



Daffodil International University

DIU_DividedByZero

khun_, tasnim07, kazi_amir

Team Reference Document

Contents

1 Setup

1.1	CP_Ubuntu	1
1.2	CP_Windows	1
1.3	StressTesting(check.sh)	1
1.4	StressTesting(gen.cpp)	1

2 Data Structures

2.1	Custom Hash	1
2.2	Fast Unordered Map	2
2.3	GP Hash Table	2
2.4	Mex of All Subarray	2
2.5	Pbds	2
2.6	Segment Tree(BSUA)	2
2.7	Segment Tree(LzP)	2
2.8	Segment Tree	3
2.9	Sparse Table	3

3 Dynamic Programming

3.1	Coin Change(Number of Ways)	3
3.2	Digit DP	3
3.3	LIS	3
3.4	Maximum Subarray Sum(Kadanes)	3

4 Graph

4.1	Articulation Point	3
4.2	BFS	4
4.3	Bellman Ford	4
4.4	Bridge Finding DFS	4
4.5	Cycle Detection in DAG	4
4.6	DSU	4
4.7	Dijkstra	4
4.8	Euler Tour	5
4.9	Floyd Warshall	5
4.10	MST	5
4.11	Max Bipartite Matching[Hopcroft Karp]	5
4.12	Max Bipartite Matching[Kuhn's]	5
4.13	Max Flow	5
4.14	Topological Sorting	6
4.15	Weighted Union Find	6

5 Math

5.1	Matrix Exponentiation	6
5.2	Matrix Rotation	6

6 Miscellaneous

6.1	Max Subarray Size Sum equal K	6
6.2	Merge Sort	7
6.3	Number of Subarray Sum is K	7

7 Number Theory

7.1	All In One NT	7
7.2	Divisor Sieve	7
7.3	Number of Pairs with GCD equal g	7
7.4	Phi(1toN)	7
7.5	Phi	7
7.6	SOD NOD	7
7.7	Segmented Sieve	7
7.8	Sieve	7
7.9	Spf	7
7.10	UniquePF of all elements till MX	8
7.11	int128	8

7.12	nCr and nPr	8
7.13	nCr v2	8

8 String

8.1	Aho Corasic	8
8.2	LCS for 3 Strings	1
8.3	Manacher Palindrome	1
8.4	String Hashing 2	1
8.5	String Hashing	1
8.6	Suffix Array	1
8.7	Suffix Automata	1
8.8	Suffix Automation	1
8.9	Trie	2

9 Tree

9.1	Centroid Decomposition	10
9.2	DSUOnTrees	10
9.3	LCA using binary Lifting	11
9.4	LCA	11

10 Notes

10.1	Geometry	11
10.2	Binomial Coefficient	11
10.3	Fibonacci Number	12
10.4	Sums	12
10.5	Series	12
10.6	Pythagorean Triples	12
10.7	Number Theory	12
10.8	Permutations	13
10.9	Partitions and subsets	13
10.10	Coloring	14
10.11	General purpose numbers	14
10.12	Ballot Theorem	14
10.13	Classical Problem	14
10.14	Matching Formula	14
10.15	Inequalities	14
10.16	Games	14
10.17	Tree Hashing	14
10.18	Permutation	14
10.19	String	14
10.20	Bit	14
10.21	Convolution	14

1 Setup

1.1 CP_Ubuntu

```
{
  "cmd": ["ulimit -s 268435456; g++ -std=c++20
    $file_name -o $file_base_name && timeout 4s
    ./ $file_base_name < inputf.in >
    outputf.in"],
  "selector": "source.cpp",
  "shell": true,
  "working_dir": "$file_path"
}
```

1.2 CP_Windows

```
{
  "cmd": ["g++.exe", "-std=c++20", "${file}",
    "-o", "${file_base_name}.exe", "&&", "${f}
    ile base name}.exe<inputf.in>outputf.in"],
  "selector": "source.cpp",
  "shell": true,
  "working_dir": "$file_path"
}
```

```
}
}

1.3 StressTesting(check.sh)
// chmod u+x check.sh
// ./check.sh
set -e
g++ gen.cpp -o gen
g++ code.cpp -o code
g++ brute.cpp -o brute
for ((i = 1; ; ++i)); do
  echo "Passed on TestCase: " $i
  ./gen $i > in
  ./code < in > out1
  ./brute < in > out2
  diff -Z out1 out2 || break
done
echo -e "WA on the following test:"
cat in
echo -e "\nExpected:"
cat out2
echo -e "\nFound:"
cat out1
```

1.4 StressTesting(gen.cpp)

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
mt19937 64 rng(chrono::steady_clock::now().time_
  _ since_epoch().count());
inline ll gen_random(ll l, ll r) {
  return uniform_int_distribution<ll>(l, r)(rng);
}
inline double gen_random_real(double l, double
  r) {
  return uniform_real_distribution<double>(l,
    r)(rng);
}
int main(int argc, char* args[]) {
  int n = atoi(args[1]);
  rng.seed(_);
  int n = gen_random(1, 5);
  vector<int> per;
  for (int i = 0; i < n; ++i) {
    per.push_back(i + 1);
  }
  shuffle(per.begin(), per.end(), rng);
  return 0;
}
```

2 Data Structures

2.1 Custom Hash

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
struct customHash {
  static uint64_t Meaw(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
      chrono::steady_clock::now().time_since_e_
        _ poch().count();
    return Meaw(x + FIXED_RANDOM);
  }
}
```

```
} } // gp_hash_table<int, int> table;
```

2.2 Fast Unordered Map

```
mp.reserve(1<<20); // about 1M buckets
mp.max_load_factor(0.7); // safe and fast
```

2.3 GP Hash Table

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
const int RANDOM = chrono::high_resolution_clock::
    ::now().time_since_epoch().count();
struct custom_hash {
    int operator()(int x) const { return x ^
        - RANDOM; }
};
//gp_hash_table<int, int, custom_hash> mp;
```

2.4 Mex of All Subarray

```
const int N = 1e5 + 9, inf = 1e9;
struct ST {
    int t[4 * N];
    ST() {}
    void build(int n, int b, int e) {
        t[n] = 0;
        if (b == e) {
            return;
        }
        int mid = (b + e) >> 1, l = n << 1, r = l |
            - 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        t[n] = min(t[l], t[r]);
    }
    void upd(int n, int b, int e, int i, int x) {
        if (b > i || e < i) return;
        if (b == e && b == i) {
            t[n] = x;
            return;
        }
        int mid = (b + e) >> 1, l = n << 1, r = l |
            - 1;
        upd(l, b, mid, i, x);
        upd(r, mid + 1, e, i, x);
        t[n] = min(t[l], t[r]);
    }
    int get_min(int n, int b, int e, int i, int j) {
        if (b > j || e < i) return inf;
        if (b >= i && e <= j) return t[n];
        int mid = (b + e) >> 1, l = n << 1, r = l |
            - 1;
        int L = get_min(l, b, mid, i, j);
        int R = get_min(r, mid + 1, e, i, j);
        return min(L, R);
    }
    int get_mex(int n, int b, int e, int i) { //
        - mex of [i... cur_id] if (b == e) return b;
        int mid = (b + e) >> 1, l = n << 1, r = l |
            - 1;
        if (t[l] >= i) return get_mex(r, mid + 1, e,
            - i);
        return get_mex(l, b, mid, i);
    }
};
```

```
} t;
int a[N], f[N];
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n;
    cin >> n;
    for (int i = 1; i <= n; i++) {
        cin >> a[i];
        --a[i];
    }
    t.build(1, 0, n);
    set<array<int, 3>> seg; // for cur_id = i,
        [x[0]... i], [x[0] + 1... i], ... [x[1]... i]
        = has mex, , x[2]
    for (int i = 1; i <= n; i++) {
        int x = a[i];
        int r = min(i - 1, t.get_min(1, 0, n, 0, x -
            - 1));
        int l = t.get_min(1, 0, n, 0, x) + 1;
        if (l <= r) {
            auto it = seg.lower_bound({l, -1, -1});
            while (it != seg.end() && (*it)[1] <= r) {
                auto x = *it;
                it = seg.erase(it);
            }
            t.upd(1, 0, n, x, i);
            for (int j = r; j >= l; j++) {
                int m = t.get_mex(1, 0, n, j);
                int L = max(l, t.get_min(1, 0, n, 0, m) +
                    - 1);
                f[m] = 1;
                seg.insert({L, j, m});
                j = L - 1;
            }
            int m = !a[i];
            seg.insert({1, i, m});
            f[m] = 1;
        }
        int ans = 0;
        while (f[ans]) ++ans;
        cout << ans + 1 << '\n';
        return 0;
    }
}
```

2.5 Pbds

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <functional>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>,
    - rb_tree_tag,
    - tree_order_statistics_node_update>
    ordered_set;
// s.order_of_key(x) = number of elements
// - strictly less than x
// *s.find_by_order(i) = ith element in set (0
// - index)
```

2.6 Segment Tree(BSUA)

```
// CSES - 1749
const int MX = 2e5 + 10;
int n;
int arr[MX], st[MX << 2];
```

```
void assign(int i, int x, int u = 1, int s = 0,
    - int e = n - 1) {
    if (s == e) {
        st[u] = x;
        return;
    }
    int v = u << 1, w = v | 1, m = (s + e) >> 1;
    if (i <= m) assign(i, x, v, s, m);
    else assign(i, x, w, m + 1, e);
    st[u] = st[v] + st[w];
}
int kth(int k, int u = 1, int s = 0, int e = n -
    - 1) {
    if (st[u] < k) return -1;
    if (s == e) {
        return s;
    }
    int v = u << 1, w = v | 1, m = (s + e) >> 1;
    if (st[v] >= k) return kth(k, v, s, m);
    else return kth(k - st[v], w, m + 1, e);
}
void solve() {
    cin >> n;
    for (int i = 0; i < n; ++i) {
        cin >> arr[i];
    }
    for (int i = 0; i < n; ++i) {
        assign(i, 1);
    }
    for (int i = 0; i < n; ++i) {
        int x;
        cin >> x;
        int ind = kth(x);
        assign(ind, 0);
        cout << arr[ind] << " ";
    }
}
```

