



Daffodil International University

DIU_DividedByZero

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Team Reference Document

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1 Code

1.1

CP_Ubuntu

```
{
  "cmd": ["ulimit -s 268435456; g++ -std=c++20 $file_name -o $file_base_name &&
    \ timeout 4s ./ $file_base_name < inputf.in > outputf.in"],
  "selector": "source.cpp",
  "shell": true,
  "working_dir": "$file_path"
}
```

1.2

CP_Windows

```
{
  "cmd": ["g++.exe", "-std=c++20", "${file}", "-o", "${file_base_name}.exe", "&&" ,
    \ "${file_base_name}.exe<inputf.in>outputf.in"],
  "selector": "source.cpp",
  "shell": true,
  "working_dir": "$file_path"
}
```

1.3 StressTesting(check.sh)

```
// chmod u+x check.sh
// ./check.sh
```

```
set -e
g++ gen.cpp -o gen
g++ code.cpp -o code
g++ brute.cpp -o brute
for ((i = 1; ; ++i)); do
  echo "Passed on TestCase: " $i
  ./gen $i > in
  ./code < in > out1
  ./brute < in > out2
  diff -Z out1 out2 || break
done
echo -e "WA on the following test:"
cat in
echo -e "\nExpected:"
```

```
cat out2
echo -e "\nFound:"
cat out1
```

1.4 StressTesting(gen.cpp)

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;

mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());

inline ll gen_random(ll l, ll r) {
    return uniform_int_distribution<ll>(l, r)(rng);
}

inline double gen_random_real(double l, double r) {
    return uniform_real_distribution<double>(l, r)(rng);
}

int main(int argc, char* args[]) {
    int _ = atoi(args[1]);
    rng.seed(_);
    int n = gen_random(1, 5);
    vector<int> per;
    for (int i = 0; i < n; ++i) {
        per.push_back(i + 1);
    }
    shuffle(per.begin(), per.end(), rng);
    return 0;
}
```

1.5 Aho Corasic

```
//number of occurence of word in a text
const ll N = 1e6+10, A = 26;
ll trie[N][A], pos[N], slink[N], dp[N], tot = 1;
vector<int> order;
void initTrie(){
    order.clear();
    while(tot--){
        memset(trie[tot], 0, sizeof(trie[tot]));
    }
    memset(pos, 0, sizeof(pos));
    memset(slink, 0, sizeof(slink));
    memset(dp, 0, sizeof(dp)); tot = 1;
}

void addStr(string &s, int ind){
    ll u = 0;
    for(auto it: s){
        ll n = it-'a';
        if(trie[u][n]==0) trie[u][n] = tot++;
        u = trie[u][n];
    } pos[ind] = u;
}

void build(){
    queue<ll> q; q.push(0);
    while(!q.empty()){
        ll p = q.front(); q.pop();
        order.push_back(p);
        for(ll c = 0; c<A; c++){
            ll u = trie[p][c];
            if(!u) continue;
            q.push(u);
            if(!p) continue;
            ll v = slink[p];
            while(v && !trie[v][c]) v = slink[v];
            slink[u] = trie[v][c];
        }
    }
}

void trav(string &s){
    ll u = 0;
    for(char c: s){
        c-='a';
        while(u && !trie[u][c]) u = slink[u];
        u = trie[u][c]; dp[u]++;
    }
}
```

```

    }
    reverse(order.begin(), order.end());
    for(auto u: order){
        dp[slink[u]]+=dp[u];
    }
}

void solve(){
    ll n; cin>>n;
    string text; cin>>text;
    string s;
    for(ll i = 0; i<n; i++){
        cin>>s; addStr(s, i);
    }
    build(); trav(text);
    for(ll i = 0; i<n; i++){
        cout<<dp[pos[i]]<<endl;
    }
}

int32_t main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    ll tc = 1;
    cin>>tc;
    for(ll i = 1; i<=tc;i++){
        cout<<"Case " <<i<<": \n";
        initTrie();
        solve();
    }
}
```

1.6 Articulation Point

```
int n; // number of nodes
vector<vector<int>> lst; // adjacency list of graph
vector<bool> vis;
vector<int> tin, low;
int timer;
void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
    int children = 0;
    for (int v : lst[u]) {
        if (v == p) continue;
        if (vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] >= tin[u] && p != -1){
                IS_CUTPOINT(u);
            }
            ++children;
        }
    }
    // if no vertex below v can reach u or higher removing u disconnects that
    // subtree
    if (p == -1 && children > 1){
        IS_CUTPOINT(u);
    }
}

void find_cutpoints() {
    timer = 0;
    vis.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!vis[i]){
            dfs(i);
        }
    }
}
```

1.7 BFS

```
vector<vector<int>> adj; // adjacency list representation
int n; // number of nodes
int s; // source vertex

void bfs() {
    queue<int> q;
    vector<int> d(n), p(n);
    vector<bool> used(n);
    q.push(s);
    used[s] = true;
    p[s] = -1;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        for (int u : adj[v]) {
            if (!used[u]) {
                used[u] = true;
                q.push(u);
                d[u] = d[v] + 1;
                p[u] = v;
            }
        }
    }
    // retrieving shortest path
    if (!used[u]) {
        cout << "No path!";
    } else {
        vector<int> path;
        for (int v = u; v != -1; v = p[v])
            path.push_back(v);
        reverse(path.begin(), path.end());
        cout << "Path: ";
        for (int v : path)
            cout << v << " ";
    }
}
```

1.8 Bellman Ford

```
#define ll long long
#define INF 1e18

void solve() {
    int n, m, v;
    cin >> n >> m >> v; // n = nodes, m = edges, v = source (0-indexed)
    vector<array<ll, 3>> edges(m); // each edge: {a, b, cost}
    for (int i = 0; i < m; i++) cin >> edges[i][0] >> edges[i][1] >> edges[i][2];
    vector<ll> d(n, INF);
    vector<int> p(n, -1);
    d[v] = 0;
    int x = -1;
    for (int i = 0; i < n; i++) {
        x = -1;
        for (auto& e : edges) {
            int a = e[0], b = e[1];
            ll cost = e[2];
            if (d[a] < INF && d[b] > d[a] + cost) {
                d[b] = max(-INF, d[a] + cost);
                p[b] = a;
                x = b;
            }
        }
    }
    if (x == -1) {
        cout << "No negative cycle from vertex " << v << '\n';
        return;
    }
    int y = x;
    for (int i = 0; i < n; i++) y = p[y];
    vector<int> path;
```

```
for (int cur = y;; cur = p[cur]) {
    path.push_back(cur);
    if (cur == y && path.size() > 1) break;
}
reverse(path.begin(), path.end());
cout << "Negative cycle: ";
for (int u : path) cout << u << ' ';
cout << '\n';
}
```

1.9 BridgeFinding

```
const int MX = 1e5 + 10;
int n, m, timer = 0;
vector<int> adj[MX];
vector<int> tin(MX, -1), low(MX, -1);
vector<bool> vis(MX, false);
void is_bridge(int u, int v) {
    // do something with the edge
}
void dfs(int u, int p = -1) {
    vis[u] = true;
    tin[u] = low[u] = timer++;
    for (int v : adj[u]) {
        if (v == p) continue;
        if (vis[v]) {
            low[u] = min(low[u], tin[v]);
        } else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (low[v] > tin[u]) {
                is_bridge(u, v);
            }
        }
    }
}
```

1.10 Centroid Decomposition

```
const int N = 2e5 + 5;
int n, k, sz[N], centered[N], ans = 0;
vector<int> adj[N];
void dfs_sz(int u, int p) {
    sz[u] = 1;
    for (auto v : adj[u]) {
        if (v != p && !centered[v]) {
            dfs_sz(v, u);
            sz[u] += sz[v];
        }
    }
}
int get_cen(int u, int p, int n) {
    for (auto v : adj[u]) {
        if (v != p && !centered[v] && sz[v] > n/2) {
            return get_cen(v, u, n);
        }
    }
    return u;
}
int t, tin[N], tout[N], nodes[N], dis[N];
void dfs(int u, int p) {
    nodes[t] = u;
    tin[u] = t++;
    for (auto v : adj[u]) {
        if (v != p && !centered[v]) {
            dis[v] = dis[u] + 1;
            dfs(v, u);
        }
    }
    tout[u] = t - 1;
}
void go(int u) {
    dfs_sz(u, u);
    int c = get_cen(u, u, sz[u]);
    centered[c] = 1;
    sz[c] = sz[u];
    t = 0;
    dis[c] = 0;
    dfs(c, c);
}
```

```

int cnt[t][1];
for(auto v: adj[c]){
    if(centred[v]) continue;
    for(int i = tin[v]; i<=tout[v]; ++i){
        int w = nodes[i];
        if(k-dis[w]>=0 && k-dis[w]<t){
            ans+=cnt[k-dis[w]];
        }
    }
    for(int i = tin[v]; i<=tout[v]; ++i){
        int w = nodes[i]; cnt[dis[w]]++;
    }
}
for(auto v: adj[c]){
    if(!centred[v]){ go(v); }
}
}

void solve() {
    cin>>n>>k;
    for(ll i = 1; i<n; i++){
        ll u, v; cin>>u>>v;
        adj[u].push_back(v);
        adj[v].push_back(u);
    } go(1);
    cout<<ans<<endl;
}

```

1.11 Coin Change(Number of Ways)

```

const int mod = 1e9+7;
void solve(){
    int n, k; cin>>n>>k;
    vector<int> coin(n);
    for(int i = 0; i<n; i++){ cin>>coin[i]; }
    vector<int> dp(k+1, 0); dp[0] = 1;
    for(int i = 1; i<=k; i++){
        for(int j = 0; j<n; j++){
            if(i-coin[j]>=0){
                dp[i] = (dp[i]+dp[i-coin[j]])%mod;
            }
        }
    } cout<<dp[k]<<endl;
}

```

1.12 Cycle Detection in DAG

