Problem set 2
MTL 763 (Introduction to Game Theory)

Q.1 Find all the Nash equilibria of the following games

(a)

$$A = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix},\ B = \begin{pmatrix} 1 & -1 \\ -1 & 2. \end{pmatrix}$$

(b)

$$\begin{pmatrix} (2,2) & (0,3) & (1,3) \\ (3,2) & (1,1) & (0,2) \end{pmatrix}.$$

(c)

$$\begin{pmatrix} (1,2) & (1,3) \\ (4,1) & (0,1) \\ (0,3) & (3,2) \end{pmatrix}.$$

Q.2 Each of two firms has a job opening. The firms offer different wages: firm i offers wage w_i where $0.5w_1 < w_2 < 2w_1$. There are two workers that want to apply for a job. Each of whom can apply to only one firm. The workers simultaneously decide whether apply to firm 1 or to firm 2. If only one worker applies to a given firm, that worker gets the job. If both workers apply to one firm, the firm hires one worker at random with probability $\frac{1}{2}$ and the other worker remains unemployed.
a) Represent this game using the normal form.
b) Solve for the Nash equilibria.

Q.3 Consider the following game

$$\begin{pmatrix} 1,2 & 3,5 & 2,1 \\ 0,4 & 2,1 & 3,0 \\ -1,1 & 4,3 & 0,2 \end{pmatrix}.$$

a) Does either player have a dominant strategy? Explain.
b) Does either player have a dominated strategy? Explain.
c) Solve the equilibrium for this game.

Q.4 Consider the game of chicken. Two players drive their cars down the center of the road directly at each other. Each player chooses SWERVE or STAY. Staying wins you the admiration of your peers (a big payoff) only if the other player swerves. Swerving loses face if the other player stays. However, clearly, the worst output is for both players to stay. The game is defined by following matrix:

P1 \ P2	Stay	Swerve
Stay	(-6, -6)	(2, -2)
Swerve	(-2, 2)	(1, 1)

a) Does either player have a dominant strategy? Explain.
b) Suppose that Player B has adopted the strategy of Staying 1/5 of the time and swerving 4/5 of the time. Show that Player A is indifferent between swerving and staying.
c) Find the Nash equilibrium of the game.

Q.5 Consider the following bimatrix game

$$\begin{pmatrix} (1,2) & (0,0) \\ (0,0) & (2,1) \end{pmatrix}.$$

a) For a fixed mixed strategy y of player 2, find the best response strategies of player 1.
b) For a fixed mixed strategy x of player 1, find the best response strategies of player 2.
c) Draw the best response functions of both the players and find the Nash equilibrium of the game graphically.

Q.6 Consider the Rock-Paper-Scissors game and compute its Nash equilibrium using Lemke-Howson algorithm.

Q.7 Using different algorithms find all pure and mixed strategy Nash equilibria in the games below.

(a)

$$\begin{pmatrix} 1,2 & 2,1 & 0,0 \\ 2,1 & 1,2 & 0,0 \\ 0,0 & 0,0 & 1,1 \end{pmatrix}.$$

(b)

$$\begin{pmatrix} 1,3 & 0,0 & 0,1 \\ 0,0 & 2,1 & 4,2 \\ 0,2 & 0,1 & 0,0 \end{pmatrix}.$$

(c)

$$\begin{pmatrix} 0,0 & 4,5 & 5,4 \\ 5,4 & 4,5 & 0,0 \\ 5,4 & 4,5 & 0,0 \end{pmatrix}.$$

(d)

$$\begin{pmatrix} 7,2 & 2,7 & 3,6 \\ 2,7 & 7,2 & 4,5 \end{pmatrix}.$$

Q.8 Two individuals go out on a hunt. Each can individually choose to hunt a stag or hunt a hare. Each player must choose an action without knowing the choice of the other. If an individual hunts a stag, he must have the cooperation of his partner in order to succeed. An individual can get a hare by himself, but a hare is worth less than a stag. So you can see in the table that if the player one choose a stag and the player two too then they will both get a payoff of 5. But if the hunter one takes a hare a the player two a stag then the first player will get 4 and the second 0 and so on. If they both choose to hunt the hare then they divide it equally.

