



Daffodil International University

# DIU\_DividedByZero

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Team Reference Document

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## 1 Code

1.1

CP\_Ubuntu

```
"cmd": ["ulimit -s 268435456; g++ -std=c++20
$file name -o $file_base_name && timeout
4s./$file_base_name < inputf.in >
outputf.in"],
"selector": "source.cpp",
"shell": true,
"working_dir": "$file_path"
```

1.2

CP\_Windows

```
"cmd": ["g++.exe", "-std=c++20", "${file}",
"-o", "${file_base_name}.exe", "&&", "${f}
ile_base_name}.exe<inputf.in>outputf.in"],
"selector": "source.cpp",
"shell": true,
"working_dir": "$file_path"
```

## 1.3 StressTesting(check.sh)

```
// chmod u+x check.sh
// ./check.sh
set -e
g++ gen.cpp -o gen
g++ code.cpp -o code
g++ brute.cpp -o brute
for ((i = 1; ; ++i)); do
echo "Passed on TestCase: " $i
./gen $i > in
./code < in > out1
./brute < in > out2
diff -Z out1 out2 || break
done
echo -e "WA on the following test:"
cat in
echo -e "\nExpected:"
cat out2
echo -e "\nFound:"
cat out1
```

## 1.4 StressTesting(gen.cpp)

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
mt19937_64 rng(chrono::steady_clock::now().time_
since_epoch().count());
inline ll gen_random(ll l, ll r) {
return uniform_int_distribution<ll>(l, r)(rng);
}
inline double gen_random_real(double l, double
r) {
return uniform_real_distribution<double>(l,
r)(rng);
}
int main(int argc, char* args[]) {
int _ = atoi(args[1]);
rng.seed(_);
int n = gen_random(1, 5);
vector<int> per;
for (int i = 0; i < n; ++i) {
per.push_back(i + 1);
}
shuffle(per.begin(), per.end(), rng);
return 0;
}
```

## 1.5 Aho Corasick

```
//number of occurrence of word in a text
const ll N = 1e6+10, A = 26;
ll trie[N][A], pos[N], slink[N], dp[N], tot = 1;
vector<int> order;
void initTrie(){
    order.clear();
    while(tot--){
        memset(trie[tot], 0, sizeof(trie[tot]));
    }
    memset(pos, 0, sizeof(pos));
    memset(slink, 0, sizeof(slink));
    memset(dp, 0, sizeof(dp)); tot = 1;
}
void addStr(string &s, int ind){
    ll u = 0;
    for(auto it: s){
        ll n = it-'a';
        if(trie[u][n]==0) trie[u][n] = tot++;
        u = trie[u][n];
    } pos[ind] = u;
}
void build(){
    queue<ll> q; q.push(0);
    while(!q.empty()){
        ll p = q.front(); q.pop();
        order.push_back(p);
        for(ll c=0; c<A; c++){
            ll u = trie[p][c];
            if(!u) continue;
            q.push(u);
            if(!p) continue;
            ll v = slink[p];
            while(v && !trie[v][c]) v = slink[v];
            slink[u] = trie[v][c];
        }
    }
}
void trav(string &s){
    ll u = 0;
    for(char c: s){
        while(u && !trie[u][c]) u = slink[u];
        u = trie[u][c]; dp[u]++;
    }
    reverse(order.begin(), order.end());
    for(auto u: order){
        dp[slink[u]]+=dp[u];
    }
}
void solve(){
    ll n; cin>>n;
    string text; cin>>text;
    string s;
    for(ll i=0; i<n; i++){
        cin>>s; addStr(s, i);
    }
    build(); trav(text);
    for(ll i=0; i<n; i++){
        cout<<dp[pos[i]]<<endl;
    }
}
int32_t main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    ll tc = 1;
```

```
cin>>tc;
for(ll i=1; i<=tc; i++){
    cout<<"Case "<<i<<": \n";
    initTrie();
    solve();
}
```

## 1.6 Articulation Point

```
int n; // number of nodes
vector<vector<int>> lst; // adjacency list of
- graph
vector<bool> vis;
vector<int> tin, low;
int timer;
void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
    int children = 0;
    for(int v : lst[u]) {
        if(v == p) continue;
        if(vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if(low[v] >= tin[u] && p != -1){
                IS_CUTPOINT(u);
            }
            ++children;
        }
    }
    // if no vertex below v can reach u or higher
    - removing u disconnects that subtree
    if(p == -1 && children > 1){
        IS_CUTPOINT(u);
    }
}
void find_cutpoints() {
    timer = 0;
    vis.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for(int i=0; i<n; ++i) {
        if(!vis[i]){
            dfs(i);
        }
    }
}
```

## 1.7 Binary Lifting using LCA

```
int n, l;
vector<vector<int>> adj;
int timer;
vector<int> tin, tout;
vector<vector<int>> up;
void dfs(int v, int p) {
    tin[v] = ++timer;
    up[v][0] = p;
    for(int i=1; i<=l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];
    for(int u : adj[v]) {
        if(u != p)
            dfs(u, v);
    }
}
```

```
tout[v] = ++timer;
}
bool is_ancestor(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}
int lca(int u, int v) {
    if(is_ancestor(u, v))
        return u;
    if(is_ancestor(v, u))
        return v;
    for(int i=l; i>=0; --i) {
        if(!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}
void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l+1));
    dfs(root, root);
}
```

## 1.8 BridgeFinding

```
const int MX = 1e5 + 10;
int n, m, timer = 0;
vector<int> adj[MX];
vector<int> tin(MX, -1), low(MX, -1);
vector<bool> vis(MX, false);
void is_bridge(int u, int v) {
    // do something with the edge
}
void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
    for(int v : adj[u]) {
        if(v == p) continue;
        if(vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if(low[v] > tin[u]) {
                is_bridge(u, v);
            }
        }
    }
}
```

## 1.9 Centroid Decomposition

```
const int N = 2e5+5;
int n, k, sz[N], centered[N], ans = 0;
vector<int> adj[N];
void dfs_sz(int u, int p) {
    sz[u] = 1;
    for(auto v: adj[u]) {
        if(v != p && !centered[v]) {
            dfs_sz(v, u); sz[u] += sz[v];
        }
    }
}
```

```

int get_cen(int u, int p, int n) {
    for (auto v: adj[u]) {
        if (v != p && !centered[v] && sz[v] > n/2) {
            return get_cen(v, u, n);
        }
    }
    return u;
}

int t, tin[N], tout[N], nodes[N], dis[N];
void dfs(int u, int p) {
    nodes[t] = u;
    tin[u] = t++;
    for (auto v: adj[u]) {
        if (v != p && !centered[v]) {
            dis[v] = dis[u] + 1;
            dfs(v, u);
        }
    }
    tout[u] = t - 1;
}

void go(int u) {
    dfs_sz(u, u);
    int c = get_cen(u, u, sz[u]);
    centered[c] = 1;
    sz[c] = sz[u];
    t = 0;
    dis[c] = 0;
    dfs(c, c);
    int cnt[t][1];
    for (auto v: adj[c]) {
        if (centered[v]) continue;
        for (int i = tin[v]; i <= tout[v]; ++i) {
            int w = nodes[i];
            if (k - dis[w] >= 0 && k - dis[w] < t) {
                ans += cnt[k - dis[w]];
            }
        }
        for (int i = tin[v]; i <= tout[v]; ++i) {
            int w = nodes[i];
            cnt[dis[w]]++;
        }
    }
    for (auto v: adj[c]) {
        if (!centered[v]) go(v);
    }
}

void solve() {
    cin >> n >> k;
    for (ll i = 1; i < n; i++) {
        ll u, v;
        cin >> u >> v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    go(1);
    cout << ans << endl;
}

```

#### 1.10 Coin Change (Number of Ways)

```

const int mod = 1e9 + 7;
void solve() {
    int n, k;
    cin >> n >> k;
    vector<int> coin(n);
    for (int i = 0; i < n; i++) {
        cin >> coin[i];
    }
    vector<int> dp(k + 1, 0);
    dp[0] = 1;
    for (int i = 1; i <= k; i++) {
        for (int j = 0; j < n; j++) {
            if (i - coin[j] >= 0) {
                dp[i] = (dp[i] + dp[i - coin[j]]) % mod;
            }
        }
    }
    cout << dp[k] << endl;
}

```

#### 1.11 DAG Cycle Detection

```

const int MX = 1e5 + 10;
bool vis[MX], pathVis[MX];
vector<int> lst[MX];
bool dfs(int u) {
    vis[u] = true;
    pathVis[u] = true;
    for (auto v: lst[u]) {
        if (!vis[v]) {
            if (dfs(v))
                return true;
        } else if (pathVis[v]) {
            return true;
        }
    }
    pathVis[u] = false;
    return false;
}

void solve() {
    // take graph input
    for (int i = 0; i < n; ++i) {
        if (!vis[i])
            dfs(i);
    }
}

```

#### 1.12 DSU

```

const int MX = 1e5 + 10;
int par[MX], sz[MX];
void init() {
    for (int i = 1; i < MX; i++) {
        par[i] = i;
        sz[i] = 1;
    }
}

int findpar(int x) {
    if (par[x] == x) return x;
    return par[x] = findpar(par[x]);
}

void unite(int u, int v) {
    u = findpar(u);
    v = findpar(v);
    if (u != v) {
        if (sz[u] < sz[v]) {
            swap(u, v);
        }
        sz[u] += sz[v];
        par[v] = u;
    }
}

```

#### 1.13 DSU On Trees

```

int n, color[MX], ans[MX];
vector<int> g[MX];
set<int> bucket[MX];
int merge(int a, int b) {
    if (bucket[a].size() < bucket[b].size())
        swap(a, b);
    bucket[a].insert(bucket[b].begin(),
        bucket[b].end());
    bucket[b].clear();
    return a;
}

int dfs(int u, int p = -1) {
    int cur = u;

```

```

    for (int v: g[u])
        if (v != p)
            cur = merge(cur, dfs(v, u));
    ans[u] = (int) bucket[cur].size();
    return cur;
}

void solve() {
    cin >> n;
    for (int i = 0; i < n; ++i) {
        cin >> color[i];
        bucket[i].insert(color[i]);
    }
    // graph input
    dfs(0);
    // print output
}

```

#### 1.14 Dijkstra

