Approximation & Heuristic Approaches for the Travelling Salesperson Problem (TSP)

CS 512 – Final Project

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What is the TSP?

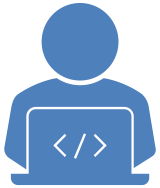
Given n cities and distances, find the shortest tour visiting each city once.

Must return to the starting city.

TSP is NP-hard; exact solutions scale factorially.

Used in routing, logistics, chip design, DNA sequencing.

Project Goals



Implement Brute Force, Nearest Neighbor, and 2-Opt.

Compare runtime, solution quality, approximation ratio.

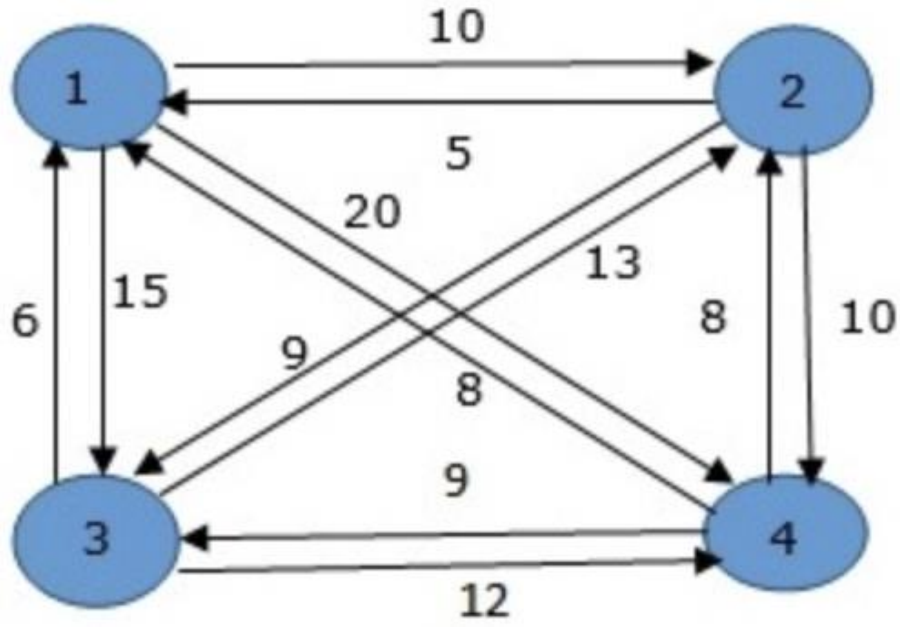
Generate visualizations and performance plots.

Data Generation & Processing

- Random (x, y) coordinates sampled in range [0, 100].
- Euclidean distance matrix computed for all city pairs.
- Small n = 6, 8, 10 used for optimal comparison.
- Large n = 20, 50, 100, 200 for heuristics.

Brute Force (Exact Solver)

- Enumerates all possible tours.
- Computes guaranteed optimal solution.
- Time complexity: O(n!).
- Feasible only for n ≤ 10.

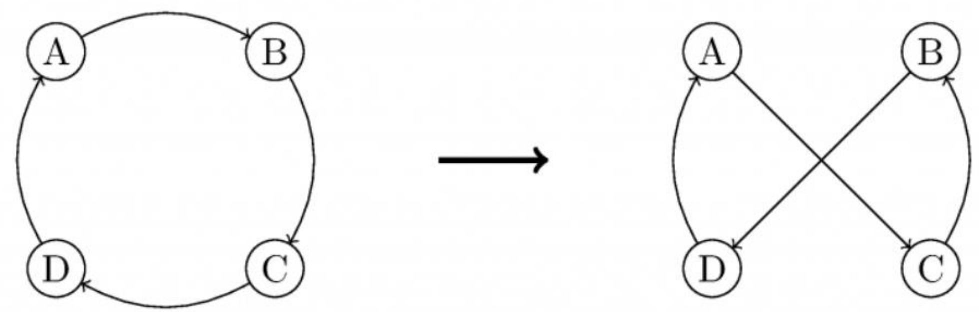


Nearest Neighbor Heuristic

- Greedy: visit closest unvisited city at each step.
- Computational complexity: $O(n^2)$.
- Very fast but may produce suboptimal global structures.

