Dynamic Modeling of Medium Size Aerostat with Tether

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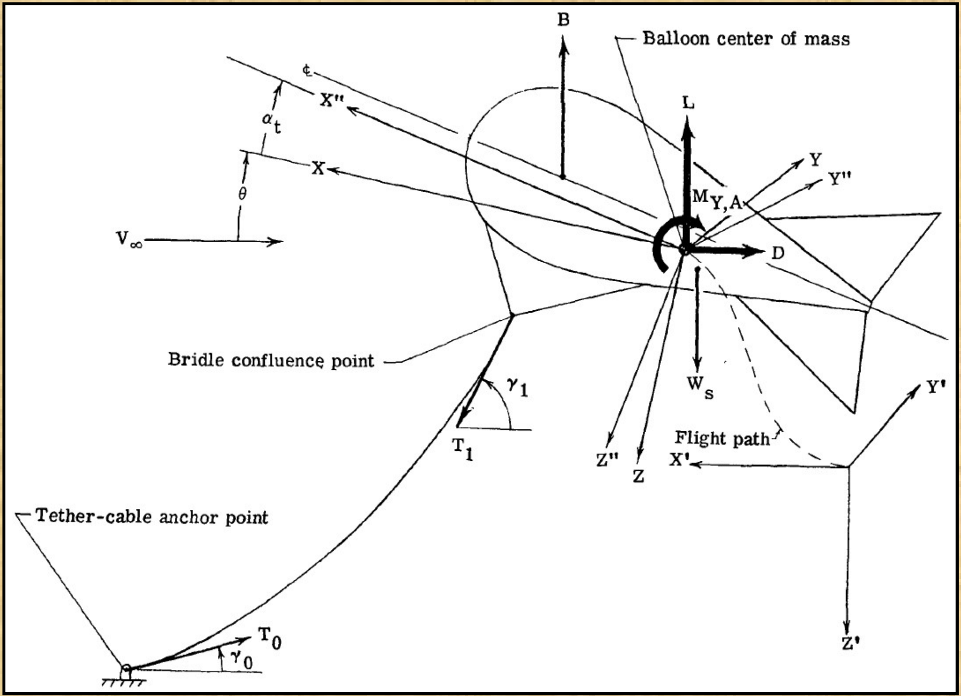
Anuj Sharma, Sc C, ADRDE, Agra, et.al

OVERVIEW OF THE WORK

- 6-DOF Dynamic Modeling of Aerostat is carried out
- Dynamic Modeling of Tether is carried out
- Dynamic Model of Tether is combined with Dynamic Model of Aerostat



Aerostat Balloon with Tether

AKASHDEEP Aerostat

(Medium Size, Volume ~ 2000 m^3)

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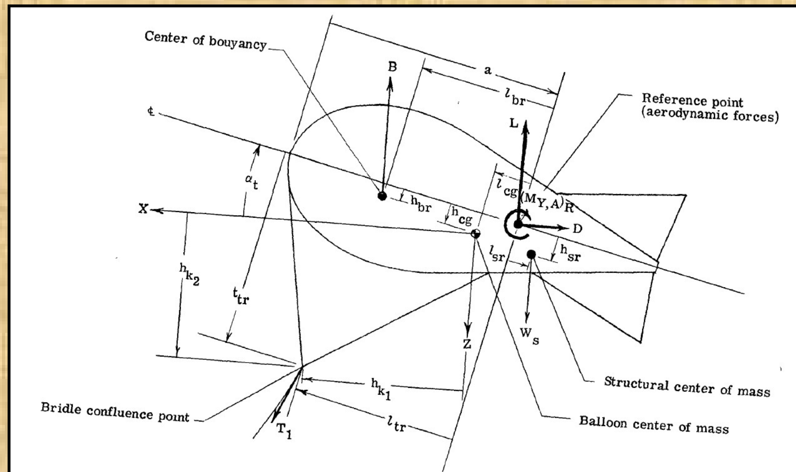
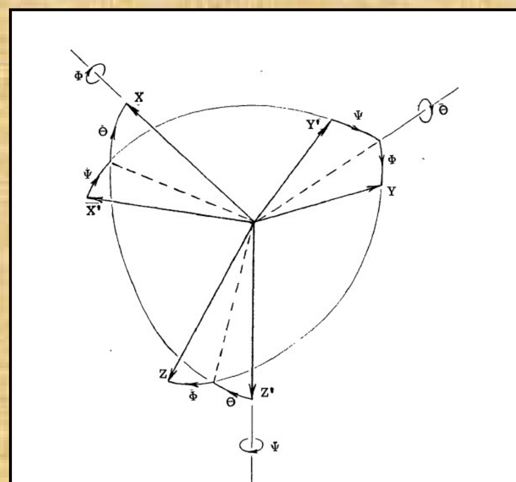
DYNAMIC MODELING OF AEROSTAT

