

# COMPILER DESIGN

**Topic: Bottom-Up parsing**

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# Introduction

- Constructs parse tree for an input string beginning at the leaves (the bottom) and working towards the root (the top)
- Example:  $\text{id} * \text{id}$

$E \rightarrow E + T \mid T$   
 $T \rightarrow T * F \mid F$   
 $F \rightarrow (E) \mid \text{id}$

$\text{id} * \text{id}$

$F * \text{id}$   
|  
 $\text{id}$

$T * \text{id}$   
|  
 $F$   
|  
 $\text{id}$

$T * F$   
| |  
 $F$   $\text{id}$   
|  
 $\text{id}$

$F$   
^  
 $T * F$   
| |  
 $F$   $\text{id}$   
|  
 $\text{id}$

$E$   
|  
 $F$   
^  
 $T * F$   
| |  
 $F$   $\text{id}$   
|  
 $\text{id}$

# Shift-reduce parser

- The general idea is to shift some symbols of input to the stack until a reduction can be applied
- At each reduction step, a specific substring matching the body of a production is replaced by the nonterminal at the head of the production
- The key decisions during bottom-up parsing are about when to reduce and about what production to apply
- A reduction is a reverse of a step in a derivation
- The goal of a bottom-up parser is to construct a derivation in reverse:
  - $E \Rightarrow T \Rightarrow T * F \Rightarrow T * id \Rightarrow F * id \Rightarrow id * id$

# Handle pruning

- A Handle is a substring that matches the body of a production and whose reduction represents one step along the reverse of a rightmost derivation

Right sentential form	Handle	Reducing production
id*id	id	$F \rightarrow id$
F*id	F	$T \rightarrow F$
T*id	id	$F \rightarrow id$
T*F	T*F	$E \rightarrow T*F$

# Shift reduce parsing

- A stack is used to hold grammar symbols
- Handle always appear on top of the stack
- Initial configuration:

Stack	Input
\$	w\$

- Acceptance configuration

Stack	Input
\$S	\$

# Shift reduce parsing (cont.)

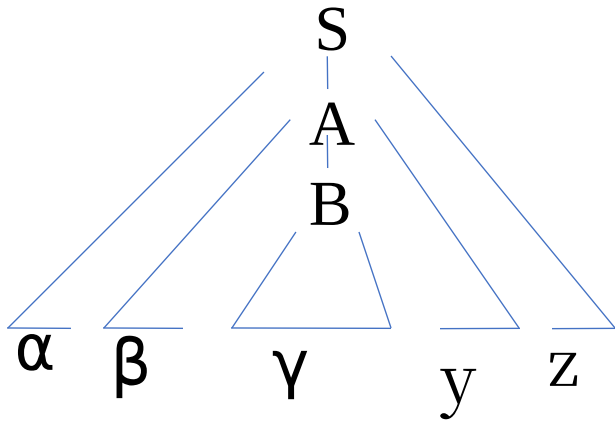
- Basic operations:

- Shift
- Reduce
- Accept
- Error

- Example:  $\text{id} * \text{id}$

Stack	Input	Action
\$	$\text{id} * \text{id} \$$	shift
$\$ \text{id}$	$* \text{id} \$$	reduce by $F \rightarrow \text{id}$
$\$ F$	$* \text{id} \$$	reduce by $T \rightarrow F$
$\$ T$	$* \text{id} \$$	shift
$\$ T *$	$\text{id} \$$	shift
$\$ T * \text{id}$	$\$$	reduce by $F \rightarrow \text{id}$
$\$ T * F$	$\$$	reduce by $T \rightarrow T * F$
$\$ T$	$\$$	reduce by $E \rightarrow T$
$\$ E$	$\$$	accept

