Q1.1 Homography

Ans. The two 3 × 4 camera projection matrices M1 and M2 corresponding to cameras C1, C2 are represented by the following:

$$x = M_1 X \qquad (1)$$

$$x' = M_2 X \qquad (2)$$

The camera matrix can be decomposed into separate matrices:

$$M_1 = K_1 R_1 [I \mid -C_1]$$
$$M_2 = K_2 R_2 [I \mid -C_2]$$

(where K_1 and R_1 are 3x3 matrices)

$$K = \begin{bmatrix} f & 0 & px \\ 0 & f & py \\ 0 & 0 & 1 \end{bmatrix}$$

If the camera and world share the same coordinate system, C=0 and no rotation exists.

Rearranging (2) and putting the value of X in (1),

$$x' = K_2 R_2 [I \mid -C_2] K_1^{-1} R_1^{-1} x \qquad (3)$$

$$\Rightarrow \qquad x' = H x \qquad (4)$$

From (4), we see that there exists a 3x3 matrix H which is the homography.

Q1.2 Homography under rotation

Ans. The two cameras separated by a pure rotation are represented by the following:

$$x = K_1 \, [I \; 0] \, X \qquad (1)$$

$$x' = K_2 \, [R \; 0] \, X \qquad (2)$$

Rearranging (2) and putting the value of X in (1),

$$x' = K_2 \, R \, K_1^{-1} \, x \qquad (3)$$

$$\Rightarrow \qquad x' = H \, x \qquad (4)$$

From (4), we see that there exists a 3x3 matrix $H = K_2 \, R \, K_1^{-1}$ which is the homography.

Q1.3 Correspondences

Ans.

1. h has 8 degrees of freedom.

2. 4 point pairs are required to solve h.

3. Let us consider $x_1^i = x'$, $y_1^i = y'$, $x_2^i = x$, $y_2^i = y$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h1 & h2 & h3 \\ h4 & h5 & h6 \\ h7 & h8 & h9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$x' = \alpha(h1 * x + h2 * y + h3)$
$y' = \alpha(h4 * x + h5 * y + h6)$
$1 = \alpha(h7 * x + h8 * y + h9)$

$x' (h7 * x + h8 * y + h9) = h1 * x + h2 * y + h3$
$y' (h7 * x + h8 * y + h9) = h4 * x + h5 * y + h6$

Rearranging terms,

$h7*x*x' + h8*y*x' + h9*x' - h1*x - h2*y - h3 = 0$
$h7*x*y' + h8*y*y' + h9*y' - h4*x - h5*y - h6 = 0$

In matrix form, this can be written as:

$$A_i h = 0$$

$$A_i = \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix}$$

$h = [h1 \quad h2 \quad h3 \quad h4 \quad h5 \quad h6 \quad h7 \quad h8 \quad h9]^T$

Q1.4 Understanding homographies under rotation

Ans. For rotation Θ, homography is given by:

$$H = \begin{bmatrix} cos\Theta & sin\Theta & 0 \\ -sin\Theta & cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H^2 = \begin{bmatrix} (cos\Theta)^2 - (sin\Theta)^2 & 2sin\Theta cos\Theta & 0 \\ -2sin\Theta cos\Theta & (cos\Theta)^2 - (sin\Theta)^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cos2\Theta & sin2\Theta & 0 \\ -sin2\Theta & cos2\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, H^2 is the homography corresponding to rotation of 2Θ

Q1.5 Limitations of the planar homography

Ans. Planar homography will fail to map the scene image to another viewpoint if they are not in the same world plane which has the homography. Also, it will not be able to handle cases where the rotation about the center is not pure.

