Date
06/9/2024

Engineering Mechanics

Basic Concepts

Newton's law of motion

① Ist law of Motion: (law of Inertia)
An object at rest remains at rest, at constant speed unless acted by forces.

② IInd law of Motion:
The acceleration of an object depends on mass of the object and amount of force applied.

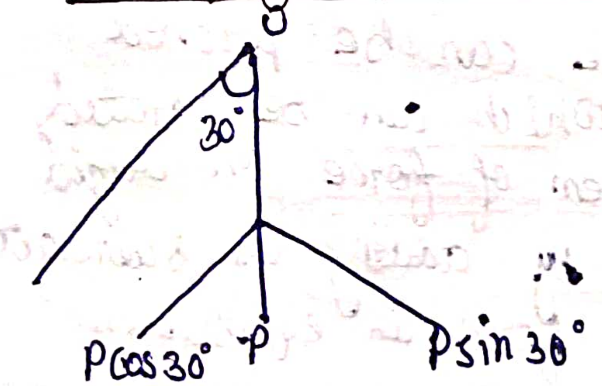
③ IIIrd law of Motion:
Whenever one object exerts a force on another, the second object exerts an equal and opposite on first.

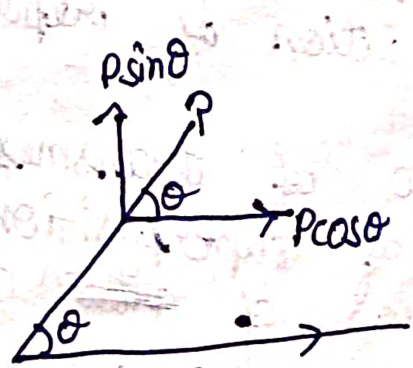
Types of forces:

- Contact forces: Contact b/w two bodies eg. (push door, play basketball)
(Hook's law)

- Non-Contact forces: Don't have contact b/w two bodies.
eg: Gravitational force; Nuclear force.

Imp: Breaking Component:





Law of parallelogram:

Total force acting on a body when two forces acting simultaneously on it.

Law of Transmissibility:

- The effect of force on a rigid body remain unchanged when force applied at any point along line of action (used for analysis of external force).
- A force is a physical quantity than can be brought into equil with gravity.

Law of Action & Reaction:

- It states that force always has contradicting force of same magnitude but of opp. direction. Force can never exist alone.
- The forces that two bodies exert upon each other are of same magnitude but of opposite direction & they lie on same line of action.

Define the principle of transmissibility of forces:

The state of the rest or motion of rigid body is unaltered if a force acting on body is replaced by another force of same magnitude and direction but acting anywhere on body along the line of action of replaced force.

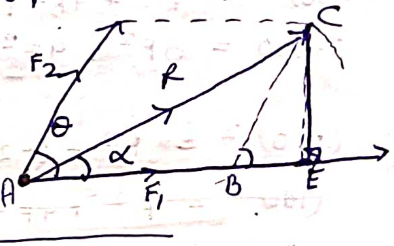
The law of transmissible can be proved using law of superposition which can be stated as action of ~~given~~ given system of force on rigid body is not changed by adding is subtract another system of force; in equilibrium.

Limitations:

(I) The law of transm. of forces can be applied only along same line of action of forces.

Parallelogram law of forces:

If two forces acting simultaneously on body at AC; are represented in magnitude and direction by two adjacent sides of llgm, their resultant is represented in magnitude and direction by the diagonal of llgm which passes through AC of intersection of two sides represent forces.



In ΔACE

$$R = AC = \sqrt{AE^2 + CE^2}$$
$$= \sqrt{(AB+BE)^2 + (CE)^2}$$
$$= \sqrt{(AB)^2 + (BE)^2 + 2AB \cdot BE + (CE)^2}$$
$$= \sqrt{(F_1)^2 + (BC\cos\theta)^2 + 2F_1 BC\cos\theta + (BC\sin\theta)^2}$$
$$= \sqrt{F_1^2 + F_2^2\cos^2\theta + 2F_1F_2\cos\theta + F_2^2\sin^2\theta}$$
$$= \sqrt{F_1^2 + F_2^2(\sin^2\theta + \cos^2\theta) + 2F_1F_2\cos\theta}$$
$$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

In ΔCAE

$$\tan\alpha = \frac{CE}{AE}$$
$$= \frac{CE}{AB+BE} = \frac{BC\sin\theta}{F_1 + BC\cos\theta} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

Que. The resultant of two forces one of which is double the other is 260N. If the direction of larger force is reversed & other remain unaltered, the resultant reduces to 180N. Determine magnitude & angle b/w forces.

$F_1 = F \quad ; F_2 = 2F$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$260 = \sqrt{F^2 + 4F^2 + 4F^2\cos\theta}$

$(260)^2 = F^2 + 4F^2 + 4F^2\cos^2\theta$ — ①

$67600 = 5F^2 + 4F^2\cos\theta$

$(180)^2 = F^2 + (-2F)^2 + 2F(-2F)\cos(180+\theta)$ — II

$\cancel{(100)^2 = F^2 + 4F^2 - 4}$

$(180)^2 = F^2 + 4F^2 - 4F^2\cos\theta$

$32400 = 5F^2 - 4F^2\cos\theta$

$67600 = 5F^2 + 4F^2\cos\theta$

$100000 = 10F^2$

$F_1 = 100$

$F_2 = 200$

$\cancel{5(100)^2 + 4(100)^2\cos\theta = 67600}$

$\boxed{\cancel{\cos\theta = 0.44}}$

$32400 = 50000 - 40000\cos\theta$

$40000\cos\theta = 17600$

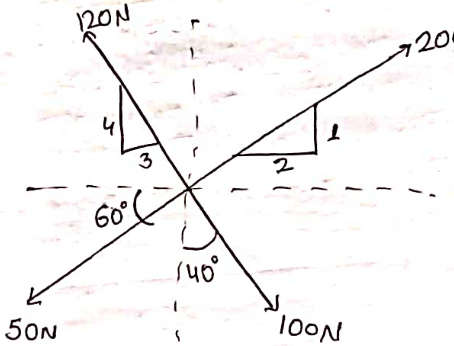
$\cos\theta = \frac{17600}{40000}$

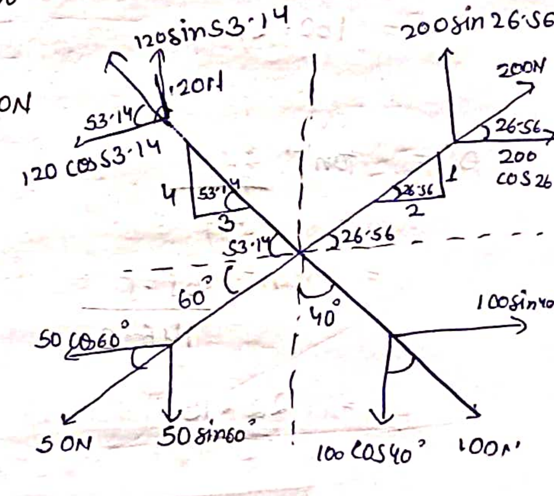
$\boxed{\theta = 63.89}$

Equilibrium Condition

$\Sigma f_x = 0 \quad \Sigma f_y = 0 \quad \Sigma M = 0$

Que. A system of four forces acting a point on a body shown in fig. Determine the resultant





$R = \sqrt{\Sigma(f_x)^2 + \Sigma(f_y)^2}$

$\Sigma f_x = 0$

$200\cos 26.56° + (-120\cos 53.14°) + (-50\cos 60°) + 100\sin 40° = 0$

$\cancel{178.8933227 - 71.98341526 =}$

$\Sigma f_x = 146.1886685 = 0$

$\boxed{\Sigma f_x = 146.2\text{ N}}$

$\Sigma f_y = 0$

$200\sin 26.56 + 120\sin 53.14 + (-50\sin 60°) + (-100\cos 40°) = 0$

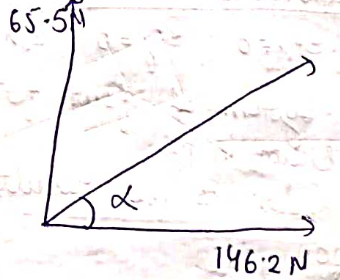
$\Sigma f_y = 65.53367009 = 0$

$\boxed{\Sigma f_y = 65.53\text{N}}$

$R = \sqrt{(146.2)^2 + (65.5)^2}$

$= \sqrt{21374.44 + 4290.25}$

$R = 160.2\,N$



$\alpha = \tan^{-1} \frac{\Sigma fx}{\Sigma fy}$

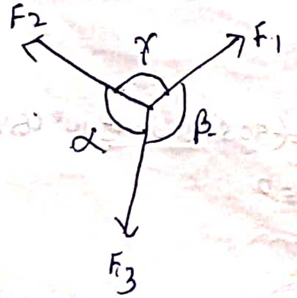
$= \tan^{-1} \frac{65.5}{146.2}$

$= \underline{22.13}$

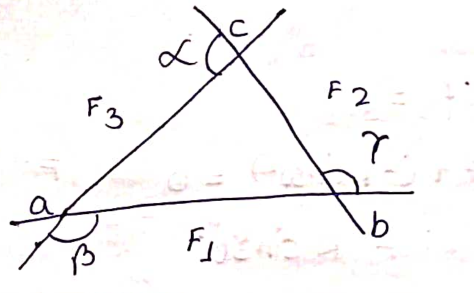
Lamie Theorem

If a body is in equillibrium under the action of only three forces, each is proportional to the sine of the angle b/w the other two forces.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



applying sinerule of Δ

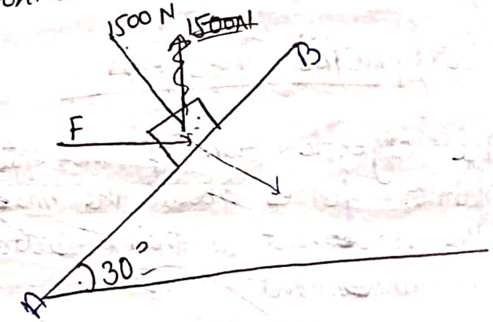


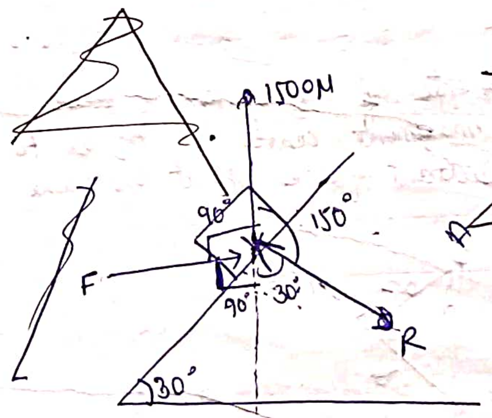
$$\frac{ab}{\sin(180^\circ-\alpha)} = \frac{bc}{\sin(180^\circ-\beta)} = \frac{ac}{\sin(180^\circ-\gamma)}$$

$$\frac{ab}{\sin\alpha} = \frac{bc}{\sin\beta} = \frac{ac}{\sin\gamma}$$

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

Qu: Determine the horizontal force F to be applied to the block weighing 1500 N to hold it in position on a smooth inclined plane AB which makes an angle of 30° with the horizontal.





$$\frac{1500}{\sin(120^\circ)} = \frac{F}{\sin(150^\circ)} = \frac{R}{\sin(90^\circ)}$$

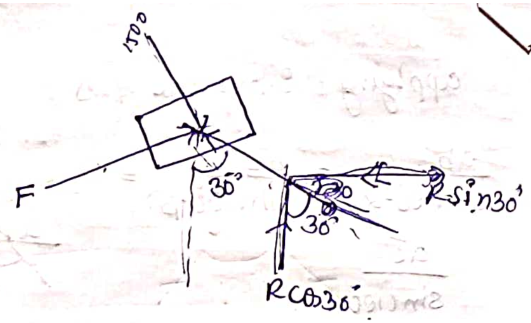
$F = 866.02\,N$

$R = 1732.05\,N$

$\Sigma f_x = 0$

$F + (-R\sin 30^\circ) = 0$

$F = R\sin 30^\circ$



$\Sigma f_y = 0$

$1500 - R\cos 30^\circ = 0$

$R\cos 30^\circ = 1500$

$R = \dfrac{1500}{\cos 30^\circ}$

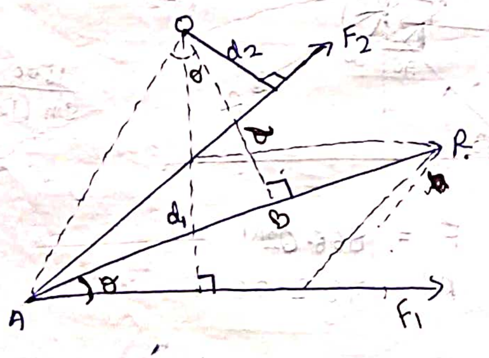
$\boxed{R = 1732.05}$

$F = 1732.05 \sin 30^\circ$

$\boxed{F = 866.025}$

* Varignon's Theorem

The algebraic sum of moments of a system of ~~co~~ co-planner forces about the moment center is equal to the moment of their resultant force about the same moment centre.



let the resultant (R) makes an angle θ with x-axis. Noting that AOB is also θ we can write

$Rd = R \times AO \cos\theta$

$= AO\,(R\cos\theta)$

$\boxed{Rd = AO\,R_x}$ — ①

Similarly,

$F_1 d_1 = AO\,F_1x$ — ②

$F_2 d_2 = AO\,F_2x$ — ③

adding eqn ② & ③

$F_1 d_1 + F_2 d_2 = AO\,F_1x + AO\,F_2x$

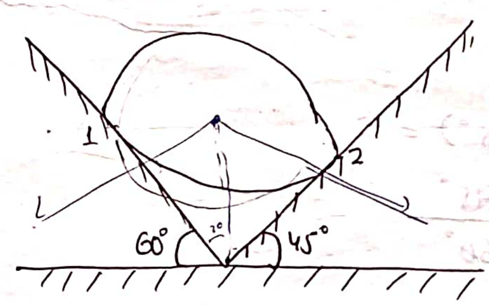
$F_1 d_1 + F_2 d_2 = AO\,(F_1x + F_2x)$

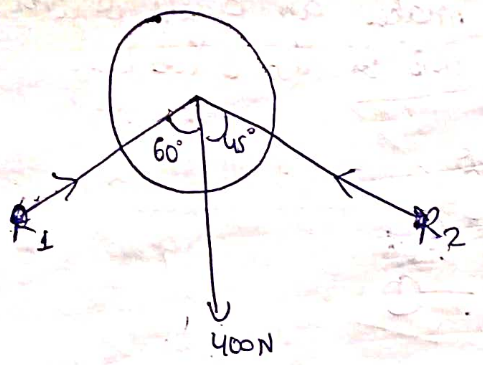
$F_1 d_1 + F_2 d_2 = AO\,R_x$

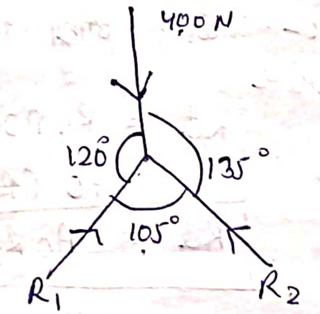
$\boxed{F_1 d_1 + F_2 d_2 = Rd}$

Hence proved

Que: A 400N sphere is resting in a through as shown in fig. determine the rean developed at the contact surfaces assume all contact surfaces are smooth.

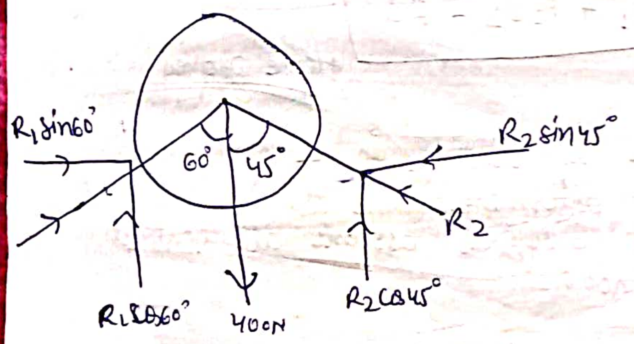






$$\frac{400}{\sin 105°} = \frac{R_1}{\sin(135°)} = \frac{R_2}{\sin 120°}$$

$$\boxed{R_1 = 292.82N}$$
$$\boxed{R_2 = 358.63\,N}$$



$\Sigma f_x = 0$

$R_1 \sin 60° + (-R_2 \sin 45°) = 0$

$$R_1 = \frac{R_2 \sin 45°}{\sin 60°}$$

$\Sigma f_y = 0$

$R_1 \cos 60° + R_2 \cos 45° - 400 = 0$

$$\cos 60° \times \frac{R_2 \sin 45°}{\sin 60°} + R_2 \cos 45° - 400 = 0$$

$$\cos 60° \, R_2 \sin 45° + R_2 \cos 45° \sin 60° - 400 \sin 60° = 0$$

$$R_2 (\sin 45° \cos 60° + \cos 45° \sin 60°) = 400 \sin 60°$$

$$R_2 = \frac{400 \sin 60°}{\cos 60° \sin 45° + \cos 45° \sin 60°}$$

$$R_2 = \frac{400 \sin 60°}{\sin(60° + 45°)}$$

$$= \frac{400 \sin 60°}{\sin(105°)}$$

$$= 358.63\,N$$

$$R_1 = 358.63 \frac{\sin 45°}{\sin 60°}$$

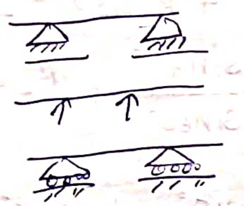
$$\boxed{R_1 = 292.82N}$$

Types of Support

① Hinge Support

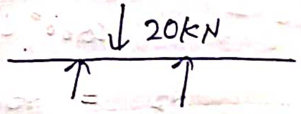
② Sinple Support

③ Roller Support

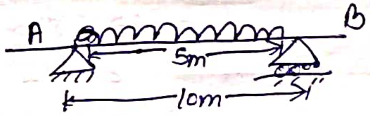


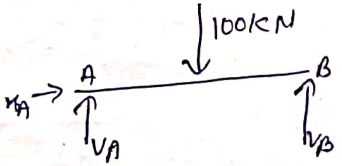
Types of load

① Point load or concentrated load

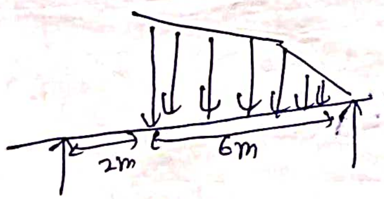


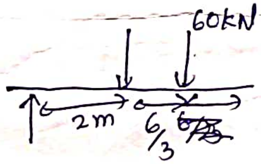
② Uniform distributed load





③ Varying ~~Vorrg~~ load





Types of Beam:

① Cantilever

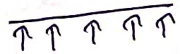
② Simply supported (One end hinged one end Roller)

④ Overhanging

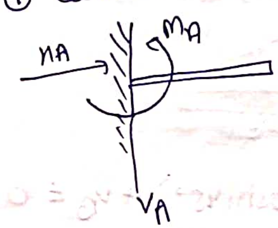
⑤ Both end hinged

⑥ Propped cantilever

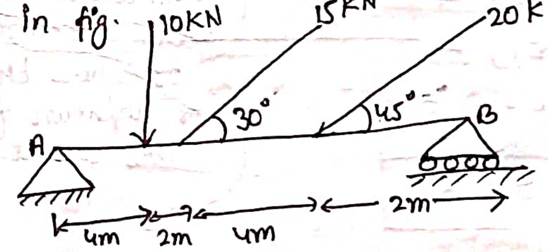
⑦ Contineous

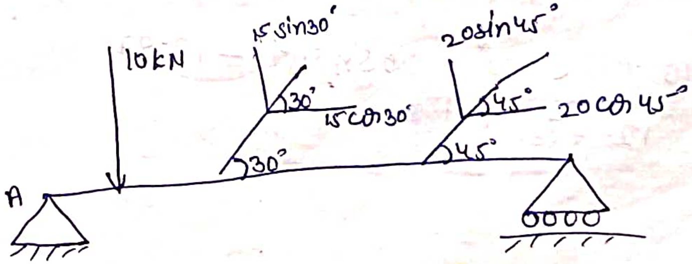


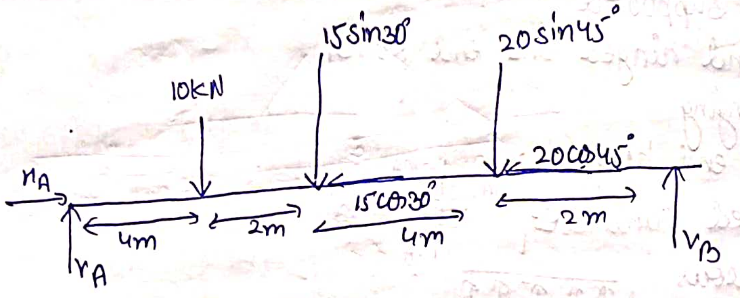
① Cantilever



Que: The beam AB of spend ~~spread~~ 12 m shown in fig. is hinge at A and is on roller at B. ~~Determine~~ the rean developed at A & B due to loading shown in fig.