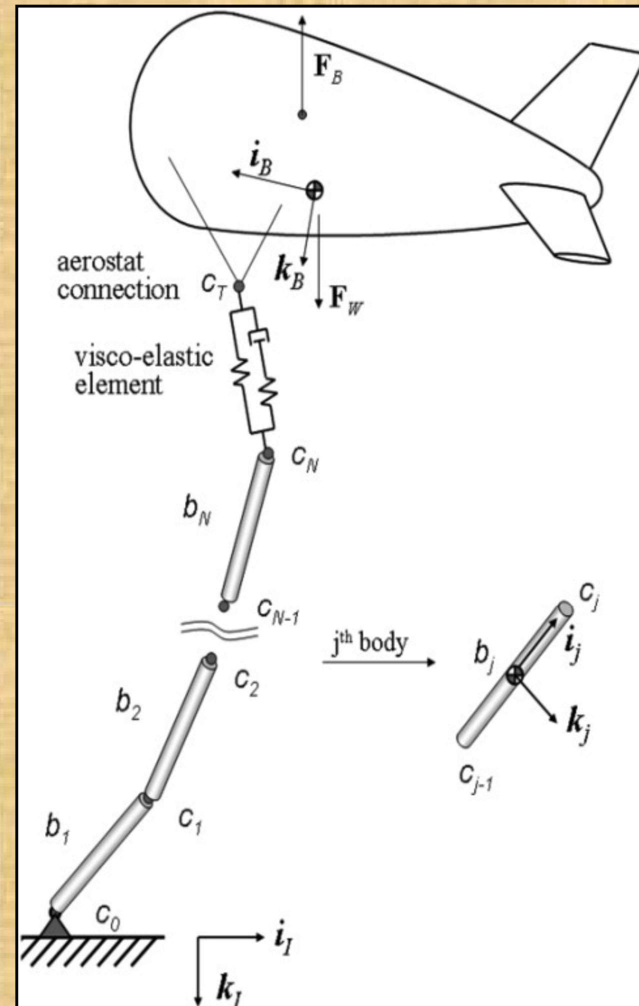
- Following Forces and Moments on the Aerostat have been modeled
  - Weight
  - Buoyancy
  - Aerodynamic
  - Tether
- Effects due to apparent mass and inertia have also been considered
- 6-DOF Equations are developed for modeling the dynamics of the aerostat in NED frame



Aerostat Balloon



Co-ordinate Frame



Aerostat with Tether

Reference: Journal of Aircraft paper *'Tethered Aerostat Modeling using an Efficient Recursive Rigid-Body Dynamics Approach'*, Vol 48, No. 2, March-April, 2011

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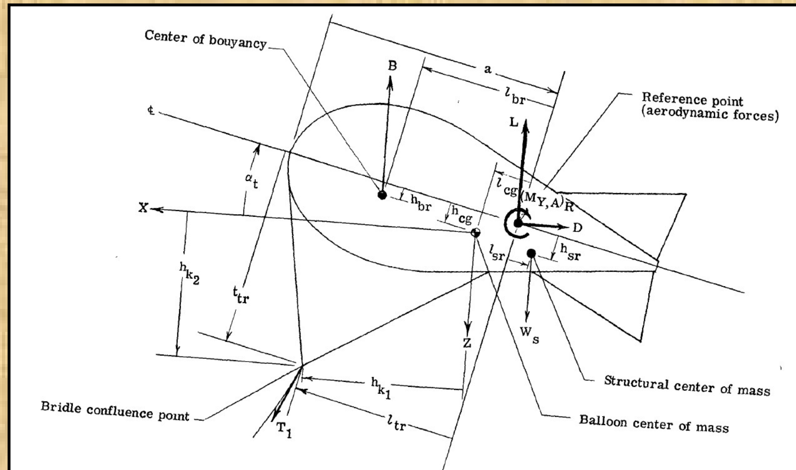
EQUATIONS OF MOTION OF AEROSTAT

$$\mathbf{F}_W = \begin{Bmatrix} 0 \\ 0 \\ m_B g \end{Bmatrix}$$

$$\mathbf{F}_B = \begin{Bmatrix} 0 \\ 0 \\ -\mathrm{Vol}(\rho_a - \rho_g)g \end{Bmatrix}$$

Weight and Buoyancy Forces



$$\mathbf{F}_{\mathrm{AM}} = -\mathbf{I}_{\mathrm{AM}} \begin{Bmatrix} \dot{u}_A \\ \dot{v}_A \\ \dot{w}_A \end{Bmatrix} - \mathbf{S}(\boldsymbol{\omega}_B)\mathbf{I}_{\mathrm{AM}} \begin{Bmatrix} u_A \\ v_A \\ w_A \end{Bmatrix}$$

$$\mathbf{M}_{\mathrm{AM}} = -\mathbf{I}_{\mathrm{AI}} \begin{Bmatrix} \dot{p}_B \\ \dot{q}_B \\ \dot{r}_B \end{Bmatrix} - \mathbf{S}(\boldsymbol{\omega}_B)\mathbf{I}_{\mathrm{AI}} \begin{Bmatrix} p_B \\ q_B \\ r_B \end{Bmatrix}$$

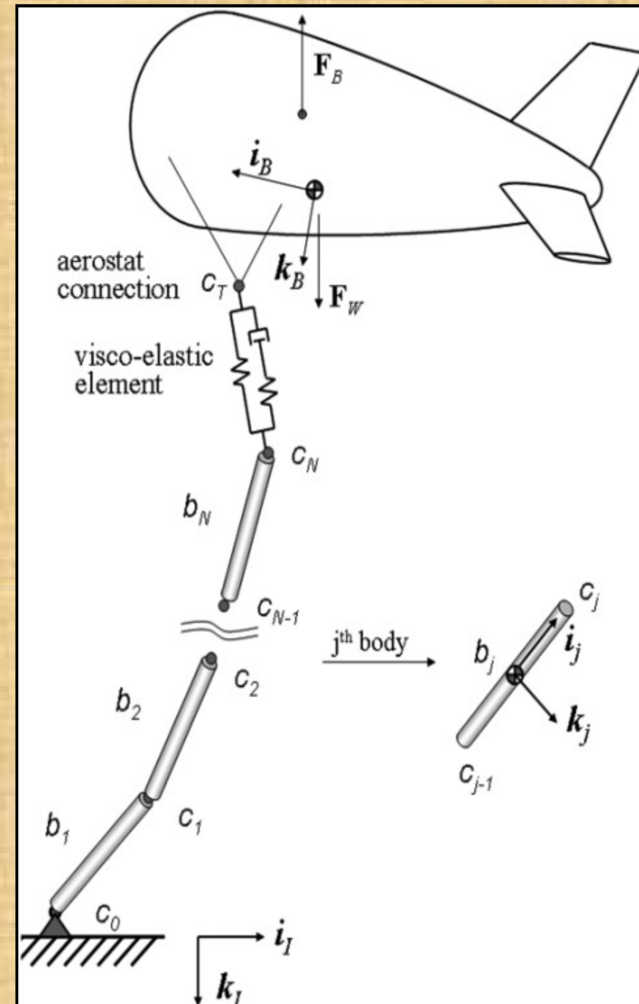
Forces and Moments due to Apparent Mass and Inertia

