ACCURATE NAVIGATION OF A UAV USING KALMAN FILTER BASED GPS / INS INTEGRATION

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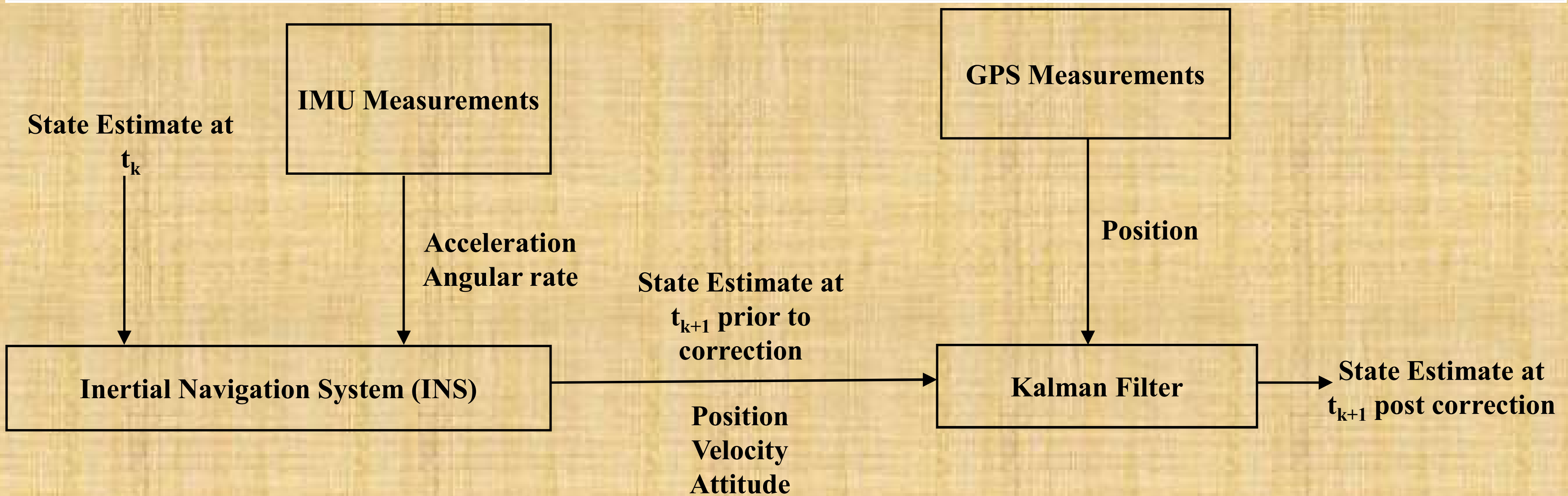
BRIEF OUTLINE OF THE WORK

Inertial Navigation System (INS)	Global Positioning System (GPS)	Kalman Filter
▪ Measures inertial angular rates and acceleration ▪ Integration of angular rates and acceleration to get attitude, position and velocity using known values of attitude, position and velocity at some reference time.	▪ Position determination using signals from GPS satellites	A filtering technique which operates on two systems of measurement and integrates the two systems into one system which has better accuracy.
Errors in INS based navigation grow with time	**Errors in GPS based navigation don't grow with time**	**Kalman Filter has been used to integrate INS and GPS**



FLIGHT PATH

- Starting location : 15^0 Latitude, 22.5^0 Longitude, 1 km Altitude
- Flight Altitude : 1 km
- Flight path : Along the constant 22.5^0 longitude at constant 5^0 angle of attack
- Speed : 60 m/sec
- Duration of Flight : 2 hours
- INS : 100 Hz
- GPS : 1 Hz

ASSUMPTIONS

- Point Mass model of the UAV has been considered
- Spherical model of the earth has been considered
- Navigation equations considered in the local NED (North, East and Down) frame attached with the UAV
- Ideal control: no control is considered and that the flight path is maintained
- Low grade INS has been considered
- Axes of accelerometers and gyroscopes aligned with the body axes
- Fixed bias and zero mean white noise in accelerometers and gyroscopes have been considered in IMU modelling
- GPS errors considered as Gauss Markov process

