B.TECH. PROJECT REPORT

GAGAN: GPS AIDED GEO AUGMENTED NAVIGATION

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INTRODUCTION

GAGAN stands for GPS Aided GEO Augmented Navigation System. It is an implementation of Satellite Based Augmentation System (SBAS) and is being developed jointly by Indian Space Research Organization (ISRO) and Airports Authority of India (AAI). GAGAN monitors the GPS satellite signals for errors and then generates correction messages to improve positioning accuracy for users. With the help of GAGAN, an aircraft will be able to make precision approach to any airport in the coverage area. There will be other uses for GAGAN as well like tracking of trains so that warning can be issued if two trains appear likely to collide.

GPS, developed by the USA, is meant for providing position and timing information for various applications. But for high safety concerns, it does not meet the requirements in terms of accuracy, integrity and availability. For this purpose, the GPS is augmented by an overlay system. U.S has put up Wide Area Augmentation System (WAAS) in its respective region. Similarly, India is working to implement SBAS. The project is known as GAGAN. There is a plan to have an operational system to provide a seamless navigation facility in the region, which is interoperable with other SBAS.

The GAGAN system will have a full complement of the SBAS inclusive of ground and onboard segment. To start with, the ground segment will consist of 8 Indian Reference Stations (INRES), one Indian Master Control Center (INMCC), and one Indian Navigation Land Uplink Station (INLUS). The onboard segment consists of a navigation payload onboard Indian geostationary satellite GSAT-4.

The GAGAN operates on the principles used by WAAS. In WAAS, there are reference stations (Wide Area Reference Stations) on the ground which receive GPS signals and send the collected information to the WAAS Master Station, where WAAS augmentation messages are generated. These messages contain information that allows GPS receivers to remove errors in the GPS signal, allowing for a significant increase in location accuracy and reliability. These augmentation messages are sent to the uplink stations which send these messages to the navigation payload on the GEO stationary satellites. The navigation payloads broadcast these augmentation messages on a GPS-like signal which is received by the GPS/WAAS receivers. The GAGAN also aims at working on the same principles as that of WAAS.

The significant errors in the GPS signals are the ionospheric propagation error, tropospheric propagation error and the multipath problem. **Ionosphere** contains gases that are ionized by the solar radiation. The ionization produces clouds of free electrons that act as a dispersive medium for GPS signals in which the propagation velocity is a function of frequency. The signal propagation speed changes as compared to that of free space. **Troposphere** is composed of dry gases and water vapor, which lengthen the propagation path due to refraction. The delay here is not frequency dependent. The **multipath problem** arises due to presence of objects in the vicinity of the antenna (generally taken to be on the ground) that can easily reflect the GPS signals and produce secondary propagation paths which are longer than the direct-path signal. These secondary-path signals superimpose on the direct-path signals and significantly distort the amplitude and phase of the direct-path signal.

Thus an algorithm needs to be developed to identify the errors in the GPS signals and mitigate them to increase the accuracy in navigation. Especially, over the Indian sky, the ionospheric effects are significant. Thus, the GAGAN involves development of an iono-model based on the observations regarding the ionospheric delays and then using this model to estimate the errors.

OBJECTIVES OF THE PROJECT

The aim of the project is to study the employment of GAGAN for aircraft navigation and develop algorithms to enable autonomous navigation. The aircraft navigation is done using the Inertial Navigation System (INS) but the errors due to the INS measurements grow with time, which need to be checked. Thus another aid of measurement is used along with the INS to result in a higher accuracy. GPS is used as another aid of measurement and is integrated with INS to result in a high accuracy in aircraft navigation. Thus the project aims at showing the improvements in the navigational accuracy in the GPS/INS integration.

SUMMARY OF THE WORK DONE IN THE PROJECT

First of all, a mathematical model of the GPS satellites has been developed. Then the GPS model is used to find out the user location. Initially, the GPS model is used to determine the position of a user which is stationary at some point on the earth's surface.

Then the flight path of an aircraft at cruise is simulated. And then the GPS model is used to determine the aircraft position throughout the flight duration.

The equations of motion of the aircraft have been studied in the navigation frame and with the knowledge of the flight path of the aircraft in the navigation frame, the specific body forces in the navigation frame are calculated.

The full state error model containing the attitude errors, the velocity errors, the position errors have been studied in the navigation frame. The values of the specific body forces computed earlier are used in the error model to determine the propagation of the errors with time.

Two examples have been simulated for the error model showing the propagation of the errors when only the INS is being used.

Lastly, the Kalman Filter algorithm has been studied to integrate the GPS and INS into one system to be used for aircraft navigation.

DEVELOPMENT OF A MATHEMATICAL MODEL OF THE GPS

1. Introduction to the GPS

The GPS has 24 satellites which move in 6 orbits, each orbit having four satellites. These satellites send signals which are received by a GPS receiver. A GPS satellite contains three different bits of information, namely the pseudorandom code, ephemeris data and almanac data. The pseudorandom code (PRN code) is used to determine which satellite signal has been received and this code is unique for every GPS satellite. Ephemeris data contains information about health of the satellite, current date and time. Almanac data is used to determine the position of the GPS satellite at the instant it transmitted the particular signal.

The GPS constellation has been designed such that at any instant of time, there are at least four satellites above the sky at any location on the earth. The signals sent by these four or more satellites are used to determine the position of the GPS receiver.

Say, for example, there are exactly four GPS satellites in the sky over a particular location. First of all, when the signals from these satellites are received, they are identified to have been sent by which of the GPS satellites using the PRN code. Then with the use of the almanac data of each of the satellites, the location of the satellites at the time of transmission of the signal is determined. The method of determining the position of the satellite at the time instant when it transmitted the signal is explained later on in the section *Determination of the satellite position at the time of transmission of the GPS signal on page* on page

The GPS signal also contains information about the time instant when the signal was transmitted from the satellite (t_t). Also, when the GPS receiver receives a GPS signal, it notes down the value of time instant at which the signal was received (t_r). Then the distance between the receiver at t_r and the satellite position at t_t can be determined from the formula,

$$d = c(t_r - t_t)$$

Thus, the distance of the receiver from all the 4 satellites can be determined as d_1, d_2, d_3, d_4. Using this information of d1, d2, d3 and d4, and the location of the satellites at the time instant of the transmission of the corresponding signal, the GPS receiver location can be determined using the formulation explained later on in the section *Determination of the receiver's position* on page

2. Mathematical Modeling of the GPS without considering any perturbations

The GPS constellation with epoch on July 1, 1993 is given by the following table.

CONSTELLATION POSITION	A (KM)	e	i (DEG)	ω (DEG)	M (DEG)	Ω (DEG)
A1	26,559.8	0	55	0	-91.874	-87.153
A2	26,559.8	0	55	0	161.786	-87.153
A3	26,559.8	0	55	0	11.676	-87.153
A4	26,559.8	0	55	0	41.806	-87.153
B1	26,559.8	0	55	0	80.956	-27.153
B2	26,559.8	0	55	0	173.336	-27.153
B3	26,559.8	0	55	0	-155.624	-27.153
B4	26,559.8	0	55	0	-50.024	-27.153
C1	26,559.8	0	55	0	111.876	32.847
C2	26,559.8	0	55	0	11.796	32.847
C3	26,559.8	0	55	0	-20.334	32.847
C4	26,559.8	0	55	0	-118.444	32.847
D1	26,559.8	0	55	0	135.226	92.847
D2	26,559.8	0	55	0	-94.554	92.847
D3	26,559.8	0	55	0	35.156	92.847
D4	26,559.8	0	55	0	167.356	92.847
E1	26,559.8	0	55	0	-162.954	152.847
E2	26,559.8	0	55	0	-57.404	152.847
E3	26,559.8	0	55	0	66.066	152.847
E4	26,559.8	0	55	0	-26.314	152.847
F1	26,559.8	0	55	0	-121.114	-147.153
F2	26,559.8	0	55	0	-14.774	-147.153
F3	26,559.8	0	55	0	105.206	-147.153
F4	26,559.8	0	55	0	135.346	-147.153

A mathematical model of the GPS has been developed without considering any perturbations in the orbit parameters.

Formulae used in the modeling of the GPS

Eccentricity e = 0,

Semi-major axis a = 26559.8 kms,

Inclination angle i = 55˚,

$M = M$ (at epoch time t_0) + $n \times (t - t_0)$,

where n = mean motion = $\sqrt{\dfrac{\mu}{a^3}}$, $\mu = G \times M_{earth} = 398600.4415$ km^3/sec^2

Ascension angle $\Omega = \Omega(at \;\; epoch)$

The radial distance from the centre of the earth to the satellite = r = a (for e = 0)

Argument of perigee = ω = 0 (since the orbit is circular for e = 0).

Thus, u = = $\upsilon + \omega = \upsilon$, where $\upsilon = M$ (for a circular orbit).

Thus, the position of the satellite at time t sec after the epoch can be computed using the following equations:

$$X = r(\cos u \cos\Omega - \sin u \cos i \sin\Omega)$$

$$Y = r(\cos u \sin\Omega + \sin u \cos i \cos\Omega)$$

$$Z = r \sin u \sin i$$

where X, Y and Z are locations in ECI frame.

In order to make a mathematical model of the entire GPS constellation, three matrices of 24 rows have been created with each row containing the position location X, Y and Z of the satellite at regular time intervals of 1 sec.

Then the values of latitude and longitude of the satellite are computed in the Greenwich Meridian frame using the following formulation:

$$\sin\phi = \frac{Z}{r} \quad \text{and} \quad \lambda = -\Theta + \tan^{-1}(\frac{Y}{X})$$

where ϕ and λ are the latitude and longitude of the satellite in the Greenwich Meridian frame.

Θ is the right ascension angle of the Greenwich Meridian at time t.

$$\Theta = \Theta(at \quad epoch) + \frac{360.9856 \times pi \times t}{86400 \times 180}, \quad \Theta(at \quad epoch) = -260.83^{\circ}$$

3. Determination of the satellite position at the time of transmission of the GPS signal

When a GPS receiver receives a GPS signal, it determines which of the satellites has sent that particular signal using the PRN code explained earlier and it then uses the data contained in the almanac data to determine first of all the position of the satellite at the time instant, the signal was transmitted.

The almanac data contains information about the time instant at which the signal was transmitted.

Almanac data contains information as illustrated in the following table.

Parameter	Description
WN_{oa}	Almanac reference epoch (part 1) : GPS week number
t_{oa}	Almanac reference epoch (part 2) : Fraction of current GPS week
$\sqrt{a}$	Square root of semi-major axis
E	Eccentricity
δ_i	Inclination offset from reference value of i_{ref}
Ω_0	Longitude of the ascending node at the weekly epoch
$\dot{\Omega}$	Rate of change of the right ascension of the ascending node
ω	Argument of perigee
M_0	Mean Anomaly at reference epoch
a_0	Spacecraft clock offset from GPS time
a_1	Clock frequency offset

The above table takes into account the orbit perturbation parameters as well.

i_{ref} is specified as 55˚.

Ω_0 defines the orientation of the orbital plane at the almanac reference epoch, but referred to the Greenwich meridian at the start of the respective GPS week.

$\Omega_0 = \Omega(t_a) - \Theta(t_0)$, where t_0 denotes the start of the GPS week and t_a is the almanac reference epoch (as denoted by WN_{0a} and t_{0a} counts).

Due to the earth's oblateness, the inertial right ascension of the ascending node experiences a secular change of $\dot{\Omega}$=-0.04˚/day.

Thus the instantaneous Greenwich longitude λ_Ω of the ascending node is computed using

$$\lambda_\Omega(t) = \Omega(t) - \Theta(t)$$
$$\approx \Omega_0 + \dot{\Omega}(t - t_a) - \omega_{earth}(t - t_0)$$

for arbitrary times t.

The Kepler's equation is solved to get the value of eccentric anomaly from the mean anomaly.

$E - e\sin(E) = M = M_0 + \sqrt{\frac{GM_{earth}}{a^3}}(t - t_a)$ using the Newton Raphson method.

Then the X, Y and Z location of the satellite is computed from the following equations:

$$X = r(\cos u \cos\Omega - \sin u \cos i \sin\Omega)$$

$$Y = r(\cos u \sin\Omega + \sin u \cos i \cos\Omega)$$

$$Z = r\sin u \sin i,$$

where,

$$\tan\frac{\upsilon}{2} = \left(\frac{1+e}{1-e}\right)^{\frac{1}{2}} \tan\frac{E}{2} \qquad n = \sqrt{\frac{\mu}{a^3}} \qquad p = a(1-e^2) \qquad h = \sqrt{p\mu}$$

$$r = \frac{p}{1 + e\cos\upsilon} \qquad u = \upsilon + \omega$$

υ is the anomaly, n is the mean motion of the satellite, p is the semi latus-rectum of the orbit, h is the specific angular momentum (angular momentum per unit mass), r is the radial distance from the earth's center to the satellite, ω is the argument of perigee.

This is how the GPS receiver calculates the position of the satellite corresponding to the signal it receives. Once, the time instant of the transmission of the signal is known, the position of the satellite can be determined by using the analysis explained in the mathematical modeling of the GPS without considering the orbit perturbations.

4. Determination of the particular satellites whose signals would be received by the GPS receiver

In normal circumstances, when the GPS satellites are functional in their orbits, they send out signals which may be received by some GPS receiver situated somewhere. It so happens in the practical conditions, but when a mathematical model has to be made which in itself would be able to determine which particular satellite's data would be received by the receiver, an assumption is made that the receiver location is known and then it is determined which of the 24 satellites would be actually visible to the receiver, i.e. which of the 24 satellites would be lying above the horizon of the receiver.

As an example, the GPS receiver is situated at some arbitrary location on the earth's surface, say $\lambda = 74.567^\circ E, \phi = 34.8083^\circ$.

The determination of the position of the GPS receiver is done at time intervals of 6 sec. Thus at every time step of 6 seconds, it is computed which of the 24 satellites lie above the horizon at that particular time instant. This can be achieved by the following formulation.

First of all , the co-ordinate system is transferred from the ECI frame to the ENZ (East, North and Zenith) frame.

$$E = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix},$$

λ and ϕ are the longitude and the latitude of the particular GPS satellite in consideration.

and $s = \begin{pmatrix} s_E \\ s_N \\ s_Z \end{pmatrix} = E(\overrightarrow{r_{ef}} - \overrightarrow{R_{ef}})$ is calculated, where $\overrightarrow{r_{ef}}$ and $\overrightarrow{R_{ef}}$ are the positions of the receiver and the satellite in the ECI frame.

s denotes the vector projections of the vector from the receiver to the satellite in ENZ frame situated at the receiver.

Then the elevation angle e1 is calculated as $e1 = \tan^{-1}\left(\frac{s_Z}{\sqrt{s_E^2 + s_N^2}}\right)$

If the elevation angle is > 5 degrees, then the data of the satellite into account, i.e. if the elevation angle that a satellite makes with a GPS receiver is < 5 degrees, the signal from that satellite has not been taken into account for determining the receiver's location.

The above procedure is repeated for all the 24 satellites and it is found out which of the satellites lie at an elevation angle > 5 degrees and once 4 such satellites are obtained, no more satellite is looked for which too would have been at an elevation angle > 5 degrees. The information of these satellites are used in the further analysis as explained later on. So far in the analysis for the determination of the user position, the satellite geometry between the selected satellites has not been taken into account. It is so because the satellite geometry becomes important when the errors in the GPS model are being studied. The selection of the visible satellites which have best geometry between them is represented by the term known as GDOP (Geometric Dilution of Precision). The minimum the GDOP, the higher is the accuracy in the user position calculations. The GDOP has been discussed at the end of the report in the section *"Integration of GPS/INS using Kalman Filter technique"*.

5. Determination of the instant of time at which the GPS signal was transmitted

The almanac data contains the information about the time instant at which the signal was transmitted. But when a mathematical model is to be made and the time instant of transmission of a GPS signal which would be received by a GPS receiver is simulated, it is assumed that the receiver location is known (as had been made in the earlier simulation in the model to determine which of the satellites' would be actually visible over the horizon).

