Error estimation in ballistic missile trajectory using Kalman Filter

Anumit Garg[1], K.S. Nagla[2]
Department of Electronics and Communication Engineering, Department of Instrumentation and Control Engineering
Dr. B.R. Ambedkar National Institute of Technology Jalandhar, India
Email:anumitgarg@gmail.com; naglaks@nitj.ac.in

Abstract – **The Kalman Filter is a very successful and popular tool for the estimation of posterior information of several engineering and non-engineering applications. In engineering applications, the estimation of the state of a ballistic missile is an important requirement for the reliable deployment of an anti ballistic missile (ABM) system. But during the measurement of its flight through radar there are many disturbances that tend to add significant amount of error in its readings thus reducing the effectiveness of ballistic missile defense system. Many researches conducted in the field estimated the state of the missile and tried to minimize the error by using various techniques such as Bayesian theorem, Kalman filter and extended Kalman filter. This research examines the Radar related errors, studies the extent of its impacton ABM system and proposes a solution to minimize the errors and increase the effectiveness of the system. The results obtained are used for error estimation in the filtered radar data and theoretically predicted trajectory of the missile. This approach considerably reduces the error and achieves a high level of accuracy in target interception.**

Keywords - Kalman filter, anti ballistic missile (ABM) system, Radar, defense, target interception, error estimation.

I. INTRODUCTION

Kalman filter is an algorithm that uses a series of sensor readings taken over a long span of time, containing noise, random variations, sensor errors and other inaccuracies, and predicts the future state of a system by taking into the account the past measurements and predicted state errors. The future state predicted by Kalman filter is more precise and accurate than those that would be based on sensor data alone [1]. It has numerous applications in technology; it is primarily used for orbit calculation, guidance, navigation and control of vehicles, tracking of maneuvering objects particularly aircraft and spacecraft [2]. Furthermore, the Kalman filter is one of the most widely applied algorithms in time series analysis used in fields such as signal processing and econometrics. Kalman filters are also one of the main topics in the field of robotic motion planning in time varying environments [3], and they are sometimes also applied for trajectory optimization of a projectile.

Ballistic missile proliferation poses a fundamentally new challenge for every country's defense planners. Ballistic missiles are used to deliver chemical, biological, nuclear, or conventional warheads in a ballistic flight trajectory [4]. Thus a missile defense system is of strategic importance to every country. An Anti- Ballistic Missile (ABM) refers to a system designed to intercept and destroy any type of ballistic threat, however it is commonly used for systems specifically designed to counter intercontinental ballistic missiles (ICBMs) [5]. Earlier ABMs had to carry nuclear warheads to stop a missile but now nearly all of our ABMs carry "Hit to kill" vehicle which physically hits the target to destroy it, thus an accurate and precise interception of a ballistic missile is of utmost importance [6]. An ABM uses Radar technology to intercept ballistic trajectory of a missile outside the atmosphere [7].

II. BALLISTIC MISSILE BASICS

Initially a ballistic missile is propelled by rockets to a desired point determined by its predefined trajectory but after that it follows an unpowered, free-falling trajectory toward their targets. The flight phase of the missile consists of three phases [8] asillustrated in Fig. 1.

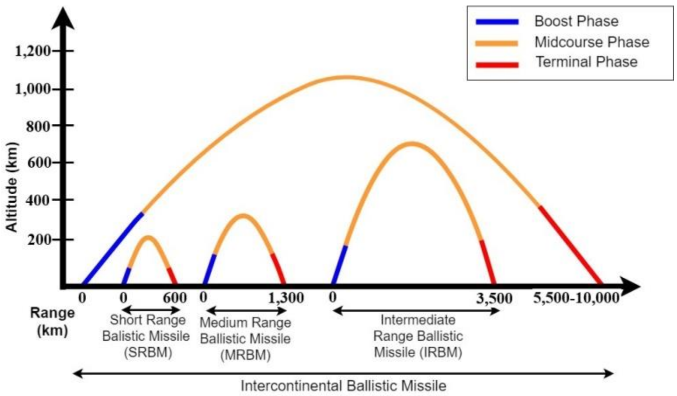


Fig. 1. Phases of a ballistic missile trajectory

A. Boost Phase

During boost phase the ballistic missile powered by an engine is continuously accelerated against gravity till it reaches its maximum velocity. Boost phase lasts for 3-4 minutes with missile attaining an altitude of 150-200 km and typical burn-out speed as 7km/s at the end of this phase. The satellites detect the launch of a missile and later transfer the control to forward deployed Radarsystems; these Radars further calculate the missile trajectory throughout its course.