2.7 Segment Tree(LzP)

```
class stree {
    vector<ll> st, lazy;
public:
    stree(int n) {
        st.assign((n << 2) + 10, 0);
        lazy.assign((n << 2) + 10, 0);
    }
    void push(int u, int s, int e) {
        if (!lazy[u]) return;
        st[u] += (e - s + 1) * 1LL * lazy[u];
        if (s != e) {
            int v = u << 1, w = v | 1, m = (s + e) >>
                - 1;
            lazy[v] += lazy[u];
            lazy[w] += lazy[u];
        }
        lazy[u] = 0;
    }
    void build(int u, int s, int e, int arr[]) {
        if (s == e) {
            st[u] = arr[s];
            return;
        }
        int v = u << 1, w = v | 1, m = (s + e) >> 1;
        build(v, s, m, arr);
        build(w, m + 1, e, arr);
        st[u] = st[v] + st[w];
    }
};
```

```

void update(int l, int r, int x, int u, int s,
            int e) {
    push(u, s, e);
    if (e < l or r < s) return;
    int v = u << 1, w = v | 1, m = (s + e) >> 1;
    if (l <= s and e <= r) {
        st[u] += (e - s + 1) * 1LL * x;
        if (s != e) {
            lazy[v] += x;
            lazy[w] += x;
        }
        return;
    }
    update(l, r, x, v, s, m);
    update(l, r, x, w, m + 1, e);
    st[u] = st[v] + st[w];
}

int query(int l, int r, int u, int s, int e) {
    push(u, s, e);
    if (e < l or r < s) return 0;
    if (l <= s and e <= r) return st[u];
    int v = u << 1, w = v | 1, m = (s + e) >> 1;
    return query(l, r, v, s, m) + query(l, r, w,
        m + 1, e);
}
};

```

2.8 Segment Tree

```

class stree {
    vector<ll> st;
public:
    stree(int n) {
        st.assign((n << 2) + 10, 0);
    }
    void build(int u, int s, int e, int arr[]) {
        if (s == e) {
            st[u] = arr[s];
            return;
        }
        int v = u << 1, w = v | 1, m = (s + e) >> 1;
        build(v, s, m, arr);
        build(w, m + 1, e, arr);
        st[u] = st[v] + st[w];
    }
    int query(int l, int r, int u, int s, int e) {
        if (e < l or r < s) return 0;
        if (l <= s and e <= r) return st[u];
        int v = u << 1, w = v | 1, m = (s + e) >> 1;
        return query(l, r, v, s, m) + query(l, r, w,
            m + 1, e);
    }
    void update(int i, int x, int u, int s, int e) {
        if (s == e) {
            st[u] = x;
            return;
        }
        int v = u << 1, w = v | 1, m = (s + e) >> 1;
        if (i <= m) update(i, x, v, s, m);
        else update(i, x, w, m + 1, e);
        st[u] = st[v] + st[w];
    }
};

```

2.9 Sparse Table

```
const int mxN = 1e5 + 10, M = 21;
```

```

int sparse[mxN][M];
void build_sparse(int n, vector<int>& v) {
    for (int i = 0; i < n; i++) sparse[i][0] =
        v[i];
    for (int k = 1; k < M; k++) {
        for (int i = 0; i + (1 << k) <= n; i++) {
            sparse[i][k] = max(sparse[i][k - 1],
                sparse[i + (1 << (k - 1))][k - 1]);
        }
    }
}

int query(int l, int r) { // 0 based index
    if (l > r) swap(l, r);
    int b = __bit_width(r - l + 1) - 1;
    return max(sparse[l][b], sparse[r - (1 << b) +
        1][b]);
}

```

3 Dynamic Programming

3.1 Coin Change (Number of Ways)

```

const int mod = 1e9+7;
void solve() {
    int n, k; cin >> n >> k;
    vector<int> coin(n);
    for (int i = 0; i < n; i++) { cin >> coin[i]; }
    vector<int> dp(k+1, 0); dp[0] = 1;
    for (int i = 1; i <= k; i++) {
        for (int j = 0; j < n; j++) {
            if (i - coin[j] >= 0) {
                dp[i] = (dp[i] + dp[i - coin[j]]) % mod;
            }
        }
    }
    cout << dp[k] << endl;
}

```

3.2 Digit DP

```

vector<int> nmbrs;
int dp[10][10][2];
int dgt_dp(int idx, int tight, int oneCnt) {
    if (idx == nmbrs.size()) {
        return oneCnt;
    }
    if (dp[idx][oneCnt][tight] != -1) return
        dp[idx][oneCnt][tight];
    int lmt = (tight ? nmbrs[idx] : 9);
    int sum = 0;
    for (int i = 0; i <= lmt; i++) {
        bool newTight = (tight and i == nmbrs[idx]);
        sum += dgt_dp(idx + 1, newTight, oneCnt + (i
            == 1));
    }
    return dp[idx][oneCnt][tight] = sum;
}

```

3.3 LIS

```

vector<int> lis(int n, vector<int>& v) {
    vector<int> parent(n, -1), ind(n);
    vector<int> lis;
    for (int i = 0; i < n; i++) {
        int it = lower_bound(lis.begin(), lis.end(),
            v[i]) - lis.begin();
        if (it == lis.size()) {
            lis.push_back(v[i]);
            ind[lis.size() - 1] = i;
        }
    }
}

```

```

parent[i] = (lis.size() == 1 ? -1 : ind[it
    - 1]);
} else {
    lis[it] = v[i];
    ind[it] = i;
    parent[i] = (it == 0 ? -1 : ind[it - 1]);
}
}
vector<int> LIS;
int it = ind[lis.size() - 1];
LIS.push_back(lis.back());
while (parent[it] != -1) {
    it = parent[it];
    LIS.push_back(v[it]);
}
return LIS;
}

```

3.4 Maximum Subarray Sum (Kadanes)

```

int max_sum_of(vector<int> &vct) {
    int mx = INT_MIN, till = 0;
    for (int i = 0; i < vct.size(); i++) {
        till = till + vct[i];
        mx = max(mx, till);
        till = max(till, 1LL * 0);
    }
    return mx;
}

```

4 Graph

4.1 Articulation Point

```

int n; // number of nodes
vector<vector<int>> lst; // adjacency list of
    graph
vector<bool> vis;
vector<int> tin, low;
int timer;
void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
    int children = 0;
    for (int v : lst[u]) {
        if (v == p) continue;
        if (vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] >= tin[u] && p != -1) {
                IS_CUTPOINT(u);
            }
            ++children;
        }
    }
    // if no vertex below v can reach u or higher
    // removing u disconnects that subtree
    if (p == -1 && children > 1) {
        IS_CUTPOINT(u);
    }
}

void find_cutpoints() {
    timer = 0;
    vis.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
}

```

```

for (int i = 0; i < n; ++i) {
    if (!vis[i]) {
        dfs(i);
    }
}
}

```

4.2 BFS

```

vector<vector<int>> adj; // adjacency list
// representation
int n; // number of nodes
int s; // source vertex

```

```

void bfs() {
    queue<int> q;
    vector<int> d(n), p(n);
    vector<bool> used(n);
    q.push(s);
    used[s] = true;
    p[s] = -1;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int u : adj[v]) {
            if (!used[u]) {
                used[u] = true;
                q.push(u);
                d[u] = d[v] + 1;
                p[u] = v;
            }
        }
    }
}

// retrieving shortest path
if (!used[u]) {
    cout << "No path!";
} else {
    vector<int> path;
    for (int v = u; v != -1; v = p[v])
        path.push_back(v);
    reverse(path.begin(), path.end());
    cout << "Path: ";
    for (int v : path)
        cout << v << " ";
}
}

```

4.3 Bellman Ford

```

#define ll long long
#define INF 1e18
void solve() {
    int n, m, v;
    cin >> n >> m >> v; // n = nodes, m = edges, v
    // = source (0-indexed)
    vector<array<ll, 3>> edges(m); // each edge:
    // {a, b, cost}
    for (int i = 0; i < m; i++) cin >> edges[i][0]
    // >> edges[i][1] >> edges[i][2];
    vector<ll> d(n, INF);
    vector<int> p(n, -1);
    d[v] = 0;
    int x = -1;
    for (int i = 0; i < n; i++) {
        x = -1;
        for (auto& e : edges) {

```

```

            int a = e[0], b = e[1];
            ll cost = e[2];
            if (d[a] < INF && d[b] > d[a] + cost) {
                d[b] = max(-INF, d[a] + cost);
                p[b] = a;
                x = b;
            }
        }
    }
    if (x == -1) {
        cout << "No negative cycle from vertex " <<
        // v << '\n';
        return;
    }
    int y = x;
    for (int i = 0; i < n; i++) y = p[y];
    vector<int> path;
    for (int cur = y; cur = p[cur]) {
        path.push_back(cur);
        if (cur == y && path.size() > 1) break;
    }
    reverse(path.begin(), path.end());
    cout << "Negative cycle: ";
    for (int u : path) cout << u << ' ';
    cout << '\n';
}

```

4.4 Bridge Finding DFS

```

const int MX = 1e5 + 10;
int n, m, timer = 0;
vector<int> adj[MX];
vector<int> tin(MX, -1), low(MX, -1);
vector<bool> vis(MX, false);
void is_bridge(int u, int v) {
    // do something with the edge
}
void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
    for (int v : adj[u]) {
        if (v == p) continue;
        if (vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u]) {
                is_bridge(u, v);
            }
        }
    }
}

```

4.5 Cycle Detection in DAG

```

const int MX = 1e5 + 10;
bool vis[MX], pathVis[MX];
vector<int> lst[MX];
bool dfs(int u) {
    vis[u] = true;
    pathVis[u] = true;
    for (auto v : lst[u]) {
        if (!vis[v]) {
            if (dfs(v))
                return true;
        } else if (pathVis[v]) {

```

```

            return true;
        }
    }
    pathVis[u] = false;
    return false;
}
void solve() {
    // take graph input
    for (int i = 0; i < n; ++i) {
        if (!vis[i])
            dfs(i);
    }
}

