Introduction to an Option Contract and Option Pricing Model

- **What is an option contract?** An Option contract is a financial derivative instrument which gives its holder a right, not an obligation, to buy (or, sell) a specified underlying asset at a pre-determined fixed price, on or before a specified date.

- **What is the price or value of an option contract?** Option price is influenced by a number of factors, such as the underlying instrument asset price, time to expiry of the contract, underlying asset price volatility, interest rates, etc.
  Under the classical Black–Scholes pricing framework, a European Call Option value, at a time t prior to expiration date T, is given by the following equation:

$$C(S(t), t; K, T, \sigma, r) = S(t)\, N(d_1) - K\, e^{-r(T-t)} N(d_2),$$

$$C(.,.): European\ Call\ Option\ Price\ Function,$$

$$S(t): Spot\ Price\ Process,$$

$$K: Strike\ of\ contract, r: Interest\ Rate, T-t: Time\ to\ expiry,$$

$$N(.): CDF\ of\ Standard\ Normal\ Random\ Variable$$

$$d_1 = \frac{1}{\left(\sigma\sqrt{T-t}\right)}\left[\ln\left(\frac{S}{K}\right) + r + \frac{\sigma^2}{2}(T-t)\right], d_2 = d_1 - \sigma\sqrt{T-t}$$

In the above pricing formula, market parameters σ, r are assumed to stay constant during the option lifetime; and the static parameters K, T define the Option contract.

Relationship between Market prices and implied volatility

- **What datasets are observed in the options market?** Investors observe Option prices for a range of strikes and a range of expiration/maturity dates. These prices can then be used for *calibrating* the Black-Scholes Model. This calibration procedure results in an estimate for the volatility parameter σ corresponding to each option price observed in the market. This estimate is known as Black-Scholes *Implied Volatility*, an estimate of σ such that the model price equals the observed market price.
  Inverse problem under Black-Scholes model makes the observed market Option prices and the Black-Scholes implied volatility interchangeable. It is common for the traders to quote option prices in terms of implied volatility levels. Such implied volatilities (or, option prices) are dynamic and evolving continually, as the market trades the options.
- **What is implied volatility?** Implied volatility is the market's forecast of likely movement or fluctuations in an underlying security's price. Black-Scholes Option pricing formula assumes that volatility parameter is constant for a particular underlying, and does not, in particular, depend on time to maturity and strike price of the option contract. Empirically, this is not what is

observed in the markets, as can inferred from the market option prices and the corresponding volatility values.

The following plots present the implied volatility as a function of expiration time, and as a function of moneyness, across a range of sample dates. As can be observed, the implied volatility changes across the expiration time (or, tenors); and the implied volatility has the popular "smile" profile when seen as a function of moneyness (i.e., ratio of $F/K, F$ is the Forward Price). Moreover, the implied volatility changes with time, and in fact in a non-deterministic manner.

