Compromising Accuracy to Encourage Regulatory Participation

Scott Baker and Anup Malani

ABSTRACT

This paper examines the value of accuracy in voluntary or opt-in regulatory regimes. We show that if welfare depends primarily on the ability to identify noncompliant firms, tolerating mistakes in concluding that firms meet regulatory standards (false positives) can improve welfare. Consumers or investors anticipate the regulatory error rate and discount the positive message of a compliance finding. Welfare is nonetheless improved because the mistaken exoneration acts as an incentive for noncompliant firms to submit to regulatory scrutiny, and as a result some noncompliant firms are unmasked.

1. INTRODUCTION

Public and private regulatory regimes are everywhere in the modern world, and each determines compliance with certain standards. The Patent and Trademark Office determines whether to award or deny a patent application. The Association of Zoos and Aquariums (AZA) certifies zoos and aquariums on criteria such as humane treatment of animals. The New York Stock Exchange (NYSE) determines whether to allow a corporation to list shares on the exchange. States administer bar exams that determine whether a lawyer can practice and environmental regulations to determine whether factories can manufacture products in their jurisdictions. Energy Star provides environmental certifications for buildings. The list goes on.

In determining compliance, every regulatory regime sets an error

SCOTT BAKER is Vice Dean for Research and Faculty Development and the William F. and Jessica L. Kirsch Professor of Law at Washington University in St. Louis. ANUP MALANI is the Lee and Brena Freeman Professor of Law at the University of Chicago. We thank Ed Morrison, the referee, and workshop participants at the University of California, Berkeley; the American Law and Economics Association meetings; Washington University in St. Louis; Tulane University; and the University of Chicago.

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rate. How much does the regulator care about improperly classifying a noncompliant firm as compliant (borrowing from criminal law, we refer to this as a mistaken exoneration or a false positive) or improperly classifying a compliant firm as being in breach (a mistaken conviction or false negative)? If participation in a regulatory regime is mandatory and the goal is deterrence, the proper balance between false positives and false negatives is well settled. The optimal degree of accuracy balances the marginal gain in deterrence against its marginal cost, as noted in Kaplow and Shavell (1994). Unless preventing one type of error is cheaper than preventing the other type of error, the social planner does not try harder to avoid mistaken convictions than mistaken exonerations. The reason resonates: deterrence turns on the difference or wedge between the sanction the individual expects if she commits the harmful act and the sanction expected if she does not. Reducing false positives increases this wedge. So does reducing false negatives.

The prior literature, however, assumes that parties must participate in the regulatory regime. This is a natural assumption in the criminal context: individuals cannot opt out of the criminal law. Here we consider situations in which, unlike criminal law, an individual or firm can opt into or out of the regulatory regime.

An entrepreneur, for example, need not file for a patent; he can protect his innovation via trade secret instead. A zoo can operate without accreditation by the AZA. A corporation need not list its shares on a public exchange but instead can rely on private equity to fund any business expansion. A lawyer need not take the bar examination in a state to provide transactional law services to clients in that state. Similarly, a firm can avoid the scrutiny of a state's environmental regime by selecting a different location for its production facilities and export its product to that state. Finally, a developer need not seek the Energy Star certification to build or sell a building.

In voluntary regulatory regimes is there any value to error, and does it depend on whether that error is a false positive or a false negative? Somewhat surprisingly, we show that certain errors can be beneficial. Our model demonstrates that mistaken findings of compliance can encourage greater participation by noncompliant or bad-quality firms. Whereas bad-quality firms avoid rigorous regulatory regimes for fear of being uncovered, they are attracted to less rigorous regimes because they might accidentally be labeled as compliant and thereby get a better reception by stakeholders such as consumers, workers, or investors. Yet because some of these firms might be identified as noncompliant, stakeholders obtain more information about them than if the firms had not participated in the regime at all. In short, false positives in voluntary regimes can, paradoxically, increase the amount of information available about noncompliant firms and thereby raise welfare.

With that said, nothing is free. The introduction of false positives dilutes the signal that stakeholders receive from the regulatory regime about good-quality firms. Because of false positives, consumers are less likely to conclude that the firm is good simply because the regulator said so. In terms of welfare, society is willing to make this trade-off—less information about good-quality firms for more information about bad-quality firms—in contexts in which ferreting out bad actors is of high import.

Our result depends on several assumptions, which are met by the examples at the top of Table 1. The first assumption is that stakeholders value identifying bad-quality firms. Lenders, for example, may want to know if a firm is involuntarily delisted from the NYSE so that they can avoid the risk of lending to that firm. Consumers who care about the social impact of the products they consume might care whether an aquarium failed AZA certification or a building failed Energy Star certification. Finally, a law firm may want to know if a lawyer failed the bar so it can avoid hiring a low-quality worker.

Second, there must be legitimate reasons for the firm or worker to avoid participating in the regulatory regime. Otherwise, firms that do not do so would be viewed as ones that would certainly have failed scrutiny. The end result is unraveling as in Grossman (1981) and Grossman and Hart (1980). This condition is met by the examples of voluntary regimes in Table 1. For example, a firm may not file for patent protection because of the high cost of preparing an application.[1] Similarly, a zoo may not file for AZA certification because the application process is costly. A firm may not list on the NYSE (and thus avoid a future delisting) because the costs of listing are substantial, especially for small firms. Many lawyers do not take the bar in a state because they do not plan to work in that state. Finally, an owner of a building might not apply for Energy Star certification because she is cash constrained.

1. In this paper, we focus on the signaling value of patent protection rather than value derived from the conveyance of an exclusive right. For an extended discussion of this function of patent law, see Long (2002, p. 627), who writes, “The ability to convey information credibly to observers at low cost is a highly valuable function of patents that has been completely overlooked in the literature.”

Table 1. Characteristics of Regulatory Regimes

Regime	Participation Voluntary	Value to Identifying Noncompliers	Legitimate Reasons Not to Participate	Failure to Qualify Is Observable	Failure Worse than Nonparticipation	Signaling Is Main Benefit
Meets the assumptions of our model:						
Association of Zoos and Aquariums	Yes	Yes	Yes	Yes	Yes	Yes
CNET reviews	Yes	Yes	Yes	Yes	Yes	Yes
Angie's List Super Service Award	Yes	Yes	Yes	Yes	Yes	Yes
College courses (nonrequired)	Yes	Yes	Yes	Yes	Yes	Yes
Energy Star certification for buildings	Yes	Yes	Yes	Yes	Yes	Yes
May meet the assumptions of our model:						
Bar exam	Yes	Yes	Yes	Yes	Yes	Maybe[a]
State environmental regulation	Yes	Yes	Yes	Yes	Yes	Maybe[b]
Airline alliance	Yes	Yes	Yes	Yes	Yes	Maybe[c]
Patent filing	Yes	Yes	Yes	Yes	Yes	Maybe[d]
Litigating a torts case	Maybe[e]	Yes	Yes	Yes	Yes	Maybe[f]
New York Stock Exchange listing	Yes	Yes	Yes	Maybe[g]	Yes	Yes
Does not meet the assumptions of our model:						
Energy Star certification for products	Yes	Yes	Yes	No	Yes	Yes
Occupational licensing for local services	No	Yes	N.A.	Yes	N.A.	No
Yelp	No	Yes	N.A.	Yes	N.A.	Yes
Michelin Star rating system	No	Yes	N.A.	Yes	N.A.	No

Note. N.A. = not applicable.
[a] May be applicable for out-of-state transactional services.
[b] In-state production may reduce transport costs for the state of registration but not costs in other states to which the firm exports.
[c] May be applicable for some lower quality airlines.
[d] Applicable when trade secret is an effective substitute.
[e] While suit is involuntary, secret settlement is voluntary.
[f] For some products, consumers may judge quality by litigation history.
[g] While evaluation of a listing application is confidential, involuntary delisting is public.

Third, stakeholders must observe a firm's failure to meet the regulatory standard. This condition is met in some cases and not others. For example, since applications to the AZA are public, zoos that fail certification—as opposed to merely not applying—can be identified. A venture capitalist can ask an inventor whether she has ever applied for a patent and been denied. A buyer of commercial real estate can ask whether the builder applied for Energy Star certification and the results of that certification process. By contrast, while delisting from the NYSE is public, applications to list on the NYSE are not public. Therefore, firms that are not listed cannot be distinguished from firms that do not apply.[2]

By focusing on voluntary regimes, our work closely relates to Dari-Mattiacci and Raskolnikov (2021). That work also relaxes the assumption that entities must participate in the regulatory regime. Among other things, Dari-Mattiacci and Raskolnikov show that the classic Becker result of higher sanctions leading to more compliance does not necessarily translate when entities are not required to participate in the regulatory regime in the first place.

Unlike Dari-Mattiacci and Raskolnikov (2021) and much of the prior work on accuracy of adjudication (Kaplow and Shavell 1994, 1996), our model is not about structuring regulation to encourage (potentially) regulated parties to behave better. Instead, the focus is on the information a regulatory regime provides to buyers of products made by those who can opt into a regulatory regime.

Gailmard and Patty (2017) also investigate the possibility that errors in adjudication might create beneficial incentive effects. Their model involves agency rulemaking subject to a notice and comment requirement. They show that a rational court might wish to affirm an agency decision that is based on no evidence whatsoever. Such judicial deference, in turn, induces the interest group to seek out strong evidence against an agency's policy position at the comment stage, which generates more information for the court.

In our model, the social planner picks the chance of false positives and false negatives to induce socially desirable behavior, knowing that buyers

2. Our model requires two additional, though less significant, conditions, which are also evaluated in Table 1. One is that failing to apply (remaining silent) is not as bad as applying and failing the test. If failing to apply is worse than applying and failing, then firms will participate in the regulatory regime regardless of the error rate. The other assumption is that there are few nonsignaling benefits from participation. Indeed, if there are enough nonsignaling benefits (or if participation is required), then a firm will participate regardless of the error rate. For example, a doctor can provide medical services in a state only if she is licensed to practice in that state.

and regulated entities will make proper inferences and decisions in light of these error rates. In this way, the model shares similarities with the Bayesian persuasion literature (Kamenica and Gentzkow 2011). In that model, the sender selects a signaling technology to persuade the receiver. The receiver understands that it is being subject to persuasion. That is to say, everyone is Bayesian. Yet by shifting, say, the composition of false negatives and false positives, the sender can alter the receiver's behavior. Similarly, our planner changes behavior by shifting the composition of errors.