Q1.6 Behavior of lines under perspective projections

Ans. Any point on the line is 3D with points (x_1',y_1',z_1') and (x_2',y_2',z_2') is given by the following equation:

$X' = \lambda x_1' + (1 - \lambda) x_2'$
$Y' = \lambda y_1' + (1 - \lambda) y_2'$
$Z' = \lambda z_1' + (1 - \lambda) z_2'$

If we project the points to 2D,

$x_1 = P x_1'$
$x_2 = P x_2'$

$x = P X'$
$x = P (\lambda x_1' + (1 - \lambda) x_2') = \lambda Px_1' + (1 - \lambda) Px_2' = \lambda x_1 + (1 - \lambda) x_2$

Therefore, $x = \lambda x_1 + (1 - \lambda) x_2$

Similarly, $y = \lambda y_1 + (1 - \lambda) y_2$

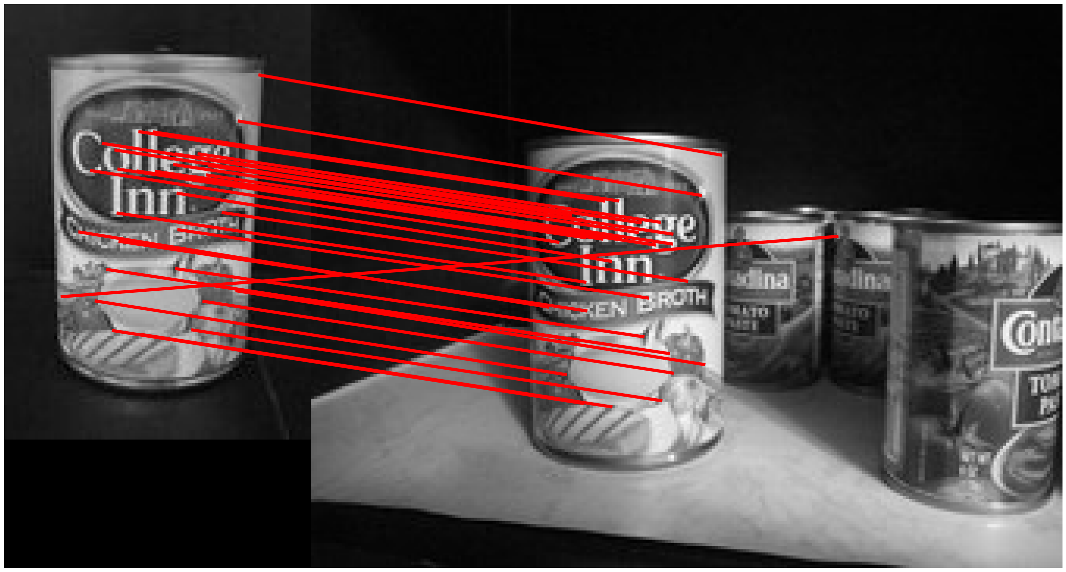
Hence, the projection P preserves the lines.

Q 2.1 See code.