Using this assumption, it has already been shown earlier how to determine which of the satellites' data needs to be taken into account in the determination of the receiver's location. Now, in order to find out the actual propagation time between the transmission of the signal and the receiving of the signal, an iterative process is used as explained below. In the analysis below, it has been taken into account that the propagation time is computed corresponding to that satellite only which lies at elevation angle more than 5 degrees.

The GPS receiver is assumed to be located at Srinagar on the earth's surface, say $\lambda = 74.567^\circ E, \phi = 34.083^\circ$.

First of all it is assumed that the time taken for the propagation of the signal from the GPS satellite to the receiver, $\tau_1 = 0$.

With the propagation time = $\Delta t = \tau$, the actual propagation time is computed had the signal been transmitted at time instant $t - \tau$ and received at instant t.

The new propagation time is computed as

$$\tau_{i+1} = \frac{\left|\overrightarrow{r(t)} - \overrightarrow{R(t-\tau_i)}\right|}{c} - \tau_i$$

where c = speed of light, $\overrightarrow{r(t)}$ and $\overrightarrow{R(t-\tau)}$ are the position locations of the receiver at time instant *t* and the satellite at time instant $t - \tau$ respectively.

The above iterative process is done so long as the propagation time τ converges.

Once τ is determined, the location of the particular satellite is computed at the instant $t - \tau$ using the analysis for the mathematical modeling of the GPS explained earlier.

Thus, the model for the instant of signal transmission time has also been developed.

6. Determination of the receiver's position

Once the information about the position and the propagation time of the signals of the 4 satellites which are to be used in the computation of the receiver's position is known, the following analysis is carried out to determine the receiver's position.

The radial distance from the earth's center to the satellite's position is computed using the following formula:

$\rho_i = c\tau_i$, where τ_i is the propagation time of the signal corresponding to the i_{th} satellite whose signal has been taken into account.

Also, $\rho_i = \sqrt{(x_i - X)^2 + (y_i - Y)^2 + (z_i - Z)^2}$, where $x_i, y_i z_i$ are the positions of the i^{th} satellite at the instant of the transmission of the corresponding GPS signal and X, Y and Z denotes the receiver's location.

When the above expression is squared on both sides, the following relationship is obtained

$$\begin{aligned}\rho_i^2 &= (x_i - X)^2 + (y_i - Y)^2 + (z_i - Z)^2 \\ &= x_i^2 + y_i^2 + z_i^2 + X^2 + Y^2 + Z^2 - 2x_i X - 2y_i Y - 2z_i Z\end{aligned}$$

$r^2 = x_i^2 + y_i^2 + z_i^2$ and $R_{earth}{}^2 = X^2 + Y^2 + Z^2 + C_{rr}$, Crr denotes the correction factor.

Thus, finally we get for all the four satellites, the following equation

$$\begin{bmatrix} \rho_1^2 - r^2 - R_{earth}^2 \\ \rho_2^2 - r^2 - R_{earth}^2 \\ \rho_3^2 - r^2 - R_{earth}^2 \\ \rho_4^2 - r^2 - R_{earth}^2 \end{bmatrix} = \begin{bmatrix} -2x_1 & -2y_1 & -2z_1 & 1 \\ -2x_2 & -2y_2 & -2z_2 & 1 \\ -2x_3 & -2y_3 & -2z_3 & 1 \\ -2x_4 & -2y_4 & -2z_4 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ C_{rr} \end{bmatrix}$$

i.e. A=BX.

Thus X = B^{-1}A is computed and this is how the receiver's position can be determined finally using the data from the GPS signals.

An example has been simulated to verify the above analysis. The receiver is assumed to be located at Srinagar and the receiver is stationary. The position is determined for a duration of 6 hours. The following plots show the location of the receiver with and without using the GPS model and the respective errors in the calculation have been shown.

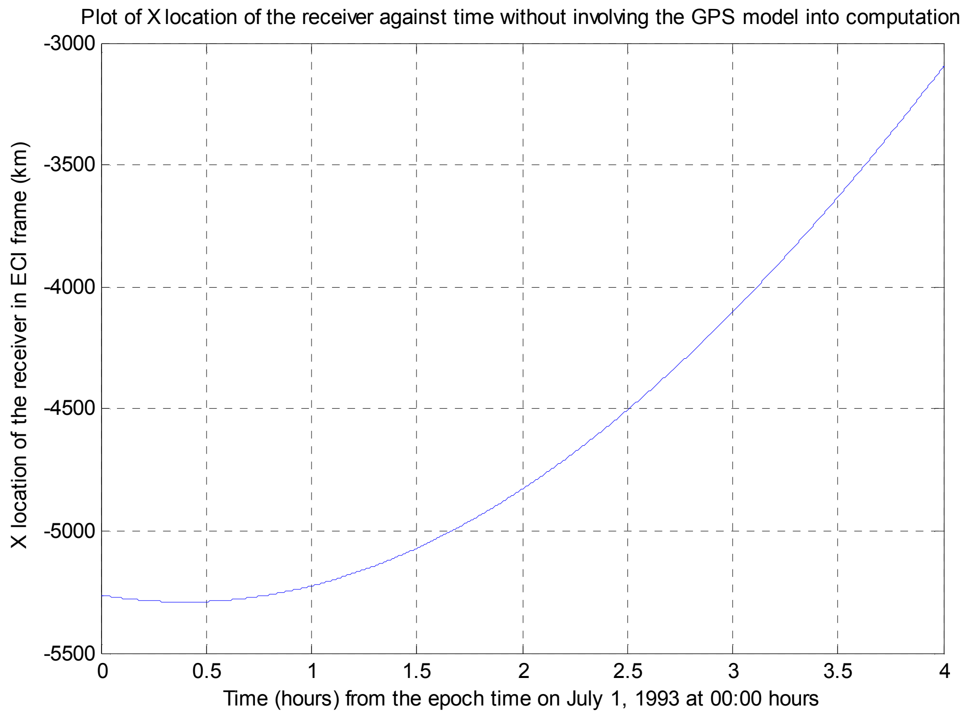

Figure 1: Plot of X location of the receiver in ECI frame without using the GPS model

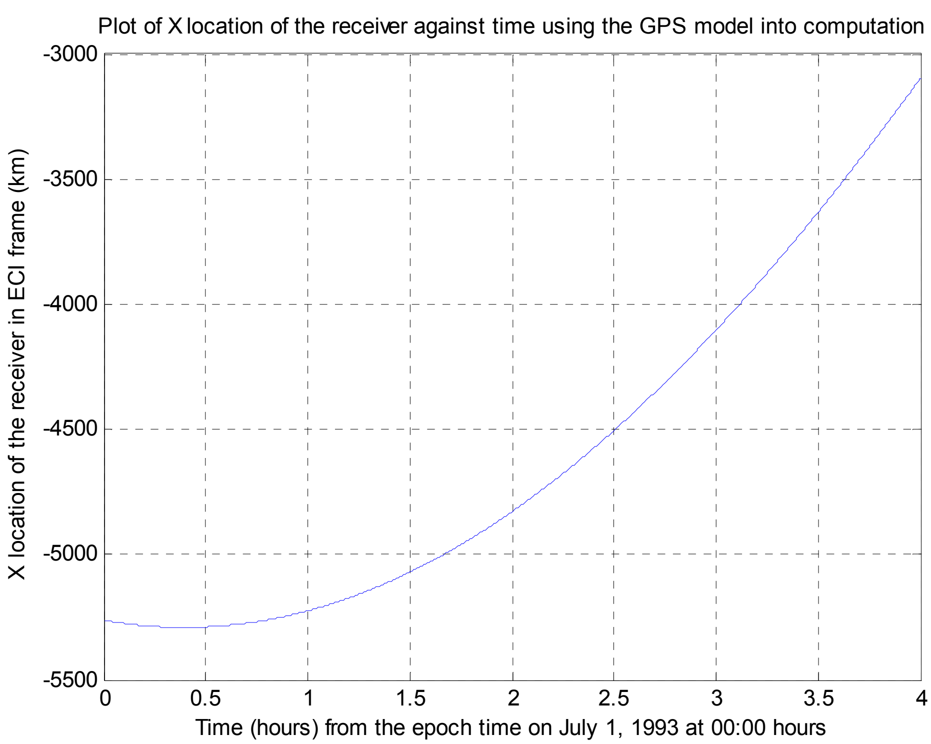

Figure 2: Plot of X location of the receiver in ECI frame using the GPS model

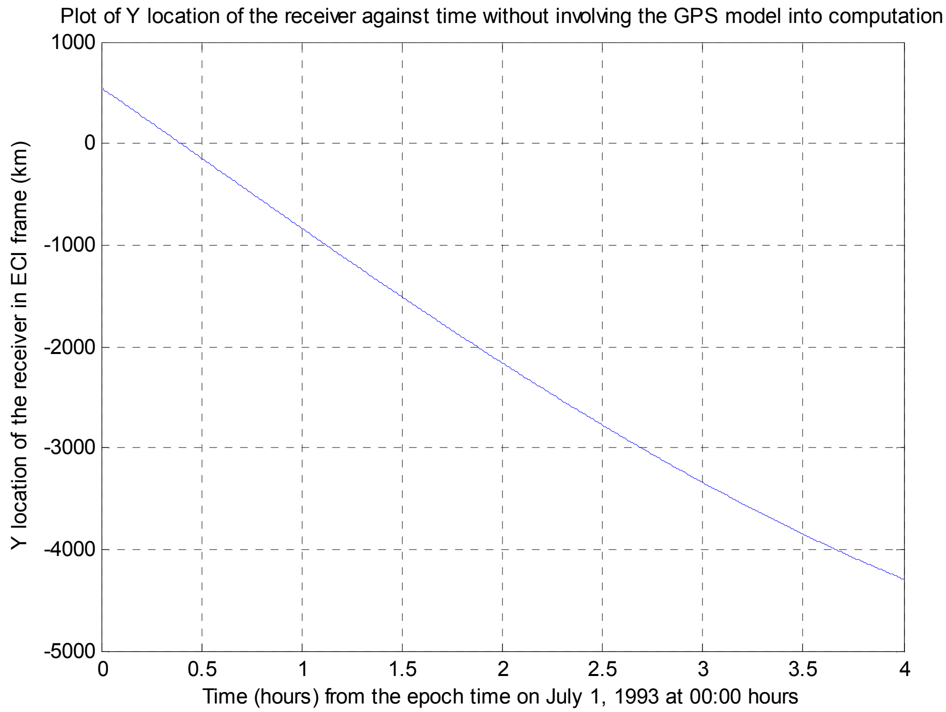

Figure 3: Plot of Y location of the receiver in ECI frame without using the GPS model

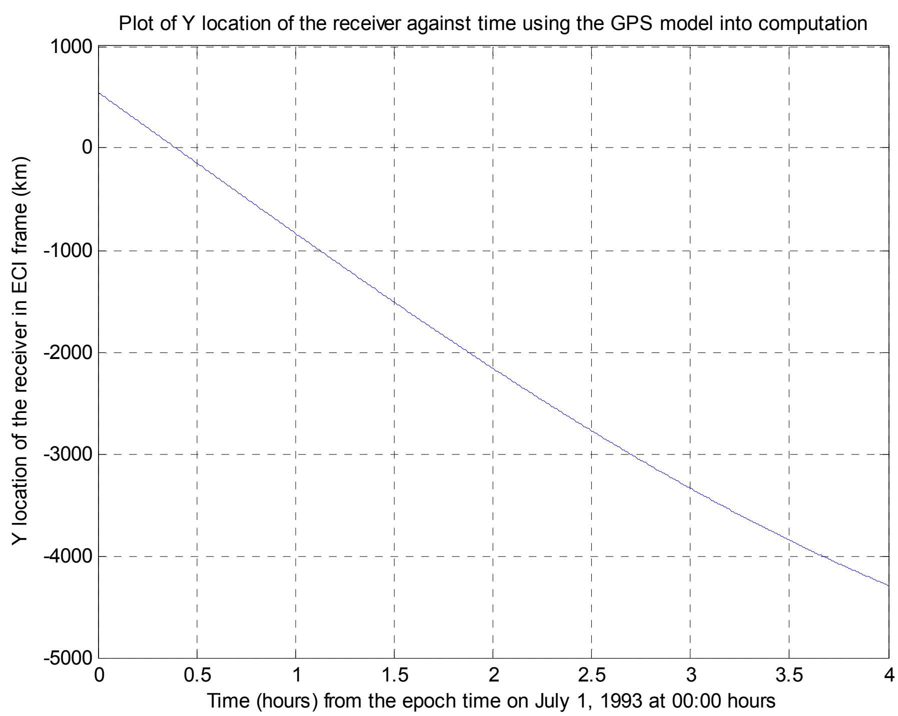

Figure 4: Plot of Y location of the receiver in ECI frame using the GPS model

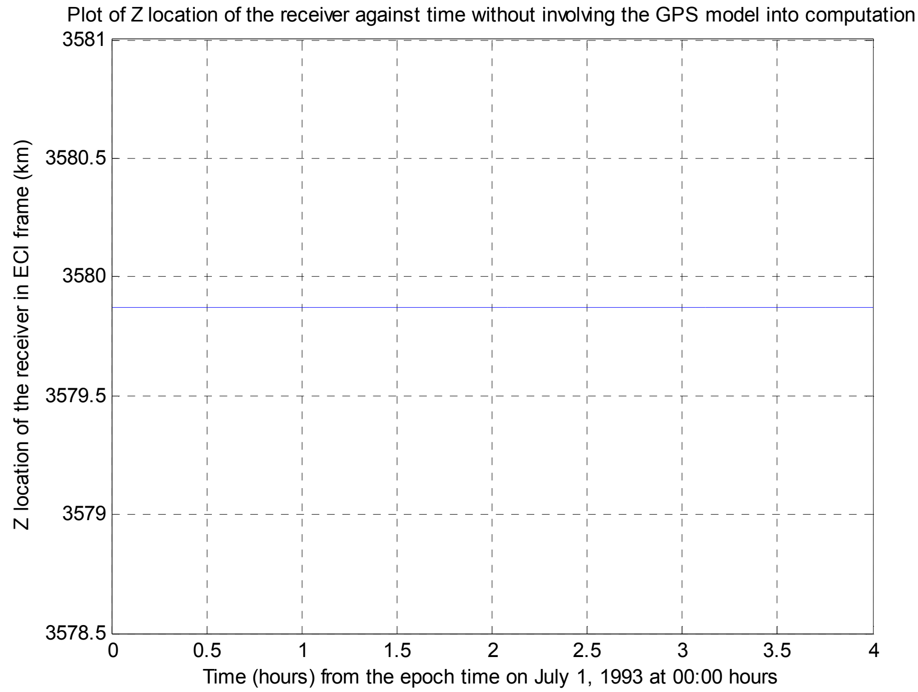

Figure 5: Plot of Z location of the receiver in ECI frame without using the GPS model

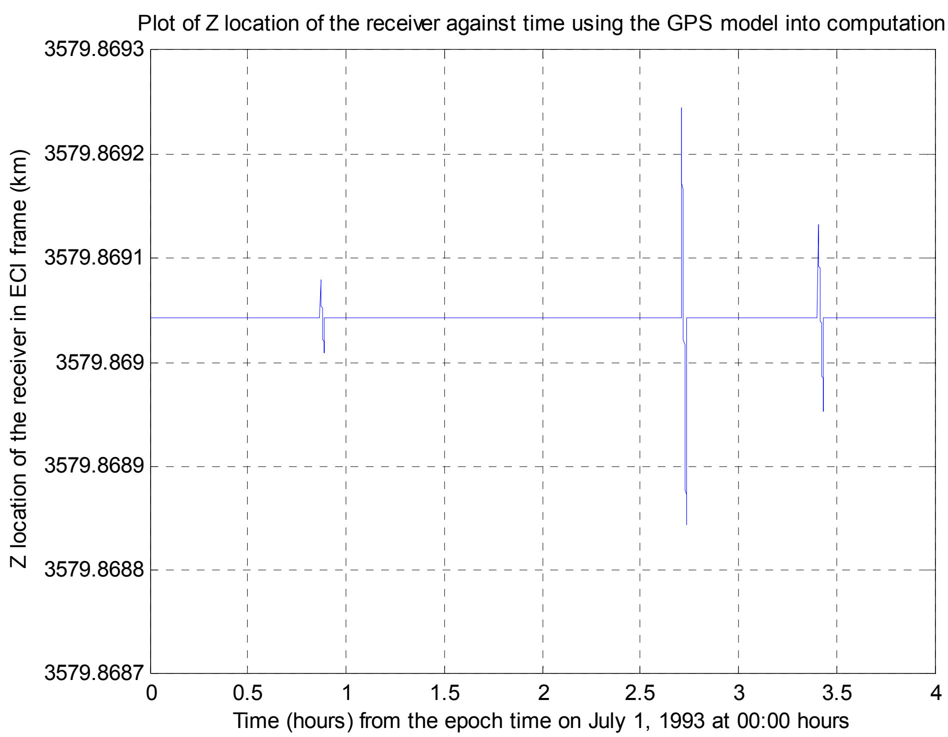

Figure 6: Plot of Z location of the receiver in ECI frame using the GPS model

From the above plots, it can be inferred that the modeling of the GPS satellites and its use to determine the location of a GPS receiver has been very accurate.