INERTIAL NAVIGATION SYSTEM (INS)

State Vector X				
Velocity (North, East, Down)	Position (Latitude, Longitude, Altitude)	Attitude (Roll, Pitch, Yaw)	Estimated Accelerometer Bias (in X, Y, Z)	Estimated Gyroscope Bias (in X, Y, Z)
V_N, V_E, V_D	L, l, h	Φ, θ, ψ	acc_{bias}	$gyro_{bias}$

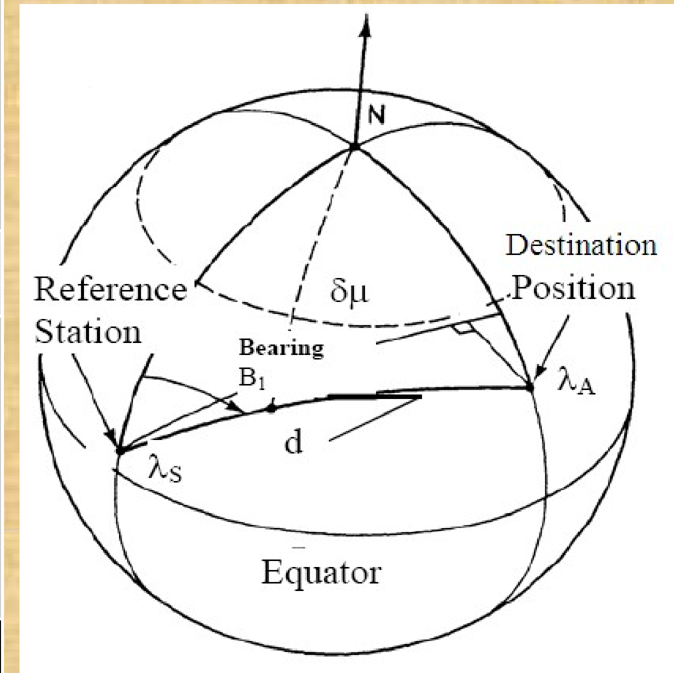
$$X = [V_N \quad V_E \quad V_D \quad L \quad l \quad h \quad \phi \quad \theta \quad \psi \quad acc_{bias}^T \quad gyro_{bias}^T]^T$$

$$acc_{bias} = [acc_{bias,X} \quad acc_{bias,Y} \quad acc_{bias,Z}]^T$$

$$gyro_{bias} = [gyro_{bias,X} \quad gyro_{bias,Y} \quad gyro_{bias,Z}]^T$$

Rate of Change of State Vector X

$$\dot{X} = [0 \quad 0 \quad 0 \quad \frac{V_N}{R_e + h} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$



NED Frame

Low Grade INS			
Accelerometer		Gyroscope	
Bias	Zero mean White Noise PSD	Bias	Zero mean White Noise PSD
0.001 g	9.6236×10^{-9} $(m/sec^2)^2/Hz$	10 deg/hr	2.3504×10^{-13} $(rad/sec)^2/Hz$

(Courtesy: Integrated Navigation and Guidance Systems by Daniel J. Biezad)

EQUATIONS OF MOTION IN NAVIGATION FRAME

Rate of change of Position

$$\begin{bmatrix}\dot{L} & \dot{l} & \dot{h}\end{bmatrix}^T = \begin{bmatrix}\dfrac{V_N}{R_e+h} & \dfrac{V_E \sec L}{R_e+h} & -V_D\end{bmatrix}^T$$