Finally, William Blackstone's famous quote "[I]t is better that ten guilty persons escape than one innocent suffer" (Blackstone [1769] 2009, p. 352) inspired our paper. There is a voluminous literature on the Blackstone principle in criminal law. Almost all of this literature either takes a moral approach or assumes that the harm from wrongful convictions outweighs the harms from letting a guilty defendant go free (Radin 1978; Solum 1994; Volokh 1997; Fallon 2008).[3] Because public and private regulatory regimes also impose punishments or grant privileges, this paper asks whether the Blackstone principle should extend beyond criminal law to other regulatory decisions. We differ from this literature by focusing on functional reasons why a society might want to prefer to permit false positives or mistaken exonerations and steer clear of false negatives or mistaken convictions.

The paper unfolds as follows. Section 2 provides a numerical example that illustrates the main themes. Section 3 sets up and solves the model. Section 4 explores the welfare implications of the model. Section 5 compares a voluntary regulatory regime that makes errors with a mandatory regulatory regime, showing the benefits of an opt-in (and error-prone) regime. Section 6 offers a short conclusion. The proofs are in the Appendix.

2. EXAMPLE

Consider an exchange between a commercial building developer (the seller) and an investor (the buyer). Suppose that the seller might have one of two types of projects: good or bad. The good project, which is envi-

3. The Supreme Court endorsed this rationale for the Blackstone principle: "It is critical that the moral force of the criminal law not be diluted by a standard of proof that leaves people in doubt whether innocent men are being condemned. It is also important in our free society that every individual going about his ordinary affairs have confidence that his government cannot adjudge him guilty of a criminal offense without convincing a proper factfinder of his guilt with utmost certainty" (*In re Winship,* 397 U.S. 358, 364 [1970]). Epps (2015) provides an illuminating alternative take on the Blackstone principle.

ronmentally friendly and cost-efficient to run, provides the buyer a return of 1. The bad project provides the buyer a return of 0. The seller knows the building's type. The buyer does not.

On the basis of prior experience, the buyer believes initially that $\frac{1}{2}$ of the projects are of good quality. Whatever its type, the seller wants to pass off the building as good so he can charge a higher price.

In making her investment decision, the buyer has the ability to invest in a safe alternative. This alternative always provides the buyer a return of $\frac{1}{4}$. Given these numbers, the buyer should invest in the seller's building when it is a good project. The buyer should invest in the safe alternative when the seller's building is a bad project.

We now explore how the existence of Energy Star certification—and its accuracy—might help the sellers with good projects signal their worth and, perhaps, help the buyer ferret out and avoid developers with bad projects. As noted in Section 1, the Energy Star program is an example of a voluntary or opt-in regulatory regime: sellers of commercial real estate do not have to obtain the Energy Star certification to sell property.[4] In this example and in the model below, we assume that, because of contract incompleteness, the buyer cannot rely on contract provisions, such as a warranty, for protection from the risk of purchasing from a bad-quality firm.

2.1. No Energy Star Certification Program

As a benchmark, consider a regime without any certification program. On average, the buyer believes that buying the seller's property will provide a payoff of

$$1 \times \Pr(\text{Good}) + 0 \times \Pr(\text{Bad}) = \frac{1}{2}.$$

Because the buyer's payoff to purchasing the seller's property exceeds the payoff to investing in the safe alternative, the buyer will do so.

4. According to data from the Department of Energy, more than 20,000 commercial buildings have certified with the Energy Star program (Environmental Protection Agency, What Is Energy Star? [https://www.nist.gov/system/files/documents/iaao/EnergyStar.pdf]). For example, Bruegge, Carrión-Flores, and Pope (2016, p. 63) find that "homeowners are willing to pay a premium for new Energy Star residences, but that this premium fades rapidly in the resale market." Eichholtz, Kok, and Quigley (2010, p. 2492) find that "buildings with a 'green rating' command rental rates that are roughly 3 percent higher per square foot than otherwise identical buildings—controlling for the quality and the specific location of office buildings. Premiums in effective rents are even higher—above 7 percent. Selling prices of green buildings are higher by about 16 percent." The Energy Star certification program for buildings is voluntary (see Energy Policy Act of 2005, sec. 131, 42 U.S.C. sec. 6294a).

Let the seller make a take-it-or-leave-it offer to sell. In other words, the seller has all the bargaining power. With such an offer, the seller can induce the buyer to pay an amount that makes the buyer indifferent between the safe alternative investment and the expected payoff from the project.

In this no-intervention benchmark, the buyer invests and pays the developer a price of $p = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$. The developer with the good project gets funded but at less than what it is really worth. At the same time, the developer with the bad project gets funded. This is a misallocation of investment resources. Society is better off if the buyer invests in the safe alternative rather than the bad project. Because the buyer always invests in the real estate project, which succeeds half of the time, expected welfare is $\frac{1}{2}$.

2.2. A Perfectly Accurate Certification Program

As noted in Section 1, the administrator of the Energy Star certification program might make two types of errors. First, the administrator might inadvertently grant certification to a developer with a bad project, a false positive or mistaken exoneration. Formally, the rate of false positives is the probability that a good score, g, is issued conditional on the underlying project being bad: $\Pr(g \mid \text{Bad})$. Second, the administrator might inadvertently deny certification to a developer with a good project, a false negative or mistaken conviction. The rate of false negatives is the probability that a bad score, b, is issued conditional on the underlying project being good: $\Pr(b \mid \text{Good})$.

Suppose that the certification system is perfectly accurate. Further, it can achieve this ideal with no administrative cost to the administrator. Kaplow and Shavell (1994) focus entirely on administrative cost as the driver of imperfectly accurate legal systems. We want to take the administrator's cost considerations off the table.

Not all developers—whether they have a good or a bad project—view Energy Star certification as a viable option. For some developers, the fees associated with certification are cost prohibitive. To capture this, suppose that only $\frac{1}{2}$ of developers—with good or bad projects—view certification as a cost-viable option. The others do not see participation in the certification process as cost justified irrespective of the benefit (in terms of being able to charge more) attached to certification.

This leaves four types of developers, each occurring in the population with a probability of $\frac{1}{4}$: developers with good projects and low costs of filing for certification, developers with good projects and high costs of filing for certification, developers with bad projects and low costs of filing for certification, and developers with bad projects and high costs of filing for certification. We now ask our first question: does a perfectly accurate regulatory system improve welfare over the no-intervention benchmark?

The first step toward an answer examines the behavior of the developer with a good project and a low cost of filing for certification. Since no mistakes are made, the developer will obtain an Energy Star certification. Seeing the good outcome (g)—and knowing that the certification system is error free—the buyer will realize that the developer has a good project for sure and will fund it. The buyer is willing to pay a price that reflects the difference in value between the value of the good project and the safe option:

$$p_{\mathrm{g}} = 1 - \frac{1}{4} = \frac{3}{4}.$$

The second, more interesting step examines the behavior of the developer with a bad project and a low cost of filing for certification. If she files for certification, given the perfect accuracy of the certification process, the project will not obtain the Energy Star mark. During due diligence, the buyer will learn this. She will therefore not fund the project.

Suppose instead that the developer with a bad project stays silent, that is, does not seek certification and hides herself among the developers with good projects who did not file because certification was too pricey. The buyer cannot readily distinguish a developer with a bad project from a developer with a good project who could not afford the application fee.

With a perfectly accurate certification system, the buyer's belief that the developer has a good project following silence is

$$\Pr(\text{Good} \mid \text{Silence}) = \frac{\Pr(\text{Silence and Good})}{\Pr(\text{Silence})}$$

$$= \frac{\Pr(\text{Good and High Cost})}{\Pr(\text{Good and High Cost}) + \Pr(\text{Bad and High Cost}) + \Pr(\text{Bad and Low Cost})}$$

$$= \frac{1/4}{(1/4) + (1/4) + (1/4)} = \frac{1}{3}.$$

The buyer understands that some developers with good projects do, in fact, apply to the Energy Star program, so she downgrades her assessment that the project is good after observing silence. This downgrade, however, is not enough to change her investment decision. She still invests but pays a lower price than if the project carried the Energy Star mark: $p_{\text{Silence}} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$.

What happens to welfare under a perfectly accurate certification system? Two outcomes are possible: the developer obtains the Energy Star certification for her building, and the buyer purchases it at a price of $\frac{3}{4}$, or the developer is silent, and the buyer purchases the building at the discounted price of 1/12. Since both types of projects are funded, and half of those funded projects fail, expected welfare is $\frac{1}{2}$. Because the informational content flowing from the verification system is insufficient to enable the buyer to avoid developers with bad projects, adoption of the perfectly accurate certification system does not improve welfare.

2.3. The Benefit of False Positives

Now consider an Energy Star program that grade inflates; that is, it occasionally grants the certification to buildings that fail to meet the standard. Assume that the certification system allows bad developers to obtain the Energy Star mark half of time ($\Pr(g \mid \text{Bad}) = \frac{1}{2}$). Everything else remains the same as above—that is to say, the Energy Star program never mistakenly denies the mark to a developer with a good project.

Consider first the behavior of the developer of a good project who can apply at low cost to the regulatory regime. She will seek certification and obtain the mark for sure.

Now consider the developer with a bad project who finds certification to be a low-cost process. If she seeks certification, half the time she will be denied the certification and go out of business. The other half of the time she will be mistakenly granted the certification. She must compare the payoff from this certification lottery with what she might make by remaining silent. Suppose that 88 percent of the bad-quality firms with low costs play the certification lottery and 12 percent remain silent. (Readers will see later why we pick these numbers.)