```

const int N = 1e5 + 5, INF = 1e18 + 7;
vector<pair<int, int>> g[N];
bool visited[N];
vector<int> dist(N, INF), parent(N);
bool dijkstra(int source) {
    priority_queue<pair<int, int>,
        vector<pair<int, int>>, greater<pair<int,
            int>>> pq;
    pq.push({0, source});
    dist[source] = 0;
    parent[source] = -1;
    while (pq.size()) {
        int x = pq.top().second;
        pq.pop();
        if (visited[x]) continue;
        visited[x] = 1;
        for (auto [child_x, child_wt]: g[x]) {
            if (dist[x] + child_wt < dist[child_x]) {
                parent[child_x] = x;
                dist[child_x] = dist[x] + child_wt;
                pq.push({dist[child_x], child_x});
            }
        }
    }
    return (dist[n] == INF);
}

```

#### 1.15 Euler Tour

```

const int MX = 2e5 + 10;
int timer = -1;
// s = start pos, e = end pos
int val[MX], s[MX], e[MX], flat[MX];
vector<int> lst[MX];
void dfs(int u, int p) {
    s[u] = ++timer;
    flat[timer] = val[u];
    for (auto v: lst[u]) {
        if (v != p)
            dfs(v, u);
    }
    e[u] = timer;
}

```

**1.16 FloydWarshall**

```
vector<vector<int>> d(n, vector<int>(n, INF));
// take graph input into d
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (d[i][k] < INF && d[k][j] < INF)
                d[i][j] = min(d[i][j], d[i][k] +
                    d[k][j]);
        }
    }
}
```

**1.17 GP Hash Table**

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
const int RANDOM = chrono::high_resolution_clock::
    ::now().time_since_epoch().count();
struct custom_hash {
    int operator()(int x) const { return x ^
        RANDOM; }
};
//gp_hash_table<int, int, custom_hash> mp;
```

**1.18 Integer Points in a Circle**

```
ll latticeInCircle(ll r){
    ll ans = (4*r) + 1; // 1 for center
    for(int i = 1; i*i<=r*r; i++){
        for(int j = 1; j*j+i*i<=r*r; j++){ ans+=4;
        }
    } return ans;
}
```

**1.19 LCA**

```
const int N = 1e5 + 5;
vector<int> g[N], parent(N), depth(N, 0);
void dfs(int vertex, int par = -1) {
    parent[vertex] = par;
    for (auto child : g[vertex]) {
        if (child != par) {
            depth[child] = depth[vertex] + 1;
            dfs(child, vertex);
        }
    }
}
int lca(int x, int y) {
    int diff = min(depth[x], depth[y]);
    while (depth[x] > diff) x = parent[x];
    while (depth[y] > diff) y = parent[y];
    while (x != y) { x = parent[x]; y = parent[y];
    } return x;
}
```

**1.20 LCS for 3 Strings**

```
string a, b, c;
ll dp[55][55][55];
ll lcs(ll i, ll j, ll k) {
    if (i == a.size() or j == b.size() or k ==
        c.size()) return 0;
    if (dp[i][j][k] != -1) return dp[i][j][k];
```

```
if (a[i] == b[j] and a[i] == c[k]) return 1 +
    lcs(i + 1, j + 1, k + 1);
ll ans = 0;
ans = max(ans, lcs(i, j, k + 1));
ans = max(ans, lcs(i, j + 1, k));
ans = max(ans, lcs(i + 1, j, k));
return dp[i][j][k] = ans;
}
```

**1.21 LIS**

```
vector<int> lis(int n, vector<int>& v) {
    vector<int> parent(n, -1), ind(n);
    vector<int> lis;
    for (int i = 0; i < n; i++) {
        int it = lower_bound(lis.begin(), lis.end(),
            v[i]) - lis.begin();
        if (it == lis.size()) {
            lis.push_back(v[i]);
            ind[lis.size() - 1] = i;
            parent[i] = (lis.size() == 1 ? -1 : ind[it
                - 1]);
        } else {
            lis[it] = v[i];
            ind[it] = i;
            parent[i] = (it == 0 ? -1 : ind[it - 1]);
        }
    }
    vector<int> LIS;
    int it = ind[lis.size() - 1];
    LIS.push_back(lis.back());
    while (parent[it] != -1) {
        it = parent[it];
        LIS.push_back(v[it]);
    }
    return LIS;
}
```

**1.22 Lazy Propagation**

```
class stree {
    vector<int> seg, lazy;
public:
    segtree(int n) {
        seg.resize(4 * n + 5);
        lazy.resize(4 * n + 5);
    }
    void propagate(int i, int low, int high) {
        if (lazy[i] != 0) {
            seg[i] += (high - low + 1) * lazy[i];
            if (low != high) {
                lazy[2 * i + 1] += lazy[i];
                lazy[2 * i + 2] += lazy[i];
            }
            lazy[i] = 0;
        }
    }
    void build(int i, int low, int high, int
        arr[]) {
        if (low == high) {
            seg[i] = arr[low];
            return;
        }
        int mid = (low + high) >> 1;
        build(2 * i + 1, low, mid, arr);
        build(2 * i + 2, mid + 1, high, arr);
```

```
seg[i] = seg[2 * i + 1] + seg[2 * i + 2];
}
void update(int i, int low, int high, int l,
    int r, int val) {
    propagate(i, low, high);
    if (high < l or r < low) return;
    if (low >= l and high <= r) {
        seg[i] += (high - low + 1) * val;
        if (low != high) {
            // has children
            lazy[2 * i + 1] += val;
            lazy[2 * i + 2] += val;
        }
        return;
    }
    int mid = (low + high) >> 1;
    update(2 * i + 1, low, mid, l, r, val);
    update(2 * i + 2, mid + 1, high, l, r, val);
    seg[i] = seg[2 * i + 1] + seg[2 * i + 2];
}
int query(int i, int low, int high, int l, int
    r) {
    propagate(i, low, high);
    if (high < l or r < low) return 0;
    if (low >= l and high <= r) return seg[i];
    int mid = (low + high) >> 1;
    int left = query(2 * i + 1, low, mid, l, r);
    int right = query(2 * i + 2, mid + 1, high,
        l, r);
    return left + right;
}
```

**1.23 MST**

```
// DSU first
void solve() {
    int n, m;
    cin >> n >> m;
    vector<tuple<int, int, int>> edges;
    for (int i = 0; i < m; ++i) {
        int u, v, wt;
        cin >> u >> v >> wt;
        edges.push_back({wt, u, v});
    }
    sort(edges.begin(), edges.end());
    init(n);
    int cost = 0;
    for (tuple& [wt, u, v] : edges) {
        if (findpar(u) == findpar(v)) continue;
        unite(u, v);
        cost += wt;
    }
    cout << cost << endl;
}
```

**1.24 Manacher\_palindrome**

```
// pal[1][i] = longest odd (half rounded down)
    palindrome around pos i and
// starts at i - pal[1][i] and ends at i +
    pal[1][i]
pal[0][i] = half length of
// longest even palindrome around pos i, i + 1
    and starts at i - par[0][i] + 1
// and ends at i + pal[0][i]
```



```

const int N = 5e5 + 10;
int pal[2][N];
void manacher(string& s) {
    int n = s.size(), idx = 2;
    while (idx--) {
        for (int l = -1, r = -1, i = 0; i < n - 1; i++) {
            if (i > r)
                l = r = i;
            else {
                int k = min(r - i, pal[idx][l + r - i]);
                l = i - k, r = i + k;
            }
            while (l - idx >= 0 and r + 1 < n and s[l - idx] == s[r + 1]) l--, r++;
            pal[idx][i] = r - i;
            // [l - 1 + idx : r] palindrome
        }
    }
}

```

## 1.25 MatExpo

```

const ll mod = 1e9;
vector<vector<ll>> matMul(vector<vector<ll>>& a,
    vector<vector<ll>>& b) {
    ll row1 = a.size(), col1 = a[0].size();
    ll row2 = b.size(), col2 = b[0].size();
    vector<vector<ll>> res(row1, vector<ll>(col2, 0));
    for (ll i = 0; i < row1; i++) {
        for (ll j = 0; j < col1; j++) {
            for (ll k = 0; k < row2; k++) {
                res[i][j] = (res[i][j] + (1LL * a[i][k] * b[k][j]) % mod) % mod;
            }
        }
    }
    return res;
}
vector<vector<ll>> matExpo(vector<vector<ll>>& Mat, ll exp) {
    ll row = Mat.size(), col = Mat[0].size();
    ll p = row;
    vector<vector<ll>> res(p, vector<ll>(p, 0));
    for (ll i = 0; i < p; i++) res[i][i] = 1;
    while (exp) {
        if (exp & 1) res = matMul(res, Mat);
        Mat = matMul(Mat, Mat);
        exp >>= 1;
    }
    return res;
}
// b = (A(i), A(i-1), A(i-2), A(i-3))
// M = Magic matrix, nth = nth term, known =
// known value
ll get_nth(ll nth, ll known, vector<ll>& b,
    vector<vector<ll>>& M) {
    if (nth <= known) return b[nth - 1] % mod;
    reverse(b.begin(), b.end());
    vector<vector<ll>> me = matExpo(M, nth - known);
    ll ans = 0;
    for (int i = 0; i < known; i++) {
        ans = (ans + (b[i] * me[i][0]) % mod) % mod;
    }
    return ans;
}