Handle will appear on top of the stack



Stack

$\$ \alpha \beta \gamma$

$\$ \alpha \beta B$

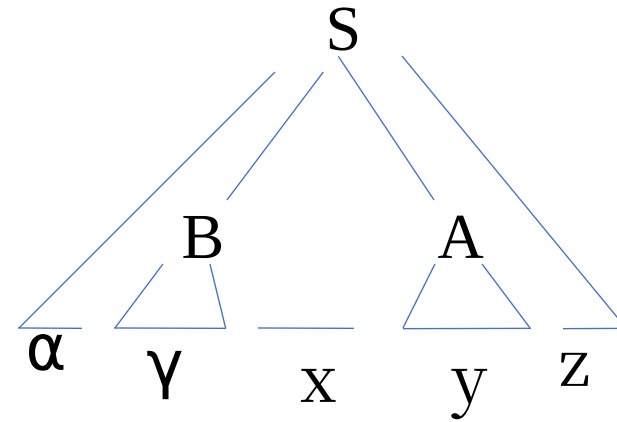
$\$ \alpha \beta B y$

Input

yz\$

yz\$

z\$



Stack

$\$ \alpha \gamma$

$\$ \alpha B x y$

Input

xyz\$

z\$

# Conflicts during shift reduce parsing

- Two kind of conflicts
  - Shift/reduce conflict
  - Reduce/reduce conflict
- Example:

```
stmt → If expr then stmt
      | If expr then stmt else stmt
      | other
```

Stack

... if expr then stmt

Input

else ...\$



# Reduce/reduce conflict

stmt -> id(parameter\_list)

stmt -> expr:=expr

parameter\_list->parameter\_list, parameter

parameter\_list->parameter

parameter->id

expr->id(expr\_list)

expr->id

expr\_list->expr\_list, expr

expr\_list->expr

Stack  
... id(id

Input  
,id) ...\$

# LR Parsing

- The most prevalent type of bottom-up parsers
- LR(k), mostly interested on parsers with  $k \leq 1$
- Why LR parsers?
  - Table driven
  - Can be constructed to recognize all programming language constructs
  - Most general non-backtracking shift-reduce parsing method
  - Can detect a syntactic error as soon as it is possible to do so
  - Class of grammars for which we can construct LR parsers are superset of those which we can construct LL parsers

# States of an LR parser

- States represent set of items
- An LR(0) item of  $G$  is a production of  $G$  with the dot at some position of the body:
  - For  $A \rightarrow XYZ$  we have following items
    - $A \rightarrow .XYZ$
    - $A \rightarrow X.YZ$
    - $A \rightarrow XY.Z$
    - $A \rightarrow XYZ.$
  - In a state having  $A \rightarrow .XYZ$  we hope to see a string derivable from  $XYZ$  next on the input.
  - What about  $A \rightarrow X.YZ$ ?

# Constructing canonical LR(0) item sets

- Augmented grammar:
  - G with addition of a production:  $S' \rightarrow S$
- Closure of item sets:
  - If I is a set of items,  $\text{closure}(I)$  is a set of items constructed from I by the following rules:
    - Add every item in I to  $\text{closure}(I)$
    - If  $A \rightarrow \alpha.B\beta$  is in  $\text{closure}(I)$  and  $B \rightarrow \gamma$  is a production then add the item  $B \rightarrow .\gamma$  to  $\text{closure}(I)$ .
- Example:

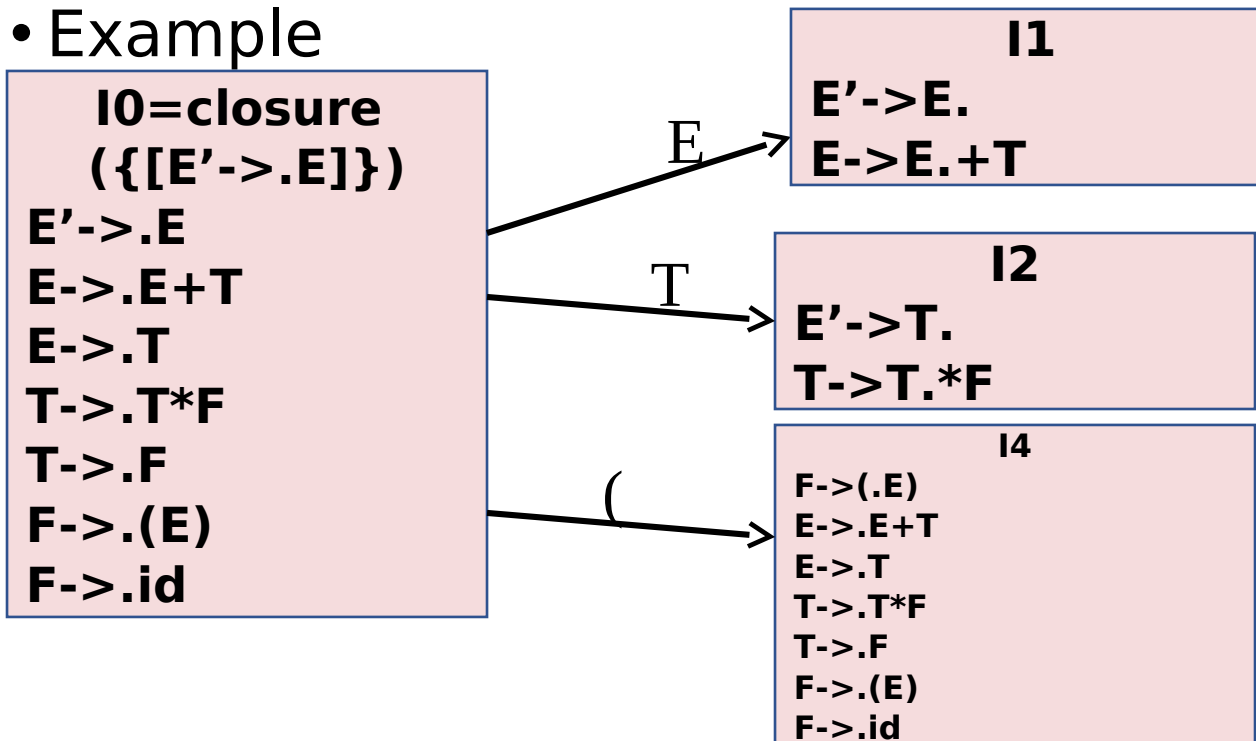
$E' \rightarrow E$   
 $E \rightarrow E + T \mid T$   
 $T \rightarrow T * F \mid F$   
 $F \rightarrow (E) \mid \text{id}$

**$I_0 = \text{closure}(\{[E' \rightarrow .E]\})$**   
 **$E' \rightarrow .E$**   
 **$E \rightarrow .E + T$**   
 **$E \rightarrow .T$**   
 **$T \rightarrow .T * F$**   
 **$T \rightarrow .F$**   
 **$F \rightarrow .(E)$**   
 **$F \rightarrow .\text{id}$**

# Constructing canonical LR(0) item sets (cont.)

- Goto ( $I, X$ ) where  $I$  is an item set and  $X$  is a grammar symbol is closure of set of all items  $[A \rightarrow \alpha X \beta]$  where  $[A \rightarrow \alpha.X \beta]$  is in  $I$

- Example



# Closure algorithm

SetOfItems CLOSURE(I) {

  J=I;

  repeat

    for (each item  $A \rightarrow \alpha.B\beta$  in J)

      for (each production  $B \rightarrow \gamma$  of G)

        if ( $B \rightarrow \gamma$  is not in J)

          add  $B \rightarrow \gamma$  to J;

  until no more items are added to J on one round;

  return J;