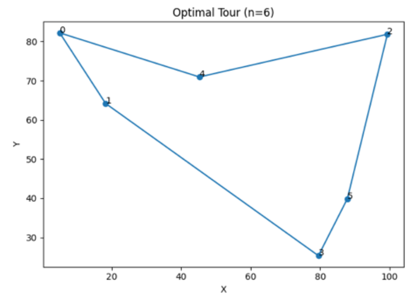
2-Opt Local Search

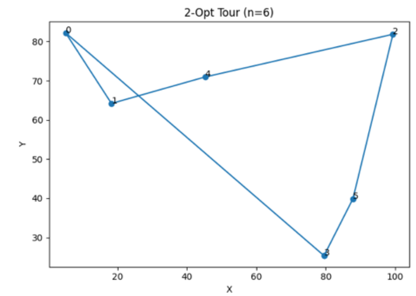
- Improves an initial tour by reversing edge segments.
- Removes crossings and reduces path length.
- Time complexity: ~$O(n^3)$.
- Produces high-quality tours at higher computation cost.

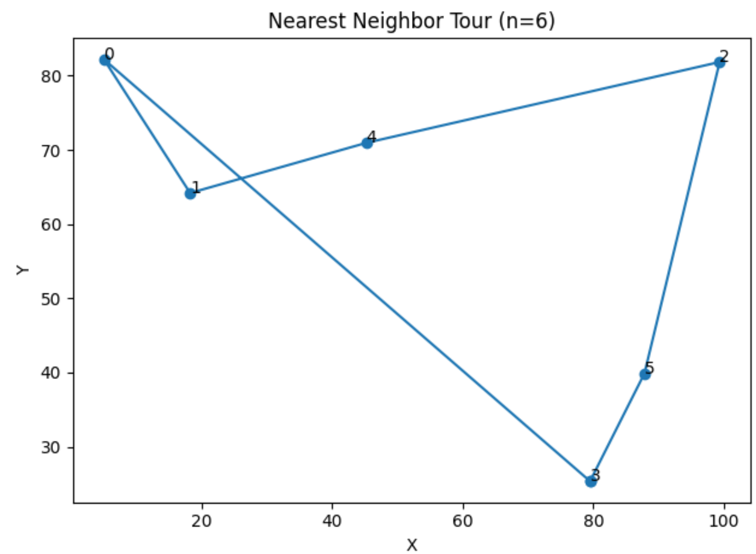


Example Tours (n = 6): Optimal, NN, 2-Opt

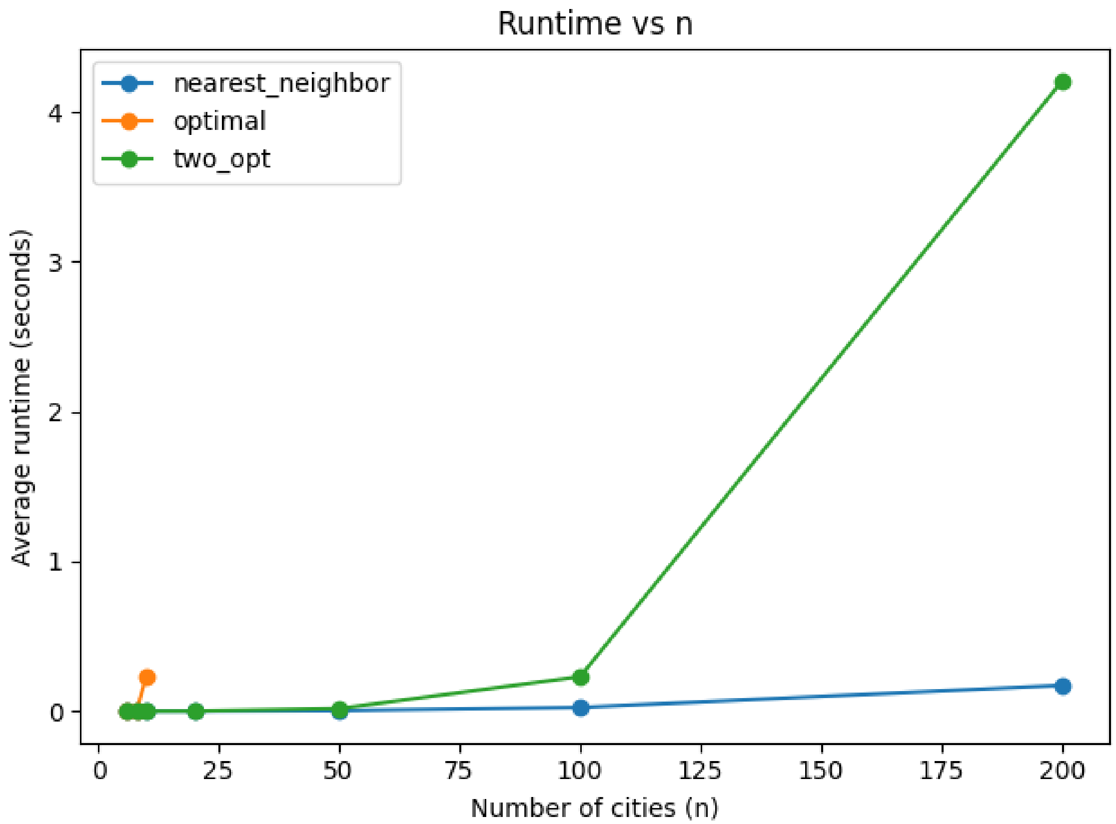
- 2-Opt improves the NN tour by removing edge crossings and reducing total distance.
- Nearest Neighbor creates a quick but sometimes suboptimal path.
- Shows how heuristics progress toward optimality on small instances.