Q 2.2 See code

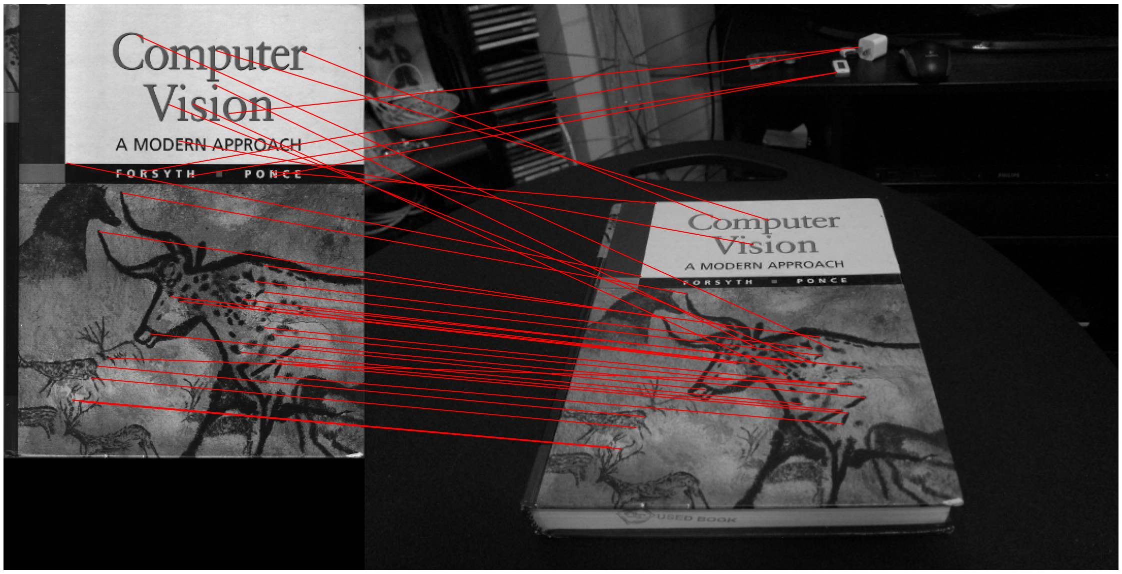
Q 2.3 See code

Q 2.4

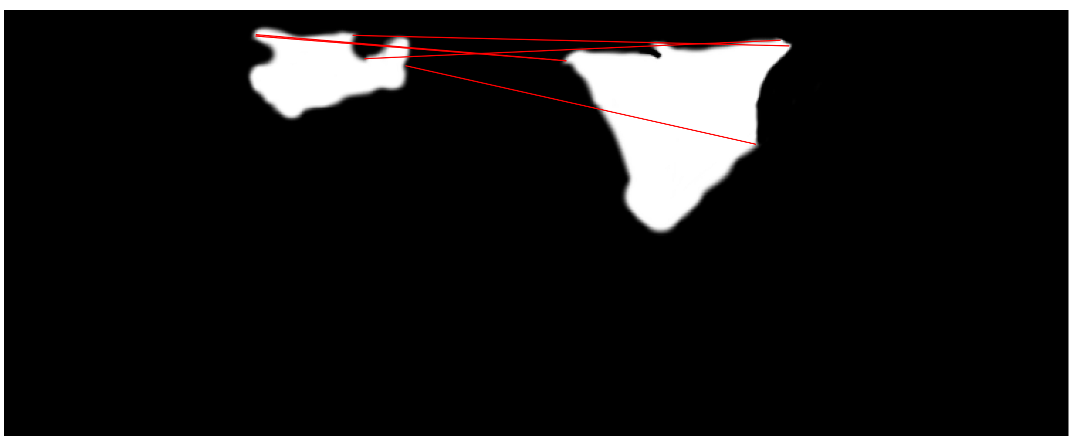


Most of the descriptors are matching for the chickenbroth images.

A very good match if the images as same as the descriptors would match for the same images.



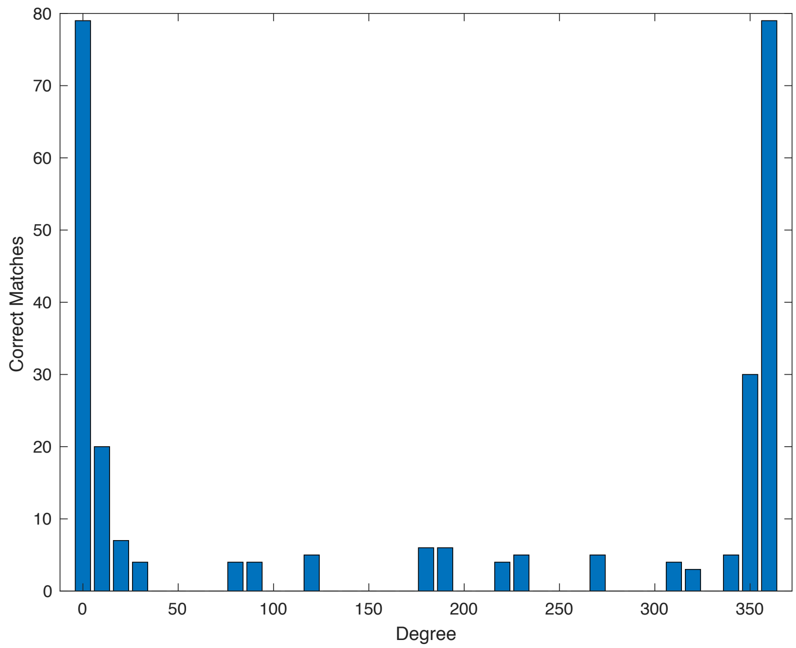
The cv_cover.jpg and cv_desk.png performs slightly worse than the chickenbroth images.