```

```

}
1.26 Max Bipartite Matching [Hopcroft Karp]
const int INF = 1e9;
void hopcroftCarp() {
    int n, m, e;
    cin >> n >> m >> e;
    vector<int> adj[n];
    for (int i = 0; i < e; ++i) {
        int u, v;
        cin >> u >> v;
        --u;
        adj[u].push_back(v);
    }
    vector<int> ml(m, -1), mr(n, -1), dist(n);
    auto bfs = [&]() -> bool {
        queue<int> q;
        for (int u = 0; u < n; ++u) {
            if (mr[u] == -1) {
                dist[u] = 0;
                q.push(u);
            } else {
                dist[u] = INF;
            }
        }
        bool foundAugmenting = false;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int v : adj[u]) {
                int pairedLeft = ml[v];
                if (pairedLeft == -1) {
                    foundAugmenting = true;
                } else if (dist[pairedLeft] == INF) {
                    dist[pairedLeft] = dist[u] + 1;
                    q.push(pairedLeft);
                }
            }
        }
        return foundAugmenting;
    };
    function<bool(int)> dfs = [&](int u) -> bool {
        for (int v : adj[u]) {
            int pairedLeft = ml[v];
            if (pairedLeft == -1 or (dist[pairedLeft] == dist[u] + 1 and dfs(pairedLeft))) {
                mr[u] = v;
                ml[v] = u;
                return true;
            }
        }
        dist[u] = INF;
        return false;
    };
    int matching = 0;
    while (bfs()) {
        for (int u = 0; u < n; ++u) {
            if (mr[u] == -1) {
                if (dfs(u)) matching++;
            }
        }
    }
    cout << matching << el;
    for (int u = 0; u < n; ++u) {
        if (mr[u] != -1) {

```

```

        cout << u << " " << mr[u] << el;
    }
}
1.27 Max Bipartite Matching [Kuhn's]
// left set size, right set size, edge count
int n, k, m, visToken = 1;
vector<int> lst[MX];
int mr[MX], ml[MX], vis[MX];
bool try_kuhn(int u) {
    if (vis[u] == visToken)
        return false;
    vis[u] = visToken;
    for (auto v : lst[u]) {
        if (ml[v] == -1 or try_kuhn(ml[v])) {
            ml[v] = u;
            mr[u] = v;
            return true;
        }
    }
    return false;
}
void solve() {
    cin >> n >> k >> m;
    for (int i = 0; i < m; ++i) {
        int u, v;
        cin >> u >> v;
        --u; --v;
        lst[u].push_back(v);
    }
    fill(mr, mr + n, -1);
    fill(ml, ml + k, -1);
    int ans = 0;
    for (int u = 0; u < n; ++u) {
        for (auto v : lst[u]) {
            if (ml[v] == -1) {
                ml[v] = u;
                mr[u] = v;
                ans++;
                break;
            }
        }
    }
    for (int u = 0; u < n; ++u) {
        if (mr[u] != -1) continue;
        visToken++;
        if (try_kuhn(u))
            ans++;
    }
    cout << ans << el;
    for (int v = 0; v < k; ++v) {
        if (ml[v] != -1) {
            cout << ml[v] + 1 << " " << v + 1 << el;
        }
    }
}

```

## 1.28 Max Subarray Size Sum equal K

```

//write gpHashTable code before this part
void solution() {
    int n, k; cin >> n >> k;
    int total sum = 0;
    vector<int> pre(n + 7, 0);
    for (int i = 1; i <= n; i++) {

```

```

int temp; cin >> temp;
total_sum += temp;
if (i == 1) pre[i] = temp;
else pre[i] = pre[i - 1] + temp;
}
if (total_sum < k) {
    cout << "-1" << endl; return;
}
if (total_sum == k) {
    cout << "0" << endl; return;
}
int maximum_subSize = 0;
gp hash table < int, int, customHash> table;
for (int i = 1; i <= n; i++) {
    if (pre[i] >= k) {
        int subSUM = pre[i] - k;
        if (subSUM == 0) {
            maximum_subSize = max(maximum_subSize, i);
        }
        else if (table[subSUM]) {
            int left = table[subSUM];
            int right = i; int subSize = right - left;
            maximum_subSize = max(subSize,
                - maximum_subSize);
        }
    }
    if (!table[pre[i]]) table[pre[i]] = i;
} cout << maximum_subSize << endl; }

```

### 1.29 Maximum Subarray Sum(Kadanes Algo)

```

int max_sum_of(vector<int> &vct){
    int mx = INT_MIN, till = 0;
    for (int i = 0; i < vct.size(); i++) {
        till = till + vct[i];
        mx = max(mx, till);
        till = max(till, 1LL*0);
    }
    return mx;
}

```

### 1.30 Mex of all subarray

```

const int N = 1e5 + 9, inf = 1e9;
struct ST {
    int t[4 * N];
    ST() {}
    void build(int n, int b, int e) {
        t[n] = 0;
        if (b == e) {
            return;
        }
        int mid = (b + e) >> 1, l = n << 1, r = l |
            - 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        t[n] = min(t[l], t[r]);
    }
    void upd(int n, int b, int e, int i, int x) {
        if (b > i || e < i) return;
        if (b == e && b == i) {
            t[n] = x;
            return;
        }
        int mid = (b + e) >> 1, l = n << 1, r = l |
            - 1;
        upd(l, b, mid, i, x);
        upd(r, mid + 1, e, i, x);
    }
}

```

```

t[n] = min(t[l], t[r]);
}
int get_min(int n, int b, int e, int i, int j) {
    if (b > j || e < i) return inf;
    if (b >= i && e <= j) return t[n];
    int mid = (b + e) >> 1, l = n << 1, r = l |
        - 1;
    int L = get_min(l, b, mid, i, j);
    int R = get_min(r, mid + 1, e, i, j);
    return min(L, R);
}
int get_mex(int n, int b, int e, int i) { //
    mex of [i... cur_id] if (b == e) return b;
    int mid = (b + e) >> 1, l = n << 1, r = l |
        - 1;
    if (t[l] >= i) return get_mex(r, mid + 1, e,
        - i);
    return get_mex(l, b, mid, i);
}
} t;
int a[N], f[N];
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n;
    cin >> n;
    for (int i = 1; i <= n; i++) {
        cin >> a[i];
        --a[i];
    }
    t.build(1, 0, n);
    set<array<int, 3>> seg; // for cur_id = i,
        [x[0]... i], [x[0] + 1... i], ...[x[1]...
        - i] has mex, x[2]
    for (int i = 1; i <= n; i++) {
        int x = a[i];
        int r = min(i - 1, t.get_min(1, 0, n, 0, x -
            - 1));
        int l = t.get_min(1, 0, n, 0, x) + 1;
        if (l <= r) {
            auto it = seg.lower_bound({l, -1, -1});
            while (it != seg.end() && (*it)[1] <= r) {
                auto x = *it;
                it = seg.erase(it);
            }
        }
        t.upd(1, 0, n, x, i);
        for (int j = r; j >= l; j++) {
            int m = t.get_mex(1, 0, n, j);
            int L = max(l, t.get_min(1, 0, n, 0, m) +
                - 1);
            f[m] = 1;
            seg.insert({L, j, m});
            j = L - 1;
        }
        int m = a[i];
        seg.insert({i, i, m});
        f[m] = 1;
    }
    int ans = 0;
    while (f[ans]) ++ans;
    cout << ans + 1 << '\n';
    return 0;
}

```

```

}
}

1.31 Number of Pairs with gcd equal g
/*a[i] <= 1e6
for all 1<=g<=n, how many pairs exist such that
- g = gcd(a[i], a[j]);
complexity : nlogn
*/
ll n; cin >> n;
ll a[n + 1];
ll cnt[n + 1]; memset(cnt, 0, sizeof cnt);
for (ll i = 1; i <= n; i++) {cin >> a[i];
    - cnt[a[i]]++;}
ll gcd[n + 1]; memset(gcd, 0, sizeof gcd);
for (ll i = n; i >= 1; i--) {
    ll pair = 0, invalid_pair = 0;
    for (ll j = i; j <= n; j += i) {
        pair += cnt[j];
        invalid_pair += gcd[j];
    }
    pair = (pair * (pair - 1)) / 2;
    gcd[i] = pair - invalid_pair;
    // how many pairs exist whose gcd is i
}

```

### 1.32 Number of Subarray Sum is K

```

//write gpHashTable code before this part
void solution(){
    int n, k; cin >> n >> k;
    int total_sum = 0;
    vector<int> pre(n + 7, 0);
    for (int i = 1; i <= n; i++) {
        int temp; cin >> temp;
        total_sum += temp;
        if (i == 1) pre[i] = temp;
        else pre[i] = pre[i - 1] + temp; }
    int cnt_subarray = 0;
    gp hash table < int, int, customHash> table;
    table[0] = 1;
    for (int i = 1; i <= n; i++) {
        cnt_subarray += table[pre[i] - k];
        table[pre[i]]++;
    } cout << cnt_subarray << endl; }

```

### 1.33 Phi(1toN)

```

const int mxN = 1e7 + 10;
vector<int> phi(mxN);
void phi_till() { //O(n.log.log(n))
    for (int i = 0; i < mxN; i++) phi[i] = i;
    for (int i = 2; i < mxN; i++) {
        if (phi[i] == i) {
            for (int j = i; j < mxN; j += i){
                phi[j] -= phi[j] / i;
            }
        }
    }
}
}

```

## 1.34 SOD\_NOD

```
// SOD = ((P^(x+1)-1)/(P-1)) *
// ((Q^(y+1)-1)/(Q-1)) * ((R^(z+1)-1)/(R-1))
// NOD = P^x * Q^y * R^z => here, P, Q, R are
// prime factors & x, y, z are
// powers NOD = (x + 1) (y + 1) (z + 1)
pair<int, int> SOD_NOD(int n) {
    int sod = 1, nod = 1;
    for (int i = 2; i * i <= n; ++i) {
        if (n % i == 0) {
            int pown = 1, pows = 0;
            while (n % i == 0) {
                pown *= i; // p^e
                pows++;
                n /= i;
            }
            pown *= i;
            sod *= (pown - 1) / (i - 1); // (p^e+1)-1
            // / p-1
            nod *= (pows + 1);
        }
    }
    if (n > 1) {
        sod *= (n + 1);
        nod *= 2;
    }
    return {sod, nod};
}
```