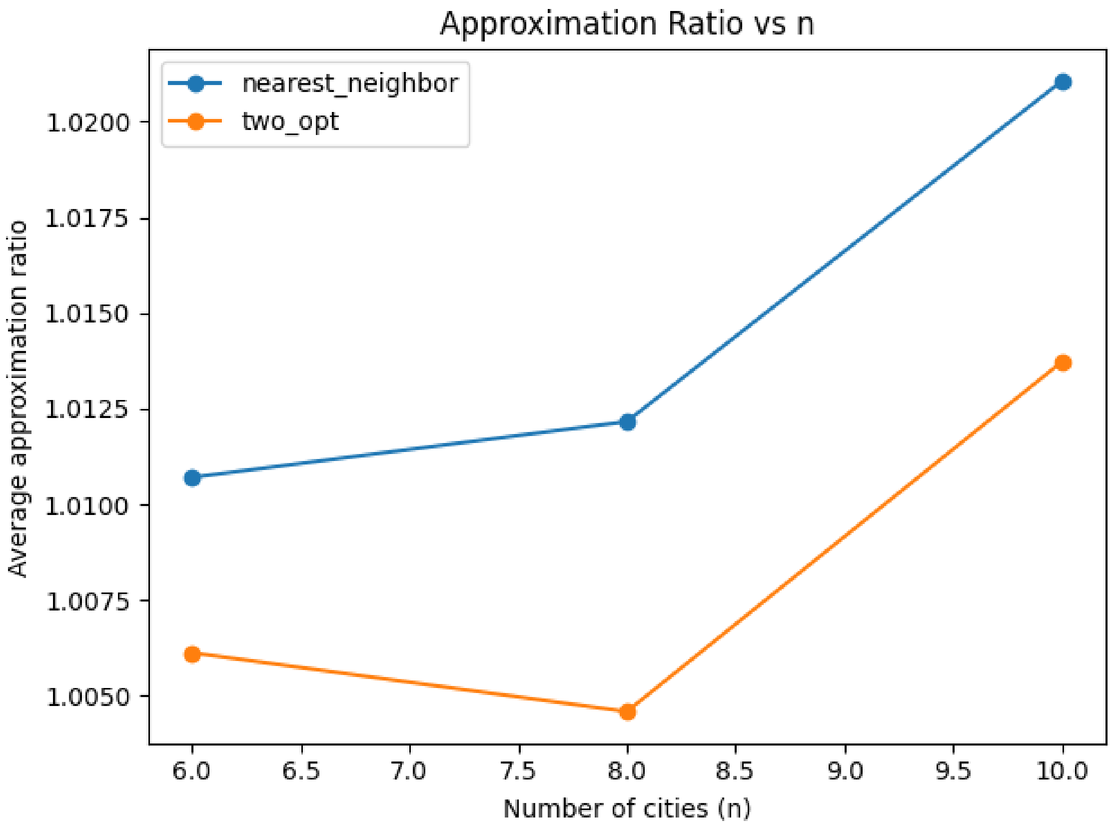


Runtime vs Number of Cities



- Brute Force grows too quickly and becomes infeasible beyond small n.
- Nearest Neighbor remains extremely fast even for large city counts.
- 2-Opt scales worse and becomes expensive for n ≥ 50 due to its cubic complexity.
- Clear trade-off: NN offers speed, 2-Opt offers higher solution quality.

Approximation Ratio vs Number of Cities



- Nearest Neighbor stays within 1–2% of the optimal solution.
- 2-Opt consistently improves the NN tour, reaching ~0.5–1.3% from optimal.
- Both heuristics perform well on small Euclidean TSP instances.
- 2-Opt offers higher accuracy but requires more computation.

Key Findings

- Brute Force is intractable beyond very small n.
- Nearest Neighbor is extremely fast and scales well.
- 2-Opt significantly improves NN solutions.
- Clear trade-off between runtime and solution quality.

Conclusion

Heuristics offer practical alternatives to exact algorithms.

2-Opt dramatically improves solution quality.

Framework can be extended with 3-Opt, SA, GA, Christofides.

TSP highlights the importance of approximation in NP-hard problems.