N, by = list(wheat\_type)]

## wheat\_type N  
## 1: 1 15  
## 2: 2 15  
## 3: 3 15

model\_mnl\_trans <-vglm(formula = cbind(wheat\_type\_1,wheat\_type\_2,wheat\_type\_3) ~ area+perimeter+compactness+length\_of\_kernel+width\_of\_kernel+asymmetry\_coefficient+length\_of\_kernel\_groove, family = multinomial, data = seeds\_trans\_train)

## Warning in checkwz(wz, M = M, trace = trace, wzepsilon =  
## control$wzepsilon): 3 diagonal elements of the working weights variable  
## 'wz' have been replaced by 1.819e-12

## Warning in checkwz(wz, M = M, trace = trace, wzepsilon =  
## control$wzepsilon): 22 diagonal elements of the working weights variable  
## 'wz' have been replaced by 1.819e-12

## Warning in checkwz(wz, M = M, trace = trace, wzepsilon =  
## control$wzepsilon): 65 diagonal elements of the working weights variable  
## 'wz' have been replaced by 1.819e-12

## Warning in slot(family, "linkinv")(eta, extra = extra): fitted  
## probabilities numerically 0 or 1 occurred

## Warning in tfun(mu = mu, y = y, w = w, res = FALSE, eta = eta, extra =  
## extra): fitted values close to 0 or 1

## Warning in checkwz(wz, M = M, trace = trace, wzepsilon =  
## control$wzepsilon): 76 diagonal elements of the working weights variable  
## 'wz' have been replaced by 1.819e-12

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## Warning in vglm.fitter(x = x, y = y, w = w, offset = offset, Xm2 = Xm2, :  
## iterations terminated because half-step sizes are very small

## Warning in vglm.fitter(x = x, y = y, w = w, offset = offset, Xm2 =  
## Xm2, : some quantities such as z, residuals, SEs may be inaccurate due to  
## convergence at a half-step

summary(model\_mnl)

## Warning in temp1@family@linkinv(eta = temp1@predictors, extra =  
## temp1@extra): fitted probabilities numerically 0 or 1 occurred

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##   
## Call:  
## vglm(formula = cbind(wheat\_type\_1, wheat\_type\_2, wheat\_type\_3) ~   
## area + perimeter + compactness + length\_of\_kernel + width\_of\_kernel +   
## asymmetry\_coefficient + length\_of\_kernel\_groove, family = multinomial,   
## data = seeds\_train)  
##   
## Pearson residuals:  
## Min 1Q Median 3Q Max  
## log(mu[,1]/mu[,3]) -0.2507 -0.0004539 -1.467e-05 6.344e-05 0.2725  
## log(mu[,2]/mu[,3]) -0.1274 -0.0008345 -5.503e-05 3.206e-05 0.1407  
##   
## Coefficients:   
## Estimate Std. Error z value Pr(>|z|)  
## (Intercept):1 -615.7521 2260.9680 -0.272 0.785  
## (Intercept):2 353.9362 3002.1203 0.118 0.906  
## area:1 -18.2152 80.2874 -0.227 0.821  
## area:2 15.5862 105.5402 0.148 0.883  
## perimeter:1 53.9929 163.4872 NA NA  
## perimeter:2 2.9887 225.8194 0.013 0.989  
## compactness:1 351.1986 1373.9877 NA NA  
## compactness:2 -282.5456 1718.4672 NA NA  
## length\_of\_kernel:1 3.4585 72.2886 NA NA  
## length\_of\_kernel:2 -57.9704 76.0597 NA NA  
## width\_of\_kernel:1 1.1885 116.8446 0.010 0.992  
## width\_of\_kernel:2 -7.3878 154.7941 NA NA  
## asymmetry\_coefficient:1 -3.1679 2.9930 -1.058 0.290  
## asymmetry\_coefficient:2 -0.2879 3.7118 -0.078 0.938  
## length\_of\_kernel\_groove:1 -39.6800 49.8832 -0.795 0.426  
## length\_of\_kernel\_groove:2 -5.1394 43.7949 NA NA  
##   
## Names of linear predictors: log(mu[,1]/mu[,3]), log(mu[,2]/mu[,3])  
##   
## Residual deviance: 0.903 on 314 degrees of freedom  
##   
## Log-likelihood: -0.4515 on 314 degrees of freedom  
##   
## Number of Fisher scoring iterations: 17   
##   
## Warning: Hauck-Donner effect detected in the following estimate(s):  
## 'perimeter:1', 'compactness:1', 'compactness:2', 'length\_of\_kernel:1', 'length\_of\_kernel:2', 'width\_of\_kernel:2', 'length\_of\_kernel\_groove:2'  
##   
##   
## Reference group is level 3 of the response

predictions\_trans <-predict(model\_mnl\_trans,newdata=seeds\_trans\_test[,2:8],type="response")