Rate of change of Velocity

$$\begin{pmatrix}\dot{V}_N \\ \dot{V}_E \\ \dot{V}_D\end{pmatrix} = \begin{pmatrix} f_N \\ f_E \\ f_D \end{pmatrix} + \begin{pmatrix} -2\omega_e V_E \sin L + \dfrac{V_N V_D - V_E^2 \tan L}{R_e+h} - \dfrac{\omega_e^2 (R_e+h)}{2}\sin(2L) \\ -2\omega_e (V_N \sin L + V_D \cos L) + \dfrac{V_E}{R_e+h}(V_D + V_D \tan L) \\ -2\omega_e V_E \cos L - \dfrac{V_E^2 + V_N^2}{R_e+h} + g - \dfrac{\omega_e^2 (R_e+h)}{2}(1+\cos(2L)) \end{pmatrix}$$

$$(f_N \quad f_E \quad f_D)^T = C_{b,NED}^T f_b^{F_b} \qquad f_{b,IMU}^{F_b} = f_b^{F_b} + acc_{bias} + acc_{white}$$

$$f_{b,corrected}^{F_b} = f_b^{F_b} - acc_{bias,estimated}$$

Rate of change of Attitude

$$\begin{pmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{pmatrix}\begin{pmatrix}\dot{\phi} \\ \dot{\theta} \\ \dot{\psi}\end{pmatrix} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} - C_{b,NED}\begin{pmatrix} \dfrac{V_E}{R_e+h} \\ -\dfrac{V_N}{R_e+h} \\ -\dfrac{V_E \tan L}{R_e+h} \end{pmatrix} - C_{b,NED}\begin{pmatrix} \omega_e \cos L \\ 0 \\ -\omega_e \sin L \end{pmatrix}$$

$$\omega_{b,I}^{F_b} = (p \quad q \quad r)^T \qquad \omega_{b,I,IMU}^{F_b} = \omega_{b,I}^{F_b} + gyro_{bias} + gyro_{white}$$

$$\omega_{b,I,corrected}^{F_b} = \omega_{b,I}^{F_b} - gyro_{bias,estimated}$$

Rate of change of Position	$\begin{bmatrix} \dot{L} & \dot{l} & \dot{h} \end{bmatrix}^T = \begin{bmatrix} \frac{V_N}{R_e + h} & \frac{V_E \sec L}{R_e + h} & -V_D \end{bmatrix}^T$
Rate of change of Velocity	$\begin{pmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{pmatrix} = C_{b,NED}^T \left(f_{b,corrected}^{F_b} \right) + \begin{pmatrix} -2\omega_e V_E \sin L + \frac{V_N V_D - V_E^2 \tan L}{R_e + h} - \frac{\omega_e^2 (R_e + h)}{2} \sin(2L) \\ -2\omega_e (V_N \sin L + V_D \cos L) + \frac{V_E}{R_e + h}(V_D + V_D \tan L) \\ -2\omega_e V_E \cos L - \frac{V_E^2 + V_N^2}{R_e + h} + g - \frac{\omega_e^2 (R_e + h)}{2}(1 + \cos(2L)) \end{pmatrix}$
Rate of change of Attitude	$\begin{pmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \omega_{b,I,corrected}^{F_b} - C_{b,NED} \begin{pmatrix} \frac{V_E}{R_e + h} \\ -\frac{V_N}{R_e + h} \\ -\frac{V_E \tan L}{R_e + h} \end{pmatrix} - C_{b,NED} \begin{pmatrix} \omega_e \cos L \\ 0 \\ -\omega_e \sin L \end{pmatrix}$
Rate of change of Estimated Accelerometer Bias	$a\dot{c}c_{bias} = [0 \quad 0 \quad 0]^T$
Rate of change of Estimated Gyroscope Bias	$g\dot{y}ro_{bias} = [0 \quad 0 \quad 0]^T$

INITIAL TRUE AND ESTIMATED STATE VECTOR

Parameters of the State Vector	True State Vector (X_0)	Estimated State Vector $(\hat{X}_0)$	Error in State Vector ΔX_0
North Velocity (V_N, m/sec)	60	40	20
East Velocity (V_E, m/sec)	0	40	40
Down Velocity (V_D, m/sec)	0	40	40
Latitude (L, deg)	15	15.1	0.1
Longitude (l, deg)	22.5	22.6	0.1
Altitude (h, m)	1000	2000	1000
Roll (deg)	0	10	10
Pitch (deg)	5	10	5
Yaw (deg)	0	10	10
Acc bias in X_b ($acc_{bias,X}$, m/sec^2)	0.001 g	0.0005 g	0.0005 g
Acc bias in Y_b ($acc_{bias,Y}$, m/sec^2)	0.001 g	0.0008 g	0.0002 g
Acc bias in Z_b ($acc_{bias,Z}$, m/sec^2)	0.001 g	0.0008 g	0.0002 g
Gyro bias in X_b ($gyro_{bias,X}$, deg/hr)	10	8	2
Gyro bias in Y_b ($gyro_{bias,Y}$, deg/hr)	10	5	5
Gyro bias in Z_b ($gyro_{bias,Z}$, deg/hr)	10	5	5

