Q 1.1

Using the fundamental matrix estimation,

$$\begin{bmatrix} x'_m & y'_m & 1 \end{bmatrix} \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ 1 \end{bmatrix} = 0$$

which translates into

$$\begin{aligned} x_m x'_m f_1 + x_m y'_m f_2 + x_m f_3 + \\ y_m x'_m f_4 + y_m y'_m f_5 + y_m f_6 + \\ x'_m f_7 + y'_m f_8 + f_9 = 0 \end{aligned}$$

where f_9 is the F_{33} element of the fundamental matrix.

Since (0,0) coincides with the principal point, one such point satisfies the above equation:

$x_m = 0$ $y_m = 0$ $x_m' = 0$ $y_m' = 0$

Putting these values in the above equation,

$f_9 = 0$ => $F_{33} = 0$

Q 1.2

Since the second camera differs from the first by a pure translation that is parallel to x-axis,

R = I

t = (T,0,0)

$$E = t \times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$$x^T E x' = 0$$

$$(u\ v\ 1) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} \mathrm{u}' \\ v' \\ 1 \end{bmatrix} = 0$$

$$(u\ v\ 1) \begin{bmatrix} 0 \\ -T \\ Tv' \end{bmatrix} = 0$$

⇨ Tv = Tv’

Therefore, the y coordinate is always the same. Hence, the epipolar lines are horizontal.

Q 1.3

Let us consider two timestamps t_1 and t_2,

$$x_1' = R_1 (x-t_1)$$
$$x_2' = R_2 (x-t_2)$$

Replacing the value of x, we get,

$$\Rightarrow \quad x_2' = R_2 R_1^T (x_1' - R_1(t_2 - t_1))$$

Hence,

$$R_{rel} = R_2 R_1^T$$
$$T_{rel} = R_1(t_2 - t_1))$$

$$E = R[t_x]$$

$$\text{Let } T_{rel} = \begin{bmatrix} txrel \\ tyrel \\ tzrel \end{bmatrix}$$

$$E = R_{rel} [T_{relx}] = R_2 R_1^T \begin{bmatrix} 0 & -tzrel & tyrel \\ tzrel & 0 & -txrel \\ -tyrel & txrel & 0 \end{bmatrix}$$

$$F = K'^T E K^{-1}$$

$$F = K'^T R_{rel} [T_{relx}] K^{-1} = K'^T R_2 R_1^T \begin{bmatrix} 0 & -tzrel & tyrel \\ tzrel & 0 & -txrel \\ -tyrel & txrel & 0 \end{bmatrix} K^{-1}$$

Q 1.4

Let P_1 and P_2 be the vectors from camera C to the real object P and it's reflection P' respectively.

Therefore, we get

$$P_1 + T = P_2$$

$$P_1 = P_2 - T$$

Since P_1, P_2 and T are coplanar,

$$(P_2 - T)^T \, T \times P_2 = 0$$

$$(P_2 - T)^T \, [t_x] \, P_2 = 0$$

$$\text{where } [t_x] = \begin{bmatrix} 0 & -Tz & Ty \\ Tz & 0 & -Tx \\ -Ty & Tx & 0 \end{bmatrix}$$

