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Team Reference Document

# Contents

## 1 Setup

1.1	CP_Ubuntu . . . . .
1.2	CP_Windows . . . . .
1.3	StressTesting(check.sh) . . . . .
1.4	StressTesting(gen.cpp) . . . . .

## 2 Data Structures

2.1	Custom Hash . . . . .
2.2	Fast Unordered Map . . . . .
2.3	GP Hash Table . . . . .
2.4	Mex of All Subarray . . . . .
2.5	Pbds . . . . .
2.6	Segment Tree(BSUA) . . . . .
2.7	Segment Tree(LzP) . . . . .
2.8	Segment Tree . . . . .
2.9	Sparse Table . . . . .

## 3 Dynamic Programming

3.1	Coin Change(Number of Ways) . . . . .
3.2	Digit DP . . . . .
3.3	LIS . . . . .
3.4	Maximum Subarray Sum(Kadanes) . . . . .

## 4 Graph

4.1	Articulation Point . . . . .
4.2	BFS . . . . .
4.3	Bellman Ford . . . . .
4.4	Bridge Finding DFS . . . . .
4.5	Cycle Detection in DAG . . . . .
4.6	DSU . . . . .
4.7	Dijkstra . . . . .
4.8	Euler Tour . . . . .
4.9	Floyd Warshall . . . . .
4.10	MST . . . . .
4.11	Max Bipartite Matching[Hopcroft Karp] . . . . .
4.12	Max Bipartite Matching[Kuhn's] . . . . .
4.13	Max Flow . . . . .
4.14	Topological Sorting . . . . .
4.15	Weighted Union Find . . . . .

## 5 Math

5.1	Matrix Exponentiation . . . . .
5.2	Matrix Rotation . . . . .

## 6 Miscellaneous

6.1	Max Subarray Size Sum equal K . . . . .
6.2	Merge Sort . . . . .
6.3	Number of Subarray Sum is K . . . . .

## 7 Number Theory

7.1	All In One NT . . . . .
7.2	Divisor Sieve . . . . .
7.3	Number of Pairs with GCD equal g . . . . .
7.4	Phi(1toN) . . . . .
7.5	Phi . . . . .
7.6	SOD NOD . . . . .
7.7	Segmented Sieve . . . . .
7.8	Sieve . . . . .
7.9	Spf . . . . .
7.10	UniquePF of all elements till MX . . . . .
7.11	int128 . . . . .

7.12	nCr and nPr . . . . .
7.13	nCr v2 . . . . .

## 8 String

8.1	Aho Corasic . . . . .
8.2	LCS for 3 Strings . . . . .
8.3	Manacher Palindrome . . . . .
8.4	String Hashing 2 . . . . .
8.5	String Hashing . . . . .
8.6	Suffix Array . . . . .
8.7	Suffix Automata . . . . .
8.8	Suffix Automation . . . . .
8.9	Trie . . . . .

## 9 Tree

9.1	Centroid Decomposition . . . . .
9.2	DSUOnTrees . . . . .
9.3	LCA using binary Lifting . . . . .
9.4	LCA . . . . .

## 10 Notes

10.1	Geometry . . . . .
10.2	Binomial Coefficient . . . . .
10.3	Fibonacci Number . . . . .
10.4	Sums . . . . .
10.5	Series . . . . .
10.6	Pythagorean Triples . . . . .
10.7	Number Theory . . . . .
10.8	Permutations . . . . .
10.9	Partitions and subsets . . . . .
10.10	Coloring . . . . .
10.11	General purpose numbers . . . . .
10.12	Ballot Theorem . . . . .
10.13	Classical Problem . . . . .
10.14	Matching Formula . . . . .
10.15	Inequalities . . . . .
10.16	Games . . . . .
10.17	Tree Hashing . . . . .
10.18	Permutation . . . . .
10.19	String . . . . .
10.20	Bit . . . . .
10.21	Convolution . . . . .

## 6 Setup

### 6.1 CP\_Ubuntu

```
6 "cmd": ["ulimit -s 268435456; g++ -std=c++20  
6   $file_name -o $file_base_name && timeout 4s  
6   ./${file_base_name} < inputf.in >  
6   ${outputf.in"]},  
7 "selector": "source.cpp",  
7 "shell": true,  
7 "working_dir": "$file_path"
```

### 6.2 CP\_Windows

```
8 "cmd": ["g++.exe", "-std=c++20", "${file}",  
8   "-o", "${file_base_name}.exe", "&&", "${f}  
8   ile_base_name}.exe<inputf.in>outputf.in"],  
8 "selector": "source.cpp",  
8 "shell": true,  
8 "working_dir": "$file_path"
```

8 }

## 8.3 StressTesting(check.sh)

```
8 // chmod u+x check.sh  
8 // ./check.sh  
9 set -e  
9 g++ gen.cpp -o gen  
9 g++ code.cpp -o code  
9 g++ brute.cpp -o brute  
9 for ((i = 1; ; ++i)); do  
10 echo "Passed on TestCase: " $i  
10 ./gen $i > in  
10 ./code < in > out1  
10 ./brute < in > out2  
10 diff -Z out1 out2 || break  
10 done  
10 echo -e "WA on the following test:"  
11 cat in  
11 echo -e "\nExpected:  
11 cat out2  
11 echo -e "\nFound:  
11 cat out1
```

## 1.4 StressTesting(gen.cpp)

```
#include <bits/stdc++.h>  
using namespace std;  
using ll = long long;  
mt19937_64 rng(std::chrono::steady_clock::now().time_<br/>  
→ since_epoch().count());  
inline ll gen_random(ll l, ll r) {  
    return uniform_int_distribution<ll>(l, r)(rng);  
}  
inline double gen_random_real(double l, double  
→ r) {  
    return uniform_real_distribution<double>(l,  
→ r)(rng);  
}  
int main(int argc, char* args[]) {  
    int _ = atoi(args[1]);  
    rng.seed(_);  
    int n = gen_random(1, 5);  
    vector<int> per;  
    for (int i = 0; i < n; ++i) {  
        per.push_back(i + 1);  
    }  
    shuffle(per.begin(), per.end(), rng);  
    return 0;  
}
```

## 2 Data Structures

### 2.1 Custom Hash

```
#include <ext/pb_ds/assoc_container.hpp>  
using namespace gnu_pbds;  
struct customHash {  
    static uint64_t Meaw(uint64_t x) {  
        x += 0x9e3779b97f4a7c15;  
        x = (x ^ (x >> 30)) * 0xbff58476d1ce4e5b9;  
        x = (x ^ (x >> 27)) * 0x94d049bb13311eb;  
        return x ^ (x >> 31);  
    }  
    size_t operator()(uint64_t x) const {  
        static const uint64_t FIXED_RANDOM =  
            chrono::steady_clock::now().time_since_e  
            → poch().count();  
        return Meaw(x + FIXED_RANDOM);  
    }
```

```
}; // gp_hash_table<int, int> table;
```

## 2.2 Fast Unordered Map

```
mp.reserve(1<<20); // about 1M buckets
mp.max_load_factor(0.7); // safe and fast
```

## 2.3 GP Hash Table

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
const int RANDOM = chrono::high_resolution_clock::now().time_since_epoch().count();
struct custom_hash {
    int operator()(int x) const { return x ^ RANDOM; }
}; //gp_hash_table<int, int, custom_hash> mp;
```

## 2.4 Mex of All Subarray

```
const int N = 1e5 + 9, inf = 1e9;
struct ST {
    int t[4 * N];
    ST() {}
    void build(int n, int b, int e) {
        t[n] = 0;
        if (b == e) {
            return;
        }
        int mid = (b + e) >> 1, l = n << 1, r = l | 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        t[n] = min(t[l], t[r]);
    }
    void upd(int n, int b, int e, int i, int x) {
        if (b > i || e < i) return;
        if (b == e && b == i) {
            t[n] = x;
            return;
        }
        int mid = (b + e) >> 1, l = n << 1, r = l | 1;
        upd(l, b, mid, i, x);
        upd(r, mid + 1, e, i, x);
        t[n] = min(t[l], t[r]);
    }
    int get_min(int n, int b, int e, int i, int j) {
        if (b > j || e < i) return inf;
        if (b >= i && e <= j) return t[n];
        int mid = (b + e) >> 1, l = n << 1, r = l | 1;
        int L = get_min(l, b, mid, i, j);
        int R = get_min(r, mid + 1, e, i, j);
        return min(L, R);
    }
    int get_mex(int n, int b, int e, int i) { // mex of [i... cur_id] if (b == e) return b;
        int mid = (b + e) >> 1, l = n << 1, r = l | 1;
        if (t[l] >= i) return get_mex(r, mid + 1, e, i);
        return get_mex(l, b, mid, i);
    }
};
```

```
} t;
int a[N], f[N];
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    cin >> n;
    for (int i = 1; i <= n; i++) {
        cin >> a[i];
        --a[i];
    }
    t.build(1, 0, n);
    set<array<int, 3>> seg; // for cur_id = i,
    // [x[0]... i], [x[0] + 1...i], ...[x[1]... i]
    for (int i = 1; i <= n; i++) {
        int x = a[i];
        int r = min(i - 1, t.get_min(1, 0, n, 0, x - 1));
        int l = t.get_min(1, 0, n, 0, x) + 1;
        if (l <= r) {
            auto it = seg.lower_bound({l, -1, -1});
            while (it != seg.end() && (*it)[1] <= r) {
                auto x = *it;
                it = seg.erase(it);
            }
        }
        t.upd(1, 0, n, x, i);
        for (int j = r; j >= l; ) {
            int m = t.get_mex(1, 0, n, j);
            int L = max(l, t.get_min(1, 0, n, 0, m) + 1);
            f[m] = 1;
            seg.insert({L, j, m});
            j = L - 1;
        }
        int m = !a[i];
        seg.insert({i, i, m});
        f[m] = 1;
    }
    int ans = 0;
    while (f[ans]) ++ans;
    cout << ans + 1 << '\n';
    return 0;
}
```

## 2.5 Pbds

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <functional>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
            tree_order_statistics_node_update>
            ordered_set;
// s.order_of_key(x) = number of elements
// strictly less than x
// *s.find_by_order(i) = ith element in set (0
// index)
```

## 2.6 Segment Tree(BSUA)

```
// CSES - 1749
const int MX = 2e5 + 10;
int n;
int arr[MX], st[MX << 2];
```

```
void assign(int i, int x, int u = 1, int s = 0,
           int e = n - 1) {
    if (s == e) {
        st[u] = x;
        return;
    }
    int v = u << 1, w = v | 1, m = (s + e) >> 1;
    if (i <= m) assign(i, x, v, s, m);
    else assign(i, x, w, m + 1, e);
    st[u] = st[v] + st[w];
}
int kth(int k, int u = 1, int s = 0, int e = n - 1) {
    if (st[u] < k) return -1;
    if (s == e) {
        return s;
    }
    int v = u << 1, w = v | 1, m = (s + e) >> 1;
    if (st[v] >= k) return kth(k, v, s, m);
    else return kth(k - st[v], w, m + 1, e);
}
void solve() {
    cin >> n;
    for (int i = 0; i < n; ++i) {
        cin >> arr[i];
    }
    for (int i = 0; i < n; ++i) {
        assign(i, 1);
    }
    for (int i = 0; i < n; ++i) {
        int x;
        cin >> x;
        int ind = kth(x);
        assign(ind, 0);
        cout << arr[ind] << " ";
    }
}
```