```

const int MX = 1e5 + 10;
bool vis[MX], pathVis[MX];
vector<int> lst[MX];
bool dfs(int u) {
    vis[u] = true;
    pathVis[u] = true;
    for (auto v : lst[u]) {
        if (!vis[v]) {
            if (dfs(v))
                return true;
        } else if (pathVis[v]) {
            return true;
        }
    }
    pathVis[u] = false;
    return false;
}

void solve() {
    // take graph input
    for (int i = 0; i < n; ++i) {
        if (!vis[i])
            dfs(i);
    }
}

```

1.13 DSU

```

const int MX = 1e5 + 10;
int par[MX], sz[MX];

```

```

void init() {
    for (int i = 1; i < MX; i++) {
        par[i] = i;
        sz[i] = 1;
    }
}

int findpar(int x) {
    if (par[x] == x) return x;
    return par[x] = findpar(par[x]);
}

void unite(int u, int v) {
    u = findpar(u);
    v = findpar(v);
    if (u != v) {
        if (sz[u] < sz[v]) {
            swap(u, v);
        }
        sz[u] += sz[v];
        par[v] = u;
    }
}

```

1.14 DSUOnTrees

```

int n, color[MX], ans[MX];
vector<int> g[MX];
set<int> bucket[MX];
int merge(int a, int b) {
    if (bucket[a].size() < bucket[b].size()) swap(a, b);
    bucket[a].insert(bucket[b].begin(), bucket[b].end());
    bucket[b].clear();
    return a;
}

int dfs(int u, int p = -1) {
    int cur = u;
    for (int v : g[u])
        if (v != p)
            cur = merge(cur, dfs(v, u));
    ans[u] = (int)bucket[cur].size();
    return cur;
}

void solve() {
    cin >> n;
    for (int i = 0; i < n; ++i) {
        cin >> color[i];
        bucket[i].insert(color[i]);
    }
    // graph input
    dfs(0);
    // print output
}

```

1.15 Digit DP

```

vector<int> nmbrs;
int dp[10][10][2];
int dgt_dp(int idx, int tight, int oneCnt) {
    if (idx == nmbrs.size()) {
        return oneCnt;
    }
    if (dp[idx][oneCnt][tight] != -1) return dp[idx][oneCnt][tight];
    int lmt = (tight ? nmbrs[idx] : 9);
    int sum = 0;
    for (int i = 0; i <= lmt; i++) {
        bool newTight = (tight & i == nmbrs[idx]);
        sum += dgt_dp(idx + 1, newTight, oneCnt + (i == 1));
    }
    return dp[idx][oneCnt][tight] = sum;
}

```

1.16 Dijkstra

```

const int N = 1e5 + 5, INF = 1e18 + 7;
vector<pair<int, int>> g[N];
bool visited[N];
vector<int> dist(N, INF), parent(N);
bool dijkstra(int source) {
    priority_queue<pair<int, int>, vector<pair<int, int>>, greater<pair<int, int>>>
        _pq;
    pq.push({0, source});
    dist[source] = 0;
    parent[source] = -1;
    while (pq.size()) {
        int x = pq.top().second;
        pq.pop();
        if (visited[x]) continue;
        visited[x] = 1;
        for (auto [child x, child wt] : g[x]) {
            if (dist[x] + child wt < dist[child_x]) {
                parent[child_x] = x;
                dist[child_x] = child wt + dist[x];
                pq.push({dist[child_x], child_x});
            }
        }
    }
    return (dist[n] == INF);
}

```

1.17 Dijkstra CP Algo

```

const int INF = 1000000000;
vector<vector<pair<int, int>>> adj;
void dijkstra(int s, vector<int>& d, vector<int>& p) {
    int n = adj.size();
    d.assign(n, INF);
    p.assign(n, -1);
    d[s] = 0;
    using pii = pair<int, int>;
    priority_queue<pii, vector<pii>, greater<pii>> q;
    q.push({0, s});
    while (!q.empty()) {
        int v = q.top().second;
        int d_v = q.top().first;
        q.pop();
        if (d_v != d[v]) continue;
        for (auto edge : adj[v]) {
            int to = edge.first;
            int len = edge.second;
            if (d[v] + len < d[to]) {
                d[to] = d[v] + len;
                p[to] = v;
                q.push({d[to], to});
            }
        }
    }
}

```

1.18 Euler Tour

```

const int MX = 2e5 + 10;
int timer = -1;
// s = start pos, e = end pos
int val[MX], s[MX], e[MX], flat[MX];
vector<int> lst[MX];
void dfs(int u, int p) {
    s[u] = ++timer;
    flat[timer] = val[u];
    for (auto v : lst[u]) {
        if (v != p)
            dfs(v, u);
    }
}

```

```

e[u] = timer;
}

```

1.19 FloydWarshall

```

vector<vector<int>> d(n, vector<int>(n, INF));
// take graph input into d
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (d[i][k] < INF && d[k][j] < INF)
                d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
        }
    }
}

```

1.20 GP Hash Table

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
const int RANDOM = chrono::high_resolution_clock::now().time_since_epoch().count();
struct custom_hash {
    int operator()(int x) const { return x ^ RANDOM; }
};
//gp_hash_table<int, int, custom_hash> mp;

```

1.21 Integer Points in a Circle

```

ll latticeInCircle(ll r){
    ll ans = (4*r) + 1; // 1 for center
    for(int i = 1; i*i<=r*r; i++){
        for(int j = 1; j*j+i*i<=r*r; j++){ ans+=4; }
    }
    return ans;
}

```

1.22 LCA using binary Lifting

```

int n, l;
vector<vector<int>> adj;
int timer;
vector<int> tin, tout;
vector<vector<int>> up;
void dfs(int v, int p) {
    tin[v] = ++timer;
    up[v][0] = p;
    for (int i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];
    for (int u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }
    tout[v] = ++timer;
}
bool is_ancestor(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}
int lca(int u, int v) {
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (int i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}
void preprocess(int root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<int>(l + 1));
}

```

```
dfs(root, root);
}
```

1.23 LCA

```
const int N = 1e5 + 5;
vector<int> g[N], parent(N), depth(N, 0);
void dfs(int vertex, int par = -1) {
    parent[vertex] = par;
    for (auto child : g[vertex]) {
        if (child != par) {
            depth[child] = depth[vertex] + 1; dfs(child, vertex);
        }
    }
}
int lca(int x, int y) {
    int diff = min(depth[x], depth[y]);
    while (depth[x] > diff) x = parent[x];
    while (depth[y] > diff) y = parent[y];
    while (x != y) { x = parent[x]; y = parent[y]; } return x;
}
```

1.24 LCS for 3 Strings

```
string a, b, c;
ll dp[55][55][55];
ll lcs(ll i, ll j, ll k) {
    if (i == a.size() or j == b.size() or k == c.size()) return 0;
    if (dp[i][j][k] != -1) return dp[i][j][k];
    if (a[i] == b[j] and a[i] == c[k]) return 1 + lcs(i + 1, j + 1, k + 1);
    ll ans = 0;
    ans = max(ans, lcs(i, j, k + 1));
    ans = max(ans, lcs(i, j + 1, k));
    ans = max(ans, lcs(i + 1, j, k));
    return dp[i][j][k] = ans;
}
```

1.25 LIS

```
vector<int> lis(int n, vector<int>& v) {
    vector<int> parent(n, -1), ind(n);
    vector<int> lis;
    for (int i = 0; i < n; i++) {
        int it = lower_bound(lis.begin(), lis.end(), v[i]) - lis.begin();
        if (it == lis.size()) {
            lis.push_back(v[i]);
            ind[lis.size() - 1] = i;
            parent[i] = (lis.size() == 1 ? -1 : ind[it - 1]);
        } else {
            lis[it] = v[i];
            ind[it] = i;
            parent[i] = (it == 0 ? -1 : ind[it - 1]);
        }
    }
    vector<int> LIS;
    int it = ind[lis.size() - 1];
    LIS.push_back(lis.back());
    while (parent[it] != -1) {
        it = parent[it];
        LIS.push_back(v[it]);
    }
    return LIS;
}
```

1.26 Lazy Propagation

```
class stree {
    vector<int> seg, lazy;
public:
    segtree(int n) {
        seg.resize(4 * n + 5);
        lazy.resize(4 * n + 5);
    }
    void propagate(int i, int low, int high) {
```

```
if (lazy[i] != 0) {
    seg[i] += (high - low + 1) * lazy[i];
    if (low != high) {
        lazy[2 * i + 1] += lazy[i];
        lazy[2 * i + 2] += lazy[i];
    }
    lazy[i] = 0;
}
}
void build(int i, int low, int high, int arr[]) {
    if (low == high) {
        seg[i] = arr[low];
        return;
    }
    int mid = (low + high) >> 1;
    build(2 * i + 1, low, mid, arr);
    build(2 * i + 2, mid + 1, high, arr);
    seg[i] = seg[2 * i + 1] + seg[2 * i + 2];
}
void update(int i, int low, int high, int l, int r, int val) {
    propagate(i, low, high);
    if (high < l or r < low) return;
    if (low >= l and high <= r) {
        seg[i] += (high - low + 1) * val;
        if (low != high) {
            // has children
            lazy[2 * i + 1] += val;
            lazy[2 * i + 2] += val;
        }
        return;
    }
    int mid = (low + high) >> 1;
    update(2 * i + 1, low, mid, l, r, val);
    update(2 * i + 2, mid + 1, high, l, r, val);
    seg[i] = seg[2 * i + 1] + seg[2 * i + 2];
}
int query(int i, int low, int high, int l, int r) {
    propagate(i, low, high);
    if (high < l or r < low) return 0;
    if (low >= l and high <= r) return seg[i];
    int mid = (low + high) >> 1;
    int left = query(2 * i + 1, low, mid, l, r);
    int right = query(2 * i + 2, mid + 1, high, l, r);
    return left + right;
}
};
```