$\Sigma f_x = 0$

$H_A + (-15\cos 30°) + (-20\cos 45°) = 0$

$\boxed{H_A = 27.13\ kN}$

$\Sigma f_y = 0$

$V_A + (-10) + (-15\sin 30°) + (-20\sin 45°) + V_B = 0$

$\Sigma M_A = 0$

$10 \times 4 + (15\sin 30°) \times (4+2) + (20\sin 45°) \times (4+2+4) - V_B \times (4+2+4+2) = 0$

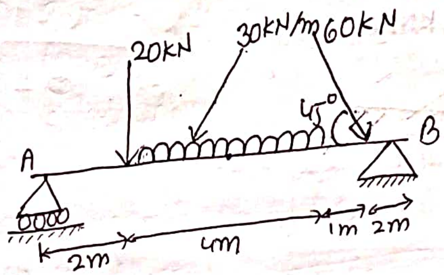
$\boxed{V_B = 18.87\ kN}$

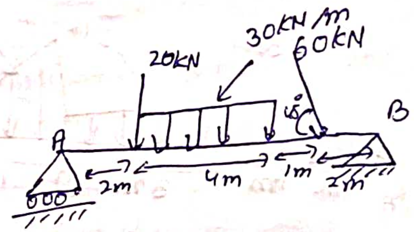
$V_A = 10 + 15\sin 30° + 20\sin 45° - 18.87$

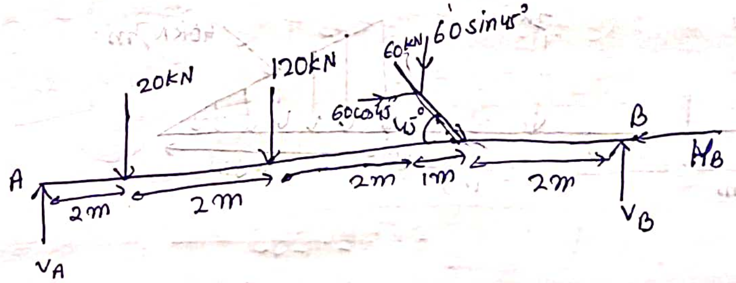
$\boxed{V_A = 12.772\ kN}$

Note:
moment always take from where line of action are more or where known or unknown forces are present

Que: Find the rear's developed at support A & B of the loaded beam shown in fig.







$\Sigma f_x = 0$

$60\cos 45° + (-H_B) = 0$

$\boxed{H_B = 42.426\ kN}$

$\Sigma f_y = 0$

$V_A + (-20) + (-120) + (-60\sin 45°) + V_B = 0$

$\boxed{V_B = 90.776\ kN}$

ΣM_B

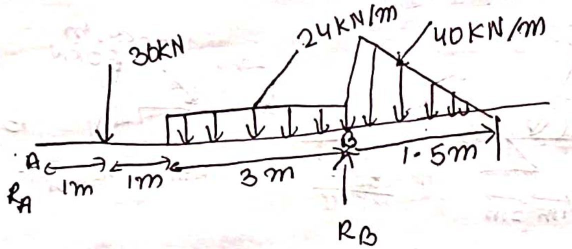
$60\sin 45° \times 2 + 120 \times (2+1+2) + 20 \times (2+2+1+2) - V_A \times (2+\cancel{2}2+2+1+2)$

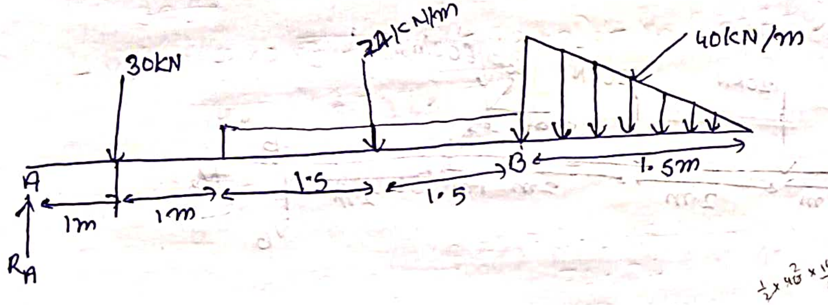
$\cancel{2V_A = 60\sin 45° \times 2 =}$

$\cancel{V_A = 412.426\ kN}$

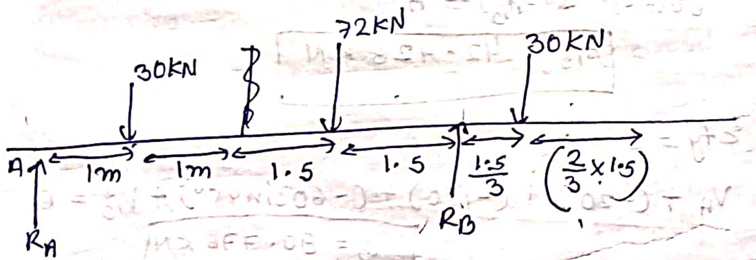
$V_A = 91.65\ kN$

Qu: Determine the reac's at support A & B of the overhanging beam shown in fig.





$\frac{1}{2} \times 40 \times \frac{1.5}{10}$



$\Sigma F_X = 0$

$R_A + (-30) + (-72) + R_B + (-30) = 0$

$\cancel{R_A + R_B - 72 = 0}$

$R_A + R_B = 72$

$R_A = 72 - 89.4$

$\Sigma M_A = 0$

$30 \times 1 + 72(1+1+1.5) - R_B(1.\hat{5}) + 30\left(\frac{1.5}{3}\right) = 0$

$1+1+1.5$

$30 + 72(1+1+1.5) - R_B(1+1+1.5+1.5) + 30\left(\frac{1.5}{3}\right) = 0$

$+1+1+1.5+1.5$

$$\boxed{R_B = 89.4\ KN}$$

$\cancel{R_A + 89.4 - 72 = 0}$

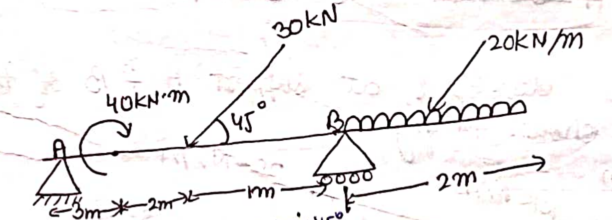
$\cancel{\boxed{R_A = 42.6\ KN}}$

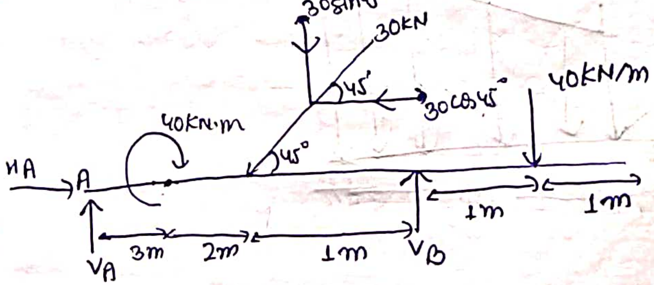
$R_A - 30 - 72 + R_B - 30 = 0$

$R_A = 30 + 72 - 89.4 + 30$

$$\boxed{R_A = 42.6\ KN}$$

Qu: Determine the reac developed at support A & B of over-hanging beam shown in fig.





$\Sigma f_x = 0$

$H_A + (-30\cos45°) = 0$

$H_A = 21.213\ kN$

$\circlearrowright^{-}\ |\ \circlearrowleft^{+}$

$\circlearrowright^{+}\ |\ \circlearrowleft^{-}$

$\Sigma f_y = 0$

$V_A + (-30\sin45°) + (-40) + V_B = 0$

$\Sigma M_A = 0$

~~30 sin~~

$-V_A \times 0 + 40\cancel{\times} + 30\sin45 (3+2) - V_B(3+2+1) + 40(3+2+1+1) = 0$

$40\cancel{\times} + 30\sin45(5) + 40(7) = V_B(6)$

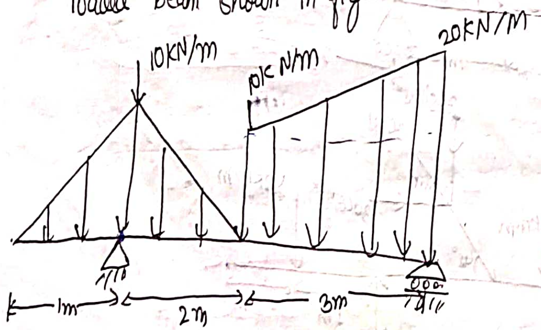
$V_B = \cancel{72.295\ kN}$ ~~60.066~~

$71.01\ kN$

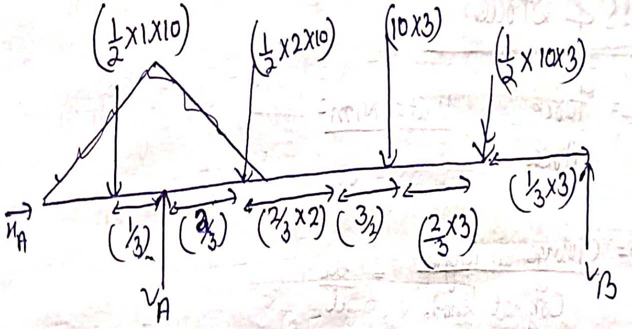
$V_A = 30\sin45° + 40 - 71.010$

$= \cancel{11.08\ kN}\ -9.796\ kN$

Que: Find the rear's developed at support A & B of the loaded beam shown in fig.



$\frac{1}{2}\times3\times10$



$\Sigma f_x = 0$

$H_A = 0$

$\Sigma f_y = 0$

$(\frac{1}{2}\times1\times10) + (-V_A) + (\frac{1}{2}\times2\times10) + (10\times3) + (\frac{1}{2}\times10\times3) - V_B = 0$

$\Sigma M_B = 0$

~~$+V_B\times0 - (\frac{1}{2}\times10\times3)\times(\frac{1}{3}\times3) - (10\times3)\times(\frac{2}{3}\times3) - (\frac{1}{2}\times2\times10)(\frac{2}{3}\times2 +$~~

33. 33 $\boxed{V_B = 34kN}$

~~$\frac{3}{2}) + V_A$~~

$V_B\times0 - (\frac{1}{2}\times10\times3)\times(\frac{1}{3}\times3) - (10\times3)\times(\frac{2}{3}\times3 + \frac{1}{3}\times3) -$

$(\frac{1}{2}\times2\times10)\times(\frac{2}{3}\times2 + \frac{3}{2} + \frac{2}{3}\times3 + \frac{1}{3}\times3) + V_A(\frac{2}{3} + \frac{2}{3}\times2 + \frac{3}{2}$

$+ \frac{2}{3}\times3 + \frac{1}{3}\times3) - (\frac{1}{2}\times10)(\frac{1}{3} + \frac{2}{3} + \frac{2}{3}\times2 + \frac{3}{2} + \frac{2}{3}\times3 + \frac{1}{3}\times3)$

$-15 - 30(2+1) - (10)(\frac{4}{3} + \frac{3}{2} + 2 + 1) + V_A(\frac{2}{3} + \frac{4}{3} + \frac{3}{2} + 2 + 1)$

$- 5(\frac{1}{3} + \frac{2}{3} + \frac{4}{3} + \frac{3}{2} + 2 + 1) = 0$

$V_A = 26kN$

Stress & Strain

(1) Stress (σ) = $\frac{\text{Force}}{\text{Area}}$ Unit: N/m^2

(E) Strain = $\frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$

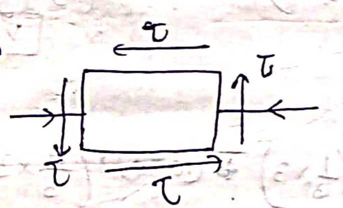
~~Explain~~ Different type of stresses.
Direct Stress or

(1) Normal stress :

Tensil stress   Compressive stress

(2) Shear Stress

Angle $\to \Delta\phi$



Different type of strain:

(1) longitudinal Strain:
The ratio of the change in length to the original length of an object caused by deforming force that changes length.

(2) Volumetric Strain:
The ratio of the change in volume to the original volume caused by deforming force that changes Volume.
$E_v = \frac{\Delta V}{V}$

(3) Shearing Strain:
Change in the orientation of an object's molecules, or the angle tilt caused by tangential stress.

(4) lateral strain
The ratio of change in the lateral dimension, such as the diameter, to the original dimension.

M. Imp (100%)

Modulus of Elasticity or Young's Modulus (E):

The ratio of tensile stress of compressive stress to the corresponding strain is a constant.

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{P}{A}}{\frac{\Delta l}{l}} = \frac{Pl}{A \times \Delta l} = \frac{\sigma}{\varepsilon}$$

Modulus of Rigidity or Shear Modulus:

The ratio of shear stress to the corresponding shear strain within the elastic limit is known as modulus of rigidity or shear modulus. It is denoted by C, G, N

$$(C, G, N) = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

Bulk Modulus (K)

The ratio of direct stress to the corresponding Volumetric strain in on the the constant for the given material when the deformation is within a certain limit. It is denoted by K.

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\varepsilon_V}$$

~~Poissons~~ Poission Ratio:

The ratio of lateral strain to the longitudnal strain is constant for given material when the material ~~is~~ stress within the elastic limit then the ratio is called poission Ratio. It is denoted by μ.

$$\mu = \frac{\text{lateral strain}}{\text{longitudnal strain}}$$

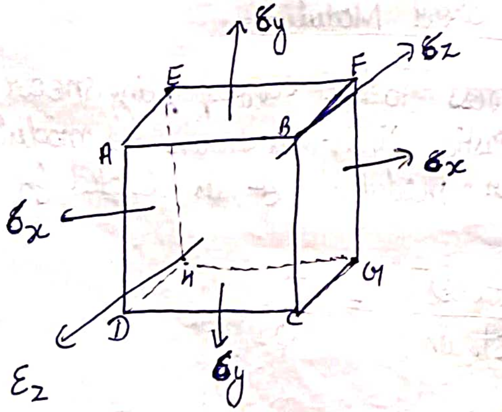
M.Imp.

Derive the following equations for the Elastic constant:

$$E = 3K(1-2\mu)$$

(or)

Derive the relation b/w Young's modulus (E) and bulk modulus (K):



$$\text{Bulk Modulus } (K) = \frac{\sigma}{\varepsilon_V}$$

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

Strain in x-direction

$$\frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \frac{\mu \sigma_z}{E}$$

$$\frac{\sigma}{E} - \frac{\mu\sigma}{E} - \frac{\mu\sigma}{E}$$

$$\varepsilon_x = \frac{\sigma}{E}(1-2\mu) \quad \text{— (1)}$$

similarly strain in y-direction

$$\frac{\sigma_y}{E} - \frac{\mu \sigma_x}{E} - \frac{\mu \sigma_z}{E}$$

$$\frac{\sigma}{E} - \frac{\mu\sigma}{E} - \frac{\mu\sigma}{E}$$

$$\varepsilon_y = \frac{\sigma}{E}(1-2\mu) \quad \text{— (2)}$$

& strain in z-direction

$$\varepsilon_z = \frac{\sigma}{E}(1-2\mu) \quad \text{— (3)}$$

Volumetric strain

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{\sigma}{E}(1-2\mu) + \frac{\sigma}{E}(1-2\mu) + \frac{\sigma}{E}(1-2\mu)$$

$$\boxed{\therefore K = \frac{\sigma}{\varepsilon_V} \quad \varepsilon_V = \frac{\sigma}{K}}$$

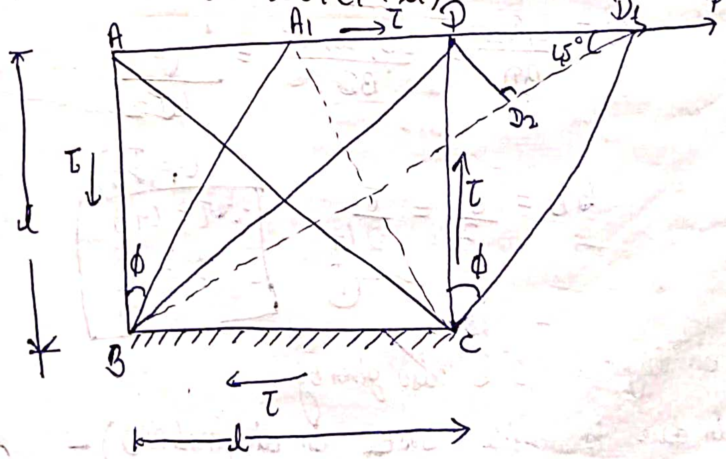
$$\frac{\sigma}{K} = \frac{3\sigma}{E}(1-2\mu)$$

$$\boxed{E = 3K(1-2\mu)}$$

Hence proved

Stablised the Relation between elasticity and Shear modulus:

$$E = 2G(1+\mu)$$



Consider a cubic element ABCD with fixed bottom BC and top face A subjected to force P.