Therefore,

$$P_1^T \, [t_x] \, P_2 = 0$$

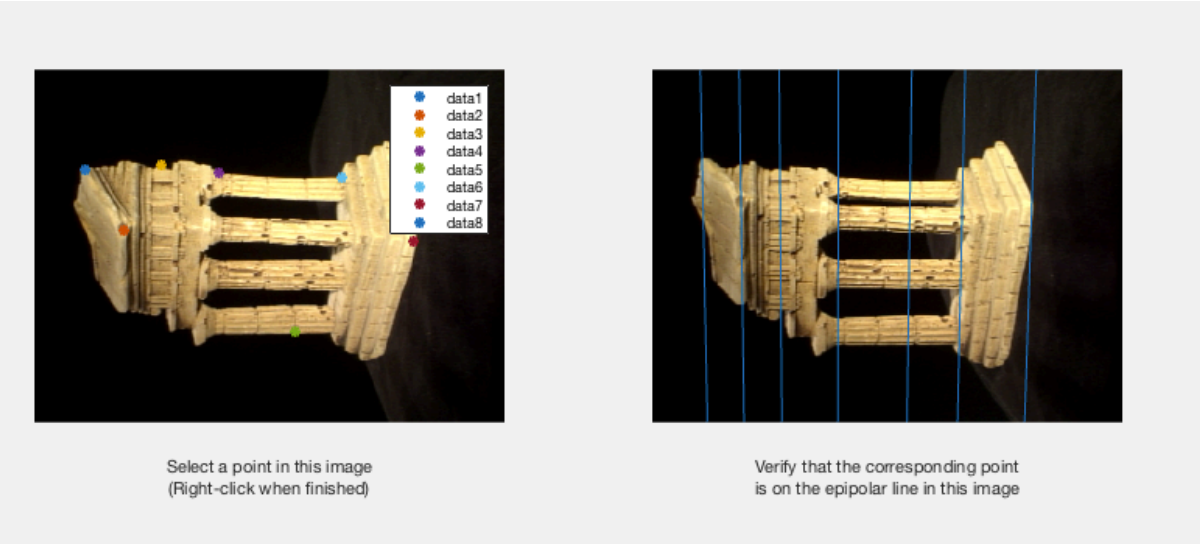
$$\text{Since } P_1^T \, E \, P_2 = 0,\ E = [t_x]$$

$$F = K^{-T} \, E \, K^{-1}$$

$$F^T = K^{-T} \, E^T \, K^{-1} = -\, K^{-T} \, E \, K^{-1} = -F$$

Since $F^T = -F$, the two images of the object are related by a skew-symmetric fundamental matrix.

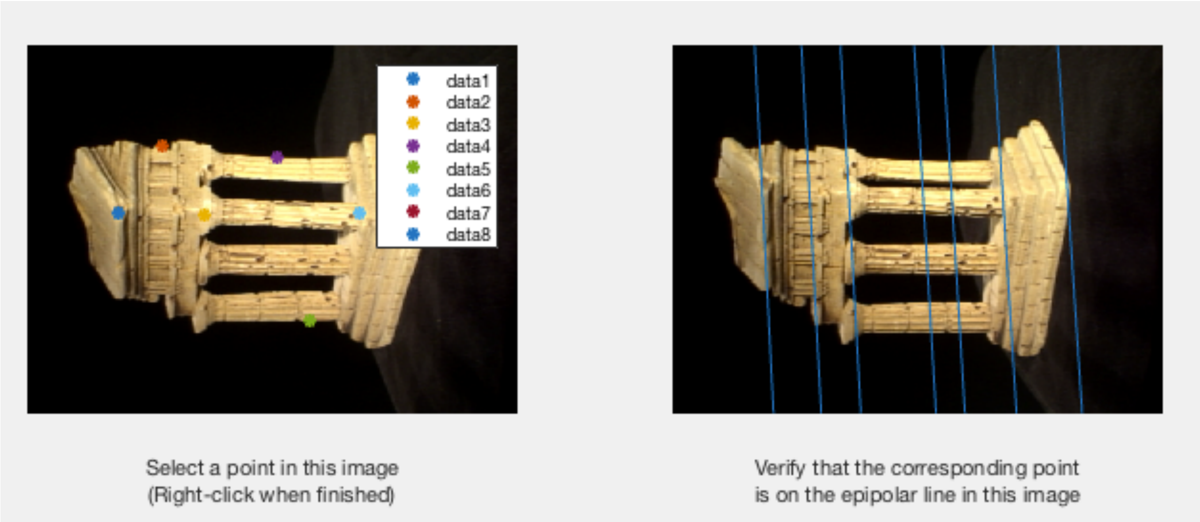
Q 2.1



F =

-0.0000	-0.0000	-0.0011
-0.0000	0.0000	0.0000
0.0011	-0.0000	-0.0042

Q 2.2



F{1}

ans =

```
  -0.0000    0.0000   -0.0014
   0.0000   -0.0000    0.0001
   0.0014   -0.0001    0.0040
```