1.27 MST

```
// DSU first
void solve() {
    int n, m;
    cin >> n >> m;
    vector<tuple<int, int, int>> edges;
    for (int i = 0; i < m; ++i) {
        int u, v, wt;
        cin >> u >> v >> wt;
        edges.push_back({wt, u, v});
    }
    sort(edges.begin(), edges.end());
    init(n);
    int cost = 0;
    for (tuple<int, int, int> & t : edges) {
        if (findpar(u) == findpar(v)) continue;
        unite(u, v);
        cost += wt;
    }
    cout << cost << endl;
}
```


1.28 Manacher_palindrome

```
// pal[1][i] = longest odd (half rounded down) palindrome around pos i and
// starts at i - pal[1][i] and ends at i + pal[1][i] pal[0][i] = half length of
// longest even palindrome around pos i, i + 1 and starts at i - par[0][i] + 1
// and ends at i + pal[0][i]
const int N = 5e5 + 10;
int pal[2][N];
void manacher(string& s) {
    int n = s.size(), idx = 2;
    while (idx--> 0) {
        for (int l = -1, r = -1, i = 0; i < n - 1; ++i) {
            if (i > r)
                l = r = i;
            else {
                int k = min(r - i, pal[idx][l + r - i]);
                l = i - k, r = i + k;
            }
            while (l - idx >= 0 and r + 1 < n and s[l - idx] == s[r + 1]) l--, r++;
            pal[idx][i] = r - i;
            // [l - 1 + idx : r] palindrome
        }
    }
}
```

1.29 MatExpo

```
const ll mod = 1e9;
vector<vector<ll>> matMul(vector<vector<ll>>& a, vector<vector<ll>>& b) {
    ll row1 = a.size(), col1 = a[0].size();
    ll row2 = b.size(), col2 = b[0].size();
    vector<vector<ll>> res(row1, vector<ll>(col2, 0));
    for (ll i = 0; i < row1; i++) {
        for (ll j = 0; j < col1; j++) {
            for (ll k = 0; k < row2; k++) {
                res[i][j] = (res[i][j] + (1LL * a[i][k] * b[k][j]) % mod) % mod;
            }
        }
    }
    return res;
}
vector<vector<ll>> matExpo(vector<vector<ll>>& Mat, ll exp) {
    ll row = Mat.size(), col = Mat[0].size();
    ll p = row;
    vector<vector<ll>> res(p, vector<ll>(p, 0));
    for (ll i = 0; i < p; i++) res[i][i] = 1;
    while (exp) {
        if (exp & 1) res = matMul(res, Mat);
        Mat = matMul(Mat, Mat);
        exp >>= 1;
    }
    return res;
}
// b = (A(i), A(i-1), A(i-2), A(i-3))
// M = Magic matrix, nth = nth term, known = known value
ll get_nth(ll nth, ll known, vector<ll>& b, vector<vector<ll>>& M) {
    if (nth <= known) return b[nth - 1] % mod;
    reverse(b.begin(), b.end());
    vector<vector<ll>> me = matExpo(M, nth - known); // MAT^(nth-known)
    ll ans = 0;
    for (int i = 0; i < known; i++) {
        ans = (ans + (b[i] * me[i][0]) % mod) % mod;
    }
    return ans;
}
```

1.30 Matrix Rotation

```
//90* clock-wise
now = {{0, 1, 0}, {-1, 0, 0}, {0, 0, 1}};
//90* anti-clock
now = {{0, -1, 0}, {1, 0, 0}, {0, 0, 1}};
//mirror with x axis at point p
now = {{-1, 0, 2 * p}, {0, 1, 0}, {0, 0, 1}};
//mirror with y axis at point p
now = {{1, 0, 0}, {0, -1, 2 * p}, {0, 0, 1}};
```

```
op[i + 1] = matMul(now, op[i]); // this
// op[i + 1] = matMul(op[i], now); //not this
```

1.31 Max Bipartite Matching [Hopcroft Karp]

```
const int INF = 1e9;
void hopcroftCarp() {
    int n, m, e;
    cin >> n >> m >> e;
    vector<int> adj[n];
    for (int i = 0; i < e; ++i) {
        int u, v;
        cin >> u >> v;
        --u;
        --v;
        adj[u].push_back(v);
    }
    vector<int> ml(m, -1), mr(n, -1), dist(n);
    auto bfs = [&]() -> bool {
        queue<int> q;
        for (int u = 0; u < n; ++u) {
            if (mr[u] == -1) {
                dist[u] = 0;
                q.push(u);
            } else {
                dist[u] = INF;
            }
        }
        bool foundAugmenting = false;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int v : adj[u]) {
                int pairedLeft = ml[v];
                if (pairedLeft == -1) {
                    foundAugmenting = true;
                } else if (dist[pairedLeft] == INF) {
                    dist[pairedLeft] = dist[u] + 1;
                    q.push(pairedLeft);
                }
            }
        }
        return foundAugmenting;
    };
    function<bool(int)> dfs = [&](int u) -> bool {
        for (int v : adj[u]) {
            int pairedLeft = ml[v];
            if (pairedLeft == -1 or (dist[pairedLeft] == dist[u] + 1 and
                ~ dfs(pairedLeft))) {
                mr[u] = v;
                ml[v] = u;
                return true;
            }
        }
        dist[u] = INF;
        return false;
    };
    int matching = 0;
    while (bfs()) {
        for (int u = 0; u < n; ++u) {
            if (mr[u] == -1) {
                if (dfs(u)) matching++;
            }
        }
    }
    cout << matching << endl;
    for (int u = 0; u < n; ++u) {
        if (mr[u] != -1) {
            cout << u << " " << mr[u] << endl;
        }
    }
}
```


1.32 Max Bipartite Matching [Kuhn's]

```
// left set size, right set size, edge count
int n, k, m, visToken = 1;
vector<int> lst[MX];
int mr[MX], ml[MX], vis[MX];
bool try_kuhn(int u) {
    if (vis[u] == visToken)
        return false;
    vis[u] = visToken;
    for (auto v : lst[u]) {
        if (ml[v] == -1 or try_kuhn(ml[v])) {
            ml[v] = u;
            mr[u] = v;
            return true;
        }
    }
    return false;
}
void solve() {
    cin >> n >> k >> m;
    for (int i = 0; i < m; ++i) {
        int u, v;
        cin >> u >> v;
        --u, --v;
        lst[u].push_back(v);
    }
    fill(mr, mr + n, -1);
    fill(ml, ml + k, -1);
    int ans = 0;
    for (int u = 0; u < n; ++u) {
        for (auto v : lst[u]) {
            if (ml[v] == -1) {
                ml[v] = u;
                mr[u] = v;
                ans++;
                break;
            }
        }
    }
    for (int u = 0; u < n; ++u) {
        if (mr[u] != -1) continue;
        visToken++;
        if (try_kuhn(u))
            ans++;
    }
    cout << ans << el;
    for (int v = 0; v < k; ++v) {
        if (ml[v] != -1) {
            cout << ml[v] + 1 << " " << v + 1 << el;
        }
    }
}
```

1.33 Max Subarray Size Sum equal K

```
//write gpHashTable code before this part
void solution() {
    int n, k; cin >> n >> k;
    int total_sum = 0;
    vector<int> pre(n + 7, 0);
    for (int i = 1; i <= n; i++) {
        int temp; cin >> temp;
        total_sum += temp;
        if (i == 1) pre[i] = temp;
        else pre[i] = pre[i - 1] + temp;
    }
    if (total_sum < k) {
        cout << "-1" << endl; return;
    }
    if (total_sum == k) {
        cout << "0" << endl; return;
    }
    int maximum_subSize = 0;
    gp_hash_table<int, int, customHash> table;
    for (int i = 1; i <= n; i++) {
```

```
    if (pre[i] >= k) {
        int subSUM = pre[i] - k;
        if (subSUM == 0) {
            maximum_subSize = max(maximum_subSize, i);
        }
        else if (table[subSUM]) {
            int left = table[subSUM];
            int right = i; int subSize = right - left;
            maximum_subSize = max(subSize, maximum_subSize);
        }
    }
    if (!table[pre[i]]) table[pre[i]] = i;
} cout << maximum_subSize << endl; }
```

1.34 Maximum Subarray Sum(Kadanes Algo)

```
int max_sum_of(vector<int> &vct){
    int mx = INT_MIN, till = 0;
    for (int i = 0; i < vct.size(); i++) {
        till = till + vct[i];
        mx = max(mx, till);
        till = max(till, 1LL*0);
    }
    return mx;
}
```

1.35 Mex of all subarray

```
const int N = 1e5 + 9, inf = 1e9;
struct ST {
    int t[4 * N];
    ST() {}
    void build(int n, int b, int e) {
        t[n] = 0;
        if (b == e) {
            return;
        }
        int mid = (b + e) >> 1, l = n << 1, r = l | 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        t[n] = min(t[l], t[r]);
    }
    void upd(int n, int b, int e, int i, int x) {
        if (b > i || e < i) return;
        if (b == e && b == i) {
            t[n] = x;
            return;
        }
        int mid = (b + e) >> 1, l = n << 1, r = l | 1;
        upd(l, b, mid, i, x);
        upd(r, mid + 1, e, i, x);
        t[n] = min(t[l], t[r]);
    }
    int get_min(int n, int b, int e, int i, int j) {
        if (b > j || e < i) return inf;
        if (b >= i && e <= j) return t[n];
        int mid = (b + e) >> 1, l = n << 1, r = l | 1;
        int L = get_min(l, b, mid, i, j);
        int R = get_min(r, mid + 1, e, i, j);
        return min(L, R);
    }
    int get_mex(int n, int b, int e, int i) { // mex of [i... cur_id] if (b == e)
        return b;
        int mid = (b + e) >> 1, l = n << 1, r = l | 1;
        if (t[l] >= i) return get_mex(r, mid + 1, e, i);
        return get_mex(l, b, mid, i);
    }
} t;
int a[N], f[N];
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n;
```

```

cin >> n;
for (int i = 1; i <= n; i++) {
    cin >> a[i];
    --a[i];
}
t.build(1, 0, n);
set<array<int, 3>> seg; // for cur_id = i, [x[0]... i], [x[0] + 1...i],
// ...[x[1]... i] has mex, , x[2]
for (int i = 1; i <= n; i++) {
    int x = a[i];
    int r = min(i - 1, t.get_min(1, 0, n, 0, x - 1));
    int l = t.get_min(1, 0, n, 0, x) + 1;
    if (l <= r) {
        auto it = seg.lower_bound({l, -1, -1});
        while (it != seg.end() && (*it)[1] <= r) {
            auto x = *it;
            it = seg.erase(it);
        }
    }
    t.upd(1, 0, n, x, i);
    for (int j = r; j >= l; j++) {
        int m = t.get_mex(1, 0, n, j);
        int L = max(l, t.get_min(1, 0, n, 0, m) + 1);
        f[m] = 1;
        seg.insert({L, j, m});
        j = L - 1;
    }
    int m = !a[i];
    seg.insert({1, i, m});
    f[m] = 1;
}
int ans = 0;
while (f[ans]) ++ans;
cout << ans + 1 << '\n';
return 0;
}

