Computational Neuroscience and Neurotheory: Homework 1

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1 Question 1.

(a) The total membrane capacitance is given by multiplying the surface area and capacitance per unit of area.

$$C_m = 0.025 \text{ mm}^2 \times 10 \text{ nF/mm}^2 = 0.25 \text{ nF}$$

(b) The total membrane resistance is given by dividing the resistance per unit area by the total surface area.

$$R_m = \frac{1 \text{ M}\Omega \cdot \text{mm}^2}{0.025 \text{ mm}^2} = 40 \text{ M}\Omega$$

(c) The membrane time constant is given by

$$\begin{aligned} \tau_m &= R_m \times C_m \\ &= 40 \text{ M}\Omega \times 0.25 \text{ nF} \\ &= 10 \text{ ms} \end{aligned}$$

(d) The amount of current required to change the voltage from -70 mV to -65 mV can be calculated using Ohm's law. The amount of current required for a $\Delta V = 5$ mV change is given by

$$I_{\text{ext}} = \frac{\Delta V}{R_m} = \frac{5 \text{ mV}}{40 \text{ M}\Omega} = 0.125 \text{ nA}$$

(e) The timecourse for the change in voltage is given by

$$V(t) = E_L + R_m I_{\text{ext}} + (V(t_0) - E_L - R_m I_{\text{ext}}) \exp\left(\frac{-t}{\tau_m}\right)$$

Set $E_L = -70 \text{ mV}, \tau_m = 10 \text{ ms}, R_m = 40 \text{ M}\Omega, I_{\text{ext}} = 0.125 \text{ nA}$ and

$$\begin{aligned} -67 &= -70 + (40)(0.125) + (-70 - (-70) - (40)(0.125)) \exp\left(\frac{-t}{10}\right) \\ 2 &= -5 \exp\left(\frac{-t}{10}\right) \\ \frac{2}{5} &= \exp\left(\frac{-t}{10}\right) \\ \log\frac{2}{5} &= \frac{-t}{10} \\ t &= 10 \log\frac{2}{5} \\ t &= 9.16 \text{ ms} \end{aligned}$$

2 Question 2.

The voltage traces for an IF neuron with different external currents are shown in figure 1. The theoretical prediction for the firing rates as a function of external current follows closely with the firing rates of the IF neuron (Figure 2).

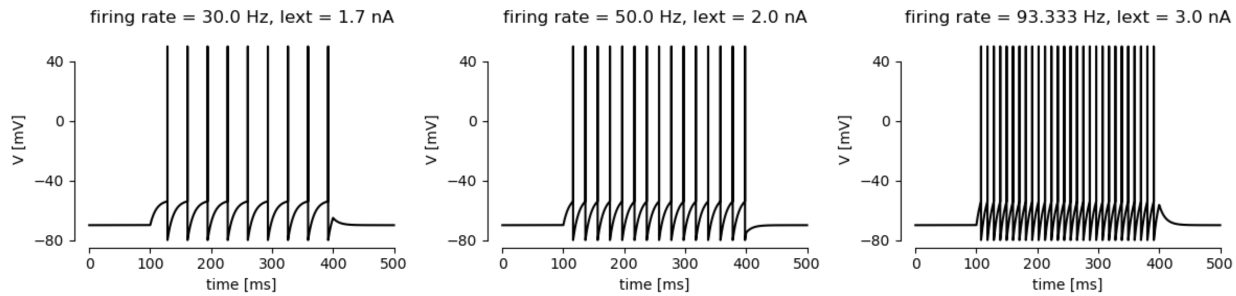

Figure 1: Voltage traces for the IF neuron with different external currents.

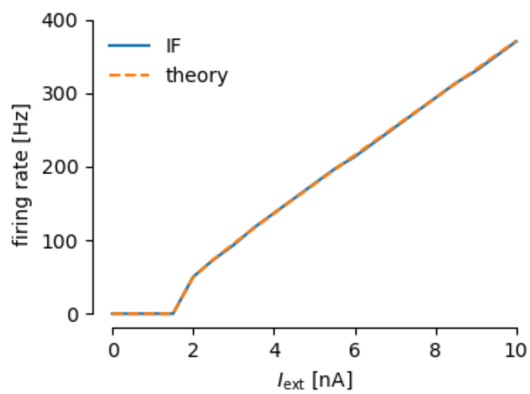

Figure 2: Relationship between external current and firing rate in the IF neuron and theoretical prediction.

3 Question 3.

(a) The plots for V, m, n and h for a $I_{\text{ext}}/A = 200$ nA/mm^2 are shown in Figure 3.

