Assignment 2 - CSC/DSC 262/462 - Fall 2017 - Due October 10

1. Two dice are tossed independently. Let X_1, X_2 be random variables representing the two outcomes, each from sample space $S_X = \{1, 2, 3, 4, 5, 6\}$. Derive the probability mass function of the following random variables:

   (a) $X = X_1 - X_2$,

   (b) $X = \begin{cases} X_1 & ; \quad X_1 = X_2 \\ 0 & ; \quad X_1 \neq X_2 \end{cases}$.

2. A random variable X possesses the following density function for some constant c:

$$f_X(x) = \begin{cases} -c(x-2)(x-4) & ; \quad x \in [2, 4] \\ 0 & ; \quad otherwise \end{cases} .$$

   (a) Determine c.

   (b) Determine the CDF $F(x) = P(X \leq x)$. Give this as a function of $x \in (-\infty, \infty)$.

3. A certain hospital delivered 10 babies during the last year. Assume that a baby is equally (and independently) likely to be a boy or girl.

   (a) What is the probability that 6 of these were boys?

   (b) What is the probability that at least 8 of these were boys?

   (c) Given that 6 of these were boys, what is the probability that the first six deliveries were all boys?

4. The distribution of the height of a certain type of plant is normally distributed with mean $\mu = 49.2$ inches and standard deviation $\sigma = 1.75$ inches. What is the probability that of 20 randomly selected plants, no more than 3 plants height of no more than 48 inches?

5. If necessary, the number $\pi = 3.141593\ldots$ can be estimated by a simple simulation experiment. The area of a square of unit sides equals 1. A circle of radius $r = 1/2$ can be embedded in this square. The area of a circle is πr^2. Write an `R` function which does the following.

   (a) Create an $n \times 2$ matrix, such that each row contains the coordinates of a point uniformly distributed in a unit square centered at $(0, 0)$.

   (b) Count the number of such points contained in the embedded circle described above. Call this number m.

   (c) Use the numbers m and n to estimate π. You can set $n = 10000$.

6. **[For graduate students]** Suppose there are m types of coupons. A collector samples them one at a time. Whenever a collector samples another coupon, assume it is of each type with equal probability, and that the selections are independent. What is the expected number of coupon samples needed to have at least one of each type? [Hint: Suppose T_j is the number of samples at which the jth new coupon is observed by the collector. What is the distribution of $T_{j+1} - T_j$, for any $j = 1, 2, \ldots, n-1$?]