## 2.7 Segment Tree(LzP)

```
class stree {
    vector<ll> st, lazy;
public:
    stree(int n) {
        st.assign((n << 2) + 10, 0);
        lazy.assign((n << 2) + 10, 0);
    }
    void push(int u, int s, int e) {
        if (!lazy[u]) return;
        st[u] += (e - s + 1) * 1LL * lazy[u];
        if (s != e) {
            int v = u << 1, w = v | 1, m = (s + e) >> 1;
            lazy[v] += lazy[u];
            lazy[w] += lazy[u];
        }
        lazy[u] = 0;
    }
    void build(int u, int s, int e, int arr[]) {
        if (s == e) {
            st[u] = arr[s];
            return;
        }
        int v = u << 1, w = v | 1, m = (s + e) >> 1;
        build(v, s, m, arr);
        build(w, m + 1, e, arr);
        st[u] = st[v] + st[w];
    }
}
```

```

void update(int l, int r, int x, int u, int s,
           int e) {
    push(u, s, e);
    if (e < l or r < s) return;
    int v = u << 1, w = v | 1, m = (s + e) >> 1;
    if (l <= s and e <= r) {
        st[u] += (e - s + 1) * 1LL * x;
        if (s != e) {
            lazy[v] += x;
            lazy[w] += x;
        }
        return;
    }
    update(l, r, x, v, s, m);
    update(l, r, x, w, m + 1, e);
    st[u] = st[v] + st[w];
}

{l query(int l, int r, int u, int s, int e) {
    push(u, s, e);
    if (e < l or r < s) return 0;
    if (l <= s and e <= r) return st[u];
    int v = u << 1, w = v | 1, m = (s + e) >> 1;
    return query(l, r, v, s, m) + query(l, r, w,
                                         m + 1, e);
}
}

```

## 2.8 Segment Tree

```

class stree {
    vector<ll> st;
public:
    stree(int n) {
        st.assign((n << 2) + 10, 0);
    }
    void build(int u, int s, int e, int arr[]) {
        if (s == e) {
            st[u] = arr[s];
            return;
        }
        int v = u << 1, w = v | 1, m = (s + e) >> 1;
        build(v, s, m, arr);
        build(w, m + 1, e, arr);
        st[u] = st[v] + st[w];
    }
    l query(int l, int r, int u, int s, int e) {
        if (e < l or r < s) return 0;
        if (l <= s and e <= r) return st[u];
        int v = u << 1, w = v | 1, m = (s + e) >> 1;
        return query(l, r, v, s, m) + query(l, r, w,
                                         m + 1, e);
    }
    void update(int i, int x, int u, int s, int e)
    {
        if (s == e) {
            st[u] = x;
            return;
        }
        int v = u << 1, w = v | 1, m = (s + e) >> 1;
        if (i <= m) update(i, x, v, s, m);
        else update(i, x, w, m + 1, e);
        st[u] = st[v] + st[w];
    }
}

```

## 2.9 Sparse Table

```
const int mxN = 1e5 + 10, M = 21;
```

```

int sparse[mxN][M];
void build_sparse(int n, vector<int>& v) {
    for (int i = 0; i < n; i++) sparse[i][0] =
        v[i];
    for (int k = 1; k < M; k++) {
        for (int i = 0; i + (1 << k) <= n; i++) {
            sparse[i][k] = max(sparse[i][k - 1],
                                sparse[i + (1 << (k - 1))][k - 1]);
        }
    }
}
int query(int l, int r) { // 0 based index
    if (l > r) swap(l, r);
    int b = bit_width(r - l + 1) - 1;
    return max(sparse[l][b], sparse[r - (1 << b) +
                                1][b]);
}

```

## 3 Dynamic Programming

### 3.1 Coin Change(Number of Ways)

```

const int mod = 1e9+7;
void solve(){
    int n, k; cin>>n>>k;
    vector<int> coin(n);
    for(int i = 0; i<n; i++){ cin>>coin[i]; }
    vector<int> dp(k+1, 0); dp[0] = 1;
    for(int i = 1; i<=k; i++){
        for(int j = 0; j<n; j++){
            if(i-coin[j]>=0){
                dp[i] = (dp[i]+dp[i-coin[j]])%mod;
            }
        }
    }
    cout<<dp[k]<<endl;
}

```

### 3.2 Digit DP

```

vector<int> nmbrs;
int dp[10][10][2];
int dgt_dp(int idx, int tight, int onecnt) {
    if (idx == nmbrs.size()) {
        return onecnt;
    }
    if (dp[idx][onecnt][tight] != -1) return
        dp[idx][onecnt][tight];
    int lmt = (tight ? nmbrs[idx] : 9);
    int sum = 0;
    for (int i = 0; i <= lmt; i++) {
        bool newTight = (tight and i == nmbrs[idx]);
        sum += dgt_dp(idx + 1, newTight, onecnt + (i
                                         == 1));
    }
    return dp[idx][onecnt][tight] = sum;
}

```

### 3.3 LIS

```

vector<int> lis(int n, vector<int>& v) {
    vector<int> parent(n, -1), ind(n);
    vector<int> lis;
    for (int i = 0; i < n; i++) {
        int it = lower_bound(lis.begin(), lis.end(),
                             v[i]) - lis.begin();
        if (it == lis.size()) {
            lis.push_back(v[i]);
            ind[lis.size() - 1] = i;
        }
        else {
            parent[i] = (lis.size() == 1 ? -1 :
                         ind[it - 1]);
            lis[it] = v[i];
            ind[i] = it;
        }
    }
    return lis;
}

```

```

parent[i] = (lis.size() == 1 ? -1 : ind[it -
                                         1]);
} else {
    lis[it] = v[i];
    ind[i] = it;
    parent[i] = (it == 0 ? -1 : ind[it - 1]);
}
vector<int> LIS;
int it = ind[lis.size() - 1];
LIS.push_back(lis.back());
while (parent[it] != -1) {
    it = parent[it];
    LIS.push_back(v[it]);
}
return LIS;
}

```

## 3.4 Maximum Subarray Sum(Kadanes)

```

int max_sum_of(vector<int> &vct){
    int mx = INT_MIN, till = 0;
    for (int i = 0; i<vct.size(); i++) {
        till = till + vct[i];
        mx = max(mx, till);
        till = max(till, 1LL*0);
    }
    return mx;
}

```

## 4 Graph

### 4.1 Articulation Point

```

int n; // number of nodes
vector<vector<int>> lst; // adjacency list of
                           graph
vector<bool> vis;
vector<int> tin, low;
int timer;
void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
    int children = 0;
    for (int v : lst[u]) {
        if (v == p) continue;
        if (vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] >= tin[u] && p != -1){
                IS_CUTPOINT(u);
            }
            ++children;
        }
    }
    // if no vertex below v can reach u or higher
    // removing u disconnects that subtree
    if (p == -1 && children > 1){
        IS_CUTPOINT(u);
    }
}
void find_cutpoints() {
    timer = 0;
    vis.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
}

```

```

for (int i = 0; i < n; ++i) {
    if (!vis[i]){
        dfs(i);
    }
}

```

**4.2 BFS**

```

vector<vector<int>> adj; // adjacency list
// representation
int n; // number of nodes
int s; // source vertex
void bfs() {
    queue<int> q;
    vector<int> d(n), p(n);
    vector<bool> used(n);
    q.push(s);
    used[s] = true;
    p[s] = -1;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int u : adj[v]) {
            if (!used[u]) {
                used[u] = true;
                q.push(u);
                d[u] = d[v] + 1;
                p[u] = v;
            }
        }
    }
    // retrieving shortest path
    if (!used[u]) {
        cout << "No path!";
    } else {
        vector<int> path;
        for (int v = u; v != -1; v = p[v])
            path.push_back(v);
        reverse(path.begin(), path.end());
        cout << "Path: ";
        for (int v : path)
            cout << v << " ";
    }
}

```

**4.3 Bellman Ford**

```

#define ll long long
#define INF 1e18
void solve() {
    int n, m, v;
    cin >> n >> m >> v; // n = nodes, m = edges, v
    // = source (0-indexed)
    vector<array<ll, 3>> edges(m); // each edge:
    // {a, b, cost}
    for (int i = 0; i < m; i++) cin >> edges[i][0]
    // >> edges[i][1] >> edges[i][2];
    vector<ll> d(n, INF);
    vector<int> p(n, -1);
    d[v] = 0;
    int x = -1;
    for (int i = 0; i < n; i++) {
        x = -1;
        for (auto& e : edges) {

```

```

            int a = e[0], b = e[1];
            ll cost = e[2];
            if (d[a] < INF && d[b] > d[a] + cost) {
                d[b] = max(-INF, d[a] + cost);
                p[b] = a;
                x = b;
            }
        }
        if (x == -1) {
            cout << "No negative cycle from vertex " <<
            ~v << '\n';
            return;
        }
        int y = x;
        for (int i = 0; i < n; i++) y = p[y];
        vector<int> path;
        for (int cur = y;; cur = p[cur]) {
            path.push_back(cur);
            if (cur == y && path.size() > 1) break;
        }
        reverse(path.begin(), path.end());
        cout << "Negative cycle: ";
        for (int u : path) cout << u << ' ';
        cout << '\n';
    }
}

```

**4.4 Bridge Finding DFS**

```

const int MX = 1e5 + 10;
int n, m, timer = 0;
vector<int> adj[MX];
vector<int> tin(MX, -1), low(MX, -1);
vector<bool> vis(MX, false);
void is_bridge(int u, int v) {
    // do something with the edge
}
void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
    for (int v : adj[u]) {
        if (v == p) continue;
        if (vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u]) {
                is_bridge(u, v);
            }
        }
    }
}

```

**4.5 Cycle Detection in DAG**

```

const int MX = 1e5 + 10;
bool vis[MX], pathVis[MX];
vector<int> lst[MX];
bool dfs(int u) {
    vis[u] = true;
    pathVis[u] = true;
    for (auto v : lst[u]) {
        if (!vis[v]) {
            if (dfs(v))
                return true;
        } else if (pathVis[v]) {

```

```

            return true;
        }
        pathVis[u] = false;
    }
    void solve() {
        // take graph input
        for (int i = 0; i < n; ++i) {
            if (!vis[i])
                dfs(i);
        }
    }

