

# CS251: Assignment 7 - Question 4

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# 1 Sample Space and Random Variable

Let  $\chi$  be a random variable such that,

$\chi$  : Outcome of roll of the die.

$\therefore \chi = \omega$  , such that  $\omega \in \{1,2,3,4,5,6\}$ .

## 2 Probability Function of the dices

### 2.1 Die:1

$$P(\chi = x) = \left\{ \frac{1}{6} \quad x \in \{1,2,3,4,5,6\} \right.$$

### 2.2 Die:2

$$P(\chi = x) = \begin{cases} \frac{1}{6} - 0.025 & x \in \{1,2,3,4\} \\ \frac{1}{6} + 0.05 & x \in \{5,6\} \end{cases}$$

### 2.3 Die:3

$$P(\chi = x) = \begin{cases} \frac{1}{6} - 0.05 & x \in \{1,2,3,4\} \\ \frac{1}{6} + 0.10 & x \in \{5,6\} \end{cases}$$

## 3 Expected Values of the Sums of the Three Dices

### 3.1 Die:1

$$E(\chi) = \sum_{x_i} P(\chi = x_i) x_i = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = 3.5$$

### 3.2 Die:2

$$E(\chi) = \sum_{x_i} P(\chi = x_i) x_i = (\frac{1}{6} - 0.025)(1 + 2 + 3 + 4) + (\frac{1}{6} + 0.05)(5 + 6) = 3.8$$

### 3.3 Die:3

$$E(\chi) = \sum_{x_i} P(\chi = x_i) x_i = (\frac{1}{6} - 0.05)(1 + 2 + 3 + 4) + (\frac{1}{6} + 0.10)(5 + 6) = 4.1$$

## 4 Observations

After rolling the three dice(each die is rolled twice), it was observed that sum on Die:1 was least and sum on Die:3 was highest, i.e., we get successively higher sums for later dice.

This can clearly be seen in Figure 1.

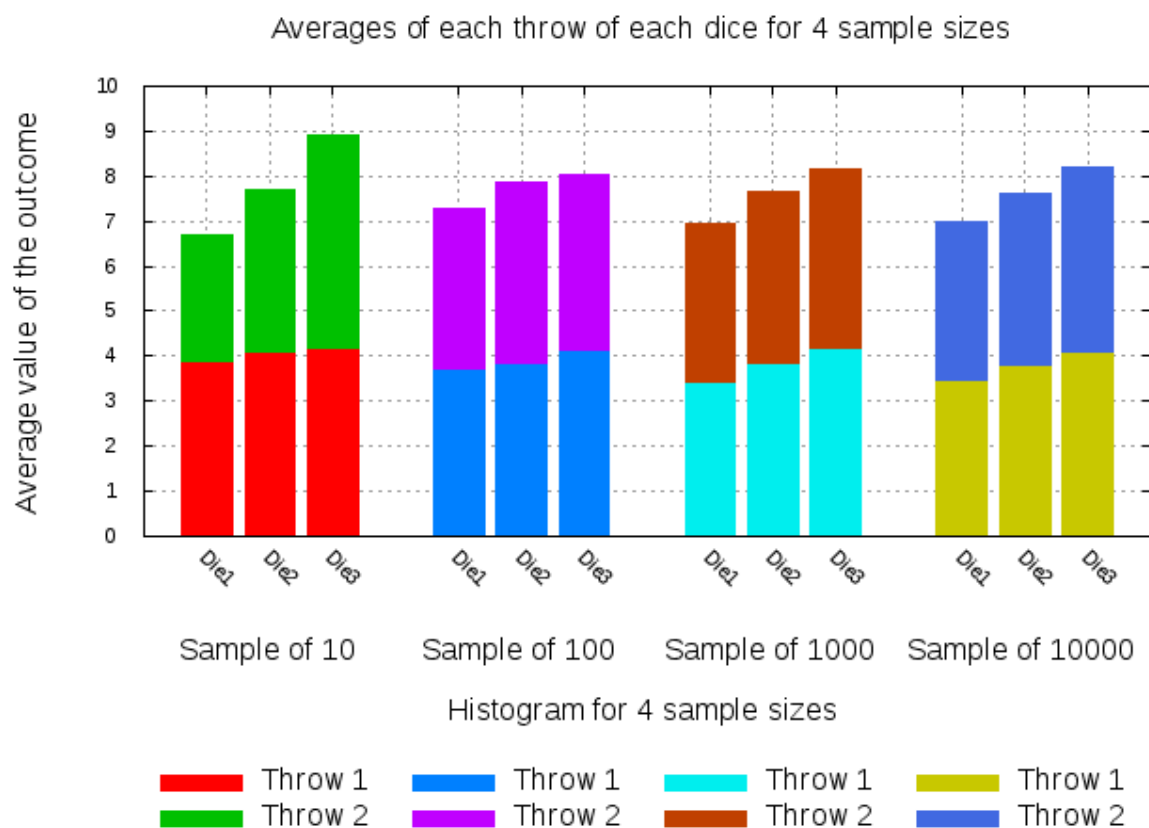


Figure 1: Representation of average of each throw of all the Dice

### 4.1 Explanation

Given below are the two possible explanations of the phenomenon observed above.

#### 4.1.1 A Mathematical Explanation

It is clear from the expected value of  $\chi$  for all the three dices that if the experiment is repeated a large number of times, the average value of outcome would be close to the expected value. Since, the expected value is lowest for Die:1 and highest for Die:3, so, we get successively higher sums for later dice.

### 4.1.2 An Intuitive Explanation

Since, all the outcomes on the first die are equally likely while the second die is biased in the favour of the outcomes 5 and 6 by 5%, so, outcome of the second die was more biased to 5 and 6 which are the highest outcomes of the die. This results in a higher sum of the outcome of the second die as compared to first.

Now, third die is biased in the favour of the outcomes 5 and 6 by 10%, so, outcome of the third die is biased more in the favour of 5 and 6 as compared to the second die. This results in a higher sum of the outcome of the third die as compared to second.

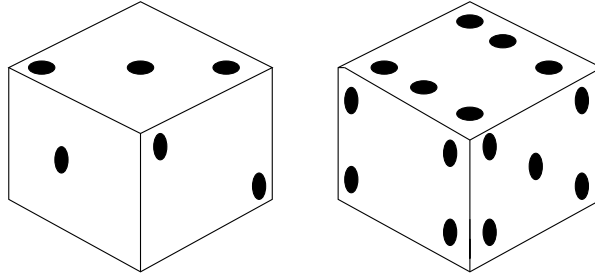


Figure 2: Two Sample Dices

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