Under this assumption, the pool of silent firms consists of all the bad-quality firms with high costs of certification, all the good-quality firms with high costs of certification, and 12 percent of the bad-quality firms with low costs of certification. Accordingly, on observing silence the buyer's assessment of the firm's quality is

$$\Pr(\text{Good} \mid \text{Silence}) = \frac{\Pr(\text{Good and High Cost})}{\Pr(\text{Silence})}$$

$$= \frac{.25}{.25 + .25 + .25 \times .12} \approx .47.$$

Given this assessment, the buyer pays the seller $p_{\text{Silence}} = .47 - .25 \approx .22$.

What should the buyer believe following certification? Some developers with bad projects who filed for certification got it, but all developers with good projects obtained the certification. Accounting for this shifting composition, the buyer's belief that the developer has a good project when the project carries the Energy Star mark is $\Pr(\text{Good} \mid g) = .69$.

Will the developer with the bad project and low costs prefer to file? If she files and obtains certification, the buyer pays the developer $p_g = .69 - .25 = .44$. Of course, this developer obtains this payoff only with a mistaken grant, a false positive. Therefore, the expected payoff from participating is

$$\Pr(\text{Mistaken Grant}) \times .44 = .5 \times .44 = .22.$$

If the developer instead remains silent, the buyer will fund the project for sure but at the discounted price of $p_{\text{Silence}} = .22$. Observe that this value is the same as the payoff obtained from filing for certification.

So the developer with a bad project and low filing costs is indifferent between risking a filing for certification and remaining silent. She is thus willing to randomize between the two actions and pursue Energy Star certification with a probability of .88. In short, the mistaken grant acts as participation credit, enticing some of the bad-quality firms into the voluntary rating system.

This analysis makes clear that a regulatory regime with false positives can generate welfare benefits. Unlike with a perfectly accurate certification system, there are three possible outcomes: the developer obtains certification, and the buyer funds the project; the developer is denied certification, and the buyer does not fund the project; and the developer remains silent, and the buyer funds the project. With a perfect regime, half of all funded projects fail. Now the buyer, on occasion, is able to identify bad projects and deny them funding, going instead with the safe

Table 2. Welfare Comparisons

	Welfare
First-best welfare	$1 \times \Pr(\text{Good}) + .25 \times \Pr(\text{Bad}) = .5 + .25 \times .5 = 6.25$
No intervention	$1 \times \Pr(\text{Good}) + 0 \times \Pr(\text{Bad}) = .5$
Perfect accuracy	$\Pr(\text{g})W(\mu_{\text{g}}) + \Pr(\text{b})W(\mu_{\text{b}}) + \Pr(\phi)W(\mu_{\phi})$ $= .25 \times 1 + 0 + .75 \times (.25/.75) = .5$
Some false positives	$\Pr(\text{g})W(\mu_{\text{g}}) + \Pr(\text{b})W(\mu_{\text{b}}) + \Pr(\phi)W(\mu_{\phi})$ $= (.25 + .25 \times .5 \times .88) \times [.25/(.25 + .25 \times .5 \times .88)]$ $+ (.5 \times .88 \times .25) \times .25$ $+ (.5 + .12 \times .25) \times [.25/(.5 + .12 \times .25)]$ $= .527$

alternative. Denials happen when the developer with a bad project tries the certification lottery and fails.

Table 2 calculates the welfare associated with different regimes. Welfare is either the buyer's assessment that the project is good after receiving some piece of information or, if the buyer forgoes investing in the developer, the value of the safe alternative; that is, $\frac{1}{4} = .25$. In the model that follows, we denote the signals the buyer might receive as s, where s might be a good score (g), a bad score (b), or silence (ϕ). We denote the buyer's belief that the firm is of good quality following signal s as μ_s. As just noted, welfare is the expected value of a trade with the firm when the buyer purchases and the value of the outside option when she does not. The error rates and equilibrium strategies of the firm types induce a distribution over the buyer's posterior beliefs. Those beliefs then determine the buyer's decision for each signal realization, which then generates a certain amount of welfare.

The first three rows in Table 2 provide benchmarks. The first is first-best welfare, that is, welfare if the buyer knows the type of project the developer offers (under no information asymmetry). In that case, the buyer will fund the good projects only. If the developer has a bad project, the buyer will invest in the safe alternative instead, reaping a return of .25. The second is the no-intervention benchmark, or welfare with no regulatory regime. The buyer invests; half the projects pan out, and half the projects fail. The third is the welfare from a perfectly accurate certification system. In such a regime, the buyer makes the purchase irrespective of the signal received, and of those projects half pan out.

The final row shows the welfare of a regime with false positives but no false negatives. Under this regime, the buyer does not always make the purchase. She purchases when the firm obtains a good score or is silent.

She buys the outside option after observing a bad signal. Notice, then, that the buyer successfully buys from all the good firms (who are either silent or obtain a good score). She avoids the bad-quality firm and reaps a return of .25 from the outside option when the bad-quality firm has a low cost of access, accesses the regime, and obtains a bad score. The probability of this event is $.25 \times .88 \times .5$. Because of this, the introduction of false positives improves welfare from .5 to .527.

The model below explores these themes in more generality—identifying conditions under which the analysis holds—but the basic thesis remains the same: with opt-in regulatory regimes, policy makers must worry about participation by the regulated entity and information revelation to third parties. A perfectly accurate system maximizes information revelation to third parties but only under the assumption of mandatory participation. Given a choice to participate, the inaccurate system—one that renders false positive but few false negatives—can enable consumers and investors to identify bad actors and avoid transacting with them, which thereby makes society better off.

3. MODEL

3.1. Setup

A firm offers a product to a buyer. When doing so, the firm makes the buyer a take-it-or-leave-it offer. The firm is one of four possible types: the firm's type, θ, is an element of $\Theta = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$. The first number in the firm's type is the quality of its product, which can be good (1) or bad (0). The second number is its cost of participating in the regulatory regime, which can be high (1) or low (0). The firm knows its type. The buyer does not. The firm decides to participate in the regulatory regime ($a_\theta = 1$) or not to participate ($a_\theta = 0$).

At the outset, the buyer believes that each type arises with a probability of $\frac{1}{4}$. Table 3 shows the initial probability distribution of types. The buyer's initial belief that the firm offers a good-quality product (the marginal probability mass function of X) is $f_X(1) = \frac{1}{2}$. Similarly, the buyer's initial belief that the firm has a high cost of accessing the regulatory regime (the marginal probability mass function of Y) is $f_Y(1) = \frac{1}{2}$.

3.1.1. Regulatory Regime. We model the regulatory regime as a test of the firm's quality that the firm can either pass or fail. We refer to the ac-

Table 3. Values of Initial Joint Probability Mass Function $f(x, y)$

	Access Costs (Y)		
Quality (X)	0	1	$f_X(x)$
0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
$f_Y(y)$	$\frac{1}{2}$	$\frac{1}{2}$	

curacy of the test as shorthand for the accuracy of the regulatory regime. As in the example, if the firm opts into the regulatory regime, the regulator might reveal a compliant, good signal (g) or a noncompliant, bad signal (b).

The regulator selects the error rate of the test. The regulator might do so through its allocation of investigation resources. The regulator, for instance, might spend more money on ferreting out noncompliant firms than avoiding false accusations of misconduct by compliant firms. In generic terms, the following conditional probabilities represent the error rate of the regulatory regime:

$$\Pr(\mathrm{g} \mid \mathrm{Good}) = 1 - \beta,$$

$$\Pr(\mathrm{g} \mid \mathrm{Bad}) = \alpha,$$

$$\Pr(\mathrm{b} \mid \mathrm{Good}) = \beta,$$

and

$$\Pr(\mathrm{b} \mid \mathrm{Bad}) = 1 - \alpha,$$

where $\beta \in [0, \frac{1}{2}]$ and $\alpha \in [0, \frac{1}{2}]$. The accuracy of the test can range from perfect ($\alpha = \beta = 0$) to uninformative ($\alpha = \beta = \frac{1}{2}$).

The probability that a good-quality firm receives a good score from the regulatory regime is the same whether that firm has a low cost or a high cost of accessing the regulatory regime. The same is true for the bad-quality firm.

3.1.2. Signals and Buyers' Inferences. If the firm participates in the regulatory regime, the buyer observes whether the firm received a good or bad signal. The buyer uses this information to update her beliefs about the firm's quality. If the firm does not participate in the regulatory regime, the buyer observes silence and must decide what to make of it. Does silence mean that it is a good-quality firm that declined regulatory scrutiny out of cost concerns, or is it a bad-quality firm that declined regulatory scrutiny fearing what that scrutiny might reveal? As noted in the example, the set of possible signals is $S = \{g, b, \phi\}$, where ϕ represents silence, and s is some element of S.

Given the firm's participation decision and the signal received, the buyer forms a posterior belief about the quality of the firm. Recall that μ_s is the buyer's posterior belief that the firm is good given a signal s. Following Frankel and Kartik (2019), we say that the quality of the product is the buyer's dimension of interest. That is to say, the buyer cares only about product quality; she does not care about the firm's cost of accessing the regulatory regime.

3.1.3. Payoffs and Purchase Decisions. As in our example, the buyer obtains a surplus of one if she purchases from a good-quality firm and zero if she buys from a bad-quality firm. In addition, the buyer has access to an outside option that provides her with a known payoff of $v > 0$. Given the availability of the outside option, a buyer purchases from the firm when

$$\mu_s > v.$$

Further, given that the firm makes a take-it-or-leave-it offer, the buyer who purchases from the firm pays $\mu_s - v$. If we combine the expressions, the firm's payoff is either 0 (the buyer does not purchase) or $\mu_s - v$ (the buyer purchases and pays her expected surplus over the outside option). The price paid to the firm is thus

$$p_s = \max\{0,\ \mu_s - v\}.$$

Accounting for the cost of accessing the regime, we see that the payoff to the firm of type θ is

$$U_\theta = a_\theta[\Pr(g \mid \theta)p_{\mathrm{g}} + \Pr(b \mid \theta)p_{\mathrm{b}}] + (1 - a_\theta)p_\phi - a_\theta y_\theta,$$

where y_θ is the cost realization for firm type θ.