```

1.36 Number of Pairs with gcd equal g

```

/*a[i] <= 1e6
for all 1<=g<=n, how many pairs exist such that g = gcd(a[i], a[j]);
complexity : nlogn
*/
ll n; cin >> n;
ll a[n + 1];
ll cnt[n + 1]; memset(cnt, 0, sizeof cnt);
for (ll i = 1; i <= n; i++) {cin >> a[i]; cnt[a[i]]++;}
ll gcd[n + 1]; memset(gcd, 0, sizeof gcd);
for (ll i = n; i >= 1; i--) {
    ll pair = 0, invalid_pair = 0;
    for (ll j = i; j <= n; j += i) {
        pair += cnt[j];
        invalid_pair += gcd[j];
    }
    pair = (pair * (pair - 1)) / 2;
    gcd[i] = pair - invalid_pair;
}
// how many pairs exist whose gcd is i

```

1.37 Number of Subarray Sum is K

```

//write gpHashTable code before this part
void solution() {
    int n, k; cin >> n >> k;
    int total_sum = 0;
    vector<int> pre(n + 7, 0);
    for (int i = 1; i <= n; i++) {
        int temp; cin >> temp;
        total_sum += temp;
        if (i == 1) pre[i] = temp;
        else pre[i] = pre[i - 1] + temp;
    }
    int cnt_subarray = 0;
    gp_hash_table<int, int, customHash> table;
    table[0] = 1;
    for (int i = 1; i <= n; i++) {

```

```

    cnt_subarray += table[pre[i] - k];
    table[pre[i]]++;
} cout << cnt_subarray << endl; }

```

1.38 Phi(1toN)

```

const int mxN = 1e7 + 10;
vector<int> phi(mxN);
void phi_till() { //O(n.log.log(n))
    for (int i = 0; i < mxN; i++) phi[i] = i;
    for (int i = 2; i < mxN; i++) {
        if (phi[i] == i) {
            for (int j = i; j < mxN; j += i) {
                phi[j] -= phi[j] / i;
            }
        }
    }
}

```

1.39 SOD_NOD

```

// SOD = ((P^(x+1)-1)/(P-1)) * ((Q^(y+1)-1)/(Q-1)) * ((R^(z+1)-1)/(R-1))
// NOD = P^x * Q^y * R^z => here, P, Q, R are prime factors & x, y, z are
// powers NOD = (x + 1) (y + 1) (z + 1)
pair<int, int> SOD_NOD(int n) {
    int sod = 1, nod = 1;
    for (int i = 2; i * i <= n; ++i) {
        if (n % i == 0) {
            int pown = 1, pows = 0;
            while (n % i == 0) {
                pown *= i; // p^e
                pows++;
                n /= i;
            }
            sod *= (pown - 1) / (i - 1); // p^e+1
            nod *= (pows + 1); // (p^e+1)-1 / p-1
        }
    }
    if (n > 1) {
        sod *= (n + 1);
        nod *= 2;
    }
    return {sod, nod};
}

```

1.40 Segment Tree(Lazy Propagation)

```

class stree {
    vector<int> seg, lazy;
public:
    segtree(int n) {
        seg.resize(4 * n + 5);
        lazy.resize(4 * n + 5);
    }
    void propagate(int i, int low, int high) {
        if (lazy[i] != 0) {
            seg[i] += (high - low + 1) * lazy[i];
            if (low != high) {
                lazy[2 * i + 1] += lazy[i];
                lazy[2 * i + 2] += lazy[i];
            }
            lazy[i] = 0;
        }
    }
    void build(int i, int low, int high, int arr[]) {
        if (low == high) {
            seg[i] = arr[low];
            return;
        }
        int mid = (low + high) >> 1;
        build(2 * i + 1, low, mid, arr);
        build(2 * i + 2, mid + 1, high, arr);
    }

```

```

    seg[i] = seg[2 * i + 1] + seg[2 * i + 2];
}
void update(int i, int low, int high, int l, int r, int val) {
    propagate(i, low, high);
    if (high < l or r < low) return;
    if (low >= l and high <= r) {
        seg[i] += (high - low + 1) * val;
        if (low != high) {
            // has children
            lazy[2 * i + 1] += val;
            lazy[2 * i + 2] += val;
        }
        return;
    }
    int mid = (low + high) >> 1;
    update(2 * i + 1, low, mid, l, r, val);
    update(2 * i + 2, mid + 1, high, l, r, val);
    seg[i] = seg[2 * i + 1] + seg[2 * i + 2];
}
int query(int i, int low, int high, int l, int r) {
    propagate(i, low, high);
    if (high < l or r < low) return 0;
    if (low >= l and high <= r) return seg[i];
    int mid = (low + high) >> 1;
    int left = query(2 * i + 1, low, mid, l, r);
    int right = query(2 * i + 2, mid + 1, high, l, r);
    return left + right;
}
};

```

1.41 Segment Tree

```

class stree {
    vector<int> seg;
public:
    segtree(int n) {
        seg.assign(4 * n + 5, 0);
    }
    void build(int ind, int low, int high, int arr[]) {
        if (low == high) {
            seg[ind] = arr[low];
            return;
        }
        int mid = (low + high) >> 1;
        build(2 * ind + 1, low, mid, arr);
        build(2 * ind + 2, mid + 1, high, arr);
        seg[ind] = min(seg[2 * ind + 1], seg[2 * ind + 2]);
    }
    int query(int ind, int low, int high, int l, int r) {
        if (r < low or high < l) return INT_MAX;
        if (low >= l and high <= r) return seg[ind];
        int mid = (low + high) / 2;
        int left = query(2 * ind + 1, low, mid, l, r);
        int right = query(2 * ind + 2, mid + 1, high, l, r);
        return min(left, right);
    }
    void update(int ind, int low, int high, int i, int val) {
        if (low == high) {
            seg[ind] = val;
            return;
        }
        int mid = (low + high) / 2;
        if (i <= mid) update(2 * ind + 1, low, mid, i, val);
        else update(2 * ind + 2, mid + 1, high, i, val);
        seg[ind] = min(seg[2 * ind + 1], seg[2 * ind + 2]);
    }
};

```

1.42 Segmented Sieve

```

void segSieve(ll low, ll high) {
    vector<bool> area((high - low) + 1, true);
    for (ll i = 0; primes[i] * primes[i] <= high; i++) {

```

```

        ll start = ((low / primes[i]) * primes[i]);
        if (start < low) start += primes[i];
        for (ll j = start; j <= high; j += primes[i]) {
            if (j == primes[i]) continue;
            area[j - low] = false;
        }
    }
    for (ll i = 0; i < (high - low) + 1; i++) {
        if (area[i]) {
            if (i + low != 1 and i + low != 0) {
                cout << i + low << endl;
            }
        }
    }
}

```

1.43 SparseTable

```

const int mxN = 1e5 + 10, M = 21;
int sparse[mxN][M];
void build_sparse(int n, vector<int>& v) {
    for (int i = 0; i < n; i++) sparse[i][0] = v[i];
    for (int k = 1; k < M; k++) {
        for (int i = 0; i + (1 << k) <= n; i++) {
            sparse[i][k] = max(sparse[i][k - 1], sparse[i + (1 << (k - 1))][k - 1]);
        }
    }
}
int query(int l, int r) { // 0 based index
    if (l > r) swap(l, r);
    int b = __bit_width(r - l + 1) - 1;
    return max(sparse[l][b], sparse[r - (1 << b) + 1][b]);
}

```

1.44 StrHash

```

const int mod1 = 911382323, mod2 = 972663749, b1 = 137, b2 = 139;
const int mxN = 5000010;
int pow_b1[mxN], pow_b2[mxN], inv_b1[mxN], inv_b2[mxN];
int binExp(int base, int power, int mod) {
    int res = 1;
    while (power) {
        if (power & 1) res = (1LL * res * base) % mod;
        base = (1LL * base * base) % mod;
        power >>= 1;
    }
    return res;
}
void pre() {
    pow_b1[0] = pow_b2[0] = 1;
    for (int i = 1; i < mxN; i++) {
        pow_b1[i] = (1LL * pow_b1[i - 1] * b1) % mod1;
        pow_b2[i] = (1LL * pow_b2[i - 1] * b2) % mod2;
    }
    inv_b1[mxN - 1] = binExp(pow_b1[mxN - 1], mod1 - 2, mod1);
    inv_b2[mxN - 1] = binExp(pow_b2[mxN - 1], mod2 - 2, mod2);
    for (int i = mxN - 2; i >= 0; i--) {
        inv_b1[i] = (1LL * inv_b1[i + 1] * b1) % mod1;
        inv_b2[i] = (1LL * inv_b2[i + 1] * b2) % mod2;
    }
}
vector<pair<int, int>> getPref(string& s) {
    int qq = s.size();
    vector<pair<int, int>> hsh(qq);
    for (int i = 0; i < qq; i++) {
        if (i == 0) {
            hsh[i].first = (1LL * s[i] * pow_b1[i]) % mod1;
            hsh[i].second = (1LL * s[i] * pow_b2[i]) % mod2;
        } else {
            hsh[i].first =
                (hsh[i - 1].first + (1LL * s[i] * pow_b1[i]) % mod1) % mod1;
            hsh[i].second =