On the mask images, a perfect match as there are very few descriptors.

Q 2.5: Brief and rotation

Ans.



As can be seen from the above graph, the number of correct matches decreases as the image is rotated. This happens because BRIEF is not rotation invariant. As the rotation increases, the interest points rotate and hence the intensity around the interest points changes as well. But since the patch doesn't rotate, the matches reduce drastically.

Q 2.6.

Ans. 1. To make the BRIEF descriptor rotation invariant, we need to rotate the patch to align it with the sampling pairs. This can be done using SIFT/SURF as the keypoint detectors. The sampling pairs can be then rotated according to the patch's orientation and hence the descriptor will become rotation invariant.

Q3.1 Computing the Homography

Ans. See code

Q3.2 Homography with normalization

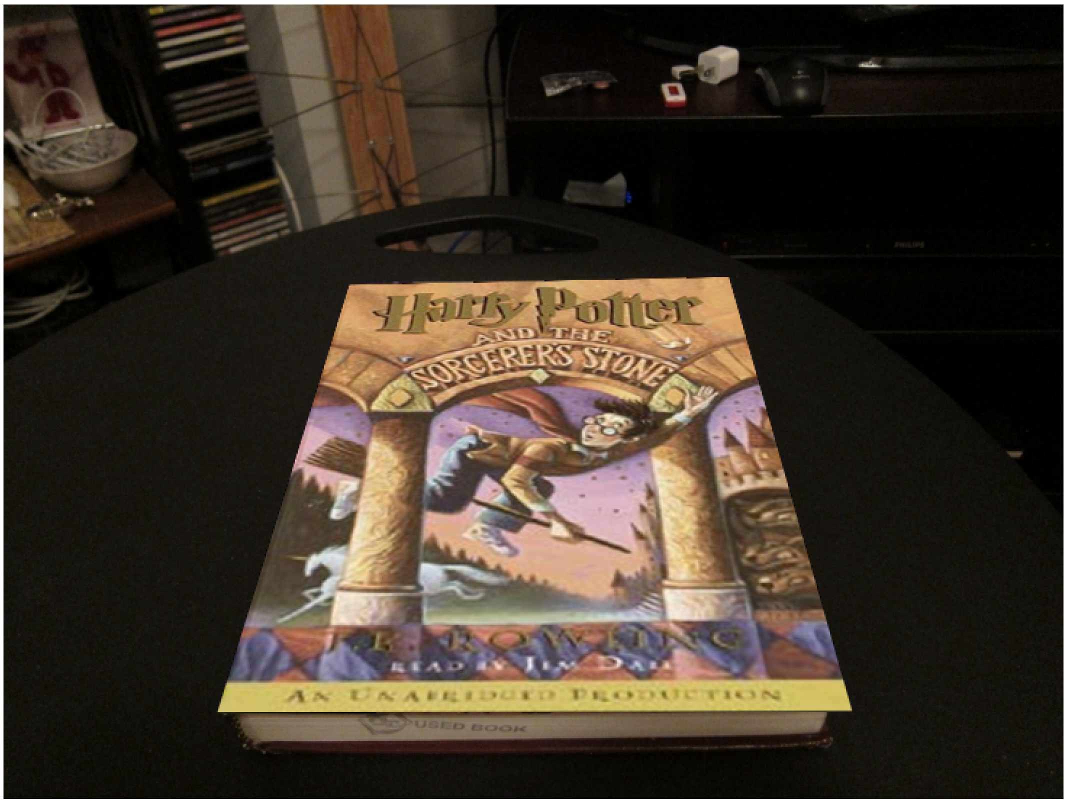
Ans. See code

Q3.3 Implement RANSAC for computing a homography

Ans. See code. Used computeH as it was working well, still implemented the computeH_norm.