# GOTO algorithm

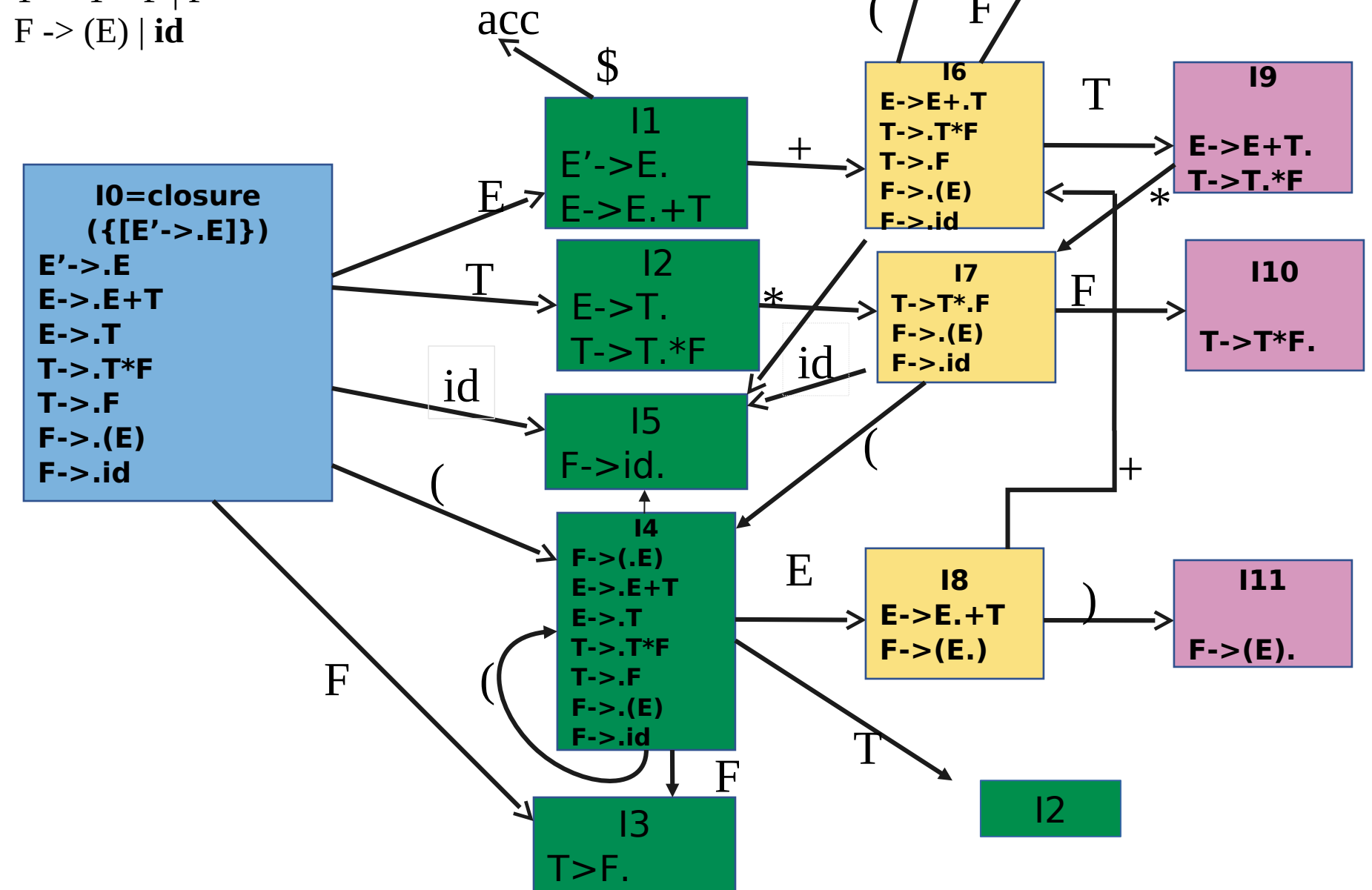
```
SetOfItems GOTO(I,X) {  
    J=empty;  
    if ( $A \rightarrow \alpha.X \beta$  is in I)  
        add CLOSURE( $A \rightarrow \alpha X. \beta$ ) to J;  
    return J;  
}
```

# Canonical LR(0) items

```
Void items( $G'$ ) {  
     $C = \text{CLOSURE}(\{[S' \rightarrow \cdot S]\})$ ;  
    repeat  
        for (each set of items  $I$  in  $C$ )  
            for (each grammar symbol  $X$ )  
                if ( $\text{GOTO}(I, X)$  is not empty and not in  $C$ )  
                    add  $\text{GOTO}(I, X)$  to  $C$ ;  
    until no new set of items are added to  $C$  on a round;  
}
```



$E' \rightarrow E$   
 $E \rightarrow E + T \mid T$   
 $T \rightarrow T * F \mid F$   
 $F \rightarrow (E) \mid id$