## Warning in object@family@linkinv(predictor, extra = new.extra): fitted  
## probabilities numerically 0 or 1 occurred

seeds\_trans\_test$pred\_mnl<-apply(predictions\_trans,1,function(i) which.max(i) )  
  
#confusion matrix  
print(table(seeds\_trans\_test$wheat\_type,seeds\_trans\_test$pred\_mnl))

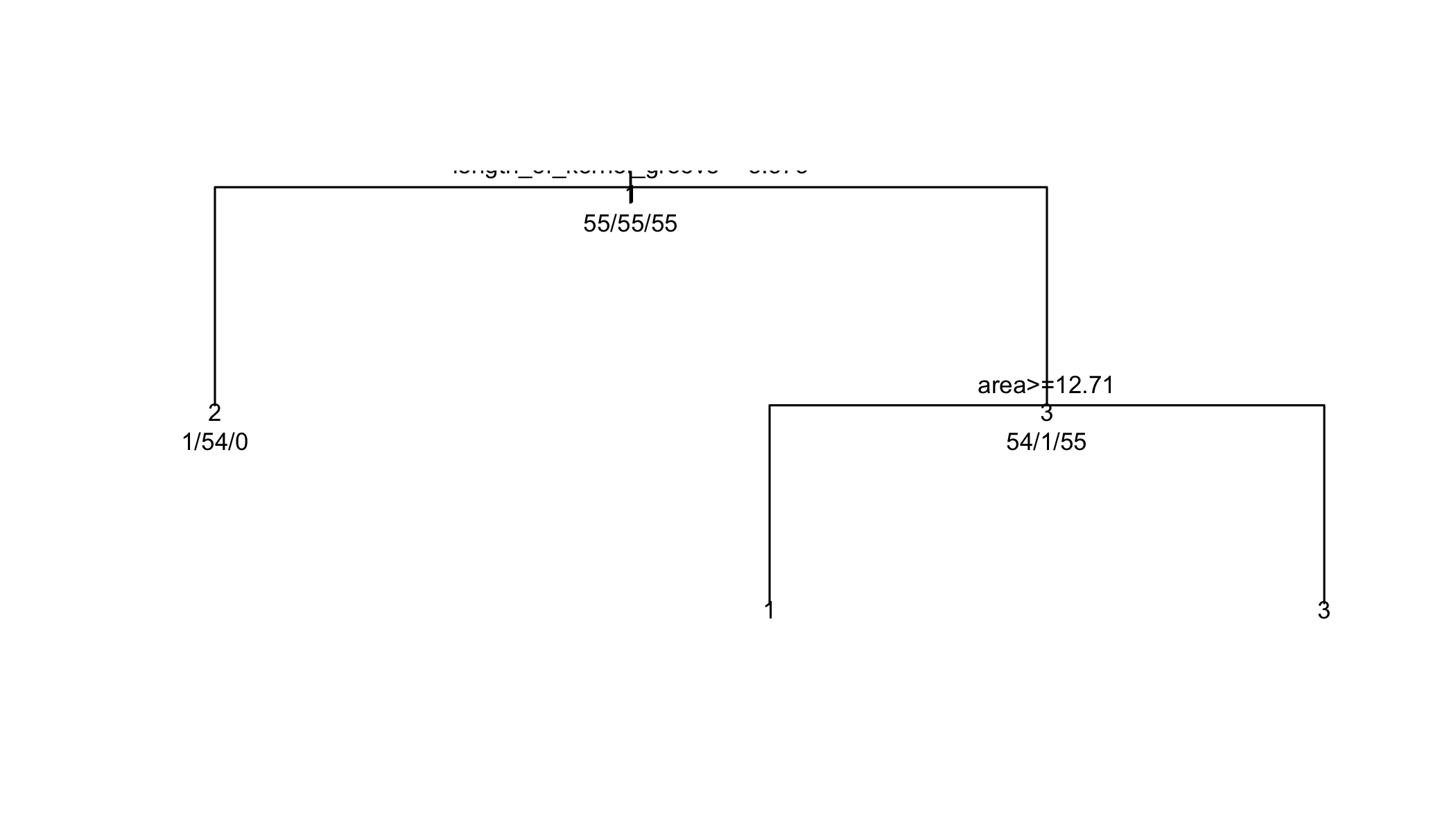
##   
## 1 2 3  
## 1 13 0 2  
## 2 0 14 1  
## 3 6 0 9