$$\mathbf{F}_A = \bar{q}\mathrm{Vol}^{2/3}\mathbf{T}(\alpha) \begin{Bmatrix} -(C_{D0} + C_{D_\alpha}\alpha + C_{D_{\alpha^2}}\alpha^2) \\ C_{Y\beta}\beta + (\bar{l}/(2V_A))(C_{Yp}p + C_{Yr}r) \\ -(C_{L0} + C_{L_\alpha}\alpha + C_{L_{\alpha^2}}\alpha^2 + (\bar{l}/(2V_A))(C_{Lq}q)) \end{Bmatrix}$$

$$\mathbf{M}_A = \bar{q}\mathrm{Vol}^{2/3}\bar{l} \begin{Bmatrix} C_{l\beta}\beta + (\bar{l}/(2V_A))(C_{lp}p + C_{lr}r) \\ C_{m0} + C_{m\alpha}\alpha + (\bar{l}/(2V_A))C_{mq}q \\ C_{n\beta}\beta + (\bar{l}/(2V_A))(C_{nr}r + C_{np}p) \end{Bmatrix}$$

$$\mathbf{T}(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

Aerodynamic Forces and Moments



$$\left((m_B + m_{gas})\mathbf{E}_3 + \mathbf{I}_{\mathrm{AM}}\right)\begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{Bmatrix} = \mathbf{F}_A + \mathbf{T}_I^B(\mathbf{F}_W + \mathbf{F}_B - \mathbf{F}_T) - \mathbf{S}(\boldsymbol{\omega}_B)\left((m + m_{gas})\mathbf{E}_3 + \mathbf{I}_{\mathrm{AM}}\right)\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} + \mathbf{I}_{\mathrm{AM}}\mathbf{T}_I^B\begin{Bmatrix} \dot{w}_x \\ \dot{w}_y \\ \dot{w}_z \end{Bmatrix} + \left(\mathbf{S}(\boldsymbol{\omega}_B)\mathbf{I}_{\mathrm{AM}} - \mathbf{I}_{\mathrm{AM}}\mathbf{S}(\boldsymbol{\omega}_B)\right)\mathbf{T}_I^B\begin{Bmatrix} w_x \\ w_y \\ w_z \end{Bmatrix}$$

Force Equation

$$(\mathbf{I}_B + \mathbf{I}_{\mathrm{AI}})\begin{Bmatrix} \dot{p}_B \\ \dot{q}_B \\ \dot{r}_B \end{Bmatrix} = \mathbf{M}_A + \mathbf{S}(\mathbf{r}_{cg}^{cb})\mathbf{T}_I^B\mathbf{F}_B - \mathbf{S}(\mathbf{r}_{cg}^{cp})\mathbf{T}_I^B\mathbf{F}_T - \mathbf{S}(\boldsymbol{\omega}_B)(\mathbf{I}_B + \mathbf{I}_{\mathrm{AI}})\begin{Bmatrix} p_B \\ q_B \\ r_B \end{Bmatrix}$$

Moment Equation

DYNAMIC MODELING OF TETHER

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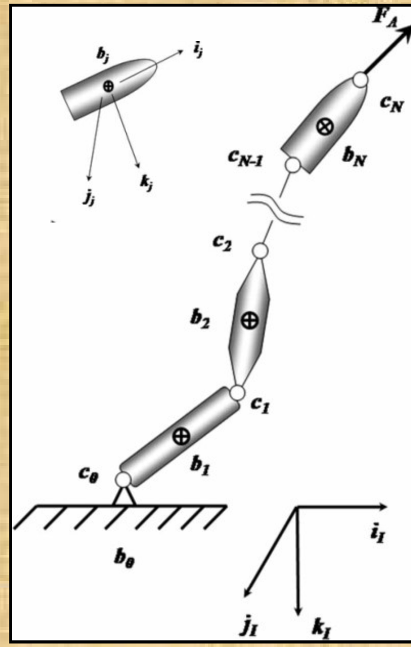
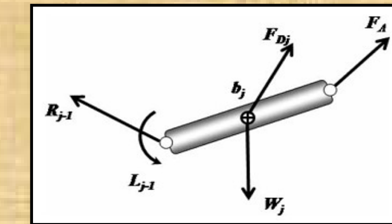
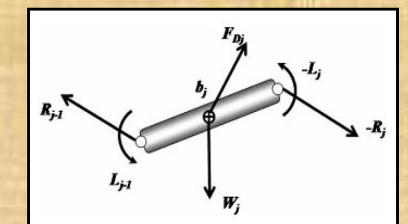
- Tether is divided into N cylindrical elements
- Tether spin is negligible
- Visco-elastic link is used to connect the tether to the aerostat. It is used to model the elastic properties of tether
- Joint damping has been modeled as viscous damping, proportional to the bending rate between two successive links
- Linear and angular accelerations in tether elements are determined recursively