```

```

        (hsh[i - 1].second + (1LL * s[i] * pow_b2[i]) % mod2) % mod2;
    }
    return hsh;
}
pair<int, int> getHash(string& str) {
    int hsh1 = 0, hsh2 = 0, sz = str.size();
    for (int i = 0; i < sz; ++i) {
        hsh1 = (hsh1 + 1LL * str[i] * pow_b1[i] % mod1) % mod1;
    }
    for (int i = 0; i < sz; ++i) {
        hsh2 = (hsh2 + 1LL * str[i] * pow_b2[i] % mod2) % mod2;
    }
    return {hsh1, hsh2};
}
pair<int, int> getSub(int l, int r, vector<pair<int, int>>& v) {
    pair<int, int> q;
    if (l == 0) {
        q = {v[r].first, v[r].second};
    } else {
        int x = (1LL * ((v[r].first - v[l - 1].first + mod1) % mod1) * inv_b1[l]) % mod1;
        int y = (1LL * ((v[r].second - v[l - 1].second + mod2) % mod2) * inv_b2[l]) % mod2;
        q = {x, y};
    }
    return q;
}

```

1.45 StrHash_2

```

const int N = 1000010, MOD = 1e9 + 7;
const ll P[] = {97, 1000003};
ll bigMod(ll a, ll e) {
    if (e == -1) e = MOD - 2;
    ll ret = 1;
    while (e) {
        if (e & 1) ret = ret * a % MOD;
        a = a * a % MOD, e >>= 1;
    }
    return ret;
}
ll pwr[2][N], inv[2][N];
void initHash() {
    for (int it = 0; it < 2; ++it) {
        pwr[it][0] = inv[it][0] = 1;
        ll INV_P = bigMod(P[it], -1);
        for (int i = 1; i < N; ++i) {
            pwr[it][i] = pwr[it][i - 1] * P[it] % MOD;
            inv[it][i] = inv[it][i - 1] * INV_P % MOD;
        }
    }
}
struct RangeHash {
    vector<ll> h[2], rev[2];
    RangeHash(const string S, bool revFlag = 0) {
        for (int it = 0; it < 2; ++it) {
            h[it].resize(S.size() + 1, 0);
            for (int i = 0; i < S.size(); ++i) {
                h[it][i + 1] = (h[it][i] + pwr[it][i + 1] * (S[i] - 'a' + 1)) % MOD;
            }
            if (revFlag) {
                rev[it].resize(S.size() + 1, 0);
                for (int i = 0; i < S.size(); ++i) {
                    rev[it][i + 1] = (rev[it][i] + inv[it][i + 1] * (S[i] - 'a' + 1)) % MOD;
                }
            }
        }
    }
}
inline ll get(int l, int r) {
    ll one = (h[0][r + 1] - h[0][l]) * inv[0][l + 1] % MOD;
    ll two = (h[1][r + 1] - h[1][l]) * inv[1][l + 1] % MOD;
}

```

```

    if (one < 0) one += MOD;
    if (two < 0) two += MOD;
    return one << 31 | two;
}
inline ll getReverse(int l, int r) {
    ll one = (rev[0][r + 1] - rev[0][l]) * pwr[0][r + 1] % MOD;
    ll two = (rev[1][r + 1] - rev[1][l]) * pwr[1][r + 1] % MOD;
    if (one < 0) one += MOD;
    if (two < 0) two += MOD;
    return one << 31 | two;
}
};

```

1.46 TopologicalSorting

```

const int N = 1e5 + 10;
vector<int> g[N], indegree(N, 0);
vector<int> topSort(int n) {
    queue<int> q;
    vector<int> order;
    for (int i = 1; i <= n; i++) {
        if (indegree[i] == 0) {
            q.push(i);
        }
    }
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        order.push_back(u);
        for (int v : g[u]) {
            indegree[v]--;
            if (indegree[v] == 0) {
                q.push(v);
            }
        }
    }
    return order;
}

```

1.47 UniquePF of all elements till MX

```

const int MX = 2e5 + 10;
vector<int> pfac[MX];
void factorize() {
    for (int i = 2; i < MX; i++) {
        if (!pfac[i].empty()) continue;
        for (int j = i; j < MX; j += i) {
            pfac[j].push_back(i);
        }
    }
}

```

1.48 WeightedUnionFind

```

const int MX = 2e5 + 10;
int par[MX], sz[MX];
ll d[MX];
void init() {
    for (int i = 0; i < MX; ++i) {
        par[i] = i;
        sz[i] = 1;
        d[i] = 0;
    }
}
int findpar(int x) {
    if (par[x] == x) return x;
    int p = par[x];
    par[x] = findpar(p);
    d[x] += d[p];
    return par[x];
}
bool unite(int a, int b, ll w) {
    int ra = findpar(a);
    int rb = findpar(b);
}

```

```

    if (ra == rb) {
        return (d[b] - d[a] == w);
    }
    if (sz[ra] < sz[rb]) {
        swap(a, b);
        swap(ra, rb);
        w = -w;
    }
    par[rb] = ra;
    d[rb] = d[a] + w - d[b];
    sz[ra] += sz[rb];
    return true;
}

dist(int a, int b) {
    findpar(a), findpar(b);
    return d[b] - d[a];
}

```

1.49 allInOneNT

```

const int MAXN = 1e6 + 9;
typedef struct info {
    int lowest_prime = 0, greatest_prime = 0, distinct_prime = 0;
    int total_prime = 0, NOD = 0, SOD = 0;
} info;
info num[MAXN];
void preStore() {
    for (int i = 2; i < MAXN; i++) {
        int n = i;
        map<int, int> factors; // Key->Factor, Val->count
        int SOD = 1, NOD = 1, total_p_factor = 0;
        if (n % 2 == 0) {
            while (n % 2 == 0) {
                n /= 2;
                factors[2]++;
                total_p_factor++;
            }
            SOD *= (1 << (factors[2] + 1)) - 1;
            NOD *= (factors[2] + 1);
        }
        for (int i = 3; i * i <= n; i += 2) {
            if (n % i == 0) {
                while (n % i == 0) {
                    n /= i;
                    factors[i]++;
                    total_p_factor++;
                }
                SOD *= (pow(i, factors[i] + 1) - 1) / (i - 1);
                NOD *= (factors[i] + 1);
            }
        }
        if (n > 1) {
            factors[n]++;
            SOD *= (pow(n, 2) - 1) / (n - 1);
            NOD *= 2;
            total_p_factor++;
        }
        num[i].distinct_prime = factors.size();
        num[i].total_prime = total_p_factor;
        num[i].NOD = NOD;
        num[i].SOD = SOD;
        auto lowest_prime = factors.begin();
        auto greatest_prime = factors.rbegin();
        num[i].lowest_prime = lowest_prime->first;
        num[i].greatest_prime = greatest_prime->first;
    }
}

```

1.50 customHash

```

#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
struct customHash {
    static uint64_t Meaw(uint64_t x) {
        x += 0x9e3779b97f4a7c15;

```

```

        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_since_epoch().count();
        return Meaw(x + FIXED_RANDOM);
    }
}; // gp_hash_table<int, int> table;

```

1.51 divisorSieve

```

const int mxN = 1e5 + 10;
vector<int> divisors[mxN];
void divisorSieve() {
    for (int i = 1; i < mxN; i++) {
        for (int j = i; j < mxN; j += i) {
            divisors[j].push_back(i);
        }
    }
}

```

1.52 int128

```

__int128 read() {
    __int128 x = 0, f = 1;
    char ch = getchar();
    while (ch < '0' || ch > '9') {
        if (ch == '-') f = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9') {
        x = x * 10 + ch - '0';
        ch = getchar();
    }
    return x * f;
}

void print(__int128 x) {
    if (x < 0) {
        putchar('-');
        x = -x;
    }
    if (x > 9) print(x / 10);
    putchar(x % 10 + '0');
}

```

1.53 mergeSort

```

// use array of elements, if multiple testcase make inv = 0 each time
// int inv = 0;
void merge(int vct[], int l, int m, int r) {
    int left = m - l + 1, right = r - m, lv[left], rv[right];
    for (int i = 0; i < left; i++) {
        lv[i] = vct[l + i];
    }
    for (int i = 0; i < right; i++) {
        rv[i] = vct[m + 1 + i];
    }
    int i = 0, j = 0, to = l;
    while (i < left && j < right) {
        if (lv[i] <= rv[j]) {
            vct[to] = lv[i];
            i++;
        } else {
            vct[to] = rv[j];
            j++;
        }
        // inversion count
        // int pore = left - i; inv += pore;
        to++;
    }
    while (i < left) {

```

```

    vct[to] = lv[i];
    i++;
    to++;
}
while (j < right) {
    vct[to] = rv[j];
    j++;
    to++;
}
}
void merge_sort(int vct[], int l, int r) {
    if (r <= l) return;
    int m = l + ((r - l) / 2);
    merge_sort(vct, l, m);
    merge_sort(vct, m + 1, r);
    merge(vct, l, m, r);
}

```

1.54 nCr and nPr

```

int fact[N], ifact[N];
void prec() {
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = 1LL * fact[i - 1] * i % mod;
    }
    ifact[N - 1] = power(fact[N - 1], -1);
    for (int i = N - 2; i >= 0; i--) {
        ifact[i] = 1LL * ifact[i + 1] * (i + 1) % mod;
    }
}
int nPr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[n - r] % mod;
}
int nCr(int n, int r) {
    if (n < r) return 0;
    return 1LL * fact[n] * ifact[r] % mod * ifact[n - r] % mod;
}