B. Midcourse Phase

Midcourse phase covers majority of the flight time of a ballistic missile and ranges from a minute to an hour (depending on the range of the missile). During this phase the payload which includes warheads, decoys and radar reflectors known as target cloud follows a ballistic trajectory to the desired target. The main objectives of this phase are to minimize miss distance and achieve a predetermined terminal flight path angle [9].

C. Terminal Phase

During this phase the missile reenter the earth's atmosphere, this phase is less than a minute long. Interception of the missile is more feasible and reliable in this phase. The impact angle and terminal speed of the missile are two of the most important factors for maximum destruction caused by the missile and thus are of strategic importance for the performance of the missile [10].

Even though the missile is steered in flight throughout the boost phase, once the rockets are spent, there is no mechanism available to control and direct the movement of the conventional missile. The midcourse phase and terminal phase are most suitable for intercepting an incoming ICBM. The mid-course phase of a missile is the longest phase of its trajectory though the terminal phase lasts for only a minute; these phases provide the most suitable time and predictable opportunities for intercepting an incoming ICBM.

III. PROBLEM FORMULATION

Radar systems are critical elements of air and missile defense systems, thus their accuracy play a strategic role in designing and deploying an effective ABM system. Radarsare designed to transmit electromagnetic waves in a format such that information about the target can be accurately extracted from its echo. Once a target is detected, the next goal is to precisely locate that target and find its radial velocity in three-dimensional space. There are many potential sources of error inherent in precision tracking of a missile. The error sources can broadly be classified in three categories [11].

- Radar platform and target state (velocity, acceleration and micro characteristics).
- Transmission characteristics including multipath and atmospheric transmission.
- Guidance radar, including radar system parameters, antenna characteristics, filters etc.

Therefore, the accuracy of the measurements is the most significant factor for an effective ABM system.

IV. KALMAN FILTER MODELLING

This section describes the filter in its original formulation (Kalman 1960) [12] where the measurements are taken at periodic intervals of time and the new, more accurate states are predicted based on those past measurements.

A. Prediction step

The state and process error of the system at time step 'k' is predicted based on its past state, control input and process covariance matrix at time step 'k-1'. The equations governing these relationships are illustrated below.

$$X_{kp}=AX_{k-1}+BU_k+W_k \tag{1}$$

$$P_{kp}=AP_{k-1}A^T+Q_k \tag{2}$$

Where,X is state matrix, P is process covariance matrix, U is control variable matrix, W is predicted state noise matrix, Q process noise covariance matrix, A and B are adaption matrices to convert input state to process state. For practical reasons the process noise and Measurement noise are assumed to be Gaussian in nature and statistically independent of each other.

B. Calculation of Kalman Gain and measurement update

1) *Kalman gain:* The Kalman gain K represents the relative importance of the predicted state estimation with respect to sensor measurement on the basis of previously observed errors in the estimation and the sensor measurement. Kalman gain is defined as follows.

Where, K is Kalman gain, H is conversion matrix and

$$K=\frac{P_{kp}H^T}{HP_{kp}H^T+R} \tag{3}$$

R is sensor noise.

Intuitively, if the predictions made are relatively more accurate, then the Kalman gain would contribute to decrease the relative importance of the new sensor reading, but if the predictions made are less accurate and more prone to errors, then it gives more importance to the new measurements to make subsequent predictions.

2) *Measurement update:* New updates of the system state and process covariance matrix are made using the trends of previously predicted system states and errors as follows –

$$X_k=X_{kp}+K[Y-HX_{kp}] \tag{4}$$

$$P_{kp}=[I-KH]P_{kp} \tag{5}$$

Where Y is sensor measurement of the state.