Tether



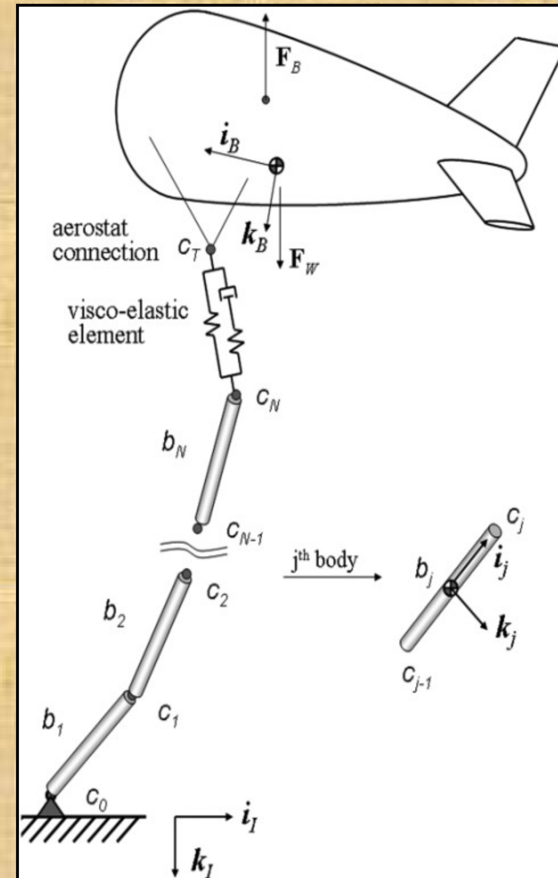
Terminal Link



Non-Terminal Link

$$\dot{F}_T + \frac{K_v}{C_v}F_T = \begin{cases} (K_v + K_S)\dot{s}_{ve} + \frac{K_v K_S}{C_v}(s_{ve} - L_{ve}), & s_{ve} - L_{ve} > 0 \\ 0 & s_{ve} - L_{ve} \le 0 \end{cases}$$

Visco-Elastic Link Force Equation



Representation of Dynamic Model of Aerostat with Tether

BOUNDARY CONDITION

For ground link, Roll, Pitch and Yaw rates are zero

Reference: '*Efficient Tether Dynamic Formulation using Recursive Rigid-Body Dynamics*', published in Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-Body Dynamics 2010 224:353

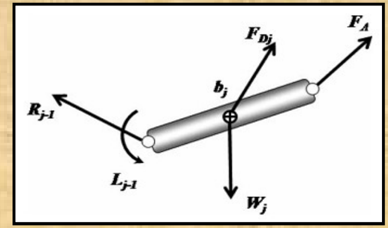
EQUATIONS OF MOTION OF TETHER

$$\boldsymbol{\omega}_{j/I} = \boldsymbol{\omega}_{j/j-1} + \mathbf{T}^{j}_{j-1}\boldsymbol{\omega}_{j-1/I}$$

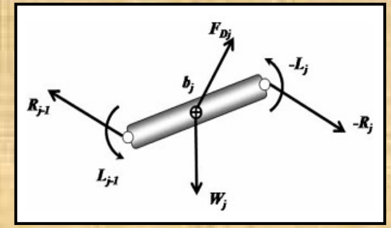
$$\omega_{x1} = 0$$

$$\tilde{\omega}_{1/I} = \begin{Bmatrix}\omega_{y1}\\ \omega_{z1}\end{Bmatrix} = \begin{Bmatrix}q_1\\ r_1\end{Bmatrix}$$

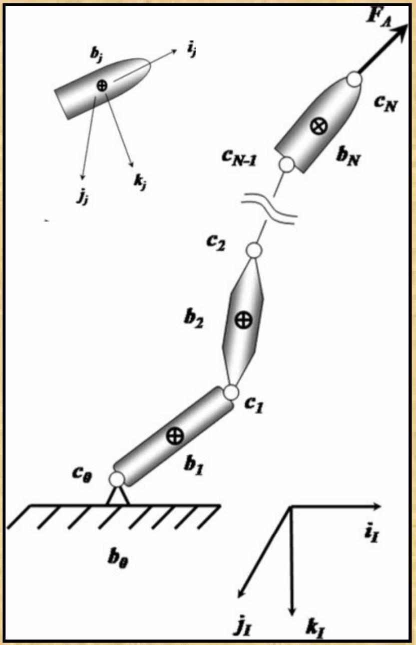
Angular Velocity



Terminal Link



Non-Terminal Link



Tether

$$\boldsymbol{\alpha}_{j/I} - \dot{\boldsymbol{\omega}}_{j/(j-1)} + \boldsymbol{\omega}_{j/I} \times \boldsymbol{\omega}_{j/j-1} + \mathbf{T}^{j}_{j-1}\boldsymbol{\alpha}_{j-1/I}$$

$$\alpha_{j/I} = \begin{Bmatrix}0\\ \dot{q}_j\\ \dot{r}_j\end{Bmatrix} = \begin{Bmatrix}\dot{\omega}_{xj}\\ \dot{\omega}_{yj}\\ \dot{\omega}_{zj}\end{Bmatrix} + \begin{Bmatrix}-r_j\omega_{yj} + q_j\omega_{zj}\\ r_j\omega_{xj}\\ -q_j\omega_{xj}\end{Bmatrix} + \mathbf{T}^{j}_{j-1}\begin{Bmatrix}0\\ \dot{q}_{j-1}\\ \dot{r}_{j-1}\end{Bmatrix}$$

$$\tilde{\alpha}_{j/I} = \begin{Bmatrix}\dot{q}_j\\ \dot{r}_j\end{Bmatrix} = \begin{Bmatrix}\dot{\omega}_{yj}\\ \dot{\omega}_{zj}\end{Bmatrix} - \begin{Bmatrix}r_j\\ -q_j\end{Bmatrix}\tilde{\mathbf{T}}^{j}_{j-1}\begin{Bmatrix}q_{j-1}\\ r_{j-1}\end{Bmatrix} + \hat{\mathbf{T}}^{j}_{j-1}\begin{Bmatrix}\dot{q}_{j-1}\\ \dot{r}_{j-1}\end{Bmatrix}$$