Q 3.1

E =

$$\begin{matrix} -0.0520 & 0.4314 & -2.1412 \\ 0.1270 & -0.0282 & 0.1256 \\ 2.1450 & -0.0511 & 0.0017 \end{matrix}$$

Q 3.2

A is represented by the following equation:

$$\begin{bmatrix} y\boldsymbol{p}_3^\top - \boldsymbol{p}_2^\top \\ \boldsymbol{p}_1^\top - x\boldsymbol{p}_3^\top \\ y'\boldsymbol{p}_3'^\top - \boldsymbol{p}_2'^\top \\ \boldsymbol{p}_1'^\top - x'\boldsymbol{p}_3'^\top \end{bmatrix} \boldsymbol{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\boldsymbol{X} = \mathbf{0}$$

In Matlab,

A = [p1(i,2)*C1(3,:)-C1(2,:);C1(1,:)-p1(i,1)*C1(3,:);p2(i,2)*C2(3,:)-C2(2,:);C2(1,:)-p2(i,1)*C2(3,:)]

Q 3.3

C2 =

1.0e+03 *

```
    1.5219    0.0205    0.2938    0.0436
    0.0589    1.5365   -0.1585    1.5710
    0.0000    0.0003    0.0010    0.0002
```

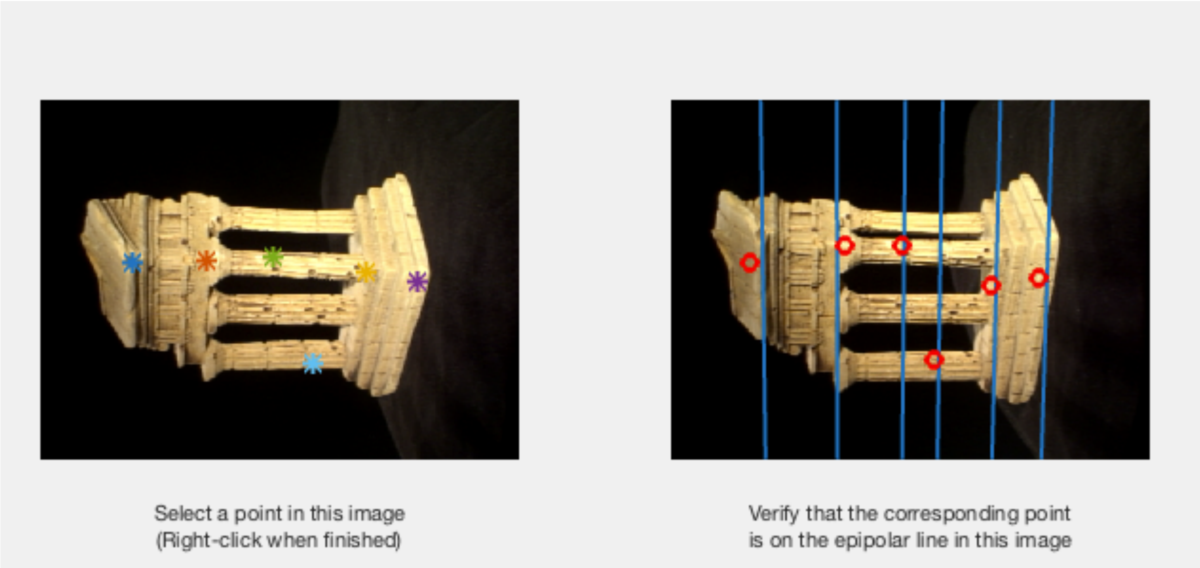
error1 =

987.5777

error2 =

945.3894

Q 4.1



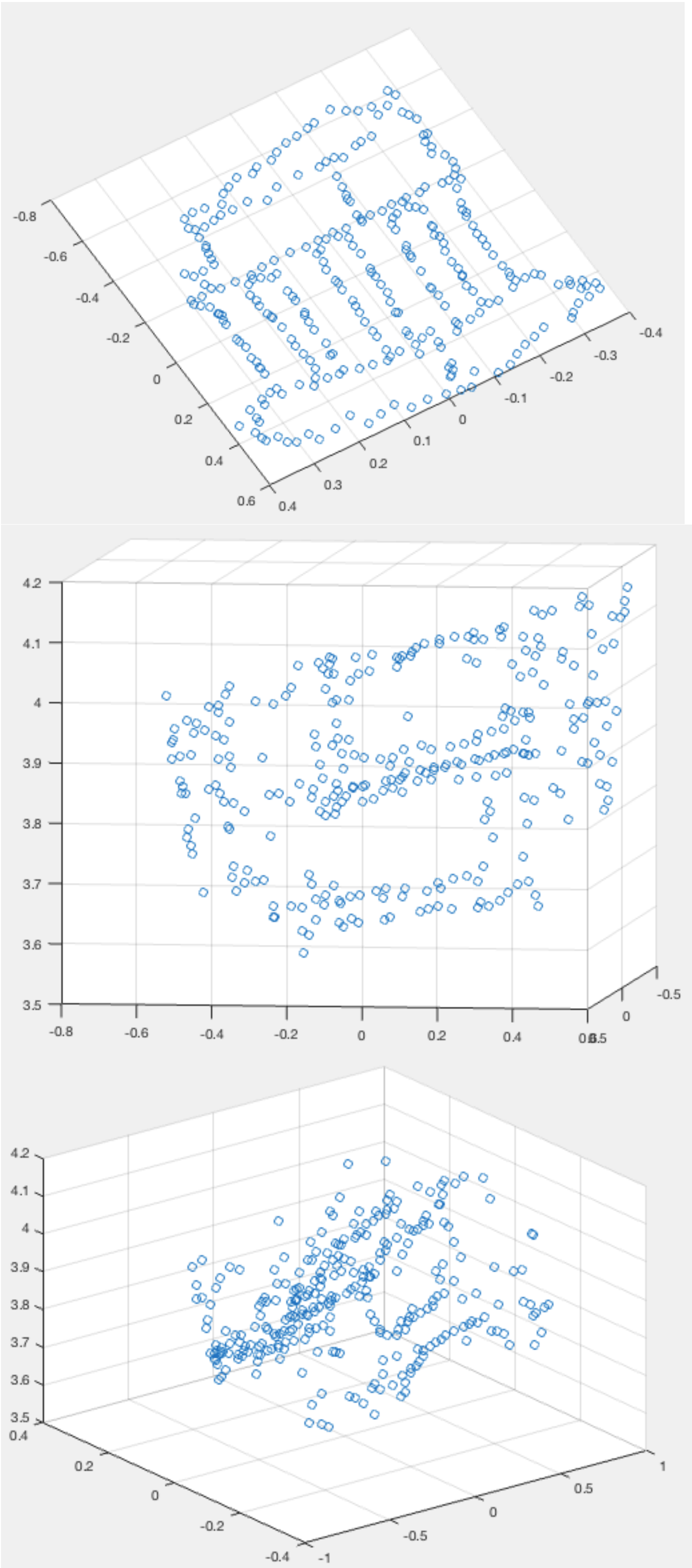
coordsIM1 =

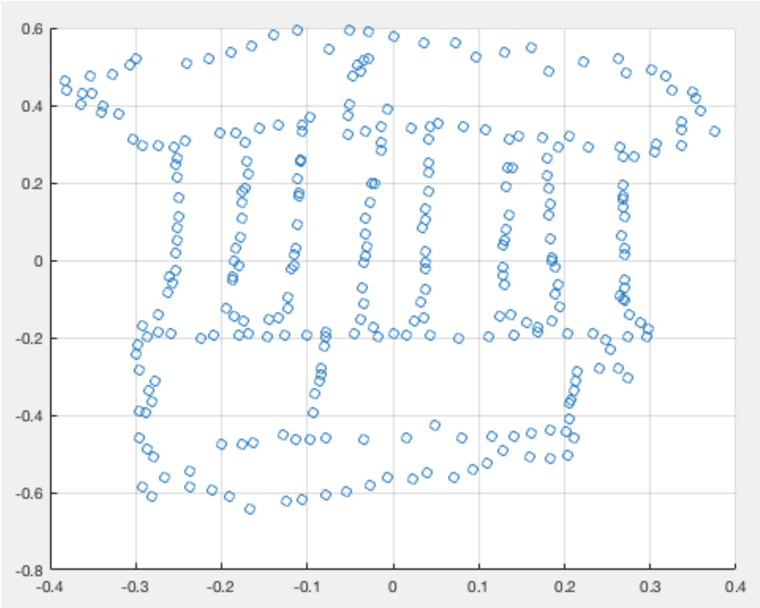
123.6746	217.6429
222.7222	215.1032
436.0556	230.3413
504.6270	243.0397
311.6111	210.0238
364.9444	352.2460

