Computational Neuroscience and Neurotheory: Homework 1

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1 Question 1.

(a) The total membrane capacitance is given by multiplying the surface area and capacitance per unit of area.

$$C_m = 0.025 \text{ mm}^2 \times 10 \text{ nF/mm}^2 = 0.25 \text{ nF}$$

(b) The total membrane resistance is given by dividing the resistance per unit area by the total surface area.

$$R_m = \frac{1 \text{ M}\Omega \cdot \text{mm}^2}{0.025 \text{ mm}^2} = 40 \text{ M}\Omega$$

(c) The membrane time constant is given by

$$\begin{aligned}
\tau_m &= R_m \times C_m \\
&= 40 \text{ M}\Omega \times 0.25 \text{ nF} \\
&= 10 \text{ ms}
\end{aligned}$$

(d) The amount of current required to change the voltage from -70 mV to -65 mV can be calculated using Ohm's law. The amount of current required for a $\Delta V = 5$ mV change is given by

$$I_{\text{ext}} = \frac{\Delta V}{R_m} = \frac{5 \text{ mV}}{40 \text{ M}\Omega} = 0.125 \text{ nA}$$

(e) The timecourse for the change in voltage is given by

$$V(t) = E_L + R_m I_{\text{ext}} + (V(t_0) - E_L - R_m I_{\text{ext}}) \exp\left(\frac{-t}{\tau_m}\right)$$

Set $E_L = -70 \text{ mV}, \tau_m = 10 \text{ ms}, R_m = 40 \text{ M}\Omega, I_{\text{ext}} = 0.125 \text{ nA}$ and

$$\begin{aligned}
-67 &= -70 + (40)(0.125) + (-70 - (-70) - (40)(0.125)) \exp\left(\frac{-t}{10}\right) \\
2 &= -5 \exp\left(\frac{-t}{10}\right) \\
\frac{2}{5} &= \exp\left(\frac{-t}{10}\right) \\
\log \frac{2}{5} &= \frac{-t}{10} \\
t &= 10 \log \frac{2}{5} \\
t &= 9.16 \text{ ms}
\end{aligned}$$

2 Question 2.

The voltage traces for an IF neuron with different external currents are shown in figure 1. The theoretical prediction for the firing rates as a function of external current follows closely with the firing rates of the IF neuron (Figure 2).

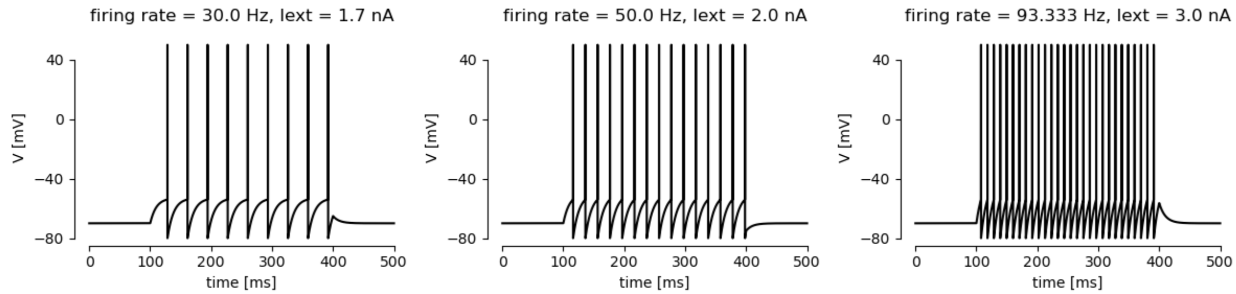

Figure 1: Voltage traces for the IF neuron with different external currents.

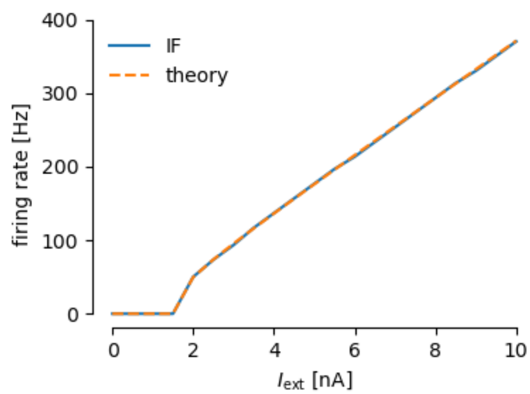

Figure 2: Relationship between external current and firing rate in the IF neuron and theoretical prediction.

3 Question 3.

(a) The plots for V, m, n and h for a $I_{\text{ext}}/A = 200$ nA/mm^2 are shown in Figure 3.