```

1.55 nCr anup

```

const int MX = 1e6 + 10;
const int M = 1e9 + 7;
int fact[MX], inv_fact[MX];
int modPow(int a, int b) {
    int ans = 1;
    while (b) {
        if (b & 1) ans = (1LL * ans * a) % M;
        a = (1LL * a * a) % M;
        b >>= 1;
    }
    return ans;
}
void precalFact() {
    fact[0] = inv_fact[0] = 1;
    for (int i = 1; i < MX; i++) {
        fact[i] = (1LL * fact[i - 1] * i) % M;
    }
    inv_fact[MX - 1] = modPow(fact[MX - 1], M - 2);
    for (int i = MX - 2; i >= 1; i--) {
        inv_fact[i] = (1LL * inv_fact[i + 1] * (i + 1)) % M;
    }
}
int nCr(int n, int r) {
    if (r < 0 or r > n) return 0;
    return 1LL * fact[n] * inv_fact[r] % M * inv_fact[n - r] % M;
}

```

1.56 pbds

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#include <functional>
using namespace __gnu_pbds;

```

```

typedef tree<int, null_type, less<int>, rb_tree_tag,
            tree_order_statistics_node_update>
ordered_set;
// s.order_of_key(x) = number of elements strictly less than x
// *s.find_by_order(i) = ith element in set (0 index)

```

1.57 phi

```

int phi(int n) { // sqrt(n)
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0) n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}

```

1.58 polynomial_interpolation

```

// P(x) = a0 + a1x + a2x^2 + ... + anx^n
// y[i] = P(i)
const int mod = 1e9 + 7;
ll BigMod(ll a, ll b) {
    ll res = 1;
    while (b) {
        if (b & 1) res = 1LL * res * a % mod;
        a = 1LL * a * a % mod;
        b >>= 1;
    }
    return res;
}
ll inv(ll x) {
    if (x < 0) x += mod;
    return BigMod(x, mod - 2);
}
ll add(ll& a, ll b) {
    a += b;
    if (a >= mod) a -= mod;
    return a;
}
ll eval(vector<ll> y, ll k) {
    int n = y.size() - 1;
    if (k <= n) {
        return y[k];
    }
    vector<ll> L(n + 1, 1);
    for (int x = 1; x <= n; ++x) {
        L[0] = L[0] * (k - x) % mod;
        L[0] = L[0] * inv(-x) % mod;
    }
    for (int x = 1; x <= n; ++x) {
        L[x] = L[x - 1] * inv(k - x) % mod * (k - (x - 1)) % mod;
        L[x] = L[x] * ((x - 1) - n + mod) % mod * inv(x) % mod;
    }
    ll yk = 0;
    for (int x = 0; x <= n; ++x) {
        yk = add(yk, L[x] * y[x] % mod);
    }
    return yk;
}

```

1.59 sieve

```

const ll MAXN = 1e7 + 10;
bool prime[MAXN];
vector<ll> prm;
void sieve() {
    prime[0] = prime[1] = true;
    for (ll i = 2; i < MAXN; i++) {
        if (!prime[i]) {
            prm.push_back(i);
            for (ll j = i + i; j < MAXN; j += i) {

```

```
int len[N], llnk[N]{-1}, last, sz = 1;
unordered_map<char, int> to[N];
void init() {
    while (sz) {
        sz--;
        to[sz].clear();
    }
    last = 0, sz = 1;
}
void add(char c) {
    int cur = sz++;
    int u = last;
```



```
len[cur] = len[last] + 1;
while (u != -1 and !to[u].count(c)) {
    to[u][c] = cur;
    u = lnk[u];
}
if (u == -1) {
    lnk[cur] = 0;
} else {
    int v = to[u][c];
    if (len[v] == len[u] + 1) {
        lnk[cur] = v;
    } else {
        int w = sz++;
        len[w] = len[u] + 1, lnk[w] = lnk[v], to[w] = to[v];
        while (u != -1 and to[u][c] == v) {
            to[u][c] = w;
            u = lnk[u];
        }
        lnk[cur] = lnk[v] = w;
    }
}
last = cur;
}
```

1.65 trie

```
const ll N = 1e6 + 5, A = 26;
ll trie[N][A], cnt[N], tot = 1, root = 1;
void initTrie() {
    cnt[tot] = 0;
    root = 1;
}
void addStr(string& s) {
    ll u = 1;
    for (auto it : s) {
        ll n = it - 'a';
        if (trie[u][n] == 0) {
            trie[u][n] = ++tot;
        }
        u = trie[u][n];
        cnt[u]++;
    }
}
ll wordCount(string& s) {
    ll u = 1;
    for (auto it : s) {
        int n = it - 'a';
        if (trie[u][n] == 0) return 0;
        u = trie[u][n];
    }
    return cnt[u];
}
```

2 Geometry

2.1 Convex Hull