```

**4.6 DSU**

```

const int MX = 1e5 + 10;
int par[MX], sz[MX];
void init() {
    for (int i = 1; i < MX; i++) {
        par[i] = i;
        sz[i] = 1;
    }
}
int findpar(int x) {
    if (par[x] == x) return x;
    return par[x] = findpar(par[x]);
}
void unite(int u, int v) {
    u = findpar(u);
    v = findpar(v);
    if (u != v) {
        if (sz[u] < sz[v]) {
            swap(u, v);
        }
        sz[u] += sz[v];
        par[v] = u;
    }
}

```

**4.7 Dijkstra**

```

const int N = 1e5 + 5, INF = 1e18 + 7;
vector<pair<int, int>> g[N];
bool visited[N];
vector<int> dist(N, INF), parent(N);
bool dijkstra(int source) {
    priority_queue<pair<int, int>,
        vector<pair<int, int>>, greater<pair<int,
        int>>> pq;
    pq.push({0, source});
    dist[source] = 0;
    parent[source] = -1;
    while (pq.size()) {
        int x = pq.top().second;
        pq.pop();
        if (visited[x]) continue;
        visited[x] = 1;
        for (auto [child_x, child_wt] : g[x]) {
            if (dist[x] + child_wt < dist[child_x]) {
                parent[child_x] = x;
                dist[child_x] = child_wt + dist[x];
                pq.push({dist[child_x], child_x});
            }
        }
    }
    return (dist[n] == INF);
}

```

## 4.8 Euler Tour

```
const int MX = 2e5 + 10;
int timer = -1;
// s = start pos, e = end pos
int val[MX], s[MX], e[MX], flat[MX];
vector<int> lst[MX];
void dfs(int u, int p) {
    s[u] = ++timer;
    flat[timer] = val[u];
    for (auto v : lst[u]) {
        if (v != p)
            dfs(v, u);
    }
    e[u] = timer;
}
```

## 4.9 Floyd Warshall

```
vector<vector<int>> d(n, vector<int>(n, INF));
// take graph input into d
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (d[i][k] < INF && d[k][j] < INF)
                d[i][j] = min(d[i][j], d[i][k] +
                                d[k][j]);
        }
    }
}
```

## 4.10 MST

```
// DSU first
void solve() {
    int n, m;
    cin >> n >> m;
    vector<tuple<int, int, int>> edges;
    for (int i = 0; i < m; ++i) {
        int u, v, wt;
        cin >> u >> v >> wt;
        edges.push_back({wt, u, v});
    }
    sort(edges.begin(), edges.end());
    init(n);
    int cost = 0;
    for (tuple& [wt, u, v] : edges) {
        if (findpar(u) == findpar(v)) continue;
        unite(u, v);
        cost += wt;
    }
    cout << cost << endl;
}
```

## 4.11 Max Bipartite Matching[Hopcroft Karp]

```
const int INF = 1e9;
void hopcroftCarp() {
    int n, m, e;
    cin >> n >> m >> e;
    vector<int> adj[n];
    for (int i = 0; i < e; ++i) {
        int u, v;
        cin >> u >> v;
        adj[u].push_back(v);
    }
    vector<int> ml(m, -1), mr(n, -1), dist(n);
    auto bfs = [&]() -> bool {
```

```
queue<int> q;
for (int u = 0; u < n; ++u) {
    if (mr[u] == -1) {
        dist[u] = 0;
        q.push(u);
    } else {
        dist[u] = INF;
    }
}
bool foundAugmenting = false;
while (!q.empty()) {
    int u = q.front();
    q.pop();
    for (int v : adj[u]) {
        int pairedLeft = ml[v];
        if (pairedLeft == -1) {
            foundAugmenting = true;
        } else if (dist[pairedLeft] == INF) {
            dist[pairedLeft] = dist[u] + 1;
            q.push(pairedLeft);
        }
    }
}
return foundAugmenting;
};
function<bool(int)> dfs = [&](int u) -> bool {
    for (int v : adj[u]) {
        int pairedLeft = ml[v];
        if (pairedLeft == -1 || (dist[pairedLeft] ==
                                dist[u] + 1 and dfs(pairedLeft))) {
            mr[u] = v;
            ml[v] = u;
            return true;
        }
    }
    dist[u] = INF;
    return false;
};
int matching = 0;
while (bfs()) {
    for (int u = 0; u < n; ++u) {
        if (mr[u] == -1) {
            if (dfs(u)) matching++;
        }
    }
}
cout << matching << el;
for (int u = 0; u < n; ++u) {
    if (mr[u] != -1) {
        cout << u << " " << mr[u] << el;
    }
}
```

## 4.12 Max Bipartite Matching[Kuhn's]

```
// left set size, right set size, edge count
int n, k, m, visToken = 1;
vector<int> lst[MX];
int mr[MX], ml[MX], vis[MX];
bool try_kuhn(int u) {
    if (vis[u] == visToken)
        return false;
    vis[u] = visToken;
    for (auto v : lst[u]) {
        if (ml[v] == -1 or try_kuhn(ml[v])) {
            ml[v] = u;
            mr[u] = v;
            return true;
        }
    }
}
```

```
return true;
}
return false;
}
void solve() {
    cin >> n >> k >> m;
    for (int i = 0; i < m; ++i) {
        int u, v;
        cin >> u >> v;
        lst[u].push_back(v);
    }
    fill(mr, mr + n, -1);
    fill(ml, ml + k, -1);
    int ans = 0;
    for (int u = 0; u < n; ++u) {
        for (auto v : lst[u]) {
            if (ml[v] == -1) {
                ml[v] = u;
                mr[u] = v;
                ans++;
                break;
            }
        }
        for (int u = 0; u < n; ++u) {
            if (mr[u] != -1) continue;
            visToken++;
            if (try_kuhn(u))
                ans++;
        }
        cout << ans << el;
        for (int v = 0; v < k; ++v) {
            if (ml[v] != -1) {
                cout << ml[v] + 1 << " " << v + 1 << el;
            }
        }
    }
}
```

## 4.13 Max Flow

```
struct Dinic {
    struct Edge {
        int to;
        ll capacity;
        int rev; // index of reverse edge
    };
    vector<vector<Edge>> adj;
    vector<int> level;
    vector<int> ptr;
    int n;
    Dinic(int nodes) : n(nodes), adj(nodes),
        level(nodes), ptr(nodes) {}
    void add_edge(int from, int to, ll cap) {
        adj[from].push_back({to, cap,
                            (int)adj[to].size()});
        adj[to].push_back({from, 0,
                            (int)adj[from].size() - 1});
    }
    bool bfs(int s, int t) {
        fill(level.begin(), level.end(), -1);
        level[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int v = q.front();
```

```

q.pop();
for (auto& edge : adj[v]) {
    if (edge.capacity > 0 && level[edge.to]
        == -1) {
        level[edge.to] = level[v] + 1;
        q.push(edge.to);
    }
}
return level[t] != -1;
}

ll_dfs(int v, int t, ll pushed) {
if (pushed == 0) return 0;
if (v == t) return pushed;
for (int& cid = ptr[v]; cid < adj[v].size(); ++cid) {
    auto& edge = adj[v][cid];
    int tr = edge.to;
    if (level[v] + 1 != level[tr] ||
        edge.capacity == 0) continue;
    ll tr_pushed = dfs(tr, t, min(pushed,
        edge.capacity));
    if (tr_pushed == 0) continue;
    edge.capacity -= tr_pushed;
    adj[tr][edge.rev].capacity += tr_pushed;
    return tr_pushed;
}
return 0;
}

ll max_flow(int s, int t) {
    ll fflow = 0;
    while (bfs(s, t)) {
        fill(ptr.begin(), ptr.end(), 0);
        while (ll pushed = dfs(s, t, 1e18)) {
            fflow += pushed;
        }
    }
    return fflow;
}

// Usage in int main():
int n, m;
cin >> n >> m;
Dinic dinic(n + 1); // Initialize with N+1 nodes
// 0 to N
for (int i = 0; i < m; i++) {
    int u, v; ll cap;
    cin >> u >> v >> cap;
    // Add directed edge u -> v with capacity
    // For bidirectional, add edge v -> u as well
    dinic.add_edge(u, v, cap);
}

// Compute flow from node 1 (Source) to node n
// (Sink)
cout << dinic.max_flow(1, n) << endl;

```

#### 4.14 Topological Sorting

```

const int N = 1e5 + 10;
vector<int> g[N], indegree(N, 0);
vector<int> topSort(int n) {
    queue<int> q;
    vector<int> order;
    for (int i = 1; i <= n; i++) {
        if (indegree[i] == 0) {
            q.push(i);
        }
    }
}
```

```

    }
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        order.push_back(u);
        for (int v : g[u]) {
            indegree[v]--;
            if (indegree[v] == 0) {
                q.push(v);
            }
        }
    }
    return order;
}

```

#### 4.15 Weighted Union Find

```

const int MX = 2e5 + 10;
int par[MX], sz[MX];
ll d[MX];
void init() {
    for (int i = 0; i < MX; ++i) {
        par[i] = i;
        sz[i] = 1;
        d[i] = 0;
    }
}
int findpar(int x) {
    if (par[x] == x) return x;
    int p = par[x];
    par[x] = findpar(p);
    d[x] += d[p];
    return par[x];
}
bool unite(int a, int b, ll w) {
    int ra = findpar(a);
    int rb = findpar(b);
    if (ra == rb) {
        return (d[rb] - d[a] == w);
    }
    if (sz[ra] < sz[rb]) {
        swap(a, b);
        swap(ra, rb);
        w = -w;
    }
    par[rb] = ra;
    d[rb] = d[a] + w - d[b];
    sz[ra] += sz[rb];
    return true;
}
ll dist(int a, int b) {
    findpar(a), findpar(b);
    return d[b] - d[a];
}

```

### 5 Math

#### 5.1 Matrix Exponentiation

```

const ll mod = 1e9;
vector<vector<ll>> matMul(vector<vector<ll>>& a,
    vector<vector<ll>>& b) {
    ll row1 = a.size(), col1 = a[0].size();
    ll row2 = b.size(), col2 = b[0].size();
    vector<vector<ll>> res(row1, vector<ll>(col2,
        0));
    for (int i = 0; i < row1; i++) {
        for (int j = 0; j < col1; j++) {
            for (int k = 0; k < row2; k++) {