There are certain instants in the plots where kinks are observed. When analysis of the above plots is done by finding out the number of visible satellites, it is observed that the kinks usually occur when the number of visible satellites changes. This can be explained by the following plot of the number of visible satellites.

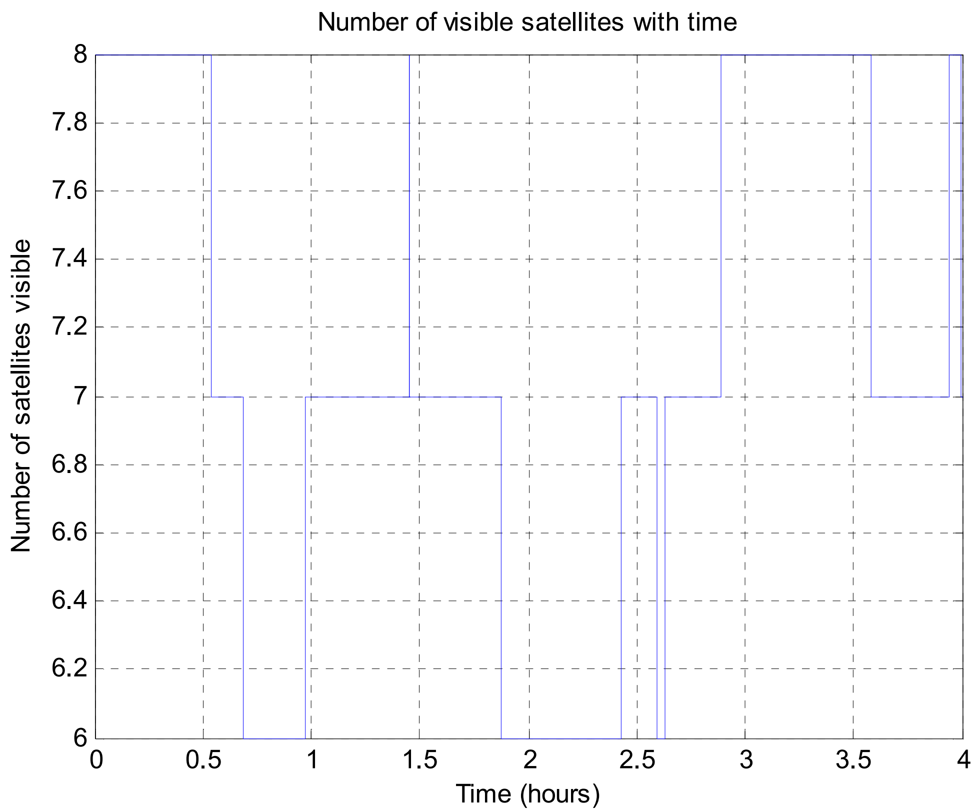

Figure 7: Plot of Number of GPS satellites visible with time

FLIGHT PATH OF AN AIRCRAFT USING THE BEARING ANGLE METHOD

When two points on the earth's surface are to be joined by the shortest distance between them, the path forms an arc of the Great circle that connects the two points. The bearing angle is the angle that the path direction makes with the north direction. The bearing angle is specified by the figure.

The bearing angle B and the distance angle d at some point S with the destination point being A are given by the following formulae:

$$\cos d = \sin \lambda_S \sin \lambda_A + \cos \lambda_S \cos \lambda_A \cos(\mu_S - \mu_A)$$

$$\sin B = \frac{\cos \lambda_A \sin|\mu_S - \mu_A|}{\sin d}$$

λ_S: latitude of S; μ_S: longitude of S;

λ_A: latitude of A; μ_A: longitude of A

Also, the velocity components in the north and east directions are given by the following formulae:

$$V_N = V\cos B$$
$$V_E = V\sin B$$

V: speed of the flight

For the above two definitions of V_N and V_E, the flight is assumed to be cruise flight so that the velocity of the flight lies in the horizontal plane.

The north and east velocities are related to the rates of changes of the latitude and longitude as follows:

$$\dot{\lambda}_S = \frac{V_N}{R_e + h}$$
$$\dot{\mu}_S = \frac{V_E}{(R_e + h)\cos L}$$

h is the altitude of flight

Thus the above equation for rates of changes of latitude and longitude can be integrated to get the latitude and longitude at the next time instant and then again the bearing angle B and the distance angle d can be calculated and the entire process is repeated. Thus the flight path can be obtained using the bearing angle method.

The bearing angle method has been used to plot the cruise flight path of an aircraft at 10 km altitude and 0.7 Mach speed between Srinagar and Kanyakumari. The co-ordinates of Srinagar and Kanyakumari are

Srinagar: 34.083˚ N, 74.8167˚ E

Kanyakumari: 8.083˚ N, 77.567˚ E

The following plots have been plotted:

1. Variation of the latitude and longitude of the flight path of the aircraft with time.
2. The flight path of the aircraft, projected on the map of India (drawn approximately).
3. The errors in the measurement of the latitude, longitude and altitude using the GPS model.
4. The number of satellites visible and the availability of the satellites with time.
5. The variation of the bearing and distance angles
6. The rates of changes of the bearing and distance angles
7. The north and east velocities.