```

4.6 DSU

```

const int MX = 1e5 + 10;
int par[MX], sz[MX];
void init() {
    for (int i = 1; i < MX; i++) {
        par[i] = i;
        sz[i] = 1;
    }
}
int findpar(int x) {
    if (par[x] == x) return x;
    return par[x] = findpar(par[x]);
}
void unite(int u, int v) {
    u = findpar(u);
    v = findpar(v);
    if (u != v) {
        if (sz[u] < sz[v]) {
            swap(u, v);
        }
        sz[u] += sz[v];
        par[v] = u;
    }
}

```

4.7 Dijkstra

```

const int N = 1e5 + 5, INF = 1e18 + 7;
vector<pair<int, int>> g[N];
bool vis[N];
vector<int> dist(N, INF), parent(N);
bool dijkstra(int source) {
    priority_queue<pair<int, int>,
    // vector<pair<int, int>>, greater<pair<int,
    // int>>> pq;
    pq.push({0, source});
    dist[source] = 0;
    parent[source] = -1;
    while (pq.size()) {
        int x = pq.top().second;
        pq.pop();
        if (vis[x]) continue;
        vis[x] = 1;
        for (auto [child_x, child_wt] : g[x]) {
            if (dist[x] + child_wt < dist[child_x]) {
                parent[child_x] = x;
                dist[child_x] = dist[x] + child_wt;
                pq.push({dist[child_x], child_x});
            }
        }
    }
    return (dist[n] == INF);
}

```


4.8 Euler Tour

```

const int MX = 2e5 + 10;
int timer = -1;
// s = start pos, e = end pos
int val[MX], s[MX], e[MX], flat[MX];
vector<int> lst[MX];
void dfs(int u, int p) {
    s[u] = ++timer;
    flat[timer] = val[u];
    for (auto v : lst[u]) {
        if (v != p)
            dfs(v, u);
    }
    e[u] = timer;
}

```

4.9 Floyd Warshall

```

vector<vector<int>> d(n, vector<int>(n, INF));
// take graph input into d
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (d[i][k] < INF && d[k][j] < INF)
                d[i][j] = min(d[i][j], d[i][k] +
                    d[k][j]);
        }
    }
}

```

4.10 MST

```

// DSU first
void solve() {
    int n, m;
    cin >> n >> m;
    vector<tuple<int, int, int>> edges;
    for (int i = 0; i < m; ++i) {
        int u, v, wt;
        cin >> u >> v >> wt;
        edges.push_back({wt, u, v});
    }
    sort(edges.begin(), edges.end());
    init(n);
    int cost = 0;
    for (tuple& [wt, u, v] : edges) {
        if (findpar(u) == findpar(v)) continue;
        unite(u, v);
        cost += wt;
    }
    cout << cost << endl;
}

```

4.11 Max Bipartite Matching[Hopcroft Karp]

```

const int INF = 1e9;
void hopcroftCarp() {
    int n, m, e;
    cin >> n >> m >> e;
    vector<int> adj[n];
    for (int i = 0; i < e; ++i) {
        int u, v;
        cin >> u >> v;
        adj[u].push_back(v);
    }
    vector<int> ml(m, -1), mr(n, -1), dist(n);
    auto bfs = [&]() -> bool {

```

```

    queue<int> q;
    for (int u = 0; u < n; ++u) {
        if (mr[u] == -1) {
            dist[u] = 0;
            q.push(u);
        } else {
            dist[u] = INF;
        }
    }
    bool foundAugmenting = false;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (int v : adj[u]) {
            int pairedLeft = ml[v];
            if (pairedLeft == -1) {
                foundAugmenting = true;
            } else if (dist[pairedLeft] == INF) {
                dist[pairedLeft] = dist[u] + 1;
                q.push(pairedLeft);
            }
        }
    }
    return foundAugmenting;
};
function<bool(int)> dfs = [&](int u) -> bool {
    for (int v : adj[u]) {
        int pairedLeft = ml[v];
        if (pairedLeft == -1 or (dist[pairedLeft]
            == dist[u] + 1 and dfs(pairedLeft))) {
            mr[u] = v;
            ml[v] = u;
            return true;
        }
    }
    dist[u] = INF;
    return false;
};
int matching = 0;
while (bfs()) {
    for (int u = 0; u < n; ++u) {
        if (mr[u] == -1) {
            if (dfs(u)) matching++;
        }
    }
}
cout << matching << el;
for (int u = 0; u < n; ++u) {
    if (mr[u] != -1) {
        cout << u << " " << mr[u] << el;
    }
}
}

```

4.12 Max Bipartite Matching[Kuhn's]

```

// left set size, right set size, edge count
int n, k, m, visToken = 1;
vector<int> lst[MX];
int mr[MX], ml[MX], vis[MX];
bool try_kuhn(int u) {
    if (vis[u] == visToken)
        return false;
    vis[u] = visToken;
    for (auto v : lst[u]) {
        if (ml[v] == -1 or try_kuhn(ml[v])) {
            ml[v] = u;
            mr[u] = v;

```

```

            return true;
        }
    }
    return false;
}
void solve() {
    cin >> n >> k >> m;
    for (int i = 0; i < m; ++i) {
        int u, v;
        cin >> u >> v;
        lst[u].push_back(v);
    }
    fill(mr, mr + n, -1);
    fill(ml, ml + k, -1);
    int ans = 0;
    for (int u = 0; u < n; ++u) {
        for (auto v : lst[u]) {
            if (ml[v] == -1) {
                ml[v] = u;
                mr[u] = v;
                ans++;
                break;
            }
        }
        for (int u = 0; u < n; ++u) {
            if (mr[u] != -1) continue;
            visToken++;
            if (try_kuhn(u))
                ans++;
        }
        cout << ans << el;
        for (int v = 0; v < k; ++v) {
            if (ml[v] != -1) {
                cout << ml[v] + 1 << " " << v + 1 << el;
            }
        }
    }
}

```

4.13 Max Flow

```

struct Dinic {
    struct Edge {
        int to;
        ll capacity;
        int rev; // index of reverse edge
    };
    vector<vector<Edge>> adj;
    vector<int> level;
    vector<int> ptr;
    int n;
    Dinic(int nodes) : n(nodes), adj(nodes),
        level(nodes), ptr(nodes) {}
    void add_edge(int from, int to, ll cap) {
        adj[from].push_back({to, cap,
            (int)adj[to].size()});
        adj[to].push_back({from, 0,
            (int)adj[from].size() - 1});
    }
    bool bfs(int s, int t) {
        fill(level.begin(), level.end(), -1);
        level[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int v = q.front();

```

```

q.pop();
for (auto& edge : adj[v]) {
    if (edge.capacity > 0 && level[edge.to]
        == -1) {
        level[edge.to] = level[v] + 1;
        q.push(edge.to);
    }
}
return level[t] != -1;
}
ll dfs(int v, int t, ll pushed) {
    if (pushed == 0) return 0;
    if (v == t) return pushed;
    for (int& cid = ptr[v]; cid < adj[v].size();
        ++cid) {
        auto& edge = adj[v][cid];
        int tr = edge.to;
        if (level[v] + 1 != level[tr] ||
            edge.capacity == 0) continue;
        ll tr_pushed = dfs(tr, t, min(pushed,
            edge.capacity));
        if (tr_pushed == 0) continue;
        edge.capacity -= tr_pushed;
        adj[tr][edge.rev].capacity += tr_pushed;
        return tr_pushed;
    }
    return 0;
}
ll max_flow(int s, int t) {
    ll flow = 0;
    while (bfs(s, t)) {
        fill(ptr.begin(), ptr.end(), 0);
        while (ll pushed = dfs(s, t, 1e18)) {
            flow += pushed;
        }
    }
    return flow;
}
// Usage in int main():
int n, m;
cin >> n >> m;
Dinic dinic(n + 1); // Initialize with N+1 nodes
// (0 to N)
for (int i = 0; i < m; i++) {
    int u, v; ll cap;
    cin >> u >> v >> cap;
    // Add directed edge u -> v with capacity
    // For bidirectional, add edge v -> u as well
    dinic.add_edge(u, v, cap);
}
// Compute flow from node 1 (Source) to node n
// (Sink)
cout << dinic.max_flow(1, n) << endl;

```

4.14 Topological Sorting

```

const int N = 1e5 + 10;
vector<int> g[N], indegree(N, 0);
vector<int> topSort(int n) {
    queue<int> q;
    vector<int> order;
    for (int i = 1; i <= n; i++) {
        if (indegree[i] == 0) {
            q.push(i);

```

```

}
while (!q.empty()) {
    int u = q.front();
    q.pop();
    order.push_back(u);
    for (int v : g[u]) {
        indegree[v]--;
        if (indegree[v] == 0) {
            q.push(v);
        }
    }
}
return order;
}

```

4.15 Weighted Union Find

```

const int MX = 2e5 + 10;
int par[MX], sz[MX];
ll d[MX];
void init() {
    for (int i = 0; i < MX; ++i) {
        par[i] = i;
        sz[i] = 1;
        d[i] = 0;
    }
}
int findpar(int x) {
    if (par[x] == x) return x;
    int p = par[x];
    par[x] = findpar(p);
    d[x] += d[p];
    return par[x];
}
bool unite(int a, int b, ll w) {
    int ra = findpar(a);
    int rb = findpar(b);
    if (ra == rb) {
        return (d[b] - d[a] == w);
    }
    if (sz[ra] < sz[rb]) {
        swap(a, b);
        swap(ra, rb);
        w = -w;
    }
    par[rb] = ra;
    d[rb] = d[a] + w - d[b];
    sz[ra] += sz[rb];
    return true;
}
ll dist(int a, int b) {
    findpar(a), findpar(b);
    return d[b] - d[a];
}

```

5 Math

5.1 Matrix Exponentiation

```

const ll mod = 1e9;
vector<vector<ll>> matMul(vector<vector<ll>>& a,
    vector<vector<ll>>& b) {
    ll row1 = a.size(), col1 = a[0].size();
    ll row2 = b.size(), col2 = b[0].size();
    vector<vector<ll>> res(row1, vector<ll>(col2,
        0));
    for (ll i = 0; i < row1; i++) {
        for (ll j = 0; j < col1; j++) {
            for (ll k = 0; k < row2; k++) {