```

```

                res[i][j] = (res[i][j] + (1LL * a[i][k]
                    * b[k][j]) % mod) % mod;
            }
        }
    }
    return res;
}
vector<vector<ll>> matExpo(vector<vector<ll>>&
    Mat, ll exp) {
    ll row = Mat.size(), col = Mat[0].size();
    ll p = row;
    vector<vector<ll>> res(p, vector<ll>(p, 0));
    for (ll i = 0; i < p; i++) res[i][i] = 1;
    while (exp) {
        if (exp & 1) res = matMul(res, Mat);
        Mat = matMul(Mat, Mat);
        exp >>= 1;
    }
    return res;
}
// b = (A(i), A(i-1), A(i-2), A(i-3))
// M = Magic matrix, nth = nth term, known =
// known value
ll get_nth(ll nth, ll known, vector<ll>& b,
    vector<vector<ll>>& M) {
    if (nth <= known) return b[nth - 1] % mod;
    reverse(b.begin(), b.end());
    vector<vector<ll>> me = matExpo(M, nth -
        known); // MAT^(nth-known)
    ll ans = 0;
    for (int i = 0; i < known; i++) {
        ans = (ans + (b[i] * me[i][0])) % mod;
    }
    return ans;
}

```

#### 5.2 Matrix Rotation

```

//90* clock-wise
now = {{0, 1, 0}, {-1, 0, 0}, {0, 0, 1}};
//90* anti-clock
now = {{0, -1, 0}, {1, 0, 0}, {0, 0, 1}};
//mirror with x axis at point p
now = {{-1, 0, 2 * p}, {0, 1, 0}, {0, 0, 1}};
//mirror with y axis at point p
now = {{1, 0, 0}, {0, -1, 2 * p}, {0, 0, 1}};
op[i + 1] = matMul(now, op[i]); // this
// op[i + 1] = matMul(op[i], now); //not this

```

### 6 Miscellaneous

#### 6.1 Max Subarray Size Sum equal K

```

//write gpHashTable code before this part
void solution(){
    int n, k; cin >> n >> k;
    int total_sum = 0;
    vector < int > pre(n + 7, 0);
    for (int i = 1; i <= n; i++) {
        int temp; cin >> temp;
        total_sum += temp;
        if (i == 1) pre[i] = temp;
        else pre[i] = pre[i - 1] + temp;
    }
    if (total_sum < k) {
        cout << "-1" << endl; return;
    }
    if (total_sum == k) {

```

```

cout << "0" << endl; return;
}
int maximum_subSize = 0;
gp_hash_table<int, int, customHash> table;
for (int i = 1; i <= n; i++) {
    if (pre[i] >= k) {
        int subSUM = pre[i] - k;
        if (subSUM == 0){
            maximum_subSize = max(maximum_subSize, i);
        } else if (table[subSUM]) {
            int left = table[subSUM];
            int right = i; int subSize = right - left;
            maximum_subSize = max(subSize,
                maximum_subSize);
        }
    } if (!table[pre[i]]) table[pre[i]] = i;
} cout << maximum_subSize << endl;
}

```

## 6.2 Merge Sort

```

// use array of elements, if multiple testcase
// make inv = 0 each time
// int inv = 0;
void merge(int vct[], int l, int m, int r) {
    int left = m - l + 1, right = r - m, lv[left],
    ~ rv[right];
    for (int i = 0; i < left; i++) {
        lv[i] = vct[l + i];
    }
    for (int i = 0; i < right; i++) {
        rv[i] = vct[m + 1 + i];
    }
    int i = 0, j = 0, to = l;
    while (i < left && j < right) {
        if (lv[i] <= rv[j]) {
            vct[to] = lv[i];
            i++;
        } else {
            vct[to] = rv[j];
            j++;
        }
        // inversion count
        // int pore = left-i; inv+=pore;
        to++;
    }
    while (i < left) {
        vct[to] = lv[i];
        i++;
        to++;
    }
    while (j < right) {
        vct[to] = rv[j];
        j++;
        to++;
    }
}
void merge_sort(int vct[], int l, int r) {
    if (r <= l) return;
    int m = l + ((r - l) / 2);
    merge_sort(vct, l, m);
    merge_sort(vct, m + 1, r);
    merge(vct, l, m, r);
}

```

## 6.3 Number of Subarray Sum is K

//write gpHashTable code before this part

```

void solution(){
    int n, k; cin >> n >> k;
    int total_sum = 0;
    vector<int> pre(n + 7, 0);
    for (int i = 1; i <= n; i++) {
        int temp; cin >> temp;
        total_sum += temp;
        if (i == 1) pre[i] = temp;
        else pre[i] = pre[i - 1] + temp;
    }
    int cnt_subarry = 0;
    gp_hash_table<int, int, customHash> table;
    table[0] = 1;
    for (int i = 1; i <= n; i++) {
        cnt_subarry += table[pre[i] - k];
        table[pre[i]]++;
    }
    cout << cnt_subarry << endl;
}

```

## 7 Number Theory

### 7.1 All In One NT

```

const int MAXN = 1e6 + 9;
typedef struct info {
    int lowest_prime = 0, greatest_prime = 0,
    ~ distinct_prime = 0;
    int total_prime = 0, NOD = 0, SOD = 0;
} info;
info num[MAXN];
void preStore() {
    for (int i = 2; i < MAXN; i++) {
        int n = i;
        map<int, int> factors; // Key->Factor,
        ~ Val->count
        int SOD = 1, NOD = 1, total_p_factor = 0;
        if (n % 2 == 0) {
            while (n % 2 == 0) {
                n /= 2;
                factors[2]++;
                total_p_factor++;
            }
            SOD *= (1 << (factors[2] + 1)) - 1;
            NOD *= (factors[2] + 1);
        }
        for (int i = 3; i * i <= n; i += 2) {
            if (n % i == 0) {
                while (n % i == 0) {
                    n /= i;
                    factors[i]++;
                    total_p_factor++;
                }
                SOD *= (pow(i, factors[i] + 1) - 1) / (i
                    ~ 1);
                NOD *= (factors[i] + 1);
            }
        }
        if (n > 1) {
            factors[n]++;
            SOD *= (pow(n, 2) - 1) / (n - 1);
            NOD *= 2;
            total_p_factor++;
        }
        num[i].distinct_prime = factors.size();
        num[i].total_prime = total_p_factor;
        num[i].NOD = NOD;
        num[i].SOD = SOD;
        auto lowest_prime = factors.begin();
        auto greatest_prime = factors.rbegin();
        num[i].lowest_prime = lowest_prime->first;
        num[i].greatest_prime =
            ~ greatest_prime->first;
    }
}

```

```

    num[i].greatest_prime =
        ~ greatest_prime->first;
}

```

### 7.2 Divisor Sieve

```

const int mxN = 1e5 + 10;
vector<int> divisors[mxN];
void divisorSeive() {
    for (int i = 1; i < mxN; i++) {
        for (int j = i; j < mxN; j += i) {
            divisors[j].push_back(i);
        }
    }
}

```

### 7.3 Number of Pairs with GCD equal g

```

/*a[i] <= 1e6
for all 1<=g<=n, how many pairs exist such that g
    ~ = gcd(a[i], a[j]);
complexity : nlogn
*/
ll n; cin >> n;
ll a[n + 1];
ll cnt[n + 1]; memset(cnt, 0, sizeof cnt);
for (ll i = 1; i <= n; i++) {cin >> a[i];
    ~ cnt[a[i]]++;}
ll gcd[n + 1]; memset(gcd, 0, sizeof gcd);
for (ll i = n; i >= 1; i--) {
    ll pair = 0, invalid_pair = 0;
    for (ll j = i; j <= n; j += i) {
        pair += cnt[j];
        invalid_pair += gcd[j];
    }
    pair = (pair * (pair - 1)) / 2;
    gcd[i] = pair - invalid_pair;
    // how many pairs exist whose gcd is i
}

```

### 7.4 Phi(1toN)

```

const int mxN = 1e7+10;
vector<int> phi(mxN);
void phi_till() { //O(n.log.log(n))
    for (int i = 0; i < mxN; i++) phi[i] = i;
    for (int i = 2; i < mxN; i++) {
        if (phi[i] == i) {
            for (int j = i; j < mxN; j += i) {
                phi[j] -= phi[j] / i;
            }
        }
    }
}

```

### 7.5 Phi

```

int phi(int n) { // sqrt(n)
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}

```

## 7.6 SOD NOD

```
// SOD = ((P^(x+1)-1)/(P-1)) *
// ((Q^(y+1)-1)/(Q-1)) * ((R^(z+1)-1)/(R-1))
// NOD = P^x * Q^y * R^z => here, P, Q, R are
// prime factors & x, y, z are
// powers NOD = (x + 1)(y + 1)(z + 1)
pair<int, int> SOD_NOD(int n) {
    int sod = 1, nod = 1;
    for (int i = 2; i * i <= n; ++i) {
        if (n % i == 0) {
            int pown = 1, pows = 0;
            while (n % i == 0) {
                pown *= i; // p^e
                pows++;
                n /= i;
            }
            pown *= i;
            sod *= (pown - 1) / (i - 1); // (p^e+1)-1 /
            nod *= (pows + 1);
        }
    }
    if (n > 1) {
        sod *= (n + 1);
        nod *= 2;
    }
    return {sod, nod};
}
```

## 7.7 Segmented Sieve

```
void segSeive(ll low, ll high) {
    vector<bool> area((high - low) + 1, true);
    for (ll i = 0; primes[i] * primes[i] <= high;
         i++) {
        ll start = ((low / primes[i]) * primes[i]);
        if (start < low) start += primes[i];
        for (ll j = start; j <= high; j +=
             primes[i]) {
            if (j == primes[i]) continue;
            area[j - low] = false;
        }
    }
    for (ll i = 0; i < (high - low) + 1; i++) {
        if (area[i]) {
            if (i + low != 1 and i + low != 0) {
                cout << i + low << endl;
            }
        }
    }
}
```

## 7.8 Sieve

```
const ll MAXN = 1e7 + 10;
bool prime[MAXN];
vector<ll> prm;
void sieve() {
    prime[0] = prime[1] = true;
    for (ll i = 2; i < MAXN; i++) {
        if (!prime[i]) {
            prm.push_back(i);
            for (ll j = i + i; j < MAXN; j += i) {
                prime[j] = true;
            }
        }
    }
}
```

## 7.9 Spf

```
const int MAXN = 1e6 + 2;
int spf[MAXN];
vector<int> prms;
void preStore() {
    for (int i = 1; i < MAXN; i++) spf[i] = i;
    for (int i = 2; i < MAXN; i++) {
        if (spf[i] == i) {
            prms.push_back(i);
            for (int j = i + i; j < MAXN; j += i) {
                spf[j] = min(spf[j], i);
            }
        }
    }
}
```

## 7.10 UniquePF of all elements till MX

```
const int MX = 2e5 + 10;
vector<int> pfac[MX];
void factorize() {
    for (int i = 2; i < MX; i++) {
        if (!pfac[i].empty()) continue;
        for (int j = i; j < MX; j += i)
            pfac[j].push_back(i);
    }
}
```

## 7.11 int128

```
int128 read() {
    int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}
void print(int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}
```