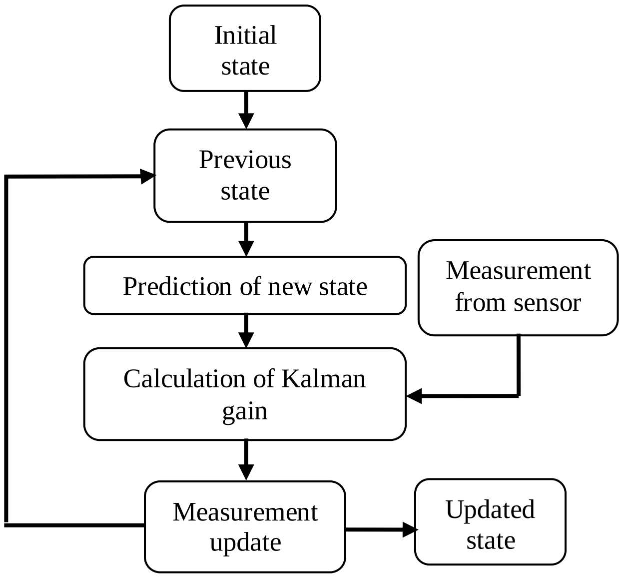
Now, the physical significance of Kalman gain and its impact on measurements taken can be demonstrated as follows –

Substitution of first limit into the measurement update equation suggests that when the magnitude of R is small, or in other words the measurements are accurate, the state estimate depends mostly on the measurements.

When the state is known accurately, then $HP_{kp}H^T$ is small compared to R, and the filter mostly ignores the measurements and relies instead on the prediction derived from the previous state (X_{kp}).

C. Iteration:

Initially we gathered all the information and required parameters to calculate the posterior information about the state of missile. Now we can iterate through the process to calculate further states of the missile. Each iteration converge the predicted state towards the actual state of the system thus reducing the error in the estimate as shown in Fig. 2.



V. Mathematical Modelling Of Proposed Solution

In this paper we propose a Kalman filter based model for the optimal trajectory prediction and error estimation of a ballistic missile before the actual impact takes place. The measurements are error prone due to various potential sources of error in radar interception [11] and thus the sensor readings cannot be considered the true measure of missile's position. Kalman Filter combines the radar measurements and past predicted values to predict a new more accurate prediction. Based on these predictions, ABMs can be deployed to destroy the missile amidst air before it hits the target.

Here,for the purpose of demonstration we have assumed the projectile to be a point mass, thus its dimension with respect to its displacement is considered negligible. According to the measured position and velocity data, the measurement equation was established. The trajectory

$$\lim_{R\to 0} \frac{P_{kp}H^T}{HP_{kp}H^T+R} = H^{-1} \tag{6}$$

$$\lim_{P_{kp}\to 0} \frac{P_{kp}H^T}{HP_{kp}H^T+R} = 0 \tag{7}$$

parameters were estimated by using linearized Kalman filter.

We define the state parameters for the projectile as follows:

$$X = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{8}$$

Where, x, y, z represent displacement in x, y and z directions and $\dot{x}, \dot{y}, \dot{z}$ represents velocities in x, y and z directions respectively.

From equation (1), the new predicted state (X_{kp}) is given by –

$$X_{kp} = AX_{k-1} + BU_k + W_k \tag{9}$$

The adaptation matrices A and B for a projectile are defined as follows -

$$A = \begin{bmatrix} 1 & 0 & 0 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 & dt & 0 \\ 0 & 0 & 1 & 0 & 0 & dt \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$B = \begin{bmatrix} dt^2/2 & 0 & 0 \\ 0 & dt^2/2 & 0 \\ 0 & 0 & dt^2/2 \\ dt & 0 & 0 \\ 0 & dt & 0 \\ 0 & 0 & dt \end{bmatrix} \tag{11}$$

Where, dt represents the time lapse between two consecutive readings.

The random variables and represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability distributions as shown below.

$$p(w) \sim N(0, \sigma_w^2) \tag{12}$$

$$p(v) \sim N(0, \sigma_v^2) \tag{13}$$

Where $N(\mu,\sigma^2)$ is the normal probability distribution function with μ as mean and σ^2 as variance.

Control variable matrix U is defined as –

$$U = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \tag{14}$$

Where, a_x , a_y , a_z represents the acceleration experienced by the missile system in x, y and z directions respectively due to gravity and variable drag during its course of flight.