```

```

                res[i][j] = (res[i][j] + (1LL * a[i][k]
                    * b[k][j]) % mod) % mod;
            }
        }
    }
    return res;
}
vector<vector<ll>> matExpo(vector<vector<ll>>&
    Mat, ll exp) {
    ll row = Mat.size(), col = Mat[0].size();
    ll p = row;
    vector<vector<ll>> res(p, vector<ll>(p, 0));
    for (ll i = 0; i < p; i++) res[i][i] = 1;
    while (exp) {
        if (exp & 1) res = matMul(res, Mat);
        Mat = matMul(Mat, Mat);
        exp >>= 1;
    }
    return res;
}
// b = (A(i), A(i-1), A(i-2), A(i-3))
// M = Magic matrix, nth = nth term, known =
// known value
ll get_nth(ll nth, ll known, vector<ll>& b,
    vector<vector<ll>>& M) {
    if (nth <= known) return b[nth - 1] % mod;
    reverse(b.begin(), b.end());
    vector<vector<ll>> me = matExpo(M, nth -
        known); // MAT^(nth-known)
    ll ans = 0;
    for (int i = 0; i < known; i++) {
        ans = (ans + (b[i] * me[i][0]) % mod) % mod;
    }
    return ans;
}

```

5.2 Matrix Rotation

```

//90* clock-wise
now = {{0, 1, 0}, {-1, 0, 0}, {0, 0, 1}};
//90* anti-clock
now = {{0, -1, 0}, {1, 0, 0}, {0, 0, 1}};
//mirror with x axis at point p
now = {{-1, 0, 2 * p}, {0, 1, 0}, {0, 0, 1}};
//mirror with y axis at point p
now = {{1, 0, 0}, {0, -1, 2 * p}, {0, 0, 1}};
op[i + 1] = matMul(now, op[i]); // this
// op[i + 1] = matMul(op[i], now); //not this

```

6 Miscellaneous

6.1 Max Subarray Size Sum equal K

```

//write gpHashTable code before this part
void solution(){
    int n, k; cin >> n >> k;
    int total_sum = 0;
    vector<int> pre(n + 7, 0);
    for (int i = 1; i <= n; i++) {
        int temp; cin >> temp;
        total_sum += temp;
        if (i == 1) pre[i] = temp;
        else pre[i] = pre[i - 1] + temp;
    }
    if (total_sum < k) {
        cout << "-1" << endl; return;
    }
    if (total_sum == k) {

```

```

    cout << "0" << endl; return;
}
int maximum_subSize = 0;
gp_hash_table < int, int, customHash> table;
for (int i = 1; i <= n; i++) {
    if (pre[i] >= k) {
        int subSUM = pre[i] - k;
        if (subSUM == 0) {
            maximum_subSize = max(maximum_subSize, i);
        }
        else if (table[subSUM]) {
            int left = table[subSUM];
            int right = i; int subSize = right - left;
            maximum_subSize = max(subSize,
                maximum_subSize);
        }
    }
    if (!table[pre[i]]) table[pre[i]] = i;
} cout << maximum_subSize << endl; }

```

6.2 Merge Sort

```

// use array of elements, if multiple testcase
// make inv = 0 each time
// int inv = 0;
void merge(int vct[], int l, int m, int r) {
    int left = m - l + 1, right = r - m, lv[left],
    rv[right];
    for (int i = 0; i < left; i++) {
        lv[i] = vct[l + i];
    }
    for (int i = 0; i < right; i++) {
        rv[i] = vct[m + 1 + i];
    }
    int i = 0, j = 0, to = l;
    while (i < left && j < right) {
        if (lv[i] <= rv[j]) {
            vct[to] = lv[i];
            i++;
        } else {
            vct[to] = rv[j];
            j++;
            // inversion count
            // int pore = left-i; inv+=pore;
        }
        to++;
    }
    while (i < left) {
        vct[to] = lv[i];
        i++;
        to++;
    }
    while (j < right) {
        vct[to] = rv[j];
        j++;
        to++;
    }
}
void merge_sort(int vct[], int l, int r) {
    if (r <= l) return;
    int m = l + ((r - l) / 2);
    merge_sort(vct, l, m);
    merge_sort(vct, m + 1, r);
    merge(vct, l, m, r);
}

```

6.3 Number of Subarray Sum is K

//write gpHashTable code before this part

```

void solution() {
    int n, k; cin >> n >> k;
    int total_sum = 0;
    vector < int > pre(n + 7, 0);
    for (int i = 1; i <= n; i++) {
        int temp; cin >> temp;
        total_sum += temp;
        if (i == 1) pre[i] = temp;
        else pre[i] = pre[i - 1] + temp; }
    int cnt_subarray = 0;
    gp_hash_table < int, int, customHash> table;
    table[0] = 1;
    for (int i = 1; i <= n; i++) {
        cnt_subarray += table[pre[i] - k];
        table[pre[i]]++;
    } cout << cnt_subarray << endl; }

```

7 Number Theory

7.1 All In One NT

```

const int MAXN = 1e6 + 9;
typedef struct info {
    int lowest_prime = 0, greatest_prime = 0,
    distinct_prime = 0;
    int total_prime = 0, NOD = 0, SOD = 0;
} info;
info num[MAXN];
void preStore() {
    for (int i = 2; i < MAXN; i++) {
        int n = i;
        map < int, int > factors; // Key->Factor,
        // Val->count
        int SOD = 1, NOD = 1, total_p_factor = 0;
        if (n % 2 == 0) {
            while (n % 2 == 0) {
                n /= 2;
                factors[2]++;
                total_p_factor++;
            }
            SOD *= (1 << (factors[2] + 1)) - 1;
            NOD *= (factors[2] + 1);
        }
        for (int i = 3; i * i <= n; i += 2) {
            if (n % i == 0) {
                while (n % i == 0) {
                    n /= i;
                    factors[i]++;
                    total_p_factor++;
                }
                SOD *= (pow(i, factors[i] + 1) - 1) / (i
                    - 1);
                NOD *= (factors[i] + 1);
            }
        }
        if (n > 1) {
            factors[n]++;
            SOD *= (pow(n, 2) - 1) / (n - 1);
            NOD *= 2;
            total_p_factor++;
        }
        num[i].distinct_prime = factors.size();
        num[i].total_prime = total_p_factor;
        num[i].NOD = NOD;
        num[i].SOD = SOD;
        auto lowest_prime = factors.begin();
        auto greatest_prime = factors.rbegin();
        num[i].lowest_prime = lowest_prime->first;
    }
}

```

```

num[i].greatest_prime =
    lowest_prime->first;
}

```

7.2 Divisor Sieve

```

const int mxN = 1e5 + 10;
vector < int > divisors[mxN];
void divisorSieve() {
    for (int i = 1; i <= mxN; i++) {
        for (int j = i; j <= mxN; j += i) {
            divisors[j].push_back(i);
        }
    }
}

```

7.3 Number of Pairs with GCD equal g

```

/*a[i] <= 1e6
for all 1<=g<=n, how many pairs exist such that g
= gcd(a[i], a[j]);
complexity : nlogn
*/
ll n; cin >> n;
ll a[n + 1];
ll cnt[n + 1]; memset(cnt, 0, sizeof cnt);
for (ll i = 1; i <= n; i++) {cin >> a[i];
    cnt[a[i]]++;}
ll gcd[n + 1]; memset(gcd, 0, sizeof gcd);
for (ll i = n; i >= 1; i--) {
    ll pair = 0, invalid_pair = 0;
    for (ll j = i; j <= n; j += i) {
        pair += cnt[j];
        invalid_pair += gcd[j];
    }
    pair = (pair * (pair - 1)) / 2;
    gcd[i] = pair - invalid_pair;
    // how many pairs exist whose gcd is i
}

```

7.4 Phi(1toN)

```

const int mxN = 1e7 + 10;
vector < int > phi(mxN);
void phi_till() { //O(n.log.log(n))
    for (int i = 0; i < mxN; i++) phi[i] = i;
    for (int i = 2; i < mxN; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= mxN; j += i) {
                phi[j] -= phi[j] / i;
            }
        }
    }
}

```

7.5 Phi

```

int phi(int n) { // sqrt(n)
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}

```


7.6 SOD NOD

```
// SOD = ((P^(x+1)-1)/(P-1)) *
// ((Q^(y+1)-1)/(Q-1)) * ((R^(z+1)-1)/(R-1))
// NOD = P^x * Q^y * R^z => here, P, Q, R are
// prime factors & x, y, z are
// powers NOD = (x + 1) (y + 1) (z + 1)
pair<int, int> SOD_NOD(int n) {
    int sod = 1, nod = 1;
    for (int i = 2; i * i <= n; ++i) {
        if (n % i == 0) {
            int pown = 1, pows = 0;
            while (n % i == 0) {
                pown *= i; // p^e
                pows++;
                n /= i;
            }
            pown *= i;
            sod *= (pown - 1) / (i - 1); // (p^(e+1)-1) /
            // p-1
            nod *= (pows + 1);
        }
    }
    if (n > 1) {
        sod *= (n + 1);
        nod *= 2;
    }
    return {sod, nod};
}
```

7.7 Segmented Sieve

```
void segSeive(ll low, ll high) {
    vector<bool> area((high - low) + 1, true);
    for (ll i = 0; primes[i]*primes[i] <= high;
        ++i) {
        ll start = ((low / primes[i]) * primes[i]);
        if (start < low) start += primes[i];
        for (ll j = start; j <= high; j +=
            primes[i]) {
            if (j == primes[i]) continue;
            area[j - low] = false;
        }
    }
    for (ll i = 0; i < (high - low) + 1; i++) {
        if (area[i]) {
            if (i + low != 1 and i + low != 0) {
                cout << i + low << endl;
            }
        }
    }
}
```

7.8 Sieve

```
const ll MAXN = 1e7 + 10;
bool prime[MAXN];
vector<ll> prm;
void sieve() {
    prime[0] = prime[1] = true;
    for (ll i = 2; i < MAXN; i++) {
        if (!prime[i]) {
            prm.push_back(i);
            for (ll j = i + i; j < MAXN; j += i) {
                prime[j] = true;
            }
        }
    }
}
```

7.9 Spf

```
const int MAXN = 1e6 + 2;
int spf[MAXN];
vector<int> prms;
void preStore() {
    for (int i = 1; i < MAXN; i++) spf[i] = i;
    for (int i = 2; i < MAXN; i++) {
        if (spf[i] == i) {
            prms.push_back(i);
            for (int j = i + i; j < MAXN; j += i) {
                spf[j] = min(spf[j], i);
            }
        }
    }
}
```