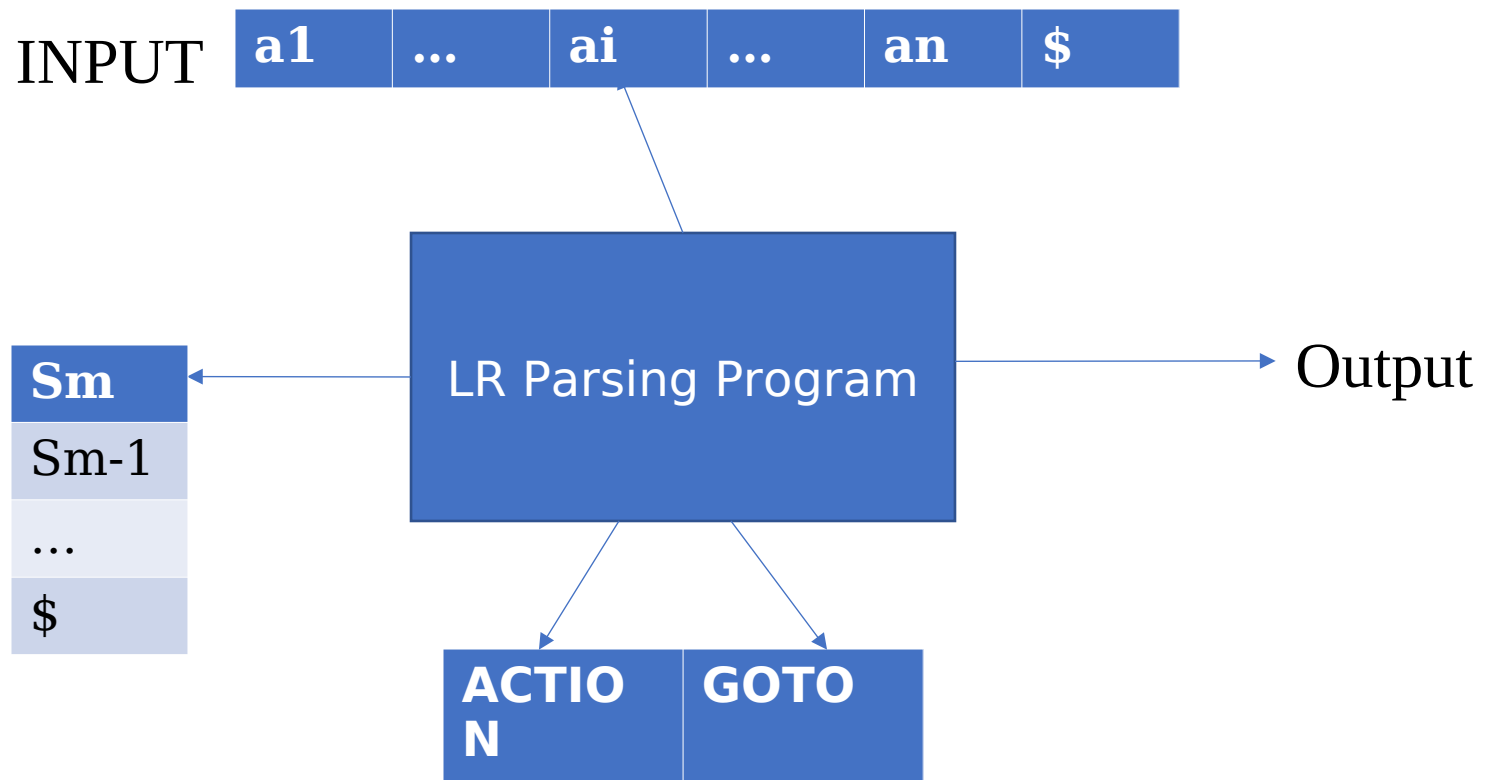
STATE	ACTON						GOTO		
	id	+	*	(	)	\$	E	T	F
0	S5			S4			1	2	3
1		S6				Acc			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

# Use of LR(0) automaton

- Example:  $\text{id} * \text{id}$

Line	Stack	Symbols	Input	Action
(1)	0	\$	$\text{id} * \text{id} \$$	Shift to 5
(2)	05	$\$ \text{id}$	$* \text{id} \$$	Reduce by $F \rightarrow \text{id}$
(3)	03	$\$ F$	$* \text{id} \$$	Reduce by $T \rightarrow F$
(4)	02	$\$ T$	$* \text{id} \$$	Shift to 7
(5)	027	$\$ T *$	$\text{id} \$$	Shift to 5
(6)	0275	$\$ T * \text{id}$	$\$$	Reduce by $F \rightarrow \text{id}$
(7)	02710	$\$ T * F$	$\$$	Reduce by $T \rightarrow T * F$
(8)	02	$\$ T$	$\$$	Reduce by $E \rightarrow T$
(9)	01	$\$ E$	$\$$	accept