Angular Acceleration

$$\mathbf{a}^{m}_{j} = \mathbf{T}^{j}_{j-1}\mathbf{a}^{c}_{j-1} + \boldsymbol{\alpha}_{j/I} \times \mathbf{r}^{m}_{j} + \boldsymbol{\omega}_{j/I} \times (\boldsymbol{\omega}_{j/I} \times \mathbf{r}^{m}_{j})$$

$$= \mathbf{T}^{j}_{j-1}\mathbf{a}^{c}_{j-1} - \mathbf{r}^{c}_{j} \times \boldsymbol{\alpha}_{j/I} + \boldsymbol{\omega}_{j/I} \times (\boldsymbol{\omega}_{j/I} \times \mathbf{r}^{c}_{j})$$

Linear Acceleration

$$\mathbf{r}^{m}_{j} \times \mathbf{T}^{j}_{I}(\mathbf{F}_{Dj} + \mathbf{W}_j) + \mathbf{r}^{c}_{j} \times \mathbf{T}^{j}_{I}\mathbf{F}_{T} + \mathbf{T}^{j}_{j-1}\mathbf{L}_{j-1} = (\mathbf{I}_j\boldsymbol{\alpha}_{j/I} + \boldsymbol{\omega}_{j/I} \times \mathbf{I}_j\boldsymbol{\omega}_{j/I}) + \mathbf{r}^{m}_{j} \times m_j\mathbf{a}^{m}_{j}$$

$$\mathbf{R}_{j-1} + \mathbf{T}^{j-1}_{I}(\mathbf{F}_{Dj} + \mathbf{W}_j + \mathbf{F}_T) = m_j\left(\mathbf{T}^{j}_{j-1}\right)^{T}\mathbf{a}^{m}_{j}$$

$$\mathbf{F}_j = \mathbf{M}_j\dot{\mathbf{v}}_j + \boldsymbol{\Gamma}_T$$

Terminal Body Recursive Dynamics

$$\mathbf{R}_{j-1} - \left(\mathbf{T}^{j}_{j-1}\right)^{T}\mathbf{R}_j + \mathbf{T}^{j-1}_{I}(\mathbf{F}_{Dj} + \mathbf{W}_j) = m_j\left(\mathbf{T}^{j}_{j-1}\right)^{T}\mathbf{a}^{m}_{j}$$

$$\mathbf{r}^{m}_{j} \times \mathbf{T}^{j}_{I}(\mathbf{F}_{Dj} + \mathbf{W}_j) - \mathbf{r}^{c}_{j} \times \mathbf{R}_j + \mathbf{T}^{j}_{j-1}\mathbf{L}_{j-1} - \mathbf{L}_j = (\mathbf{I}_j\boldsymbol{\alpha}_{j/I} + \boldsymbol{\omega}_{j/I} \times \mathbf{I}_j\boldsymbol{\omega}_{j/I}) + \mathbf{r}^{m}_{j} \times m_j\mathbf{a}^{m}_{j}$$

$$\mathbf{F}_j = \mathbf{M}_j\dot{\mathbf{v}}_j + \boldsymbol{\Gamma}_j + D^{T}_{j+1}\mathbf{F}_{j+1}$$