7.10 UniquePF of all elements till MX

```
const int MX = 2e5 + 10;
vector<int> pfac[MX];
void factorize() {
    for (int i = 2; i < MX; i++) {
        if (!pfac[i].empty()) continue;
        for (int j = i; j < MX; j += i) {
            pfac[j].push_back(i);
        }
    }
}
```

7.11 int128

```
__int128 read() {
    __int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}
void print(__int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}
```

7.12 nCr and nPr

```
int fact[N], ifact[N];
void prec() {
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = 1LL * fact[i - 1] * i % mod;
    }
    ifact[N - 1] = power(fact[N - 1], -1);
    for (int i = N - 2; i >= 0; i--) {
        ifact[i] = 1LL * ifact[i + 1] * (i + 1) %
            mod;
    }
}
int nPr(int n, int r) {
    if (n < r) return 0;
```

```
return 1LL * fact[n] * ifact[n - r] % mod;
}
int nCr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[r] % mod *
        ifact[n - r] % mod;
}
```

7.13 nCr v2

```
const int MX = 1e6 + 10;
const int M = 1e9 + 7;
int fact[MX], inv_fact[MX];
int modPow(int a, int b) {
    int ans = 1;
    while (b) {
        if (b & 1) ans = (1LL * ans * a) % M;
        a = (1LL * a * a) % M;
        b >>= 1;
    }
    return ans;
}
void precalFact() {
    fact[0] = inv_fact[0] = 1;
    for (int i = 1; i < MX; i++) {
        fact[i] = (1LL * fact[i - 1] * i) % M;
    }
    inv_fact[MX - 1] = modPow(fact[MX - 1], M - 2);
    for (int i = MX - 2; i >= 1; i--) {
        inv_fact[i] = (1LL * inv_fact[i + 1] * (i +
            1)) % M;
    }
}
int nCr(int n, int r) {
    if (r < 0 or r > n) return 0;
    return 1LL * fact[n] * inv_fact[r] % M *
        inv_fact[n - r] % M;
}
```

8 String**8.1 Aho Corasic**

```
//number of occurrence of word in a text
const ll N = 1e6+10, A = 26;
ll trie[N][A], pos[N], slink[N], dp[N], tot = 1;
vector<int> order;
void initTrie() {
    order.clear();
    while(tot--){
        memset(trie[tot], 0, sizeof(trie[tot]));
    }
    memset(pos, 0, sizeof(pos));
    memset(slink, 0, sizeof(slink));
    memset(dp, 0, sizeof(dp)); tot = 1;
}
void addStr(string &s, int ind){
    ll u = 0;
    for(auto it: s){
        ll n = it-'a';
        if(trie[u][n]==0) trie[u][n] = tot++;
        u = trie[u][n];
    } pos[ind] = u;
}
void build(){
    queue<ll> q; q.push(0);
    while(!q.empty()){
        ll p = q.front(); q.pop();
        order.push_back(p);
```

```

for(ll c = 0; c<A; c++){
    ll u = trie[p][c];
    if(!u) continue;
    q.push(u);
    if(!p) continue;
    ll v = slink[p];
    while(v && !trie[v][c]) v = slink[v];
    slink[u] = trie[v][c];
}
}

void trav(string &s){
    ll u = 0;
    for(char c: s){
        c-='a';
        while(u && !trie[u][c]) u = slink[u];
        u = trie[u][c]; dp[u]++;
    }
    reverse(order.begin(), order.end());
    for(auto u: order){
        dp[slink[u]]+=dp[u];
    }
}

void solve(){
    ll n; cin>>n;
    string text; cin>>text;
    string s;
    for(ll i = 0; i<n; i++){
        cin>>s; addStr(s, i);
    }
    build(); trav(text);
    for(ll i = 0; i<n; i++){
        cout<<dp[pos[i]]<<endl;
    }
}

int32_t main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    ll tc = 1;
    cin>>tc;
    for(ll i = 1; i<=tc; i++){
        cout<<"Case "<<i<<": \n";
        initTrie();
        solve();
    }
}

```

8.2 LCS for 3 Strings

```

string a, b, c;
ll dp[55][55][55];
ll lcs(ll i, ll j, ll k) {
    if (i == a.size() || j == b.size() || k == c.size()) return 0;
    if (dp[i][j][k] != -1) return dp[i][j][k];
    if (a[i] == b[j] and a[i] == c[k]) return 1 + lcs(i + 1, j + 1, k + 1);
    ll ans = 0;
    ans = max(ans, lcs(i, j, k + 1));
    ans = max(ans, lcs(i, j + 1, k));
    ans = max(ans, lcs(i + 1, j, k));
    return dp[i][j][k] = ans;
}

```

8.3 Manacher Palindrome

```

// pal[1][i] = longest odd (half rounded down)
// palindrome around pos i and
// starts at i - pal[1][i] and ends at i +
// pal[1][i] pal[0][i] = half length of

```

```

// longest even palindrome around pos i, i + 1
// and starts at i - par[0][i] + 1
// and ends at i + pal[0][i]
const int N = 5e5 + 10;
int pal[2][N];
void manacher(string& s) {
    int n = s.size(), idx = 2;
    while (idx--) {
        for (int l = -1, r = -1, i = 0; i < n - 1; ++i) {
            if (i > r) l = r = i;
            else {
                int k = min(r - i, pal[idx][l + r - i]);
                l = i - k, r = i + k;
            }
            while (l - idx >= 0 and r + 1 < n and s[l - idx] == s[r + 1]) l--, r++;
            pal[idx][i] = r - i;
            // [l - 1 + idx : r] palindrome
        }
    }
}

```

8.4 String Hashing 2

```

const int N = 1000010, MOD = 1e9 + 7;
const ll P[] = {97, 1000003};
ll bigMod(ll a, ll e) {
    if (e == -1) e = MOD - 2;
    ll ret = 1;
    while (e) {
        if (e & 1) ret = ret * a % MOD;
        a = a * a % MOD, e >>= 1;
    }
    return ret;
}

ll pwr[2][N], inv[2][N];
void initHash() {
    for (int it = 0; it < 2; ++it) {
        pwr[it][0] = inv[it][0] = 1;
        ll INV_P = bigMod(P[it], -1);
        for (int i = 1; i < N; ++i) {
            pwr[it][i] = pwr[it][i - 1] * P[it] % MOD;
            inv[it][i] = inv[it][i - 1] * INV_P % MOD;
        }
    }
}

struct RangeHash {
    vector<ll> h[2], rev[2];
    RangeHash(const string &s, bool revFlag = 0) {
        for (int it = 0; it < 2; ++it) {
            h[it].resize(s.size() + 1, 0);
            for (int i = 0; i < s.size(); ++i) {
                h[it][i + 1] = (h[it][i] + pwr[it][i + 1] * (S[i] - 'a' + 1)) % MOD;
            }
            if (revFlag) {
                rev[it].resize(s.size() + 1, 0);
                for (int i = 0; i < s.size(); ++i) {
                    rev[it][i + 1] = (rev[it][i] + inv[it][i + 1] * (S[i] - 'a' + 1)) % MOD;
                }
            }
        }
    }

    inline ll get(int l, int r) {

```

```

    ll one = (h[0][r + 1] - h[0][l]) * inv[0][l + 1] % MOD;
    ll two = (h[1][r + 1] - h[1][l]) * inv[1][l + 1] % MOD;
    if (one < 0) one += MOD;
    if (two < 0) two += MOD;
    return one << 31 | two;
}

inline ll getReverse(int l, int r) {
    ll one = (rev[0][r + 1] - rev[0][l]) * pwr[0][r + 1] % MOD;
    ll two = (rev[1][r + 1] - rev[1][l]) * pwr[1][r + 1] % MOD;
    if (one < 0) one += MOD;
    if (two < 0) two += MOD;
    return one << 31 | two;
}
};

```

8.5 String Hashing

```

const int mod1 = 911382323, mod2 = 972663749, b1 = 137, b2 = 139;
const int mxN = 5000010;
int pow_b1[mxN], pow_b2[mxN], inv_b1[mxN], inv_b2[mxN];
int binExp(int base, int power, int mod) {
    int res = 1;
    while (power) {
        if (power & 1) res = (1LL * res * base) % mod;
        base = (1LL * base * base) % mod;
        power >>= 1;
    }
    return res;
}

void pre() {
    pow_b1[0] = pow_b2[0] = 1;
    for (int i = 1; i < mxN; i++) {
        pow_b1[i] = (1LL * pow_b1[i - 1] * b1) % mod1;
        pow_b2[i] = (1LL * pow_b2[i - 1] * b2) % mod2;
    }
    inv_b1[mxN - 1] = binExp(pow_b1[mxN - 1], mod1 - 2, mod1);
    inv_b2[mxN - 1] = binExp(pow_b2[mxN - 1], mod2 - 2, mod2);
    for (int i = mxN - 2; i >= 0; i--) {
        inv_b1[i] = (1LL * inv_b1[i + 1] * b1) % mod1;
        inv_b2[i] = (1LL * inv_b2[i + 1] * b2) % mod2;
    }
}

vector<pair<int, int>> getPref(string& s) {
    int qq = s.size();
    vector<pair<int, int>> hsh(qq);
    for (int i = 0; i < qq; i++) {
        if (i == 0) {
            hsh[i].first = (1LL * s[i] * pow_b1[i]) % mod1;
            hsh[i].second = (1LL * s[i] * pow_b2[i]) % mod2;
        }
    }
}