GLOBAL POSITIONING SYSTEM (GPS)

- GPS constellation is modelled with the following errors [1, 2]:
  - Bias
    - Clock offset
    - Ephemeris errors
    - Atmospheric propagation errors through troposphere and ionosphere
  - Zero mean white noise
    - Multipath
    - Receiver's noise errors
- GPS errors modelled as Gauss Markov processes

$$b_k = b_{k-1} \exp(-\beta_b \Delta t) + w_{b,k-1}$$

- True Range from the UAV to the GPS satellites is determined

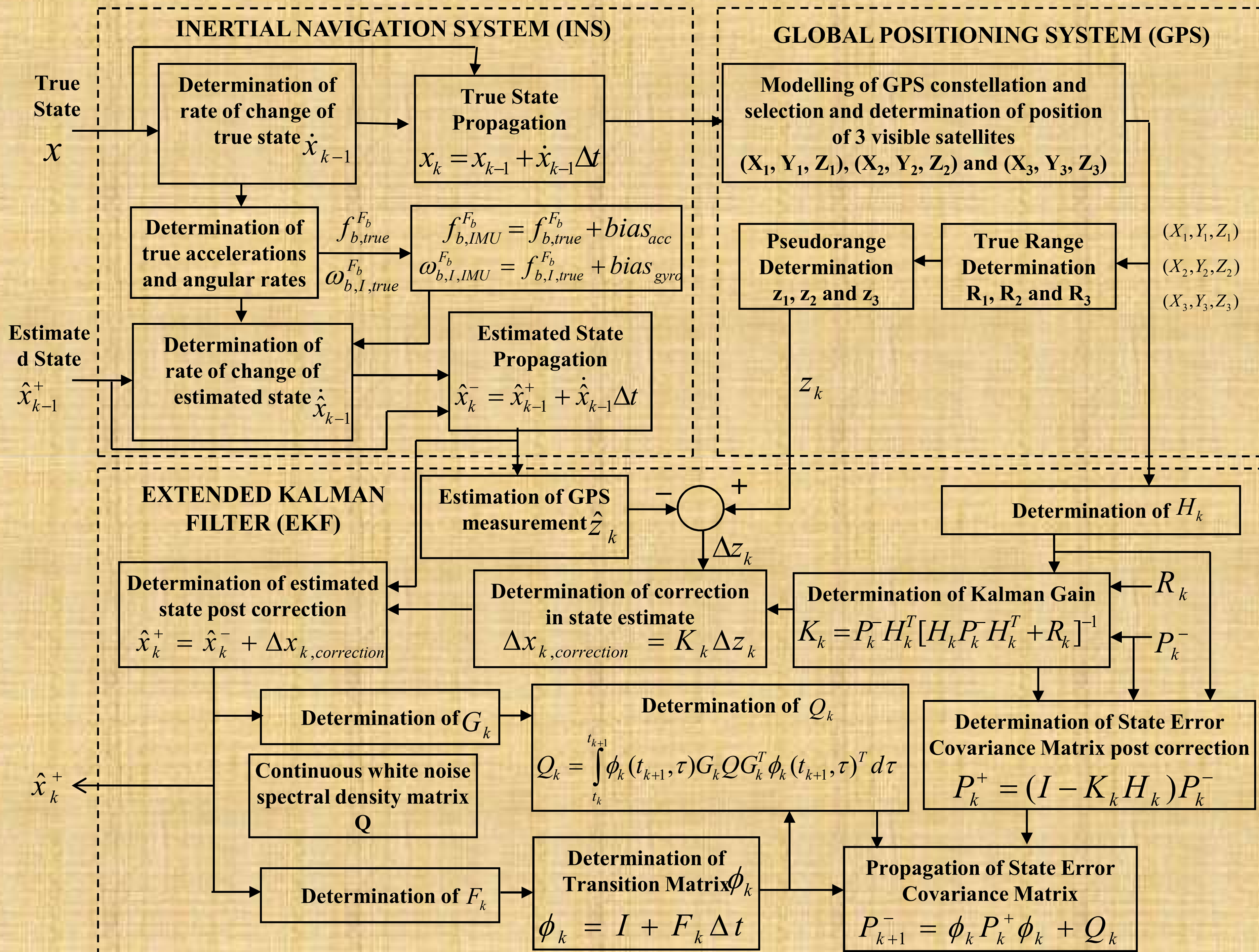
$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} = \begin{pmatrix} \sqrt{(X_1 - X)^2 + (Y_1 - Y)^2 + (Z_1 - Z)^2} \\ \sqrt{(X_2 - X)^2 + (Y_2 - Y)^2 + (Z_2 - Z)^2} \\ \sqrt{(X_3 - X)^2 + (Y_3 - Y)^2 + (Z_3 - Z)^2} \end{pmatrix}$$