# LR-Parsing model



# LR parsing algorithm

```
let a be the first symbol of w$;
while(1) { /*repeat forever */
    let s be the state on top of the stack;
    if (ACTION[s,a] = shift t) {
        push t onto the stack;
        let a be the next input symbol;
    } else if (ACTION[s,a] = reduce A-> $\beta$ ) {
        pop  $|\beta|$  symbols of the stack;
        let state t now be on top of the stack;
        push GOTO[t,A] onto the stack;
        output the production A-> $\beta$ ;
    } else if (ACTION[s,a]=accept) break; /* parsing is done */
    else call error-recovery routine;
}
```

# Example: Moves of SLR parser

Moves of SLR parser for the given grammar:-

- (0)  $E' \rightarrow E$
- (1)  $E \rightarrow E + T$
- (2)  $E \rightarrow T$
- (3)  $T \rightarrow T * F$
- (4)  $T \rightarrow F$
- (5)  $F \rightarrow (E)$
- (6)  $F \rightarrow \mathbf{id}$

# Example $id*id+id?$

STATE	ACTION						GOTO		
	id	+	*	(	)	\$	E	T	F
0	S5			S4			1	2	3
1		S6				Acc			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

Line	Stack	Symbols	Input	Action
(1)	0		id*id+id\$	Shift to 5
(2)	05	id	*id+id\$	Reduce by F->id
(3)	03	F	*id+id\$	Reduce by T->F
(4)	02	T	*id+id\$	Shift to 7
(5)	027	T*	id+id\$	Shift to 5
(6)	0275	T*id	+id\$	Reduce by F->id
(7)	02710	T*F	+id\$	Reduce by T->T*F
(8)	02	T	+id\$	Reduce by E->T
(9)	01	E	+id\$	Shift
(10)	016	E+	id\$	Shift
(11)	0165	E+id	\$	Reduce by F->id
(12)	0163	E+F	\$	Reduce by T->F
(13)	0169	E+T`	\$	Reduce by E->E+T
(14)	01	E	\$	accept

# Constructing SLR parsing table

- Method

- Construct  $C = \{I_0, I_1, \dots, I_n\}$ , the collection of LR(0) items for  $G'$
- State  $i$  is constructed from state  $li$ :
  - If  $[A \rightarrow \alpha.a\beta]$  is in  $li$  and  $\text{Goto}(li, a) = lj$ , then set  $\text{ACTION}[i, a]$  to “shift  $j$ ”
  - If  $[A \rightarrow \alpha.]$  is in  $li$ , then set  $\text{ACTION}[i, a]$  to “reduce  $A \rightarrow \alpha$ ” for all  $a$  in  $\text{follow}(A)$
  - If  $\{S' \rightarrow .S\}$  is in  $li$ , then set  $\text{ACTION}[i, \$]$  to “Accept”
- If any conflicts appears then we say that the grammar is not SLR(1).
- If  $\text{GOTO}(li, A) = lj$  then  $\text{GOTO}[i, A] = j$
- All entries not defined by above rules are made “error”
- The initial state of the parser is the one constructed from the set of items containing  $[S' \rightarrow .S]$



# Example grammar which is not SLR(1)

$S \rightarrow L=R \mid R$

$L \rightarrow *R \mid \text{id}$

$R \rightarrow L$

I0  
 $S' \rightarrow .S$   
 $S \rightarrow .L=R$   
 $S \rightarrow .R$   
 $L \rightarrow .*R \mid$   
 $L \rightarrow .\text{id}$   
 $R \rightarrow .L$

I1  
 $S' \rightarrow S.$

I2  
 $S \rightarrow L.=R$   
 $R \rightarrow L.$

I3  
 $S \rightarrow R.$

I4  
 $L \rightarrow *.R$   
 $R \rightarrow .L$   
 $L \rightarrow .*R$   
 $L \rightarrow .\text{id}$