## 1.35 Segment Tree(Lazy Propagation)

```
class stree {
    vector<int> seg, lazy;
public:
    segtree(int n) {
        seg.resize(4 * n + 5);
        lazy.resize(4 * n + 5);
    }
    void propagate(int i, int low, int high) {
        if (lazy[i] != 0) {
            seg[i] += (high - low + 1) * lazy[i];
            if (low != high) {
                lazy[2 * i + 1] += lazy[i];
                lazy[2 * i + 2] += lazy[i];
            }
            lazy[i] = 0;
        }
    }
    void build(int i, int low, int high, int
        _ arr[]) {
        if (low == high) {
            seg[i] = arr[low];
            return;
        }
        int mid = (low + high) >> 1;
        build(2 * i + 1, low, mid, arr);
        build(2 * i + 2, mid + 1, high, arr);
        seg[i] = seg[2 * i + 1] + seg[2 * i + 2];
    }
    void update(int i, int low, int high, int l,
        _ int r, int val) {
        propagate(i, low, high);
        if (high < l or r < low) return;
        if (low >= l and high <= r) {
            seg[i] += (high - low + 1) * val;

```

```
        if (low != high) {
            // has children
            lazy[2 * i + 1] += val;
            lazy[2 * i + 2] += val;
        }
        return;
    }
    int mid = (low + high) >> 1;
    update(2 * i + 1, low, mid, l, r, val);
    update(2 * i + 2, mid + 1, high, l, r, val);
    seg[i] = seg[2 * i + 1] + seg[2 * i + 2];
}
int query(int i, int low, int high, int l, int
    _ r) {
    propagate(i, low, high);
    if (high < l or r < low) return 0;
    if (low >= l and high <= r) return seg[i];
    int mid = (low + high) >> 1;
    int left = query(2 * i + 1, low, mid, l, r);
    int right = query(2 * i + 2, mid + 1, high,
        _ l, r);
    return left + right;
}
};
```

## 1.36 Segment Tree

```
class stree {
    vector<int> seg;
public:
    segtree(int n) {
        seg.assign(4 * n + 5, 0);
    }
    void build(int ind, int low, int high, int
        _ arr[]) {
        if (low == high) {
            seg[ind] = arr[low];
            return;
        }
        int mid = (low + high) >> 1;
        build(2 * ind + 1, low, mid, arr);
        build(2 * ind + 2, mid + 1, high, arr);
        seg[ind] = min(seg[2 * ind + 1], seg[2 * ind
            _ + 2]);
    }
    int query(int ind, int low, int high, int l,
        _ int r) {
        if (r < low or high < l) return INT_MAX;
        if (low >= l and high <= r) return seg[ind];
        int mid = (low + high) / 2;
        int left = query(2 * ind + 1, low, mid, l,
            _ r);
        int right = query(2 * ind + 2, mid + 1,
            _ high, l, r);
        return min(left, right);
    }
    void update(int ind, int low, int high, int i,
        _ int val) {
        if (low == high) {
            seg[ind] = val;
            return;
        }
        int mid = (low + high) / 2;
        if (i <= mid) update(2 * ind + 1, low, mid,
            _ i, val);

```

```
        else update(2 * ind + 2, mid + 1, high, i,
            _ val);
        seg[ind] = min(seg[2 * ind + 1], seg[2 * ind
            _ + 2]);
    }
};
```

## 1.37 Segmented Sieve

```
void segSeive(ll low, ll high) {
    vector<bool> area((high - low) + 1, true);
    for (ll i = 0; primes[i] * primes[i] <= high;
        _ i++) {
        ll start = ((low / primes[i]) * primes[i]);
        if (start < low) start += primes[i];
        for (ll j = start; j <= high; j +=
            _ primes[i]) {
            if (j == primes[i]) continue;
            area[j - low] = false;
        }
    }
    for (ll i = 0; i < (high - low) + 1; i++) {
        if (area[i]) {
            if (i + low != 1 and i + low != 0) {
                cout << i + low << endl;
            }
        }
    }
}
```

## 1.38 SparseTable

```
const int mxN = 1e5 + 10, M = 21;
int sparse[mxN][M];
void build_sparse(int n, vector<int>& v) {
    for (int i = 0; i < n; i++) sparse[i][0] =
        _ v[i];
    for (int k = 1; k < M; k++) {
        for (int i = 0; i + (1 << k) <= n; i++) {
            sparse[i][k] = max(sparse[i][k - 1],
                _ sparse[i + (1 << (k - 1))][k - 1]);
        }
    }
}
int query(int l, int r) { // 0 based index
    if (l > r) swap(l, r);
    int b = __bit_width(r - l + 1) - 1;
    return max(sparse[l][b], sparse[r - (1 << b) +
        _ 1][b]);
}
```

## 1.39 StrHash

```
const int mod1 = 911382323, mod2 = 972663749, b1
    _ = 137, b2 = 139;
const int mxN = 5000010;
int pow_b1[mxN], pow_b2[mxN], inv_b1[mxN],
    _ inv_b2[mxN];
int binExp(int base, int power, int mod) {
    int res = 1;
    while (power) {
        if (power & 1) res = (1LL * res * base) %
            _ mod;
        base = (1LL * base * base) % mod;

```



```

    power >= 1;
}
return res;
}
void pre() {
    pow_b1[0] = pow_b2[0] = 1;
    for (int i = 1; i < mxN; i++) {
        pow_b1[i] = (1LL * pow_b1[i - 1] * b1) %
            mod1;
        pow_b2[i] = (1LL * pow_b2[i - 1] * b2) %
            mod2;
    }
    inv_b1[mxN - 1] = binExp(pow_b1[mxN - 1], mod1
        - 2, mod1);
    inv_b2[mxN - 1] = binExp(pow_b2[mxN - 1], mod2
        - 2, mod2);
    for (int i = mxN - 2; i >= 0; i--) {
        inv_b1[i] = (1LL * inv_b1[i + 1] * b1) %
            mod1;
        inv_b2[i] = (1LL * inv_b2[i + 1] * b2) %
            mod2;
    }
}
vector<pair<int, int>> getPref(string& s) {
    int qq = s.size();
    vector<pair<int, int>> hsh(qq);
    for (int i = 0; i < qq; i++) {
        if (i == 0) {
            hsh[i].first = (1LL * s[i] * pow_b1[i]) %
                mod1;
            hsh[i].second = (1LL * s[i] * pow_b2[i]) %
                mod2;
        } else {
            hsh[i].first =
                (hsh[i - 1].first + (1LL * s[i] *
                    pow_b1[i]) % mod1) % mod1;
            hsh[i].second =
                (hsh[i - 1].second + (1LL * s[i] *
                    pow_b2[i]) % mod2) % mod2;
        }
    }
    return hsh;
}
pair<int, int> getHash(string& str) {
    int hsh1 = 0, hsh2 = 0, sz = str.size();
    for (int i = 0; i < sz; ++i) {
        hsh1 = (hsh1 + 1LL * str[i] * pow_b1[i] %
            mod1) % mod1;
    }
    for (int i = 0; i < sz; ++i) {
        hsh2 = (hsh2 + 1LL * str[i] * pow_b2[i] %
            mod2) % mod2;
    }
    return {hsh1, hsh2};
}
pair<int, int> getSub(int l, int r,
    vector<pair<int, int>>& v) {
    pair<int, int> q;
    if (l == 0) {
        q = {v[r].first, v[r].second};
    } else {
        int x = (1LL * ((v[r].first - v[l - 1].first
            - + mod1) % mod1) * inv_b1[l]) %
            mod1;

```

```

    int y =
        (1LL * ((v[r].second - v[l - 1].second +
            mod2) % mod2) * inv_b2[l]) %
        mod2;
    q = {x, y};
}
return q;
}

```

#### 1.40 StrHash\_2

```

const int N = 1000010, MOD = 1e9 + 7;
const ll P[] = {97, 1000003};
ll bigMod(ll a, ll e) {
    if (e == -1) e = MOD - 2;
    ll ret = 1;
    while (e) {
        if (e & 1) ret = ret * a % MOD;
        a = a * a % MOD, e >= 1;
    }
    return ret;
}
ll pwr[2][N], inv[2][N];
void initHash() {
    for (int it = 0; it < 2; ++it) {
        pwr[it][0] = inv[it][0] = 1;
        ll INV_P = bigMod(P[it], -1);
        for (int i = 1; i < N; ++i) {
            pwr[it][i] = pwr[it][i - 1] * P[it] % MOD;
            inv[it][i] = inv[it][i - 1] * INV_P % MOD;
        }
    }
}
struct RangeHash {
    vector<ll> h[2], rev[2];
    RangeHash(const string S, bool revFlag = 0) {
        for (int it = 0; it < 2; ++it) {
            h[it].resize(S.size() + 1, 0);
            for (int i = 0; i < S.size(); ++i) {
                h[it][i + 1] = (h[it][i] + pwr[it][i +
                    1] * (S[i] - 'a' + 1)) % MOD;
            }
            if (revFlag) {
                rev[it].resize(S.size() + 1, 0);
                for (int i = 0; i < S.size(); ++i) {
                    rev[it][i + 1] =
                        (rev[it][i] + inv[it][i + 1] *
                            (S[i] - 'a' + 1)) % MOD;
                }
            }
        }
    }
    inline ll get(int l, int r) {
        ll one = (h[0][r + 1] - h[0][l]) * inv[0][l
            + 1] % MOD;
        ll two = (h[1][r + 1] - h[1][l]) * inv[1][l
            + 1] % MOD;
        if (one < 0) one += MOD;
        if (two < 0) two += MOD;
        return one << 31 | two;
    }
    inline ll getReverse(int l, int r) {
        ll one = (rev[0][r + 1] - rev[0][l]) *
            pwr[0][r + 1] % MOD;
        ll two = (rev[1][r + 1] - rev[1][l]) *
            pwr[1][r + 1] % MOD;
        if (one < 0) one += MOD;