```

```

    } else {
        hsh[i].first =
            (hsh[i - 1].first + (1LL * s[i] *
            pow_b1[i]) % mod1) % mod1;
        hsh[i].second =
            (hsh[i - 1].second + (1LL * s[i] *
            pow_b2[i]) % mod2) % mod2;
    }
}
return hsh;
}
pair<int, int> getHash(string& str) {
    int hsh1 = 0, hsh2 = 0, sz = str.size();
    for (int i = 0; i < sz; ++i) {
        hsh1 = (hsh1 + 1LL * str[i] * pow_b1[i] %
            mod1) % mod1;
        hsh2 = (hsh2 + 1LL * str[i] * pow_b2[i] %
            mod2) % mod2;
    }
    return {hsh1, hsh2};
}
pair<int, int> getSub(int l, int r,
    vector<pair<int, int>>& v) {
    pair<int, int> q;
    if (l == 0) {
        q = {v[r].first, v[r].second};
    } else {
        int x = (1LL * ((v[r].first - v[l - 1].first
            + mod1) % mod1) * inv_b1[l]) %
            mod1;
        int y =
            (1LL * ((v[r].second - v[l - 1].second +
            mod2) % mod2) * inv_b2[l]) %
            mod2;
        q = {x, y};
    }
    return q;
}

```

8.6 Suffix Array

```

// fahimcp495
array<vector<int>, 2> get_sa(string& s, int lim
    = 128) { // for integer, just change string
    to vector<int> and minimum value of vector
    must be >= 1
    int n = s.size() + 1, k = 0, a, b;
    vector<int> x(begin(s), end(s) + 1), y(n),
        sa(n), lcp(n), ws(max(n, lim)), rank(n);
    x.back() = 0;
    iota(begin(sa), end(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j *
        2), lim = p) {
        p = j, iota(begin(y), end(y), n - j);
        for (int i = 0; i < n; ++i)
            if (sa[i] >= j) y[p++] = sa[i] - j;
        fill(begin(ws), end(ws), 0);
        for (int i = 0; i < n; ++i) ws[x[i]]++;
        for (int i = 1; i < lim; ++i) ws[i] += ws[i
            - 1];
        for (int i = n; i--;) sa[--ws[x[y[i]]]] =
            y[i];
        swap(x, y), p = 1, x[sa[0]] = 0;
    }
}

```

```

    for (int i = 1; i < n; ++i) a = sa[i - 1], b
        = sa[i], x[b] = (y[a] == y[b] && y[a +
        1] == y[b + 1]) ? p - 1 : p++;
    }
    for (int i = 1; i < n; ++i) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] =
        k)
        for (k && k--, j = sa[rank[i] - 1]; s[i + k]
            == s[j + k]; k++);
    sa.erase(sa.begin()), lcp.erase(lcp.begin());
    return {sa, lcp};
}

```

8.7 Suffix Automata

```

const int N = 2e5 + 10; // max string size
int len[N], lnk[N]{-1}, last, sz = 1;
unordered_map<char, int> to[N];
void add(char c) {
    int cur = sz++;
    len[cur] = len[last] + 1;
    int u = last;
    while (u != -1 and !to[u].count(c)) {
        to[u][c] = cur;
        u = lnk[u];
    }
    if (u == -1) {
        lnk[cur] = 0;
    } else {
        int v = to[u][c];
        if (len[v] == len[u] + 1) {
            lnk[cur] = v;
        } else {
            int w = sz++;
            len[w] = len[u] + 1, lnk[w] = lnk[v],
            to[w] = to[v];
            while (u != -1 and to[u][c] == v) {
                to[u][c] = w;
                u = lnk[u];
            }
            lnk[cur] = lnk[v] = w;
        }
    }
    last = cur;
}

```

8.8 Suffix Automation

```

int len[N], lnk[N]{-1}, last, sz = 1;
unordered_map<char, int> to[N];
void init() {
    while (sz) {
        sz--;
        to[sz].clear();
    }
    last = 0, sz = 1;
}
void add(char c) {
    int cur = sz++;
    int u = last;
    len[cur] = len[last] + 1;
    while (u != -1 and !to[u].count(c)) {
        to[u][c] = cur;
        u = lnk[u];
    }
    if (u == -1) {
        lnk[cur] = 0;
    } else {
        int v = to[u][c];
    }
}

```

```

    if (len[v] == len[u] + 1) {
        lnk[cur] = v;
    } else {
        int w = sz++;
        len[w] = len[u] + 1, lnk[w] = lnk[v],
        to[w] = to[v];
        while (u != -1 and to[u][c] == v) {
            to[u][c] = w;
            u = lnk[u];
        }
        lnk[cur] = lnk[v] = w;
    }
    last = cur;
}

```

8.9 Trie

```

const ll N = 1e6 + 5, A = 26;
ll trie[N][A], cnt[N], tot = 1, root = 1;
void initTrie() {
    cnt[tot] = 0;
    root = 1;
}
void addStr(string& s) {
    ll u = 1;
    for (auto it : s) {
        ll n = it - 'a';
        if (trie[u][n] == 0) {
            trie[u][n] = ++tot;
        }
        u = trie[u][n];
        cnt[u]++;
    }
}
ll wordCount(string& s) {
    ll u = 1;
    for (auto it : s) {
        int n = it - 'a';
        if (trie[u][n] == 0) return 0;
        u = trie[u][n];
    }
    return cnt[u];
}

```

9 Tree

9.1 Centroid Decomposition

```

const int N = 2e5 + 5;
int n, k, sz[N], centered[N], ans = 0;
vector<int> adj[N];
void dfs_sz(int u, int p) {
    sz[u] = 1;
    for (auto v : adj[u]) {
        if (v != p && !centered[v]) {
            dfs_sz(v, u);
            sz[u] += sz[v];
        }
    }
}
int get_cen(int u, int p, int n) {
    for (auto v : adj[u]) {
        if (v != p && !centered[v] && sz[v] > n/2) {
            return get_cen(v, u, n);
        }
    }
    return u;
}
int t, tin[N], tout[N], nodes[N], dis[N];
void dfs(int u, int p) {
    nodes[t] = u;
    tin[u] = t++;
}

```

```

    for(auto v: adj[u]){
        if(v!=p && !centered[v]){
            dis[v] = dis[u]+1; dfs(v, u);
        }
    } tout[u] = t-1;
}
void go(int u){
    dfs_sz(u, u);
    int c = get_cen(u, u, sz[u]);
    centered[c] = 1; sz[c] = sz[u];
    t = 0; dis[c] = 0; dfs(c, c);
    int cnt[t+1];
    for(auto v: adj[c]){
        if(centered[v]) continue;
        for(int i = tin[v]; i<=tout[v]; ++i){
            int w = nodes[i];
            if(k-dis[w]>=0 && k-dis[w]<t){
                ans+=cnt[k-dis[w]];
            }
        }
        for(int i = tin[v]; i<=tout[v]; ++i){
            int w = nodes[i]; cnt[dis[w]]++;
        }
    }
    for(auto v: adj[c]){
        if(!centered[v]){ go(v); }
    }
}
void solve() {
    cin>>n>>k;
    for(ll i = 1; i<n; i++){
        ll u, v; cin>>u>>v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    } go(1);
    cout<<ans<<endl;
}

```

9.2 DSUOnTrees

```

int n, color[MX], ans[MX];
vector<int> g[MX];
set<int> bucket[MX];
int merge(int a, int b) {
    if (bucket[a].size() < bucket[b].size())
        swap(a, b);
    bucket[a].insert(bucket[b].begin(),
        bucket[b].end());
    bucket[b].clear();
    return a;
}
int dfs(int u, int p = -1) {
    int cur = u;
    for (int v : g[u])
        if (v != p)
            cur = merge(cur, dfs(v, u));
    ans[u] = (int)bucket[cur].size();
    return cur;
}
void solve() {
    cin >> n;
    for (int i = 0; i < n; ++i) {
        cin >> color[i];
        bucket[i].insert(color[i]);
    }
    // graph input
    dfs(0);
    // print output
}

```

```

}
}
9.3 LCA using binary Lifting
int n, l;
vector<vector<int>> adj;
int timer;
vector<int> tin, tout;
vector<vector<int>> up;
void dfs(int v, int p) {
    tin[v] = ++timer;
    up[v][0] = p;
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];
    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }
    tout[v] = ++timer;
}
bool is_ancestor(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}
int lca(int u, int v) {
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}
void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
    dfs(root, root);
}

```

9.4 LCA

```

const int N = 1e5 + 5;
vector<int> g[N], parent(N), depth(N, 0);
void dfs(int vertex, int par = -1) {
    parent[vertex] = par;
    for (auto child : g[vertex]) {
        if (child != par) {
            depth[child] = depth[vertex] + 1;
            dfs(child, vertex);
        }
    }
}
int lca(int x, int y) {
    int diff = min(depth[x], depth[y]);
    while (depth[x] > diff) x = parent[x];
    while (depth[y] > diff) y = parent[y];
    while (x != y) { x = parent[x]; y = parent[y]; }
    return x;
}

```

10 Notes

10.1 Geometry

10.1.1 Triangles

Circumradius: $R = \frac{abc}{4A}$, Inradius: $r = \frac{A}{s}$

The area of a triangle using two sides and the included angle can be given as:

$$A = \frac{1}{2}ab \sin \angle C$$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a =$

$$\sqrt{bc \left[1 - \left(\frac{a}{b+c} \right)^2 \right]}$$

$$\text{Law of tangents: } \frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$$

10.1.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

10.1.3 Spherical coordinates

$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \text{atan2}(y, x) \end{aligned}$$

10.1.4 Pick's Theorem:

Given a lattice polygon with non-zero area, we define: S as the area of the polygon, I as the number of integer-coordinate points strictly inside the polygon, B as the number of integer-coordinate points on the boundary of the polygon. Then, Pick's Theorem states:

$$S = I + \frac{B}{2} - 1$$

The number of lattice points on segments (x_1, y_1) to (x_2, y_2) is: $\gcd(\text{abs}(x_2 - x_1), \text{abs}(y_2 - y_1)) + 1$

10.1.5 Polygon

For a regular polygon with n sides and side length a , the circumradius R is given by:

$$R = \frac{a}{2 \sin(\frac{\pi}{n})}$$

10.1.6 Area of a Circular Segment

The area of a circular segment, which is the region enclosed by a chord and the corresponding arc, can be calculated using the formula:

$$A = \frac{R^2}{2} (\theta - \sin \theta)$$

where: R is the radius of the circle, θ is the central angle subtended by the chord, in radians.