Therefore, from equations (1), (8), (10), (11) and (14) the new predicted state (X_{kp}) is given by:

$$X_{kp} = AX_{k-1} + BU_k + W_k \tag{15}$$

$$X = \begin{bmatrix} 1 & 0 & 0 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 & dt & 0 \\ 0 & 0 & 1 & 0 & 0 & dt \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \tag{16}$$

$$\begin{bmatrix} dt^2/2 & 0 & 0 \\ 0 & dt^2/2 & 0 \\ 0 & 0 & dt^2/2 \\ dt & 0 & 0 \\ 0 & dt & 0 \\ 0 & 0 & dt \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Therefore, the predicted state is:

$$X = \begin{bmatrix} x + (\dot{x} \times dt) + (0.5 \times a_x \times dt^2) \\ y + (\dot{y} \times dt) + (0.5 \times a_y \times dt^2) \\ z + (\dot{z} \times dt) + (0.5 \times a_z \times dt^2) \\ \dot{x} + (a_x \times dt) \\ \dot{y} + (a_y \times dt) \\ \dot{z} + (a_z \times dt) \end{bmatrix} \tag{17}$$

Process Variance - Covariance matrix (P):

$$P = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{x\dot{x}} & \sigma_{x\dot{y}} & \sigma_{x\dot{z}} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} & \sigma_{y\dot{x}} & \sigma_{y\dot{y}} & \sigma_{y\dot{z}} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 & \sigma_{z\dot{x}} & \sigma_{z\dot{y}} & \sigma_{z\dot{z}} \\ \sigma_{x\dot{x}} & \sigma_{y\dot{x}} & \sigma_{z\dot{x}} & \sigma_{\dot{x}}^2 & \sigma_{\dot{x}\dot{y}} & \sigma_{\dot{x}\dot{z}} \\ \sigma_{x\dot{y}} & \sigma_{y\dot{y}} & \sigma_{z\dot{y}} & \sigma_{\dot{x}\dot{y}} & \sigma_{\dot{y}}^2 & \sigma_{\dot{y}\dot{z}} \\ \sigma_{x\dot{z}} & \sigma_{y\dot{z}} & \sigma_{z\dot{z}} & \sigma_{\dot{x}\dot{z}} & \sigma_{\dot{y}\dot{z}} & \sigma_{\dot{z}}^2 \end{bmatrix} \tag{18}$$

Where variance (σ_x^2) is a measure of the variability or spread in a set of data (here the data being x). Mathematically, it is the average squared deviation from the mean score and Covariance (σ_{xy}) is a measure of the extent to which corresponding elements from two sets of ordered data (here the data being x and y) move in the same direction.

Therefore, from equation (2) the new predicted state (P_{kp}) is given by –

$$P_{kp} = AP_{k-1}A^T + Q_k \tag{19}$$

Once the predicted values are obtained, the Kalman gain matrix, K is calculated by:

$$K = \frac{P_{kp}H^T}{HP_{kp}H^T + R} \tag{20}$$

here, matrix H is defined as:

$$H = \begin{bmatrix} 100000 \\ 010000 \\ 001000 \\ 000100 \\ 000010 \\ 000001 \end{bmatrix} \tag{21}$$

and matrix R is the sensor noise covariance matrix that depends upon the error in sensor measurement.

Based on the value of Kalman Gain (K), a new state and covariance matrix is predicted as follows –

$$X_k = X_{kp} + K[Y - HX_k] \tag{22}$$

$$P_{kp} = [I - KH]P_{kp} \tag{23}$$

Now we can iterate through the estimates on discrete time scale. Each iteration converge the state predicted by the model towards the actual state of the system thus reducing error in the estimate and providing a more promising and accurate posterior information about the state of the missile.

VI. Simulation Results And Discussion

The mathematical model and the associated computer simulations of the radar data demonstrate the reduction in the target dependent errors upon the application of Kalman Filter on the sensor readings. Fig. 3 depicts radar measurement compared to the theoretically obtained trajectory. In this illustration a large variation can be observed between the radar data and theoretically determined trajectory of the missile. This error is undesirable for effective deployment of ABM. Fig. 4 depicts the filtered sensor readings obtained after the application of Kalman filter compared with true sensor values and Fig. 5 depicts the filtered sensor readings compared with theoretically predicted values. Clearly upon application of Kalman filter the radar measurements converge to the true trajectory of the missile, thus reducing the error in missile interception.

The effectiveness of the solution is analyzed based on different sets of synthesized radar data prone to errors with variable means and standard deviations. Table 1 represents the corresponding means and variances of the sensor and the results of the Kalman filter, the readings clearly suggests the decrease of 60.51% in the mean sensor error and 84% decrease in the variance of the sensor errors. The result can also be realized by observing the probability distribution curve of the sensor and filtered readings (shown in Fig. 6).