```

```

    if (two < 0) two += MOD;
    return one << 31 | two;
}
}

```

#### 1.41 TopologicalSorting

```

const int N = 1e5 + 10;
vector<int> g[N], indegree(N, 0);
vector<int> topSort(int n) {
    queue<int> q;
    vector<int> order;
    for (int i = 1; i <= n; i++) {
        if (indegree[i] == 0) {
            q.push(i);
        }
    }
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        order.push_back(u);
        for (int v : g[u]) {
            indegree[v]--;
            if (indegree[v] == 0) {
                q.push(v);
            }
        }
    }
    return order;
}

```

#### 1.42 UniquePF of all elements till MX

```

const int MX = 2e5 + 10;
vector<int> pfac[MX];
void factorize() {
    for (int i = 2; i < MX; i++) {
        if (!pfac[i].empty()) continue;
        for (int j = i; j < MX; j += i)
            pfac[j].push_back(i);
    }
}

```

#### 1.43 WeightedUnionFind

```

const int MX = 2e5 + 10;
int par[MX], sz[MX];
ll d[MX];
void init() {
    for (int i = 0; i < MX; ++i) {
        par[i] = i;
        sz[i] = 1;
        d[i] = 0;
    }
}
int findpar(int x) {
    if (par[x] == x) return x;
    int p = par[x];
    par[x] = findpar(p);
    d[x] += d[p];
    return par[x];
}
bool unite(int a, int b, ll w) {
    int ra = findpar(a);
    int rb = findpar(b);

```

```

if (ra == rb) {
    return (d[b] - d[a] == w);
}
if (sz[ra] < sz[rb]) {
    swap(a, b);
    swap(ra, rb);
    w = -w;
}
par[rb] = ra;
d[rb] = d[a] + w - d[b];
sz[ra] += sz[rb];
return true;
}
dist(int a, int b) {
    findpar(a), findpar(b);
    return d[b] - d[a];
}

```

## 1.44 allInOneNT

```

const int MAXN = 1e6 + 9;
typedef struct info {
    int lowest_prime = 0, greatest_prime = 0,
    distinct_prime = 0;
    int total_prime = 0, NOD = 0, SOD = 0;
} info;
info num[MAXN];
void preStore() {
    for (int i = 2; i < MAXN; i++) {
        int n = i;
        map<int, int> factors; // Key->Factor,
        Val->count
        int SOD = 1, NOD = 1, total_p_factor = 0;
        if (n % 2 == 0) {
            while (n % 2 == 0) {
                n /= 2;
                factors[2]++;
                total_p_factor++;
            }
            SOD *= (1 << (factors[2] + 1)) - 1;
            NOD *= (factors[2] + 1);
        }
        for (int i = 3; i * i <= n; i += 2) {
            if (n % i == 0) {
                while (n % i == 0) {
                    n /= i;
                    factors[i]++;
                    total_p_factor++;
                }
                SOD *= (pow(i, factors[i] + 1) - 1) / (i - 1);
                NOD *= (factors[i] + 1);
            }
        }
        if (n > 1) {
            factors[n]++;
            SOD *= (pow(n, 2) - 1) / (n - 1);
            NOD *= 2;
            total_p_factor++;
        }
        num[i].distinct_prime = factors.size();
        num[i].total_prime = total_p_factor;
        num[i].NOD = NOD;
        num[i].SOD = SOD;
        auto lowest_prime = factors.begin();
        auto greatest_prime = factors.rbegin();
        num[i].lowest_prime = lowest_prime->first;
    }
}

```

```

num[i].greatest_prime =
    lowest_prime->first;
}
}
1.45 customHash
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
struct customHash {
    static uint64_t Meaw(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_since_epoch().count();
        return Meaw(x + FIXED_RANDOM);
    }
}; // gp_hash_table<int, int> table;

```

## 1.46 divisorSieve

```

const int mxN = 1e5 + 10;
vector<int> divisors[mxN];
void divisorSieve() {
    for (int i = 1; i < mxN; i++) {
        for (int j = i; j < mxN; j += i) {
            divisors[j].push_back(i);
        }
    }
}

```

## 1.47 int128

```

__int128 read() {
    __int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}
void print(__int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}

```

## 1.48 mergeSort

```

// use array of elements, if multiple testcase
// make inv = 0 each time
// int inv = 0;
void merge(int vct[], int l, int m, int r) {
    int left = m - l + 1, right = r - m, lv[left],
    rv[right];
}

```

```

for (int i = 0; i < left; i++) {
    lv[i] = vct[l + i];
}
for (int i = 0; i < right; i++) {
    rv[i] = vct[m + 1 + i];
}
int i = 0, j = 0, to = l;
while (i < left && j < right) {
    if (lv[i] <= rv[j]) {
        vct[to] = lv[i];
        i++;
    } else {
        vct[to] = rv[j];
        j++;
    }
    // inversion count
    // int pore = left - i; inv += pore;
    to++;
}
while (i < left) {
    vct[to] = lv[i];
    i++;
    to++;
}
while (j < right) {
    vct[to] = rv[j];
    j++;
    to++;
}
}
void merge_sort(int vct[], int l, int r) {
    if (r <= l) return;
    int m = l + ((r - l) / 2);
    merge_sort(vct, l, m);
    merge_sort(vct, m + 1, r);
    merge(vct, l, m, r);
}

```

## 1.49 nCr and nPr

```

int fact[N], ifact[N];
void prec() {
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = 1LL * fact[i - 1] * i % mod;
    }
    ifact[N - 1] = power(fact[N - 1], -1);
    for (int i = N - 2; i >= 0; i--) {
        ifact[i] = 1LL * ifact[i + 1] * (i + 1) %
        mod;
    }
}
int nPr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[n - r] % mod;
}
int nCr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[r] % mod *
    ifact[n - r] % mod;
}

```

## 1.50 pbds

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <functional>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>,
    _ rb_tree_tag,
    tree_order_statistics_node_update>
    ordered_set;
// s.order_of_key(x) = number of elements
// strictly less than x
// *s.find_by_order(i) = ith element in set (0
// index)
```

## 1.51 phi

```
int phi(int n) { // sqrt(n)
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}
```

## 1.52 polynomial\_interpolation

```
// P(x) = a0 + a1x + a2x^2 + ... + anx^n
// y[i] = P(i)
const int mod = 1e9 + 7;
ll BigMod(ll a, ll b) {
    ll res = 1;
    while (b) {
        if (b & 1) res = 1ll * res * a % mod;
        a = 1ll * a * a % mod;
        b >>= 1;
    }
    return res;
}
ll inv(ll x) {
    if (x < 0) x += mod;
    return BigMod(x, mod - 2);
}
ll add(ll& a, ll b) {
    a += b;
    if (a >= mod) a -= mod;
    return a;
}
ll eval(vector<ll> y, ll k) {
    int n = y.size() - 1;
    if (k <= n) {
        return y[k];
    }
    vector<ll> L(n + 1, 1);
    for (int x = 1; x <= n; ++x) {
        L[0] = L[0] * (k - x) % mod;
        L[0] = L[0] * inv(-x) % mod;
    }
    for (int x = 1; x <= n; ++x) {
        L[x] = L[x - 1] * inv(k - x) % mod * (k - (x
            - 1)) % mod;
        L[x] = L[x] * ((x - 1) - n + mod) % mod *
            inv(x) % mod;
    }
}
```

```
ll yk = 0;
for (int x = 0; x <= n; ++x) {
    yk = add(yk, L[x] * y[x] % mod);
}
return yk;
}
```

## 1.53 sieve

```
const ll MAXN = 1e7 + 10;
bool prime[MAXN];
vector<ll> prm;
void sieve() {
    prime[0] = prime[1] = true;
    for (ll i = 2; i < MAXN; i++) {
        if (!prime[i]) {
            prm.push_back(i);
            for (ll j = i + i; j < MAXN; j += i) {
                prime[j] = true;
            }
        }
    }
}
```

## 1.54 spf

```
const int MAXN = 1e6 + 2;
int spf[MAXN];
vector<int> prms;
void preStore() {
    for (int i = 1; i < MAXN; i++) spf[i] = i;
    for (int i = 2; i < MAXN; i++) {
        if (spf[i] == i) {
            prms.push_back(i);
            for (int j = i + i; j < MAXN; j += i) {
                spf[j] = min(spf[j], i);
            }
        }
    }
}
```

## 1.55 sqrt Distinct Floor

```
//1st problem
const ll mod = 1e9+7;
void solution(){
    ll n; cin>>n;
    ll i = 1;
    ll l = 0, r = 0;
    ll sum = 0;
    while(i<=n){
        ll p = n/i;
        ll l = i-1;
        i = (n/p)+1;
        ll r;
        if(i<=n){
            r = i-1;
        }
        else{
            r = n;
        }
        ll s1 = (__int128(l)*(l+1)/2)%mod;
        ll s2 = (__int128(r)*(r+1)/2)%mod;
        // cout<<"<=r<<" "<=s1<<" "<=s2<<endl;
        sum = ((sum%mod) +
            (((s2-s1+mod)%mod)*(p%mod))%mod)%mod;
    }
    cout<<sum<<endl;
}
```

```
}
//2nd problem
void solution(){
    ll n; cin>>n;
    vector<ll> v;
    ll i = 1;
    ll sum = 0;
    while(i<=n){
        ll p = n/i;
        ll prev = i;
        v.push_back(p);
        i = (n/p)+1;
        ll q;
        if(i<=n){
            q = i-prev;
        }
        else{
            q = n-prev+1;
        }
        sum+=p*q;
    }
    cout<<sum<<endl;
}
```

## 1.56 suffixArray