10.2 Binomial Coefficient

- Factoring in: $\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}$
- Sum over k : $\sum_{k=0}^n \binom{n}{k} = 2^n$
- Alternating sum: $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$
- Even and odd sum: $\sum_{k=0}^n \binom{n}{2k} = \sum_{k=0}^n \binom{n}{2k+1} = 2^{n-1}$
- The Hockey Stick Identity
 - (Left to right) Sum over n and k : $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m-1}{m}$
 - (Right to left) Sum over n : $\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$
- Sum of the squares: $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$
- Weighted sum: $\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$
- Connection with the fibonacci numbers: $\sum_{k=0}^n \binom{n-k}{k} = F_{n+1}$
- Vandermonde's Identity: $\sum_{i=0}^k \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}$
- If $f(n, k) = C(n, 0) + C(n, 1) + \dots + C(n, k)$, Then $f(n+1, k) = 2 * f(n, k) - C(n, k)$ [For multiple $f(n, k)$ queries, use Mo's algo]

Lucas Theorem

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$$

- $\binom{m}{n}$ is divisible by p if and only if at least one of the base- p digits of n is greater than the corresponding base- p digit of m .
- The number of entries in the n th row of Pascal's triangle that are not divisible by $p = \prod_{i=0}^k (n_i + 1)$
- All entries in the $(p^k - 1)th$ row are not divisible by p .
- $\binom{n}{m} \equiv \lfloor \frac{n}{p} \rfloor! \pmod{p}$

10.3 Fibonacci Number

- $k = A - B, F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1}$
- $\sum_{i=0}^n F_i^2 = F_{n+1} F_n$
- $\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^{n-1} F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^n F_i F_{i-1} = F_{n+1}^2 - (-1)^n$
- $\gcd(F_m, F_n) = F_{\gcd(m, n)}$
- $\sum_{0 \leq k \leq n} \binom{n-k}{k} = F_{n+1}$
- $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = \gcd(F_{n+1}, F_{n+2}) = 1$

10.4 Sums

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \left[(n+1)^{m+1} - 1 - \sum_{i=1}^n ((i+1)^{m+1} - i^{m+1} - (m+1)i^m) \right]$$

$$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k}$$

$$\sum_{k=0}^n kx^k = (x - (n+1)x^{n+1} + nx^{n+2}) / (x-1)^2$$

10.5 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

$$(x+a)^{-n} = \sum_{k=0}^{\infty} (-1)^k \binom{n+k-1}{k} x^k a^{-n-k}$$

Generating Function

$$1/(1-x) = 1 + x + x^2 + x^3 + \dots$$

$$1/(1-ax) = 1 + ax + (ax)^2 + (ax)^3 + \dots$$

$$1/(1-x)^2 = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$1/(1-x)^3 = C(2,2) + C(3,2)x + C(4,2)x^2 + C(5,2)x^3 + \dots$$

$$1/(1-ax)^{k+1} = 1 + C(1+k,k)(ax) + C(2+k,k)(ax)^2 + C(3+k,k)(ax)^3 + \dots$$

$$x(x+1)(1-x)^{-3} = 1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + \dots$$

$$e^x = 1 + x + (x^2)/2! + (x^3)/3! + (x^4)/4! + \dots$$

10.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

10.7 Number Theory

- HCN: 1e6(240), 1e9(1344), 1e12(6720), 1e14(17280), 1e15(26880), 1e16(41472)
- $\gcd(a, b, c, d, \dots) = \gcd(a, b-a, c-b, d-c, \dots)$
- $\gcd(a+k, b+k, c+k, d+k, \dots) = \gcd(a+k, b-a, c-b, d-c, \dots)$
- Primitive root exists iff $n = 1, 2, 4, p^k, 2 \times p^k$, where p is an odd prime.
- If primitive root exists, there are $\phi(\phi(n))$ primitive roots of n .
- The numbers from 1 to n have in total $O(n \log \log n)$ unique prime factors.
- $x \equiv r_1 \pmod{m_1}$ and $x \equiv r_2 \pmod{m_2}$ has a solution iff $\gcd(m_1, m_2) | (r_1 - r_2)$ Solution of $x^2 \equiv a \pmod{p}$
- $ca \equiv cb \pmod{m} \iff a \equiv b \pmod{\frac{n}{\gcd(n, c)}}$
- $ax \equiv b \pmod{m}$ has a solution $\iff \gcd(a, m) | b$
- If $ax \equiv b \pmod{m}$ has a solution, then it has $\frac{m}{\gcd(a, m)}$ solutions and they are separated by $\frac{m}{\gcd(a, m)}$
- $ax \equiv 1 \pmod{m}$ has a solution or a is invertible $\pmod{m} \iff \gcd(a, m) = 1$
- $x^2 \equiv 1 \pmod{p}$ then $x \equiv \pm 1 \pmod{p}$
- There are $\frac{p-1}{2}$ has no solution.
- There are $\frac{p-1}{2}$ has exactly two solutions.
- When $p \% 4 = 3$, $x \equiv \pm a^{\frac{p+1}{4}}$
- When $p \% 8 = 5$, $x \equiv a^{\frac{p+3}{8}}$ or $x \equiv 2^{\frac{p-1}{4}} a^{\frac{p+3}{8}}$

10.7.1 Primes

$p = 962592769$ is such that $2^{21} | p-1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

10.7.2 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

10.7.3 Perfect numbers

$n > 1$ is called perfect if it equals sum of its proper divisors and 1. Even n is perfect iff $n = 2^{p-1}(2^p - 1)$ and $2^p - 1$ is prime (Mersenne's). No odd perfect numbers are yet found.

10.7.4 Carmichael numbers

A positive composite n is a Carmichael number ($a^{n-1} \equiv 1 \pmod{n}$ for all $a: \gcd(a, n) = 1$), iff n is square-free, and for all prime divisors p of n , $p-1$ divides $n-1$.

10.7.5 Totient

- If p is a prime $(p^k) = p^k - p^{k-1}$
- If a, b are relatively prime, $\phi(ab) = \phi(a)\phi(b)$
- $\phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})(1 - \frac{1}{p_3}) \dots (1 - \frac{1}{p_k})$
- Sum of coprime to $n = n * \frac{\phi(n)}{2}$
- If $n = 2^k, \phi(n) = 2^{k-1} = \frac{n}{2}$
- For $a, b, \phi(ab) = \phi(a)\phi(b) \frac{d}{\phi(d)}$
- $\phi(ip) = p\phi(i)$ whenever p is a prime and it divides i
- The number of $a(1 \leq a \leq N)$ such that $\gcd(a, N) = d$ is $\phi(\frac{N}{d})$
- If $n > 2, \phi(n)$ is always even
- Sum of $\gcd, \sum_{i=1}^n \gcd(i, n) = \sum_{d|n} d\phi(\frac{n}{d})$
- Sum of $\text{lcm}, \sum_{i=1}^n \text{lcm}(i, n) = \frac{n}{2} (\sum_{d|n} (d\phi(d)) + 1)$
- $\phi(1) = 1$ and $\phi(2) = 1$ which two are only odd ϕ
- $\phi(3) = 2$ and $\phi(4) = 2$ and $\phi(6) = 2$ which three are only prime ϕ
- Find minimum n such that $\frac{\phi(n)}{n}$ is maximum- Multiple of small primes- $2 * 3 * 5 * 7 * 11 * 13 * \dots$

10.7.6 Mobius function

$\mu(1) = 1$. $\mu(n) = 0$, if n is not squarefree. $\mu(n) = (-1)^s$, if n is the product of s distinct primes. Let f, F be functions on positive integers. If for all $n \in \mathbb{N}$, $F(n) = \sum_{d|n} f(d)$, then $f(n) = \sum_{d|n} \mu(d)F(\frac{n}{d})$, and vice versa. $\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$. $\sum_{d|n} \mu(d) = 1$. If f is multiplicative, then $\sum_{d|n} \mu(d)f(d) = \prod_{p|n} (1 - f(p))$, $\sum_{d|n} \mu(d)^2 f(d) = \prod_{p|n} (1 + f(p))$.

$$\sum_{i=1}^n \sum_{j=1}^n [\gcd(i, j) = 1] = \sum_{k=1}^n \mu(k) \lfloor \frac{n}{k} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n k \sum_{l=1}^{\lfloor \frac{n}{k} \rfloor} \mu(l) \lfloor \frac{n}{kl} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n \left(\frac{\lfloor \frac{n}{k} \rfloor (1 + \lfloor \frac{n}{k} \rfloor)}{2} \right)^2 \sum_{d|k} \mu(d)kd$$

10.7.7 Legendre symbol

If p is an odd prime, $a \in \mathbb{Z}$, then $\left(\frac{a}{p}\right)$ equals 0, if $p|a$; 1 if a is a quadratic residue modulo p ; and -1 otherwise. Euler's criterion: $\left(\frac{a}{p}\right) = a^{\left(\frac{p-1}{2}\right)} \pmod{p}$.

10.7.8 Jacobi symbol

If $n = p_1^{a_1} \dots p_k^{a_k}$ is odd, then $\left(\frac{a}{n}\right) = \prod_{i=1}^k \left(\frac{a}{p_i}\right)^{a_i}$.

10.7.9 Primitive roots

If the order of g modulo m ($\min n > 0: g^n \equiv 1 \pmod{m}$) is $\phi(m)$, then g is called a primitive root. If Z_m has a primitive root, then it has $\phi(\phi(m))$ distinct primitive roots. Z_m has a primitive root iff m is one of $2, 4, p^k, 2p^k$, where p is an odd prime. If Z_m has a primitive root g , then for all a coprime to m , there exists unique integer $i = \text{ind}_g(a)$ modulo $\phi(m)$, such that $g^i \equiv a \pmod{m}$. $\text{ind}_g(a)$ has logarithm-like properties: $\text{ind}(1) = 0$, $\text{ind}(ab) = \text{ind}(a) + \text{ind}(b)$.

If p is prime and a is not divisible by p , then congruence $x^n \equiv a \pmod{p}$ has $\gcd(n, p-1)$ solutions if $a^{(p-1)/\gcd(n, p-1)} \equiv 1 \pmod{p}$, and no solutions otherwise. (Proof sketch: let g be a primitive root, and $g^i \equiv a \pmod{p}$, $g^u \equiv x \pmod{p}$. $x^n \equiv a \pmod{p}$ iff $g^{nu} \equiv g^i \pmod{p}$ iff $nu \equiv i \pmod{p}$.)