Non-Terminal Body Recursive Dynamics

RESULTS

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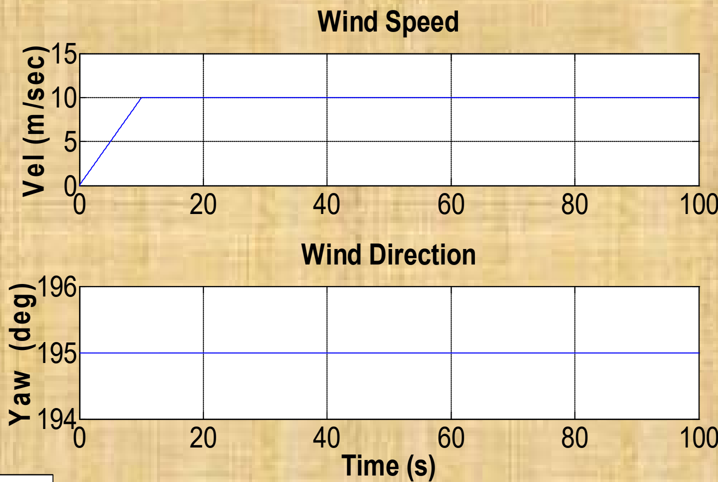
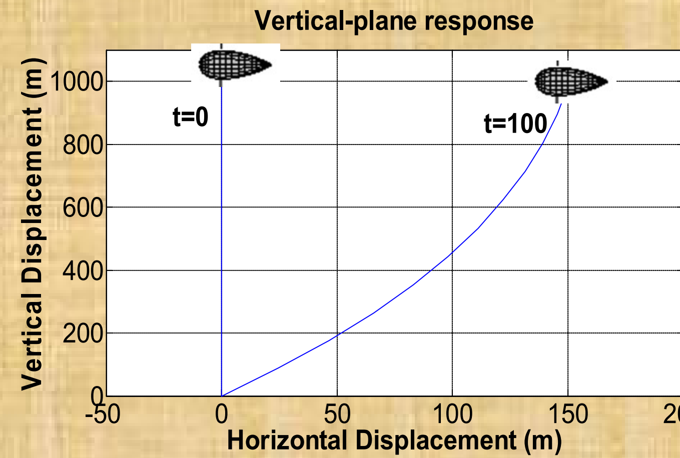
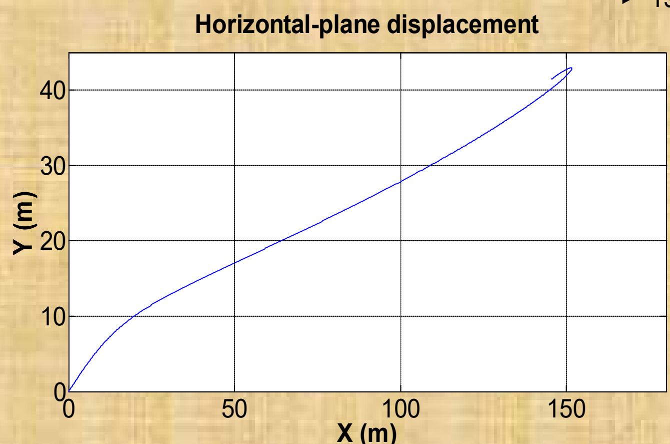
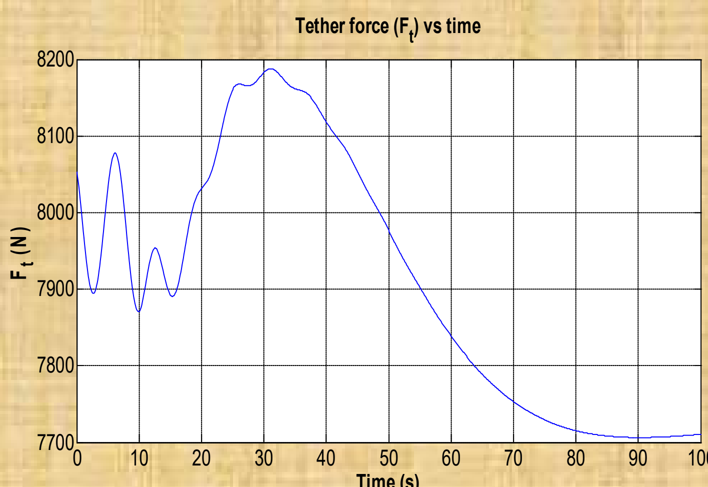
INITIAL CONDITIONS:

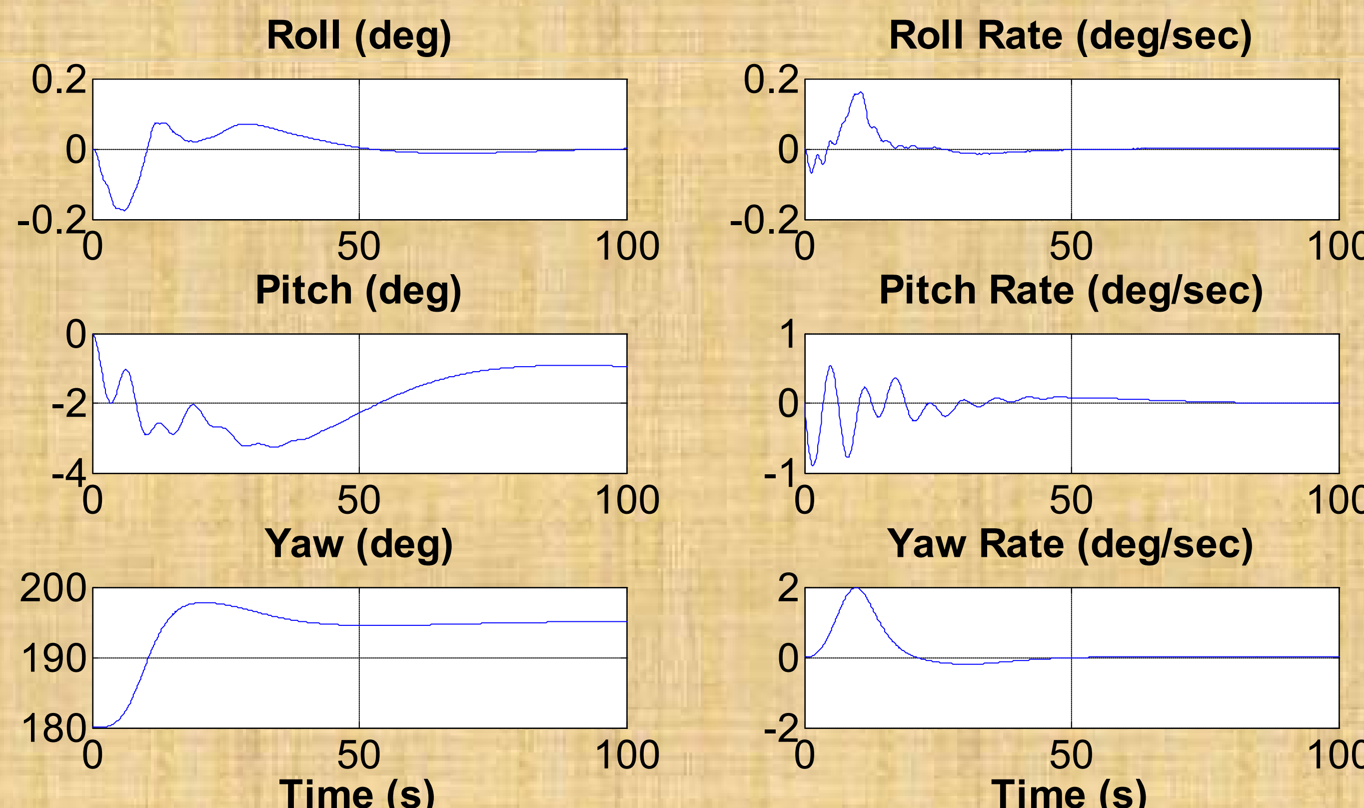
1. Zero Wind
2. Roll = Pitch = 0 deg , Yaw = 180 deg
3. X = Y = 0 m, Z = 1000 m
4. Tether is vertical