```
// fahimcp495
array<vector<int>, 2> get_sa(string& s, int lim
    = 128) { // for integer, just change string
    to vector<int> and minimum value of vector
    must be >= 1
    int n = s.size() + 1, k = 0, a, b;
    vector<int> x(begin(s), end(s) + 1), y(n),
        sa(n), lcp(n), ws(max(n, lim)), rank(n);
    x.back() = 0;
    iota(begin(sa), end(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j *
        2), lim = p) {
        p = j, iota(begin(y), end(y), n - j);
        for (int i = 0; i < n; ++i)
            if (sa[i] >= j) y[p++] = sa[i] - j;
        fill(begin(ws), end(ws), 0);
        for (int i = 0; i < n; ++i) ws[x[i]]++;
        for (int i = 1; i < lim; ++i) ws[i] += ws[i
            - 1];
        for (int i = n; i--;) sa[--ws[x[y[i]]]] =
            y[i];
        swap(x, y), p = 1, x[sa[0]] = 0;
        for (int i = 1; i < n; ++i) a = sa[i - 1], b
            = sa[i], x[b] = (y[a] == y[b] && y[a +
                1] == y[b + 1]) ? p - 1 : p++;
    }
    for (int i = 1; i < n; ++i) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] =
        k)
        for (k &&k--, j = sa[rank[i] - 1]; s[i + k]
            == s[j + k]; k++);
    sa.erase(sa.begin()), lcp.erase(lcp.begin());
    return {sa, lcp};
}
```

## 1.57 suffixAutomata

```

const int N = 2e5 + 10; // max string size
int len[N], lnk[N]{-1}, last, sz = 1;
unordered_map<char, int> to[N];
void add(char c) {
    int cur = sz++;
    len[cur] = len[last] + 1;
    int u = last;
    while (u != -1 and !to[u].count(c)) {
        to[u][c] = cur;
        u = lnk[u];
    }
    if (u == -1) {
        lnk[cur] = 0;
    } else {
        int v = to[u][c];
        if (len[v] == len[u] + 1) {
            lnk[cur] = v;
        } else {
            int w = sz++;
            len[w] = len[u] + 1, lnk[w] = lnk[v],
            to[w] = to[v];
            while (u != -1 and to[u][c] == v) {
                to[u][c] = w;
                u = lnk[u];
            }
            lnk[cur] = lnk[v] = w;
        }
    }
    last = cur;
}

```

## 1.58 suffixAutomation

```

int len[N], lnk[N]{-1}, last, sz = 1;
unordered_map<char, int> to[N];
void init() {
    while (sz) {
        sz--;
        to[sz].clear();
    }
    last = 0, sz = 1;
}
void add(char c) {
    int cur = sz++;
    int u = last;
    len[cur] = len[last] + 1;
    while (u != -1 and !to[u].count(c)) {
        to[u][c] = cur;
        u = lnk[u];
    }
    if (u == -1) {
        lnk[cur] = 0;
    } else {
        int v = to[u][c];
        if (len[v] == len[u] + 1) {
            lnk[cur] = v;
        } else {
            int w = sz++;
            len[w] = len[u] + 1, lnk[w] = lnk[v],
            to[w] = to[v];
            while (u != -1 and to[u][c] == v) {
                to[u][c] = w;
                u = lnk[u];
            }
            lnk[cur] = lnk[v] = w;
        }
    }
}

```

```

}
last = cur;
}

```

## 1.59 trie

```

const ll N = 1e6 + 5, A = 26;
ll trie[N][A], cnt[N], tot = 1, root = 1;
void initTrie() {
    cnt[tot] = 0;
    root = 1;
}
void addStr(string& s) {
    ll u = 1;
    for (auto it : s) {
        ll n = it - 'a';
        if (trie[u][n] == 0) {
            trie[u][n] = ++tot;
        }
        u = trie[u][n];
        cnt[u]++;
    }
}
ll wordCount(string& s) {
    ll u = 1;
    for (auto it : s) {
        int n = it - 'a';
        if (trie[u][n] == 0) return 0;
        u = trie[u][n];
    }
    return cnt[u];
}

```

## 2 Geometry

## 2.1 Convex Hull