Q3.4 Putting it together

Ans.

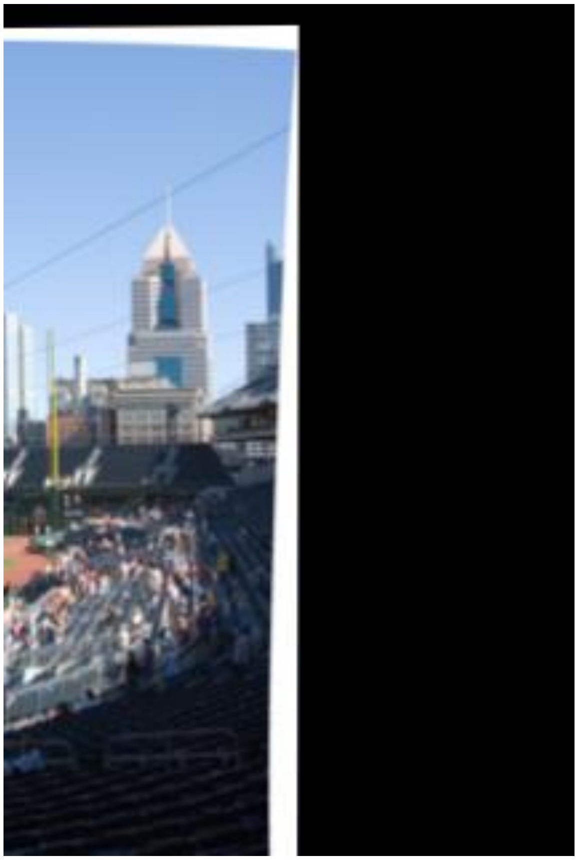


H matrix with bottom right value 1:

$$
\begin{matrix}
0.7249 & -0.3407 & 238.6304 \\
-0.0239 & 0.2268 & 193.7461 \\
-0.0000 & -0.0009 & 1.0000
\end{matrix}
$$

Q 4.1

Ans.



(pnc0.png,pnc1.png)

H matrix with bottom right value 1:

0.9872	-0.0001	-100.7460
-0.0102	0.9969	10.0143
-0.0000	0.0000	1.0000

Q 4.2

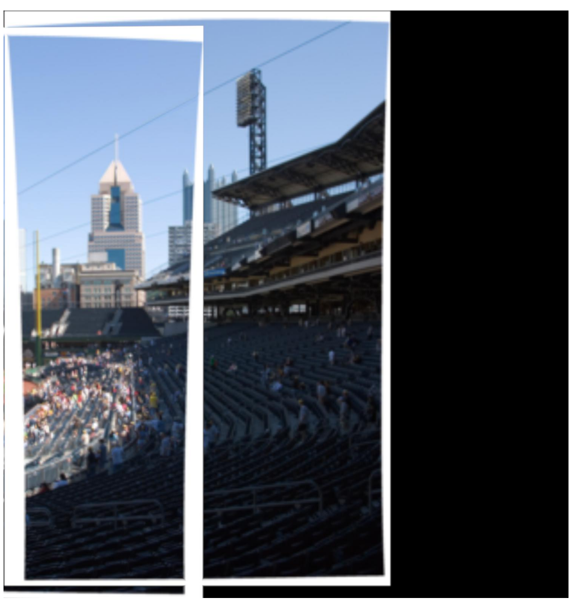
Ans.

(incline_L.png,incline_R.png)

Q 4.3

Ans.

$$H = \begin{matrix} -0.0018 & 0.0001 & -0.9991 \\ 0.0002 & -0.0024 & 0.0429 \\ 0.0000 & 0.0000 & -0.0027 \end{matrix}$$



$$H = \begin{matrix} -0.0097 & 0.0000 & 0.9950 \\ 0.0001 & -0.0098 & -0.0989 \\ 0.0000 & -0.0000 & -0.0099 \end{matrix}$$

Q. 5.1

Ans. Code attached

Using fg1, bg1 and mask1 :

Using bg2, fg2 and mask2 :