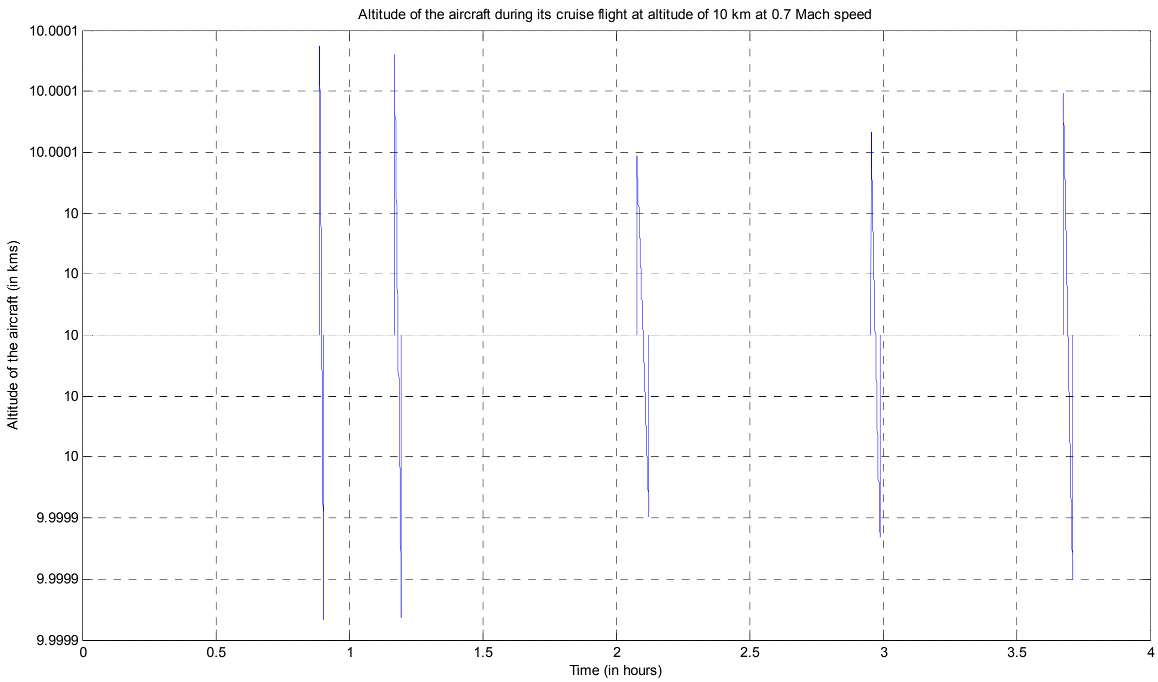

Figure 9: Plot of altitude measurement using the GPS model

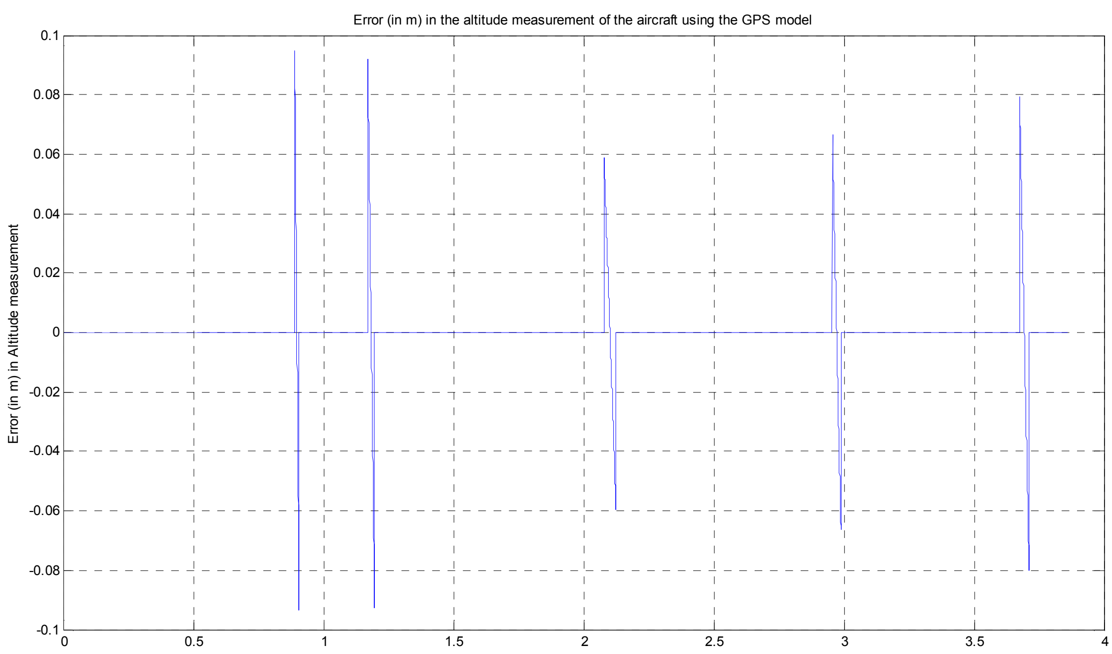

Figure 10: Plot of errors in the altitude measurement using the GPS model

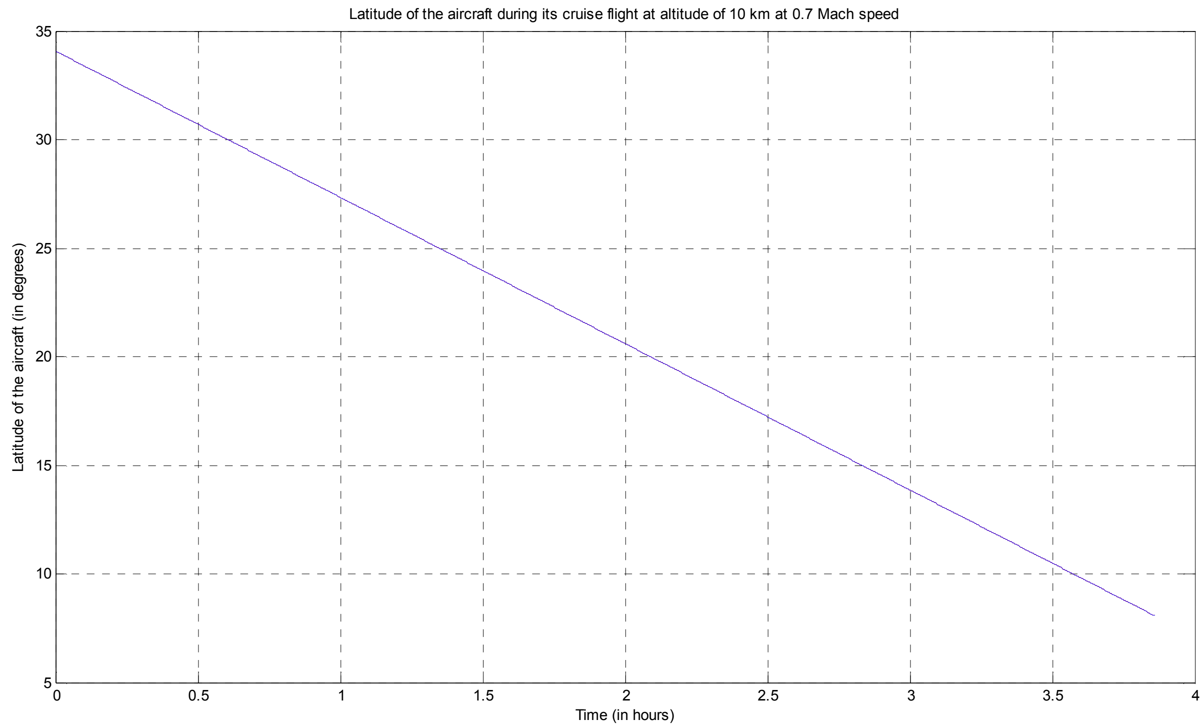

Figure 11: Plot of latitude against time using the GPS model

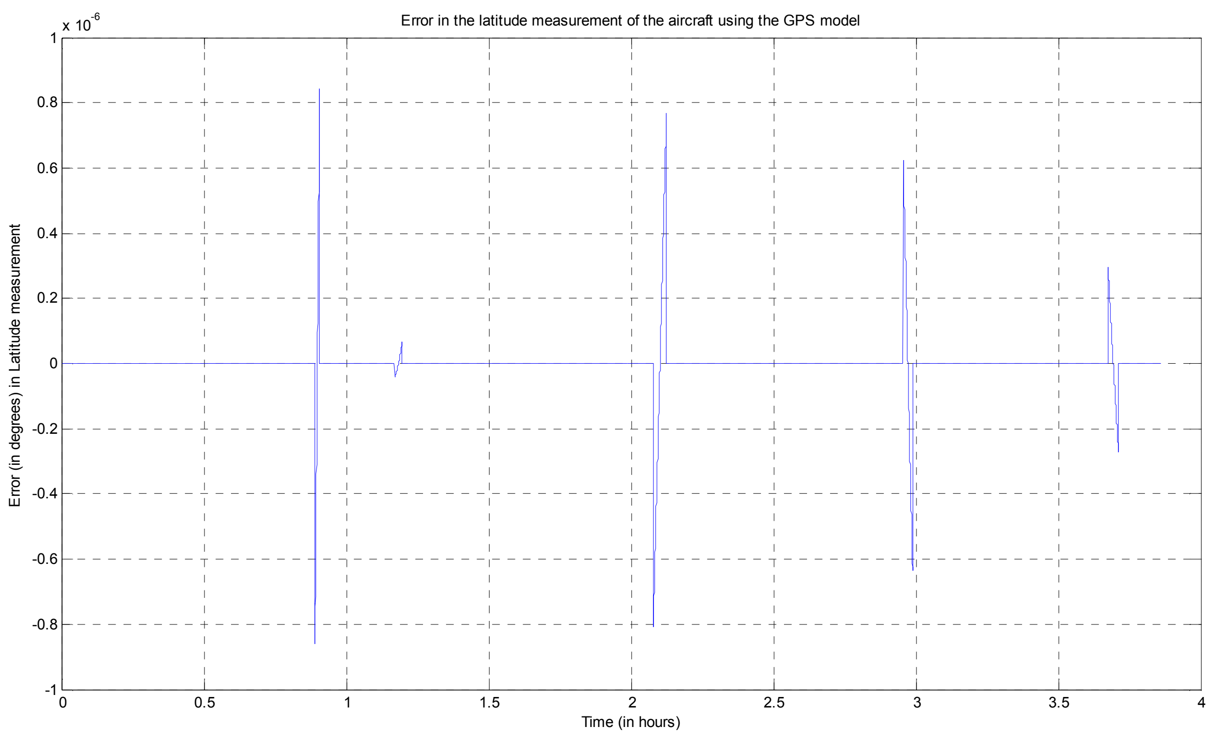

Figure 12: Plot of errors in the latitude measurement using the GPS model

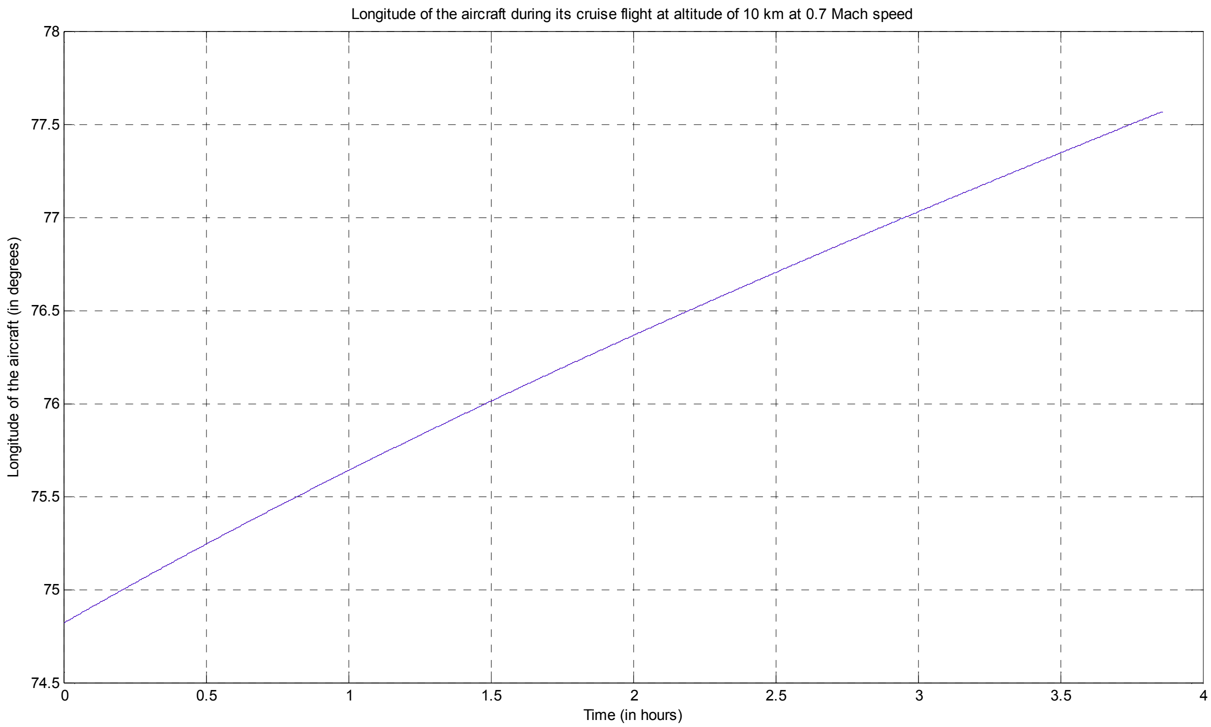

Figure 13: Plot of longitude measurement using the GPS model

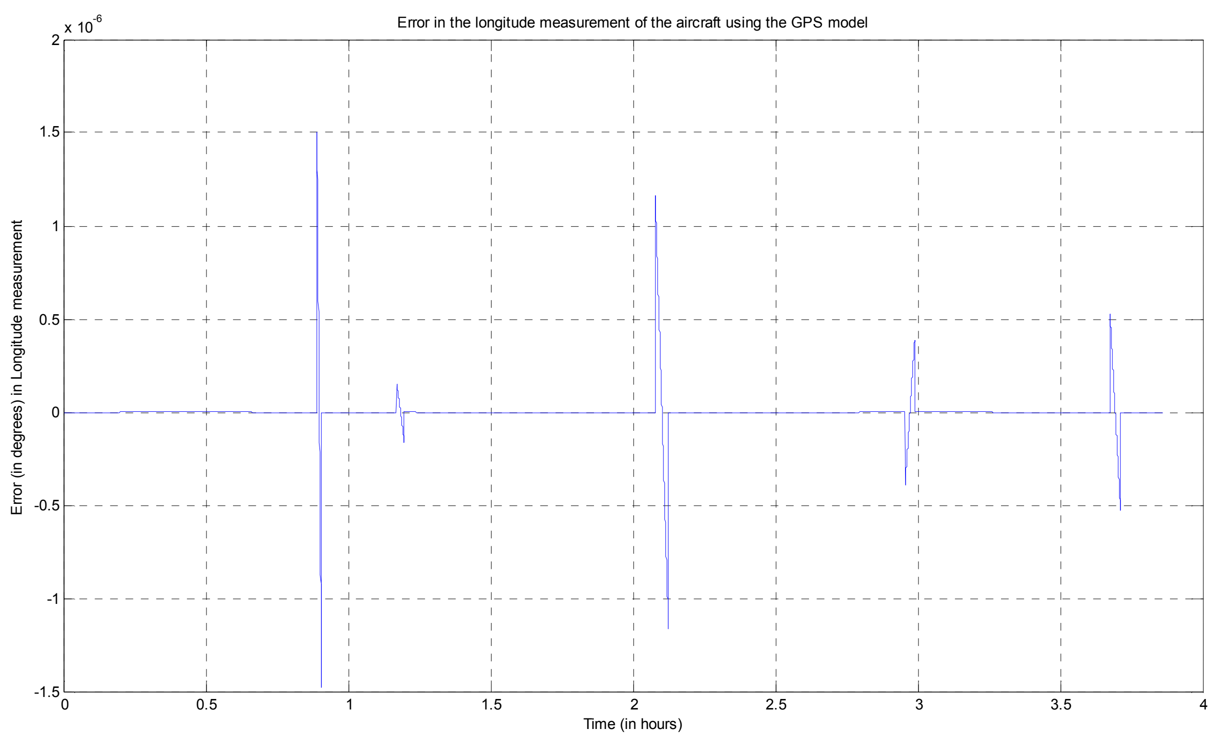

Figure 14: Plot of errors in the longitude measurement using the GPS model

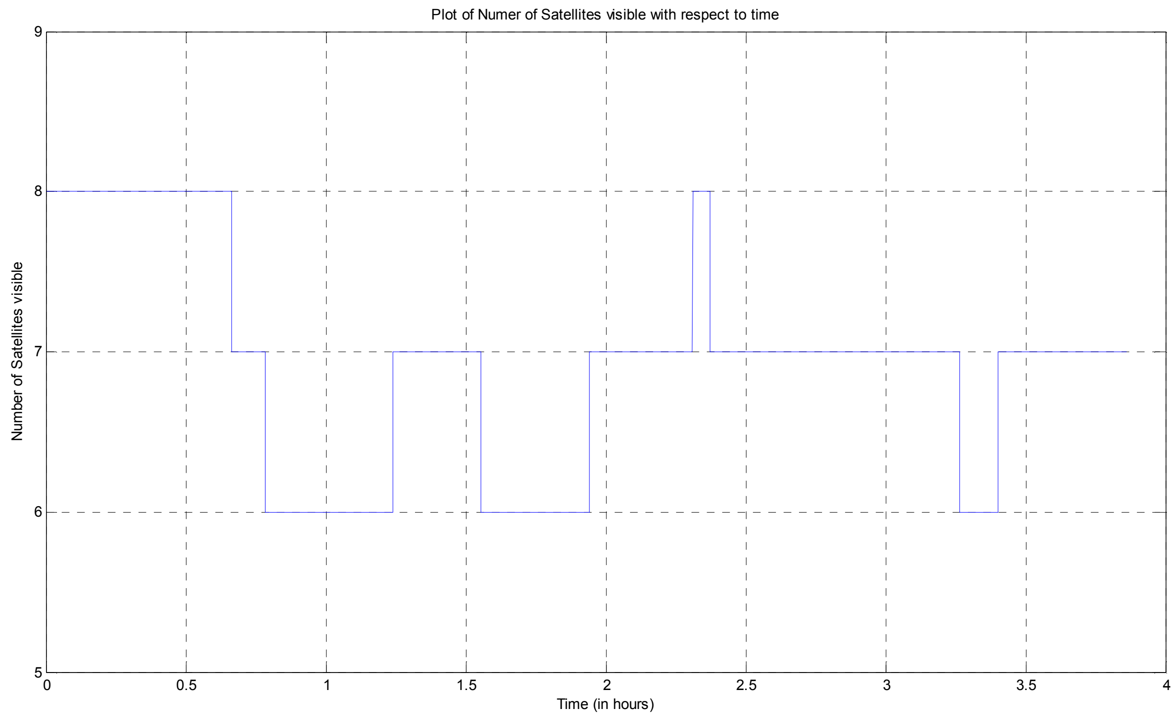

Figure 15: Plot of Number of GPS satellites visible with time

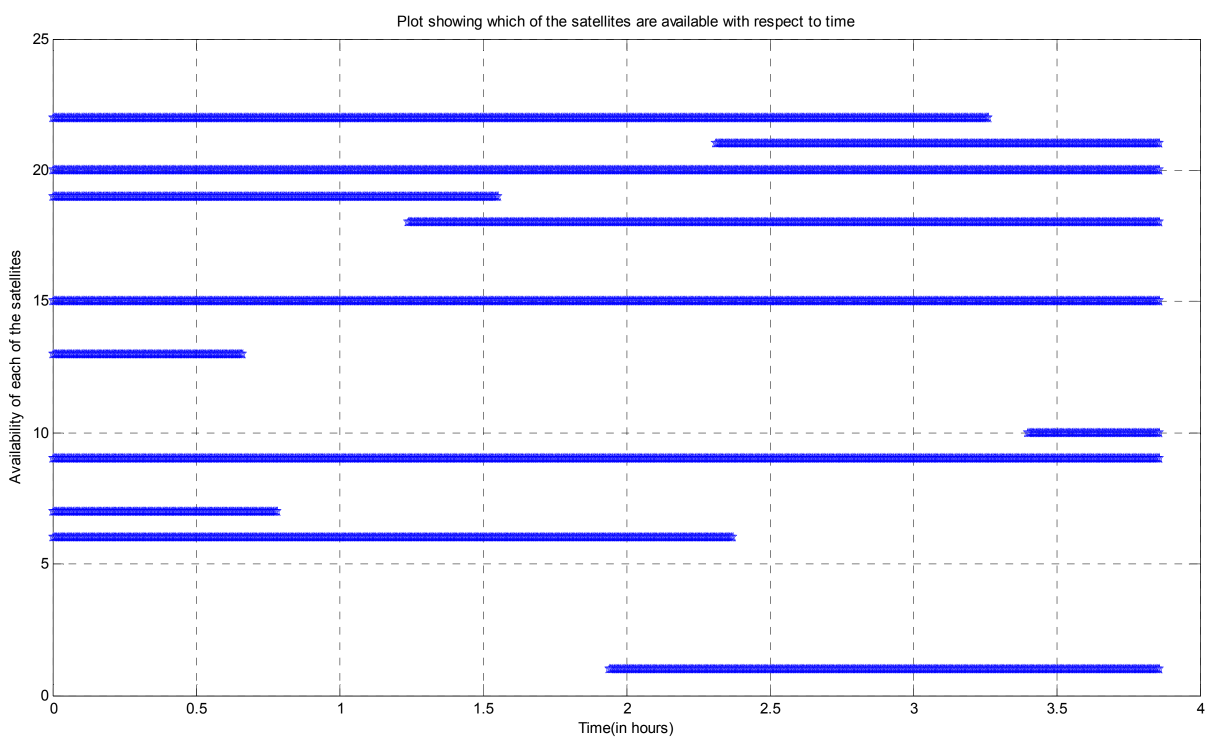

Figure 16: Plot showing which of the GPS satellites are visible during the flight