```

vector<PT> convexHull(vector<PT> p) {
    int n = p.size(), m = 0;
    if (n < 3) return p;
    vector<PT> hull(n + n);
    sort(p.begin(), p.end(), [&] (PT a, PT b) {
        return (a.x == b.x ? a.y < b.y : a.x < b.x);
    });
    for (int i = 0; i < n; ++i) {
        while (m > 1 and cross(hull[m - 2] - p[i],
            hull[m - 1] - p[i]) <= 0) --m;
        hull[m++] = p[i];
    }
    for (int i = n - 2, j = m + 1; i >= 0; --i) {
        while (m >= j and cross(hull[m - 2] - p[i],
            hull[m - 1] - p[i]) <= 0) --m;
        hull[m++] = p[i];
    }
    hull.resize(m - 1); return hull;
}

```

## 3 Notes

## 3.1 Geometry

## 3.1.1 Triangles

Circumradius:  $R = \frac{abc}{4A}$ , Inradius:  $r = \frac{A}{s}$

The area of a triangle using two sides and the included angle can be given as:

$$A = \frac{1}{2}ab \sin \angle C$$

Length of median (divides triangle into two equal-area triangles):  $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):  $s_a =$

$$\sqrt{bc \left[ 1 - \left( \frac{a}{b+c} \right)^2 \right]}$$

$$\text{Law of tangents: } \frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$$

## 3.1.2 Quadrilaterals

With side lengths  $a, b, c, d$ , diagonals  $e, f$ , diagonals angle  $\theta$ , area  $A$  and magic flux  $F = b^2 + d^2 - a^2 - c^2$ :

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is  $180^\circ$ ,  $ef = ac + bd$ , and  $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ .

## 3.1.3 Spherical coordinates

$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z / \sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \arctan2(y, x) \end{aligned}$$

## 3.1.4 Pick's Theorem:

Given a lattice polygon with non-zero area, we define:  $S$  as the area of the polygon,  $I$  as the number of integer-coordinate points strictly inside the polygon,  $B$  as the number of integer-coordinate points on the boundary of the polygon. Then, Pick's Theorem states:

$$S = I + \frac{B}{2} - 1$$

The number of lattice points on segments  $(x_1, y_1)$  to  $(x_2, y_2)$  is:  $\gcd(\text{abs}(x_2 - x_1), \text{abs}(y_2 - y_1)) + 1$

## 3.1.5 Polygon

For a regular polygon with  $n$  sides and side length  $a$ , the circumradius  $R$  is given by:

$$R = \frac{a}{2 \sin \left( \frac{\pi}{n} \right)}$$

## 3.1.6 Area of a Circular Segment

The area of a circular segment, which is the region enclosed by a chord and the corresponding arc, can be calculated using the formula:

$$A = \frac{R^2}{2} (\theta - \sin \theta)$$

where:  $R$  is the radius of the circle,  $\theta$  is the central angle subtended by the chord, in radians.

**3.2 Binomial Coefficient**

- Factoring in:  $\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}$
- Sum over  $k$ :  $\sum_{k=0}^n \binom{n}{k} = 2^n$
- Alternating sum:  $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$
- Even and odd sum:  $\sum_{k=0}^n \binom{n}{2k} = \sum_{k=0}^n \binom{n}{2k+1} = 2^{n-1}$
- The Hockey Stick Identity
  - (Left to right) Sum over  $n$  and  $k$ :  $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m-1}{m}$
  - (Right to left) Sum over  $n$ :  $\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$
- Sum of the squares:  $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$
- Weighted sum:  $\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$
- Connection with the fibonacci numbers:  $\sum_{k=0}^n \binom{n-k}{k} = F_{n+1}$
- Vandermonde's Identity:  $\sum_{i=0}^k \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}$
- If  $f(n, k) = C(n, 0) + C(n, 1) + \dots + C(n, k)$ , Then  $f(n+1, k) = 2 * f(n, k) - C(n, k)$  [For multiple  $f(n, k)$  queries, use Mo's algo]

**Lucas Theorem**

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$$

- $\binom{m}{n}$  is divisible by  $p$  if and only if at least one of the base- $p$  digits of  $n$  is greater than the corresponding base- $p$  digit of  $m$ .
- The number of entries in the  $n$ th row of Pascal's triangle that are not divisible by  $p = \prod_{i=0}^k (n_i + 1)$
- All entries in the  $(p^k - 1)th$  row are not divisible by  $p$ .
- $\binom{n}{m} \equiv \lfloor \frac{n}{p} \rfloor \pmod{p}$

**3.3 Fibonacci Number**

- $k = A - B, F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1}$
- $\sum_{i=0}^n F_i^2 = F_{n+1} F_n$
- $\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^{n-1} F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^n F_i F_{i-1} = F_{n+1}^2 - (-1)^n$
- $\gcd(F_m, F_n) = F_{\gcd(m, n)}$
- $\sum_{0 \leq k \leq n} \binom{n-k}{k} = F_{n+1}$
- $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = \gcd(F_{n+1}, F_{n+2}) = 1$

**3.4 Sums**

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \left[ (n+1)^{m+1} - 1 - \sum_{i=1}^n ((i+1)^{m+1} - i^{m+1} - (m+1)i^m) \right]$$

$$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k}$$

$$\sum_{k=0}^n kx^k = (x - (n+1)x^{n+1} + nx^{n+2}) / (x-1)^2$$

**3.5 Series**

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

$$(x+a)^{-n} = \sum_{k=0}^{\infty} (-1)^k \binom{n+k-1}{k} x^k a^{-n-k}$$

**Generating Function**

$$1/(1-x) = 1 + x + x^2 + x^3 + \dots$$

$$1/(1-ax) = 1 + ax + (ax)^2 + (ax)^3 + \dots$$

$$1/(1-x)^2 = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$1/(1-x)^3 = C(2,2) + C(3,2)x + C(4,2)x^2 + C(5,2)x^3 + \dots$$

$$1/(1-ax)^{k+1} = 1 + C(1+k,k)(ax) + C(2+k,k)(ax)^2 + C(3+k,k)(ax)^3 + \dots$$

$$x(x+1)(1-x)^{-3} = 1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + \dots$$

$$e^x = 1 + x + (x^2)/2! + (x^3)/3! + (x^4)/4! + \dots$$

**3.6 Pythagorean Triples**

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with  $m > n > 0$ ,  $k > 0$ ,  $m \perp n$ , and either  $m$  or  $n$  even.

**3.7 Number Theory**

- HCN: 1e6(240), 1e9(1344), 1e12(6720), 1e14(17280), 1e15(26880), 1e16(41472)
- $\gcd(a, b, c, d, \dots) = \gcd(a, b-a, c-b, d-c, \dots)$
- $\gcd(a+k, b+k, c+k, d+k, \dots) = \gcd(a+k, b-a, c-b, d-c, \dots)$
- Primitive root exists iff  $n = 1, 2, 4, p^k, 2 \times p^k$ , where  $p$  is an odd prime.
- If primitive root exists, there are  $\phi(\phi(n))$  primitive roots of  $n$ .
- The numbers from 1 to  $n$  have in total  $O(n \log \log n)$  unique prime factors.
- $x \equiv r_1 \pmod{m_1}$  and  $x \equiv r_2 \pmod{m_2}$  has a solution iff  $\gcd(m_1, m_2) | (r_1 - r_2)$  Solution of  $x^2 \equiv a \pmod{p}$
- $ca \equiv cb \pmod{m} \iff a \equiv b \pmod{\frac{n}{\gcd(n, c)}}$
- $ax \equiv b \pmod{m}$  has a solution  $\iff \gcd(a, m) | b$
- If  $ax \equiv b \pmod{m}$  has a solution, then it has  $\frac{m}{\gcd(a, m)}$  solutions and they are separated by  $\frac{m}{\gcd(a, m)}$
- $ax \equiv 1 \pmod{m}$  has a solution or  $a$  is invertible  $\pmod{m} \iff \gcd(a, m) = 1$
- $x^2 \equiv 1 \pmod{p}$  then  $x \equiv \pm 1 \pmod{p}$
- There are  $\frac{p-1}{2}$  has no solution.
- There are  $\frac{p-1}{2}$  has exactly two solutions.
- When  $p \% 4 = 3$ ,  $x \equiv \pm a^{\frac{p+1}{4}}$
- When  $p \% 8 = 5$ ,  $x \equiv a^{\frac{p+3}{8}}$  or  $x \equiv 2^{\frac{p-1}{4}} a^{\frac{p+3}{8}}$

**3.7.1 Primes**

$p = 962592769$  is such that  $2^{21} | p-1$ , which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power  $p^a$ , except for  $p = 2, a > 2$ , and there are  $\phi(\phi(p^a))$  many. For  $p = 2, a > 2$ , the group  $\mathbb{Z}_{2^a}^\times$  is instead isomorphic to  $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$ .

**3.7.2 Estimates**

$\sum_{d|n} d = O(n \log \log n)$ .

The number of divisors of  $n$  is at most around 100 for  $n < 5e4$ , 500 for  $n < 1e7$ , 2000 for  $n < 1e10$ , 200 000 for  $n < 1e19$ .

**3.7.3 Perfect numbers**

$n > 1$  is called perfect if it equals sum of its proper divisors and 1. Even  $n$  is perfect iff  $n = 2^{p-1}(2^p - 1)$  and  $2^p - 1$  is prime (Mersenne's). No odd perfect numbers are yet found.

**3.7.4 Carmichael numbers**

A positive composite  $n$  is a Carmichael number ( $a^{n-1} \equiv 1 \pmod{n}$  for all  $a$ :  $\gcd(a, n) = 1$ ), iff  $n$  is square-free, and for all prime divisors  $p$  of  $n$ ,  $p-1$  divides  $n-1$ .



### 3.7.5 Totient

- If  $p$  is a prime  $(p^k) = p^k - p^{k-1}$
- If  $a, b$  are relatively prime,  $\phi(ab) = \phi(a)\phi(b)$
- $\phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})(1 - \frac{1}{p_3}) \dots (1 - \frac{1}{p_k})$
- Sum of coprime to  $n = n * \frac{\phi(n)}{2}$
- If  $n = 2^k, \phi(n) = 2^{k-1} = \frac{n}{2}$
- For  $a, b, \phi(ab) = \phi(a)\phi(b) \frac{d}{\phi(d)}$
- $\phi(ip) = p\phi(i)$  whenever  $p$  is a prime and it divides  $i$
- The number of  $a(1 \leq a \leq N)$  such that  $\gcd(a, N) = d$  is  $\phi(\frac{N}{d})$
- If  $n > 2, \phi(n)$  is always even
- Sum of  $\gcd, \sum_{i=1}^n \gcd(i, n) = \sum_{d|n} d\phi(\frac{n}{d})$
- Sum of  $\text{lcm}, \sum_{i=1}^n \text{lcm}(i, n) = \frac{n}{2}(\sum_{d|n} (d\phi(d)) + 1)$
- $\phi(1) = 1$  and  $\phi(2) = 1$  which two are only odd  $\phi$
- $\phi(3) = 2$  and  $\phi(4) = 2$  and  $\phi(6) = 2$  which three are only prime  $\phi$
- Find minimum  $n$  such that  $\frac{\phi(n)}{n}$  is maximum- Multiple of small primes-  $2 * 3 * 5 * 7 * 11 * 13 * \dots$

### 3.7.6 Mobius function

$\mu(1) = 1$ .  $\mu(n) = 0$ , if  $n$  is not squarefree.  $\mu(n) = (-1)^s$ , if  $n$  is the product of  $s$  distinct primes. Let  $f, F$  be functions on positive integers. If for all  $n \in \mathbb{N}, F(n) = \sum_{d|n} f(d)$ , then  $f(n) = \sum_{d|n} \mu(d)F(\frac{n}{d})$ , and vice versa.  $\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$ .  $\sum_{d|n} \mu(d) = 1$ . If  $f$  is multiplicative, then  $\sum_{d|n} \mu(d)f(d) = \prod_{p|n} (1 - f(p))$ ,  $\sum_{d|n} \mu(d)^2 f(d) = \prod_{p|n} (1 + f(p))$ .

$$\sum_{i=1}^n \sum_{j=1}^n [\gcd(i, j) = 1] = \sum_{k=1}^n \mu(k) \lfloor \frac{n}{k} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n k \sum_{l=1}^{\lfloor \frac{n}{k} \rfloor} \mu(l) \lfloor \frac{n}{kl} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n \left( \frac{\lfloor \frac{n}{k} \rfloor (1 + \lfloor \frac{n}{k} \rfloor)}{2} \right)^2 \sum_{d|k} \mu(d)kd$$

### 3.7.7 Legendre symbol

If  $p$  is an odd prime,  $a \in \mathbb{Z}$ , then  $\left(\frac{a}{p}\right)$  equals 0, if  $p|a$ ; 1 if  $a$  is a quadratic residue modulo  $p$ ; and -1 otherwise. Euler's criterion:  $\left(\frac{a}{p}\right) = a^{\left(\frac{p-1}{2}\right)} \pmod{p}$ .

### 3.7.8 Jacobi symbol

If  $n = p_1^{a_1} \dots p_k^{a_k}$  is odd, then  $\left(\frac{a}{n}\right) = \prod_{i=1}^k \left(\frac{a}{p_i}\right)^{a_i}$ .

### 3.7.9 Primitive roots

If the order of  $g$  modulo  $m$  ( $\min n > 0: g^n \equiv 1 \pmod{m}$ ) is  $\phi(m)$ , then  $g$  is called a primitive root. If  $Z_m$  has a primitive root, then it has  $\phi(\phi(m))$  distinct primitive roots.  $Z_m$  has a primitive root iff  $m$  is one of  $2, 4, p^k, 2p^k$ , where  $p$  is an odd prime. If  $Z_m$  has a primitive root  $g$ , then for all  $a$  coprime to  $m$ , there exists unique integer  $i = \text{ind}_g(a)$  modulo  $\phi(m)$ , such that  $g^i \equiv a \pmod{m}$ .  $\text{ind}_g(a)$  has logarithm-like properties:  $\text{ind}(1) = 0$ ,  $\text{ind}(ab) = \text{ind}(a) + \text{ind}(b)$ .

If  $p$  is prime and  $a$  is not divisible by  $p$ , then congruence  $x^n \equiv a \pmod{p}$  has  $\gcd(n, p-1)$  solutions if  $a^{(p-1)/\gcd(n, p-1)} \equiv 1 \pmod{p}$ , and no solutions otherwise. (Proof sketch: let  $g$  be a primitive root, and  $g^i \equiv a \pmod{p}$ ,  $g^u \equiv x \pmod{p}$ .  $x^n \equiv a \pmod{p}$  iff  $g^{nu} \equiv g^i \pmod{p}$  iff  $nu \equiv i \pmod{p}$ .)

### 3.7.10 Discrete logarithm problem

Find  $x$  from  $a^x \equiv b \pmod{m}$ . Can be solved in  $O(\sqrt{m})$  time and space with a meet-in-the-middle trick. Let  $n = \lfloor \sqrt{m} \rfloor$ , and  $x = ny - z$ . Equation becomes  $a^{ny} \equiv ba^z \pmod{m}$ . Precompute all values that the RHS can take for  $z = 0, 1, \dots, n-1$ , and brute force  $y$  on the LHS, each time checking whether there's a corresponding value for RHS.

### 3.7.11 Pythagorean triples

Integer solutions of  $x^2 + y^2 = z^2$  All relatively prime triples are given by:  $x = 2mn, y = m^2 - n^2, z = m^2 + n^2$  where  $m > n, \gcd(m, n) = 1$  and  $m \not\equiv n \pmod{2}$ . All other triples are multiples of these. Equation  $x^2 + y^2 = 2z^2$  is equivalent to  $(\frac{x+y}{2})^2 + (\frac{x-y}{2})^2 = z^2$ .

### 3.7.12 Postage stamps/McNuggets problem

Let  $a, b$  be relatively-prime integers. There are exactly  $\frac{1}{2}(a-1)(b-1)$  numbers *not* of form  $ax + by$  ( $x, y \geq 0$ ), and the largest is  $(a-1)(b-1) - 1 = ab - a - b$ .

### 3.7.13 Fermat's two-squares theorem

Odd prime  $p$  can be represented as a sum of two squares iff  $p \equiv 1 \pmod{4}$ . A product of two sums of two squares is a sum of two squares. Thus,  $n$  is a sum of two squares iff every prime of form  $p = 4k + 3$  occurs an even number of times in  $n$ 's factorization.

### 3.8 Permutations

#### 3.8.1 Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$\frac{n}{n!}$	1	11	12	13	14	15	16	17		
$\frac{n!}{n}$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$\frac{n!}{n!}$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

#### 3.8.2 Cycles

Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

#### 3.8.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

#### 3.8.4 Burnside's lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts "configurations" (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k)$$

### 3.9 Partitions and subsets

#### 3.9.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$\frac{n}{p(n)}$	0	1	2	3	4	5	6	7	8	9	20	50	100
	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

#### 3.9.2 Partition Number

- Time Complexity:  $O(n\sqrt{n})$