3.1.4. Equilibrium Definition. A perfect Bayesian equilibrium consists of a profile of strategies and the buyer's beliefs such that each firm type acts optimally given the buyer's beliefs. The buyer pays the firm—if this number is positive—the surplus over the outside option she expects from the transaction, and the buyer's beliefs are consistent with Bayes's rule whenever possible.

3.1.5. Welfare. Welfare depends on the buyer's purchase decisions and the firm's cost of participating in the regime. As noted, the surplus equals one if the buyer buys and the firm is of good quality and zero if the buyer buys and the firm is of bad quality. If the buyer goes with the outside option instead, her benefit is v. We can thus define welfare as

$$W \equiv \sum_{\theta \in \Theta} \{\Pr(\text{Purchase} \mid \theta) x_\theta + [1 - \Pr(\text{Purchase} \mid \theta)] v - a_\theta y_\theta\} \Pr(\theta),$$

where $x_\theta \in \{0, 1\}$ is the surplus obtained from the purchase of a product from a firm of type θ.

The social planner wants to help the buyer ferret out the bad-quality firm so she can choose the more profitable, outside option. The social planner also wants to ensure that the firm types with high access costs do not participate in the regulatory regime.

3.1.6. Benchmarks. We compare different regimes with two benchmarks. In our first benchmark, the buyer knows the firm's quality. In that case, no firm will bother with regulatory review. The buyer will purchase from the good-quality firm and select the outside option if she encounters a bad-quality firm. In this first-best case, expected welfare is

$$W^{\text{FB}} = \frac{1}{2} + \frac{v}{2}.$$

Our second benchmark is when the social planner offers no regulatory regime whatsoever. To make matters interesting, we assume throughout that $v < \frac{1}{2}$, so the buyer purchases the product in this no-intervention benchmark, that is, no regulatory regime. As in the example, the expected welfare from this regime is $\frac{1}{2}$.

3.2. The Perfectly Accurate Regulatory Regime

Should the regulator choose a perfect regulatory regime, with no errors, to identify compliant and noncompliant firms? To answer this question, proposition 1 lays out the equilibrium with a perfectly accurate regulatory regime and identifies its welfare properties.

Proposition 1. Suppose that the regime is perfectly accurate; that is, $\beta = \alpha = 0$.

i) There is no equilibrium in which a good-quality firm with no access costs ($\theta = (1, 0)$) and a bad-quality firm with low access costs ($\theta = (0, 0)$) both participate in the regulatory regime.

ii) An equilibrium exists in which a good-quality firm with low access costs ($\theta = (1, 0)$) participates in the regulatory regime and all other types do not. In this equilibrium, the buyer's beliefs about the quality of the product are $\mu_g = 1$, $\mu_b = 0$, and $\mu_\phi = \frac{1}{3}$.

iii) If $v \leq \frac{1}{3}$, welfare is the same under a perfectly accurate regime as in the no-intervention benchmark. If $v \in (\frac{1}{3}, \frac{1}{2}]$, welfare is higher with a perfectly accurate regulatory regime than in the no-intervention benchmark.

Given a perfectly accurate regime, a good-quality firm with low access costs opts into the regulatory regime. The bad-quality firm with low access costs does not. Instead, this type hides among the good-quality firm types that did not participate because they faced a high cost of accessing the regime.

Proposition 1.iii reveals that a perfectly accurate regime improves welfare over the no-intervention benchmark only when the value of the outside option is high. The intuition is as follows: In the no-intervention benchmark, the buyer purchases from the firm. To change welfare, the regulatory process must generate information that changes the buyer's behavior following one of the signal realizations. If the buyer obtains a small value from the outside option, that does not happen. The buyer purchases after a good signal or silence, while the bad signal is never sent. In this setting, the buyer buys after observing silence despite knowing that the silent pool is composed of $\frac{2}{3}$ bad-quality firms and $\frac{1}{3}$ good-quality firms. She does so because going with the outside option is so unattractive.

On the other hand, if the buyer obtains high value from the outside option, she switches to the outside option after observing silence. As a result, she does not purchase unless the firm participates in the regime and obtains a good score. Of course, by failing to purchase following silence, the buyer misses the chance to purchase from the good-quality firm with a high cost of access. At the same time, she avoids purchasing from the bad-quality firm altogether. The condition that $v > \frac{1}{3}$ means that the gains in welfare from successfully avoiding a purchase from the bad-quality firm outweigh the loss in welfare from missing out, on occasion, on the opportunity to buy from a good-quality firm.

3.3. The Inaccurate Regulatory Regime

We now ask if the planner might do better with an inaccurate regime. And if so, should the planner be more concerned about mistakenly giving a bad-quality firm a good signal or mistakenly giving a good-quality firm a bad signal?

Irrespective of the accuracy of the regime, the behavior of firms whose type has a high cost of access is the same: they do not participate. After all, the most these firms could make from participating in the regime is $1 - v$. This sales revenue does not cover the cost of participation for a firm with high access costs. So, by assumption, we have determined the behavior of two of the four types of firms. Lemma 1 goes a step further and establishes that we can focus solely on the behavior the low-quality firm with low access costs ($\theta = (0, 0)$).

Lemma 1. In an inaccurate regulatory regime, if the bad-quality firm with low access costs participates in the regulatory regime or is indifferent between participating and not participating, the good-quality firm with low access costs participates for sure.

A bad-quality firm understands that it is more apt to receive a bad signal than a good signal and that the buyer pays more following a good signal than a bad signal. A good-quality firm, by contrast, anticipates obtaining a good signal more often than a bad signal. And so if a bad-quality firm prefers to access the regulatory regime, the good-quality firm must prefer accessing it too. The reason resonates: the good-quality firm is more likely to receive a good score and, as a result, the more substantial payment from the buyer.[5] With lemma 1 in hand, proposition 2 characterizes the equilibrium associated with an inaccurate regulatory regime.

Proposition 2. Suppose the regulatory regime is inaccurate; that is, $\alpha > 0$ and $\beta > 0$.

i) There does not exist an equilibrium in which a good-quality firm with no access costs ($\theta = (1, 0)$) participates in the regime and a bad-quality firm with no access costs ($\theta = (0, 0)$) does not.

ii) An equilibrium exists with the following properties:

5. This same idea arises in Hörner and Skrzypacz (2016). In that model, the agent sets the difficulty of a test. Hörner and Skrzypacz (2016, p. 1526) write, "Given her knowledge of the type, [the agent] assigns different probabilities to these posterior beliefs than the firm. . . . If she is incompetent, the expectation of the posterior belief is below p_0 [the prior belief], as she does not know whether she will be lucky in taking the test (the process is then a supermartingale)."

a) The good-quality firm with no access costs participates in the regulatory regime ($a_{(0,0)} = 1$).

b) The good-quality and bad-quality firm types with high access costs do not participate ($a_{(0,1)} = a_{(1,1)} = 0$).

c) The bad-quality firm with no access costs either participates in the regime ($a_{(0,0)} = 1$) or randomizes between participating and not participating ($a_{(0,0)} = \gamma$, where $\gamma \in (0, 1)$).

d) In the equilibrium in which firms of type $\theta = (0, 0)$ participate for sure, the buyer's beliefs are

$$\mu_{\mathrm{g}} = \frac{(1-\beta)f(1,\ 0)}{(1-\beta)f(1,\ 0) + \alpha f(0,\ 0)} = \frac{1-\beta}{1-\beta+\alpha},$$

$$\mu_{\mathrm{b}} = \frac{\beta f(1,\ 0)}{\beta f(1,\ 0) + (1-\alpha)f(0,\ 0)} = \frac{\beta}{\beta+(1-\alpha)},$$

and

$$\mu_{\phi} = \frac{f(1,\ 1)}{f(1,\ 1) + f(0,\ 1)} = \frac{1}{2}.$$

e) In the equilibrium in which firms of type $\theta = (0, 0)$ mix, the buyer's beliefs are

$$\mu_{\mathrm{g}} = \frac{(1-\beta)f(1,\ 0)}{(1-\beta)f(1,\ 0) + \alpha\gamma f(0,\ 0)} = \frac{1-\beta}{1-\beta+\alpha\gamma},$$

$$\mu_{\mathrm{b}} = \frac{\beta f(1,\ 0)}{\beta f(1,\ 0) + (1-\alpha)\gamma f(0,\ 0)} = \frac{\beta}{\beta+(1-\alpha)\gamma},$$

and

$$\mu_{\phi} = \frac{f(1,\ 1)}{f(1,\ 1) + f(0,\ 1) + (1-\gamma)f(0,\ 0)} = \frac{1}{3-\gamma}.$$

f) The buyer's payment is given by $p_s = \max\{0, \mu_s - v\}$.

Proposition 2.i is unsurprising. Given the possibility of false negatives—a good-quality firm obtaining a bad score—if only good-quality firms participate in the regulatory regime, the buyer will ignore the signal generated by the regulatory process and assume that the firm is of good quality. And so the buyer will pay $1 - v$ to any participating firm. The bad-quality firm with no access costs, then, will prefer to deviate and participate rather than remain silent.

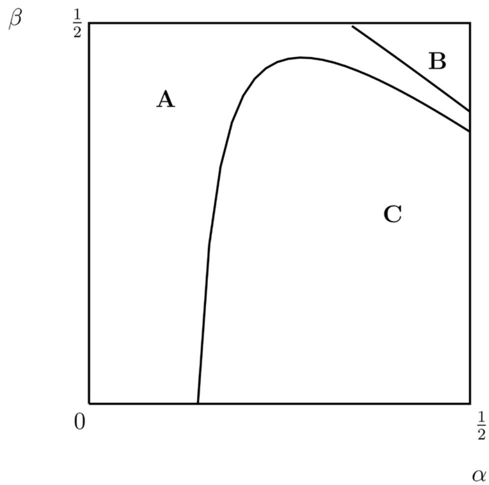

Figure 1. Equilibrium behavior where $v = 7/16$

Proposition 2.ii describes an equilibrium. Following each possible signal realization, the error probabilities and equilibrium strategies induce beliefs in the buyer. The buyer then makes a purchase decision on the basis of these beliefs. For some error rates, the bad-quality firm with no access costs expects to make more by participating than not participating, and so she does. Notably, this type participates while understanding that her participation degrades what the buyer is willing to pay after the good and bad signal realizations generated by the regulatory process.