- Pseudorange is determined using the true range and the modelled GPS errors and is used as GPS measurement

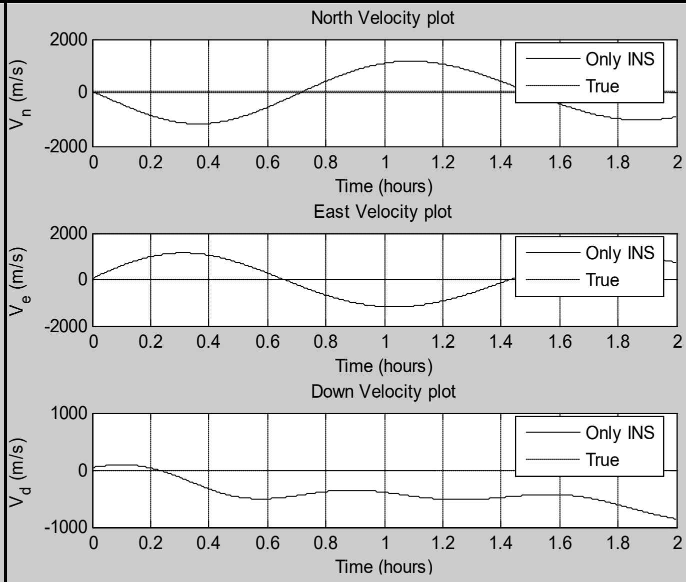
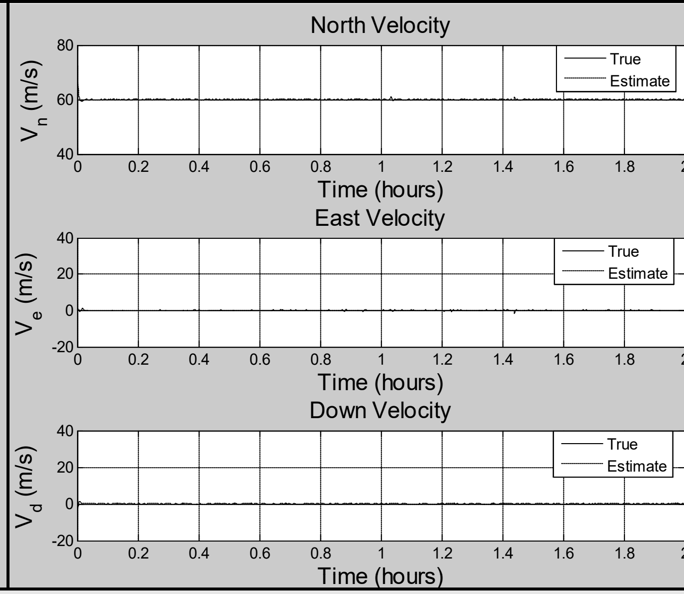
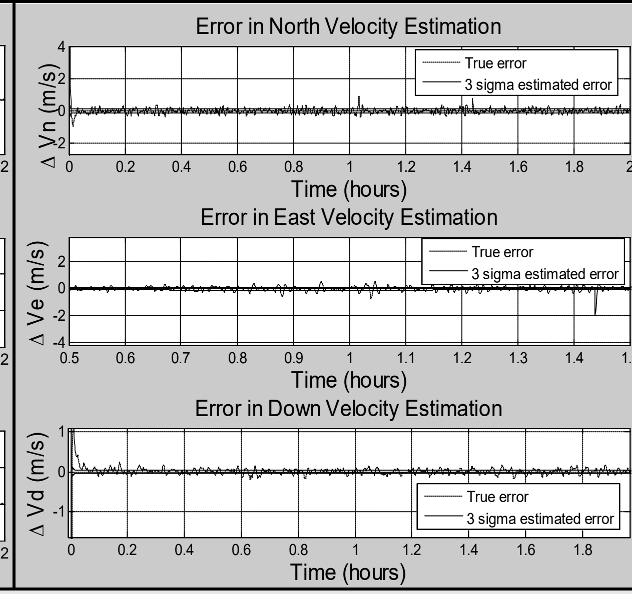
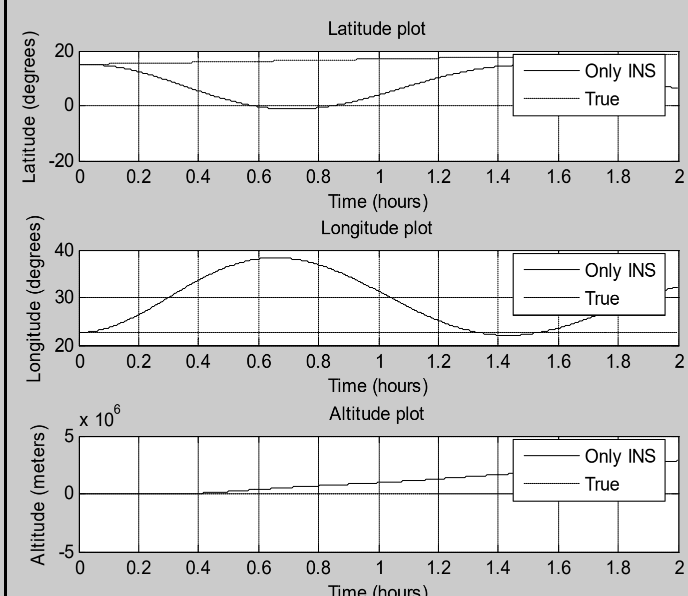
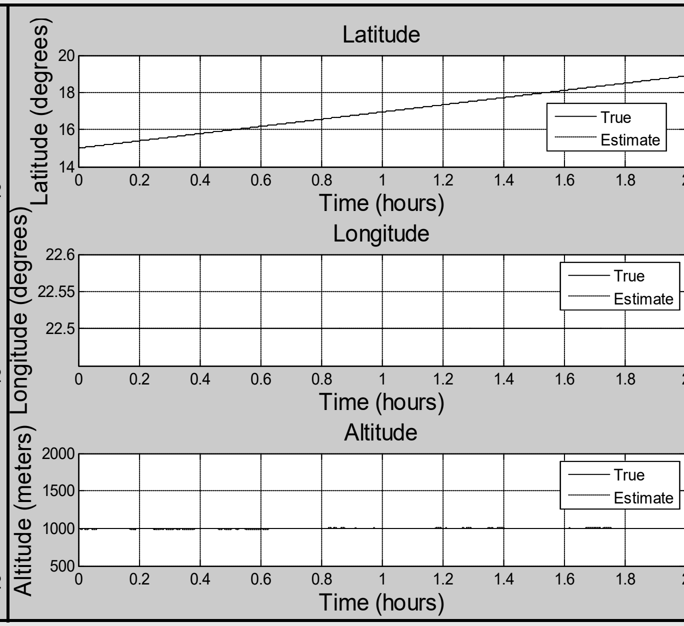