```
for (int i = 1; i <= n; ++i) {
    pent[2 * i - 1] = i * (3 * i - 1) / 2;
    pent[2 * i] = i * (3 * i + 1) / 2;
}
p[0] = 1;
for (int i = 1; i <= n; ++i) {
    p[i] = 0;
    for (int j = 1, k = 0; pent[j] <= i; ++j) {
        if (k < 2) p[i] = add(p[i], p[i - pent[j]]);
        else p[i] = sub(p[i], p[i - pent[j]]); ++k, k &
    }
}
```

- The number of partitions of a positive integer  $n$  into exactly  $k$  parts equals the number of partitions of  $n$  whose largest part equals  $k$

$$p_k(n) = p_k(n-k) + p_{k-1}(n-1)$$

#### 3.9.3 2nd Kaplansky's Lemma

The number of ways of selecting  $k$  objects, no two consecutive, from  $n$  labelled objects arrayed in a circle is  $\frac{n}{k} \binom{n-k-1}{k-1} = \frac{n-k}{n-k} \binom{n-k}{k}$

#### 3.9.4 Distinct Objects into Distinct Bins

-  $n$  distinct objects into  $r$  distinct bins  $= r^n$   
 - Among  $n$  distinct objects, exactly  $k$  of them into  $r$  distinct bins  $= \binom{n}{k} r^k$   
 -  $n$  distinct objects into  $r$  distinct bins such that each bin contains at least one object  $= \sum_{i=0}^r (-1)^i \binom{r}{i} (r-i)^n$

**3.10 Coloring**

The number of labeled undirected graphs with  $n$  vertices,  $G_n = 2^{\binom{n}{2}}$

The number of labeled directed graphs with  $n$  vertices,  $G_n = 2^{n(n-1)}$

The number of connected labeled undirected graphs with  $n$  vertices,  $C_n = 2^{\binom{n}{2}} - \frac{1}{n} \sum_{k=1}^{n-1} k \binom{n}{k} 2^{\binom{n-k}{2}} C_k = 2^{\binom{n}{2}} - \sum_{k=1}^{n-1} \frac{n-1}{k-1} 2^{\binom{n-k}{2}} C_k$

The number of  $k$ -connected labeled undirected graphs with  $n$  vertices,  $D[n][k] = \sum_{s=1}^n \binom{n-1}{s-1} C_s D[n-s][k-1]$

Cayley's formula: the number of trees on  $n$  labeled vertices = the number of spanning trees of a complete graph with  $n$  labeled vertices =  $n^{n-2}$

Number of ways to color a graph using  $k$  color such that no two adjacent nodes have same color

Complete graph =  $k(k-1)(k-2)\dots(k-n+1)$

Tree =  $k(k-1)^{n-1}$

Cycle =  $(k-1)^n + (-1)^n(k-1)$

Number of trees with  $n$  labeled nodes:  $n^{n-2}$

**3.11 General purpose numbers****3.11.1 Eulerian numbers**

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

**3.11.2 Bell numbers**

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ . For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

**3.11.3 Bernoulli numbers**

$\sum_{j=0}^m \binom{m+1}{j} B_j = 0$ .  $B_0 = 1, B_1 = -\frac{1}{2}$ .  $B_n = 0$ , for all odd  $n \neq 1$ .

**3.11.4 Catalan numbers**

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, C_{n+1} = \frac{2(2n+1)}{n+2} C_n, C_{n+1} = \sum C_i C_{n-i}$$

- $C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$
- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with  $n+1$  leaves (0 or 2 children).
- ordered trees with  $n+1$  vertices.
- ways a convex polygon with  $n+2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.

- Find the count of balanced parentheses sequences consisting of  $n+k$  pairs of parentheses where the first  $k$  symbols are open brackets.

$$C_n^{(k)} = \frac{k+1}{n+k+1} \binom{2n+k}{n}$$

- Recursive formula of Catalan Numbers:

$$C_n^{(k)} = \frac{(2n+k-1) \cdot (2n+k)}{n \cdot (n+k+1)} C_{n-1}^{(k)}$$

**3.11.5 Lucas Number**

Number of edge cover of a cycle graph  $C_n$  is  $L_n$

$$L(n) = L(n-1) + L(n-2); L(0) = 2, L(1) = 1$$

**3.12 Ballot Theorem**

Suppose that in an election, candidate A receives  $a$  votes and candidate B receives  $b$  votes, where  $a > b$  for some positive integer  $k$ . Compute the number of ways the ballots can be ordered so that A maintains more than  $k$  times as many votes as B throughout the counting of the ballots.

The solution to the ballot problem is  $\frac{a-kb}{a+b} \times C(a+b, a)$

**3.13 Classical Problem**

$F(n, k)$  = number of ways to color  $n$  objects using exactly  $k$  colors  
Let  $G(n, k)$  be the number of ways to color  $n$  objects using no more than  $k$  colors.

Then,  $F(n, k) = G(n, k) - C(k, 1) * G(n, k-1) + C(k, 2) * G(n, k-2) - C(k, 3) * G(n, k-3) \dots$

**Determining  $G(n, k)$ :**

Suppose, we are given a  $1 * n$  grid. Any two adjacent cells can not have same color. Then,  $G(n, k) = k * ((k-1)^{n-1})$

If no such condition on adjacent cells. Then,  $G(n, k) = k^n$

**3.14 Matching Formula****3.14.1 Normal Graph**

$MM + MEC = n$  (excluding vertex),  $IS + VC = G$ ,  $MIS + MVC = G$

**3.14.2 Bipartite Graph**

$MIS = n - MBM$ ,  $MVC = MBM$ ,  $MEC = n - MBM$

**3.15 Inequalities****3.15.1 Titu's Lemma**

For positive reals  $a_1, a_2, \dots, a_n$  and  $b_1, b_2, \dots, b_n$ ,

$$\frac{a_1^2}{b_1} + \frac{a_2^2}{b_2} + \dots + \frac{a_n^2}{b_n} \geq \frac{a_1 + a_2 + \dots + a_n^2}{b_1 + b_2 + \dots + b_n}$$

Equality holds if and only if  $a_i = kb_i$  for a non-zero real constant  $k$ .

**3.16 Games****3.16.1 Grundy numbers**

For a two-player, normal-play (last to move wins) game on a graph  $(V, E)$ :  $G(x) = \text{mex}(\{G(y) : (x, y) \in E\})$ , where  $\text{mex}(S) = \min\{n \geq 0 : n \notin S\}$ .  $x$  is losing iff  $G(x) = 0$ .

**3.16.2 Sums of games**

- *Player chooses a game and makes a move in it* Grundy number of a position is xor of grundy numbers of positions in summed games.

- *Player chooses a non-empty subset of games (possibly, all) and makes moves in all of them* A position is losing iff each game is in a losing position.

- *Player chooses a proper subset of games (not empty and not all), and makes moves in all chosen ones.* A position is losing iff grundy numbers of all games are equal.

- *Player must move in all games, and loses if can't move in some game* A position is losing if any of the games is in a losing position.

**3.16.3 Misère Nim**

A position with pile sizes  $a_1, a_2, \dots, a_n \geq 1$ , not all equal to 1, is losing iff  $a_1 \oplus a_2 \oplus \dots \oplus a_n = 0$  (like in normal nim.) A position with  $n$  piles of size 1 is losing iff  $n$  is odd.

**3.17 Tree Hashing**

$$f(u) = sz[u] * \sum_{i=0} f(v) * p^i; f(v) \text{ are sorted } f(child) = 1$$

**3.18 Permutation**

To maximize the sum of adjacent differences of a permutation, it is necessary and sufficient to place the smallest half numbers in odd position and the greatest half numbers in even position. Or, vice versa.

**3.19 String**

- If the sum of length of some strings is  $N$ , there can be at most  $\sqrt{N}$  distinct length.

- A Text can have at most  $O(N \times \sqrt{N})$  distinct substrings that match with given patterns where the sum of the length of the given patterns is  $N$ .

- Period =  $n \% (n - \text{pi.back}() == 0)? n - \text{pi.back}(): n$

- The first (*period*) cyclic rotations of a string are distinct. Further cyclic rotations repeat the previous strings.

- $S$  is a palindrome if and only if its period is a palindrome.

- If  $S$  and  $T$  are palindromes, then the periods of  $S \ T$  are same if and only if  $S + T$  is a palindrome.

**3.20 Bit**

- $(a \text{ xor } b)$  and  $(a + b)$  has the same parity
- $(a + b) = (a \text{ xor } b) + 2(a \& b)$
- $\text{gcd}(a, b) \leq a - b \leq \text{xor}(a, b)$

**3.21 Convolution**

- Hamming Distance: Replace 0 with -1 - SQRT Decomposition: Find block size,  $B = \sqrt{8 * n}$