Due to load P,

$AA_1 = DD_1$

$\tan\phi = \frac{DD_1}{l}$ ($\tan\phi$ is very small)

$DD_1 = l\phi$

Longitudinal strain in

$BD = \frac{BD_1 - BD}{BD} = \frac{BD_1 - BD_2}{BD}$

[DD_2 is $\perp$ from D to BD_1]

DD_1 is very small, therefore $\angle BDC = \angle BD_1C = 45°$

$\angle DD_1D_2 \approx 45°$

Longitudinal Strain

$BD = \frac{D_1D_2}{BD} = \frac{DD_1 \cos 45°}{BD} = \frac{l\phi(\frac{1}{\sqrt{2}})}{l\sqrt{2}}$

$BD = \frac{\phi}{2} = \frac{\tau}{2G}$ —(1)

$\left(G = \frac{\tau}{\phi}\right)$

$\boxed{\therefore \tau = G\phi \quad \phi = \frac{\tau}{G}}$

Strain in diagonal BD is also given by ;

(Strain due to tensile stress in diagonal BD) − (Strain due to complimentary stress in diagonal AC)

$= \frac{\tau}{E} - \left(-\mu\frac{\tau}{E}\right) = \frac{\tau}{E}(1+\mu)$ —(2)

from eqn (1) & (2)

$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$

$E = 2G(1+\mu)$

In terms of $\frac{1}{m}$,

$E = 2G\left(1 + \frac{1}{m}\right)$

$E = 2G\left(\frac{m+1}{m}\right)$

$E = \frac{2Gm + 2G}{m}$

$mE = G(2m+2)$

$G = \frac{mE}{2(m+1)}$

Established a relation b/w all the three modulus of elasticity: <OR>

Show the E, G, K are related by the following expression

$E = \frac{9KG}{3K+G}$

We know, $E = 2G(1+\mu)$ —(1)

$E = 3K(1-2\mu)$ —(2)

from eqn (1)

$\frac{E}{2G} = 1+\mu$ —(3)

$\mu = \frac{E}{2G} - 1$

putting eqn (3) in (2)

$$E = 3K\left(1 - 2\left(\frac{E}{2G} - 1\right)\right)$$

$$E = 3K\left(1 - \frac{2E}{2G} + 2\right)$$

$$E = 3K\left(\frac{-E + 3G}{G}\right)$$

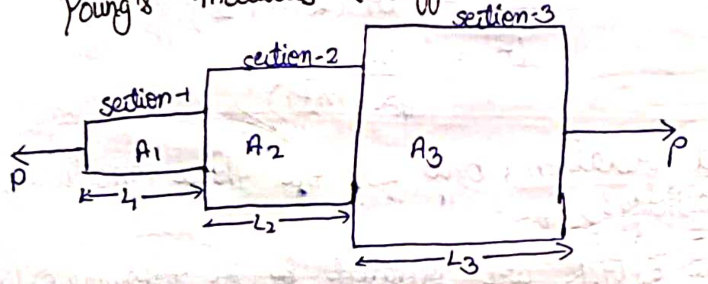
$$E = -\frac{3KE}{G} + 9KG$$

$$EG + 3KE = 9KG$$

$$E(G + 3K) = 9KG$$

$$E = \frac{9KG}{3K + G}$$

Que: Three sections of a bar having different lengths & different diameters. The bare is subjected to an axial load P. Determines the total change in length of the bar. Take Young's modulus a different sections same.



It is given that bar of diff. lengths & of different diameters & hence different cross-sectional area.

<u>Stress in sec- I, II & III</u>

$$\sigma_1 = \frac{P}{A_1} \qquad \sigma_2 = \frac{P}{A_2} \qquad \sigma_3 = \frac{P}{A_3}$$

~~By~~ using $E = \frac{\sigma}{\varepsilon}$

$\cancel{\sigma = E\varepsilon}$

Strain, $\varepsilon = \frac{\sigma}{E} = \cancel{\frac{P}{A}}$

$$\varepsilon_1 = \frac{P}{A_1 E}, \quad \varepsilon_2 = \frac{P}{A_2 E} \;\&\; \varepsilon_3 = \frac{P}{A_3 E}$$

But strain in sec-1 = $\frac{\text{change in length}}{\text{original length}}$

$$\varepsilon_1 = \frac{dL_1}{L_1}$$

$$\cancel{dL} = dL_1 = \varepsilon_1 L_1$$

$$dL_1 = \frac{PL_1}{A_1 E} \quad -(1)$$

Similarly in sec-2 & 3

$$dL_2 = \frac{PL_2}{A_2 E} \quad -(2) \qquad dL_3 = \frac{PL_3}{A_3 E} \quad -(3)$$

Total change in length of the bar

$$dL = dL_1 + dL_2 + dL_3$$

$$= \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

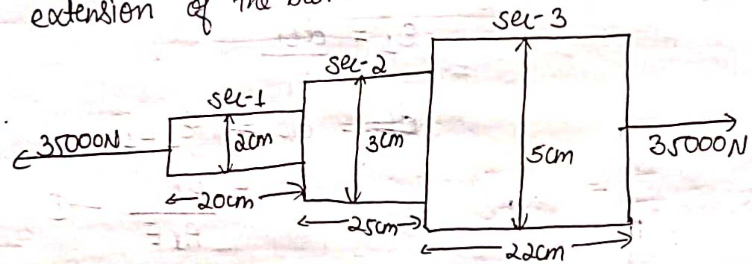
$$\boxed{dL = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)}$$

If the Young's modulus of different sections is different, the total change in length

$$\boxed{dL = P\left(\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} + \frac{L_3}{E_3 A_3}\right)}$$

Que: An axial pull of 35000 N is acting on a bar consisting of three lengths as shown in fig. If the Young's modulus = 2.1×10^5 N/mm², determine:

(a) stresses in each section and
(b) Total extension of the bar.



$F = 35000$ N
$E = 2.1 \times 10^3$ N/mm²

Stress $(\sigma) = \frac{F}{A}$

In sec – I

Area $= (20 \times 2)$ cm²
$= 40$ cm²
$= 4000$ mm²

$\sigma_1 = \frac{35000}{4000}$ N/mm²

$= 8.75$ N/mm²

In sec – II

Area $= (25 \times 3)$ cm²
$= 75$ cm²
$= 7.5$ mm²

$\sigma_2 = \frac{35000}{7.5}$ N/mm²

$= 4666.66$ N/mm²

In sec – III

Area $= (22 \times 5)$ cm²
$= 110$ cm²
$= 11$ mm²

$\sigma_3 = \frac{35000}{11}$ N/mm²

$= 3181.81$ N/mm²

~~Strain $(\epsilon) = \frac{\sigma}{E}$~~

$F = 35000$ N
$E = 2.1 \times 10^3$ N/mm²

In section – 1

$D = 2$ cm
$r = 1$ cm
$= 10$ mm

Area $= \pi r^2$
$= \pi \times (10)^2 = 100\pi$

$\sigma = \frac{P}{Area} = \frac{35000}{\pi \times (100)} = 111.464$ N/mm²

In sec – 2

$D = 3$ cm
$r = 1.5$ cm
$= \cancel{0.15 \text{ mm}}$
$= 15$ mm

Area $= \pi r^2$
$= \pi \times (15)^2$
$= 225\pi$

$\sigma = \frac{35000}{\pi \times (15)^2} = \cancel{49477.65}$
$= 49.539$ N/mm²

In sec-3

$D = 5$ cm
$r = 2.5$ cm
$= 25$ mm

Area $= \pi r^2$
$= \pi (25)^2$
$= 625\pi$

$\sigma = \frac{35000}{\pi \times (25)^2} = 17.834$ N/mm²

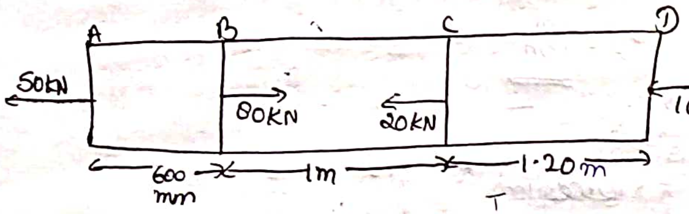
Total extension $= \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$

$= \frac{35000}{2.1 \times 10^5}\left(\frac{200}{100\pi} + \frac{250}{225\pi} + \frac{220}{625\pi}\right)$

$= \frac{35000}{2.1 \times 10^5}\left(0.6366 + 0.3536 + 0.1120\right)$

$= 0.1835$ mm

Q: A brass bar, having cross sectional area of 1000 mm^2, is subjected to axial forces as shown in fig.



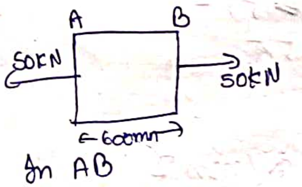
Take $E = 1.05 \times 10^5 \, N/mm^2$

Area (A) = 1000 mm^2

$E = 1.05 \times 10^5 \, N/mm^2$

We know,

$E = \frac{\text{stress}}{\text{strain}} = \frac{P/A}{\delta l / l} = \frac{P \times l}{A \times \delta l}$

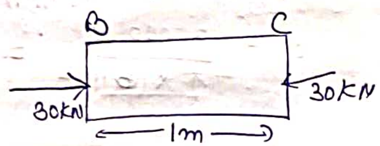


In AB

$\delta l = \frac{P \times l}{A \times E}$

$= \frac{(50 \times 1000) \times 600}{1000 \times 1.05 \times 10^5}$

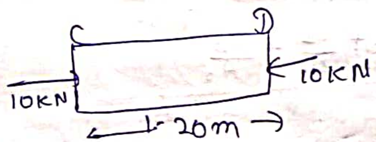
$= 0.285 mm$



$\delta l = \frac{P \times l}{A \times E}$

$= \frac{30 \times 1000 \times 1000}{1000 \times 1.05 \times 10^5}$

$= 0.285 mm$



$\delta l = \frac{P \times l}{A \times E}$

$= \frac{10 \times 1000 \times 1.20 \times 1000}{1000 \times 1.05 \times 10^5}$

$= 0.114 mm$

~~Total l~~

Total elongation of bar = $[0.285 + (-0.285) + (-0.114)] mm$

we take -ve sing because the forces that are applied is in opposite direction of force applied on AB.

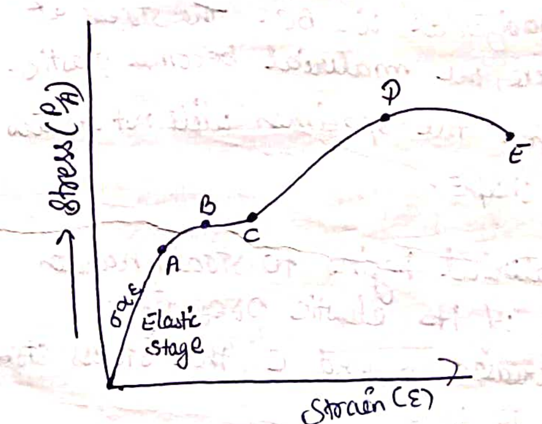
Hence

Total elongation = $\cancel{0.285}_m + (-\cancel{0.285})_m + (-0.114) mm$

$= -0.114 mm$

-ve sing indicates the compression takes place.

Discuss the tensile test diagram for ductile materials:-



A = Limit of proportionality
B = Yield point
D = Ultimate strength
E = Breaking point

A material is said to be ductile, if it elongates appreciably before fracture. One such material is mild steel.

Fig. shows a stress - strain diagram for a mild steel, in which the axial strain $\left(\frac{\delta L}{L}\right)$ are plotted along x-axis and corresponding stresses $\left(\frac{P}{A} = \frac{\text{load}}{\text{Area}}\right)$ are plotted along Y-axis

This diagram is obtained by performing a tensile test ~~and the~~

① Now, when the load on the test piece is increased slowly & corresponding extension is measured. From these readings, the curve OABCDE is obtained.

On this curve from O to A the stress is proportional to the strain and this is known as elastic stage & upto point A, Hooke's law is applicable.

② If the load is increased further the elongation becomes more rapid and diagram becomes curved. The stress will not be proportional to the strain.

Beyond point B, a sudden elongation of bar takes place without increase in load. This phenomenon is known as yielding of the material & it is shown by a horizontal line BC. The stress at yield point here, the material becomes plastic.

⊗ If load is removed the specimen will not return to its original shape.

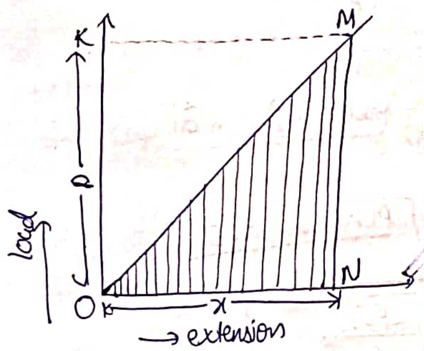
③ At point C, the material begins to strain harden & recovers some of its elastic property.

If the load increased beyond C, the stress strain curve climbs to point D.

the load at D divided by initial areas gives ultimate strength.

④ Beyond point D, further stretching of the bar is accompanied by a decrease in load and fracture takes place at point E, suddenly.

Que Give the expression for strain energy in a body when the load is applied gradually.



Work done by load = Area

$= \text{Area of } \Delta OMN$

$= \frac{1}{2} \times P \times x$

$= \frac{1}{2} \times (\sigma \times A) \times (\varepsilon \times l)$

$= \frac{1}{2} (\sigma \times A) \times (\frac{\sigma}{E} \times l)$

$= \frac{1}{2} \frac{\sigma^2 A l}{E}$

Stored in energy (U) = Workdone $= \frac{1}{2} \frac{\sigma^2 \times V}{E}$ $\boxed{A \times l = \text{Volume}}$

Que: Derive an expression for strain energy stored in a body when the load is applied with impact.

We know Young's Modulus,

$E = \frac{\text{Stress}}{\text{Strain}}$

$\text{Strain} = \frac{\text{Stress}}{E}$

$\frac{\delta L}{L} = \frac{\sigma}{E}$

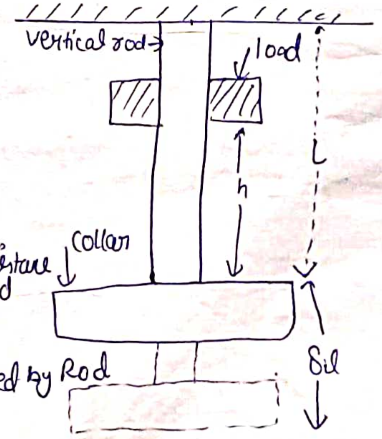
Work done by load = load × distance moved

$= P(h + \delta L)$

Straing energy ~~from above we know~~ stored by Rod

Energy $(U) = \frac{1}{2} \frac{\sigma^2}{2E}$

$= \frac{\sigma^2}{} \times A \times L$



equating eqn (1) & (2)

$$p(h+\delta L) = \frac{\sigma^2}{2E} \times A \times L$$

~~p·h + p·δL~~

$$\sigma^2 = \frac{p(h+\delta L) \times 2E}{AL}$$

$$\sigma = \sqrt{\frac{p(h+(\delta L)) \times 2E}{AL}}$$

$$\boxed{\sigma = \sqrt{\frac{2PEh}{AL}}}$$

Truss

Date
24/9/2024

A truss is a structure made up of cylinder member pin connected (hinged) ~~out~~ at ends and is capable of taking loads at joints.

Types of Truss:

(1) Perfect Truss
(Just rigid Truss)

(2) Imperfect Truss

$m = 2j - 3$ for 2-D
$m = 3j - 6$ for space

where m = no. of member
j = no. of joints.

(2) Imperfect Truss

	2D	3D
→ Defecent Truss	$m < 2j - 3$	$m < 3j - 6$
→ Redundant Truss (Cover rigid Truss)	$m > 2j - 3$	$m > 3j - 6$

Assumption:

1. The truss is statically determinate or perfect one.
2. Loads at joints only.
3. Ends of the members are pin jointed (hinged) and joints are smooth.
4. Self weight of members is negligible.

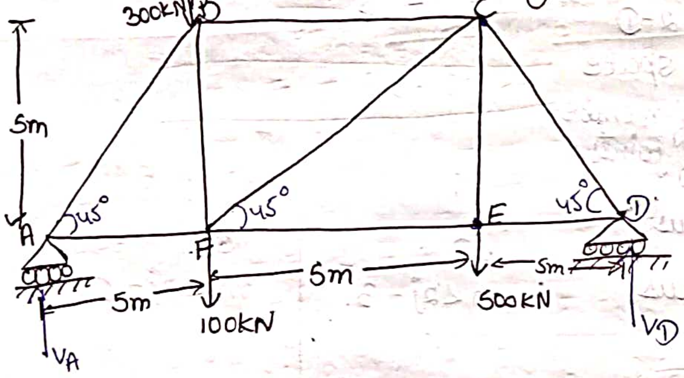
⑤ Cross - section of members is uniform.

Method:

① Method of Joint.

② Method of Section.

Ques: Determine forces in all the members in truss shown in fig. all the horizontal & vertical members are 5m long.



① $2J = 2j - 3$

$m = 2j - 3$

$9 = 2\times6 - 3$

$9 = 12 - 3$

$9 = 9$

perfect

② Angle find

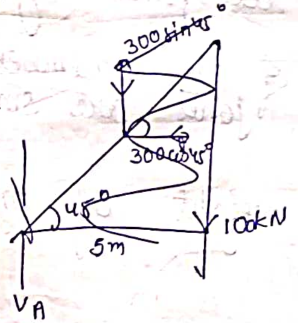
③ find of the reaction & reaction values.