For other values of the error rates, this type mixes between participating and not participating. The probability of mixing induces posterior beliefs (and purchase decisions) by the buyer such that this type is indifferent between accessing the regulatory regime and remaining silent and thus is willing to play that strategy. The equilibriums can be grouped into two categories: one in which the buyer purchases following a bad signal and one in which the buyer purchases the outside option after observing the bad signal. Furthermore, we focus on the equilibrium where the buyer always purchases if she observes silence. To further fix ideas, we provide two examples, one with a high-valued outside option and one with a low-valued outside option.

Example 1: High-Valued Outside Option ($v = 7/16$). Figure 1 represents the possible error pairs (α, β). Depending on the value of the errors, we can identify the equilibrium behavior of the bad-quality firm with low access costs and the buyer's purchase decisions.

If the errors lie in area A, then the bad-quality firm with low access costs mixes between participating and not participating, and the buyer refrains from purchasing when she observes the bad signal. If the errors lie in area B, then the bad-quality firm with low access costs mixes between participating and not participating, and the buyer purchases after observing a bad signal. If the errors lie in area C, then the bad-quality firm with low access costs participates for sure, and the buyer refrains from purchasing following a bad signal.

First, consider the error pair $(\alpha, \beta) = (\frac{1}{2}, \frac{1}{10})$. This pair lies in area C, so the equilibrium is characterized by the bad-quality firm with low access costs participating in the regime for sure. Given the small value of β, a bad score almost always reveals that the firm is of bad quality. And so the buyer refrains from purchasing following a bad score. Yet the bad-quality firm with low access costs still wants to risk it and access the regime. Why? After a good score, the buyer pays

$$\begin{aligned}\mu_g - v &= \frac{1-\beta}{1-\beta+\alpha} - v \\ &= \frac{9}{14} - \frac{7}{16} \quad \left(\text{by substituting } (\alpha, \beta) = \left(\frac{1}{2}, \frac{1}{10}\right), v = \frac{7}{16}\right) \\ &\approx .20.\end{aligned}$$

Following silence, the buyer pays

$$\mu_\phi - v = \frac{f(1, 1)}{f(1, 1) + f(0, 1)} - \frac{7}{16} = \frac{1/4}{(1/4) + (1/4)} - \frac{7}{16} = \frac{1}{16} \approx .06.$$

In accessing the regime, the bad-quality firm with low access costs understands that it will get a good signal with a probability of $\frac{1}{2}$ and thereafter make a hefty margin on the sale. On the other hand, with a probability of $\frac{1}{2}$ the firm will obtain a bad score and make no sale.

This type's expected payoff from participation is thus

$$\alpha(\mu_g - v) = (.5)(.20) = .1.$$

By contrast, the payoff to remaining silent is less—the payoff is only .06. Because the payoff to participation exceeds the payoff to silence, the bad-quality firm with low access costs prefers participation. And the firm does so while understanding that after a bad signal no sale will be made.

Second, take the pair of errors $(\alpha, \beta) = (\frac{1}{10}, \frac{1}{4})$. This pair lands in area A, so the bad-quality firm with no access costs mixes, and the buyer refrains from purchasing following a bad signal. To see why this is an equilibrium, first assume that the buyer refrains from purchasing after a bad signal. To mix, the bad-quality type with low access costs must be indifferent between participating and not participating. In other words, the probability of mixing must solve

$$\begin{aligned} 0 &= \alpha(\mu_{\mathrm{g}} - v) - (\mu_{\phi} - v) \\ &= \left\{\frac{1}{10}\left[\frac{3/4}{(3/4) + (1/10)\gamma} - \frac{7}{16}\right]\right\} - \left(\frac{1}{3-\gamma} - \frac{7}{16}\right) \\ &= \frac{7}{16} + \frac{14\gamma^2 - 497\gamma - 1{,}995}{160(3-\gamma)(2\gamma+15)} \end{aligned}$$

(the second line results from using $(\alpha, \beta) = (\frac{1}{10}, \frac{1}{4})$ and $v = 7/16$) from which we have a positive solution between 0 and 1 of $\gamma^* = 92/100$. Given this mixing value, it is easy to see the buyer will not, in fact, purchase after observing a bad signal.[6]

Finally, in area B, the buyer buys even when she observes a bad signal. The reason is that the values of β and α are so high that the regulatory regime is, in effect, uninformative.

Example 2: A Low-Valued Outside Option ($v = \frac{1}{3}$). Unlike the setting with a high-valued outside option, the equilibrium with a low-valued outside option can be partitioned into only two groups. As in the first example, in area A in Figure 2 the bad-quality firm with low access costs participates with positive probability, and the buyer refrains from purchasing following a bad signal. In area B, this type participates with positive probability, and the buyer purchases following a bad signal. There does not exist an equilibrium in which this type participates with a probability of 1. Why not?

The reason runs as follows: If v is low, the planner cannot create a carrot large enough to attract the bad-quality firm to participate all the time. Yet remaining silent for sure for this type is not a best response either, as noted in proposition 2.i. We are left with the equilibrium involving a mixed strategy by the bad-quality firm with a low cost of access.

Figure 3 shows the partition of the error space where the buyer buys following silence and where she does not for even smaller values of v. Note that the range of error parameters where the buyer prefers the outside option following a bad signal—area A—shrinks.

6. Observe that $\mu_{\mathrm{b}} - v = \beta/[\beta + \gamma^*(1-\alpha)] - v = (1/4)/[(1/4) + (92/100)(9/10)] - 7/16 = -.205$. In words, the buyer's posterior belief is less than the outside option, and thus she prefers the outside option after observing a bad signal.

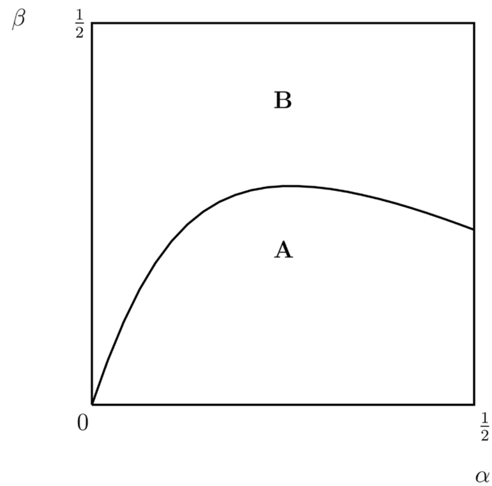

Figure 2. Equilibrium behavior where $\upsilon = \frac{1}{3}$

As υ falls, the buyer's posterior beliefs must fall by still greater amounts before she wishes to forgo a purchase from the firm and go with the outside option. Notably, even when υ is small area A exists, and it clusters around the point $(\alpha, \beta) = (\frac{1}{2}, 0)$. This set of errors ensures that the buyer forgoes a purchase following a bad signal. Because the good-quality firm with low access costs never gets the bad signal ($\beta \approx 0$), the buyer refuses to buy from the firm if she observes the bad signal, reasoning that the firm is of bad quality with low access costs.

One more aspect of these examples bears mentioning. For many values of α, when β is large, the buyer purchases following a bad signal; when β is small, she does not. This suggests that the planner might want to treat the error rates asymmetrically, setting β low so that the buyer refrains from purchasing after observing the bad signal. Section 4 confirms that this is, in fact, the case.

4. WELFARE

Do the lessons from the example (welfare can be improved by introducing some false positives) extend to the more general model? Note that although mistakes are made, the regulatory process on occasion unmasks some bad types because not all participating bad types are exonerated. If unmasked, the consumer bypasses the inefficient purchase. Fleshing this out formally yields our main welfare result.

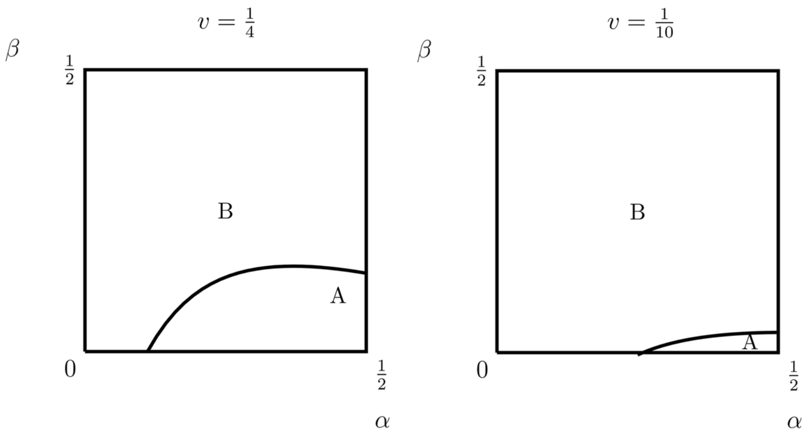

Figure 3. Equilibrium behavior with small outside options

Proposition 3

i) There always exists an imperfectly accurate regulatory regime that generates higher welfare than the perfectly accurate regulatory regime.

ii) The optimal degree of inaccuracy exhibits the fewest number of false negatives.

Why does the introduction of error increase welfare? We repeat the reason: the inclusion of false positives weakens the inference about quality that consumers can make following a good signal. A good signal could arise from a good-quality firm or a mistakenly identified bad-quality firm. As a result, false positives dilute the strength of the positive message. That loss of information, however, does not change behavior. The buyer purchases anyway. Sure, the buyer pays less following this good signal than following a positive signal generated from a perfectly accurate test. That lower payment, however, has no welfare consequences. The good-quality firm with low access costs still wants to participate because participation offers it the chance to separate from silent firms.

Yet by introducing false positives, the bad-quality firm might want to opt for regulatory scrutiny too. It seeks an improper classification as good (and the resulting increase in the price it can charge relative to remaining silent). And it is willing to risk being uncloaked for that chance.