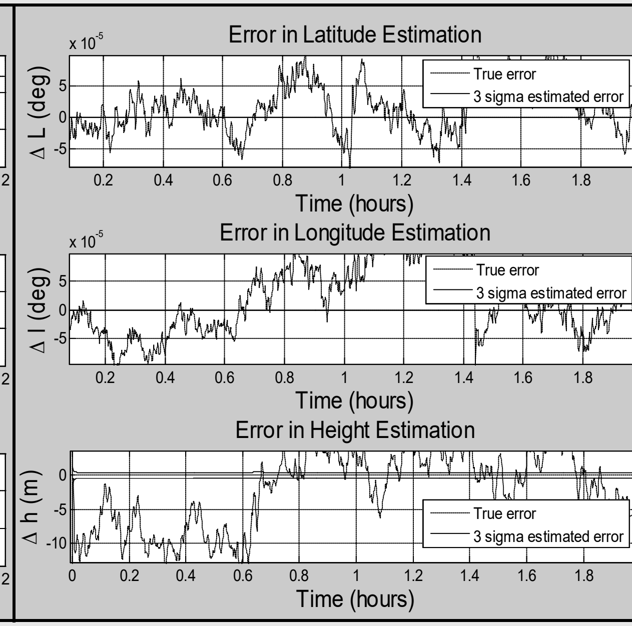
$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} \sqrt{(X_1 - X)^2 + (Y_1 - Y)^2 + (Z_1 - Z)^2} + b_1 + v_1 \\ \sqrt{(X_2 - X)^2 + (Y_2 - Y)^2 + (Z_2 - Z)^2} + b_2 + v_2 \\ \sqrt{(X_3 - X)^2 + (Y_3 - Y)^2 + (Z_3 - Z)^2} + b_3 + v_3 \end{pmatrix}$$

