. Q2.1 Show that if you use the line equation $x\cos\theta + y\sin\theta - \varrho = 0$, each image point (x, y) results in a sinusoid in (ϱ, θ) Hough space. Relate the amplitude and phase of the sinusoid to the point (x, y).

. Ans. For every point (x,y),

. $x\cos\theta + y\sin\theta = \varrho$

. Sinusoid (ϱ, θ) can be represented as $\varrho = A\sin(\theta+\Phi)$

. Hence, $x\cos\theta + y\sin\theta = A\sin(\theta+\Phi)$

. $x\cos\theta + y\sin\theta = A\sin\theta\cos\Phi + A\cos\theta\sin\Phi$

. Equating LHS and RHS,

$y = A\sin\Phi$

. $x = A\cos\Phi$

. Hence, amplitude $A = (x^2+y^2)^{1/2}$

. Phase $\Phi = \tan^{-1}(y/x)$

. Therefore, every point (x,y) can be represented as a sinusoid in (ϱ, θ).

. Q2.2 Why do we parameterize the line in terms of ϱ, θ instead of slope and intercept (m, c)? Express the slope and intercept in terms of ϱ and θ.

. Ans. On parameterizing in terms of (m,c), we can get unbounded values of m and hence computational issues. ($m \in (-inf,+inf)$) Hence, (ϱ, θ) is preferred.

. For every point (x,y),

. $x\cos\theta + y\sin\theta = \varrho$

. $y = -x * \cot\theta + \varrho * \operatorname{cosec}\theta$

. Since this is of the form y=mx+c,

. Slope, $m = -\cot\theta$

. Intercept, $c = \varrho * \operatorname{cosec}\theta$

.

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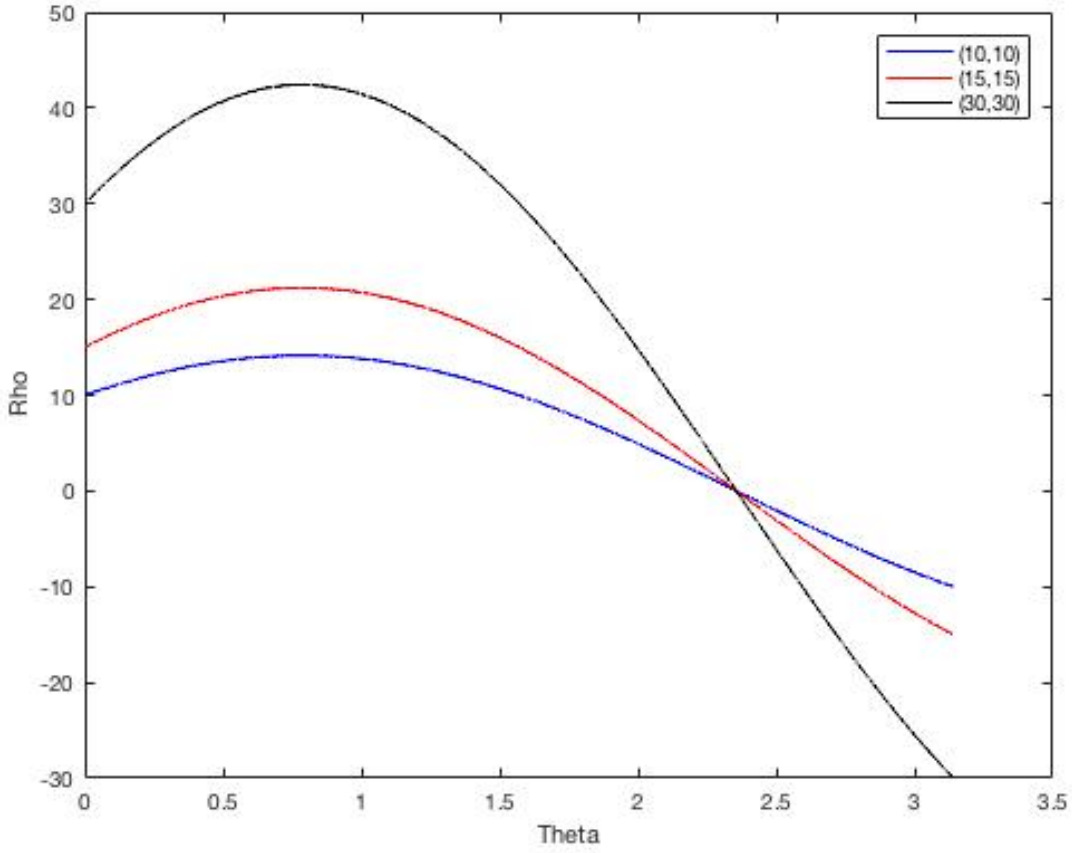
. Q2.3 Assuming that the image points (x, y) are in an image of width W and height H (i.e., $x \in [1, W]$, $y \in [1, H]$), what is the maximum absolute value of ϱ and what is the range of θ?