# error rate  
mean(seeds\_trans\_test$pred\_mnl != seeds\_trans\_test$wheat\_type)

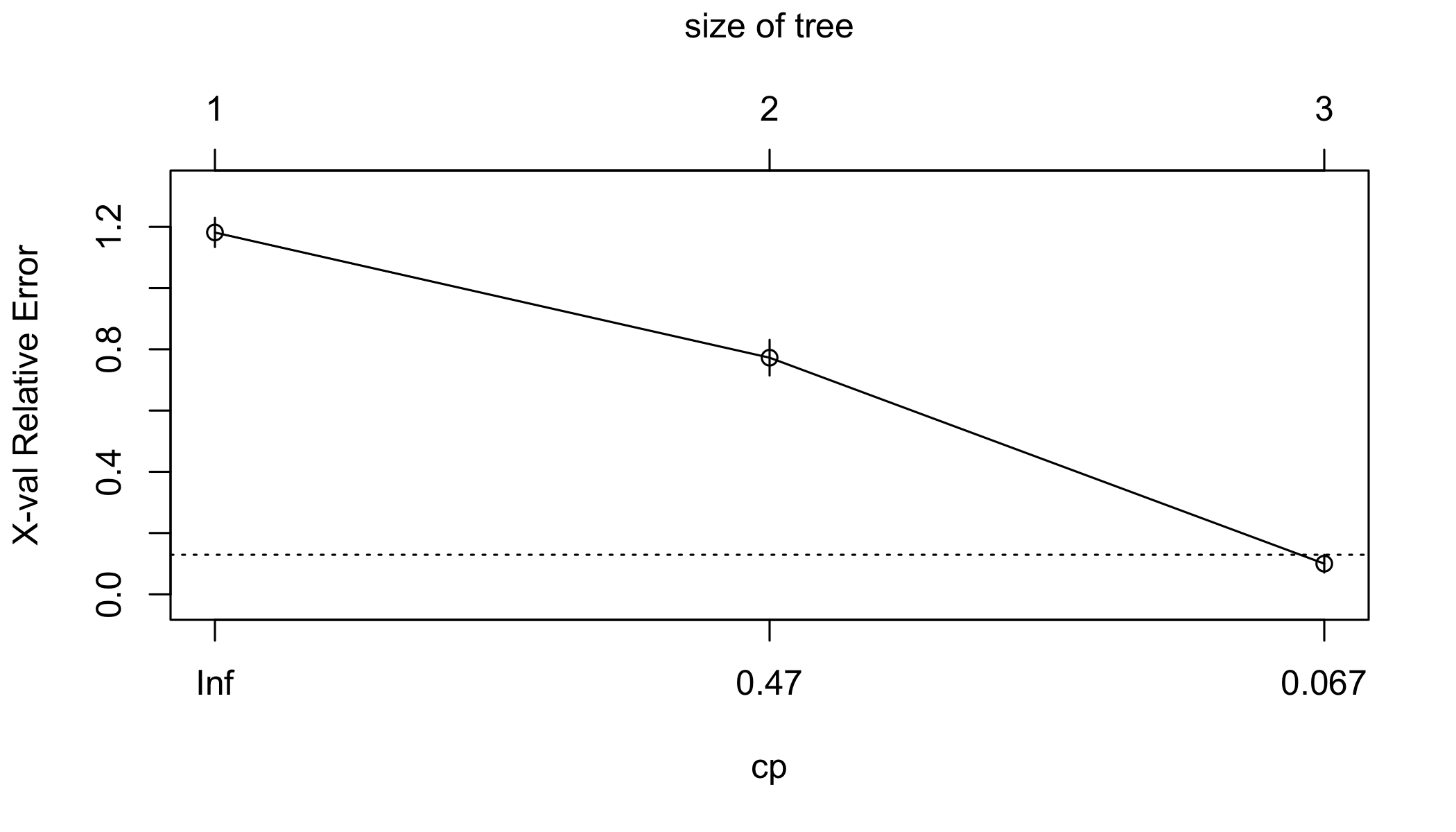
## [1] 0.2

###CART

######## CART  
model\_ct <- rpart(wheat\_type ~ area+perimeter+compactness+length\_of\_kernel+width\_of\_kernel+asymmetry\_coefficient+length\_of\_kernel\_groove, data = seeds\_train, method="class")  
plot(model\_ct)  
text(model\_ct, use.n=TRUE, all=TRUE, cex=.7)



plotcp(model\_ct)



seeds\_test$pred\_ct<-predict(model\_ct,seeds\_test,type="vector")  
#confusion matrix  
table(seeds\_test$wheat\_type,seeds\_test$pred\_ct)