## 7.12 nCr and nPr

```
int fact[N], ifact[N];
void prec() {
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = 1LL * fact[i - 1] * i % mod;
    }
    ifact[N - 1] = power(fact[N - 1], -1);
    for (int i = N - 2; i >= 0; i--) {
        ifact[i] = 1LL * ifact[i + 1] * (i + 1) %
                    mod;
    }
}
int nPr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[n - r] % mod;
}
```

```
int nCr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[r] % mod *
           ifact[n - r] % mod;
}
```

## 7.13 nCr v2

```
const int MX = 1e6 + 10;
const int M = 1e9 + 7;
int fact[MX], inv_fact[MX];
int modPow(int a, int b) {
    int ans = 1;
    while (b) {
        if (b & 1) ans = (1LL * ans * a) % M;
        a = (1LL * a * a) % M;
        b >>= 1;
    }
    return ans;
}
void precalFact() {
    fact[0] = inv_fact[0] = 1;
    for (int i = 1; i < MX; i++) {
        fact[i] = (1LL * fact[i - 1] * i) % M;
    }
    inv_fact[MX - 1] = modPow(fact[MX - 1], M - 2);
    for (int i = MX - 2; i >= 1; i--) {
        inv_fact[i] = (1LL * inv_fact[i + 1] * (i +
                                                1)) % M;
    }
}
int nCr(int n, int r) {
    if (r < 0 or r > n) return 0;
    return 1LL * fact[n] * inv_fact[r] % M *
           inv_fact[n - r] % M;
}
```

## 8 String

## 8.1 Aho Corasic

```
//number of occurence of word in a text
const ll N = 1e6+10, A = 26;
ll trie[N][A], pos[N], slink[N], dp[N], tot = 1;
vector<int> order;
void initTrie(){
    order.clear();
    while(tot--){
        memset(trie[tot], 0, sizeof(trie[tot]));
    }
    memset(pos, 0, sizeof(pos));
    memset(slink, 0, sizeof(slink));
    memset(dp, 0, sizeof(dp)); tot = 1;
}
void addStr(string &s, int ind){
    ll u = 0;
    for(auto it: s){
        ll n = it - 'a';
        if(trie[u][n]==0) trie[u][n] = tot++;
        u = trie[u][n];
    }
    pos[ind] = u;
}
void build(){
    queue<ll> q; q.push(0);
    while(!q.empty()){
        ll p = q.front(); q.pop();
        order.push_back(p);
    }
}
```

```

for(ll c = 0; c<A; c++){
    ll u = trie[p][c];
    if(!u) continue;
    q.push(u);
    if(!p) continue;
    ll v = slink[p];
    while(v && !trie[v][c]) v = slink[v];
    slink[u] = trie[v][c];
}
void trav(string &s){
    ll u = 0;
    for(char c: s){
        c-='a';
        while(u && !trie[u][c]) u = slink[u];
        u = trie[u][c]; dp[u]++;
    }
    reverse(order.begin(), order.end());
    for(auto u: order){
        dp[slink[u]]+=dp[u];
    }
}
void solve(){
    ll n; cin>>n;
    string text; cin>>text;
    string s;
    for(ll i = 0; i<n; i++){
        cin>>s; addStr(s, i);
    }
    build(); trav(text);
    for(ll i = 0; i<n; i++){
        cout<<dp[pos[i]]<<endl;
    }
}
int32_t main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    ll tc = 1;
    cin>>tc;
    for(ll i = 1; i<=tc; i++){
        cout<<"Case "<<i<<": \n";
        initTrie();
        solve();
    }
}

```

## 8.2 LCS for 3 Strings

```

string a, b, c;
ll dp[55][55][55];
ll lcs(ll i, ll j, ll k) {
    if (i == a.size() or j == b.size() or k ==
        c.size()) return 0;
    if (dp[i][j][k] != -1) return dp[i][j][k];
    if (a[i] == b[j] and a[i] == c[k]) return 1 +
        lcs(i + 1, j + 1, k + 1);
    ll ans = 0;
    ans = max(ans, lcs(i, j, k + 1));
    ans = max(ans, lcs(i, j + 1, k));
    ans = max(ans, lcs(i + 1, j, k));
    return dp[i][j][k] = ans;
}

```

## 8.3 Manacher Palindrome

```

// pal[1][i] = longest odd (half rounded down)
// palindrome around pos i and
// starts at i - pal[1][i] and ends at i +
// pal[1][i] pal[0][i] = half length of

```

```

// longest even palindrome around pos i, i + 1
// and starts at i - par[0][i] + 1
// and ends at i + pal[0][i]
const int N = 5e5 + 10;
int pal[2][N];
void manacher(string& s) {
    int n = s.size(), idx = 2;
    while (idx--) {
        for (int l = -1, r = -1, i = 0; i < n - 1;
             ++i) {
            if (i > r)
                l = r = i;
            else {
                int k = min(r - i, pal[idx][l + r - i]);
                l = i - k, r = i + k;
            }
            while (l - idx >= 0 and r + 1 < n and s[l -
                idx] == s[r + 1]) l--, r++;
            pal[idx][i] = r - i;
            // [l - 1 + idx : r] palindrome
        }
    }
}

```

## 8.4 String Hashing 2

```

const int N = 10000010, MOD = 1e9 + 7;
const ll P[] = {97, 1000003};
ll bigMod(ll a, ll e) {
    if (e == -1) e = MOD - 2;
    ll ret = 1;
    while (e) {
        if (e & 1) ret = ret * a % MOD;
        a = a * a % MOD, e >>= 1;
    }
    return ret;
}
ll pwr[2][N], inv[2][N];
void initHash() {
    for (int it = 0; it < 2; ++it) {
        pwr[it][0] = inv[it][0] = 1;
        ll INV_P = bigMod(P[it], -1);
        for (int i = 1; i < N; ++i) {
            pwr[it][i] = pwr[it][i - 1] * P[it] % MOD;
            inv[it][i] = inv[it][i - 1] * INV_P % MOD;
        }
    }
}

```

```

struct RangeHash {
    vector<ll> h[2], rev[2];
    RangeHash(const string S, bool revFlag = 0) {
        for (int it = 0; it < 2; ++it) {
            h[it].resize(S.size() + 1, 0);
            for (int i = 0; i < S.size(); ++i) {
                h[it][i + 1] = (h[it][i] + pwr[it][i + 1] *
                    (S[i] - 'a' + 1)) % MOD;
            }
            if (revFlag) {
                rev[it].resize(S.size() + 1, 0);
                for (int i = 0; i < S.size(); ++i) {
                    rev[it][i + 1] =
                        (rev[it][i] + inv[it][i + 1] *
                            (S[i] - 'a' + 1)) % MOD;
                }
            }
        }
    }
    inline ll get(int l, int r) {

```

```

        ll one = (h[0][r + 1] - h[0][l]) * inv[0][l +
            1] % MOD;
        ll two = (h[1][r + 1] - h[1][l]) * inv[1][l +
            1] % MOD;
        if (one < 0) one += MOD;
        if (two < 0) two += MOD;
        return one << 31 | two;
    }
    inline ll getReverse(int l, int r) {
        ll one = (rev[0][r + 1] - rev[0][l]) * pwr[0][r + 1] % MOD;
        ll two = (rev[1][r + 1] - rev[1][l]) * pwr[1][r + 1] % MOD;
        if (one < 0) one += MOD;
        if (two < 0) two += MOD;
        return one << 31 | two;
    }
}

```

## 8.5 String Hashing

```

const int mod1 = 911382323, mod2 = 972663749, b1
    = 137, b2 = 139;
const int mxN = 5000010;
int pow_b1[mxN], pow_b2[mxN], inv_b1[mxN],
inv_b2[mxN];
int binExp(int base, int power, int mod) {
    int res = 1;
    while (power) {
        if (power & 1) res = (1LL * res * base) %
            mod;
        base = (1LL * base * base) % mod;
        power >>= 1;
    }
    return res;
}
void pre() {
    pow_b1[0] = pow_b2[0] = 1;
    for (int i = 1; i < mxN; i++) {
        pow_b1[i] = (1LL * pow_b1[i - 1] * b1) %
            mod1;
        pow_b2[i] = (1LL * pow_b2[i - 1] * b2) %
            mod2;
    }
    inv_b1[mxN - 1] = binExp(pow_b1[mxN - 1], mod1
        - 2, mod1);
    inv_b2[mxN - 1] = binExp(pow_b2[mxN - 1], mod2
        - 2, mod2);
    for (int i = mxN - 2; i >= 0; i--) {
        inv_b1[i] = (1LL * inv_b1[i + 1] * b1) %
            mod1;
        inv_b2[i] = (1LL * inv_b2[i + 1] * b2) %
            mod2;
    }
}
vector<pair<int, int>> getPref(string& s) {
    int qq = s.size();
    vector<pair<int, int>> hsh(qq);
    for (int i = 0; i < qq; i++) {
        if (i == 0) {
            hsh[i].first = (1LL * s[i] * pow_b1[i]) %
            mod1;
            hsh[i].second = (1LL * s[i] * pow_b2[i]) %
            mod2;
        } else {
            hsh[i].first = (hsh[i - 1].first * b1 + s[i]) %
            mod1;
            hsh[i].second = (hsh[i - 1].second * b2 + s[i]) %
            mod2;
        }
    }
}

```

```

} else {
    hsh[i].first =
        (hsh[i - 1].first + (1LL * s[i] *
        ~ pow_b1[i]) % mod1) % mod1;
    hsh[i].second =
        (hsh[i - 1].second + (1LL * s[i] *
        ~ pow_b2[i]) % mod2) % mod2;
}
return hsh;
}

pair<int, int> getHash(string& str) {
    int hsh1 = 0, hsh2 = 0, sz = str.size();
    for (int i = 0; i < sz; ++i) {
        hsh1 = (hsh1 + 1LL * str[i] * pow_b1[i] %
        ~ mod1) % mod1;
    }
    for (int i = 0; i < sz; ++i) {
        hsh2 = (hsh2 + 1LL * str[i] * pow_b2[i] %
        ~ mod2) % mod2;
    }
    return {hsh1, hsh2};
}

pair<int, int> getSub(int l, int r,
                     vector<pair<int, int>>& v) {
    pair<int, int> q;
    if (l == 0) {
        q = {v[r].first, v[r].second};
    } else {
        int x = (1LL * ((v[r].first - v[l - 1].first +
        ~ mod1) % mod1) * inv_b1[l]) %
        mod1;
        int y =
            (1LL * ((v[r].second - v[l - 1].second +
            ~ mod2) % mod2) * inv_b2[l]) %
            mod2;
        q = {x, y};
    }
    return q;
}