. Ans The maximum value of ϱ is $(H^2+W^2)^{1/2}$

. Range of θ is $(0,2\Pi)$

. Q2.4 For points (10,10),(15,15)and(30,30) in the image, plot the corresponding sinusoid waves in Hough space (ϱ, θ) and visualize how their intersection point defines the line (what is (m, c) for this line?).

. Ans.



.

The intersection point (ϱ, θ) defines a line whose slope and intercept can be defined by :

m = -cotθ = -cot(3*pi/4) = 1

c = ϱ * cosecθ = 0

Hence, the line is y = x

. Q2.5 **Extra Credit extra credit: 10 points** How does the dimension of parameter space affects Hough Transform method? What would you do when the parameter space is high? Briefly explain your method in the write-up.

. Ans. As the dimensions of parameter space increases, the complexity increases in the order of S^{n-2} where S is size of space and n is the number of parameters. For a very high dimensional space, computations might not be feasible for the Hough transform method. To overcome this, the inputs to the Hough transform need to be modified. This can be done using certain constraints on the data that significantly reduces the size. This can include cropping of the image to reduce it to only certain areas of interest and passing it through Hough transform. Another way could be compromising the accuracy to reduce complexity. This could mean keeping a margin on the shape prediction to reduce the size of the accumulator array. The main aim is to reduce the size of the accumulator array so that it is able to meet the computational feasibilities.

Q3.1 Write a function that convolves an image with a given convolution filter

Ans. Please see code

Image after convolution with Gaussian filter for img09.jpg

Q3.2 Convolution with one for loop

Ans For convolution, the kernel has to be flipped in both horizontal and vertical directions. This is achieved by using rot90. Then, it has to be moved to the input array and exactly be aligned with the center and finally multiplied with the overlapping data. To achieve this, the kernel is converted into a column vector. Also, the input array data is rearranged into column vectors with im2col using the size of kernel as the input. These matrices are then multiplied element-wise and then summed up using reshape to get the final output matrix. (Code attached)

Q3.3 Edge detection

Ans. Please see code

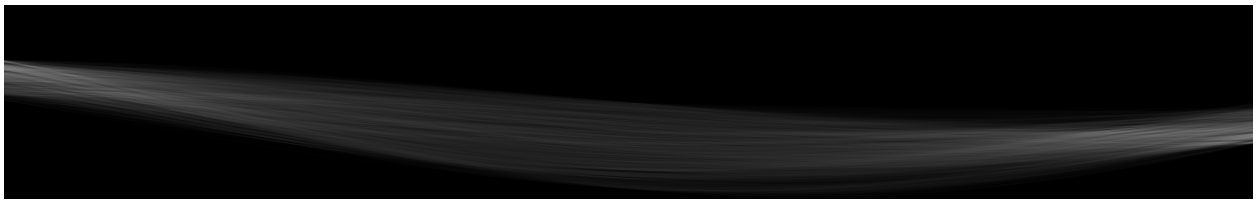
Im for img09.jpg after NMS

Io for img09.jpg after NMS

Q3.4 Hough transform

Ans. Please see code

Hough transform for img09.jpg



Q3.5 Finding lines

Ans. Please see code

Q3.6 Fitting Line Segments for Visualization

Ans. Please see code

Line segments for img09.jpg

Q4 Experiments

Ans. The code doesn't work equally well on all images. On few images, there is a noise observed with random lines. The non-maximum suppression causes most differences in the output and needs to be accurate to attain the desired outcome. Changing the binning affects the outcome drastically. Increasing the maximum number of lines from 50 results in a better output. Also, adding multiple layers of non-maximum suppressions including further neighbors till 2, 3 etc. results in better noise cancellation though runtime increases.

Experiment	**Result**
Changing the threshold value	Does reduce the noise in the images but not much impact with small changes
Changing theta resolution	Better accuracy but increase in runtime even with a factor of 2 reduction in thetaRes
Changing rho resolution	More noise, less accurate with increase in resolution
Changing number of lines	More lines observed in the images, with some increase in runtime as lines increased.