RESULTS

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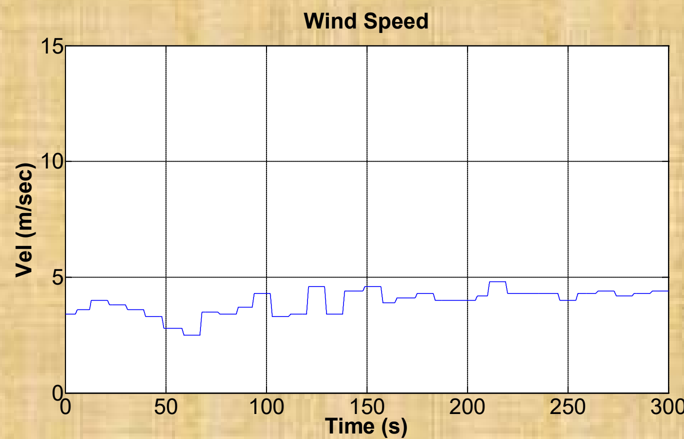
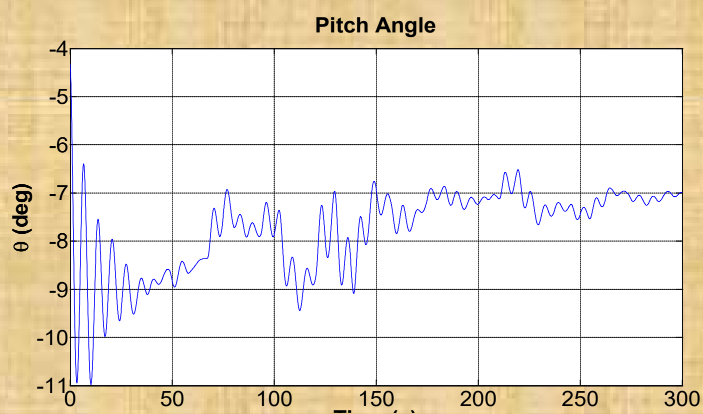
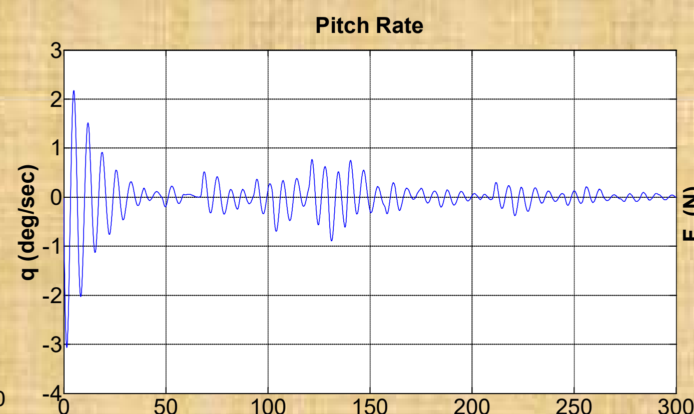
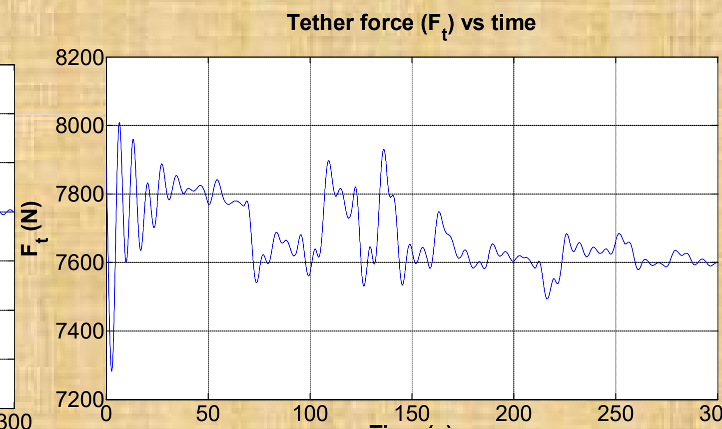
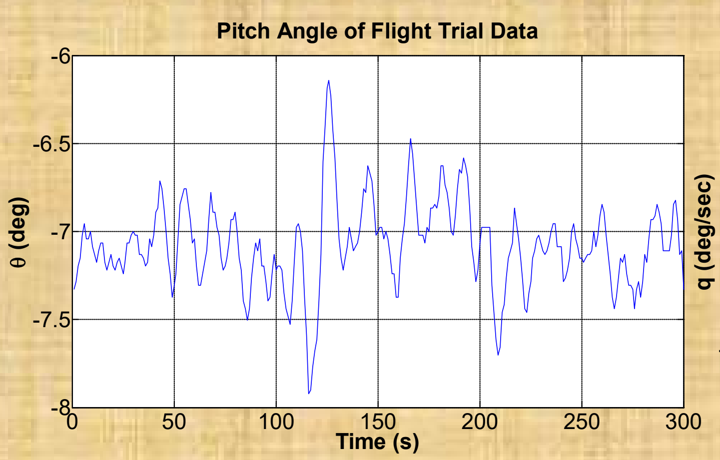
COMPARISON OF RESULTS WITH FLIGHT TRIAL DATA