$\Sigma fx = 0$

$\Sigma fy = 0$

$\Sigma M = 0$

~~$\Sigma fx = 0$~~



~~V_A~~ ~~$300\cos45°$~~ =

~~Σfy~~

~~$V_A + (-100)$~~ + (-

$\Sigma fx = 0$

$\Sigma fy = 0$

$V_A - 100 - 300 - 500 + V_D = 0$

$\Sigma M_A = 0$

$-V_A \times 0 + (100\times5) + (300\times5) + (500\times10) - V_D \times 15 = 0$

$15 V_D = 500 + 1500 + 5000$

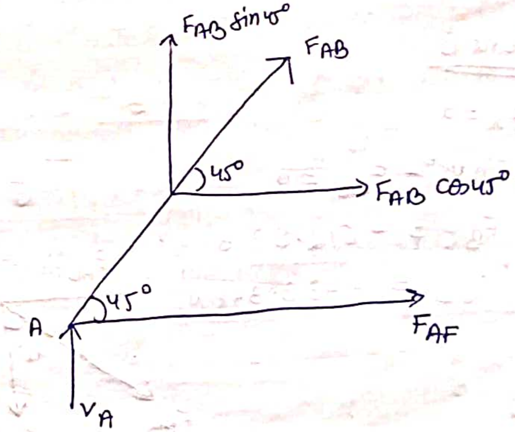
$V_D = \frac{7000}{15}$

$V_D = 466.66 KN$

$V_A = 100 + 300 + 500 - 466.66$

$= 433.34 \text{ kN}$

At Joint A



$\Sigma fx = 0$

$F_{AF} + F_{AB}\cos45° = 0$

$\Sigma fy = 0$

$V_A + F_{AB}\sin45° = 0$

$F_{AB} = -\frac{433.34}{\sin45°}$

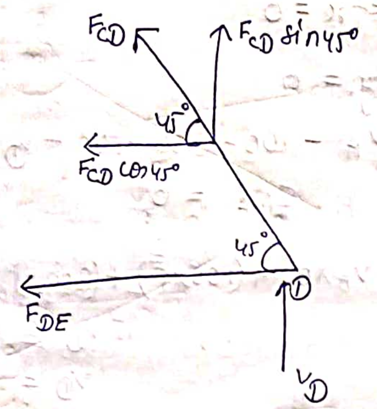
$\boxed{F_{AB} = -612.83 \text{ KN}}$

$F_{AF} = -F_{AB}\cos45°$

$= -(-612.83\cos45°)$

$\boxed{F_{AF} = 433.33 \text{ KN}}$

At Joint D



$\Sigma f_x = 0$

$F_{CD}\cos 45° + F_{DE} = 0$

$F_{DE} = -F_{CD}\cos 45°$

$\boxed{F_{DE} = 466.65\ kN}$

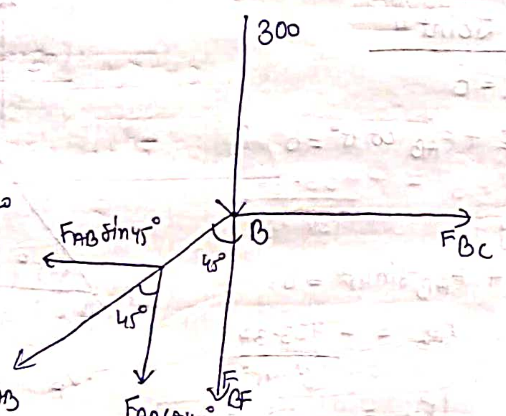
$\Sigma f_y = 0$

$V_D + F_{CD}\sin 45° = 0$

$F_{CD} = \frac{-V_D}{\sin 45°} = \frac{466.66}{\sin 45°}$

$\boxed{F_{CD} = -659.95\ kN}$

At Joint B



$\Sigma f_x = 0$

$F_{AB}\sin 45° - F_{BC} = 0$

$F_{BC} = -612.83\sin 45°$

$\boxed{F_{BC} = -433.33\ kN}$

$\Sigma f_y = 0$

$F_{AB}\cos 45° + F_{BF} + 300 = 0$

$F_{BF} = 612.83\cos 45° - 300$

$\boxed{F_{BF} = 133.33\ kN}$

At Joint E

$\Sigma f_x = 0$

$F_{EF} - F_{ED} = 0$

$F_{EF} = F_{DE}$

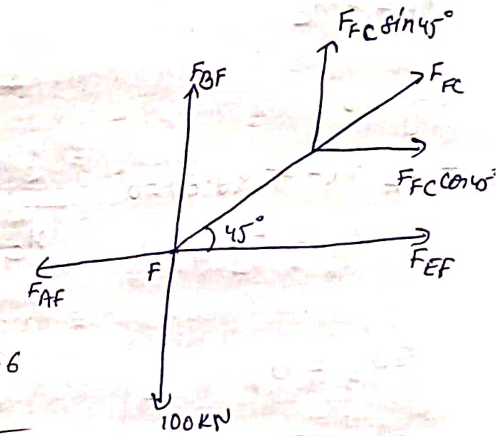
$F_{EF} = 466.66\ kN$

$\Sigma f_y = 0$

$F_{EC} - 500 = 0$

$F_{EC} = 500$

Joint F



$\Sigma f_x = 0$

$F_{AF} - F_{EF} - F_{FC}\cos 45° = 0$

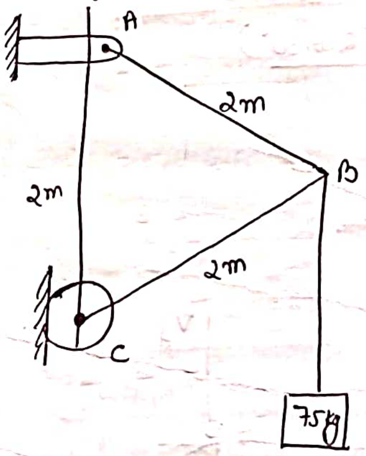
$F_{FC}\cos 45° = F_{AF} - F_{EF}$

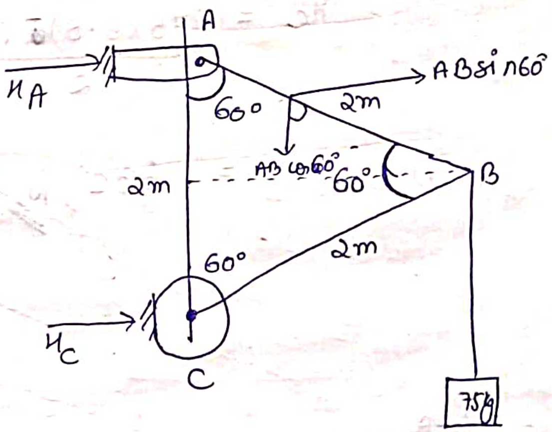
$= \frac{433.33 - \cancel{466.66}}{\cos 45}$

$= \frac{-47.13\ kN}{\cos 45°}$

66.65

Que: Determine the force in each member of the simple equilateral truss as shown in figure, $g = 9.8\ m/s^2$





Step ① find Angle

② Draw FBD and find out the rxn forces

③ find the value of rxn forces

$\Sigma f_x = 0$

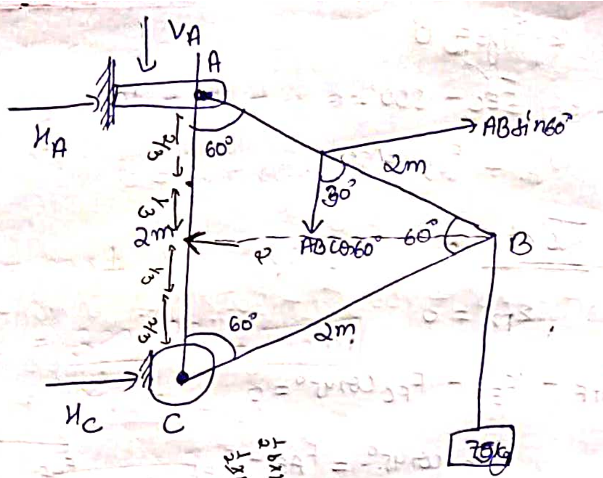
$\Sigma f_y = 0$

$\Sigma M = 0$

$\Sigma f_x = 0$

$H_A + H_C = 0$

$H_A = -H_C$



$\Sigma f_y = 0$

$-V_A + (-75 \times 9.8) = 0$

$V_A = -735\ kN$

$\Sigma M_A = 0$

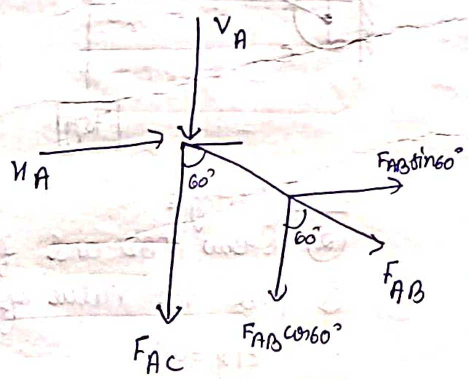
$-V_A \times 0 + H_A \times 0 + H_C \times 2 + \cancel{(2\cos 30°)} - \frac{(75 \times 9.8)}{\cancel{75 \times 2}} = 0$

$\cancel{2 H_C = \frac{x\cos 30°}{2}} + 75 \times 2$ $\quad \frac{75 \times 9.8 \times 3}{2}$ $\quad \frac{75 \times 9.8 \times 1}{2 \times 2}$

$H_C = \frac{(75 \times 9.8) \times (2\cos 30°)}{2}$

$H_C = 632\ kN$

At Joint A



$\Sigma f_x = 0$

$H_A + F_{AB} \sin 60° = 0$

$F_{AB} = \frac{636}{\sin 60}$

$\boxed{F_{AB} = 734.38\ kN}$

$\Sigma f_y = 0$

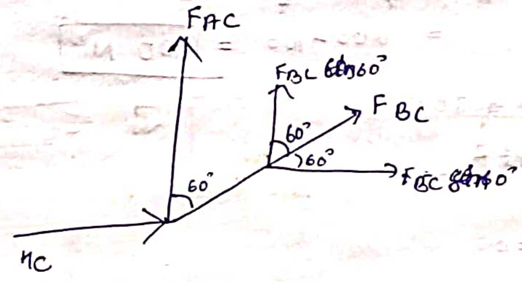
$V_A + F_{AC} + F_{AB} \cos 60° = 0$

$-735 + F_{AC} + 734.38 \cos 60° = 0$

$F_{AC} = 735 - 734.38 \cos 60°$

$\boxed{F_{AC} = 367.81\ kN}$

Joint C



$\Sigma f_x = 0$

$H_C + F_{BC} \cos 60° = 0$

$F_{BC} = -\frac{636}{\cos 60°}$

$\boxed{F_{BC} = 734.38\ kN}$

$\Sigma f_y = 0$

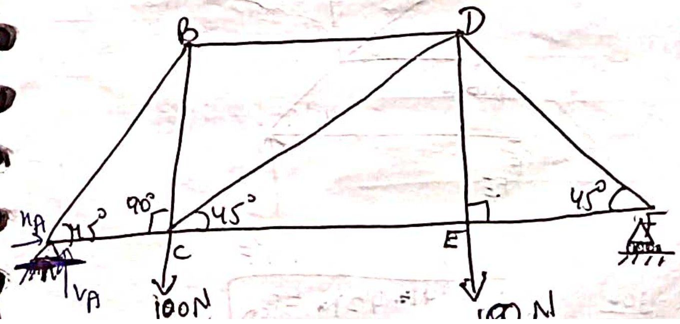
$F_{AC} + F_{BC} \cos 60° = 0$

$F_{AC} = -F_{BC} \cos 60°$

$= 734.38 \cos 60°$

$\boxed{F_{AC} = 367.19\ kN}$

Que: for the trussified force in all members, all angle are 45° & 90°



$\Sigma f_x = 0$

$H_A = 0$

$\Sigma f_y = 0$

$V_A - 100 - 100 + V_f = 0$

$V_A + V_f = 200$

$\Sigma M_A = 0$

$\cancel{100}(-H_A \times 0) + (-V_A \times 0) + (100 \times 1) + (100 \times 2) + (-3 \times V_f) = 0$

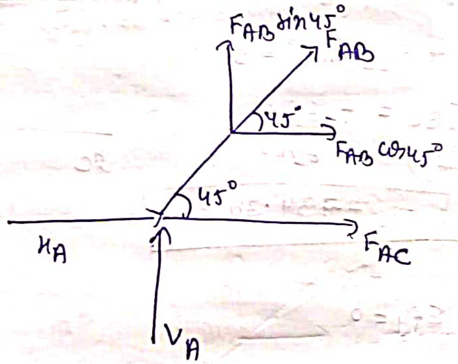
$V_f = \frac{300}{3} \; 100 = \boxed{100\ N}$

$V_A = \cancel{V_f}\, 200 - V_f$

$= 200 - 100 = \boxed{100\ N}$

Method of Joint

At Joint A



$\Sigma f_x = 0$

$H_A + F_{AC} + F_{AB}\cos 45^\circ = 0$

$0 + F_{AC} + F_{AB}\cos 45^\circ = 0$

$\Sigma f_y = 0$

$V_A + F_{AB}\sin 45^\circ = 0$

$F_{AB} = \frac{-100}{\sin 45^\circ}$

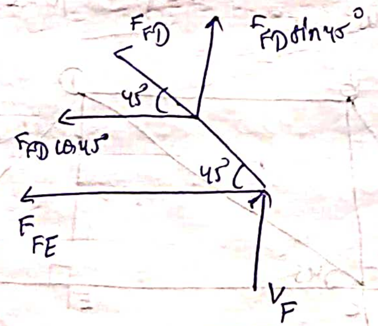
$\boxed{F_{AB} = -141.42\ N}$

$\cancel{\Sigma f_x}$

$F_{AC} = -F_{AB}\cos 45^\circ$

$\boxed{F_{AC} = +100\ N}$

Joint F



$\Sigma f_x = 0$

$F_{FE} + F_{FD}\cos 45^\circ = 0$

$\Sigma f_y = 0$

$V_F + F_{FD}\sin 45^\circ = 0$

$= \frac{-V_F}{\sin 45^\circ} = \frac{-100}{\sin 45^\circ} = \boxed{-141.42\,N = F_{FD}}$

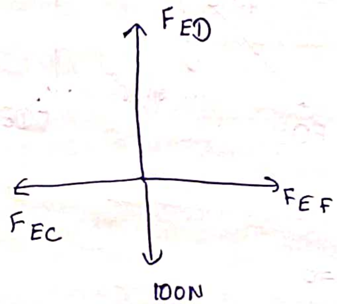
$F_{FE} = -F_{FD}\cos 45^\circ$

$= 141.42 \cos 45^\circ$

$F_{FE} = 99.99\ N$

$\boxed{F_{FE} \approx 100\ N}$

At Joint E



$\Sigma f_x = 0$

$F_{EC} - F_{EF} = 0$

$F_{EC} = F_{EF}$

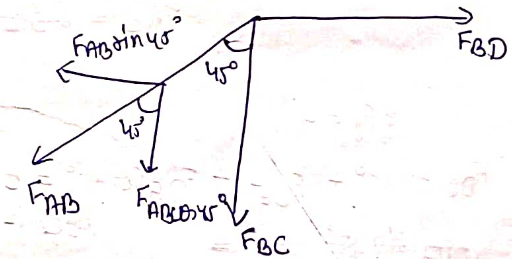
$\boxed{F_{EC} = 100\,N}$

Σf_y

$F_{ED} - 100 = 0$

$\boxed{F_{ED} = 100\,N}$

Joint B



$\Sigma f_x = 0$

$F_{AB}\sin 45^\circ - F_{BD} = 0$

$F_{BD} = F_{AB}\sin 45^\circ$

$= -141.42 \sin 45^\circ$

$= -100\ N$

$\Sigma f_y = 0$

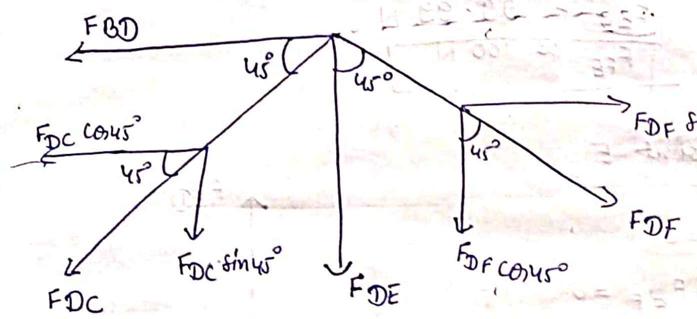
$F_{AB}\cos 45^\circ + F_{BC} = 0$

$F_{BC} = -F_{AB}\cos 45^\circ$

$= +141.42 \cos 45^\circ$

$F_{BC} = +100\ N$

At Joint D



$\Sigma f_x = 0$

$F_{BD} + F_{DC}\cos 45° - F_{DF}\sin 45° = 0$

$-100 + F_{DC}\cos 45° - (-141.42 \sin 45°) = 0$

$F_{DC}\cos 45° = -141.42 \sin 45° + 100$

$F_{DC} = \frac{\cancel{+200}}{\cos 45°} \; 9.59 \times 10^{-4}$

$F_{DC} = \cancel{202.84N} \; \cancel{+35N} \; 1.35 \times 10^{-3} N$

$\Sigma f_y = 0$

$F_{DC}\sin 45° + F_{DE} + F_{DF}\cos 45° = 0$

$F_{DC}\sin 45° = -F_{DE} - F_{DF}\cos 45°$

$\cancel{F_{DC}\sin 45°}$

$F_{DC} = \frac{-100 + 141.42\cos 45°}{\sin 45}$

$F_{DF}\cos 45° = -[F_{DC}\sin 45° - F_{DE}]$

$F_{DF} = \frac{\cancel{202.84}\sin 45 \; 1.35 \quad 100}{\cos 45} = -1.35 \times 10^{-3}$

$= \cancel{-424.26N}$

$= -142.77N$

Zero force Member

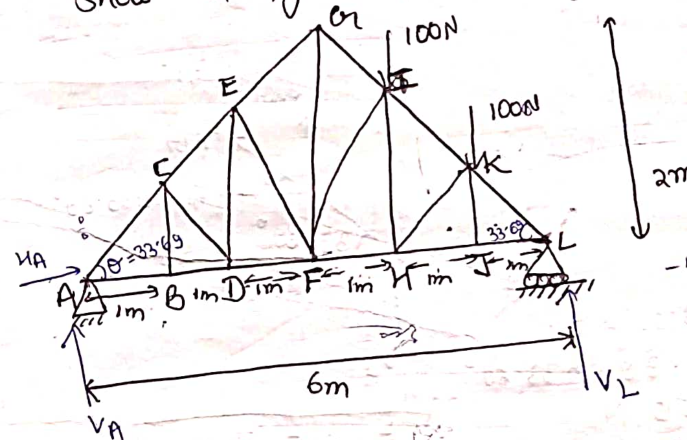
Zero force member are the members in which their is no force.

After the knowing the members of zero forces, they can't be eliminated while calculating the forces in the members.

Conditions:

1. Only three members are connected to a pin.
2. No external force acts on this pin.
3. Out of the three members, two are colinear.

Que: Identify the zero force members in the truss as show in fig. & find the force in member GF.