P2 / P1	Stag	Hare
Stag	(5, 5)	(0, 4)
Hare	(4, 0)	(2, 2)

Find all the pure and mixed strategy Nash equilibrium by drawing the best response functions. Which Nash equilibrium is also Pareto optimal.

Q.9 By drawing the best response functions, find the mixed strategy Nash equilibrium for following games:

(a) Matching Pennies.

(b)

$$\begin{pmatrix} 1,1 & 0,1 \\ 1,0 & 0,0 \end{pmatrix}.$$

Q.10 Using Lemke-Howson algorithm find all the Nash equilibria of the game given below for all values of α and β

$$A = \begin{pmatrix} 1 & 3 \\ \alpha & 2 \end{pmatrix}, \ B = \begin{pmatrix} 2 & 3 \\ 4 & \beta \end{pmatrix},$$

where $\alpha \in [1, 4], \beta \in [1, 4]$.

Q.11 Find the pure strategy Nash equilibria for the following games:

$$\begin{pmatrix} 6,6 & 8,20 & 0,8 \\ 10,0 & 5,5 & 2,8 \\ 8,0 & 20,0 & 4,4 \end{pmatrix}, \begin{pmatrix} 5,2 & 2,6 & 1,4 & 0,4 \\ 0,0 & 3,2 & 2,1 & 1,1 \\ 7,0 & 2,2 & 1,5 & 5,1 \\ 9,5 & 1,3 & 0,2 & 4,8 \end{pmatrix}, \begin{pmatrix} 0,7 & 2,5 & 7,0 & 0,1 \\ 5,2 & 3,3 & 5,2 & 0,1 \\ 7,0 & 2,5 & 0,7 & 0,1 \\ 0,0 & 0,-2 & 0,0 & 10,-1 \end{pmatrix}.$$

Q.12 Give examples of two player pure strategy games for the following situations
(a) The game has a unique Nash equilibrium which is not a weakly dominant strategy equilibrium.
(b) The game has a unique Nash equilibrium which is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.
(c) The game has one strongly dominant or one weakly dominant strategy equilibrium and a second one which is only a Nash equilibrium.

Q.13 Consider the following game

P2 / P1	NC	C
NC	(-4, -4)	(-2, $-x$)
C	($-x$, -2)	($-x$, $-x$)

Find the values of x for which:

(a) the profile (C,C) is a strongly dominant strategy equilibrium.
(b) the profile (C,C) is a weakly dominant strategy equilibrium (of type 2) but not a strongly dominant strategy equilibrium.
(c) the profile (C,C) is a Nash equilibrium but not a dominant strategy equilibrium.
(d) the profile (C,C) is not even a Nash equilibrium.

In each case, say whether it is possible to find such an x. Justify your answer in each case.

Q.14 The following payoff matrix corresponds to a modified version of the Prisoner's Dilemma problem called the DA's brother problem. In this problem prisoner 1 is related to the District Attorney. Does it have a strongly dominant or a weakly dominant equilibrium?

P1 \ P2	NC	C
NC	(0, -2)	(-10, -1)
C	(-1, -10)	(-5, -5)

Q.15 There are n-farmers in a village who decides whether to keep a sheep or not. Let 1 corresponds to keeping a sheep, and 0 corresponds to not keeping a sheep. Keeping a sheep gives a benefit of 1. However, when a sheep is kept, damage to the environment is 5. This damage is equally shared by all the farmers. Formulate this problem as a non-cooperative game and find all pure strategy Nash equilibria. Find strongly/weakly dominant Nash equilibrium profiles if they exist ? If the government decides to impose a pollution tax of 5 units for each sheep kept, find all the Nash equilibria. Does there exist a strongly/weakly dominant strategy Nash equilibrium? Give proper justifications to your arguments.

Q.16 With proper justifications either construct an example or give proper arguments why such an example does not exist in the following questions.

(a) A 3×3 bimatrix game which has two pure strategy Nash equilibrium where one of the Nash equilibrium is weakly dominant Nash equilibrium.

(b) A 3×3 bimatrix game with two pure strategy Nash equilibrium where one is strongly dominant Nash equilibrium and other one is just a Nash equilibrium.

(c) A 3×3 bimatrix game whose diagonal entries correspond to Nash equilibrium.