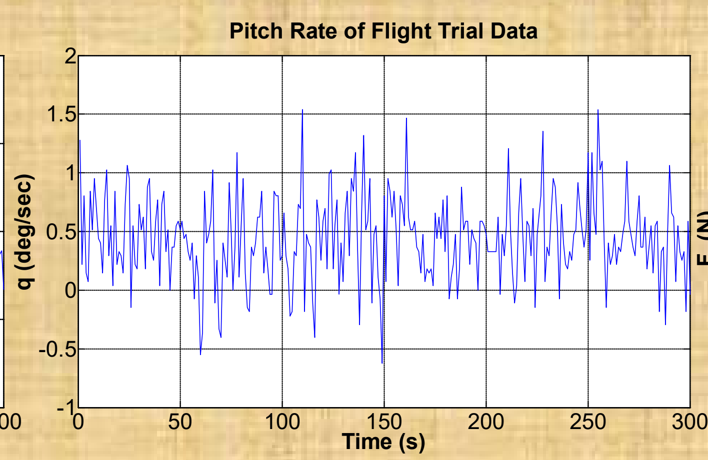


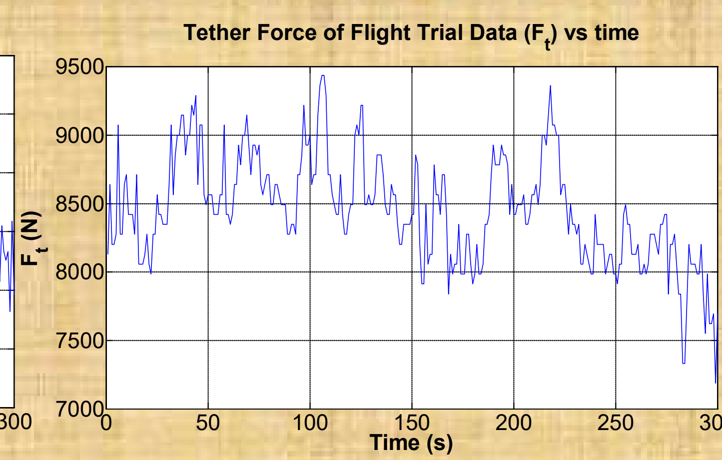




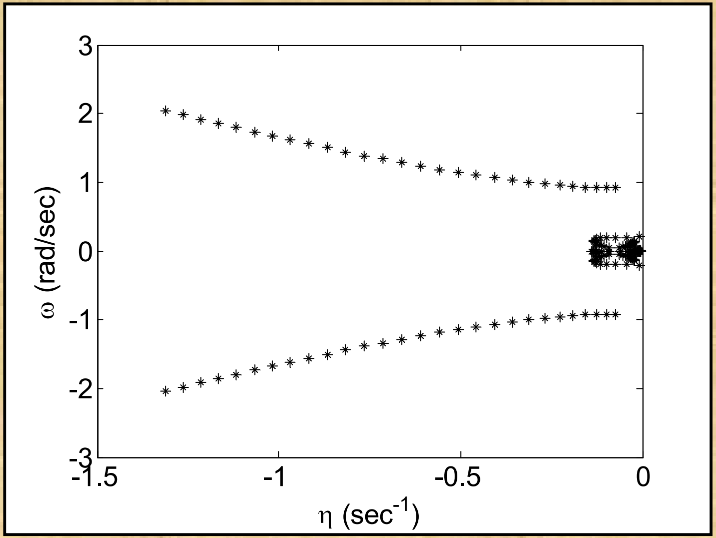


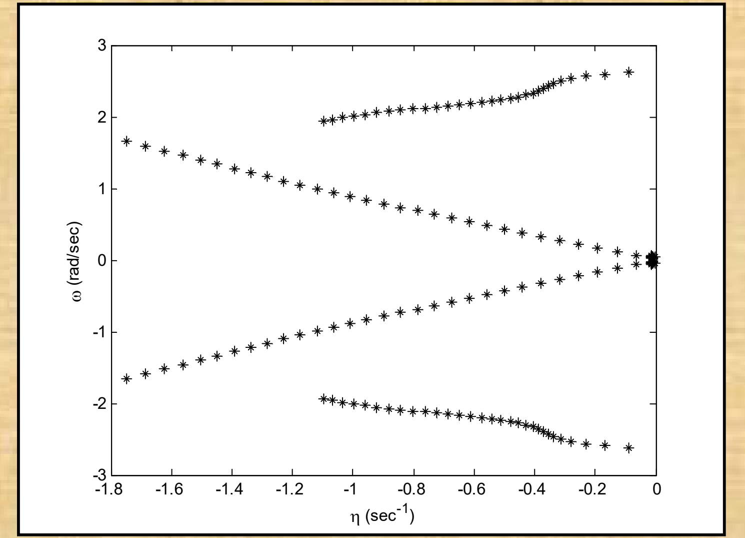






CONCLUSION





Root Locus Plot

- Results simulated against the actual wind vary within 10% of the actual flight trial data
- Dynamic Stability of the Aerostat with Tether against the wind disturbances is established

THANK YOU