$\tan\theta = \frac{2}{3}$

$\theta = \tan^{-1}\left(\frac{2}{3}\right)$

$\theta = 33.69$

$\Sigma f_x = 0$

$\cancel{H_A + 100} \cdot H_A = 0$

$\Sigma f_y = 0 \quad V_A - 100 - 100 + V_L = 0$

$V_A + V_L = 200$

$\cancel{V_A - 100 - 100 + V_L} = 0 \quad V_A = 200 - 150$

$\cancel{V_A = 200 - 900}$ $\quad = 50N$

$-H_A \times 0 - V_A \times 0 + 100 \times 4 + 100 \times 5 - V_L \times 6 = 0$

$6V_L = 400 + 500$

$V_L = \frac{\cancel{900}}{\cancel{6}} \; \cancel{300} \; 150$

$V_L = 150$

$\Sigma M_A = 0$

$\cancel{-H_A \times 0 - V_A \times 0 + 100}(4) + 100(5) - V_L = 0$

$V_L = 100 \times 4 + 100 \times 5$

$= 400 + 500$

$= \frac{\cancel{900} \; 900}{6} \; 150$

$V_A = 50 N$

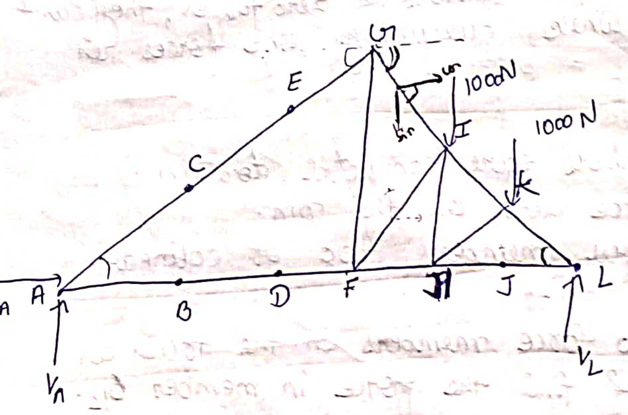
$V_L = 150 N$

$F_{BC} = 0$

$F_{CD} = 0$

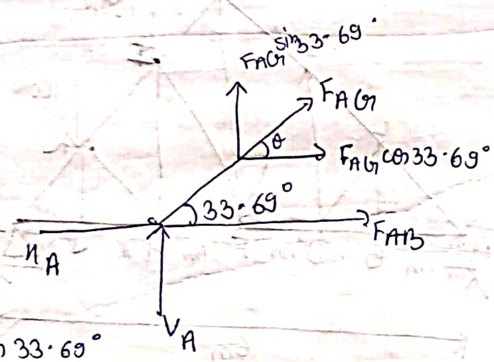
$F_{DE} = 0$

$F_{FE} = 0$



by Method of Joint

At A



$\Sigma f_x = 0$

$H_A + F_{AB} + F_{AG}\cos 33.69° = 0$

$0 + F_{AB} = 901.30 \cos 33.69°$

$F_{AB} = 750\ N$

$\boxed{F_{AF} = 750N}$

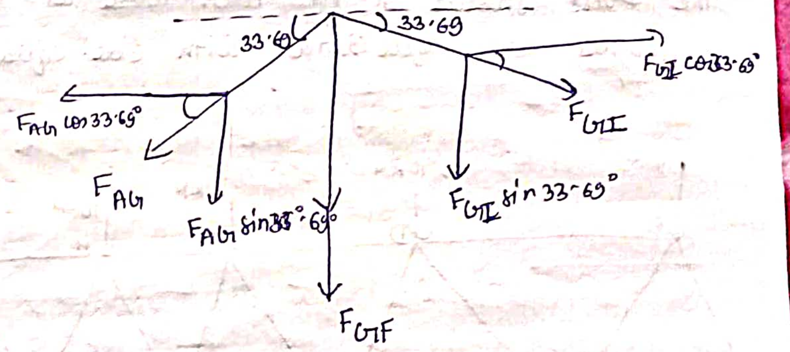
$\Sigma f_y = 0$

$F_{AG}\sin 33.69° + V_A = 0$

$F_{AG} = \dfrac{-500}{\sin 33.69°}$

$\boxed{F_{AG} = \cancel{901.30\ N} \\ -90.130\ N}$

At ~~G~~ G



$\Sigma f_x = 0$

$F_{AG}\cos 33.69 - F_{GI}\cos 33.69 = 0$

$F_{GI} = \dfrac{F_{AG}\cos 33.69}{\cos 33.69}$

$F_{GI} = \cancel{901.30\ N}$

$= -90.30\ N$

$\Sigma f_y = 0$

$F_{AG}\sin 33.69° + F_{GI}\sin 33.69° + F_{GF} = 0$

$F_{GF} = -F_{AG}\sin 33.69° - F_{GI}\sin 33.69°$

$= -(-90.1\text{-}38\ \sin 33.69°) - (-90.1\text{-}38)\ \sin 33.69°)$

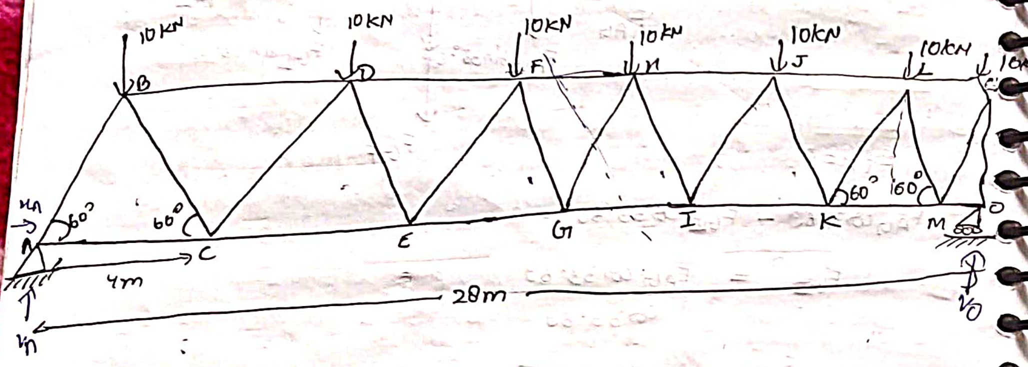
$= 901.38 \sin 33.69° + 901.38 \sin 33.69°$

$= 999.88\ N$

$= 1000\ N$

Method of Section

Qus: Determine the forces in the members F_H, H_G & G_I in the truss shown in fig. Each load is 10 kN & all the triangles are equilateral with side equal to 4 m.



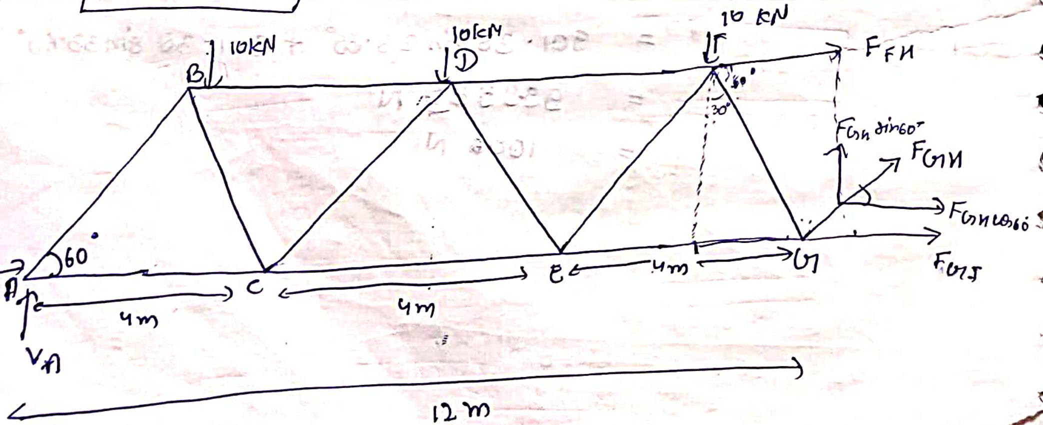
$\Sigma f_x = 0$

$H_A = 0$

$\Sigma f_y = 0$

$V_A + V_O = 70$

$\boxed{V_A = 35 \quad V_O = 35}$



<u>At Joint A</u>

$\Sigma f_x = 0$

$H_A + F_{GH} \cos 60° + F_{GI} + F_{FH} = 0$

$F_{GH} \cos 60° + F_{GI} + F_{FH} = 0$

$\Sigma f_y = 0$

$V_A + F_{GH} \sin 60° - 10 - 10 - 10 = 0$

$F_{GH} = \dfrac{30 - 35}{\sin 60°}$

$= -5.77 \text{ kN}$

$\Sigma M_{G} = 0$

$(F_{GI} \times 0) + (F_{GH} \cos 60° \times 0) + (F_{GH} \sin 60° \times 0) + (F_{FH} \times 5.65)(2)$

$+ (10 \times 10) + 10(6)$

$\tan 60° = \dfrac{10}{mx}$, $mx = \dfrac{10}{\tan 60°}$

$(F_{FH} \times 0) + 10\left(\frac{1}{2} \times 4\right) \times 10 + 10(8)$

$\frac{1}{2} \times 2 \times 10$

$\cos 60° = \dfrac{h}{10}$

$\cos 60° = \dfrac{10}{x}$

$(F_{FH} \times 4) + (10 \times 4) \times$

$V_A \times 12 + (-10 \times 10) + (-10 \times 6) + (-10 \times 2) + (-F_H \times 3.46) = 0$

$35 \times 12 - 100 - 60 - 20 = F_{FH}(3.46)$

$\dfrac{240}{3.46} = F_H$

$F_{FH} = 69.36$

$F_{GH} \cos 60° + F_{GJ} + F_{FH} = 0$

$F_{GJ} = -F_{GH} \cos 60° - F_{FH}$

$= 5.77 \cos 60° - 69.36 = -66.39$

Date 07/10/2024

Centroid

Center of Gravity:
It is the point where the entire weight of the body is to be concentrated.

Centroid:
It is defined as that point in the body at which the entire volume is assumed to be concentrated.

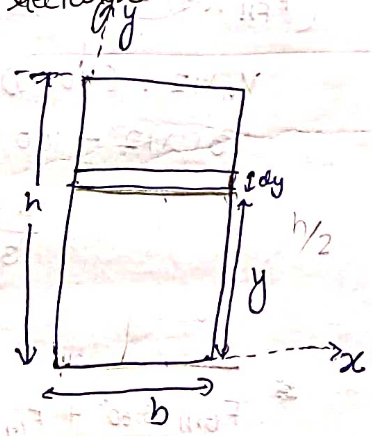
$$\bar{x} = \frac{\int_0^a x \cdot dA}{A} \qquad \bar{y} = \frac{\int_0^a y \cdot dA}{A}$$

$$\boxed{\bar{x} \cdot A = \int_0^a x \cdot dA}$$

Centroid is defined as that point where whole line/area/volume is assumed to be concentrated.

$$\bar{x} = \frac{\int_0^L x \cdot dL}{L} \qquad \bar{x} = \frac{\int_0^V x \cdot dv}{V}$$

Determine the centroid of the rectangle.



$$\bar{y} = \frac{\int_0^a y \cdot dA}{A}$$

$$\bar{y} = \frac{\int_0^h y \cdot (b \cdot dy)}{b \times h}$$

$$= \frac{y^2}{2} \int_0^h b \cdot \quad \frac{\cancel{b}\int_0^h y \cdot dy}{\cancel{b} \times h}$$

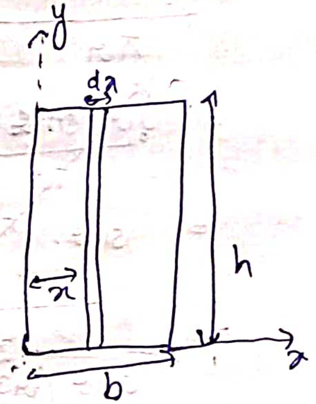
$$\bar{y} = \frac{\left(\frac{y^2}{2}\right)_0^h}{h}$$

$$\bar{y} = \left(\frac{\frac{h^2}{2} - \frac{0^2}{2}}{h}\right)$$

$$\bar{y} = \frac{h^{\cancel{2}}}{2 \times \cancel{h}}$$

$$\bar{y} = \frac{h}{2}$$

$$\frac{y^{n+1}}{n+1}$$



$$\bar{x} = \frac{\int_0^b x \cdot dA}{A}$$

$$\bar{x} = \frac{\int_0^b x \cdot (h \cdot dx)}{h \times b}$$

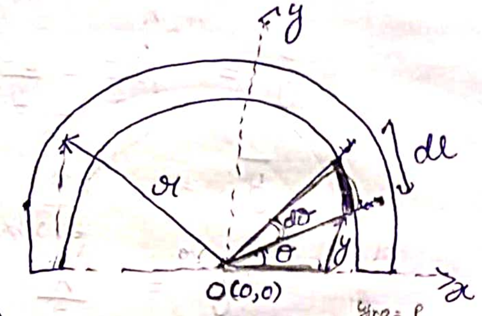
$$= \frac{\cancel{h}\int_0^b x \cdot dx}{b \times \cancel{h}}$$

$$= \frac{\left(\frac{x^2}{2}\right)_0^b}{b}$$

$$= \frac{b^{\cancel{2}}}{2 \times \cancel{b}}$$

$$\bar{x} = \frac{b}{2}$$

Derive an expression for the centroid of a thin semi-circular arc of mean radius 'r'.



$$\bar{y} = \frac{\int_0^{L} y \cdot dL}{L}$$

$\boxed{dL = r\,d\theta}$

$x = r\cos\theta$

$y = r\sin\theta$

$$\bar{y} = \frac{\int_0^{l} r\sin\theta \cdot r\,d\theta}{\pi r}$$

$\cos\theta = \frac{b}{h}$ $\quad \sin\theta = \frac{P}{H}$

$\frac{arc}{radius} = \theta$ $\quad \sin\theta = \frac{y}{r}$

$\frac{dl}{r} = \theta$ $\quad y = r\sin\theta$

$dl = r\theta$

$$= \frac{\int_0 r^2 \sin\theta\, d\theta}{\pi r}$$

$$= \frac{\sin\theta \cdot \int_0^{\pi} r^2 \cdot d\theta - \int^{\pi} \sin\theta\, d\theta \cdot \int_0^{\pi} r^2 \cdot d\theta}{\pi r}$$

$\frac{x}{r} = \cos\theta$

$x = r\cos\theta$

$$\frac{\sin\theta \left(\frac{r^3}{3}\right)_0^{\pi} - \int_0^{\pi} \cos\theta \cdot \frac{r^3}{3}}{\pi r}$$

$$= \frac{\sin\theta \left(\frac{r^3}{3}\right)_0^{\pi} - \left[\cos\theta \cdot \int_0^{\pi} \frac{r^3}{3} \cdot d\theta - \int_0^{\pi} \cos\theta\, dx \int_0^{\pi} \frac{r^3}{3\,d\theta}\right]}{\pi r}$$

$$= \frac{r\cancel{r} \int_0^{\pi} \sin\theta \cdot d\theta}{\pi \cancel{r}}$$

$\sin y\, dy$

$\sin x\, dy$

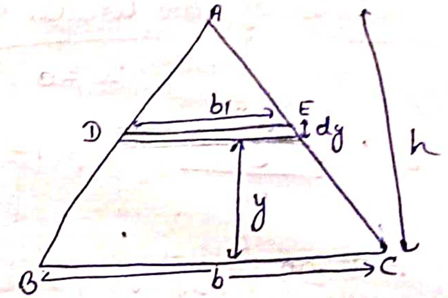
$$= \frac{r\,(-\cos\theta)_0^{\pi}}{\pi}$$

$\cos\pi = -1$

$$= \frac{r\,(\cos\pi + \cos 0)}{\pi}$$

$$= \frac{r\,(-1+1)}{\pi} \Rightarrow \boxed{\frac{2r}{\pi}}$$

Centroid of triangle:



$$\bar{y} = \frac{\int_0^{a} y\, dA}{A}$$

$$\bar{y} = \frac{\int_0^{h} y \cdot b_1\, dy}{\frac{1}{2} \times b \times h}$$

In Δ ADE & ABC due to similar Δ

$$\frac{b_1}{h-y} = \frac{b}{h} = \frac{b_1}{b}$$

$$b_1 = \frac{b}{h}(h-y)$$

$$\frac{b_1}{h} = \frac{h-y}{h}$$

$$\frac{b_1}{h-y} = \frac{b}{h}$$

$$\bar{y} = \frac{\int_0^{h} y \times \frac{b}{h}(h-y)\, dy}{\frac{1}{2} \times b \times h}$$

$\frac{yb}{h}(h-y)$ $\quad -\frac{yb}{h} \times y$ $\quad \frac{y^2 b}{h}$

$$= \frac{\int_0^{h} \left(by - \frac{y^2 b}{h}\right) dy}{\frac{1}{2} \times b \times h}$$

$$= \frac{b\int_0^{h} y\, dy - \frac{b}{h}\int_0^{h} y^2 \cdot dy}{\frac{1}{2} \times b \times h}$$

$$= \frac{b\left(\frac{y^2}{2}\right)_0^{h} - \frac{b}{h}\left(\frac{y^3}{3}\right)_0^{h}}{\frac{1}{2} \times b \times h}$$

$$= \frac{\frac{b \times h^2}{2} - \frac{b}{\cancel{h}} \times \frac{h^3}{3}}{\frac{1}{2} \times b \times h}$$

$\frac{bh^2}{2} - \frac{bh^2}{3}$ $\quad \frac{3bh^2 - 2bh^2}{6}$

$$\frac{\frac{bh^2}{6}}{\frac{1}{2} b \times h}$$

$$= b \times h\left(\frac{h}{2} - \frac{h}{3}\right) \times \frac{2}{b \times h}$$

$$= \left(\frac{3h - 2h}{6}\right) 2$$

$\frac{2 \times bh^2}{6 \times bh}$ $\quad \frac{h}{3}$

$$= \frac{h}{3}$$

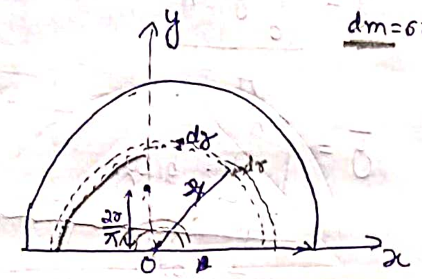
~~Centroid~~ Centre of mass of semi circular disc:

$x = 0$

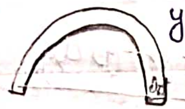
$\sigma = \frac{1}{A}$ $\quad \frac{dm}{dA} = \sigma$

$dm = \sigma dA$

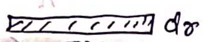
$$y = \frac{\int y\, dm}{m}$$



$$= \frac{\frac{2m}{R^2}\int_0^R \boxed{r\,dr}\left(\frac{2r}{\pi}\right)}{m}$$



$y =$



πr $\quad dr$

$$= \frac{4}{\pi R^2}\left(\frac{r^3}{3}\right)_0^R$$

$dm = \sigma \cdot dA$

$= \frac{m}{\frac{\pi R^2}{2}}\pi r \cdot dr$

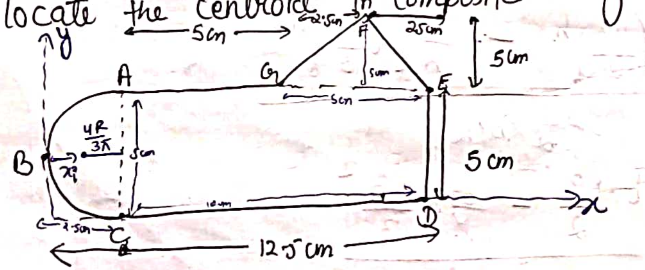
$$= \frac{4}{\pi R^2} \times \frac{R^3}{3}$$

$$= \frac{4R}{3\pi}$$

$= \frac{2m}{\pi R^2} \cdot \pi r \cdot dr.$

$\frac{2m}{R^2}\int$

Que: locate the centroid in composite body.