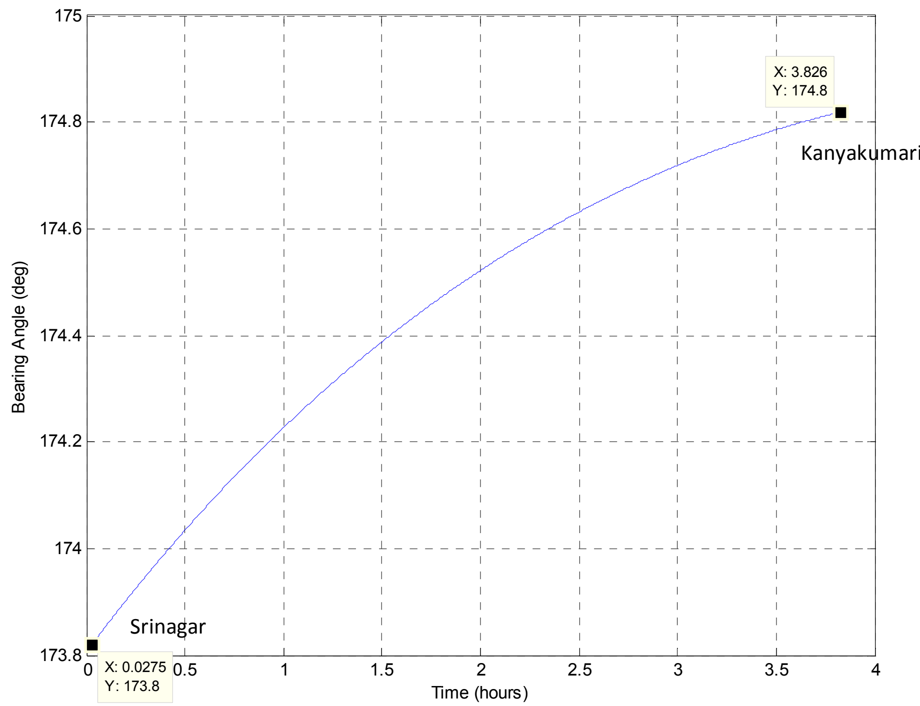

Figure 17: Plot of bearing angle variation with time

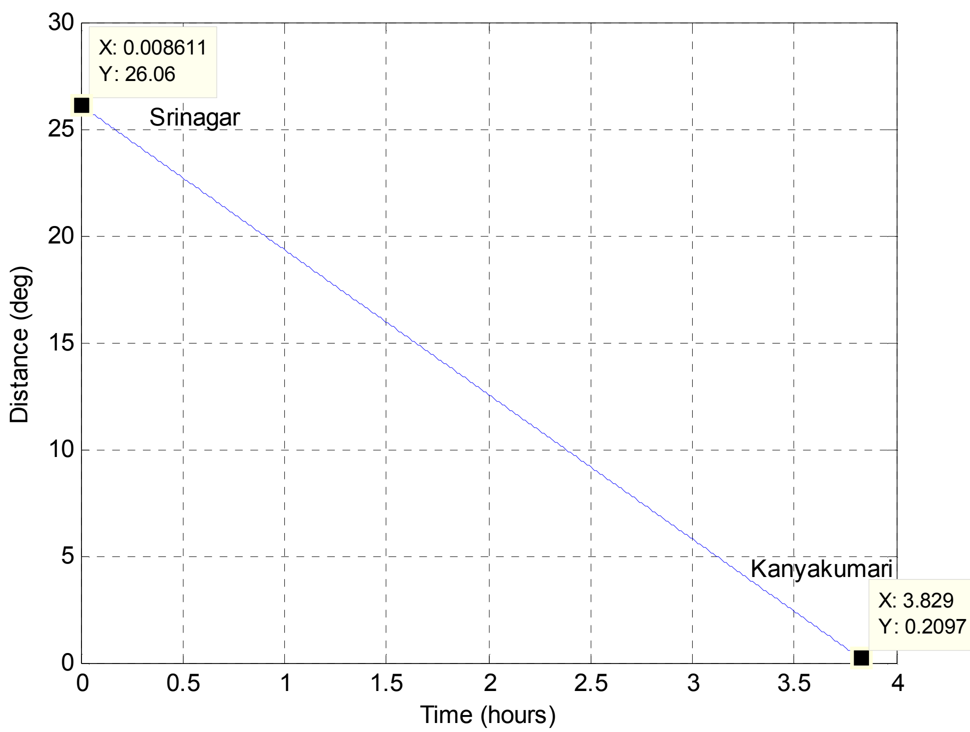

Figure 18: Plot of distance angle variation with time

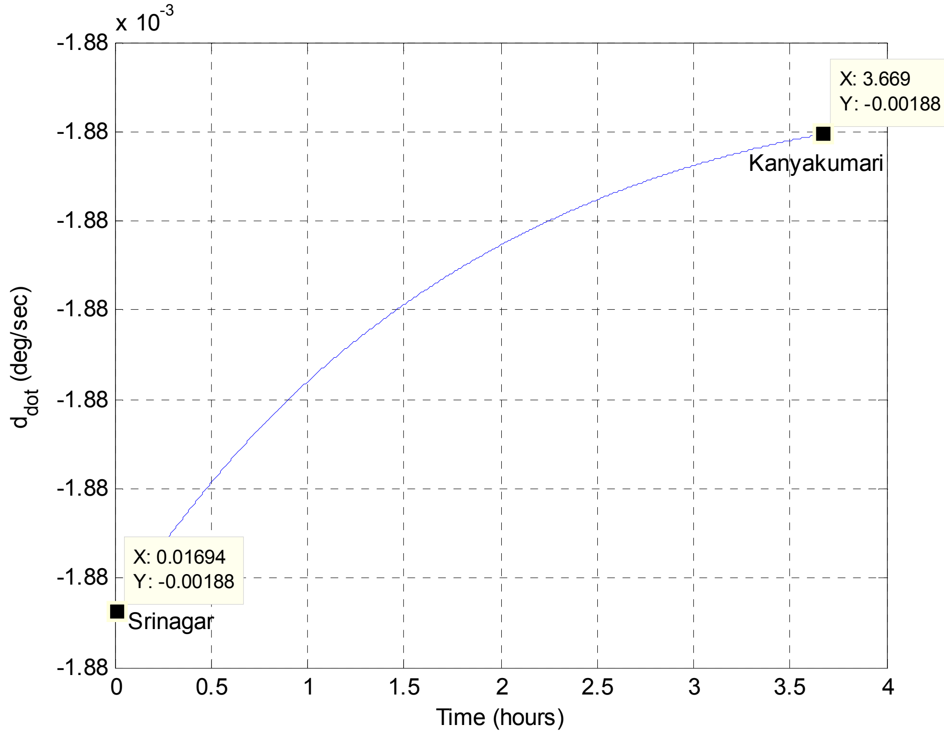

Figure 19: Plot of rate of change of distance angle with time

Since the cruise flight is taking place at a constant velocity and along the Great Circle arc, thus the rate of change of d should be zero. This is verified from the above plot for the rate of change of the distance angle d. The values of d_dot is almost constant with very small changes.

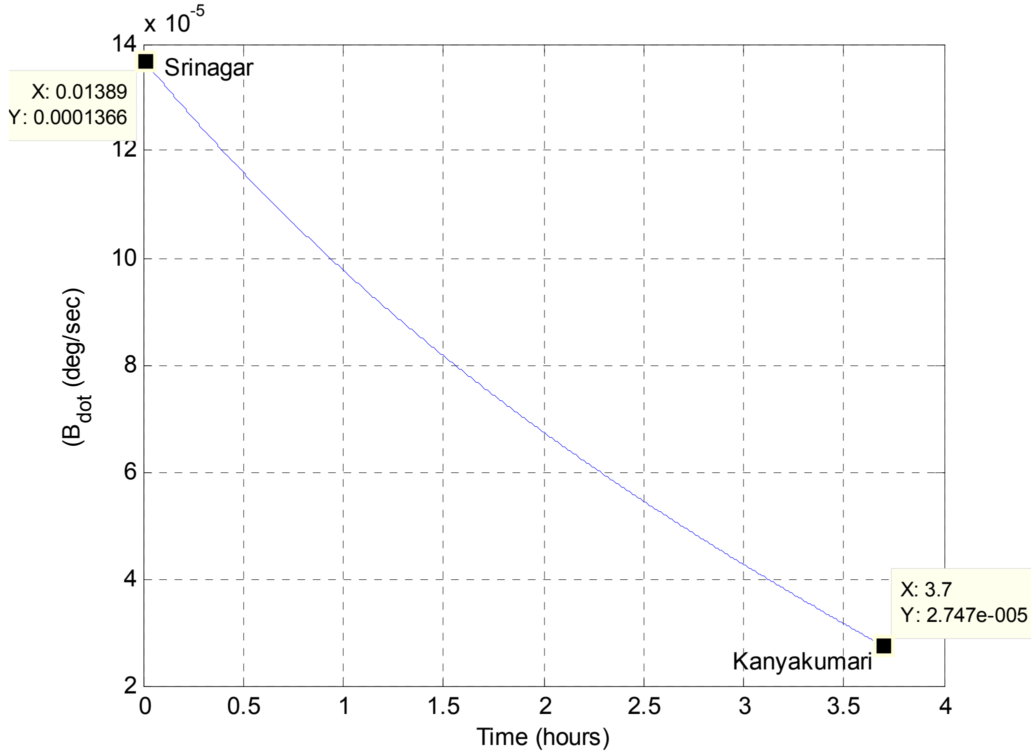

Figure 20: Plot of rate of change of bearing angle with time

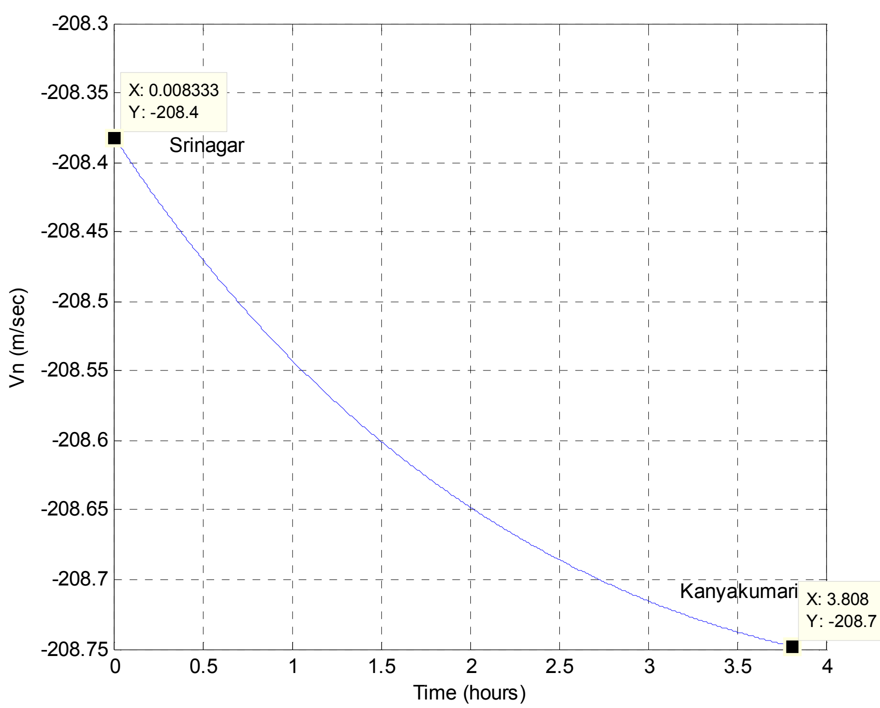

Figure 21: Plot of North Velocity variation with time

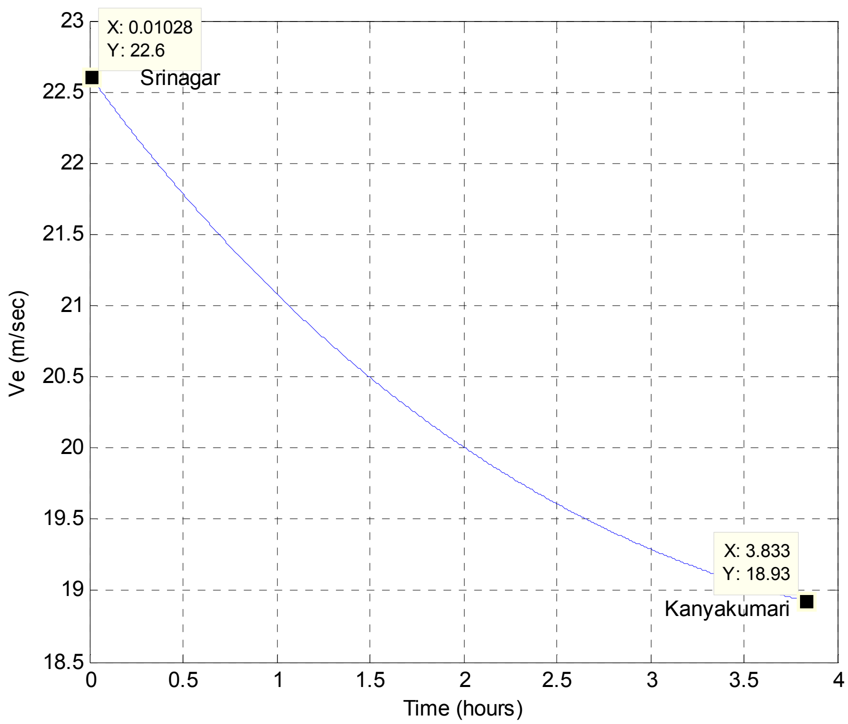

Figure 22: Plot of East Velocity variation with time

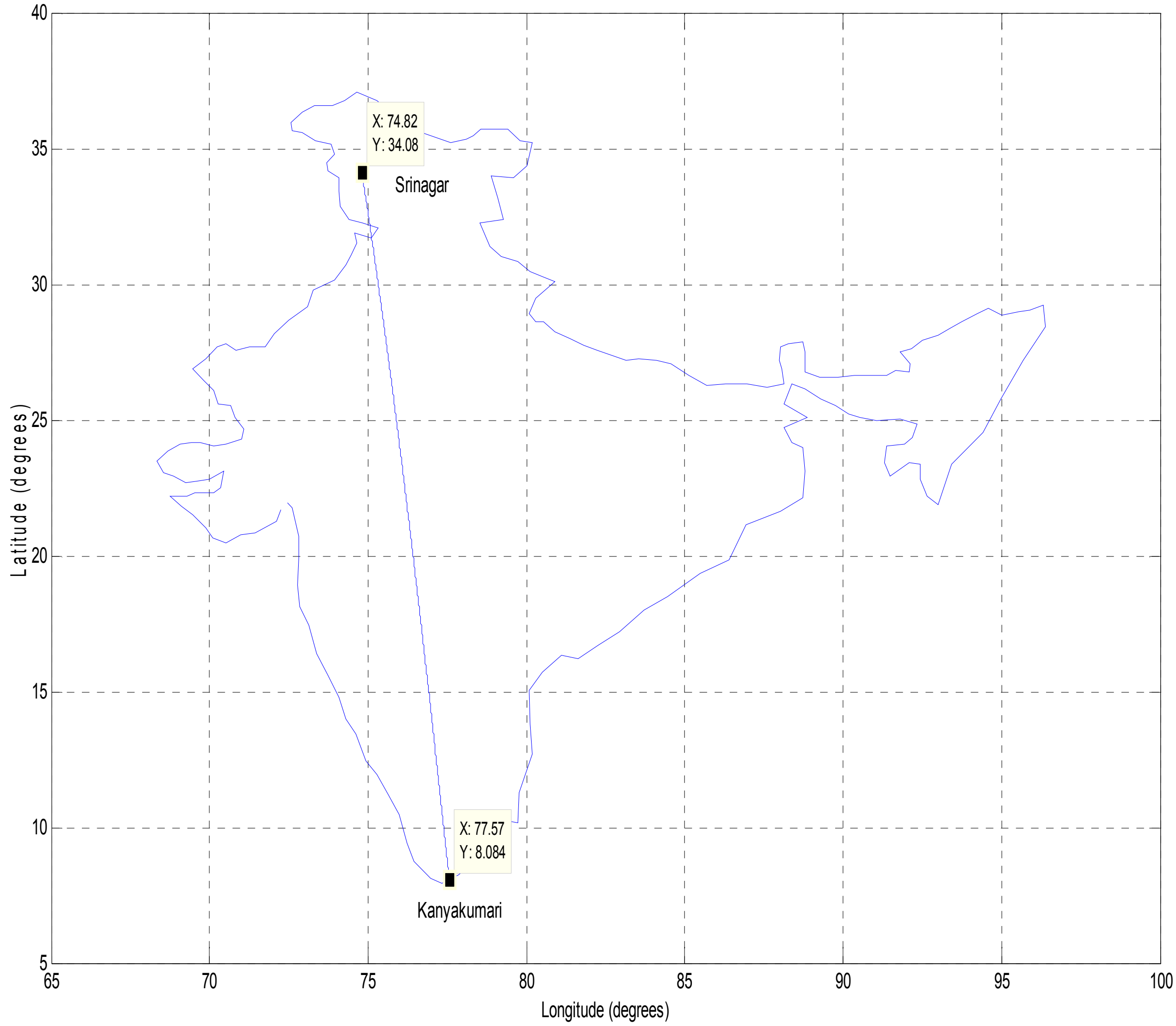

Figure 23: Flight path on the map of India

The above plot of the map of India is generated by finding out the co-ordinates of the boundary of India using the googlemap and since the boundary co-ordinates are chosen randomly with some finite spacing between every two successive points, the continuity of the boundary is not achieved and hence the map is not very accurate and is only an approximate representation of the map of India.

EQUATIONS OF MOTION OF AN AIRCRAFT IN NAVIGATION FRAME

The navigation frame is defined by the local North, local East and local Down frame. The equation of motion in the navigation frame is given by the following formulae (Chapter 3, [4]):

$$\begin{pmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{pmatrix} = \begin{pmatrix} f_N - 2\Omega V_E \sin L + \dfrac{V_N V_D - V_E^2 \tan L}{R_e + h} - \dfrac{\Omega^2 (R_e + h)}{2} \sin(2L)) \\ f_E - 2\Omega(V_N \sin L + V_D \cos L) + \dfrac{V_E}{R_e + h}(V_D + V_D \tan L) \\ f_D - 2\Omega V_E \cos L - \dfrac{V_E^2 + V_N^2}{R_e + h} + g - \dfrac{\Omega^2 (R_e + h)}{2}(1 + \cos(2L)) \end{pmatrix}$$

where V_N, V_E, V_D are the velocities in the north, east and down directions respectively.

Re is the radius of the earth, h is the altitude, L and l are latitude and longitude respectively.

f_N, f_E and f_D are the specific body forces sensed by the accelerometer,

and Ω is the rate of rotation of the earth about its spin axis.

The above equation of motion can be used to determine the fN, fE and fD throughout the flight path by using the information of $\dot{V}_N, \dot{V}_E, \dot{V}_D$ from the kinematics model stated earlier.

$\dot{V}_D = 0$ since the flight path considered is a cruise flight.