I5  
 $L \rightarrow \text{id}.$

I6  
 $S \rightarrow L=.R$   
 $R \rightarrow .L$   
 $L \rightarrow .*R$   
 $L \rightarrow .\text{id}$

I7  
 $L \rightarrow *R.$

I8  
 $R \rightarrow L.$

I9  
 $S \rightarrow L=R.$

Action

=

# More powerful LR parsers

- Canonical-LR or just LR method
  - Use lookahead symbols for items: LR(1) items
  - Results in a large collection of items
- LALR: lookaheads are introduced in LR(0) items

# Canonical LR(1) items

- In LR(1) items each item is in the form:  $[A \rightarrow \alpha.\beta, a]$
- An LR(1) item  $[A \rightarrow \alpha.\beta, a]$  is valid for a viable prefix  $\gamma$  if there is a derivation  $S \xRightarrow{*} \delta A w \xRightarrow{rm} \delta \alpha \beta w$ , where
  - $\Gamma = \delta \alpha$
  - Either  $a$  is the first symbol of  $w$ , or  $w$  is  $\epsilon$  and  $a$  is  $\$$
- Example:
  - $S \rightarrow BB$
  - $B \rightarrow aB | b$

$$S \xRightarrow{*} aaBab \xRightarrow{rm} aaaBab$$

Item  $[B \rightarrow a.B, a]$  is valid for  $\gamma = aaa$   
and  $w = ab$

# Constructing LR(1) sets of items

```
SetOfItems Closure(I) {  
    repeat  
        for (each item  $[A \rightarrow \alpha.B\beta, a]$  in I)  
            for (each production  $B \rightarrow \gamma$  in  $G'$ )  
                for (each terminal  $b$  in  $\text{First}(\beta a)$ )  
                    add  $[B \rightarrow \gamma, b]$  to set I;  
    until no more items are added to I;  
    return I;  
}
```

```
SetOfItems Goto(I, X) {  
    initialize J to be the empty set;  
    for (each item  $[A \rightarrow \alpha.X\beta, a]$  in I)  
        add item  $[A \rightarrow \alpha.X\beta, a]$  to set J;  
    return closure(J);  
}
```

```
void items( $G'$ ) {  
    initialize C to  $\text{Closure}(\{[S' \rightarrow \cdot S, \$]\})$ ;  
    repeat  
        for (each set of items I in C)  
            for (each grammar symbol X)  
                if ( $\text{Goto}(I, X)$  is not empty and not in C)  
                    add  $\text{Goto}(I, X)$  to C;  
    until no new sets of items are added to C;  
}
```

# Example

$S' \rightarrow S$

$S \rightarrow CC$

$C \rightarrow cC$

$C \rightarrow d$

# Canonical LR(1) parsing table

- Method

- Construct  $C = \{I_0, I_1, \dots, I_n\}$ , the collection of LR(1) items for  $G'$
- State  $i$  is constructed from state  $I_i$ :
  - If  $[A \rightarrow \alpha.a\beta, b]$  is in  $I_i$  and  $\text{Goto}(I_i, a) = I_j$ , then set  $\text{ACTION}[i, a]$  to “shift  $j$ ”
  - If  $[A \rightarrow \alpha., a]$  is in  $I_i$ , then set  $\text{ACTION}[i, a]$  to “reduce  $A \rightarrow \alpha$ ”
  - If  $\{S' \rightarrow .S, \$\}$  is in  $I_i$ , then set  $\text{ACTION}[i, \$]$  to “Accept”
- If any conflicts appears then we say that the grammar is not LR(1).
- If  $\text{GOTO}(I_i, A) = I_j$  then  $\text{GOTO}[i, A] = j$
- All entries not defined by above rules are made “error”
- The initial state of the parser is the one constructed from the set of items containing  $[S' \rightarrow .S, \$]$

