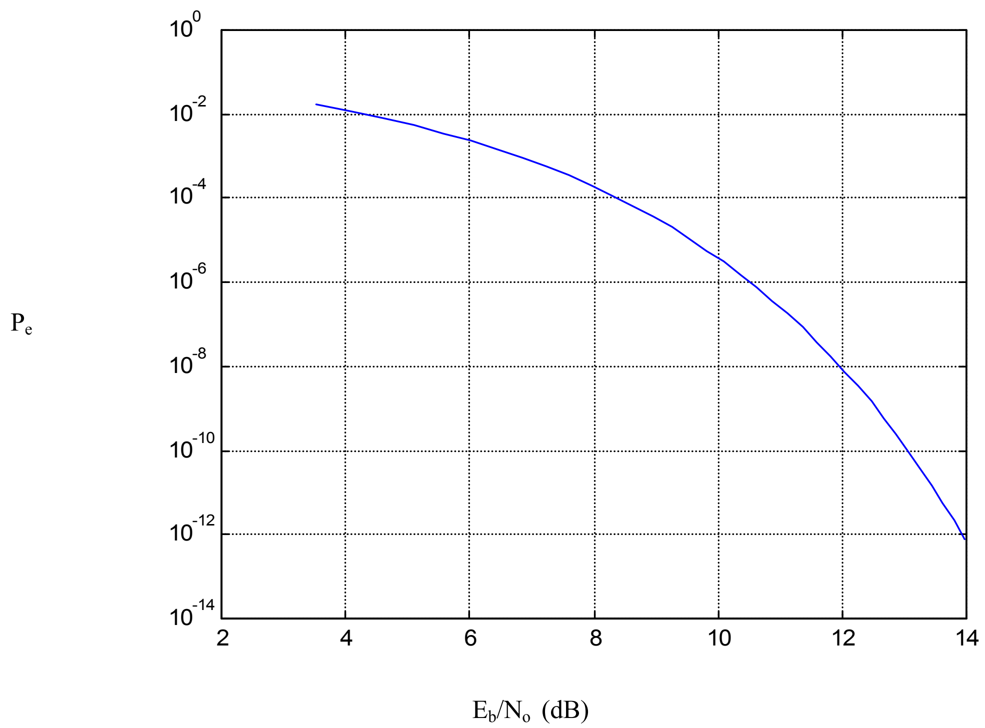
Error Rate

1. A binary PCM system using polar NRZ signaling operates just above the error threshold with an average probability of error equal to 10^{-6}. Suppose that the signaling rate is doubled. Find the new value of the average probability of error.

2. A continuous-time signal is sampled and then transmitted as a PCM signal. The random variable at the input of the decision device in the receiver has a variance of 0.01 volts^2. Assuming the use of polar NRZ signaling, determine the pulse amplitude that must be transmitted for the average error rate not to exceed 1 bit in 10^8 bits.

3. A binary PCM wave uses unipolar NRZ signaling to transmit symbols 1 and 0; symbol 1 is represented by a rectangular pulse of amplitude A and duration T_b. The channel noise is modeled as additive, white and Gaussian, with zero mean and power spectral density $N_o/2$. Assuming that the symbols 1 and 0 occur with equal probability, find an expression for the average probability of error at the receiver output, using a matched filter implemented by the integrate-and-dump circuit.



Solution

1.

Pe = 10^{-6}

From the graph, $\sqrt{E_b / N_o} = 3.3$

If the signaling rate is doubled, the bit duration is reduced by half. Correspondingly, E_b is reduced by half.

New Pe = 1/2 erfc(3.3/$\sqrt{2}$)=10e^{-3}

2.

The approach is similar to question 1.

(The variance is $N_o / 2T_b$)

3.

This is only a solution outline.

Assumptions:

The system uses an on-off format, symbol 1 is represented by A volts and symbol 0 is represented by zero volt

The symbols 1 and 0 occurs with equal probability

The channel noise is white and Gaussian with zero mean and power spectral density $N_o / 2$

When symbol 0 was sent, the matched filter output is

$$y(t) = \int_0^{T_b} s(t)x(t)dt$$

Its probability density function is

$$f(y|0) = \frac{1}{\sqrt{\pi N_o T_b A}} \exp\left(-\frac{y^2}{N_o T_b A^2}\right)$$

The corresponding error probability is

$$P_{10} = \int_{A/2}^{\infty} f(y|0)dy$$

$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{A^2 T_b / 2N_o}}^{\infty} \exp(-z^2)dz$$

$$= \frac{1}{2} erfc\left(\sqrt{\frac{A^2 T_b}{4N_o}}\right)$$

The error probability $= \frac{1}{2} erfc\left(\sqrt{\frac{A^2 T_b}{4N_o}}\right)$