Thus the specific forces can be calculated using the kinematics model. The plots of the specific forces are as shown below:

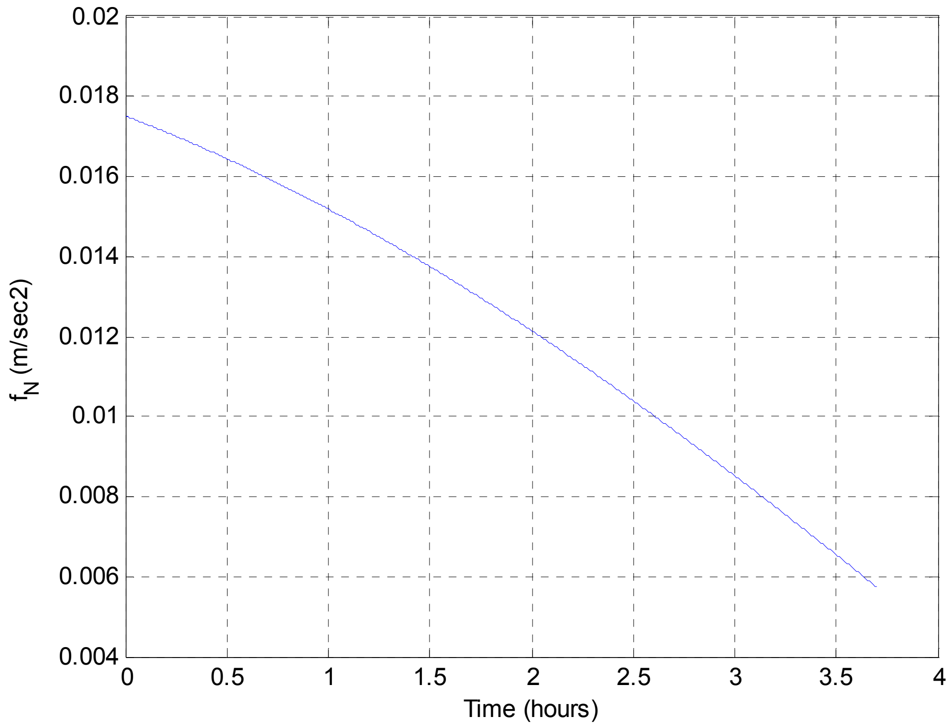

Figure 24: Plot of f_N variation with time

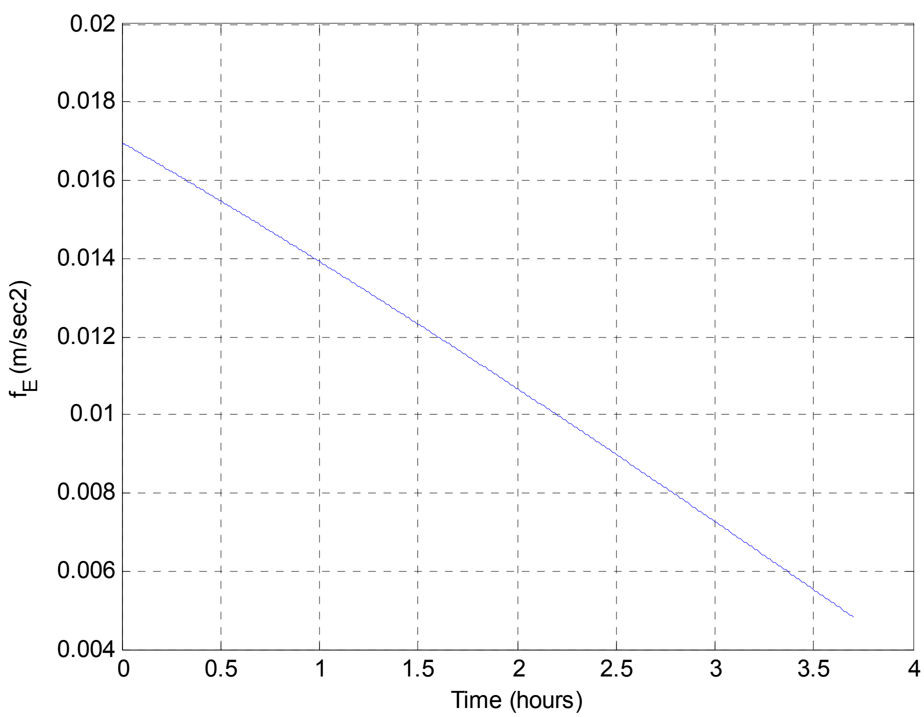

Figure 25: Plot of f_E variation with time

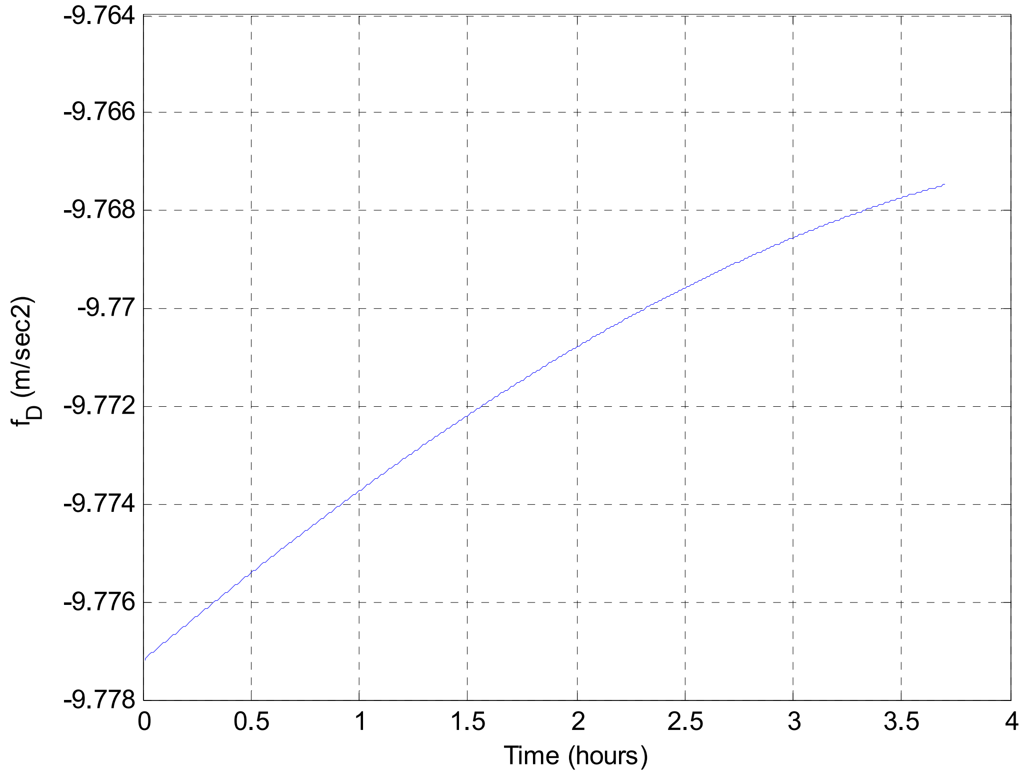

Figure 26: Plot of f_D variation with time

FULL-STATE ERROR MODEL

The equations of motion in the navigation frame are used to derive the error model and the error equations are given by the following formulae:

$$\delta\ddot{x} = F\delta x + Gu\ ,$$

where

$\boldsymbol{\delta}\mathbf{x} = [\delta\alpha\ \delta\beta\ \delta\gamma\ \delta V_N\ \delta V_E\ \delta V_D\ \delta L\ \delta l\ \delta h]^T$

$\mathbf{u} = [\delta\omega\ \delta\omega\ \delta\omega\ \delta fx\ \delta fy\ \delta fz]^T$

$\delta\alpha$, $\delta\beta$, $\delta\gamma$ are the misalignment errors in the roll angle φ, pitch angle θ and yaw angle ψ respectively.

δV_N, δV_E, δV_D correspond to the errors in the north, east and down velocities respectively

δL, δl, δh correspond to the errors in the latitude, longitude and height respectively

$\delta\omega_x$, $\delta\omega_y$, $\delta\omega_z$ correspond to the gyroscope bias about the north, east and down axes respectively

δf_x, δf_y, δf_z correspond to the accelerometer biases in the north, east and down directions respectively

and the matrices F and G are given as follows:

$$F = \left(0 \quad \begin{matrix}
0 & -\left(\Omega\sin L + \frac{V_E}{R}\tan L\right) & \frac{V_N}{R} & 0 & \frac{1}{R} & 0 & -\Omega\sin L & 0 & -\frac{V_E}{R^2} \\
\left(\Omega\sin L + \frac{V_E}{R}\tan L\right) & 0 & \left(\Omega\cos L + \frac{V_E}{R}\right) & -\frac{1}{R} & 0 & 0 & 0 & 0 & \frac{V_N}{R^2} \\
-\frac{V_N}{R} & -\left(\Omega\cos L + \frac{V_E}{R}\right) & 0 & 0 & -\frac{\tan L}{R} & 0 & \begin{matrix}-\Omega\cos L \\ -\frac{V_E}{R\cos^2 L}\end{matrix} & 0 & \frac{V_E\tan L}{R^2} \\
0 & -f_D & f_E & \frac{V_D}{R} & -2\left(\Omega\sin L + \frac{V_E}{R}\tan L\right) & \frac{V_N}{R} & -V_E\left(2\Omega\cos L + \frac{V_E}{R\cos^2 L}\right) & 0 & \frac{1}{R^2}\left(V_E^2\tan L - V_N V_D\right) \\
f_D & 0 & -f_N & \left(2\Omega\sin L + \frac{V_E}{R}\tan L\right) & \frac{1}{R}\left(V_N\tan L + V_D\right) & 2\Omega\cos L + \frac{V_E}{R} & \begin{pmatrix}2\Omega\left(V_N\cos L - V_D\sin L\right) \\ +\frac{V_N V_E}{R\cos^2 L}\end{pmatrix} & 0 & -\frac{V_E}{R^2}\left(V_N\tan L + V_D\right) \\
-f_E & f_N & 0 & -\frac{2V_N}{R} & -2\left(\Omega\cos L + \frac{V_E}{R}\right) & 0 & 2\Omega V_E\sin L & 0 & \frac{1}{R^2}\left(V_N^2 + V_E^2\right) \\
0 & 0 & 0 & \frac{1}{R} & 0 & 0 & 0 & 0 & -\frac{V_N}{R^2} \\
0 & 0 & 0 & 0 & \frac{1}{R\cos L} & 0 & \frac{V_E\tan L}{R\cos L} & 0 & -\frac{V_E}{R^2\cos L} \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0
\end{matrix} \right)$$

and $G = \begin{pmatrix} -c_{11} & -c_{12} & -c_{13} & 0 & 0 & 0 \\ -c_{21} & -c_{22} & -c_{23} & 0 & 0 & 0 \\ -c_{31} & -c_{32} & -c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{11} & c_{12} & c_{13} \\ 0 & 0 & 0 & c_{21} & c_{22} & c_{23} \\ 0 & 0 & 0 & c_{31} & c_{32} & c_{33} \end{pmatrix}$

where $\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$ is the transformation matrix from the body frame to the navigation frame.

The above error model has been used to plot the propagation of the errors i.e. $\boldsymbol{\delta x}$ for the following two cases:

<u>Case I: Aircraft Navigation using INS only with constant gyroscope and accelerometer biases</u>

The flight path considered for this simulation is the cruise flight path from Srinagar to Kanyakumari at an altitude of 10 km and 0.7 Mach speed.

Initial alignment errors: $\delta\alpha_0$: 0.1 mrad

$\delta\beta_0$: 0.1 mrad

$\delta\gamma_0$: 1.0 mrad

Gyroscopic bias: $\delta\omega_x$: 0.01˚/h

$\delta\omega_y$: 0.01˚/h

$\delta\omega_z$: 0.01˚/h

Accelerometer bias: δf_x : 0.1 milli-g

δf_y : 0.1 milli-g

δf_z : 0.1 milli-g

These conditions have been used to plot the propagation of the errors when only the INS is being used for aircraft navigation and the figures are shown below:

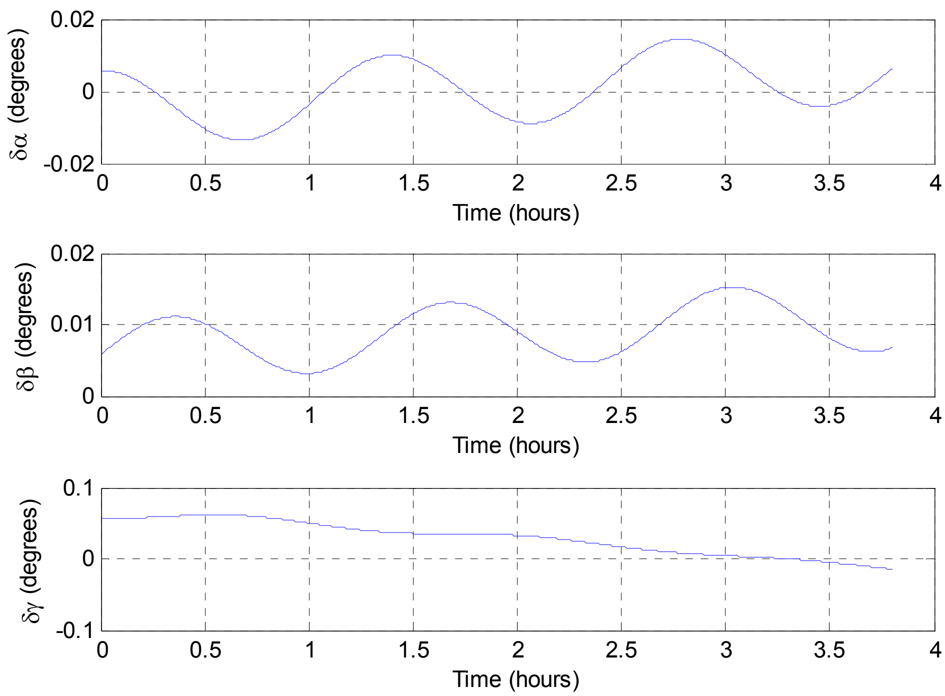

Figure 27: Plot of attitude errors variation with time

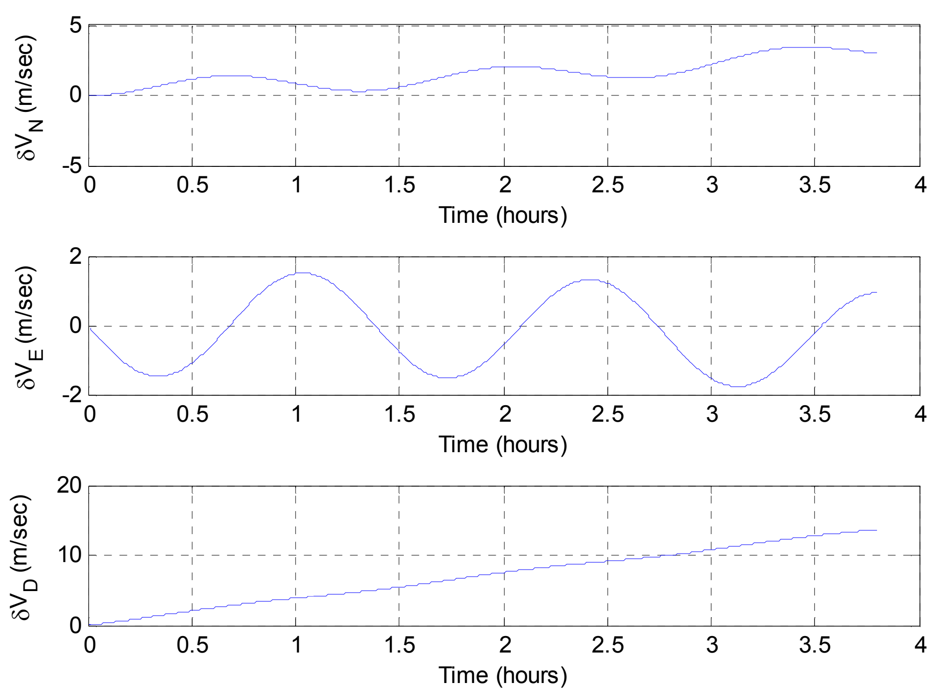

Figure 28: Plot of velocity errors variation with time

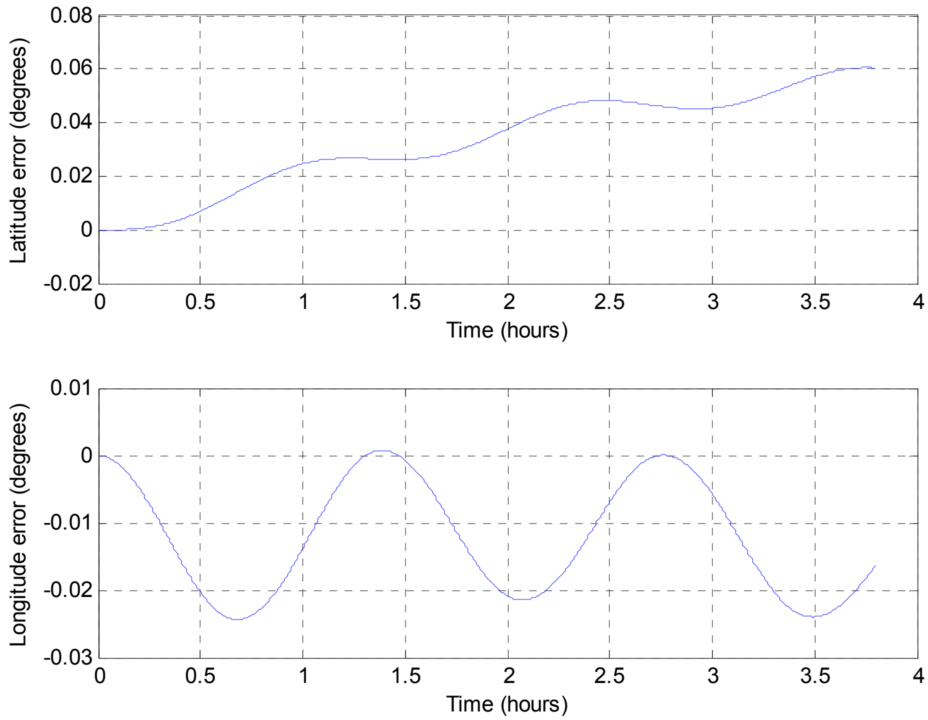

Figure 29: Plot of latitude and longitude errors variation with time

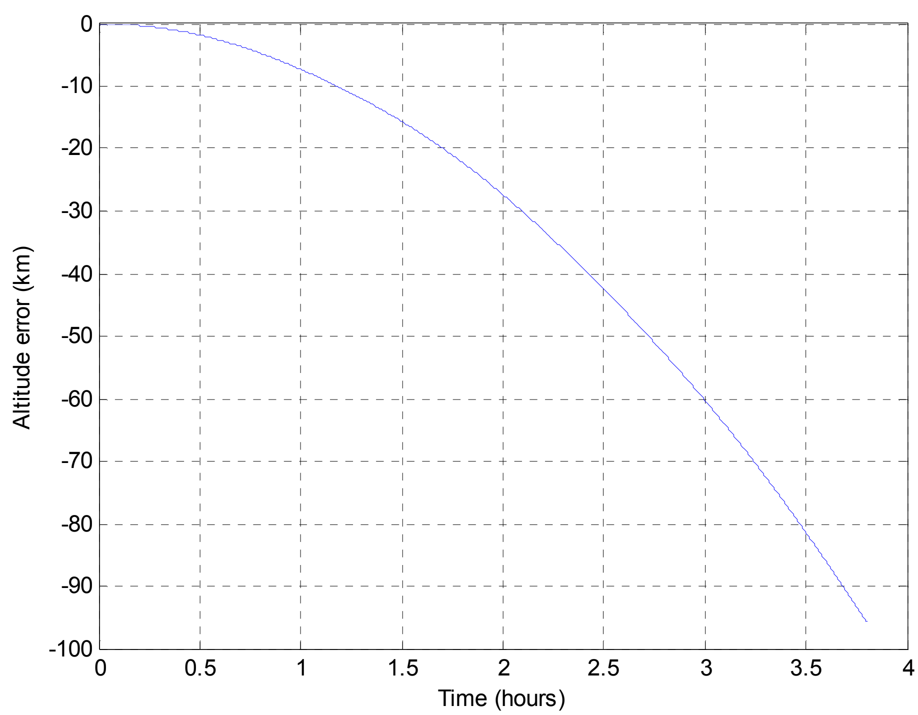

Figure 30: Plot of altitude error variation with time

Case II: Aircraft Navigation using INS only with gyroscopic and accelerometer biases being white noise

The flight path considered for this simulation is a cruise flight path at an altitude of 10 km and 0.7 Mach speed with the starting point being Srinagar and the flight is considered to be following a constant longitude path.

The initial misalignments are the same as in the previous simulation. The accelerometer and gyroscopic noises are represented as:

Accelerometer white-noise spectral density = 0.0036 $(m/sec^2)^2/(rad/sec)$

Gyro white-noise spectral density = 2.35 (10^{-11}) $(rad/sec)^2/(rad/sec)$

The error equation is:

$$\delta\dot{x} = F\delta x + Gu$$

and if δx is represented as x, then

$$\dot{x} = Fx + Gu$$

Thus, if the x is known at some time instant t_k, then the error at time instant t_{k+1} is given by the following relation

$$x_{k+1} = \phi_k x_k + w_k$$

where $\phi_k = \phi(t_{k+1}, t_k) = \int_{t_k}^{t_{k+1}} e^{F\xi} d\xi$ and for small $\Delta t = t_{k+1} - t_k$ is approximated as $\phi_k = e^{F\Delta t}$ assuming F to be constant over the Δt.

and $w_k = \int_{t_k}^{t_{k+1}} \phi(t_{k+1}, \xi) G(\xi) u(\xi) d\xi$

and $Q_k = E\left[w_k w_k^T\right] = \int_{t_k}^{t_{k+1}} \int_{t_k}^{t_{k+1}} \phi(t_{k+1}, \xi) G(\xi) E\left[u(\xi) u(\tau)^T\right] G(\tau)^T \phi(t_k, \tau)^T \, d\xi d\tau$

with $E\left[u(\xi) u(\tau)^T\right] = \delta(\xi - \tau)$, δ being the Dirac delta function over here.

Thus the Q_k can be calculated and then w_k can be calculated as:

$w_k(i) = \sqrt{Q_k(i,i)} \times random\_number$ where i relates to the $(i,i)^{th}$ coefficient in Q_k.

Thus wk can be calculated and x_{k+1} can be calculated using the expression for x_{k+1}.

The above process is repeated recursively to get the error in the state space vector at every time instant (with frequency 1 Hz) of the flight path. The figures showing the propagation of the errors for this model are shown below:

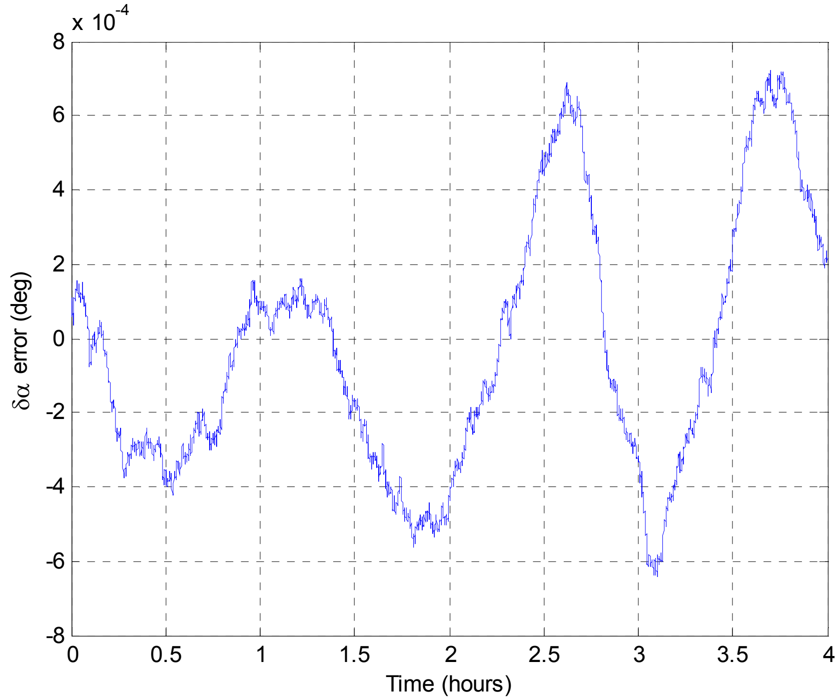

Figure 31: Plot of δα variation with time

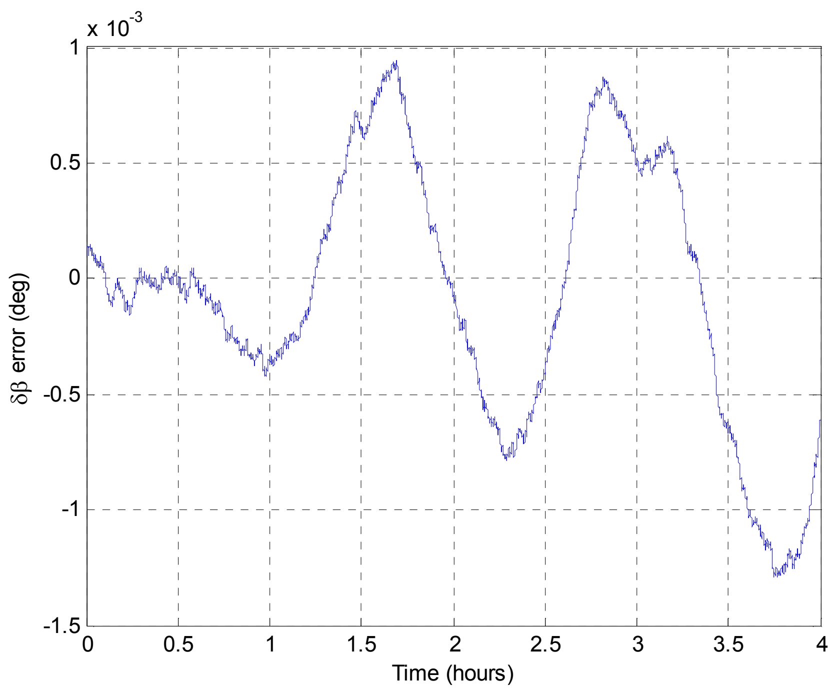

Figure 32: Plot of δβ variation with time

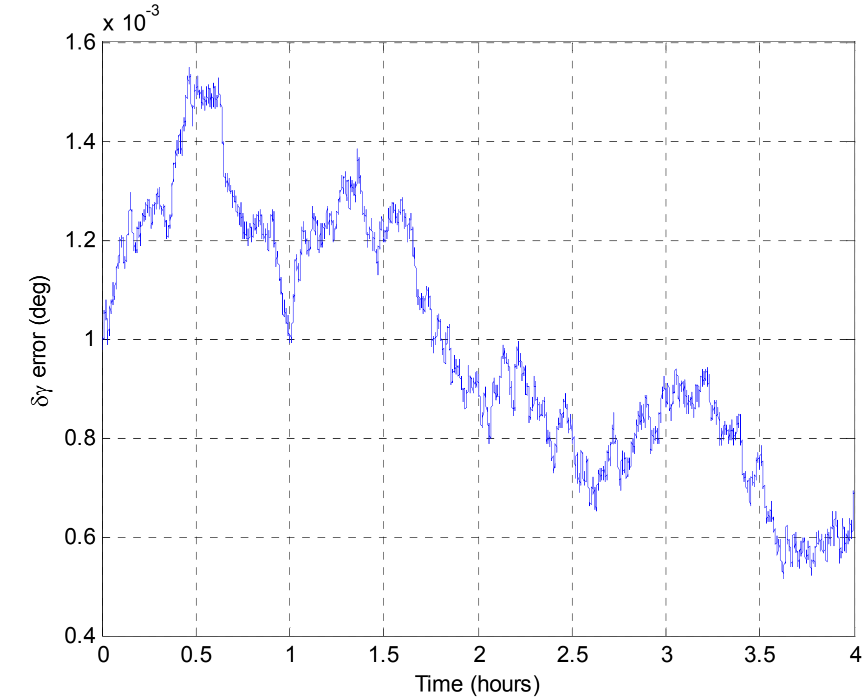

Figure 33: Plot of $\delta\gamma$ variation with time

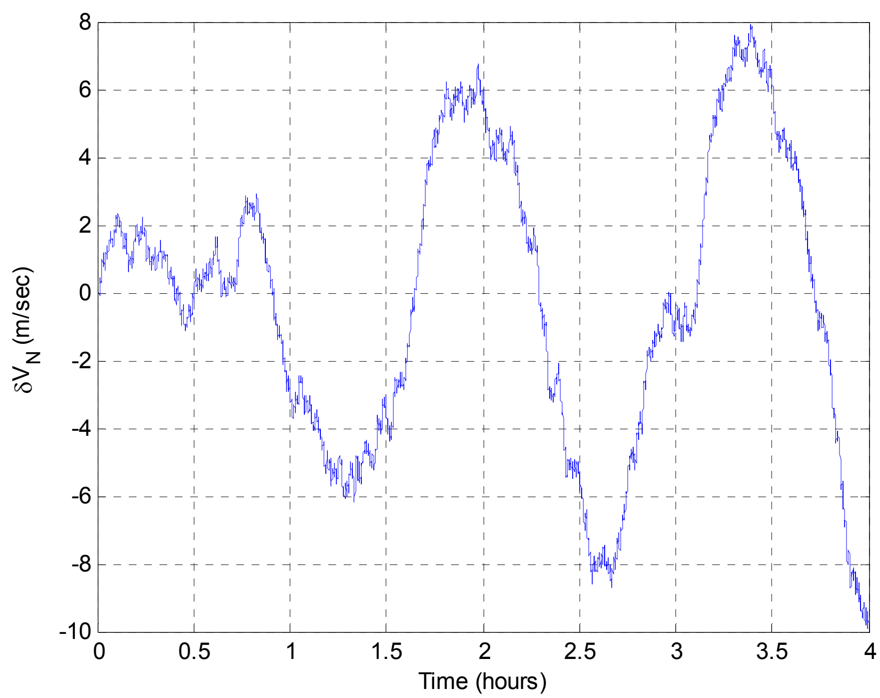

Figure 34: Plot of δV_N variation with time

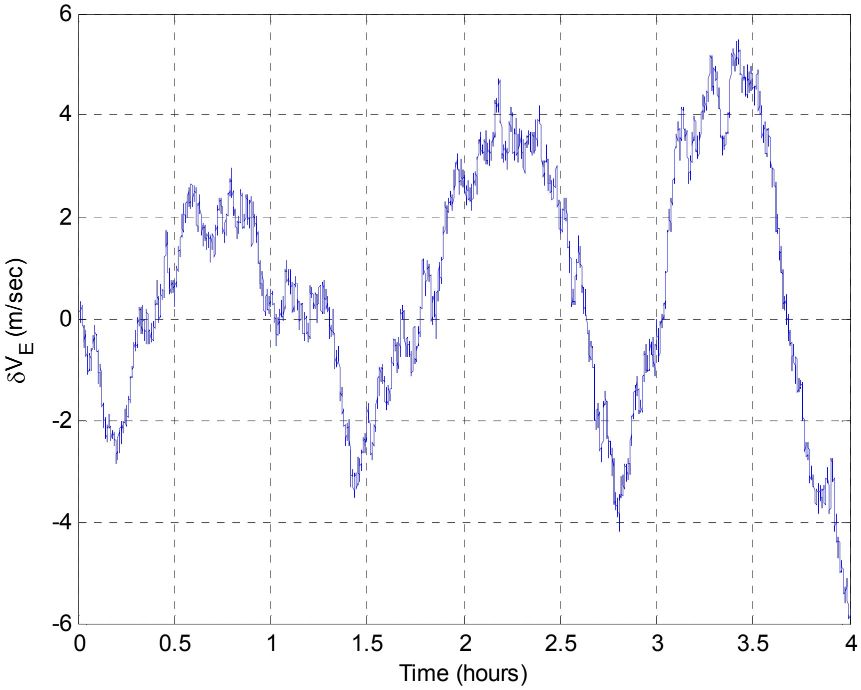

Figure 35: Plot of δV_E variation with time

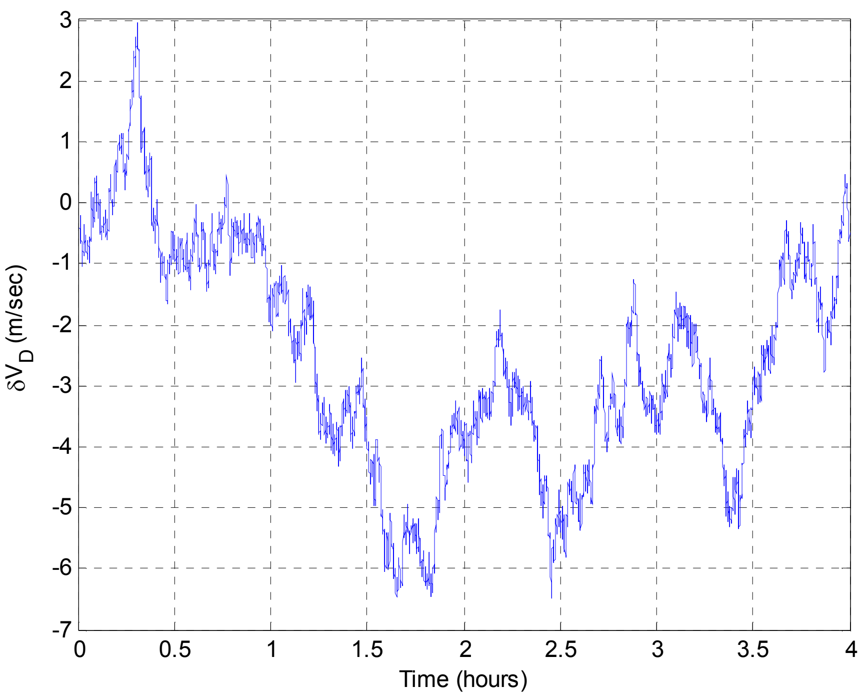

Figure 36: Plot of δV_D variation with time

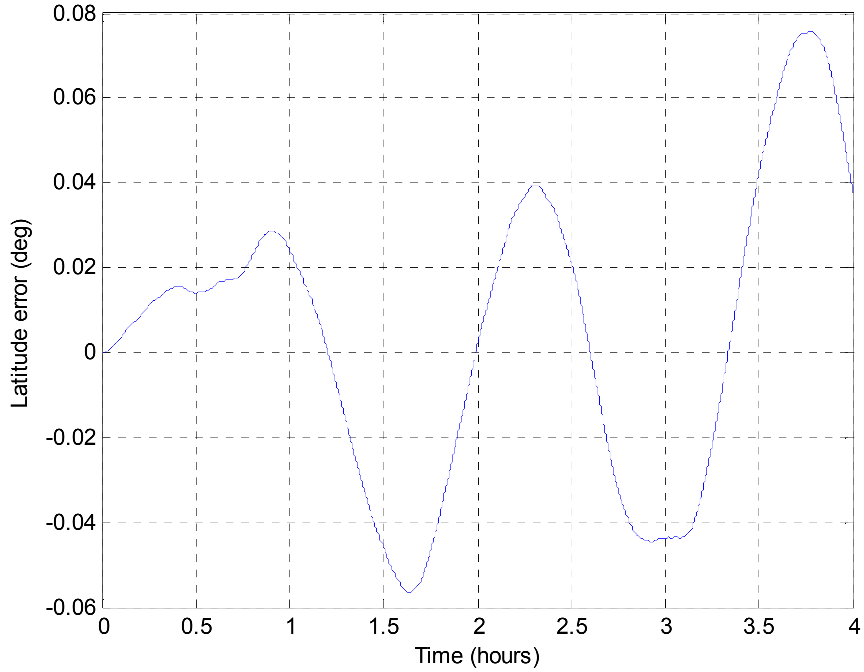

Figure 37: Plot of latitude error variation with time

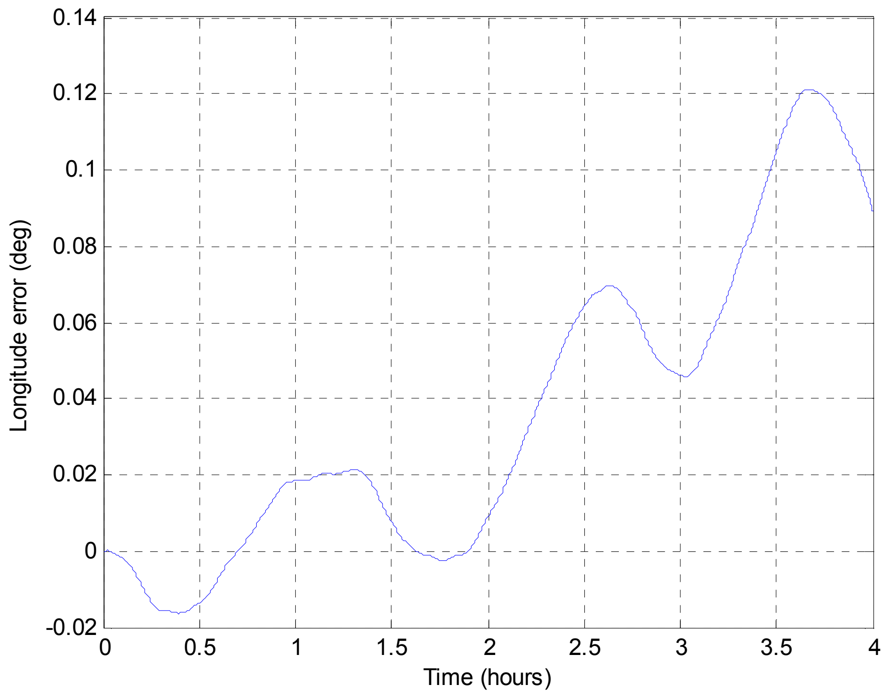

Figure 38: Plot of longitude error variation with time

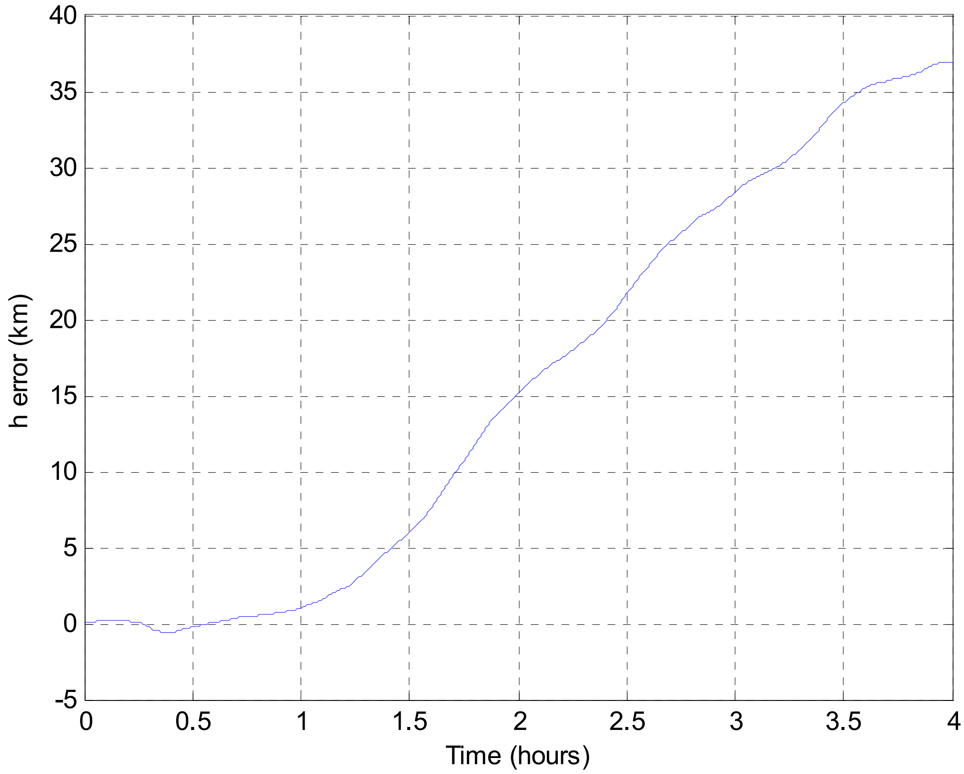

Figure 39: Plot of altitude error variation with time

APPLICATION OF DISCRETE KALMAN FILTERING TO AIRCRAFT NAVIGATION

The Discrete Kalman Filtering technique has been used to integrate the GPS and INS together into one system to be used for the aircraft navigation.

From the earlier simulations for aircraft navigation when only INS is being used, it is observed that the errors grow with time and thus to have an accurate navigation, one more system is used which would provide the measurements of the position. This measurement will be used to update the estimate of the error at the next time step. Thus with the aid of another system, the error is reduced. This integration of the two systems is done using the Kalman Filter. Thus, the INS is integrated with the GPS for a higher accuracy.

The Kalman filter algorithm

The INS model is defined as the process model and the GPS model is defined as the measurement model.

$$\mathbf{x}_{k+1} = \boldsymbol{\varphi}_k * \mathbf{x}_k + \mathbf{w}_k$$

$$\mathbf{z}_k = \mathbf{H}_k * \mathbf{x}_k + \mathbf{v}_k$$

where $E[\mathbf{w}_k \mathbf{w}_k^T] = \mathbf{Q}_k$ and $E[\mathbf{v}_k \mathbf{v}_k^T] = \mathbf{R}_k$

The method to calculate Q_k and then w_k has already been shown earlier in the report. The same method is used here as well. For vk, R_k has been taken as constant as

$$R_k = \begin{bmatrix} 225 & 0 & 0 & 0 \\ 0 & 225 & 0 & 0 \\ 0 & 0 & 225 & 0 \\ 0 & 0 & 0 & 225 \end{bmatrix} \mathrm{m}^2$$

$\mathbf{x}_k$ represents the INS model and $\mathbf{z}_k$ represents the GPS model.

The measurement $\mathbf{z}_k$ is done using the value $\mathbf{x}_k$ (the projected value of the error at the k^{th} step using the INS model).

Since the GPS model gives the measurement as pseudoranges, thus the errors in the pseudoranges have to be related to the $\mathbf{x}_k$.

Pseudorange to the ith visible satellite is given by

$$\psi_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + C\Delta t$$

where X_i, Y_i and Z_i are the co-ordinates of the i_{th} visible satellite in the ECEF frame and x, y and z corresponds to the user location in the ECEF frame.

Four satellites are used to estimate the user position location as stated earlier in the report.

$\mathbf{H_k}$ is given by the following expression:

$$H_k = \begin{pmatrix} \frac{\partial \psi_1}{\partial x} & \frac{\partial \psi_1}{\partial y} & \frac{\partial \psi_1}{\partial z} & 1 \\ \frac{\partial \psi_2}{\partial x} & \frac{\partial \psi_2}{\partial y} & \frac{\partial \psi_2}{\partial z} & 1 \\ \frac{\partial \psi_3}{\partial x} & \frac{\partial \psi_3}{\partial y} & \frac{\partial \psi_3}{\partial z} & 1 \\ \frac{\partial \psi_4}{\partial x} & \frac{\partial \psi_4}{\partial y} & \frac{\partial \psi_4}{\partial z} & 1 \end{pmatrix} = \begin{pmatrix} -\frac{X_1 - x}{\psi_1} & -\frac{Y_1 - y}{\psi_1} & -\frac{Z_1 - z}{\psi_1} & 1 \\ -\frac{X_2 - x}{\psi_2} & -\frac{Y_2 - y}{\psi_2} & -\frac{Z_2 - z}{\psi_2} & 1 \\ -\frac{X_3 - x}{\psi_3} & -\frac{Y_3 - y}{\psi_3} & -\frac{Z_3 - z}{\psi_3} & 1 \\ -\frac{X_4 - x}{\psi_4} & -\frac{Y_4 - y}{\psi_4} & -\frac{Z_1 - z}{\psi_4} & 1 \end{pmatrix}$$

The matrix Hk depends on the GPS satellites that have been selected for estimating the user position. The equation relating $\mathbf{z_k}$ to $\mathbf{x_k}$ can be approximately written as:

$\mathbf{z_k = H_k\, x_k}$

Thus $\mathbf{x_k = (H_k^T\, H_k)^{-1}\, H_k^T\, z_k}$

and $E[\mathbf{x\, x^T}] = \mathbf{(H_k^T\, H_k)^{-1}\, H_k^T\, E[z_k\, z_k^T]\, H_k\, (H_k^T\, H_k)^{-1}}$

with $E\mathbf{[z_k\, z_k^T]} = \sigma^2\mathbf{I}$

Thus $E[\mathbf{x\, x^T}] = \sigma^2\mathbf{(H_k^T\, H_k)^{-1}}$

Therefore to make the estimation of the user position as accurate as possible, the trace of the sum of the diagonal elements of $\mathbf{(H_k^T\, H_k)^{-1}}$ is made minimum which is done as follows:

GDOP = sqrt(trace($\mathbf{(H_k^T\, H_k)^{-1}}$))

Since using the GPS model, all the satellites that are visible at any time instant t with elevation angle > 5 degrees are known, the GDOP corresponding to each of the 4 satellites selection from the entire set of visible satellites is done, i.e. if there are N visible satellites, then there are ${}^{N}C_4$ ways of selecting 4 satellites from the set of N satellites and thus the GDOP corresponding to each of the selections is calculated and then the minimum GDOP of them all is used to select the 4 satellites which would be used to estimate the user position.

This is how the satellite selection is done to minimize the errors due to the GPS measurements.

But the above $\mathbf{H_k}$ corresponds to the $\mathbf{x_k}$ being the errors in the measurement of the position in terms of the ECEF co-ordinates. But the error state vector that is being used in the error model, it does not correspond to the ECEF model; rather it corresponds to the errors in the attitude, velocity, position, latitude and longitude. Thus the above $\mathbf{H_k}$ matrix has to be transformed into the $\mathbf{x_k}$ being used in the model which can be done as follows:

$X_{ECEF} = (R_e+h) \cos L \cos l$

$Y_{ECEF} = (R_e+h) \cos L \sin l$

$Z_{ECEF} = (R_e+h) \sin L$

$$\text{Thus, } \begin{aligned} \partial X_{ECEF} &= -(R_e + h)\sin L \cos l \partial L - (R_e + h)\cos L \sin l \partial l + \cos L \cos l \partial h \\ \partial Y_{ECEF} &= -(R_e + h)\sin L \sin l \partial L + (R_e + h)\cos L \cos l \partial l + \cos L \sin l \partial h \\ \partial Z_{ECEF} &= (R_e + h)\cos L \partial L + \sin L \partial h \end{aligned}$$

Thus the $\mathbf{H_k}$ for the error model being used can be calculated by transforming from the ECEF to the error model by multiplying the $\mathbf{H_k}$ matrix above with the transformation matrix relating ECEF measurement frame to the error state vector.

Thus $\mathbf{z_k}$ is calculated using the equation $\mathbf{z_k = H_k * x_k + v_k}$

Also, there is an estimate $\hat{x}_k^-$ of the error $\mathbf{x_k}$ at the instant just before the measurement $\mathbf{z_k}$ is done. And then this estimate $\hat{x}_k^-$ is made better by using the Kalman Gain times the difference between the measurement $\mathbf{z_k}$ and the $\hat{x}_k^-$. The better estimate is obtained by the following relation:

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k x_k^-)$$

In the above expression the $\mathbf{K_k}$ represents the Kalman Gain at the k^{th} step and $\mathbf{H_k}$ relates the error in the measurement of the GPS model i.e. $\mathbf{z_k}$ to the error in the INS model i.e. $\mathbf{x_k}$.

The Kalman Gain $\mathbf{K_k}$ is given by the relationship:

$$K_k = \left(P_k^- H_k^T\right)\left[H_k P_k^- H_k^T + R_k\right]^{-1}$$

where $\mathbf{P_k} = E[\mathbf{e_k e_k}^T]$ and $e_k = \hat{x}_k - \hat{x}_k^-$

The matrix P is then updated as $P_k = \left(I - K_k H_k\right) P_k^-$

and the estimate just an instant before the next measurement is given by $\hat{x}_{k+1}^- = \phi_k \hat{x}_k$

And the P^- matrix is updated as $P_{k+1}^- = \phi_k P_k \phi_k^T + Q_k$

Now the $\hat{x}_k$ is used as the better estimate at the k^{th} step and then the projected value of the error using the INS model at the $(k+1)^{th}$ step is calculated as

$$x_{k+1} = \phi_k \hat{x}_k + w_k$$

The entire process as above is repeated throughout the duration of the flight in the GPS/INS integration system.

I still don't have the results to show the improvement in the navigational accuracy using the GPS/INS integration using the Kalman Filter algorithm. I have written down the code for the same but am getting some errors which I have to correct and then I shall be able to show the improvements in the results.

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