What is the response time $T_{1/2}$ to go from $Y=0$ to $Y=Y_{ST}$ when a signal S_x appears?
($\beta=0 \rightarrow \beta=\beta$)

$$\frac{dY}{dt} = \beta - \alpha Y$$

$$\int \frac{\frac{dY}{dt}}{\beta - \alpha Y}\, dt = \int dt$$

Let $u = Y$, and $du = \frac{dY}{dt} dt$

$$\int \frac{du}{\beta - \alpha u} = \int dt$$

Let $s = \beta - \alpha u$

$\therefore ds = -\alpha\, du$

$du = -\frac{1}{\alpha} ds$

$$-\frac{1}{\alpha}\int \frac{1}{s}\, ds = \int dt$$

$$-\frac{1}{\alpha} \ln s + c_1 = t + c_2$$

$$-\frac{1}{\alpha} \ln(\beta - \alpha Y) = t + c_3$$

$$\ln(\beta - \alpha Y) = -\alpha t + c_3$$

$$\beta - \alpha Y = c_3 e^{-\alpha t}$$

$$-\alpha Y = -\beta + c_3 e^{-\alpha t}$$

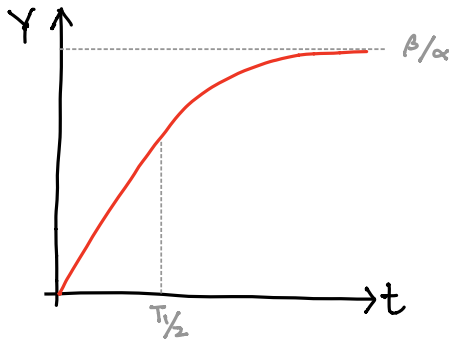
$$Y = \frac{\beta}{\alpha} - c_4 e^{-\alpha t}$$

$$0 = \frac{\beta}{\alpha} - c_4 e^{-\alpha(0)}$$

$$\frac{\beta}{\alpha} = c_4$$

$$Y = \frac{\beta}{\alpha} - \frac{\beta}{\alpha} e^{-\alpha t}$$

$$\boxed{Y = \frac{\beta}{\alpha}\left(1 - e^{-\alpha t}\right)}$$



at $T_{1/2}$, $Y = \frac{1}{2} Y_{ST}$ $\rightarrow$ $\frac{1}{2} Y_{ST} = \frac{\beta}{\alpha}\left(1 - e^{-\alpha T_{1/2}}\right)$

$$Y_{ST} = \frac{\beta}{\alpha}$$

$$\frac{1}{2}\frac{\beta}{\alpha} = \frac{\beta}{\alpha}\left(1 - e^{-\alpha T_{1/2}}\right)$$

$$\frac{1}{2} = 1 - e^{-\alpha T_{1/2}}$$

$$\frac{1}{2} = e^{-\alpha T_{1/2}}$$

$$-\ln 2 = -\alpha T_{1/2}$$

$$\boxed{\frac{\ln 2}{\alpha} = T_{1/2}}$$