~~$\bar{x}_i$ = particular part is x-axis se y-axis tk.~~

x_i = part ke individual ke centroid se x-axis ~~ke duri se~~ y-axis ka duri

$\frac{105}{x}$

$\bar{x}$ = distance of centroid from y-axis

$$\bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + \cdots A_nx_n}{A_n} = \frac{\sum_{i=1}^{n=\infty} A_nx_n}{\Sigma A_n}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + \cdots A_ny_n}{A_n} = \frac{\sum_{i=1}^{n=\infty} A_ny_n}{\Sigma A_n}$$

Part/Shape	Area	$\bar{x}_i$	$\bar{y}_i$	A_ix_i	A_iy_i
Semi circular disk	$\frac{\pi(2.5)^2}{2}$	$\left(2.5-\frac{4R}{3\pi}\right)$	2.5	14.12	24.544
Rectange	10×5	$2.5+5 = 7.5$	2.5	375	125
Triangle	$\frac{1}{2}\times5\times5$	10	$5+\frac{5}{3}$	125	83.333
Total	72.314			514.127	232.877

12.5

$$\bar{X} = \frac{\Sigma A_ix_i}{\Sigma A_i} = \frac{514.127}{72.317} = 7.109\text{ cm}$$

$$\bar{Y} = \frac{\Sigma A_iy_i}{\Sigma A_i} = \frac{232.877}{72.314} = 3.220$$

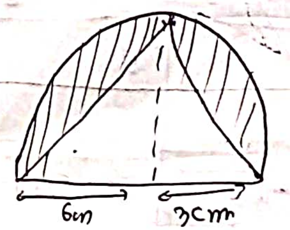
$\begin{array}{r} 2 \\ 7.5 \\ 0.5 \\ \hline 37.5 \end{array}$

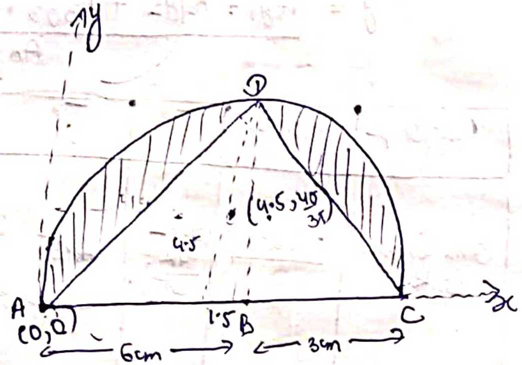
1.44

~~75~~

98.125
9.81 $\quad$ 1.06

Que: A triangle is removed from a semicircular disk as shown in fig. locate the centroid of the remaining part.

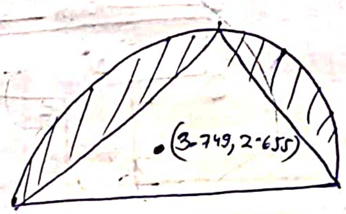




Shape	Area	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
Semicircular disk	$\frac{\pi(4.5)^2}{2}$ 31.808	4.5	$\frac{4r}{3\pi}$ 1.909	143.136	60.721
~~Triangle~~	~~$\frac{1}{2} \times 9 \times$~~ 4.2 ~~20.25~~ 18.9				
Triangle ABD	$\frac{1}{2} \times 6 \times 4.2$ 12.6	4	$\frac{4.232}{3}$ 1.410	-50.4	-18.003
Triangle DBC	$\frac{1}{2} \times 3 \times 4.2$ 6.3	$6 + \frac{3}{3}$	1.410	-44.1	9.007
	12.715			47.671	33.745

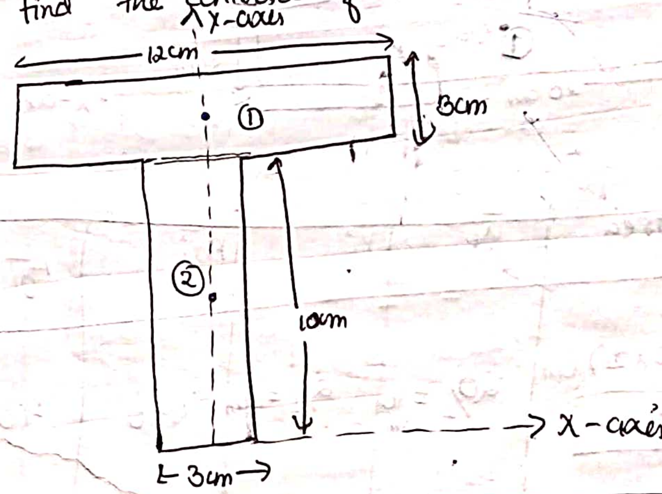
$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma x_i} = \frac{47.671}{12.715} \Rightarrow 3.749$$

$$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma y_i} = \frac{33.745}{12.715} = 2.655$$



Que: find the centroid of section - T.

T, I, H, Z section

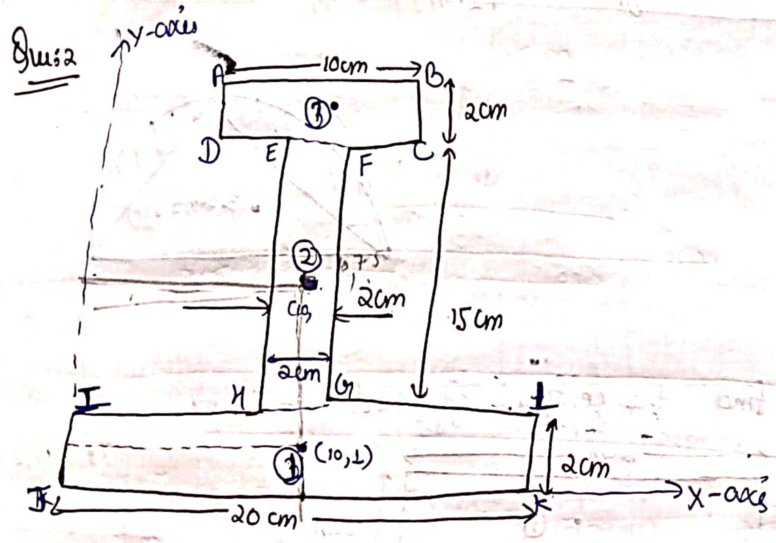


Shape	Area	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
Rectangle 1	(12x3) cm 36 cm	0	10 + 1.5	0	414
Rectangle 2	(3x10) cm 30 cm	0	5 cm	0	150
	66 cm				564

$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} = 0$$

$$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{564}{66} = 8.545 \text{ cm}$$

Ques:2

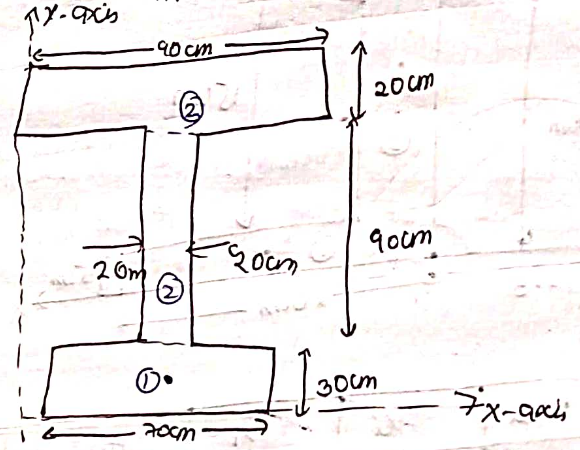


Shape	Area	x_i	y_i	$A x_i$	$A y_i$
Rectangle ①	(20x2) cm^2 $\boxed{40\ cm^2}$	$20/2 = 10$ cm	$2/2 = 1$ cm	400	40
Rectangle ②	(15x2) cm 30 cm^2	$20/2 = 10$ cm	$\cancel{1+\frac{15}{2}=8.5}$ $2+\frac{15}{2}=9.5$	300	285
Rectangle	(10x2) cm 20 cm^2	$20/2 = 10$ cm	$2+15+\frac{2}{2}$ = 18 cm	200	360
	90 cm^2			900	685 cm

$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{900}{90} = 10$$

$$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{685}{90} = 7.611 \text{ cm}$$

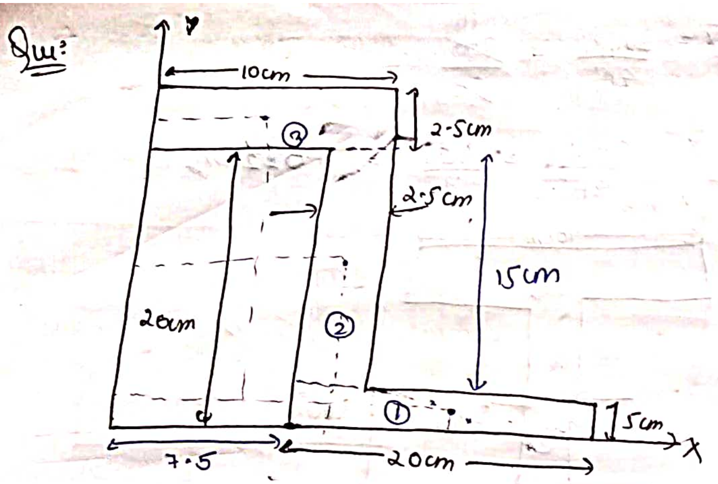
Ques:3



Shape	Area	y_i	$A_i y_i$
Rectangle ①	70X30 2100cm	15cm	31500
Rectangle ②	20X90 1800cm	75 cm	135000
Rectangle ③	20X90 1800cm	130cm	234000
	5700		400500

$$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = 70.263 \text{ cm}$$

Qu:



Shape	Area	$\bar{x}_i$	$\bar{y}_i$	$A_i x_i$	$A_i y_i$
Rectangle ①	(20×5) cm $100 cm^2$	$7.5+10$ 17.5 cm	$5/2 = 2.5$ cm	175 cm	250 cm
Rectangle ②	(15×2.5) $37.5 cm^2$	$7.5+\frac{2.5}{2}$ 8.75 cm	$\frac{15}{2}+5$ 12.5 cm	~~3234.375 cm~~ 328.125	468.75 cm
Rectangle ③	10×2.5 cm $25 cm^2$	$10/2 = 5$ cm	$15+5+\frac{2.5}{2}$ 21.25 cm	125 cm	531.25 cm
	$162.5 cm^2$ ~~$72.5 cm^2$~~			2203.125 cm ~~534.375 cm~~ ~~2109.375 cm~~	1250 ~~1022.5~~ cm

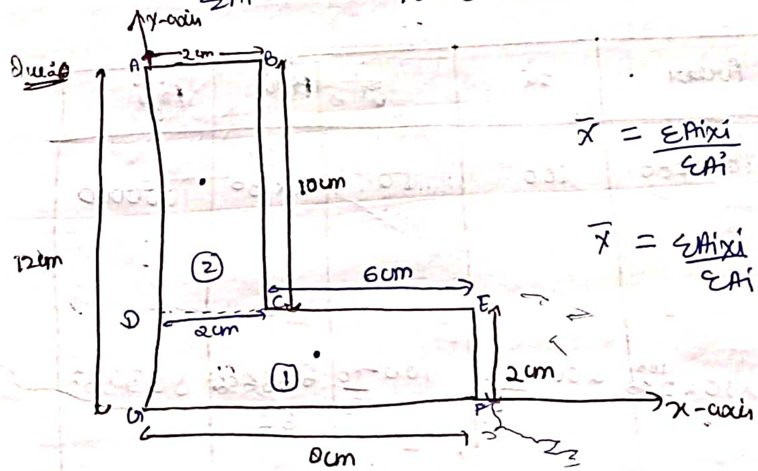
$\bar{x} = \frac{\Sigma A_i x_i}{A_i} = \frac{\cancel{2109.375}}{\cancel{534.375}} \big/ \cancel{72.5} = \cancel{7.370 cm}$ ~~29.094 cm~~

$\cancel{\bar{y} = \frac{\Sigma A_i y_i}{A_i}} = \frac{\cancel{1022.5}}{\cancel{72.5}} = \cancel{14.137 cm}$ ~~17.241 cm~~

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{2203.125}{162.5} = 13.557 \text{ cm}$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{1250}{162.5} = 7.692 \text{ cm}$$

Qu:

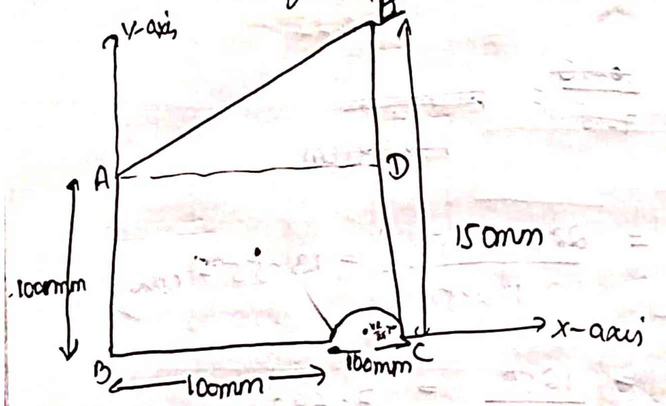


$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{84}{36} = 2.333$$

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{156}{36} = 4.333$$

Shape	Area	$\bar{x}_i$	$\bar{y}_i$	$A_i x_i$	$A_i y_i$
Rectangle ①	(8×2) cm 16 cm	4 cm	1 cm	64 cm	16 cm
Rectangle ②	(10×2) cm 20 cm.	1 cm	$2+\frac{10}{2}$ 7 cm	20 cm	140 cm
	36 cm			84 cm	156 cm

Que: A semicircular area is removed from a trapezoid as shown in fig. determine the centroid of the remaining area.



Part	Area	x_i	y_i	$A_i x_i$	$A_i y_i$
Rectangle	100×200	100	50	2×10^6	1000000
Triangle	$\frac{1}{2} \times 50 \times \cancel{200}\ 100$	$200 - \frac{200}{3}$	$100 + \frac{50}{3}$	666666.66	583333
Semicircular	$\frac{\pi (50)^2}{2}$ -3926.990	$100 + 50$ 150	$\frac{4(50)}{3\pi}$ 21.220	-58904.62	-83333.33
	~~20926.49~~ 21073			~~2725657.28~~ 2077618	~~916666~~ 1500000

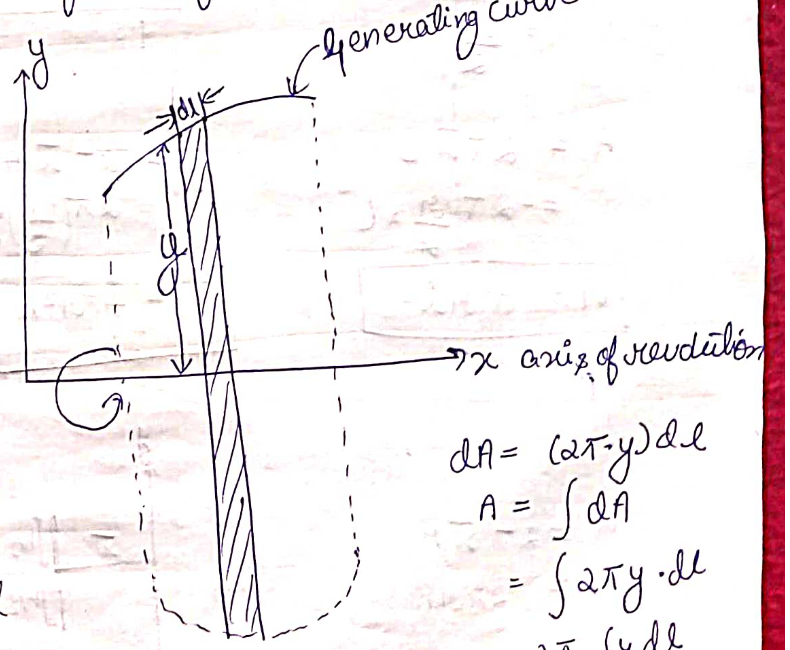
$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{2077618}{21073} = 98.59\ mm$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{1500000}{21073} = 71.181\ mm$$

M. Imp.

Pappus' Theorem:

The surface of revolution developed by revolving a co-planner generating curve about an axis of revolution has an area equal to the length of generating curve times the circumference of the circle form by the centroid of the generating curve in the process of generating the surface of revolution.



$$\bar{y} = \frac{\int y\,dl}{L}$$

$$\bar{y}L = \int y\,dl$$

$$dA = (2\pi y)\,dl$$

$$A = \int dA$$

$$= \int 2\pi y \cdot dl$$

$$= 2\pi \int y\,dl$$

$$\boxed{A = 2\pi \bar{y} L}$$

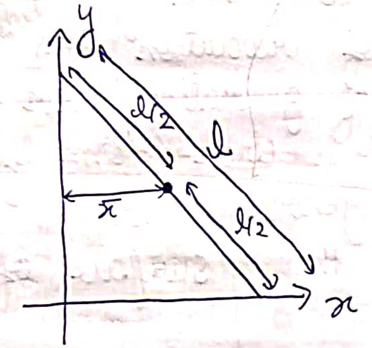
$$dv = (2\pi y)\cdot dA$$

$$V = \int 2\pi y \, dA$$

$$= 2\pi \int y \, dA$$

$$V = 2\pi \bar{y} A$$

Que: ~~compute~~ Calculate the lateral area of cone of base radius small r & slant height (l) using pappu's theorem.



similar Δ

$$\frac{\bar{x}}{r} = \frac{l/2}{l}$$

$$\bar{x} = \frac{l/2 \times r}{l}$$

$$= \frac{r}{2}$$

$$A = 2\pi \bar{x} L$$

$$= \cancel{2}\pi \frac{r}{\cancel{2}} \times L$$

$$\boxed{A = \pi r l}$$

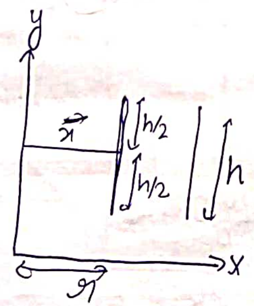
$$V = 2\pi \bar{x} A$$

$$= 2\pi \left(\frac{r}{3}\right) \times \frac{1}{2} \times r \times h$$

$$\boxed{V = \frac{1}{3}\pi r^2 h}$$

$$A = 2\pi \bar{x} h$$

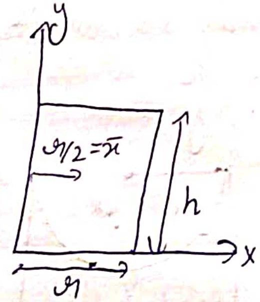
$$A = 2\pi r h$$



$$V = 2\pi \bar{x} A$$

$$= 2\pi \left(\frac{r}{2}\right) \times r \times h$$

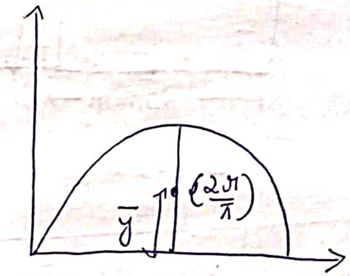
$$= \pi r^2 h$$



$$A = 2\pi \bar{y} \cdot L$$

$$= 2\pi \left(\frac{2r}{\cancel{\pi}}\right) \cancel{\pi} r$$

$$= 4\pi r^2$$

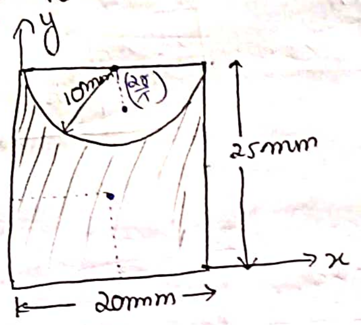


$$V = 2\pi \bar{y} \cdot A$$

$$= \cancel{2}\cancel{\pi} \left(\frac{4r}{3\cancel{\pi}}\right) \times \left(\frac{\cancel{\pi} r^3}{\cancel{2}}\right)$$

$$= \frac{4}{3}\pi r^3$$

Que: Determine the centroid of the shaded area as shown in fig.