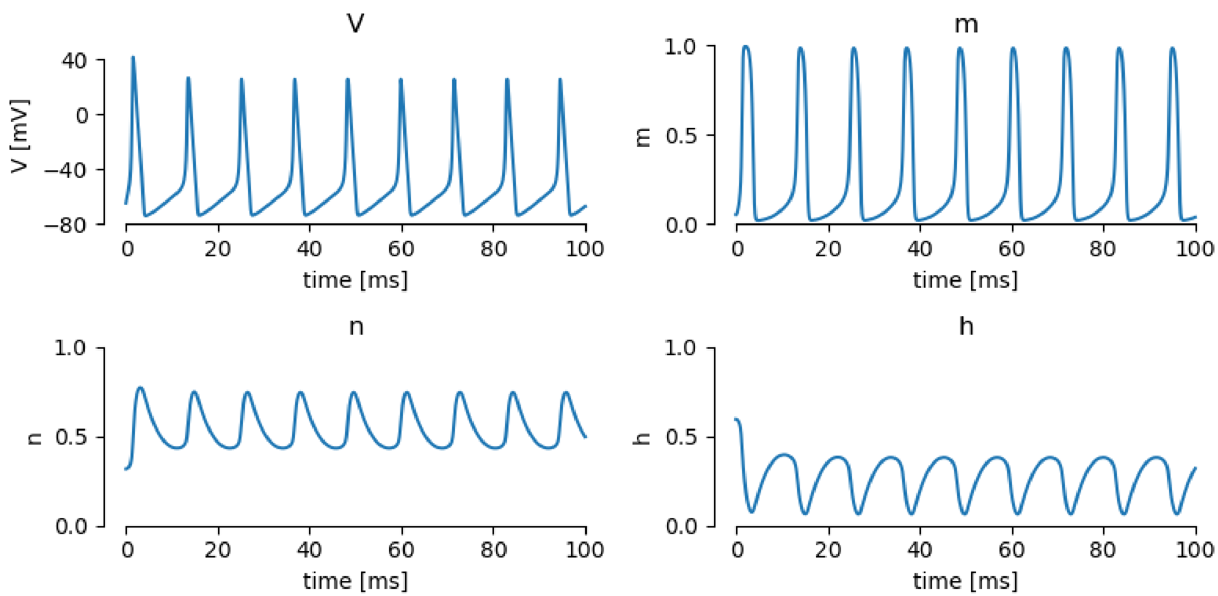

Figure 3: Hodgkin-Huxley model for an external current of $I_{\text{ext}}/A = 200$ nA/mm^2

(b) The relationship between the external current and the firing rate is shown in Figure 4. The firing rate is low for low currents, and then there is a sharp increase at $I_e/A = 100$ nA/mm^2. After which, the firing rate appears to increase linearly, with a shallow slope. This slow increase is due to the small number of spikes the HH neuron can add due to the presence of a refractory period. In contrast, the firing rate of the IF model increases linearly as a function of current after a minimum current is added to the neuron.

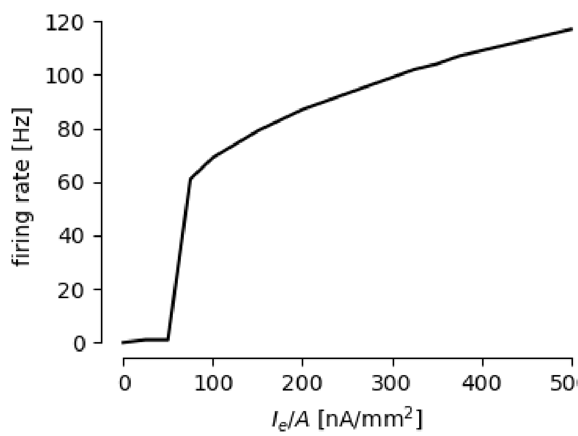

Figure 4: Relationship between external current and firing rate in the HH neuron and theoretical prediction.

(c) Injection of a -50 nA/mm^2 into an HH neuron for 5 ms first leads to hyperpolarisation, followed by the production of a spike. This is because the channel gating lags behind compared to the voltage, which leads to the spike production. This can also be seen in the coefficients that drive the gating of the channel, m, n and h.

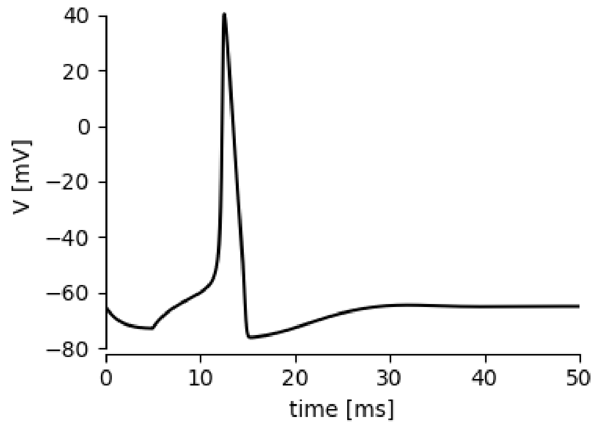

Figure 5: Hodgkin-Huxley model for an external current of $I_{\text{ext}}/A = -50$ nA/mm^2.