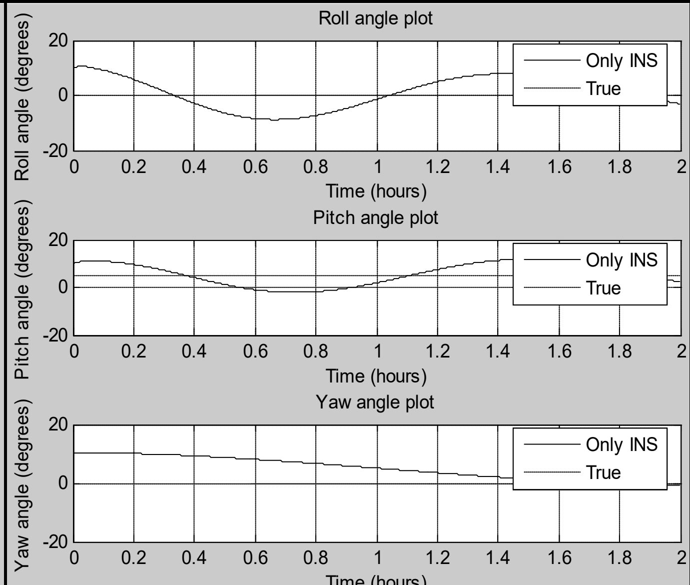
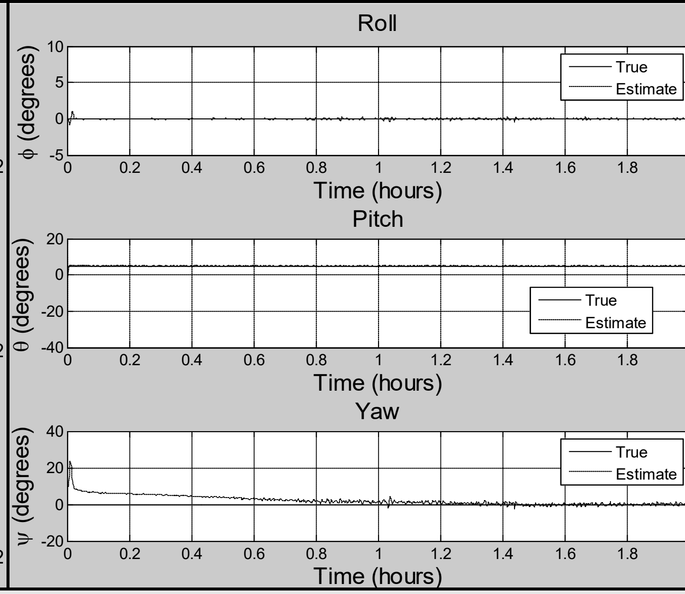
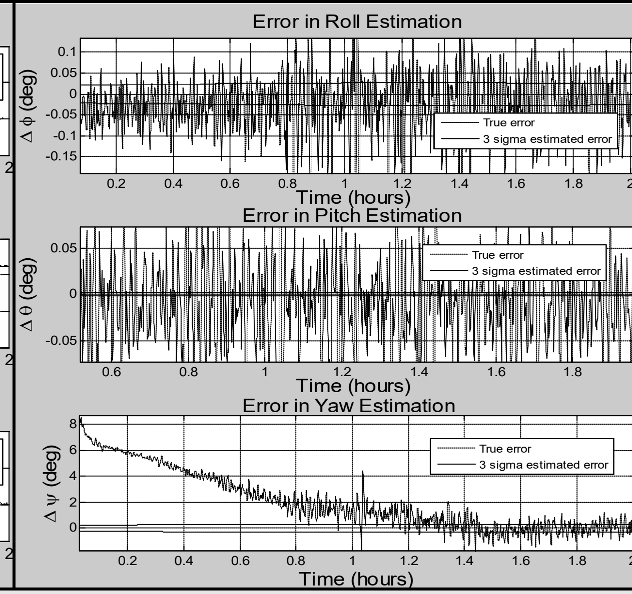
1. **Misra, Pratap, Enge, Per (2006): '*Global Positioning System : Signals, Measurements, and Performance*'**
2. **Hablani, Hari B., (2009), '*Autonomous Inertial Relative Navigation with Sight-Line-Stabilized Integrated Sensors for Spacecraft Rendezvous*', Journal of Guidance, Control, and Dynamics, Vol. 32, No.1, January-February, 2009**