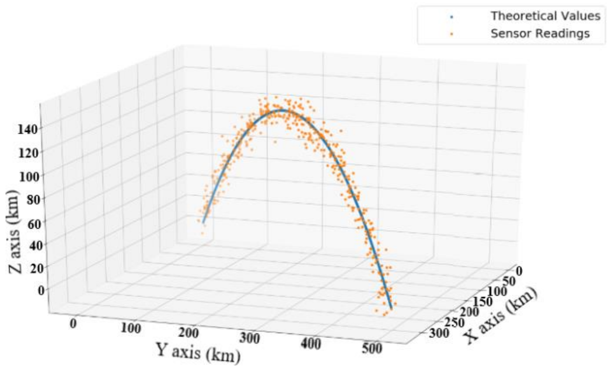


Fig. 3. Sensor Readings compared with theoretically predicted trajectory of the missile.

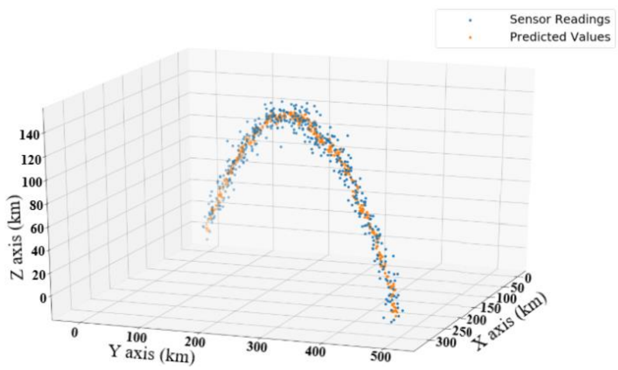


Fig. 4. Predicted values (based on Kalman Filter) compared with sensor readings

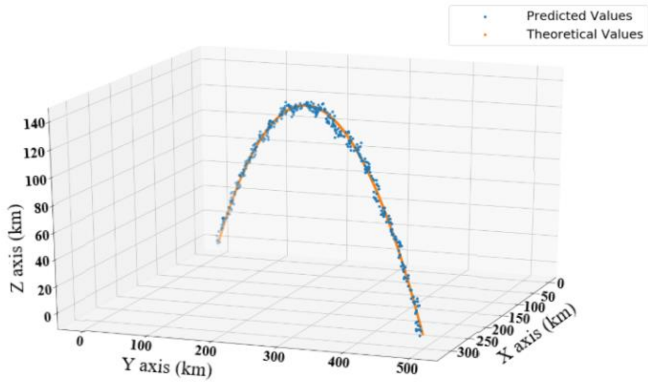


Fig. 5. Predicted values (based on Kalman Filter) compared with theoretically predicted trajectory of the missile.