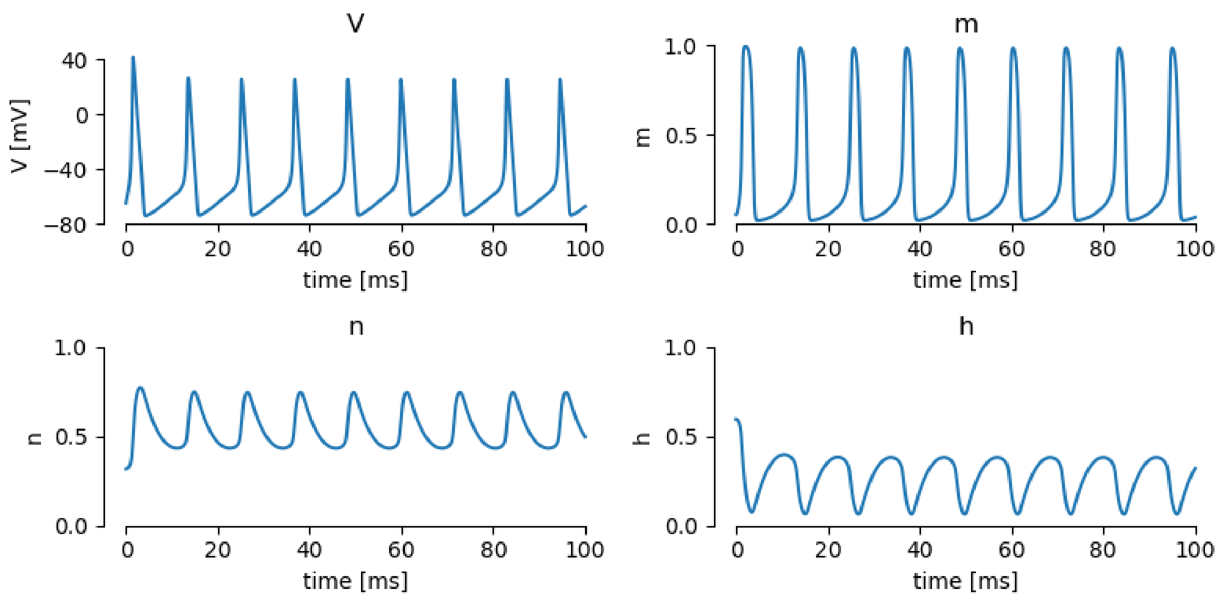

Figure 3: Hodgkin-Huxley model for an external current of $I_{\text{ext}}/A = 200$ nA/mm^2

(b) The relationship between the external current and the firing rate is shown in Figure 4. The relationship here is not linear, and the function appears to be stepwise, where there are significant jumps in the firing rate when the external current is increased. Compared to the IF model, the HH model does not follow a linear relationship between current and firing rate.

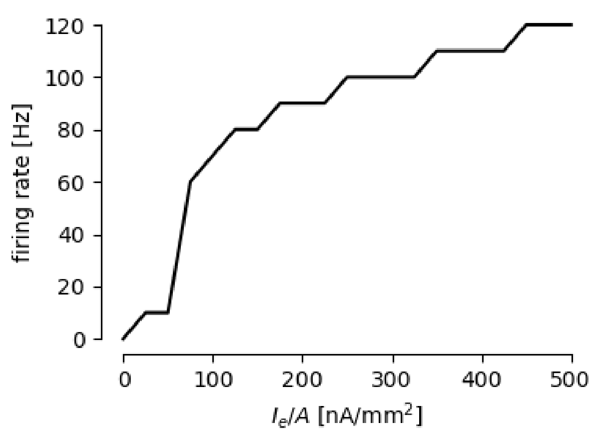

Figure 4: Relationship between external current and firing rate in the HH neuron and theoretical prediction.

(c) Injection of a -50 nA/mm^2 into an HH neuron for 5 ms first leads to hyperpolarisation, followed by the production of a spike. This is because the channel gating lags behind compared to the voltage, which leads to the spike production.

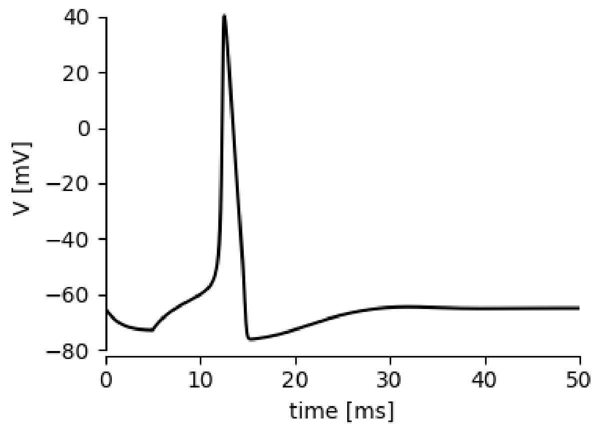

Figure 5: Hodgkin-Huxley model for an external current of $I_{\text{ext}}/A = -50$ nA/mm^2.