1. Construct an integrate-and-fire model with an excitatory synaptic conductance similar to the model used in the previous assignment:

$$c_m \frac{dV}{dt} = -\overline{g}_L(V - E_L) - g_{ex}(V - E_{ex})$$

with $c_m = 10$ nF/mm^2, $\overline{g}_L = 1.0$ μS/mm^2, $E_L = -70$ mV, and $E_{ex} = 0$. Also, as before, the threshold and reset potentials for the model are $V_{th} = -54$ mV and $V_{reset} = -80$ mV, and the excitatory synaptic conductance decays exponentially to zero:

$$\tau_{ex} \frac{dg_{ex}}{dt} = -g_{ex}$$

However, this time use $\tau_{ex} = 5$ ms (last time it was 10 ms).

We imagine that the neuron receives synaptic input from two sources. First, whenever an unconditioned stimulus is presented,

$$g_{ex} \rightarrow g_{ex} + \Delta g_{US}$$

with $\Delta g_{US} = 1.2$ μS/mm^2. Whenever a conditioned stimulus is presented,

$$g_{ex} \rightarrow g_{ex} + \Delta g_{CS} .$$

The quantity Δg_{CS} is subject to spike-timing dependent synaptic plasticity. This means that at the time of every postsynaptic spike (a spike fired by the integrate-and-fire neuron that you are modeling), Δg_{CS} gets incremented by

$$\Delta g_{CS} \rightarrow \Delta g_{CS} + A_{LTP} \exp(-\Delta t_{LTP}/\tau_{LTP})$$

where Δt_{LTP} is the time since the most recent previous presynaptic spike activated by the conditioned stimulus. Similarly, every time there is a presynaptic spike activated by the conditioned stimulus, Δg_{CS} gets decremented by

$$\Delta g_{CS} \rightarrow \Delta g_{CS} - A_{LTD} \exp(-\Delta t_{LTD}/\tau_{LTD})$$

where Δt_{LTD} is the time since the most recent previous postsynaptic spike. Take $A_{LTP} = 0.35$ μS/mm^2, $A_{LTD} = 0.4$ μS/mm^2, $\tau_{LTP} = 25$ ms, and $\tau_{LTD} = 35$ ms. Restrict Δg_{CS} to the range $0 \leq \Delta g_{CS} \leq 1.2$ μS/mm^2. This means that if the LTP rule above tries to set Δg_{CS} to a value greater than 1.2 μS/mm^2, you should just set $\Delta g_{CS} = 1.2$ μS/mm^2. Similarly, if the LTD rule above tries to set Δg_{CS} to a value less than 0, you should just set $\Delta g_{CS} = 0$.

Simulate the following two 1-second long runs. In both cases, set V initially to E_L and g_{ex} initially to 0. For the first run, set Δg_{CS} initially to 0 and simulate the presentation of the unconditioned stimulus at times 100, 200, 300, 400, 500, and 600 ms after the start of the run. Simulate the presentation of the conditioned stimulus at times 90, 190, 290, 390, 490, 590, 690, 790, and 890 ms after the start of the run. Describe what happens and relate it to classical conditioning. For the second run, set Δg_{CS} initially to 1 μS/mm^2 and simulate the presentation of the unconditioned stimulus at times 100, 200, 300, 400, 500, and 600 ms after the start of the run. Simulate the presentation of the conditioned stimulus at times 110, 210, 310, 410, 510, 610, 710, 810, and 910 ms after the start of the run. Describe what happens and relate it to classical conditioning.

2. Consider a spike train being transmitted from two different synapses coming from the same presynaptic neuron. Generate the presynaptic spike train by choosing a rate r (see below for values) and generate a spike on every time step Δt of your program with probability $r\Delta t$ (be careful about units: if r is in Hz, Δt must be expressed in seconds not milliseconds), Everytime there is a presynaptic spike, each synapses releases transmitter with a probability P, computed as described below. For both synapses, P recovers exponentially to a resting value P_0,

$$\tau_P \frac{dP}{dt} = P_0 - P\,.$$

However, the synapses are different.

One synapse is a depressing synapse, with $\tau_P = 300$ ms and $P_0 = 1$. For this synapse, P gets reduced to the value 0,

$$P \to 0$$

every time there is a release of transmitter from the synapse.

The other synapse is a facilitating synapse, with $\tau_P = 100$ ms and $P_0 = 0$. For this synapse, P gets increased by an amount

$$P \to P + 0.1(1 - P)$$

every time there is a presynaptic spike (whether or not it results in a transmittion).

Make a stacked set of plots showing the following five quantities: 1) an indication of when the presynaptic spikes occurred, and indication of when transmissions occurred from the depressing synapse, 2) an indication of when transmissions occurred from the facilitating synapse, 3) the release probability at the depressing synapses as a function of time, and 4) the release probability at the facilitating synapse as a function of time. At each start of the run, set the P value for each synapse to its corresponding P_0 value (in other words, set the initial values as $P = 1$ for the depressing synapse and $P = 0$ for the facilitating synapse). Show a 1 second long sequence for constant rates of 10, 50, and 100 Hz. Also show the results for a firing rate that is zero except for a burst at $r = 100$ Hz between times 500 and 600 ms after the start of the run. Then, plot the rate of transmission (the number of transmissions divided by the length of time over which you count them) for both synapses as a function of the presynaptic firing rate over the range from $r = 0$ to $r = 100$ Hz. You will need to collect enough random events for this plot so that the points are not excessively scattered, which make take runs longer than 1 second. Describe the differences in the signals transmitted by these two types of synapses in relation to the underlying presynaptic spike train, using these and other examples that you create with other kinds of presynaptic spike sequences to illustrate your points.