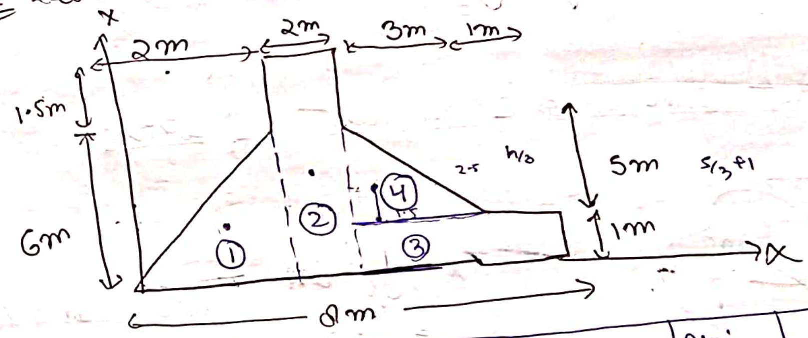


Shape	Area	$\bar{X}_i$	$\bar{Y}_i$	$A_i\bar{X}_i$	$A_i\bar{Y}_i$
Rectangle	(25×20)mm 500mm	10mm	$\frac{25}{2}$mm $= 12.5$mm	5000	6250
Semicircular disk	$-\frac{\pi(10)^2}{2}$ 157.079	10mm	~~$\frac{2\times10}{\pi}$~~ ~~6.366~~mm $\left(25-\frac{4r}{3\pi}\right)$ $25-12.732$ $25-4.244$ ~~12.268~~ 20.756	1570.79	2927.010 1927.045
	~~657.079~~ 342.921			~~6540.09~~ 3429.21	3322.899

$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma x_i} = \frac{3429.21}{342.921} = 10$

$\bar{Y} = \frac{\Sigma A_i x_i}{\Sigma y_i} = \frac{3322.899}{342.921} = \cancel{9.689}$ 8.417

Que: Determine the ~~section~~ centroid of concreate dam



Shape	Area	$\bar{x}_i$	$\bar{x}_i$	A_ix_i	A_iy_i
~~Rectangle~~ Triangle ①	$\frac{1}{2}\times2\times6$ 6m	1	$\frac{6}{3} = 2$m	6	12
Rectangle ②	7.5×2 15m	3	$\frac{7.5}{2} = 3.75$	45	56.25
Rectangle ③	4×1 4m	6	$\frac{1}{2} = 0.5$	24	2
Triangle ④	$\frac{1}{2}\times3\times5$ 7.5	5.5	$\frac{5}{3}+1 =$ 2.666	41.25	19.995

Area = 32.5

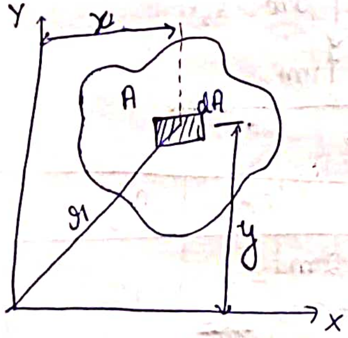
$A_i x_i = 116.25$

$A_i y_i = 90.245$

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{116.25}{32.5} = 3.576$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{90.245}{32.5} = 2.776$$

The first and second moment of area:



$$\left.\begin{aligned} I_x &= \int y^2\, dA \\ I_y &= \int x^2\, dA \end{aligned}\right] \text{rectangular axis}$$

$$I_z = \int r^2 \cdot dA \rightarrow \text{polar axis}$$

Consider a plane area A in the xy plane. shown in fig. Moments of inertia of area A about x & y axis.

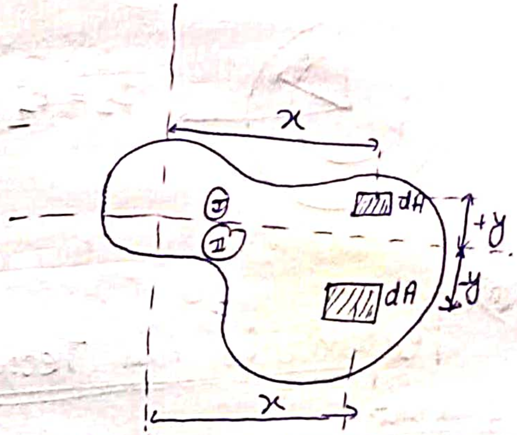
$\int y \cdot dA$ = first moment of area

$\int y^2 \cdot dA$ = second moment of area /inertia.

x-axis & y-axis are called triangular axes or polar axis.

~~Total~~ Product moment inertia:

$$I_{xy} = \int_A xy \cdot dA$$



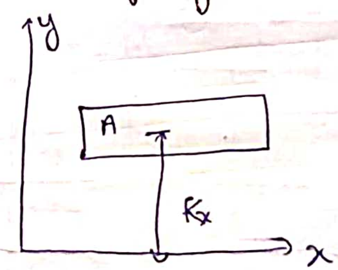
$$I_1 xy = \int xy \cdot dA$$

$$I_2 xy = -\int xy \cdot dA$$

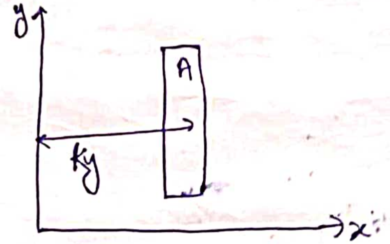
$$I_{xy} = I_1 xy + I_2 xy$$
$$= \int xy \cdot dA + \left(-\int xy \cdot dA\right)$$
$$= 0$$

The product moment of inertia is zero when either (or both) of the two axis is an axis of symmetry

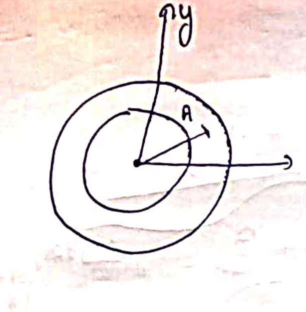
Radius of Gyration:



$I_x = K_x^2 A \qquad k_x = \sqrt{\frac{I_x}{A}}$



$I_y = k_y^2 A \qquad k_y = \sqrt{\frac{I_y}{A}}$



$I_z = k_z^2 A \Rightarrow k_z = \sqrt{\frac{I_z}{A}}$

M.Imp

Perpendicular Axis Theorem:

Statement:

" The sum of moment of inertia of an area about two orthogonal axes lying in the plane of the area is equal to the polar moment of inertia of the area about the third axis of the chosen quardinate system. "

$I_x + I_y = I_z$

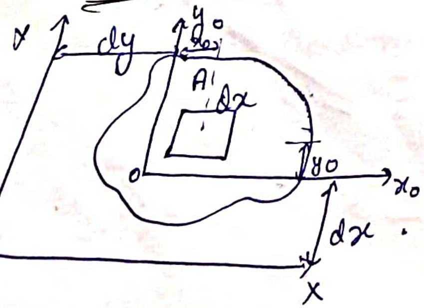
$\int_A y^2 dA + \int_A x^2 dA = I_z$

$\int_A (y^2 + x^2) dA = I_z$

$\int_A r^2 \, dA = I_z$

$\boxed{I_x + I_y = I_z}$

Parallel Axis Theorem:



centroidal axis,

Self axis

I_{x_0} or $\bar{I}_x$ (centroidal moment of Inertia)

I_x (moment of Inertia)

$I_x = \int (y_0 + d_x)^2 dA$

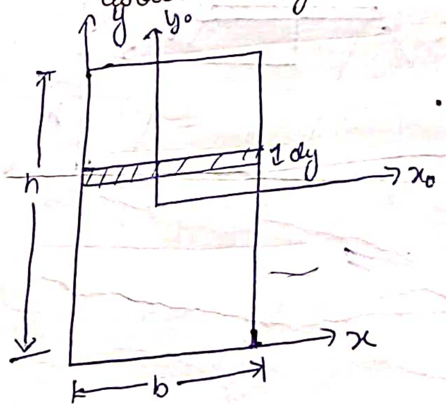
$= \int (y_0^2 + d_x^2 + 2y_0 d_x) dA$

$= \int y_0^2 dA + d_x^2 \int dA + 2d_x \int y_0 dA$

$= \bar{I}_{x_0} + d_x^2 A + 2d_x (\bar{y}_0)$

$\boxed{I_x = \bar{I}_x + d_x^2 A}$

Qu: Determine the moment of inertia of rectangular area about its edges and the centroidal axes ∥ to edges.



$I_x = \int y^2 dA$

$= \int_0^h y^2 (b \times dy)$

$= b \int_0^h y^2 dy$

$= b \left(\frac{y^3}{3}\right)_0^h$

$I_x = b\frac{h^3}{3}$

$I_x = \bar{I}_x + d_x^2 A$

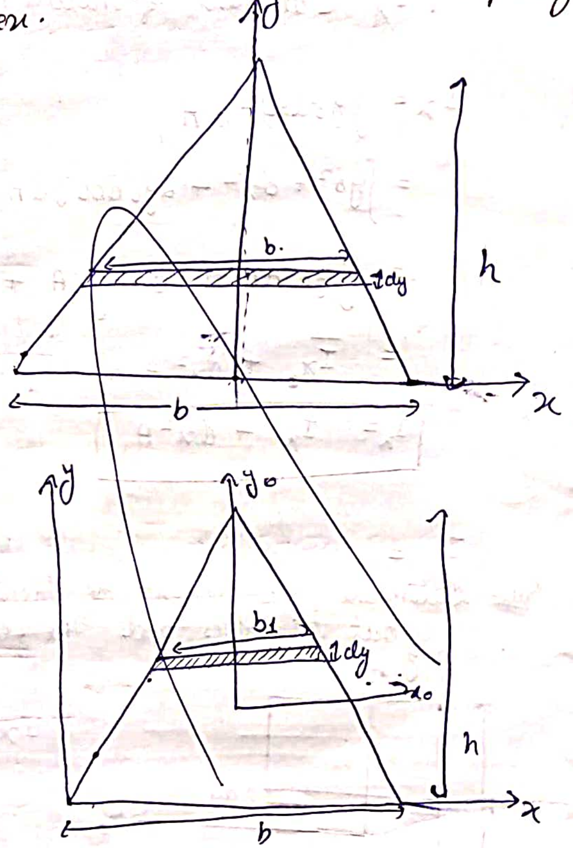
$\frac{bh^3}{3} = \bar{I}_x + \left(\frac{h}{2}\right)^2 \times (b \times h)$

$\bar{I}_x = \frac{bh^3}{3} - \frac{bh^3}{4}$

$\boxed{\bar{I}_x = \frac{bh^3}{12}}$

$\boxed{\bar{I}_y = \frac{hb^3}{12}}$

Que: Determine the moment of inertia of triangular area of height h & base b about its base and about the ‖ axis passing through its vertex.



$I_x = \int y^2 dA$

$I_x = \int y^2 \times \frac{1}{2} \times \frac{b}{2} \times dy$

$I_x = \frac{b}{4} \int_0^h y^2 dy$

$I_x = \frac{b}{4} \left(\frac{y^3}{3}\right)_0^h$

$I_x = \frac{b}{4} \left(\frac{h^3}{3}\right)$

$I_x = \frac{bh^3}{12}$

$I_x = \bar{I}_x + dx^2 A$

$\frac{bh^3}{12} = \bar{I}_x + \left(\frac{h}{2}\right)^2 \times \frac{1}{2} \times b \times h$

$\frac{bh^3}{12} = \bar{I}_x + \frac{bh^3}{8}$

$\bar{I}_x = \frac{bh^3}{12} - \frac{bh^3}{8}$

$= \frac{2bh^3 - 3bh^3}{24}$

$I_x = \int y^2 dA$

$I_x = \int y^2 \times \frac{1}{2} b_1 dy$

$= \int y^2 \frac{b}{h} (h-y) dy$

$I_x = \int \left(y^2 b - \frac{by^3}{h}\right) dy$

$I_x = \int_0^h y^2 b\, dy - \int_0^h \frac{by^3}{h} dy$

$= b\left(\frac{y^3}{3}\right)_0^h - \frac{b}{h}\left(\frac{y^4}{4}\right)_0^h$

$= \frac{bh^3}{3} - \frac{bh^3}{4}$

$= \frac{bh^3}{12}$

Moment of inertia about an axis passing through parallel to the base.

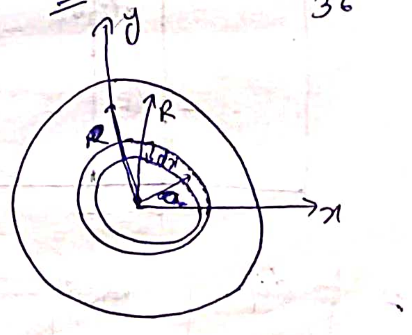
$I_x = \bar{I}_x + dx^2 A$

$\frac{bh^3}{12} = \bar{I}_x + \left(\frac{h}{3}\right)^2 \times \frac{1}{2} \times b \times h$

$\bar{I}_x = \frac{bh^3}{12} - \frac{h^3 b}{18}$

$= \cancel{\frac{3bh^3}{36}} \; \frac{bh^3}{36}$

$I_y = \frac{hb^3}{36}$



$I_z = \int_0^R a^2 dA$

$= \int_0^R a^2 \cdot 2\pi a\, da$

$= 2\pi \int_0^R a^3 da$

$= 2\pi \left(\frac{a^4}{4}\right)_0^R$

$= 2\pi \left(\frac{R^4}{4}\right)$

$= \frac{\pi R^4}{2}$

πr^2

$R = \frac{d}{2}$

$\frac{R}{2} = d$

$R = 2d$

$R^4 = (2d)^4 = 16 d^4$ ....

$$I_z = \frac{\pi D^4}{32}$$

by $\perp$ axis theorem.

$$I_x + I_y = I_z$$

$$I_x = I_y$$

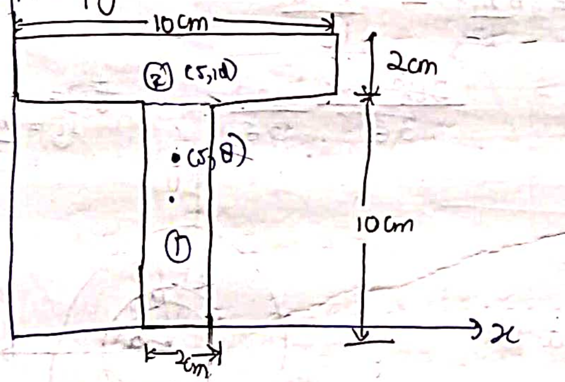
$$2I_x = I_z$$

$$2I_x = \frac{\pi D^4}{32}$$

$$I_x = \frac{\pi D^4}{64}$$

$$I_x = I_y = \frac{\pi D^4}{64}$$

Sue: Determine the moment of inertia of the T-section shown in fig. ~~about the horizontal axis~~ y



Shape	Area	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
Triangle	10x2 20cm	$\frac{10}{2}$ 5cm	$\frac{10}{2}$ 5 cm	100cm	100 cm
Triangle	10x2 20cm	5cm	$\frac{2}{2}+10$ 11cm	100cm	220
	40			200	320

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{200}{40} = 5\,cm$$

(5-5)

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{320}{40} = 8\,cm$$

hb^3

$$I_{x_1} = \bar{I}_{x_1} + d{x_1}^2 A_1$$

$\frac{b'h^3}{12}$

$$= \frac{(2)\times(10)^3}{12} + (8-5)^2 \times (10\times 2)$$

$$= \frac{2\times 1000}{12} + 9\times 10\times 2$$

166.666

$$= 346.666\ cm^4$$

$$I_{y_1} = \bar{I}_{y_1} + d{y_1}^2 A_1$$

$=$

$$I_{x_2} = \bar{I}_{x_2} + d{x_2}^2 A_2$$

$$= \frac{b_2 h_2^3}{12} + (8-11)^2 \times 20$$

$$= \frac{10\times 2^3}{12} + 9\times 20$$

$$= 6.666 + 180$$

$$= 186.666\ cm^4$$

$dx = (\bar{y} - \bar{y}_i)$	dy

$$I_x = I_{x_1} + I_{x_2}$$

$$533.332\ cm^4$$

$$I_{y_1} = \bar{I}_{y_1} + d_{y_1}^2 A_1$$

$$= \frac{h_1 b_1^3}{12} + (\bar{x}_1 - x_1)^2 A_1$$

$$= \cancel{6.666\ cm^4}$$

$$\frac{10 \times 2^3}{12} + (5-5)^2\, 10 \times 2$$

$$= 6.666\ cm^4$$

$$I_{y_2} = \bar{I}_{y_2} + d_{y_2}^2 A_2$$

$$= \frac{h_2 b_2^3}{12} + (\bar{x}_2 - x_2)^2 A_2$$

$$= \frac{12 \times 10^3}{12} + (5-5)^2 A_2$$

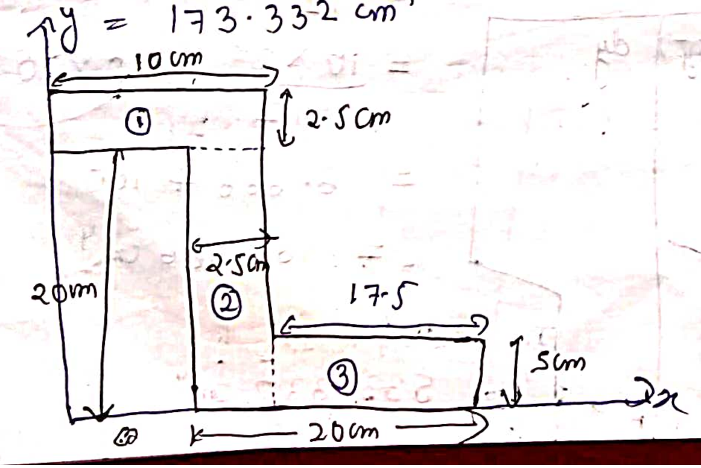
$$= 166.66\ cm^4$$

$$I_y = I_{y_1} + I_{y_2}$$

$$= 6.666 + 166.666$$

$$I_y = 173.332\ cm^4$$

Que:



Shape	Area	$\bar{x}_i$	$\bar{y}_i$	$A_i x_i$	$A_i y_i$	$d_x = (\bar{y} - \bar{y}_i)$	$d_y = (\bar{x} - \bar{x}_i)$
Rectangle ①	10×2.5 25 cm²	$\frac{10}{2}$ 5 cm	$20 + \frac{2.5}{2}$ 21.25 cm	125 cm³	531.25 cm³	(7.692 − 21.25) −13.558	(13.55 − 5) 8.55 cm
Rectangle ②	20×2.5 50 cm²	$7.5 + \frac{2.5}{2}$ 8.75 cm	$\frac{20}{2}$ 10 cm	~~250~~ 437.5	500	(7.692 − 10) −2.308	(13.55 − 8.75) 4.8 cm
Rectangle ③	17.5×5 87.5	$\frac{17.5}{2} + 10$ 18.75	$\frac{5}{2}$ 2.5	1640.625	218.75	(7.692 − 2.5) 5.192	(13.55 − 18.75) −5.2 cm
	162.5			2203.125	1250		

$$x_i = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{2203.125}{162.5} = 13.557\ cm$$

$$\bar{y}_i = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{1250}{162.5} = 7.692\ cm$$

$$I_{x_1} = \bar{I}_{x_1} + d_{x_1}^2 A_1$$

$$= \frac{bh^3}{12} + (7.692 - 21.25)^2\, 25$$

$$= \frac{10 \times (2.5)^3}{12} + (13.558)^2 \times 25$$

$$= \cancel{10} \; 13.020 + 4595.484$$

$$= 4608.504\ cm^4$$

$$I_{x_2} = \bar{I}_{x_2} + d_{x_2}^2 A_2$$

$$= \frac{bh^3}{12} + (-2.308)^2 \times 50$$

$$= \frac{2.5 \times (20)^3}{12} + 5.326 \times 50$$

$$= 1666.666 + 266.343$$

$$= 1933.0098 \text{ cm}^4$$

$$I_{x_3} = \bar{I}_{x_3} + d_3 x_3^2 A_2$$

$$= \frac{bh^3}{12} + (7.692 - 2.5)^2 \, 87.5$$

$$= \frac{17.5 \times (5)^3}{12} + (5.192)^2 \times 87.5$$

$$= 182.291 + 2358.725$$

$$= 2541.016 \text{ cm}^4$$

$$I_x = I_{x_1} + I_{x_2} + I_{x_3}$$

$$= (4608.504 + 1933.009 + 2541.016) \text{ cm}^4$$

$$= 9082.529 \text{ cm}^4$$

$$I_{y_1} = \bar{I}_{y_1} + d_1 y_1^2 A_1$$

$$= \frac{hb^3}{12} + (13.55 - 5)^2 \times 25$$

$$= \frac{2.5 (10)^3}{12} + (8.55)^2 \times 25$$

$$= 208.333 + 1827.562$$

$$= 2035.895 \text{ cm}^4$$

$$I_{y_2} = \bar{I}_{y_2} + d_2 y_2^2 A_2$$

$$= \frac{hb^3}{12} + (13.55 - 8.75)^2 \, 50$$

$$= \frac{20 (2.5)^3}{12} + (4.8)^2 \times 50$$

$$= 26.041 + 1152$$

$$= 1178.041 \text{ cm}^4$$

$$I_{y_3} = \bar{I}_{y_3} + d_3 y_3^2 A_3$$

$$= \frac{hb^3}{12} + (13.55 - 18.75)^2 \times 87.5$$

$$= \frac{5 \times (17.5)^3}{12} + (-5.2)^2 \times 87.5$$

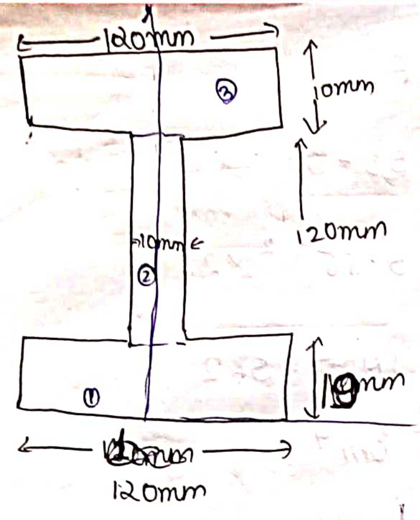
$$= 2233.072 + 2366$$

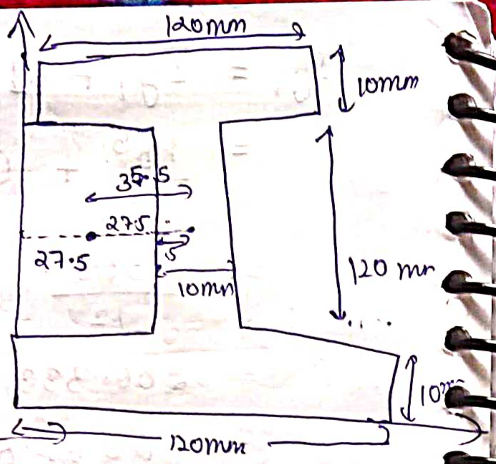
$$= 4599.072 \text{ cm}^4$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3}$$

$$= (2035.895 + 1178.041 + 4599.072)$$

$$= 7813.008 \text{ cm}^4$$





Shape/Size	Area	$\bar{x}_i$	$\bar{y}_i$	$A_i x_i$	$A_i y_i$	$d_x^2 = (\bar{y}_i - \bar{y})$	$d_y^2 = (\bar{x}_i - \bar{x})$
Rectangle ①	120X10 1200		5		6000	70-5 65	0
②	120X10 1200		60+10 70mm		84000	70-70 0	0
③	120X10 1200		130+5 135mm		162000	70-135 -65	0
	3600				252000		

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{252000}{3600}$$

$$= 70$$

$$I_{x_1} = \frac{bh^3}{12} + d_{x_1}^2 A_1$$

$$= \frac{120 \times (10)^3}{12} + (65)^2 \times 1200$$

$$= 10000 + 5070000$$

$$= 5080000 \text{ mm}^4$$

$$I_{x_2} = \frac{bh^3}{12} + d_2 x_2^2 A_2$$

$$= \frac{10 \times (120)^3}{12} + 0$$

$$= 1440000 \text{ mm}^4$$

$$I_{x_3} = \frac{bh^3}{12} + d_3 x_3^2 A_3$$

$$= \frac{120 \times 10^3}{12} + (65)^2 \times 1200$$

$$= 10000 + 5070000$$

$$= 5080000 \text{ mm}^4$$

$$I_x = I_{x_1} + I_{x_2} + I_{x_3}$$

$$= (5080000 \times 2 + 1440000)$$

$$= 11600000 \text{ mm}^4$$

$$I_{y_1} = \frac{hb^3}{12} = \frac{10 \times 120^3}{12} = 1440000 \text{ mm}^4$$

$$I_{y_2} = \frac{hb^3}{12} = \frac{120 \times 10^3}{12} = 10000 \text{ mm}^4$$

$$I_{y_3} = \frac{hb^3}{12} = \frac{10 \times 120^3}{12} = 1440000 \text{ mm}^4$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3}$$

$$= (1440000 \times 2 + 10000)$$

$$= 2890000 \text{ mm}^4$$

method-2

Area of full rectangle/part

$$I_x = \frac{b_1h_1^3}{12} - 2\left(\frac{b_1h_1^3}{12}\right)$$

$$= \frac{120\times(120+10+10)^3}{12} - 2\frac{(55\times120^3)}{12}$$

$$= \cancel{120}\ 10\times(140)^3 - 2\frac{(95040000)}{12}$$

$$= 27440000 - 15840000$$

$$= 11600000\ mm^4$$

$$I_y = \frac{h_1b_1^3}{12} - 2\left(\frac{h_2b_2^3}{12} + (32.5)^2\times55\times120\right)$$

$$= \frac{140\times120^3}{12} - 2\left(\frac{120\times55^3}{12} + \cancel{3}\ 6971250\right)$$

$$= 20160000 - 2\left(1663750 + 6971250\right)$$

$$= 20160000 - 17270000$$

$$= 2890000$$

Date 22/11/2024

Unit:4

~~Kinetic energy~~

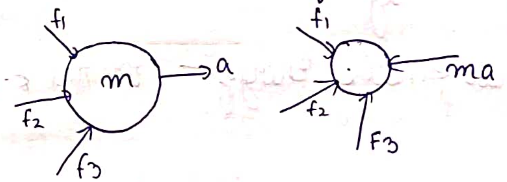
De-Alembert's Principle:

The system of forces acting on a moving body is in dinamic equillibrium with ~~on~~ the inertia forces of the body -

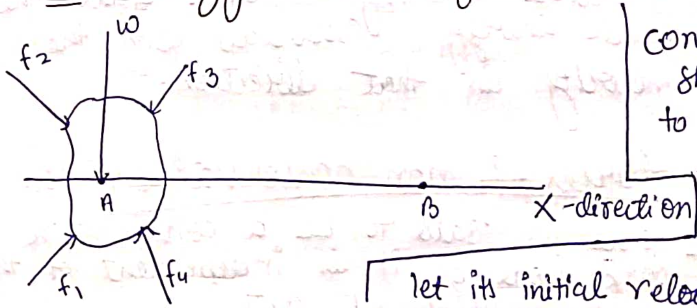
$$F = ma$$

$$\boxed{F-(ma)=0}$$

External force + inertia force $=0$



Work Energy Equation for translation Motion:



Consider the body shown in fig. subjected to system forces $f_1, f_2,$ ~~f_3, f_4~~ and moving with acceleration a in x-direction.

let its initial velocity 'u' & 'v' final velocity when it move distance $AB = S$ and ~~v~~ velocity v then the resultant of system of forces must be in x-direction

Newton II^{nd} law of motion

$$R = ma$$

$$R = \frac{\omega}{g}a$$

multiply distance ds in both side.

$$R\,ds = \frac{\omega}{g}a\,ds$$

$$R\,ds = \frac{\omega}{g}v\frac{dv}{ds}ds$$

$$R\,ds = \frac{w}{g} v\,dv$$

Integrating both side for the motion from A to B

$$\int_0^S R \cdot ds = \int_u^v \frac{w}{g} v\,dv$$

$$Rs = \frac{w}{g}\left[\frac{v^2}{2}\right]_u^v$$

$$Rs = \frac{w}{2g}v^2 - \frac{w}{2g}u^2$$

Work done = final kinetic Energy − Initial K.E.

Note:

Statement:

Work done on a particle in any direction equals the change in kinetic energy associate with the component velocity in that direction.

Conservative forces & non conservative forces:

- A ~~conservati~~ force is said to be a conservative force if the work done by it is independent of the path followed by particles it acts upon.
- In other words the work done depends only on the initial and final position of the particle.
- A scalar quantity called the Potential energy can be defined for conservative force such that the work done by the force is equal to the negative of the change in the potential energy.
- Potential energy is the point function.

- The work done by the all conservative force acting on the particle would be zero if moving particles comes back to its starting position.

Conservation of mechanical energy:

~~Newton's law can be used~~

If sum of the forces contributing to the work done on the particle are conservative forces, then the kinetic energy theorem would be more convenient because the work done by conservative forces can be computed without any integration. ~~On~~

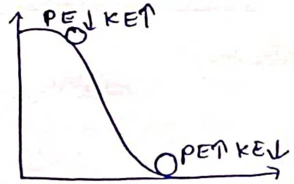
On the other hand, their might be cases when all forces which do work are conservative.

In such case

$$PE_1 - PE_2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = KE_2 - KE_1$$

$$PE_1 - PE_2 = KE_2 - KE_1$$

$$PE_1 + KE_1 = KE_2 + PE_2$$

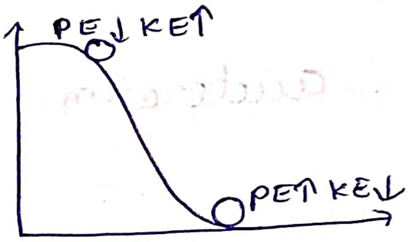


> The work done by the all conservative force acting on the particle would be zero if moving Particles cames back to its Starting position.

Conservation of mechanical energy:

~~Newton's law can be used~~
If sum of the forces contributing to the work done on the particle are conservative forces, then the kinetic energy theorem would be more convenient because the work done by conservative forces can be computed without any integration. ~~On~~
On the other hand, their might be cases when all forces which do work are conservative.
In such case

$$PE_1 - PE_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = KE_2 - KE_1$$



$$PE_1 - PE_2 = KE_2 - KE_1$$
$$PE_1 + KE_1 = KE_2 + PE_2$$

Kinematics:

It is the branch of dynamics which deals with the motion of the bodies without refering to the forces causing the motion.

Kinetics:

It is the branch of dynamics which deals with the motion of the bodies w

Qu: A particle moves along a straight line so that its displacement in metre from a fixed point is given by equation $s = t^3 + 3t^2 + 4t + 5$. find the velocity at the start and after 4 sec. The acceleration at start and at after 4 sec.

$s = t^3 + 3t^2 + 4t + 5$

$s' = 3t^2 + 6t + 4$

at 4 sec

Velocity $= s' = 3(4)^2 + 6 \times 4 + 4$

$= 48 + 24 + 4$

$= 76\ m$

acceleration $= v' = 6t + 6$

$= 24 + 6$

$= 30\ m$

Qu: The motion of the particle is described by the equation $x = 2t^3 - 6t^2 - 18t + 24$. where x is in metre & t is in sec. Determine the time, position, displacement & acceleration of the particle when its velocity becomes zero.

$\frac{dx}{dt} = v = 6t^2 - 12t - 18$

$0 = 6t^2 - 12t - 18$

$= 6(t^2 - 2t - 3)$

$= \cancel{t^2}$

(2+1)

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{2 \pm \sqrt{4 - 4(1 \times -3)}}{2 \times 1}$

$\frac{2 \pm \sqrt{4 + 12}}{2}$

$\frac{2 \pm \sqrt{16}}{2}$ $\qquad \frac{2 \pm 4}{2}$

$\frac{\cancel{6}\,3}{\cancel{2}}$ $\qquad \frac{2 - 4}{2} = \frac{-\cancel{2}\,1}{\cancel{2}}$

$3, -1$

at $v = 0$ $\qquad t = 3$

$\cancel{x = 2(27)}$

$= 6(3)^2 - 12 \times 3 - 18$

$= 54 - 36 - 18$

$= \cancel{54} - \cancel{54}$

$= 0$

acceleration $= 12t - 12$

$= 12 \times 3 - 12$

$= 36 - 12$

$= 24$

$v = \frac{\text{disp}}{\text{tin}} = m/s$

acc. $= \frac{v}{t}$ m/s^2

$p = mv$

$= g \times m/s$

$= kg\,m$

$x = 2 \times (3)^3 - 6(3)^2 - 18 \times 3 + 24$

$= \cancel{54} - \cancel{54} - 54 + 24$

$= -30$

Qu: The motion of the particle is given by $a = t^3 - 3t^2 + 5$ where a is the acceleration in m/s^2 & t is the time in sec the velocity of the particle at $t = 1$ sec is $6.25\ m/s$ & the displacement is $8.3\ m$. calculate the displacement & velocity at $t = 2$ sec.

$$a = t^3 - 3t^2 + 5$$

$$\frac{dv}{dt} = a$$

$$v = \int t^3 - 3t^2 + 5$$

$$\cancel{6.25} = \left[\frac{t^4}{4} - \frac{3t^3}{3} + 5t\right]$$

$$v\ \cancel{6.25} = \cancel{\left[\frac{1}{4} - \frac{3}{3} + 5\right]}$$

2·2·2·2

$$\cancel{v = \left[\frac{(2)^4}{4} - \frac{3(2^3)}{3} - 5\times 2\right] - \left[\frac{1}{4} - \frac{3}{3} + 5\right]}$$

$$\cancel{= [4 - 8 - 10] - \left[\frac{1}{4} - 1 + 5\right]}$$

$$\cancel{= [4 - 18] - \left[\frac{1}{4} + 4\right]}$$

$$\cancel{v = -14}$$

$$\cancel{= -12 - \frac{17}{4}}$$

$$\cancel{=}$$

$$v = \int$$

$$v = \frac{t^4}{4} - t^3 + 5t + c_1$$

$$6.25 = \frac{1}{4} - 1 + 5 + c_1$$

$$6.25 = \frac{1 + 16}{4} + c_1$$

$$6.25 - \frac{17}{4} = c_1$$

$$c_1 = 2$$

$$v = \frac{t^4}{4} - t^3 + 5t + 2$$

$$v = \frac{(2)^4}{4} - (2)^3 + 5\times 2 + 2$$

$$= 4 - 8 + 10 + 2$$

$$= 6 + 2$$

$$\boxed{v = 8}$$

$$\frac{ds}{dt} = \frac{t^4}{4} - t^3 + 5t + 2$$

$$= \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + c_2$$

$$8.3 = \frac{1}{20} - \frac{1}{4} + \frac{5}{2} + 2 + c_2$$

$$8.3 = \frac{1 - 5 + 50 + 40}{20} + c_2$$

$$c_2 = 8.3 - \frac{86}{20}$$

$$c_2 = 4$$

$$\left(\frac{ds}{dt}\right)_{at\ t=2} = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + 4$$

$$= \frac{(2)^5}{20} - \frac{(2)^4}{4} + \frac{5(2)^2}{2} + 2\times2+4$$

$$= 1.6 - 4 + 10 + 4 + 4$$

$$= 15.6$$

Impulse & Momentum theorem:

According to Newton's second law of motion rate of change of linear momentum is directly proportional to force $\vec{F} = \frac{d\vec{P}}{dt}$ —①

as Impulse $\vec{I} = \vec{F} \times dt$ —②

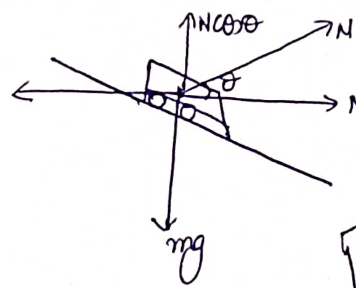
from ① & ②

$\vec{I} = d\vec{P}$

It means Impulse produce is equal to the total change in momentum produced during the impact.

~~Condition of~~ Motion of car on circular track maximum & minimum speed of safe turn on banking of road:

without friction



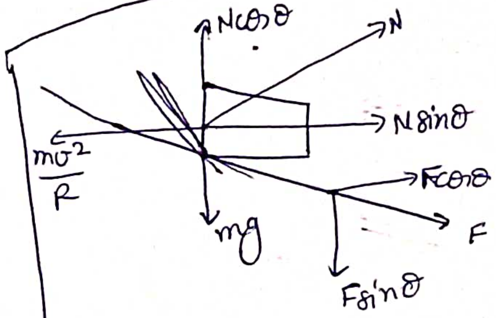
$N\cos\theta = mg$ —①

$N\sin\theta = \frac{mv^2}{R}$ —②

$$\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{R}}{mg}$$

$$\tan\theta = \frac{v^2}{Rg}$$

$$v = \sqrt{Rg\tan\theta}$$



$N\cos\theta - f\sin\theta - mg = 0$

$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$

$F = \mu N$

$$\cancel{\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} = mg}$$

$$\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta} = \frac{\frac{mv^2}{R}}{mg}$$

$$\frac{v^2}{Rg} = \frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta}$$

$$\frac{v^2}{Rg} = \frac{\cancel{N}}{\cancel{N}}\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$$

$$v^2 = \frac{Rg\,\cos\theta\left(\frac{\sin\theta}{\cos\theta} + \mu\right)}{\cos\theta(1 - \mu\tan\theta)}$$

$$v^2 = Rg\frac{(\tan\theta + \mu)}{1 - \mu\tan\theta}$$

$$v_{max} = \sqrt{\frac{Rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$$