```
vector <PT> convexHull (vector <PT> p) {
    int n = p.size(), m = 0;
    if (n < 3) return p;
    vector <PT> hull(n + n);
    sort(p.begin(), p.end(), [&] (PT a, PT b) {
        return (a.x==b.x? a.y<b.y: a.x<b.x);
    });
    for (int i = 0; i < n; ++i) {
        while (m > 1 and cross(hull[m - 2] - p[i], hull[m - 1] - p[i]) <= 0) --m;
        hull[m++] = p[i];
    }
    for (int i = n - 2, j = m + 1; i >= 0; --i) {
        while (m >= j and cross(hull[m - 2] - p[i], hull[m - 1] - p[i]) <= 0) --m;
        hull[m++] = p[i];
    }
    hull.resize(m - 1); return hull;
}
```

3 Notes

3.1 Geometry

3.1.1 Triangles

Circumradius: $R = \frac{abc}{4A}$, Inradius: $r = \frac{A}{s}$

The area of a triangle using two sides and the included angle can be given as:

$$A = \frac{1}{2}ab \sin \angle C$$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc \left[1 - \left(\frac{a}{b+c} \right)^2 \right]}$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$

3.1.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

3.1.3 Spherical coordinates

$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

3.1.4 Pick's Theorem:

Given a lattice polygon with non-zero area, we define: S as the area of the polygon, I as the number of integer-coordinate points strictly inside the polygon, B as the number of integer-coordinate points on the boundary of the polygon. Then, Pick's Theorem states:

$$S = I + \frac{B}{2} - 1$$

The number of lattice points on segments (x1, y1) to (x2, y2) is: gcd(abs(x2 - x1), abs(y2 - y1)) + 1

3.1.5 Polygon

For a regular polygon with n sides and side length a , the circumradius R is given by:

$$R = \frac{a}{2\sin\left(\frac{\pi}{n}\right)}$$

3.1.6 Area of a Circular Segment

The area of a circular segment, which is the region enclosed by a chord and the corresponding arc, can be calculated using the formula:

$$A = \frac{R^2}{2} (\theta - \sin \theta)$$

where: R is the radius of the circle, θ is the central angle subtended by the chord, in radians.

3.2 Binomial Coefficient

- Factoring in: $\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}$
- Sum over k : $\sum_{k=0}^n \binom{n}{k} = 2^n$
- Alternating sum: $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$
- Even and odd sum: $\sum_{k=0}^n \binom{n}{2k} = \sum_{k=0}^n \binom{n}{2k+1} = 2^{n-1}$
- The Hockey Stick Identity
 - (Left to right) Sum over n and k : $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m-1}{m}$
 - (Right to left) Sum over n : $\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$
- Sum of the squares: $\sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$
- Weighted sum: $\sum_{k=1}^n k \binom{n}{k} = n 2^{n-1}$
- Connection with the fibonacci numbers: $\sum_{k=0}^n \binom{n-k}{k} = F_{n+1}$
- Vandermonde's Identity: $\sum_{i=0}^k \binom{m}{i} \binom{n}{k-i} = \binom{m+n}{k}$
- If $f(n, k) = C(n, 0) + C(n, 1) + \dots + C(n, k)$, Then $f(n+1, k) = 2 * f(n, k) - C(n, k)$ [For multiple $f(n, k)$ queries, use Mo's algo]

Lucas Theorem

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$$

- $\binom{m}{n}$ is divisible by p if and only if at least one of the base- p digits of n is greater than the corresponding base- p digit of m .
- The number of entries in the n th row of Pascal's triangle that are not divisible by $p = \prod_{i=0}^k (n_i + 1)$
- All entries in the $(p^k - 1)th$ row are not divisible by p .
- $\binom{n}{m} \equiv \lfloor \frac{n}{p} \rfloor! \pmod{p}$

3.3 Fibonacci Number

- $k = A - B, F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1}$
- $\sum_{i=0}^n F_i^2 = F_{n+1} F_n$
- $\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^n F_i F_{i+1} = F_{n+1}^2 - (-1)^n$
- $\sum_{i=0}^n F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1}$
- $\gcd(F_m, F_n) = F_{\gcd(m, n)}$
- $\sum_{0 \leq k \leq n} \binom{n-k}{k} = F_{n+1}$
- $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = \gcd(F_{n+1}, F_{n+2}) = 1$

3.4 Sums

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \left[(n+1)^{m+1} - 1 - \sum_{i=1}^n ((i+1)^{m+1} - i^{m+1} - (m+1)i^m) \right]$$

$$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k}$$

$$\sum_{k=0}^n k x^k = (x - (n+1)x^{n+1} + n x^{n+2}) / (x-1)^2$$

3.5 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

$$(x+a)^{-n} = \sum_{k=0}^{\infty} (-1)^k \binom{n+k-1}{k} x^k a^{-n-k}$$

Generating Function

$$1/(1-x) = 1 + x + x^2 + x^3 + \dots$$

$$1/(1-ax) = 1 + ax + (ax)^2 + (ax)^3 + \dots$$

$$1/(1-x)^2 = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$1/(1-x)^3 = C(2, 2) + C(3, 2)x + C(4, 2)x^2 + C(5, 2)x^3 + \dots$$

$$1/(1-ax)^k + 1 = 1 + C(1+k, k)(ax) + C(2+k, k)(ax)^2 + C(3+k, k)(ax)^3 + \dots$$

$$x(x+1)(1-x)^{-3} = 1 + x + 4x^2 + 9x^3 + 16x^4 + 25x^5 + \dots$$

$$e^x = 1 + x + (x^2)/2! + (x^3)/3! + (x^4)/4! + \dots$$

3.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0, k > 0, m \perp n$, and either m or n even.

3.7 Number Theory

- HCN: 1e6(240), 1e9(1344), 1e12(6720), 1e14(17280), 1e15(26880), 1e16(41472)
- $\gcd(a, b, c, d, \dots) = \gcd(a, b-a, c-b, d-c, \dots)$
- $\gcd(a+k, b+k, c+k, d+k, \dots) = \gcd(a+k, b-a, c-b, d-c, \dots)$
- Primitive root exists iff $n = 1, 2, 4, p^k, 2 \times p^k$, where p is an odd prime.
- If primitive root exists, there are $\phi(\phi(n))$ primitive roots of n .
- The numbers from 1 to n have in total $O(n \log \log n)$ unique prime factors.
- $x \equiv r_1 \pmod{m_1}$ and $x \equiv r_2 \pmod{m_2}$ has a solution iff $\gcd(m_1, m_2) | (r_1 - r_2)$ Solution of $x^2 \equiv a \pmod{p}$
- $ca \equiv cb \pmod{m} \iff a \equiv b \pmod{\frac{n}{\gcd(n, c)}}$
- $ax \equiv b \pmod{m}$ has a solution $\iff \gcd(a, m) | b$
- If $ax \equiv b \pmod{m}$ has a solution, then it has $\gcd(a, m)$ solutions and they are separated by $\frac{m}{\gcd(a, m)}$
- $ax \equiv 1 \pmod{m}$ has a solution or a is invertible $\pmod{m} \iff \gcd(a, m) = 1$
- $x^2 \equiv 1 \pmod{p}$ then $x \equiv \pm 1 \pmod{p}$
- There are $\frac{p-1}{2}$ has no solution.
- There are $\frac{p-1}{2}$ has exactly two solutions.
- When $p \% 4 = 3, x \equiv \pm a^{\frac{p+1}{4}}$
- When $p \% 8 = 5, x \equiv a^{\frac{p+3}{8}}$ or $x \equiv 2^{\frac{p-1}{4}} a^{\frac{p+3}{8}}$

3.7.1 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000. Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

3.7.2 Estimates

$\sum_{d \mid n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

3.7.3 Perfect numbers

$n > 1$ is called perfect if it equals sum of its proper divisors and 1. Even n is perfect iff $n = 2^{p-1}(2^p - 1)$ and $2^p - 1$ is prime (Mersenne’s). No odd perfect numbers are yet found.

3.7.4 Carmichael numbers

A positive composite n is a Carmichael number ($a^{n-1} \equiv 1 \pmod n$ for all a : $\gcd(a, n) = 1$), iff n is square-free, and for all prime divisors p of n , $p - 1$ divides $n - 1$.

3.7.5 Totient

- If p is a prime $(p^k) = p^k - p^{k-1}$
- If a, b are relatively prime, $\phi(ab) = \phi(a)\phi(b)$
- $\phi(n) = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})(1 - \frac{1}{p_3}) \dots (1 - \frac{1}{p_k})$
- Sum of coprime to $n = n * \frac{\phi(n)}{2}$
- If $n = 2^k, \phi(n) = 2^{k-1} = \frac{n}{2}$
- For $a, b, \phi(ab) = \phi(a)\phi(b) \frac{d}{\phi(d)}$
- $\phi(ip) = p\phi(i)$ whenever p is a prime and it divides i
- The number of $a(1 \leq a \leq N)$ such that $\gcd(a, N) = d$ is $\phi(\frac{N}{d})$
- If $n > 2, \phi(n)$ is always even
- Sum of gcd, $\sum_{i=1}^n \gcd(i, n) = \sum_{d \mid n} d \phi(\frac{n}{d})$
- Sum of lcm, $\sum_{i=1}^n \text{lcm}(i, n) = \frac{n}{2} (\sum_{d \mid n} (d \phi(d)) + 1)$
- $\phi(1) = 1$ and $\phi(2) = 1$ which two are only odd ϕ
- $\phi(3) = 2$ and $\phi(4) = 2$ and $\phi(6) = 2$ which three are only prime ϕ
- Find minimum n such that $\frac{\phi(n)}{n}$ is maximum- Multiple of small primes- $2 * 3 * 5 * 7 * 11 * 13 * \dots$

3.7.6 Mobius function

$\mu(1) = 1$. $\mu(n) = 0$, if n is not squarefree. $\mu(n) = (-1)^s$, if n is the product of s distinct primes. Let f, F be functions on positive integers. If for all $n \in \mathbb{N}, F(n) = \sum_{d \mid n} f(d)$, then $f(n) = \sum_{d \mid n} \mu(d)F(\frac{n}{d})$, and vice versa.

$\phi(n) = \sum_{d \mid n} \mu(d) \frac{n}{d}$. $\sum_{d \mid n} \mu(d) = 1$.

If f is multiplicative, then $\sum_{d \mid n} \mu(d)f(d) = \prod_{p \mid n} (1 - f(p))$, $\sum_{d \mid n} \mu(d)^2 f(d) = \prod_{p \mid n} (1 + f(p))$.

$$\sum_{i=1}^n \sum_{j=1}^n [\gcd(i, j) = 1] = \sum_{k=1}^n \mu(k) \lfloor \frac{n}{k} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n k \sum_{l=1}^{\lfloor \frac{n}{k} \rfloor} \mu(l) \lfloor \frac{n}{kl} \rfloor^2$$

$$\sum_{i=1}^n \sum_{j=1}^n \gcd(i, j) = \sum_{k=1}^n (\frac{\lfloor \frac{n}{k} \rfloor (\lfloor \frac{n}{k} \rfloor + 1)}{2})^2 \sum_{d \mid k} \mu(d)kd$$

3.7.7 Legendre symbol

If p is an odd prime, $a \in \mathbb{Z}$, then $(\frac{a}{p})$ equals 0, if $p \mid a$; 1 if a is a quadratic residue modulo p ; and -1 otherwise. Euler’s criterion: $(\frac{a}{p}) = a^{(\frac{p-1}{2})} \pmod p$.

3.7.8 Jacobi symbol

If $n = p_1^{a_1} \dots p_k^{a_k}$ is odd, then $(\frac{a}{n}) = \prod_{i=1}^k (\frac{a}{p_i})^{k_i}$.

3.7.9 Primitive roots

If the order of g modulo m (min $n > 0$: $g^n \equiv 1 \pmod m$) is $\phi(m)$, then g is called a primitive root. If Z_m has a primitive root, then it has $\phi(\phi(m))$ distinct primitive roots. Z_m has a primitive root iff m is one of 2, 4, $p^k, 2p^k$, where p is an odd prime. If Z_m has a primitive root g , then for all a coprime to m , there exists unique integer $i = \text{ind}_g(a)$ modulo $\phi(m)$, such that $g^i \equiv a \pmod m$. $\text{ind}_g(a)$ has logarithm-like properties: $\text{ind}(1) = 0, \text{ind}(ab) = \text{ind}(a) + \text{ind}(b)$.

If p is prime and a is not divisible by p , then congruence $x^n \equiv a \pmod p$ has $\gcd(n, p - 1)$ solutions if $a^{(p-1)/\gcd(n, p-1)} \equiv 1 \pmod p$, and no solutions otherwise. (Proof sketch: let g be a primitive root, and $g^i \equiv a \pmod p, g^u \equiv x \pmod p$. $x^n \equiv a \pmod p$ iff $g^{nu} \equiv g^i \pmod p$ iff $nu \equiv i \pmod p$.)

3.7.10 Discrete logarithm problem

Find x from $a^x \equiv b \pmod m$. Can be solved in $O(\sqrt{m})$ time and space with a meet-in-the-middle trick. Let $n = \lceil \sqrt{m} \rceil$, and $x = ny - z$. Equation becomes $a^{ny} \equiv ba^z \pmod m$. Precompute all values that the RHS can take for $z = 0, 1, \dots, n - 1$, and brute force y on the LHS, each time checking whether there’s a corresponding value for RHS.

3.7.11 Pythagorean triples

Integer solutions of $x^2 + y^2 = z^2$ All relatively prime triples are given by: $x = 2mn, y = m^2 - n^2, z = m^2 + n^2$ where $m > n, \gcd(m, n) = 1$ and $m \not\equiv n \pmod 2$. All other triples are multiples of these. Equation $x^2 + y^2 = 2z^2$ is equivalent to $(\frac{x+y}{2})^2 + (\frac{x-y}{2})^2 = z^2$.

3.7.12 Postage stamps/McNuggets problem

Let a, b be relatively-prime integers. There are exactly $\frac{1}{2}(a - 1)(b - 1)$ numbers *not* of form $ax + by$ ($x, y \geq 0$), and the largest is $(a - 1)(b - 1) - 1 = ab - a - b$.

3.7.13 Fermat’s two-squares theorem

Odd prime p can be represented as a sum of two squares iff $p \equiv 1 \pmod 4$. A product of two sums of two squares is a sum of two squares. Thus, n is a sum of two squares iff every prime of form $p = 4k + 3$ occurs an even number of times in n ’s factorization.

3.8 Permutations

3.8.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$\frac{n}{n!}$		$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{120}$	$\frac{1}{5040}$	$\frac{1}{40320}$	$\frac{1}{362880}$	$\frac{1}{3628800}$
$\frac{n}{n!}$		4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14		
$\frac{n}{n!}$		20	25	30	40	50	100	150	171	
$\frac{n}{n!}$		2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX	

3.8.2 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left(\sum_{n \in S} \frac{x^n}{n} \right)$$

3.8.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n - 1)(D(n - 1) + D(n - 2)) = nD(n - 1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

3.8.4 Burnside’s lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$). If $f(n)$ counts “configurations” (of some sort) of length n , we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k \mid n} f(k)\phi(n/k)$$

3.9 Partitions and subsets

3.9.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0)=1, \, p(n)=\sum_{k\in \mathbb{Z}\setminus\{0\}}(-1)^{k+1}p(n-k(3k-1)/2)$$

$$p(n)\sim 0.145/n\cdot \exp(2.56\sqrt{n})$$

$\frac{n}{p(n)}$		0	1	2	3	4	5	6	7	8	9	20	50	100
		1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

3.9.2 Partition Number

- Time Complexity: $O(n\sqrt{n})$