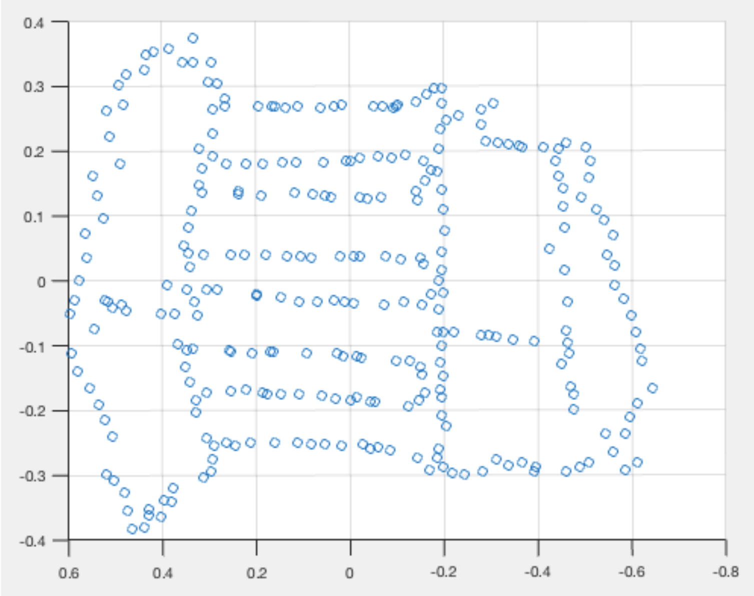
coordsIM2 =

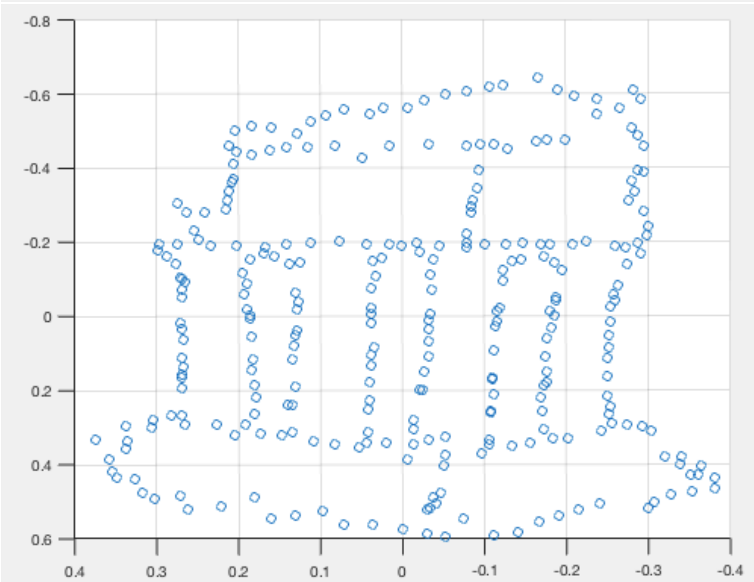
107	217
233	195
428	248
491	238
309	196
353	347