Unlike false positives, the social planner cannot improve welfare via the introduction of false negatives. There are two problems with false negatives. First, by occasionally incorrectly marking a good-quality firm with a bad signal, the planner runs the risk that the buyer will purchase the outside option rather than the product from the good-quality firm that opts for regulatory scrutiny.

Second, the introduction of false negatives lessens what the buyer is willing to pay following a good signal. A lower payment following a good signal makes regulatory scrutiny less attractive to the bad-quality firm with low access costs. And so fewer bad-quality firms participate, which means that fewer bad-quality firms are in the mix to be marked with a bad signal.

5. MANDATORY REGIMES

At this point, a reader might wonder why the social planner does not make participation in the regulatory regime mandatory. Why not force any firm that wants to sell products or raise capital to obtain regulatory approval before doing so and ensure that the approval process never misfires?

Regulators, of course, can decide whether a firm must comply with a regulatory scheme before sending goods or services into the marketplace. For example, under the Consumer Safety Improvement Act of 2008, many children's products must be tested and certified as in compliance with safety standards (16 C.F.R. sec. 1250.2). Similarly, automobiles must meet minimum safety emissions standards.

The model enables us to explore the costs and benefits of mandatory versus voluntary regulatory regimes. Before we do so, there is a caveat. Many mandatory regulations, like emissions standards, are designed to limit negative externalities. The model does not speak to that issue. The central concern of the model is, instead, regulations that ensure a minimum quality of the good and provide information for consumers.

Notably, a mandatory regime could take two forms. One form (test to sell) requires that any firm that wishes to sell a good submit to the regulatory test, which determines whether the product is good or bad. The other form (pass to sell) requires that any firm that wishes to sell a good both submit to the test and pass it. In our theoretical world, where the outside option is better for consumers than buying a bad product and administrative costs to the government of increasing accuracy are 0, these two forms end up as the same policy. Moreover, the optimal level of error is 0.

To see that the two forms are the same, observe that there is no benefit to allowing firms that fail the test to sell their products: consumers would prefer the outside option. To see that the optimal error level is 0, note that there are no gains to consumers from allowing bad firms to pass as good firms by participating in the test and passing. Unlike in a voluntary regime, in a mandatory regime all firms must participate to sell products, so there is no need to offer false positives as a participation credit. Going forward, therefore, we equate a mandatory regime with the pass-to-sell variant.

Under a past-to-sell regime, the good-quality firm with low access costs prefers to participate in the regime, obtaining a good score. Fearing a perfectly accurate test or the expenditure of high access costs, no other firm type wants to participate, so no other type is allowed to make a sale. The consumer is forced to buy the outside option. The welfare from the mandatory regime is thus

$$W^{\mathrm{MR}} = 1f(1,\ 0) + v[f(0,\ 0) + f(0,\ 1) + f(1,\ 1)] = \frac{1}{4} + \frac{3v}{4}.$$

The trouble with the mandatory regime is that the good-quality firm with high access costs does not find it cost justified to participate. It is de facto barred from the market.

By contrast, the good-quality firm with high access costs makes a sale in the imperfectly accurate voluntary regime, while the regime also ferrets out some bad-quality firms by issuing some bad signals. This alternative course is the better one. Proposition 4 presents this observation in formal terms.

Proposition 4. Welfare is higher in an inaccurate voluntary regime than in a mandatory (and perfectly accurate) regulatory regime.

6. CONCLUSION

Normatively, our model shows how regulators can use the error rate of the regulatory regime to improve welfare. Further, altering the error rate is likely to be more effective than forcing firms to opt into the regulatory regime. It is also cheaper than subsidizing firms that opt for enhanced scrutiny. In short, the ideal voluntary regulatory regime does not have an error rate of 0, even if that rate is achievable at no cost. The message for policy makers from the prior literature on accuracy is that they should be concerned with not punishing firms that comply with regulation and punishing firms in breach. We show that the regulator should also be aware of how the error rate affects participation in the regulated regime in the first place. In particular, a regime that exhibits false positives can serve to maximize the participation incentive, which improves welfare.

Before closing, we note two limitations of the results presented here. First, the introduction of inaccuracy will reduce deterrence. Because the consumer pays less following a good signal, the good-quality firm makes less money, which shrinks the returns from investing in quality. This effect must be balanced against the gains realized when buyers avoid purchasing from a bad-quality firm. We explore a variant of that model in a working paper (Baker and Malani 2011).

Second, we have assumed that buyers care more about avoiding bad-quality firms than, say, finding out just how good a good-quality firm is. We might think of the buyer wanting to take an action to match the seller's type as in Shavell (1994). The consumer might want to take more precautions when she buys from a bad-quality firm than from a good-quality firm. In this environment, the inaccurate regulatory regime will be less useful than a perfectly accurate regulatory regime. In our model, the buyer's precaution takes the form of a threshold rule. If the consumer's beliefs exceed some value, she buys the good; otherwise, she does not. Given this type of behavior, it does not matter by how much the consumer's beliefs exceed the threshold. As a result, the regulator could adopt the Blackstone approach to error minimization (which results in a loss of information) without changing the buyer's behavior while at the same time attracting some of the bad types into the regulatory regime. That is no longer true if the consumer's precaution needs to be tied closely to the firm's type.

APPENDIX: PROOFS

A1. Proof of Proposition 1

Proof of Proposition 1.i. For proposition 1 and the ones that follow, observe that firms with high access costs—types $\theta = (0, 1)$ and $\theta = (1, 1)$—never find it optimal to participate in the regulatory regime. At most, the buyer will be willing to pay $1 - v$, which happens when she is convinced that the firm is of good quality. Yet this price—which is the highest possible payment—does not cover the access costs when the firm's access costs are 1.

Next suppose, contrary to the statement in proposition 1, that an equilibrium exists in which types $\theta = (1, 0)$ and $\theta = (0, 0)$ both participate in the regulatory regime. Before proceeding, it is useful to note that the buyer's belief that the firm is good conditional on a signal s is $\mu_s = \Pr(X = 1, Y = 0 \mid s) + \mu(X = 1, Y = 1) \mid s$. Yet because the type $\theta = (1, 1)$ firm never participates, the belief that the firm is of good quality following $s \in \{\mathrm{b}, \mathrm{g}\}$ is $\mu_s = \Pr(X = 1, Y = 0 \mid s)$. Because type $\theta = (1, 0)$ participates for sure, she is never silent. As a result, the buyer's belief that the firm is good following silence is just the belief that the firm is a good type with high access costs: $\mu_\phi = \Pr(X = 1, Y = 0 \mid \phi)$. In the perfectly accurate regime, $\alpha = \Pr(\mathrm{g} \mid X = 0) = 0$ and $1 - \beta = \Pr(\mathrm{g} \mid X = 1) = 1$.

Thus, in the proposed equilibrium, the buyer's belief that the firm is good after a good signal is

$$\mu_{\mathrm{g}} = \Pr(1, 0 \mid g) = \frac{\Pr(g \mid 1, 0)f(1, 0)}{\Pr(g \mid 1, 0)f(0, 0) + \Pr(g \mid 0, 0)f(0, 0)}$$
$$= \frac{1 \times (1/4)}{1 \times (1/4) + 0 \times (1/4)} = 1.$$

The buyer's belief that the firm is good given a bad signal is

$$\mu_{\mathrm{b}} = \Pr(1, 0 \mid b) = 0.$$

Finally, the buyer's belief that the firm is good after observing silence is

$$\mu_\phi = \Pr(1, 1 \mid \phi) = \frac{\Pr(\phi \mid 1, 1)f(1, 1)}{\Pr(\phi \mid 1, 1)f(1, 1) + \Pr(\phi \mid 0, 1)f(0, 1)}$$
$$= \frac{1 \times (1/4)}{1 \times (1/4) + 1 \times (1/4)} = \frac{1}{2}.$$

On participating, the perfectly accurate regime always identifies type $\theta = (0, 1)$ as a bad-quality firm. That type thus obtains a payoff of 0. If the firm deviates and refuses to participate, the buyer will pay a price of $\mu_\phi - v = \frac{1}{2} - v > 0$, which makes the deviation profitable. Because the bad-quality firm with low costs can profit from a single deviation, this equilibrium cannot exist.

Proof of Proposition 1.ii. We construct an equilibrium in which type $\theta = (1, 0)$ opts for regulatory scrutiny and all other types do not. In light of these strategies and the perfect regulatory regime, the buyer observes only the good and silence signals in equilibrium. Since the bad signal is off the path, we have freedom in assigning the buyer's beliefs to that signal realization. Suppose that the buyer believes that the firm is bad following a bad signal. So, following a good and bad signal realization, the buyer's beliefs are

$$\mu_{\mathrm{g}} = 1$$

and

$$\mu_{\mathrm{b}} = 0.$$

Following silence, the buyer's belief is

$$\begin{aligned}\mu_\phi &= \Pr(X = 1,\ Y = 1) \mid \phi) \\ &= \frac{\Pr(\phi \mid 1,\ 1) f(1,\ 1)}{\Pr(\phi \mid 1,\ 1) f(1,\ 1) + \Pr(\phi \mid 0,\ 1) f(0,\ 1) \Pr(\phi \mid 0,\ 0) f(0,\ 0)} \\ &= \frac{f(1,\ 0)}{f(1,\ 0) + f(0,\ 0) + f(1,\ 1)} = \frac{1}{3}.\end{aligned}$$

Given these beliefs, the buyer pays $1 - v$ following a good signal and 0 following a bad signal.

What the buyer pays following silence depends on the value of the outside option. If the value of the outside option exceeds $\frac{1}{3}$, the buyer does not purchase following silence. If the value of the outside option is less than $\frac{1}{3}$, the buyer purchases following silence, paying $\frac{1}{3} - v$. Putting these two cases together, we see that the paid price following silence is $\max\{0,\ \frac{1}{3} - v\}$.