```

## 8.6 Suffix Array

```

// fahimcp495
array<vector<int>, 2> get_sa(string& s, int lim
    = 128) { // for integer, just change string
    to vector<int> and minimum value of vector
    must be >= 1
    int n = s.size() + 1, k = 0, a, b;
    vector<int> x(begin(s), end(s) + 1), y(n),
    ~ sa(n), lcp(n), ws(max(n, lim)), rank(n);
    x.back() = 0;
    iota(begin(sa), end(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j *
    ~ 2), lim = p) {
        p = j, iota(begin(y), end(y), n - j);
        for (int i = 0; i < n; ++i)
            if (sa[i] >= j) y[p++] = sa[i] - j;
        fill(begin(ws), end(ws), 0);
        for (int i = 0; i < n; ++i) ws[x[i]]++;
        for (int i = 1; i < lim; ++i) ws[i] += ws[i -
        ~ 1];
        for (int i = n; i--;) sa[--ws[x[y[i]]]] =
        ~ y[i];
        swap(x, y), p = 1, x[sa[0]] = 0;
    }
}

```

```

for (int i = 1; i < n; ++i) a = sa[i - 1], b
= sa[i], x[b] = (y[a] == y[b] && y[a +
~ j] == y[b + j]) ? p - 1 : p++;
}
for (int i = 1; i < n; ++i) rank[sa[i]] = i;
for (int i = 0, j; i < n - 1; lcp[rank[i++]] =
~ k)
    for (k &&k--, j = sa[rank[i] - 1]; s[i + k]
    ~ == s[j + k]; k++);
sa.erase(sa.begin()), lcp.erase(lcp.begin());
return {sa, lcp};
}

```

## 8.7 Suffix Automata

```

const int N = 2e5 + 10; // max string size
int len[N], lnk[N]{-1}, last, sz = 1;
unordered_map<char, int> to[N];
void add(char c) {
    int cur = sz++;
    len[cur] = len[last] + 1;
    int u = last;
    while (u != -1 and !to[u].count(c)) {
        to[u][c] = cur;
        u = lnk[u];
    }
    if (u == -1) {
        lnk[cur] = 0;
    } else {
        int v = to[u][c];
        if (len[v] == len[u] + 1) {
            lnk[cur] = v;
        } else {
            int w = sz++;
            len[w] = len[u] + 1, lnk[w] = lnk[v],
            ~ to[w] = to[v];
            while (u != -1 and to[u][c] == v) {
                to[u][c] = w;
                u = lnk[u];
            }
            lnk[cur] = lnk[v] = w;
        }
    }
    last = cur;
}

```

## 8.8 Suffix Automation

```

int len[N], lnk[N]{-1}, last, sz = 1;
unordered_map<char, int> to[N];
void init() {
    while (sz) {
        sz--;
        to[sz].clear();
    }
    last = 0, sz = 1;
}
void add(char c) {
    int cur = sz++;
    int u = last;
    len[cur] = len[last] + 1;
    while (u != -1 and !to[u].count(c)) {
        to[u][c] = cur;
        u = lnk[u];
    }
    if (u == -1) {
        lnk[cur] = 0;
    } else {
        int v = to[u][c];

```

```

if (len[v] == len[u] + 1) {
    lnk[cur] = v;
} else {
    int w = sz++;
    len[w] = len[u] + 1, lnk[w] = lnk[v],
    ~ to[w] = to[v];
    while (u != -1 and to[u][c] == v) {
        to[u][c] = w;
        u = lnk[u];
    }
    lnk[cur] = lnk[v] = w;
}
last = cur;
}

```

## 8.9 Trie

```

const ll N = 1e6 + 5, A = 26;
ll trie[N][A], cnt[N], tot = 1, root = 1;
void initTrie() {
    cnt[tot] = 0;
    root = 1;
}
void addStr(string& s) {
    ll u = 1;
    for (auto it : s) {
        ll n = it - 'a';
        if (trie[u][n] == 0) {
            trie[u][n] = ++tot;
        }
        u = trie[u][n];
        cnt[u]++;
    }
}
ll wordCount(string& s) {
    ll u = 1;
    for (auto it : s) {
        int n = it - 'a';
        if (trie[u][n] == 0) return 0;
        u = trie[u][n];
    }
    return cnt[u];
}

```

## 9 Tree

### 9.1 Centroid Decomposition

```

const int N = 2e5+5;
int n, k, sz[N], centered[N], ans = 0;
vector<int> adj[N];
void dfs_sz(int u, int p) {
    sz[u] = 1;
    for (auto v: adj[u]) {
        if (v != p && !centered[v]) {
            dfs_sz(v, u); sz[u] += sz[v];
        }
    }
}
int get_cen(int u, int p, int n) {
    for (auto v: adj[u]) {
        if (v != p && !centered[v] && sz[v] > n/2) {
            return get_cen(v, u, n);
        }
    }
    return u;
}
int t, tin[N], tout[N], nodes[N], dis[N];
void dfs(int u, int p) {
    nodes[t] = u;
    tin[u] = tout[u] = t++;
}

```

```

for(auto v: adj[u]){
    if(v!=p && !centered[v]){
        dis[v] = dis[u]+1; dfs(v, u);
    }
} tout[u] = t-1;
}

void go(int u){
    dfs_sz(u, u);
    int c = get_cen(u, u, sz[u]);
    centered[c] = 1; sz[c] = sz[u];
    t = 0; dis[c] = 0; dfs(c, c);
    int cnt[t+1];
    for(auto v: adj[c]){
        if(centered[v]) continue;
        for(int i = tin[v]; i<=tout[v]; ++i){
            int w = nodes[i];
            if(k-dis[w]>=0 && k-dis[w]<t){
                ans+=cnt[k-dis[w]];
            }
        }
        for(int i = tin[v]; i<=tout[v]; ++i){
            int w = nodes[i]; cnt[dis[w]]++;
        }
    }
    for(auto v: adj[c]){
        if(!centered[v]) go(v);
    }
}
void solve(){
    cin>>n>>k;
    for(ll i = 1; i<n; i++){
        ll u, v; cin>>u>>v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    go(1);
    cout<<ans<<endl;
}

```

## 9.2 DSUOnTrees

```

int n, color[MX], ans[MX];
vector<int> g[MX];
set<int> bucket[MX];
int merge(int a, int b) {
    if (bucket[a].size() < bucket[b].size())
        swap(a, b);
    bucket[a].insert(bucket[b].begin(),
                     bucket[b].end());
    bucket[b].clear();
    return a;
}
int dfs(int u, int p = -1) {
    int cur = u;
    for (int v : g[u])
        if (v != p)
            cur = merge(cur, dfs(v, u));
    ans[u] = (int)bucket[cur].size();
    return cur;
}
void solve() {
    cin >> n;
    for (int i = 0; i < n; ++i) {
        cin >> color[i];
        bucket[i].insert(color[i]);
    }
    // graph input
    dfs(0);
    // print output
}

```

---

```

9.3 LCA using binary Lifting
int n, l;
vector<vector<int>> adj;
int timer;
vector<int> tin, tout;
vector<vector<int>> up;
void dfs(int v, int p) {
    tin[v] = ++timer;
    up[v][0] = p;
    for (int i = 1; i <= l; i++)
        up[v][i] = up[up[v][i - 1]][i - 1];
    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }
    tout[v] = ++timer;
}
bool is_ancestor(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}
int lca(int u, int v) {
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}
void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
    dfs(root, root);
}

```

---

```

9.4 LCA
const int N = 1e5 + 5;
vector<int> g[N], parent(N), depth(N, 0);
void dfs(int vertex, int par = -1) {
    parent[vertex] = par;
    for (auto child : g[vertex]) {
        if (child != par) {
            depth[child] = depth[vertex] + 1;
            dfs(child, vertex);
        }
    }
}
int lca(int x, int y) {
    int diff = min(depth[x], depth[y]);
    while (depth[x] > diff) x = parent[x];
    while (depth[y] > diff) y = parent[y];
    while (x != y) { x = parent[x]; y = parent[y];
    }
    return x;
}

```

---

## 10 Notes

### 10.1 Geometry

#### 10.1.1 Triangles

$$\text{Circumradius: } R = \frac{abc}{4A}, \text{ Inradius: } r = \frac{A}{s}$$

The area of a triangle using two sides and the included angle can be given as:

$$A = \frac{1}{2}ab \sin C$$

Length of median (divides triangle into two equal-area triangles):  $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):  $s_a = \sqrt{bc \left[ 1 - \left( \frac{a}{b+c} \right)^2 \right]}$

$$\text{Law of tangents: } \frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$$

#### 10.1.2 Quadrilaterals

With side lengths  $a, b, c, d$ , diagonals  $e, f$ , diagonals angle  $\theta$ , area  $A$  and magic flux  $F = b^2 + d^2 - a^2 - c^2$ :

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is  $180^\circ$ ,  $ef = ac + bd$ , and  $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ .

#### 10.1.3 Spherical coordinates

$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \arctan(y/x) \end{aligned}$$

#### 10.1.4 Pick's Theorem:

Given a lattice polygon with non-zero area, we define:  $S$  as the area of the polygon,  $I$  as the number of integer-coordinate points strictly inside the polygon,  $B$  as the number of integer-coordinate points on the boundary of the polygon. Then, Pick's Theorem states:

$$S = I + \frac{B}{2} - 1$$

The number of lattice points on segments  $(x_1, y_1)$  to  $(x_2, y_2)$  is:  $\gcd(\text{abs}(x_2 - x_1), \text{abs}(y_2 - y_1)) + 1$

#### 10.1.5 Polygon

For a regular polygon with  $n$  sides and side length  $a$ , the circumradius  $R$  is given by:

$$R = \frac{a}{2 \sin(\frac{\pi}{n})}$$

#### 10.1.6 Area of a Circular Segment

The area of a circular segment, which is the region enclosed by a chord and the corresponding arc, can be calculated using the formula:

$$A = \frac{R^2}{2} (\theta - \sin \theta)$$

where:  $R$  is the radius of the circle,  $\theta$  is the central angle subtended by the chord, in radians.