Q 4.2



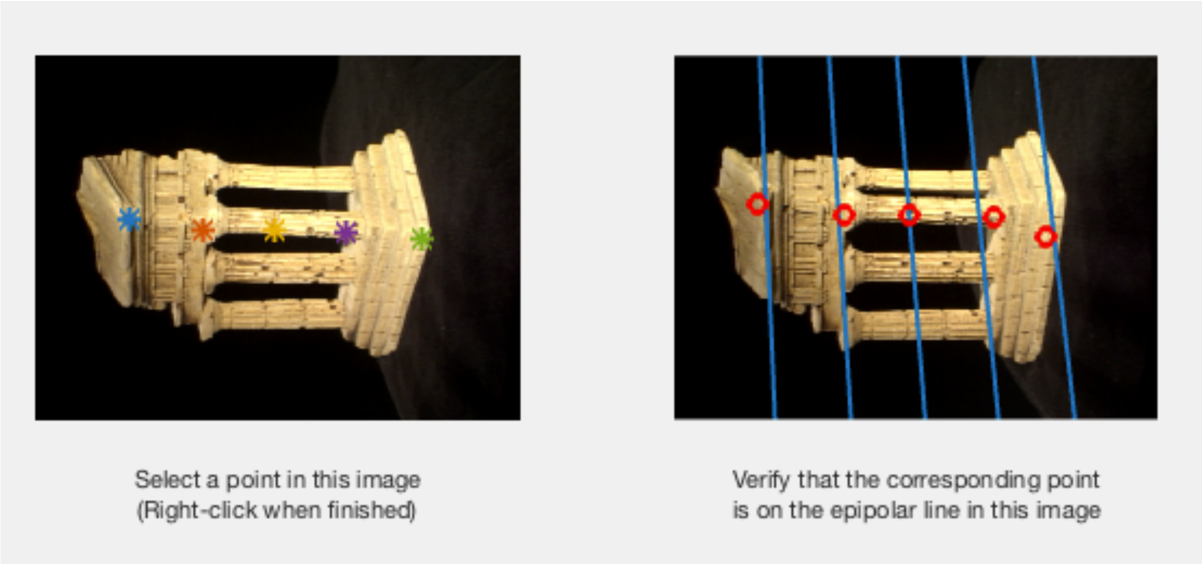

0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8

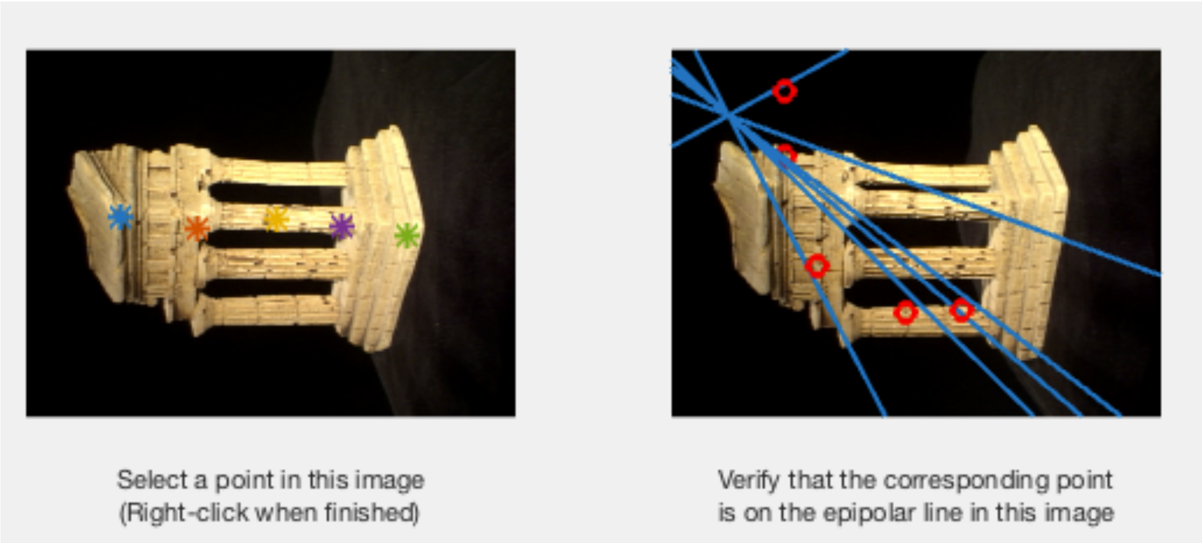

-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4

Q 5.1

Output from RANSAC with noisy correspondence



Output from Eightpoint



As it can be observed, output from RANSAC is much better than eightpoint.
For RANSAC, the metric for computing error used is the distance between the point and the mapped epipolar line found using sevenpoint for each F. The threshold used for inliers is 0.03 and the loop is run for 75% inliers. Since RANSAC has mostly correct matches, it gives a better result.