# Example

$S' \rightarrow S$

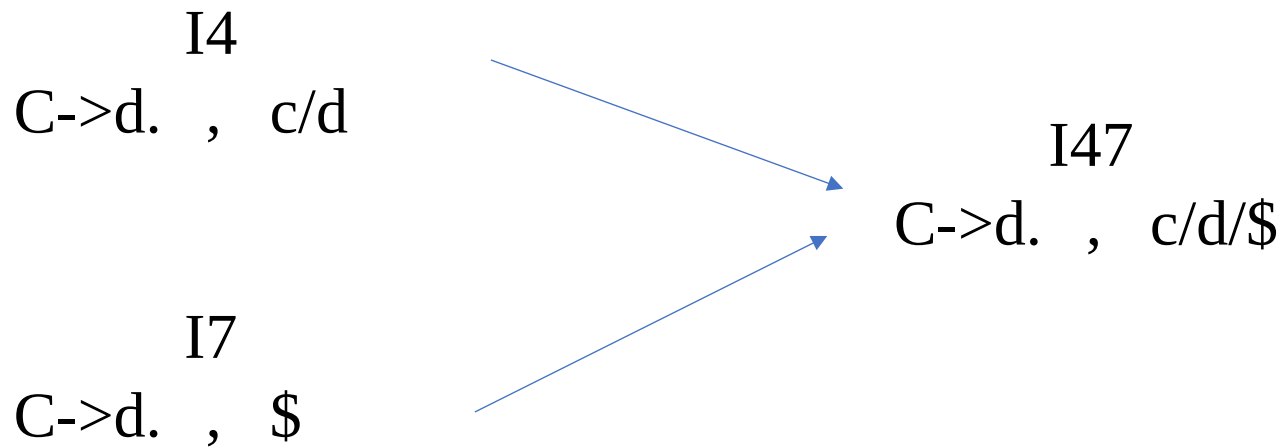
$S \rightarrow CC$

$C \rightarrow cC$

$C \rightarrow d$

# LALR Parsing Table

- For the previous example we had:



- State merges can't produce Shift-Reduce conflicts. Why?
- But it may produce reduce-reduce conflict



# Example of RR conflict in state merging

$S' \rightarrow S$

$S \rightarrow aAd \mid bBd \mid aBe \mid bAe$

$A \rightarrow c$

$B \rightarrow c$

# An easy but space-consuming LALR table construction

- Method:

1. Construct  $C = \{I_0, I_1, \dots, I_n\}$  the collection of LR(1) items.
2. For each core among the set of LR(1) items, find all sets having that core, and replace these sets by their union.
3. Let  $C' = \{J_0, J_1, \dots, J_m\}$  be the resulting sets. The parsing actions for state  $i$ , is constructed from  $J_i$  as before. If there is a conflict grammar is not LALR(1).
4. If  $J$  is the union of one or more sets of LR(1) items, that is  $J = I_1 \cup I_2 \dots \cup I_k$  then the cores of  $\text{Goto}(I_1, X)$ , ...,  $\text{Goto}(I_k, X)$  are the same and is a state like  $K$ , then we set  $\text{Goto}(J, X) = K$ .

- This method is not efficient, a more efficient one is discussed in the book

# Compaction of LR parsing table

- Many rows of action tables are identical
  - Store those rows separately and have pointers to them from different states
  - Make lists of (terminal-symbol, action) for each state
  - Implement Goto table by having a link list for each nonterminal in the form (current state, next state)

# Using ambiguous grammars

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow (E)$

$E \rightarrow id$

I0:  $E' \rightarrow .E$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

I1:  $E' \rightarrow E.$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

I4:  $E \rightarrow E + .E$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

$E \rightarrow E. (E)$

$E \rightarrow E. id$

I2:  $E \rightarrow (.E)$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

I5:  $E \rightarrow E * .E$

$E \rightarrow (.E)$

$E \rightarrow .E + E$

$E \rightarrow .E * E$

$E \rightarrow .(E)$

$E \rightarrow .id$

STATE	ACTION						GO TO
	id	+	*	(	)	\$	
0	S3			S2			1
1		S4	S5			Acc	
2	S3		S2				6
3		R4	R4		R4	R4	
4	S3			S2			7
5	S3			S2			8
6		S4	S5				
7		R1	S5		R1	R1	
8		R2	R2		R2	R2	
9		R3	R3		R3	R3	

I3:  $E \rightarrow .id$

I6:  $E \rightarrow (E.)$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

I8:  $E \rightarrow E * E.$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

I7:  $E \rightarrow E + E.$

$E \rightarrow E. + E$

$E \rightarrow E. * E$

I9:  $E \rightarrow (E).$

**Thank You**