##   
## 1 2 3  
## 1 11 0 4  
## 2 1 14 0  
## 3 5 0 10

# error rate  
mean(seeds\_test$pred\_ct != seeds\_test$wheat\_type)

## [1] 0.2222222

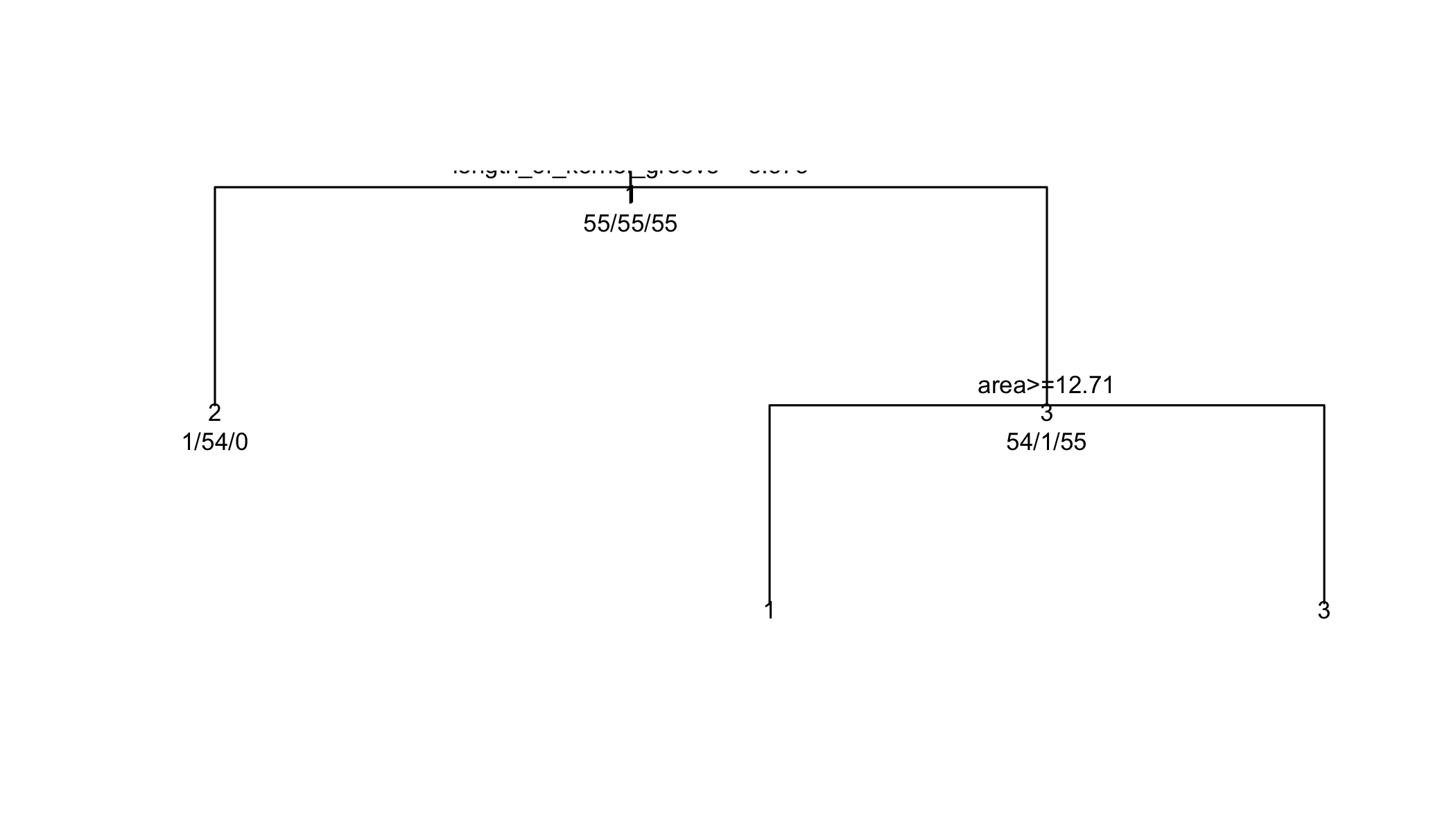
model\_ct$cptable

## CP nsplit rel error xerror xstd  
## 1 0.4909091 0 1.00000000 1.1818182 0.04773875  
## 2 0.4454545 1 0.50909091 0.7727273 0.05836061  
## 3 0.0100000 2 0.06363636 0.1000000 0.02912876

model\_ct\_fit<- prune(model\_ct, cp=model\_ct$cptable[which.min(model\_ct$cptable[,"xerror"]),"CP"])  
  
summary(model\_ct\_fit)