Q5.2

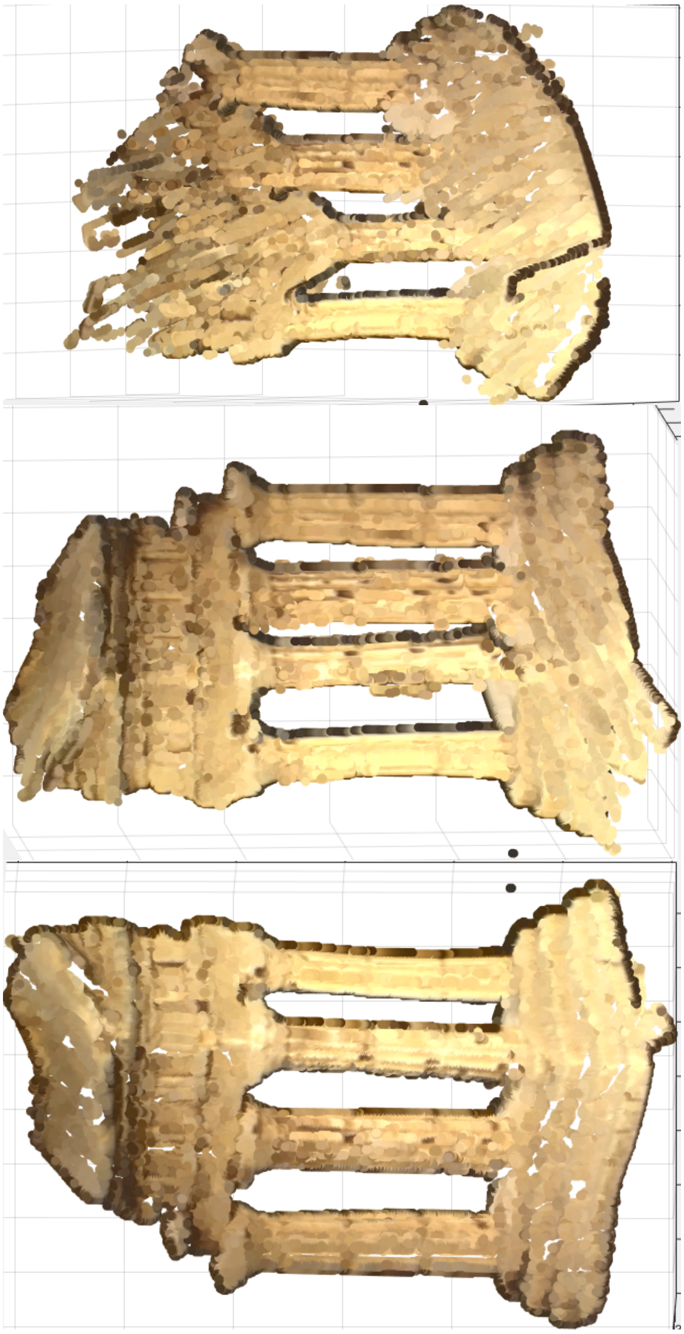
See code

Case tested:

```
r = [2 1 2]
R =
  -0.1056    0.5363    0.8374
   0.3481   -0.7689    0.5363
   0.9315    0.3481   -0.1056
```

Verified with the in-built functions rotationMatrixToVector() and rotationVectorToMatrix()

Q 6.1



These images were taken using 3-D Point Cloud Processing in the Computer Vision System Toolbox in Matlab. ([http://www.mathworks.com/help/vision/examples/structure-from-motion-from-two-views.html)](http://www.mathworks.com/help/vision/examples/structure-from-motion-from-two-views.html)

The key idea is to generate more points for the 3D reconstruction of the image and record each pixel value. The output P of triangulation along with the pixels are sent to the pointCloud function which generates the given plots.