EXTENDED KALMAN FILTER (EKF) BASED GPS/INS INTEGRATION



RESULTS

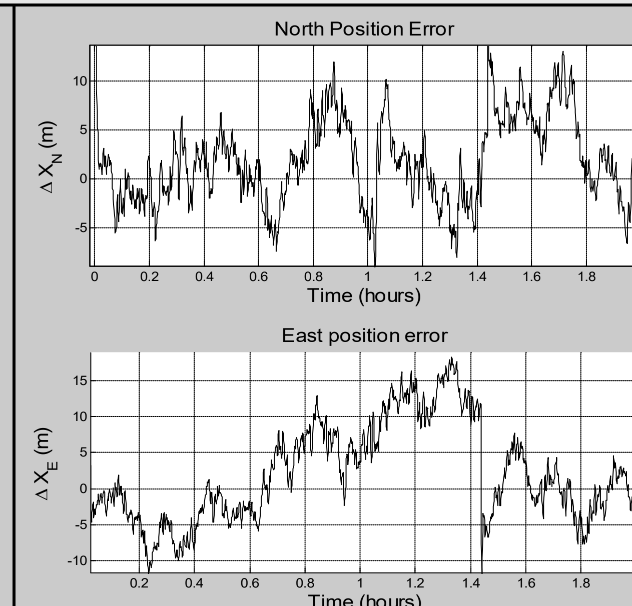
	Only INS	EKF Based GPS/INS Integration	Error in Estimated State from True State
Velocity			
	0.2 m/sec error in Velocity estimation		
Position			
	15 m error in Position estimation		

	Only INS	EKF Based GPS/INS Integration	Error in Estimated State from True State
Attitude			

0.1 deg error in Attitude estimations

North and East Position Error

15 m error in north and east position estimation



THANK YOU