10.7.10 Discrete logarithm problem

Find x from $a^x \equiv b \pmod{m}$. Can be solved in $O(\sqrt{m})$ time and space with a meet-in-the-middle trick. Let $n = \lfloor \sqrt{m} \rfloor$, and $x = ny - z$. Equation becomes $a^{ny} \equiv ba^z \pmod{m}$. Precompute all values that the RHS can take for $z = 0, 1, \dots, n-1$, and brute force y on the LHS, each time checking whether there's a corresponding value for RHS.

10.7.11 Pythagorean triples

Integer solutions of $x^2 + y^2 = z^2$ All relatively prime triples are given by: $x = 2mn, y = m^2 - n^2, z = m^2 + n^2$ where $m > n, \gcd(m, n) = 1$ and $m \not\equiv n \pmod{2}$. All other triples are multiples of these. Equation $x^2 + y^2 = 2z^2$ is equivalent to $(\frac{x+y}{2})^2 + (\frac{x-y}{2})^2 = z^2$.

10.7.12 Postage stamps/McNuggets problem

Let a, b be relatively-prime integers. There are exactly $\frac{1}{2}(a-1)(b-1)$ numbers *not* of form $ax + by$ ($x, y \geq 0$), and the largest is $(a-1)(b-1) - 1 = ab - a - b$.

10.7.13 Fermat's two-squares theorem

Odd prime p can be represented as a sum of two squares iff $p \equiv 1 \pmod{4}$. A product of two sums of two squares is a sum of two squares. Thus, n is a sum of two squares iff every prime of form $p = 4k + 3$ occurs an even number of times in n 's factorization.

10.8 Permutations

10.8.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$\frac{n!}{n}$		1	11	12	13	14	15	16	17	
$\frac{n!}{n!}$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$\frac{n!}{n!}$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

10.8.2 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left(\sum_{n \in S} \frac{x^n}{n} \right)$$

10.8.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

10.8.4 Burnside's lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n , we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k)$$

10.9 Partitions and subsets

10.9.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$\frac{n}{p(n)}$	0	1	2	3	4	5	6	7	8	9	20	50	100
	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

10.9.2 Partition Number

- Time Complexity: $O(n\sqrt{n})$

```
for (int i = 1; i <= n; ++i) {
    pent[2 * i - 1] = i * (3 * i - 1) / 2;
    pent[2 * i] = i * (3 * i + 1) / 2;
}
p[0] = 1;
for (int i = 1; i <= n; ++i) {
    p[i] = 0;
    for (int j = 1, k = 0; pent[j] <= i; ++j) {
        if (k < 2) p[i] = add(p[i], p[i - pent[j]]);
        else p[i] = sub(p[i], p[i - pent[j]]); ++k, k &
    }
}
```

- The number of partitions of a positive integer n into exactly k parts equals the number of partitions of n whose largest part equals k

$$p_k(n) = p_k(n-k) + p_{k-1}(n-1)$$

10.9.3 2nd Kaplansky's Lemma

The number of ways of selecting k objects, no two consecutive, from n labelled objects arrayed in a circle is $\frac{n}{k} \binom{n-k-1}{k-1} = \frac{n-k}{n-k} \binom{n-k}{k}$

10.9.4 Distinct Objects into Distinct Bins

- n distinct objects into r distinct bins $= r^n$
 - Among n distinct objects, exactly k of them into r distinct bins $= \binom{n}{k} r^k$
 - n distinct objects into r distinct bins such that each bin contains at least one object $= \sum_{i=0}^r (-1)^i \binom{r}{i} (r-i)^n$

10.10 Coloring

The number of labeled undirected graphs with n vertices, $G_n = 2^{\binom{n}{2}}$

The number of labeled directed graphs with n vertices, $G_n = 2^{n(n-1)}$

The number of connected labeled undirected graphs with n vertices, $C_n = 2^{\binom{n}{2}} - \frac{1}{n} \sum_{k=1}^{n-1} k \binom{n}{k} 2^{\binom{n-k}{2}} C_k = 2^{\binom{n}{2}} - \sum_{k=1}^{n-1} \binom{n-1}{k-1} 2^{\binom{n-k}{2}} C_k$

The number of k -connected labeled undirected graphs with n vertices, $D[n][k] = \sum_{s=1}^n \binom{n-1}{s-1} C_s D[n-s][k-1]$

Cayley's formula: the number of trees on n labeled vertices = the number of spanning trees of a complete graph with n labeled vertices = n^{n-2}

Number of ways to color a graph using k color such that no two adjacent nodes have same color

Complete graph = $k(k-1)(k-2)\dots(k-n+1)$

Tree = $k(k-1)^{n-1}$

Cycle = $(k-1)^n + (-1)^n(k-1)$

Number of trees with n labeled nodes: n^{n-2}

10.11 General purpose numbers**10.11.1 Eulerian numbers**

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j :s s.t. $\pi(j) \geq j$, k j :s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

10.11.2 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

10.11.3 Bernoulli numbers

$\sum_{j=0}^m \binom{m+1}{j} B_j = 0$. $B_0 = 1$, $B_1 = -\frac{1}{2}$. $B_n = 0$, for all odd $n \neq 1$.

10.11.4 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, C_{n+1} = \frac{2(2n+1)}{n+2} C_n, C_{n+1} = \sum C_i C_{n-i}$$

- $C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$
- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parentheses, correctly nested.
- binary trees with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.

- permutations of $[n]$ with no 3-term increasing subseq.
- Find the count of balanced parentheses sequences consisting of $n+k$ pairs of parentheses where the first k symbols are open brackets.

$$C_n^{(k)} = \frac{k+1}{n+k+1} \binom{2n+k}{n}$$

- Recursive formula of Catalan Numbers:

$$C_n^{(k)} = \frac{(2n+k-1) \cdot (2n+k)}{n \cdot (n+k+1)} C_{n-1}^{(k)}$$

10.11.5 Lucas Number

Number of edge cover of a cycle graph C_n is L_n

$$L(n) = L(n-1) + L(n-2); L(0) = 2, L(1) = 1$$

10.12 Ballot Theorem

Suppose that in an election, candidate A receives a votes and candidate B receives b votes, where $a > b$ for some positive integer k . Compute the number of ways the ballots can be ordered so that A maintains more than k times as many votes as B throughout the counting of the ballots.

The solution to the ballot problem is $\frac{a-kb}{a+b} \times C(a+b, a)$

10.13 Classical Problem

$F(n, k)$ = number of ways to color n objects using exactly k colors

Let $G(n, k)$ be the number of ways to color n objects using no more than k colors.

Then, $F(n, k) = G(n, k) - C(k, 1) * G(n, k-1) + C(k, 2) * G(n, k-2) - C(k, 3) * G(n, k-3) \dots$

Determining $G(n, k)$:

Suppose, we are given a $1 * n$ grid. Any two adjacent cells can not have same color. Then, $G(n, k) = k * ((k-1)^{n-1})$

If no such condition on adjacent cells. Then, $G(n, k) = k^n$

10.14 Matching Formula**10.14.1 Normal Graph**

$MM + MEC = n$ (excluding vertex), $IS + VC = G$, $MIS + MVC = G$

10.14.2 Bipartite Graph

$MIS = n - MBM$, $MVC = MBM$, $MEC = n - MBM$

10.15 Inequalities**10.15.1 Titu's Lemma**

For positive reals a_1, a_2, \dots, a_n and b_1, b_2, \dots, b_n ,

$$\frac{a_1^2}{b_1} + \frac{a_2^2}{b_2} + \dots + \frac{a_n^2}{b_n} \geq \frac{a_1 + a_2 + \dots + a_n}{b_1 + b_2 + \dots + b_n}$$

Equality holds if and only if $a_i = kb_i$ for a non-zero real constant k .

10.16 Games**10.16.1 Grundy numbers**

For a two-player, normal-play (last to move wins) game on a graph (V, E) : $G(x) = \text{mex}(\{G(y) : (x, y) \in E\})$, where $\text{mex}(S) = \min\{n \geq 0 : n \notin S\}$. x is losing iff $G(x) = 0$.

10.16.2 Sums of games

- *Player chooses a game and makes a move in it* Grundy number of a position is xor of grundy numbers of positions in summed games.

- *Player chooses a non-empty subset of games (possibly, all) and makes moves in all of them* A position is losing iff each game is in a losing position.

- *Player chooses a proper subset of games (not empty and not all), and makes moves in all chosen ones.* A position is losing iff grundy numbers of all games are equal.

- *Player must move in all games, and loses if can't move in some game* A position is losing if any of the games is in a losing position.

10.16.3 Misère Nim

A position with pile sizes $a_1, a_2, \dots, a_n \geq 1$, not all equal to 1, is losing iff $a_1 \oplus a_2 \oplus \dots \oplus a_n = 0$ (like in normal nim.) A position with n piles of size 1 is losing iff n is odd.

10.17 Tree Hashing

$f(u) = sz[u] * \sum_{i=0} f(v) * p^i$; $f(v)$ are sorted $f(child) = 1$

10.18 Permutation

To maximize the sum of adjacent differences of a permutation, it is necessary and sufficient to place the smallest half numbers in odd position and the greatest half numbers in even position. Or, vice versa.

10.19 String

- If the sum of length of some strings is N , there can be at most \sqrt{N} distinct length.

- A Text can have at most $O(N \times \sqrt{N})$ distinct substrings that match with given patterns where the sum of the length of the given patterns is N .

- Period = $n \% (n - \text{pi.back()} == 0)? n - \text{pi.back}(): n$

- The first (*period*) cyclic rotations of a string are distinct. Further cyclic rotations repeat the previous strings.

- S is a palindrome if and only if it's period is a palindrome.

- If S and T are palindromes, then the periods of $S \ T$ are same if and only if $S + T$ is a palindrome.

10.20 Bit

- $(a \text{ xor } b)$ and $(a + b)$ has the same parity
- $(a + b) = (a \text{ xor } b) + 2(a \ \& \ b)$
- $\text{gcd}(a, b) \leq a - b \leq \text{xor}(a, b)$

10.21 Convolution

- Hamming Distance: Replace 0 with -1 - SQRT Decomposition: Find block size, $B = \sqrt{8 * n}$