**10.2 Binomial Coefficient**

- Factoring in:  $\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}$
- Sum over  $k$ :  $\sum_{k=0}^n \binom{n}{k} = 2^n$
- Alternating sum:  $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$
- Even and odd sum:  $\sum_{k=0}^n \binom{n}{2k} = \sum_{k=0}^n \binom{n}{2k+1} 2^{n-1}$
- The Hockey Stick Identity
  - (Left to right) Sum over  $n$  and  $k$ :  $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m-1}{m}$
  - (Right to left) Sum over  $n$ :  $\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$
- Sum of the squares:  $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$
- Weighted sum:  $\sum_{k=1}^n k \binom{n}{k} = n 2^{n-1}$
- Connection with the fibonacci numbers:  $\sum_{k=0}^n \binom{n-k}{k} = F_{n+1}$
- Vandermonde's Identity:  $\sum_{i=0}^k \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}$
- If  $f(n, k) = C(n, 0) + C(n, 1) + \dots + C(n, k)$ , Then  $f(n+1, k) = 2 * f(n, k) - C(n, k)$  [For multiple  $f(n, k)$  queries, use Mo's algo]

**Lucas Theorem**

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$$

- $\binom{m}{n}$  is divisible by  $p$  if and only if at least one of the base- $p$  digits of  $n$  is greater than the corresponding base- $p$  digit of  $m$ .
- The number of entries in the  $n$ th row of Pascal's triangle that are not divisible by  $p = \prod_{i=0}^k (n_i + 1)$
- All entries in the  $(p^k - 1)$ th row are not divisible by  $p$ .
- $\binom{n}{m} \equiv \lfloor \frac{n}{p} \rfloor (\pmod{p})$

**10.3 Fibonacci Number**

- $k = A - B, F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1}$
- $\sum_{i=0}^n F_i^2 = F_{n+1} F_n$
- $\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^n F_i F_{i+1} = 5 \sum_{i=0}^n F_i F_{i-1}$
- $\gcd(F_m, F_n) = F_{\gcd(m, n)}$
- $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = \gcd(F_{n+1}, F_{n+2}) = 1$

**10.4 Sums**

$$\begin{aligned} 1^2 + 2^2 + 3^2 + \dots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \dots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \dots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \\ \sum_{i=1}^n i^m &= \frac{1}{m+1} \left[ (n+1)^{m+1} - 1 - \sum_{i=1}^n ((i+1)^{m+1} - i^{m+1} - (m+1)i^m) \right] \\ \sum_{i=1}^{n-1} i^m &= \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k} \\ \sum_{k=0}^n kx^k &= (x - (n+1)x^{n+1} + nx^{n+2})/(x-1)^2 \end{aligned}$$

**10.5 Series**

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty) \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1) \\ \sqrt{1+x} &= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty) \\ (x+a)^{-n} &= \sum_{k=0}^{\infty} (-1)^k \binom{n+k-1}{k} x^k a^{-n-k} \end{aligned}$$

**Generating Function**

$$\begin{aligned} 1/(1-x) &= 1 + x + x^2 + x^3 + \dots \\ 1/(1-ax) &= 1 + ax + (ax)^2 + (ax)^3 + \dots \\ 1/(1-x)^2 &= 1 + 2x + 3x^2 + 4x^3 + \dots \\ 1/(1-x)^3 &= C(2, 2) + C(3, 2)x + C(4, 2)x^2 + C(5, 2)x^3 + \dots \\ 1/(1-ax)^{k+1} &= 1 + C(1+k, k)(ax) + C(2+k, k)(ax)^2 + C(3+k, k)(ax)^3 + \dots \\ x(x+1)(1-x)^{-3} &= 1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + \dots \\ e^x &= 1 + x + (x^2)/2! + (x^3)/3! + (x^4)/4! + \dots \end{aligned}$$

**10.6 Pythagorean Triples**

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with  $m > n > 0$ ,  $k > 0$ ,  $m \perp n$ , and either  $m$  or  $n$  even.

**10.7 Number Theory**

- HCN: 1e6(240), 1e9(1344), 1e12(6720), 1e14(17280), 1e15(26880), 1e16(41472)
- $\gcd(a, b, c, d, \dots) = \gcd(a, b-a, c-b, d-c, \dots)$
- $\gcd(a+k, b+k, c+k, d+k, \dots) = \gcd(a+k, b-a, c-b, d-c, \dots)$
- Primitive root exists iff  $n = 1, 2, 4, p^k, 2 \times p^k$ , where  $p$  is an odd prime.
- If primitive root exists, there are  $\phi(\phi(n))$  primitive roots of  $n$ .
- The numbers from 1 to  $n$  have in total  $O(n \log \log n)$  unique prime factors.
- $x \equiv r_1 \pmod{m_1}$  and  $x \equiv r_2 \pmod{m_2}$  has a solution iff  $\gcd(m_1, m_2) | (r_1 - r_2)$  Solution of  $x^2 \equiv a \pmod{p}$
- $ca \equiv cb \pmod{m} \iff a \equiv b \pmod{\frac{n}{\gcd(n, c)}}$
- $ax \equiv b \pmod{m}$  has a solution  $\iff \gcd(a, m) | b$
- If  $ax \equiv b \pmod{m}$  has a solution, then it has  $\gcd(a, m)$  solutions and they are separated by  $\frac{m}{\gcd(a, m)}$
- $ax \equiv 1 \pmod{m}$  has a solution or  $a$  is invertible  $\pmod{m} \iff \gcd(a, m) = 1$
- $x^2 \equiv 1 \pmod{p}$  then  $x \equiv \pm 1 \pmod{p}$
- There are  $\frac{p-1}{2}$  has no solution.
- There are  $\frac{p-1}{2}$  has exactly two solutions.
- When  $p \% 4 = 3$ ,  $x \equiv \pm a^{\frac{p+1}{4}}$
- When  $p \% 8 = 5$ ,  $x \equiv a^{\frac{p+3}{8}}$  or  $x \equiv 2^{\frac{p-1}{4}} a^{\frac{p+3}{8}}$

**10.7.1 Primes**

$p = 962592769$  is such that  $2^{21} | p-1$ , which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power  $p^a$ , except for  $p = 2, a > 2$ , and there are  $\phi(\phi(p^a))$  many. For  $p = 2, a > 2$ , the group  $\mathbb{Z}_{2^a}^\times$  is instead isomorphic to  $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$ .

**10.7.2 Estimates**

$$\sum_{d|n} d = O(n \log \log n).$$

The number of divisors of  $n$  is at most around 100 for  $n < 5e4$ , 500 for  $n < 1e7$ , 2000 for  $n < 1e10$ , 200 000 for  $n < 1e19$ .

**10.7.3 Perfect numbers**

$n > 1$  is called perfect if it equals sum of its proper divisors and 1. Even  $n$  is perfect iff  $n = 2^{p-1}(2^p - 1)$  and  $2^p - 1$  is prime (Mersenne's). No odd perfect numbers are yet found.

**10.7.4 Carmichael numbers**

A positive composite  $n$  is a Carmichael number ( $a^{n-1} \equiv 1 \pmod{n}$  for all  $a: \gcd(a, n) = 1$ ), iff  $n$  is square-free, and for all prime divisors  $p$  of  $n$ ,  $p-1$  divides  $n-1$ .

**10.7.5 Totient**

- If  $p$  is a prime ( $p^k = p^k - p^{k-1}$ )
- If  $a, b$  are relatively prime,  $\phi(ab) = \phi(a)\phi(b)$
- $\phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})(1 - \frac{1}{p_3})...(1 - \frac{1}{p_k})$
- Sum of coprime to  $n = n * \frac{\phi(n)}{2}$
- If  $n = 2^k, \phi(n) = 2^{k-1} = \frac{n}{2}$
- For  $a, b, \phi(ab) = \phi(a)\phi(b) \frac{d}{\phi(d)}$
- $\phi(ip) = p\phi(i)$  whenever  $p$  is a prime and it divides  $i$
- The number of  $a (1 \leq a \leq N)$  such that  $\gcd(a, N) = d$  is  $\phi(\frac{n}{d})$
- If  $n > 2, \phi(n)$  is always even
- Sum of gcd,  $\sum_{i=1}^n \gcd(i, n) = \sum_{d|n} d\phi(\frac{n}{d})$
- Sum of lcm,  $\sum_{i=1}^n \text{lcm}(i, n) = \frac{n}{2}(\sum_{d|n} d\phi(d)) + 1$
- $\phi(1) = 1$  and  $\phi(2) = 1$  which two are only odd  $\phi$
- $\phi(3) = 2$  and  $\phi(4) = 2$  and  $\phi(6) = 2$  which three are only prime  $\phi$
- Find minimum  $n$  such that  $\frac{\phi(n)}{n}$  is maximum- Multiple of small primes-  $2 * 3 * 5 * 7 * 11 * 13 * ...$

**10.7.6 Möbius function**

$\mu(1) = 1$ .  $\mu(n) = 0$ , if  $n$  is not squarefree.  $\mu(n) = (-1)^s$ , if  $n$  is the product of  $s$  distinct primes. Let  $f, F$  be functions on positive integers. If for all  $n \in N$ ,  $F(n) = \sum_{d|n} f(d)$ , then  $f(n) = \sum_{d|n} \mu(d)F(\frac{n}{d})$ , and vice versa.  $\phi(n) = \sum_{d|n} \mu(d)\frac{n}{d}$ .  $\sum_{d|n} \mu(d) = 1$ .

If  $f$  is multiplicative, then  $\sum_{d|n} \mu(d)f(d) = \prod_{p|n} (1 - f(p))$ ,  $\sum_{d|n} \mu(d)^2 f(d) = \prod_{p|n} (1 + f(p))$ .

$$\sum_{i=1}^n \sum_{j=1}^n [\gcd(i, j) = 1] = \sum_{k=1}^n \mu(k) \lfloor \frac{n}{k} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n k \sum_{l=1}^{\lfloor \frac{n}{k} \rfloor} \mu(l) \lfloor \frac{n}{kl} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n (\frac{\lfloor \frac{n}{k} \rfloor}{2})(1 + \frac{\lfloor \frac{n}{k} \rfloor}{2})^2 \sum_{d|k} \mu(d)kd$$

**10.7.7 Legendre symbol**

If  $p$  is an odd prime,  $a \in \mathbb{Z}$ , then  $\left(\frac{a}{p}\right)$  equals 0, if  $p|a$ ; 1 if  $a$  is a quadratic residue modulo  $p$ ; and  $-1$  otherwise. Euler's criterion:  $\left(\frac{a}{p}\right) = a^{\left(\frac{p-1}{2}\right)} \pmod{p}$ .

**10.7.8 Jacobi symbol**

If  $n = p_1^{a_1} \cdots p_k^{a_k}$  is odd, then  $\left(\frac{a}{n}\right) = \prod_{i=1}^k \left(\frac{a}{p_i}\right)^{k_i}$ .

**10.7.9 Primitive roots**

If the order of  $g$  modulo  $m$  ( $\min n > 0: g^n \equiv 1 \pmod{m}$ ) is  $\phi(m)$ , then  $g$  is called a primitive root. If  $Z_m$  has a primitive root, then it has  $\phi(\phi(m))$  distinct primitive roots.  $Z_m$  has a primitive root iff  $m$  is one of 2, 4,  $p^k$ ,  $2p^k$ , where  $p$  is an odd prime. If  $Z_m$  has a primitive root  $g$ , then for all  $a$  coprime to  $m$ , there exists unique integer  $i = \text{ind}_g(a)$  modulo  $\phi(m)$ , such that  $g^i \equiv a \pmod{m}$ .  $\text{ind}_g(a)$  has logarithm-like properties:  $\text{ind}(1) = 0$ ,  $\text{ind}(ab) = \text{ind}(a) + \text{ind}(b)$ .