For this equilibrium to exist, the following incentive constraints must hold:

$$1 - v \geq \left[\frac{1}{3} - v\right]^+, \tag{A1}$$

$$\left[\frac{1}{3} - v\right]^+ \geq 0, \tag{A2}$$

$$\left[\frac{1}{3} - v\right]^+ \geq (1 - v) - 1, \tag{A3}$$

and

$$\left[\frac{1}{3} - v\right]^+ \geq -1, \tag{A4}$$

where $z^+ := \max\{0,\ z\}$. Type $\theta = (1,\ 0)$ prefers to participate since $1 - v > \frac{1}{3} - v$. Constraint (A2) reports that the bad-quality firm with low access costs ($\theta = (1,\ 0)$) prefers to opt out of regulatory scrutiny. That constraint holds since $\left[\frac{1}{3} - v\right]^+$ must be greater than or equal to 0. Constraints (A3) and (A4) say that firms with high costs prefer to avoid scrutiny, and these constraints always hold.

Proof of Proposition 1.iii. Under the assumption that $v < \frac{1}{2}$, in the no-intervention benchmark, the buyer always purchases the good. Expected welfare is thus

$$1f(1,\ 0) + 1f(1,\ 1) + 0f(0,\ 0) + 0f(0,\ 1) = \frac{1}{2}.$$

With a perfectly accurate regime, in equilibrium type $\theta = (1,\ 0)$ accesses the regime and always obtains a good signal. Thereafter, the buyer purchases from this type. Turning to what happens when the buyer observes silence, we must consider two cases.

Case 1: $v < \frac{1}{3}$*—Buyer Purchases after Silence.* As noted above, the buyer purchases following a good signal. In this case, she purchases following silence. In terms of welfare, the buyer buys from type $\theta = (1, 0)$; the firm sends a good signal that generates a surplus of 1. She also buys from type $\theta = (1, 1)$; that firm sends a silent signal that generates a surplus of 1. With that said, she also buys from types $\theta = (0, 0)$ and $\theta = (0, 1)$, given both these types remain silent. Welfare is thus

$$1f(0,\ 1) + 1f(1,\ 1)1 + 0f(0,\ 1) + 0f(0,\ 0) = \frac{1}{2},$$

which is the same as in the no-intervention benchmark.

Case 2: $v \in [\frac{1}{3}, \frac{1}{2})$*—Buyer Does Not Buy following Silence.* Because the value of the outside option is high, the buyer refrains from purchasing following silence. She still purchases following a good signal. She therefore buys from type $\theta = (1, 0)$, a firm that sends a good signal, and buys from no one else, going with the outside option instead. Welfare is thus

$$1f(1,\ 0) + vf(1,\ 1) + vf(0,\ 0) + vf(0,\ 1) = \frac{1}{4} + \frac{3}{4}v \geq \frac{1}{2},$$

where the inequality follows from the restriction in this case to values of $v \geq \frac{1}{3}$.

A2. Proof of Lemma 1

Suppose that type $\theta = (1, 0)$ participates and type $\theta = (0, 0)$ participates with a probability of $\gamma \in [0, 1]$. Given the probabilities of error β and α and the fact that types $\theta = (0, 1)$ and $\theta = (1, 1)$ never participate, the buyer's beliefs following a good and bad signal are

$$\mu_{\mathrm{g}} = \Pr(X = 1,\ Y = 0 \mid g) = \frac{1 - \beta}{1 - \beta + \gamma\alpha}$$

and

$$\mu_{\mathrm{b}} = \Pr(X = 1,\ Y = 0 \mid b) = \frac{\beta}{\beta + \gamma(1 - \alpha)}.$$

Following silence, the buyer's belief is

$$\begin{aligned}\mu_{\phi} &= \Pr(X = 1,\ Y = 1 \mid \phi) = \frac{f(1,\ 1)}{f(1,\ 1) + f(0,\ 1) + (1 - \gamma)f(0,\ 0)}\\ &= \frac{1/4}{(1/4) + (1/4) + (1 - \gamma)/4} = \frac{1}{3 - \gamma}.\end{aligned}$$

The incentive constraint for type $\theta = (0, 0)$ is satisfied if $H(\gamma) \geq 0$, where this function is defined as

$$H(\gamma) := \alpha(\mu_{\mathrm{g}} - v) + (1-\alpha)[\mu_{\mathrm{b}} - v]^{+} - [\mu_{\phi} - v]^{+}.$$

Similarly, the incentive constraint for type $\theta = (1, 0)$ is satisfied if $G(\gamma) \geq 0$, where

$$G(\gamma) := (1-\beta)[(\mu_{\mathrm{g}} - v) + \beta[\mu_{\mathrm{b}} - v]^{+} - [\mu_{\phi} - v]^{+}.$$

Importantly,

$$G(\gamma) - H(\gamma) = (1 - \beta - \alpha)\{(\mu_{\mathrm{g}} - v) - [\mu_{\mathrm{b}} - v]^{+}\}.$$

It must be that

$$G(\gamma) - H(\gamma) > 0,$$

since $1 - \beta - \alpha > 0$ and $\mu_{\mathrm{g}} > \mu_{\mathrm{b}}$, which completes the proof.

A3. Proof of Proposition 2

Proof of Proposition 2.i. Suppose that an equilibrium exists in which type $\theta = (1, 0)$ opts into the regulatory regime and type $\theta = (0, 0)$ does not. As noted in the proof of proposition 1, types $\theta = (1, 1)$ and $\theta = (0, 1)$ will not participate no matter the error rate set by the social planner.

Because $\beta > 0$, the regulatory regime on occasion gives a bad signal to type $\theta = (1, 0)$. As a result, the buyer might observe both good and bad signals in equilibrium. Yet, because only type $\theta = (1, 0)$ participates, the buyer infers—irrespective of the signal realization—that the firm is of good quality. She therefore is willing to pay $1 - v$. Knowing this, type $\theta = (0, 0)$ prefers to deviate and participate. The deviation yields a payoff of $1 - v$, which is the highest price the firm can receive and, as a result, higher than the price this type obtains by remaining silent. Because a bad-quality firm with low access costs can gain from a single deviation, this equilibrium cannot exist.

Proof of Proposition 2.ii. In the candidate equilibrium, type $\theta = (1, 0)$ participates for sure, and type $\theta = (0, 0)$ either participates for sure or randomizes with a probability of γ, whereas types $\theta = (0, 1)$ and $\theta = (1, 1)$ always remain silent. Combined with the error rates, the equilibrium strategies yield the following beliefs by the buyer:

$$\mu_{\mathrm{g}} = \Pr(1,\ 0 \mid g) = \frac{(1-\beta)f(1,\ 0)}{(1-\beta)f(1,\ 0) + \alpha\gamma f(0,\ 0)} = \frac{1-\beta}{1-\beta+\alpha\gamma},$$

$$\mu_{\mathrm{b}} = \Pr(1,\ 0 \mid b) = \frac{\beta f(1,\ 0)}{\beta f(1,\ 0) + (1-\alpha)\gamma f(0,\ 0)} = \frac{\beta}{\beta + \gamma(1-\alpha)},$$

and

$$\mu_{\phi} = \Pr(1,\ 1 \mid \phi) = \frac{f(1,\ 1)}{f(1,\ 1) + f(0,\ 1) + (1-\gamma)f(0,\ 0)} = \frac{1}{3-\gamma},$$

where the last line in each derivation follows from $f(0, 0) = f(1, 0) = f(1, 1) = f(0, 1) = \frac{1}{4}$. The buyer gets her best response by setting $p_s = [\mu_s - v]^+$.

Next we show that for type $\theta = (0, 0)$, randomization or a strict preference for participation are the best responses to these prices. To see this, define two functions. These functions represent the difference in the payoff to type $\theta = (0, 0)$ between accessing the regulatory regime and not participating under various assumptions about the buyer's behavior.

Let

$$\begin{aligned} h(\gamma) &= \alpha(\mu_g - v) + (1-\alpha)(\mu_b - v) - (\mu_\phi - v) \\ &= \alpha\left(\frac{1-\beta}{1-\beta+\gamma\alpha}\right) + (1-\alpha)\left[\frac{\beta}{\beta+\gamma(1-\alpha)}\right] - \left(\frac{1}{3-\gamma}\right). \end{aligned}$$

The function $h(\gamma)$ represents the difference in payoffs from participating and if the buyer purchases following every signal realization: good, bad, or silence.

Let

$$r(\gamma) = \alpha(\mu_g - v) - (\mu_\phi - v) = \alpha\left(\frac{1-\beta}{1-\beta+\gamma\alpha} - v\right) - \left(\frac{1}{3-\gamma} - v\right).$$

The function $r(\gamma)$ represents the difference in payoffs for the bad-quality firm with low access costs from participating and not participating when the buyer purchases following silence and a good signal but refrains from doing so following a bad signal.

Both $r(\gamma)$ and $h(\gamma)$ are continuous, differentiable, and monotonically decreasing in $\gamma \in (0, 1)$. Further, it is the case that $h(0) > 0$ and $r(0) > 0$.

Next define $\hat{\gamma} = \beta(1-v) / v(1-\alpha)$. This mixing value for $\theta = (0, 0)$ solves

$$0 = \mu_b - v.$$

At this mixing value, the buyer obtains the same payoff from buying from the firm with a bad signal (μ_b) as she does from the outside option (v). Thus, she will buy but pay nothing. Clearly, if the equilibrium mixing value exceeds $\hat{\gamma}$, then the buyer goes with the outside option after observing a bad signal, and if the mixing probability is less than $\hat{\gamma}$, the buyer buys from the firm after observing a bad signal.

Observe that $r(\hat{\gamma}) = h(\hat{\gamma})$. For all values (α, β), Table A1 presents the equilibrium behavior for firm type $\theta = (0, 0)$. Consider the first row, where firm type $\theta = (0, 0)$ strictly prefers to participate since $r(1) > 0$. Further, given that this type's participation rate exceeds $\hat{\gamma}$, the buyer refrains from purchasing following a bad signal.