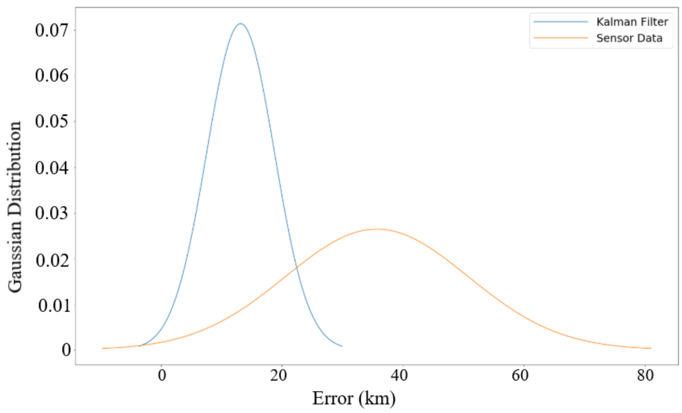


Fig. 6. Probability distribution function

TABLE I. THE CORRESPONDING MEANS AND VARIANCES OF THE SENSOR MEASUREMENT AND THE RESULTS OF THE KALMAN FILTER

Sr. no.	Sensor Readings		Prediction Values		Percentage change	
	Sensor error mean (km)	***Sensor error variance (km^2)***	***Prediction error mean (km)***	***Prediction error variance (km^2)***	***Percentage change in mean***	***Percentage change in variance***
1	1.636	0.494	0.665	0.083	59.352%	83.198%
2	4.984	4.534	1.986	0.671	60.152%	85.201%
3	8.078	11.686	3.066	1.823	62.045%	84.400%
4	11.255	19.958	4.612	3.73	59.023%	81.311%
5	14.572	33.569	5.47	5.468	62.462%	83.711%
6	16.507	47.503	6.664	7.98	59.629%	83.201%
7	19.315	62.598	7.728	10.609	59.990%	83.052%
8	23.496	90.12	8.502	12.775	63.815%	85.824%
9	26.1514	118.903	10.707	17.299	59.058%	85.451%
10	29.982	158.157	12.11	23.844	59.609%	84.924%
				Average	**60.514%**	**84.027%**

VII. CONCLUSION

This paper analyzes the effect of inherent radar errors on the trajectory interception of a ballistic missile. In this paper we proposed a Kalman filter based solution to estimate and decrease the error to achieve high level of accuracy in target interception. Firstly, the sources of errors in radar measurements are analyzed, subsequently a mathematical model of the target - radar system is modeled to analyze the effect of Kalman filtering on corresponding sensor noise. Upon application of filter a decrease of approximately 60.5% in mean error and 84% in error variance is observed.

This error analysis can further be used to carry out actual in field testing of the proposed solution and validate the simulation results. Thus after further research it can be implemented on existing ABM systems to achieve high level of accuracy in target interception and launch of hit to kill vehicles.

REFERENCES

[1] Mohammed, Yassir Obeid, and Abdelrasoul Jabar Alzubaidi, "Missile position tracking using Kalman filter," IOSR J. Eng 4, vol. 02, pp. 43-45, 2014

[2] Li, Qiang, Ranyang Li, Kaifan Ji, and Wei Dai, "Kalman filter and its application," 8th IEEE International Conference on Intelligent Networks and Intelligent Systems (ICINIS), pp. 74-77, 2015.

[3] Elnagar, Ashraf, "Prediction of moving objects in dynamic environments using Kalman Filters," IEEE International Symposium on Computational Intelligence in Robotics and Automation, pp. 414-419, 2001.

[4] Hull AW, "Role of ballistic missiles in Third World defense strategies," INSTITUTE FOR DEFENSE ANALYSES ALEXANDRIA VA; June 1991.

[5] Ann, Sungjun, Seokwon Lee, and Youdan Kim, "Intercept Point Prediction for Midcourse Guidance of Anti-Ballistic Missile." 26th Mediterranean Conference on Control and Automation (MED), IEEE, pp. 1-9, 2018.

[6] Shtessel, Yuri B., Ilya A. Shkolnikov, and Arie Levant, "Guidance and control of missile interceptor using second-order sliding modes," IEEE Transactions on Aerospace and Electronic Systems, vol. 45 no. 1, pp.110-124, 2009

[7] Camp, William W., Joseph T. Mayhan, and Robert M. O'Donnell, "Wideband radar for ballistic missile defense and range-Doppler imaging of satellites," Lincoln Laboratory Journal, vol. 12 no. 2, pp. 267-280, 2000

[8] Moon, Kyujin, Hojun Kwon, Chang-Kyung Ryoo, and Hongchul Sim, "Trajectory Estimation for a Ballistic Missile in Ballistic Phase using IR Images," 9th International Conference on Mechanical and Aerospace Engineering (ICMAE), IEEE, pp. 173-177, 2018.

[9] Morgan, Robert W., and Aaron P. Noseck, "Generalized optimal midcourse guidance," 53rd Annual Conference on Decision and Control (CDC), IEEE, pp. 260-264, 2014.

[10] Moon, Kyujin, Sang-Sup Park, and Chang-Kyung Ryoo , "Guidance algorithm for satisfying terminal speed and Impact Angle constraints with the neural network technique," 15th International Conference on Control,

Automation and Systems (ICCAS), IEEE, pp. 466-471, 2015.

[11] Zheng, Liu, Tang Xing, and Fu Qiang, "An analysis on error sources of guidance radar detection performance," IEEE CIE International Conference on Radar (Radar), IEEE, vol. 2, 2011.

[12] Kalman, Rudolph Emil, "A new approach to linear filtering and prediction problems," Journal of basic Engineering, vol.82 no.1, pp. 35-45, 1960.

978-1-7281-1528-3/19/$31.00 ©2019 Crown