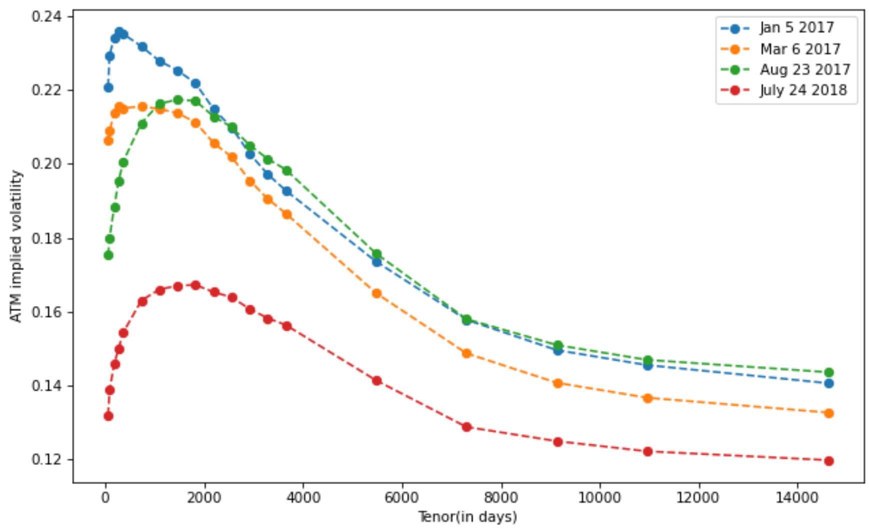

Figure: Evolution of ATM Implied volatility versus tenors

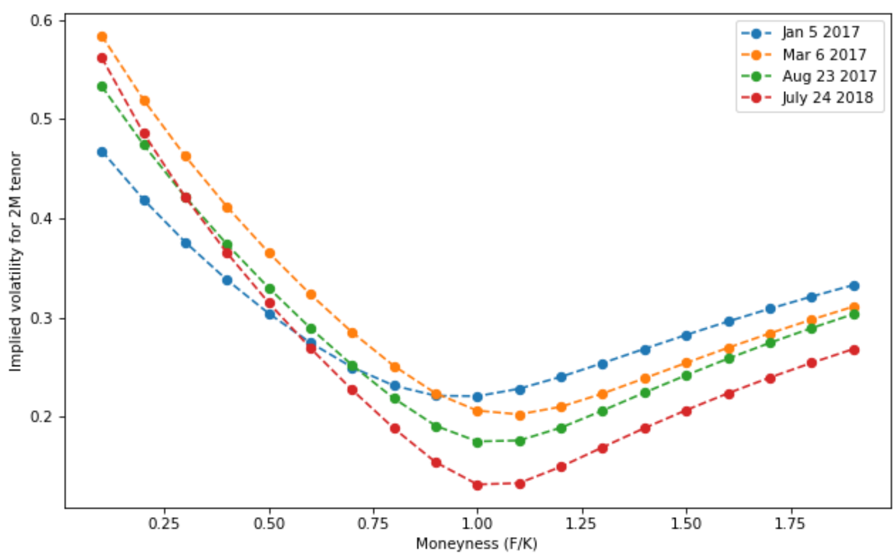

Figure: Evolution of Implied volatility versus Moneyness

- **What is volatility surface?** Volatility surface refers to a 3-dimensional plot of implied volatility of a stock option across tenors and strikes (or, moneyness). A sample plot is provided below.

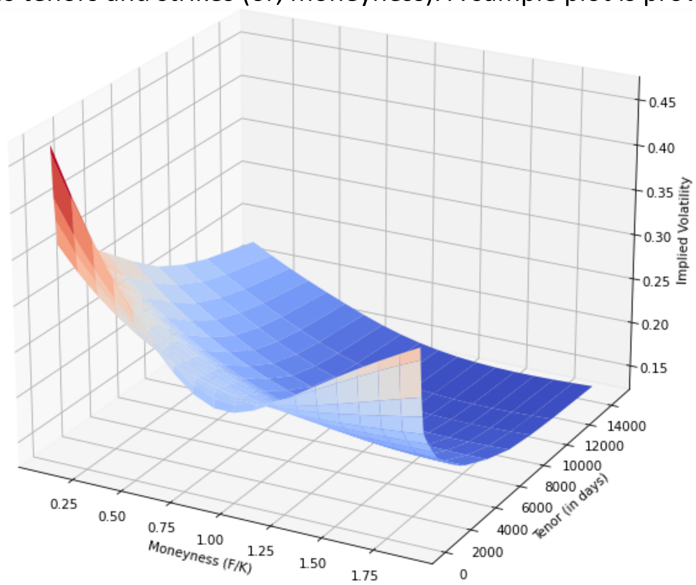

Figure: Sample volatility surface

Problem Statement – Model Implied Volatility Surface Dynamics using AI/ML techniques

1. Model *Implied Volatility Surface* dynamics based on the training dataset (**training_data.csv**), i.e., 2.5 years of volatility surface data is made available for model calibration. Participants are encouraged to do exploratory data analysis on the training data to find any patterns, trends, dependencies among tenors and use any dimension reduction methods before modelling the temporal dynamics of implied volatility surface**.** More generally, the problem calls for an ML technique to represent dynamics of a matrix-valued stochastic process.
2. The training data consists of *Implied Volatility* values across a range of maturities, and strikes (or, moneyness). Dataset has 19 tenors (from 2 months to 40 years), and 19 Strikes (represented as fraction of asset spot price) for each historical date (a total of 3 years of data, with 2.5 years of training sample and 2 months of testing sample).
3. The participants are required **to forecast the volatility surface for the next 60 trading days** (10/15/19- 1/6/20) using training data set. They're required to make their submission in the **prediction_template.csv.**
4. Selected model will be judged based on accuracy of predictions (e.g., based on the metric Root-mean-squared-error or RMSE), complexity of model (Model Selection criteria, Specific ML technique), code quality (e.g., computational time), pitch book and presentation (for finalist). Teams can make reasonable assumptions as part of model building, if needed.
5. Participants are expected to prepare a pitch book with up to 5 slides. Shortlisted teams would be asked to present this in the final round.
6. Participants can form teams of up to 3 members. Each team should finalize a name, this will be used to identify their submissions, ppt etc.

7. During the competition, teams can make a maximum of 3 submissions against the actual test data, and receive a RMSE score. Organizers will share the schedule for these submissions. And lastly the team will also submit final predictions close to the competition deadline.
8. **Final deliverables:** 1 final_prediction csv, pitch book, (preferable Jupyter notebook) Python/R code