```
for (int i = 1; i <= n; ++i) {
    pent[2 * i - 1] = i * (3 * i - 1) / 2;
    pent[2 * i] = i * (3 * i + 1) / 2;
}
p[0] = 1;
for (int i = 1; i <= n; ++i) {
    p[i] = 0;
    for (int j = 1, k = 0; pent[j] <= i; ++j) {
        if (k < 2) p[i] = add(p[i], p[i - pent[j]]);
        else p[i] = sub(p[i], p[i - pent[j]]); ++k, k &= 3;
    }
}
```

- The number of partitions of a positive integer n into exactly k parts equals the number of partitions of n whose largest part equals k

$$p_k(n)=p_k(n-k)+p_{k-1}(n-1)$$

3.9.3 2nd Kaplansky’s Lemma

The number of ways of selecting k objects, no two consecutive,from n labelled objects arrayed in a circle is $\frac{n}{k}\binom{n-k-1}{k-1}=\frac{n}{n-k}\binom{n-k}{k}$

3.9.4 Distinct Objects into Distinct Bins

- n distinct objects into r distinct bins = r^n

- Among n distinct objects, exactly k of them into r distincts bins = $\binom{n}{k}r^k$

- n distinct objects into r distinct bins such that each bin contains at least one object = $\sum_{i=0}^r(-1)^i\binom{r}{i}(r-i)^n$

3.10 Coloring

The number of labeled undirected graphs with n vertices, $G_n=2^{\binom{n}{2}}$

The number of labeled directed graphs with n vertices, $G_n=2^{n(n1)}$

The number of connected labeled undirected graphs with n vertices, $C_n=2^{\binom{n}{2}}-\frac{1}{n}\sum_{s=1}^{n-1}k\binom{n}{k}2^{\binom{n-k}{2}}C_k=2^{\binom{n}{2}}-\sum_{k=1}^{n-1}\binom{n-1}{k-1}2^{\binom{n-k}{2}}C_k$

The number of k -connected labeled undirected graphs with n vertices, $D[n][k]=\sum_{s=1}^n\binom{n-1}{s-1}C_sD[n-s][k-1]$

Cayley’s formula: the number of trees on n labeled vertices = the number of spanning trees of a complete graph with n labeled vertices = n^{n-2}

Number of ways to color a graph using k color such that no two adjacent nodes have same color

Complete graph = $k(k-1)(k-2)...\textcolor{teal}{k-n+1}$

Tree = $k(k-1)^{n-1}$

Cycle = $(k-1)^n+(-1)^n(k-1)$

Number of trees with n labeled nodes: n^{n-2}

3.11 General purpose numbers

3.11.1 Eulerian numbers

Number of permutations $\pi\in S_n$ in which exactly k elements are greater than the previous element. $k\,j$:s s.t. $\pi(j)>\pi(j+1)$, $k+1\,j$:s s.t. $\pi(j)\geq j$, $k\,j$:s s.t. $\pi(j)>j$.

$$E(n,k)=(n-k)E(n-1,k-1)+(k+1)E(n-1,k)$$

$$E(n,0)=E(n,n-1)=1$$

$$E(n,k)=\sum_{j=0}^k(-1)^j\binom{n+1}{j}(k+1-j)^n$$

3.11.2 Bell numbers

Total number of partitions of n distinct elements. $B(n)=1,1,2,5,15,52,203,877,4140,21147,\dots$. For p prime,

$$B(p^m+n)\equiv mB(n)+B(n+1)\pmod{p}$$

3.11.3 Bernoulli numbers

$$\sum_{j=0}^m\binom{m+1}{j}B_j=0. \quad B_0=1, B_1=-\frac{1}{2}. \, B_n=0, \text{ for all odd } n\neq 1.$$

3.11.4 Catalan numbers

$$C_n=\frac{1}{n+1}\binom{2n}{n}=\binom{2n}{n}-\binom{2n}{n+1}=\frac{(2n)!}{(n+1)!n!}$$

$$C_0=1, \, C_{n+1}=\frac{2(2n+1)}{n+2}C_n, \, C_{n+1}=\sum C_iC_{n-i}$$

- $C_n=1,1,2,5,14,42,132,429,1430,4862,16796,58786,\dots$
- sub-diagonal monotone paths in an $n\times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.
- Find the count of balanced parentheses sequences consisting of $n+k$ pairs of parentheses where the first k symbols are open brackets.

$$C_n^{(k)}=\frac{k+1}{n+k+1}\binom{2n+k}{n}$$

- Recursive formula of Catalan Numbers:

$$C_n^{(k)}=\frac{(2n+k-1)\cdot(2n+k)}{n\cdot(n+k+1)}C_{n-1}^{(k)}$$

3.11.5 Lucas Number

Number of edge cover of a cycle graph C_n is L_n

$$L(n)=L(n-1)+L(n-2);L(0)=2,L(1)=1$$

3.12 Ballot Theorem

Suppose that in an election, candidate A receives a votes and candidate B receives b votes, where $a>b$ for some positive integer k . Compute the number of ways the ballots can be ordered so that A maintains more than k times as many votes as B throughout the counting of the ballots.

The solution to the ballot problem is $\frac{a-kb}{a+b}\times C(a+b,a)$

3.13 Classical Problem

$F(n,k)$ = number of ways to color n objects using exactly k colors

Let $G(n,k)$ be the number of ways to color n objects using no more than k colors.

Then, $F(n,k)=G(n,k)-C(k,1)*G(n,k-1)+C(k,2)*G(n,k-2)-C(k,3)*G(n,k-3)...$

Determining $G(n,k)$:

Suppose, we are given a $1\times n$ grid. Any two adjacent cells can not have same color. Then, $G(n,k)=k*((k-1)^{n-1})$

If no such condition on adjacent cells. Then, $G(n,k)=k^n$

3.14 Matching Formula

3.14.1 Normal Graph

MM + MEC = n (exculding vertex), IS + VC = G, MIS + MVC = G

3.14.2 Bipartite Graph

MIS = n - MBM, MVC = MBM, MEC = n - MBM

3.15 Inequalities

3.15.1 Titu’s Lemma

For positive reals a_1, a_2, \dots, a_n and b_1, b_2, \dots, b_n ,

$$\frac{a_1^2}{b_1} + \frac{a_2^2}{b_2} + \dots + \frac{a_n^2}{b_n} \geq \frac{a_1 + a_2 + \dots + a_n^2}{b_1 + b_2 + \dots + b_n}$$

Equality holds if and only if $a_i = k b_i$ for a non-zero real constant k .

3.16 Games

3.16.1 Grundy numbers

For a two-player, normal-play (last to move wins) game on a graph (V, E) : $G(x) = \text{mex}(\{G(y) : (x, y) \in E\})$, where $\text{mex}(S) = \min\{n \geq 0 : n \notin S\}$. x is losing iff $G(x) = 0$.

3.16.2 Sums of games

- *Player chooses a game and makes a move in it* Grundy number of a position is xor of grundy numbers of positions in summed games.
- *Player chooses a non-empty subset of games (possibly, all) and makes moves in all of them* A position is losing iff each game is in a losing position.
- *Player chooses a proper subset of games (not empty and not all), and makes moves in all chosen ones.* A position is losing iff grundy numbers of all games are equal.
- *Player must move in all games, and loses if can’t move in some game* A position is losing if any of the games is in a losing position.

3.16.3 Misère Nim

A position with pile sizes $a_1, a_2, \dots, a_n \geq 1$, not all equal to 1, is losing iff $a_1 \oplus a_2 \oplus \dots \oplus a_n = 0$ (like in normal nim.) A position with n piles of size 1 is losing iff n is odd.

3.17 Tree Hashing

$f(u) = sz[u] * \sum_{i=0} f(v) * p^i$; $f(v)$ are sorted $f(child) = 1$

3.18 Permutation

To maximize the sum of adjacent differences of a permuation, it is necessary and sufficient to place the smallest half numbers in odd position and the greatest half numbers in even position. Or, vice versa.

3.19 String

- If the sum of length of some strings is N , there can be at most \sqrt{N} distinct length.
- A Text can have at most $O(N \times \sqrt{N})$ distinct substrings that match with given patterns where the sum of the length of the given patterns is N .
- `Period = n % (n - pi.back() == 0)? n - pi.back(): n`
- The first (*period*) cyclic rotations of a string are distinct. Further cyclic rotations repeat the previous strings.
- S is a palindrome if and only if it’s period is a palindrome.
- If S and T are palindromes, then the periods of $S \cdot T$ are same if and only if $S + T$ is a palindrome.

3.20 Bit

- $(a \text{ xor } b)$ and $(a + b)$ has the same parity
- $(a + b) = (a \text{ xor } b) + 2(a \cdot b)$
- $\text{gcd}(a, b) \leq a - b \leq \text{xor}(a, b)$

3.21 Convolution

- Hamming Distance: Replace 0 with -1 - SQRT Decomposition: Find block size, $B = \sqrt{(8 * n)}$