## Call:  
## rpart(formula = wheat\_type ~ area + perimeter + compactness +   
## length\_of\_kernel + width\_of\_kernel + asymmetry\_coefficient +   
## length\_of\_kernel\_groove, data = seeds\_train, method = "class")  
## n= 165   
##   
## CP nsplit rel error xerror xstd  
## 1 0.4909091 0 1.00000000 1.1818182 0.04773875  
## 2 0.4454545 1 0.50909091 0.7727273 0.05836061  
## 3 0.0100000 2 0.06363636 0.1000000 0.02912876  
##   
## Variable importance  
## area perimeter width\_of\_kernel   
## 20 20 18   
## length\_of\_kernel length\_of\_kernel\_groove compactness   
## 17 11 8   
## asymmetry\_coefficient   
## 5   
##   
## Node number 1: 165 observations, complexity param=0.4909091  
## predicted class=1 expected loss=0.6666667 P(node) =1  
## class counts: 55 55 55  
## probabilities: 0.333 0.333 0.333   
## left son=2 (55 obs) right son=3 (110 obs)  
## Primary splits:  
## length\_of\_kernel\_groove < 5.5755 to the right, improve=52.05455, (0 missing)  
## perimeter < 15.305 to the right, improve=49.21818, (0 missing)  
## area < 16.305 to the right, improve=47.91776, (0 missing)  
## width\_of\_kernel < 3.0935 to the right, improve=47.35340, (0 missing)  
## length\_of\_kernel < 5.8415 to the right, improve=46.49091, (0 missing)  
## Surrogate splits:  
## perimeter < 15.305 to the right, agree=0.988, adj=0.964, (0 split)  
## area < 16.305 to the right, agree=0.982, adj=0.945, (0 split)  
## length\_of\_kernel < 5.8415 to the right, agree=0.976, adj=0.927, (0 split)  
## width\_of\_kernel < 3.436 to the right, agree=0.927, adj=0.782, (0 split)  
## compactness < 0.8745 to the right, agree=0.691, adj=0.073, (0 split)  
##   
## Node number 2: 55 observations  
## predicted class=2 expected loss=0.01818182 P(node) =0.3333333  
## class counts: 1 54 0  
## probabilities: 0.018 0.982 0.000   
##   
## Node number 3: 110 observations, complexity param=0.4454545  
## predicted class=3 expected loss=0.5 P(node) =0.6666667  
## class counts: 54 1 55  
## probabilities: 0.491 0.009 0.500   
## left son=6 (58 obs) right son=7 (52 obs)  
## Primary splits:  
## area < 12.71 to the right, improve=44.74442, (0 missing)  
## width\_of\_kernel < 3.0935 to the right, improve=43.20296, (0 missing)  
## perimeter < 13.795 to the right, improve=39.34958, (0 missing)  
## compactness < 0.866 to the right, improve=34.63515, (0 missing)  
## asymmetry\_coefficient < 4.124 to the left, improve=23.38504, (0 missing)  
## Surrogate splits:  
## perimeter < 13.745 to the right, agree=0.973, adj=0.942, (0 split)  
## width\_of\_kernel < 2.9715 to the right, agree=0.973, adj=0.942, (0 split)  
## compactness < 0.86 to the right, agree=0.864, adj=0.712, (0 split)  
## length\_of\_kernel < 5.3405 to the right, agree=0.845, adj=0.673, (0 split)  
## asymmetry\_coefficient < 3.5915 to the left, agree=0.782, adj=0.538, (0 split)  
##   
## Node number 6: 58 observations  
## predicted class=1 expected loss=0.0862069 P(node) =0.3515152  
## class counts: 53 1 4  
## probabilities: 0.914 0.017 0.069   
##   
## Node number 7: 52 observations  
## predicted class=3 expected loss=0.01923077 P(node) =0.3151515  
## class counts: 1 0 51  
## probabilities: 0.019 0.000 0.981

plot(model\_ct\_fit)  
text(model\_ct\_fit, use.n=TRUE, all=TRUE, cex=.7)



seeds\_test$pred\_ct\_fit<-predict(model\_ct\_fit,seeds\_test,type="vector")  
#confusion matrix  
table(seeds\_test$wheat\_type,seeds\_test$pred\_ct\_fit)

##   
## 1 2 3  
## 1 11 0 4  
## 2 1 14 0  
## 3 5 0 10

# error rate  
mean(seeds\_test$pred\_ct\_fit != seeds\_test$wheat\_type)

## [1] 0.2222222