If  $p$  is prime and  $a$  is not divisible by  $p$ , then congruence  $x^n \equiv a \pmod{p}$  has  $\gcd(n, p-1)$  solutions if  $a^{(p-1)/\gcd(n,p-1)} \equiv 1 \pmod{p}$ , and no solutions otherwise. (Proof sketch: let  $g$  be a primitive root, and  $g^i \equiv a \pmod{p}$ ,  $g^u \equiv x \pmod{p}$ .  $x^n \equiv a \pmod{p}$  iff  $g^{nu} \equiv g^i \pmod{p}$  iff  $nu \equiv i \pmod{p}$ .)

**10.7.10 Discrete logarithm problem**

Find  $x$  from  $x^n \equiv b \pmod{m}$ . Can be solved in  $O(\sqrt{m})$  time and space with a meet-in-the-middle trick. Let  $n = \lceil \sqrt{m} \rceil$ , and  $x = ny - z$ . Equation becomes  $a^{ny} \equiv ba^z \pmod{m}$ . Precompute all values that the RHS can take for  $z = 0, 1, \dots, n-1$ , and brute force  $y$  on the LHS, each time checking whether there's a corresponding value for RHS.

**10.7.11 Pythagorean triples**

Integer solutions of  $x^2 + y^2 = z^2$  All relatively prime triples are given by:  $x = 2mn, y = m^2 - n^2, z = m^2 + n^2$  where  $m > n, \gcd(m, n) = 1$  and  $m \not\equiv n \pmod{2}$ . All other triples are multiples of these. Equation  $x^2 + y^2 = 2z^2$  is equivalent to  $(\frac{x+y}{2})^2 + (\frac{x-y}{2})^2 = z^2$ .

**10.7.12 Postage stamps/McNuggets problem**

Let  $a, b$  be relatively-prime integers. There are exactly  $\frac{1}{2}(a-1)(b-1)$  numbers not of form  $ax+by$  ( $x, y \geq 0$ ), and the largest is  $(a-1)(b-1) - 1 = ab - a - b$ .

**10.7.13 Fermat's two-squares theorem**

Odd prime  $p$  can be represented as a sum of two squares iff  $p \equiv 1 \pmod{4}$ . A product of two sums of two squares is a sum of two squares. Thus,  $n$  is a sum of two squares iff every prime of form  $p = 4k+3$  occurs an even number of times in  $n$ 's factorization.

**10.8 Permutations****10.8.1 Factorial**

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n!$	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n!$	20	25	30	40	50	100	150	171		
	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

**10.8.2 Cycles**

Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

**10.8.3 Derangements**

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

**10.8.4 Burnside's lemma**

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts "configurations" (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k)$$

**10.9 Partitions and subsets****10.9.1 Partition function**

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

**10.9.2 Partition Number**

- Time Complexity:  $O(n\sqrt{n})$

```
for (int i = 1; i <= n; ++i) {
    pent[2 * i - 1] = i * (3 * i - 1) / 2;
    pent[2 * i] = i * (3 * i + 1) / 2;
}
p[0] = 1;
for (int i = 1; i <= n; ++i) {
    p[i] = 0;
    for (int j = 1, k = 0; pent[j] <= i; ++j) {
        if (k < 2) p[i] = add(p[i], p[i - pent[j]]);
        else p[i] = sub(p[i], p[i - pent[j]]); ++k, k &
```

- The number of partitions of a positive integer  $n$  into exactly  $k$  parts equals the number of partitions of  $n$  whose largest part equals  $k$

$$p_k(n) = p_k(n-k) + p_{k-1}(n-1)$$

**10.9.3 2nd Kaplansky's Lemma**

The number of ways of selecting  $k$  objects, no two consecutive, from  $n$  labelled objects arrayed in a circle is  $\frac{n}{k} \binom{n-k-1}{k-1} = \frac{n}{n-k} \binom{n-k}{k}$

**10.9.4 Distinct Objects into Distinct Bins**

- $n$  distinct objects into  $r$  distinct bins =  $r^n$
- Among  $n$  distinct objects, exactly  $k$  of them into  $r$  distinct bins =  $\binom{n}{k} r^k$
- $n$  distinct objects into  $r$  distinct bins such that each bin contains at least one object =  $\sum_{i=0}^r (-1)^i \binom{r}{i} (r-i)^n$

### 10.10 Coloring

The number of labeled undirected graphs with  $n$  vertices,  $G_n = 2^{\binom{n}{2}}$

The number of labeled directed graphs with  $n$  vertices,  $G_n = 2^{n^n}$   
 $1)$

The number of connected labeled undirected graphs with  $n$  vertices,  $C_n = 2^{\binom{n}{2}} - \frac{1}{n} \sum_{k=1}^{n-1} k \binom{n}{k} 2^{\binom{n-k}{2}} C_k = 2^{\binom{n}{2}} - \sum_{k=1}^{n-1} \binom{n-1}{k-1} 2^{\binom{n-k}{2}} C_k$

The number of  $k$ -connected labeled undirected graphs with  $n$  vertices,  $D[n][k] = \sum_{s=1}^n \binom{n-1}{s-1} C_s D[n-s][k-1]$

Cayley's formula: the number of trees on  $n$  labeled vertices = the number of spanning trees of a complete graph with  $n$  labeled vertices =  $n^{n-2}$

Number of ways to color a graph using  $k$  colors such that no two adjacent nodes have same color

Complete graph =  $k(k-1)(k-2)\dots(k-n+1)$

Tree =  $k(k-1)^{n-1}$

Cycle =  $(k-1)^n + (-1)^n (k-1)$

Number of trees with  $n$  labeled nodes:  $n^{n-2}$

### 10.11 General purpose numbers

#### 10.11.1 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$  j:s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$  j:s s.t.  $\pi(j) \geq j$ ,  $k$  j:s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

#### 10.11.2 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ . For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

#### 10.11.3 Bernoulli numbers

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0. \quad B_0 = 1, B_1 = -\frac{1}{2}. \quad B_n = 0, \text{ for all odd } n \neq 1.$$

#### 10.11.4 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

- $C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$
- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with  $n+1$  leaves (0 or 2 children).
- ordered trees with  $n+1$  vertices.
- ways a convex polygon with  $n+2$  sides can be cut into triangles by connecting vertices with straight lines.

- permutations of  $[n]$  with no 3-term increasing subseq.
- Find the count of balanced parentheses sequences consisting of  $n+k$  pairs of parentheses where the first  $k$  symbols are open brackets.

$$C_n^{(k)} = \frac{k+1}{n+k+1} \binom{2n+k}{n}$$

- Recursive formula of Catalan Numbers:

$$C_n^{(k)} = \frac{(2n+k-1) \cdot (2n+k)}{n \cdot (n+k+1)} C_{n-1}^{(k)}$$

#### 10.11.5 Lucas Number

Number of edge cover of a cycle graph  $C_n$  is  $L_n$

$$L(n) = L(n-1) + L(n-2); L(0) = 2, L(1) = 1$$

#### 10.12 Ballot Theorem

Suppose that in an election, candidate A receives  $a$  votes and candidate B receives  $b$  votes, where  $a > b$  for some positive integer  $k$ . Compute the number of ways the ballots can be ordered so that A maintains more than  $k$  times as many votes as B throughout the counting of the ballots.

The solution to the ballot problem is  $\frac{a-kb}{a+b} \times C(a+b, a)$

#### 10.13 Classical Problem

$F(n, k)$  = number of ways to color  $n$  objects using exactly  $k$  colors

Let  $G(n, k)$  be the number of ways to color  $n$  objects using no more than  $k$  colors.

Then,  $F(n, k) = G(n, k) - C(k, 1)*G(n, k-1) + C(k, 2)*G(n, k-2) - C(k, 3)*G(n, k-3) \dots$

#### Determining $G(n, k)$ :

Suppose, we are given a  $1 \times n$  grid. Any two adjacent cells can not have same color. Then,  $G(n, k) = k * ((k-1)^{n-1})$

If no such condition on adjacent cells. Then,  $G(n, k) = k^n$

#### 10.14 Matching Formula

##### 10.14.1 Normal Graph

MM + MEC =  $n$  (excluding vertex), IS + VC =  $G$ , MIS + MVC =  $G$

##### 10.14.2 Bipartite Graph

MIS =  $n - MBM$ , MVC =  $MBM$ , MEC =  $n - MBM$

#### 10.15 Inequalities

##### 10.15.1 Titu's Lemma

For positive reals  $a_1, a_2, \dots, a_n$  and  $b_1, b_2, \dots, b_n$ ,

$$\frac{a_1^2}{b_1} + \frac{a_2^2}{b_2} + \dots + \frac{a_n^2}{b_n} \geq \frac{a_1 + a_2 + \dots + a_n}{b_1 + b_2 + \dots + b_n}^2$$

Equality holds if and only if  $a_i = kb_i$  for a non-zero real constant  $k$ .

#### 10.16 Games

##### 10.16.1 Grundy numbers

For a two-player, normal-play (last to move wins) game on a graph  $(V, E)$ :  $G(x) = \text{mex}(\{G(y) : (x, y) \in E\})$ , where  $\text{mex}(S) = \min\{n \geq 0 : n \notin S\}$ .  $x$  is losing iff  $G(x) = 0$ .

#### 10.16.2 Sums of games

- Player chooses a game and makes a move in it Grundy number of a position is xor of grundy numbers of positions in summed games.

- Player chooses a non-empty subset of games (possibly, all) and makes moves in all of them. A position is losing iff each game is in a losing position.

- Player chooses a proper subset of games (not empty and not all), and makes moves in all chosen ones. A position is losing iff grundy numbers of all games are equal.

#### 10.16.3 Misère Nim

#### 10.17 Tree Hashing

$f(u) = sz[u] * \sum_{i=0} f(v) * p^i$ ;  $f(v)$  are sorted  $f(\text{child}) = 1$

#### 10.18 Permutation

To maximize the sum of adjacent differences of a permutation, it is necessary and sufficient to place the smallest half numbers in odd position and the greatest half numbers in even position. Or, vice versa.

#### 10.19 String

- If the sum of length of some strings is  $N$ , there can be at most  $\sqrt{N}$  distinct length.

- A Text can have at most  $O(N \times \sqrt{N})$  distinct substrings that match with given patterns where the sum of the length of the given patterns is  $N$ .

- Period =  $n \% (n - pi.back() == 0) ? n - pi.back() : n$

- The first (*period*) cyclic rotations of a string are distinct. Further cyclic rotations repeat the previous strings.

- $S$  is a palindrome if and only if its period is a palindrome.

- If  $S$  and  $T$  are palindromes, then the periods of  $S \cdot T$  are same if and only if  $S + T$  is a palindrome.

#### 10.20 Bit

- $(a \text{ xor } b)$  and  $(a + b)$  has the same parity
- $(a + b) = (a \text{ xor } b) + 2(a \cdot b)$
- $\text{gcd}(a, b) \leq a - b \leq \text{xor}(a, b)$

#### 10.21 Convolution

- Hamming Distance: Replace 0 with  $-1$  - SQRT Decomposition: Find block size,  $B = \sqrt{(8 * n)}$