In the second row, the equilibrium strategy of firm type $\theta = (0, 0)$ solves $r(\gamma) = 0$. This value always exists since r is continuous and decreasing with $r(\hat{\gamma}) > 0$ and $r(1) < 0$. Such mixing induces beliefs by the buyer via Bayes's rule that this type is indifferent between participating and not participating. The solution value is such

Table A1. Equilibrium Behavior

Conditions	Type $\theta = (0, 0)$ Behavior	Buyer's Behavior
$\hat{\gamma} \in (0, 1)$ and $r(1) > 0$	Participates for sure	Purchases following good signal and silence; takes outside option following bad signal
$\hat{\gamma} \in (0, 1)$, $h(\hat{\gamma}) = r(\hat{\gamma}) > 0$, and $r(1) < 0$	Mixing probability solves $r(\gamma) = 0$, where $\gamma \in (\hat{\gamma}, 1)$	Purchases following good signal and silence; takes outside option following bad signal
$\hat{\gamma} \in (0, 1)$ and $h(\hat{\gamma}) = r(\hat{\gamma}) < 0$	Mixing probability solves $h(\gamma) = 0$, where $\gamma \in (0, \hat{\gamma})$	Always purchases
$\hat{\gamma} > 1$	Mixing probability solves $h(\gamma) = 0$, where $\gamma \in (0, 1)$	Always purchases

that the buyer is willing to purchase following silence and not willing to purchase following a bad signal.

In the third row, note that $h(\hat{\gamma}) < 0$. This means that firm type $\theta = (0, 0)$ refrains from participation if the buyer believes that its participation rate is $\hat{\gamma}$. So mixing with a probability of $\hat{\gamma}$ is not a best response by this type.

Instead, the equilibrium strategy solves $h(\gamma) = 0$, where $\gamma < \hat{\gamma}$. Note that h is decreasing and $h(0) > 0$ (that is, this type prefers to participate if the buyer believes it is not doing so, and so avoiding scrutiny with a probability of one is not a best response). As a result, a solution to $h(\gamma) = 0$ always exists. The solution is less than $\hat{\gamma}$, and thus the buyer purchases following a bad signal.

Finally, take the last row of Table A1. Since $\hat{\gamma} > 1$, the buyer will purchase following a bad signal even when firm type $\theta = (0, 0)$ participates with a probability of one. Observe that $h(1) < 0$. The reason follows:

$$
\begin{aligned}
h(1) &= \alpha(\mu_g - v) + (1-\alpha)(\mu_b - v) - (\mu_\phi - v) = \alpha\mu_g + (1-\alpha)\mu_b - \frac{1}{2} \\
&= \left(\frac{\alpha}{2} + \frac{\alpha}{2}\right)\mu_g + \left(\frac{1-\alpha}{2} + \frac{1-\alpha}{2}\right)\mu_b - \frac{1}{2} \\
&< \frac{\alpha}{2}\mu_g + \frac{1-\beta}{2}\mu_g + \frac{1-\alpha}{2}\mu_b + \frac{\beta}{2}\mu_b - \frac{1}{2} = 0,
\end{aligned}
$$

where the inequality follows because $\alpha < 1 - \beta$ and $\mu_g > \mu_b$. The last equality follows from the definitions of the posterior beliefs. Notice also that $\mu_\phi = \frac{1}{2}$ when $\gamma = 1$, a fact used in the second line.

A4. Proof of Proposition 3

Proof of Proposition 3.i. Welfare is the same under the imperfect regime in which the buyer purchases following a bad signal. We thus restrict our search for a welfare-enhancing inaccurate regime to error pairs in which the buyer refrains from purchasing following a bad signal (area A in Figures 1–3). It can be shown that such a set of errors exists for any value of v. There are two cases.

Case 1: $v \in [0, \frac{1}{3}]$. For these errors, denote W^{IP} the welfare from the imperfect regime. It consists of three properties.

The first is the probability of a good signal times the buyer's belief that $X = 1$ given a good signal. The probability of a good signal is the probability that the firm is of type $\theta = (1, 0)$ times the probability that this type receives a good signal plus the probability that the firm is type $\theta = (0, 0)$ times the probability that this type mistakenly receives a good signal. The latter arises because there are false positives.

The second is the probability of a bad signal times the value of the outside option (since the buyer chooses the outside option following a bad signal). The probability of a bad signal is the probability that the firm is of type $\theta = (1, 0)$ times the probability that this type mistakenly receives a bad signal plus the probability that the firm is of type $\theta = (0, 0)$ times the probability that this type receives a bad signal.

The third is the probability of silence times the buyer's belief that $X = 1$ following silence. Formally,

$$\begin{aligned} W^{\mathrm{IP}} &= \Pr(g)W(\mu^{g}) + \Pr(b)W(\mu^{b}) + \Pr(\phi)W(\mu^{\phi}) \\ &= \left(\frac{1-\beta}{4} + \frac{\alpha\gamma}{4}\right) \times \frac{(1-\beta)/4}{[(1-\beta)/4] + (\alpha\gamma/4)} \\ &\quad + \frac{[\gamma(1-\alpha)+\beta]v}{4} + \left(\frac{1}{2} + \frac{1-\gamma}{4}\right) \times \frac{1/4}{(1/2) + (1-\gamma)/4}. \end{aligned}$$

Simplifying,

$$W^{\mathrm{IP}} = \frac{1}{2} - \frac{\beta(1-v)}{4} + \frac{\gamma(1-\alpha)v}{4}. \tag{A5}$$

Choose $\beta = v/a$, where a is a constant. Remember that when $v \in [0, \frac{1}{3}]$, welfare under a perfectly accurate regime is $\frac{1}{2}$.

So welfare is higher under the imperfect regime if

$$\frac{1}{2} - \frac{\beta(1-v)}{4} + \frac{\gamma(1-\alpha)v}{4} > \frac{1}{2}$$

or

$$\gamma(1-\alpha)v > \beta(1-v). \tag{A6}$$

Notice that

$$\lim_{a\to\infty} \frac{v}{a} = 0. \tag{A7}$$

Since $\beta = v/a$, as a gets larger the right-hand side of expression (A6) goes to 0. Proposition 2.i teaches that any equilibrium with imperfect accuracy involves some positive participation by the bad-quality firm with low access costs; that is, $\gamma > 0$. As a result, the left-hand side of expression (A6) must be positive, which completes the proof of proposition 2.i.

Case 2: $v \in (\frac{1}{3}, \frac{1}{2}]$. For these values of v, we can always find an equilibrium in which the bad-quality firm with low access costs participates with a probability of 1. Substituting $\gamma = 1$ in expression (A5), we see that welfare under the imperfect regime with full participation by firm type $\theta = (0, 0)$ is

$$W^{\mathrm{IP}} = \frac{1}{2} - \frac{\beta(1-v)}{4} + \frac{(1-\alpha)v}{4}. \tag{A8}$$

Given the values of the outside option in this interval, the buyer purchases following silence in the perfectly accurate regime. Accordingly, welfare from the perfectly accurate regime is

$$W = \frac{1}{4} + \frac{3}{4}v.$$

Notice that W^{IP} decreases in β and α. Set $\beta = v/a$ and let a go to infinity.

Next, recall that

$$\cdot(1) := \alpha\left(\frac{1-\beta}{1-\beta+\alpha} - v\right) - \left(\frac{1}{2} - v\right). \tag{A9}$$

Letting β go to 0 and solving $r(1) = 0$ for α yields a positive solution of

$$\alpha^* = -\frac{-1+\sqrt{1+8v(2v-1)}}{4v}.$$

Substituting $\beta = 0$ and $\alpha = \alpha^*$ in equation (A8) results in

$$\begin{aligned} W^{\mathrm{IP}} &= \frac{1}{2} + \frac{1}{4}(1-\alpha^*)v = \frac{1}{2} + \frac{1}{4}\times\left(\frac{4v-1+\sqrt{1+8v(2v-1)}}{4v}\right)v \\ &= \frac{1}{2} + \frac{1}{4}\times\left(\frac{4v+1+\sqrt{(4v-1)^2}}{4v}\right)v = \frac{1}{2} + \frac{1}{4}\times\left(\frac{4v-1+4v-1}{4}\right) \\ &= \frac{1}{2} + \frac{1}{4}\times\left(\frac{8v-2}{4}\right) = \frac{1}{2} + \frac{v}{2} - \frac{1}{8}. \end{aligned} \tag{A10}$$

This exceeds the welfare from a perfectly accurate regime (namely, $W = \frac{1}{4} + 3v/4$) whenever $v < \frac{1}{2}$, which means that we have found a pair of errors $(\alpha^*, 0)$ where welfare is higher in the imperfect regime.

Proof of Proposition 3.ii. Again restrict attention to the values of α and β when the buyer does not purchase following a bad signal. There is no impact of changing β when the buyer buys following a bad signal; then welfare is always $\frac{1}{2}$.

Case 1: $v \in [0, \frac{1}{3}]$. In this case, the bad-quality firm with no access costs mixes with a probability of γ. In that setting, we may write the (implicitly defined) probability of mixing as

$$r(\gamma) := \alpha\left(\frac{1-\beta}{1-\beta+\gamma\alpha} - v\right) - \left(\frac{1}{3-\gamma} - v\right) = 0. \tag{A11}$$

By totally differentiating expression (A11), it is immediate that $\partial\gamma/\partial\beta = -r_\beta/r_\gamma$.

Since $r_\gamma < 0$, the sign of $\partial\gamma/\partial\beta$ is the same as the sign of r_β, which equals

$$-\frac{\gamma\alpha^2}{(\gamma\alpha+1-\beta)^2} < 0.$$

So $\partial\gamma/\partial\beta < 0$.

In case 1, expression (A5) defines the welfare. Observe that the derivative of welfare with respect to β is

$$-\frac{(1-v)}{4}+\frac{1}{4}\times\frac{\partial\gamma}{\partial\beta}\times(1-\alpha)\times v<0.$$

Since welfare decreases in β, the optimal regulatory regime sets the fewest number of false negatives.

Case 2: $v\in(\frac{1}{3},\frac{1}{2}]$. In this case, expression (A8) defines the welfare. This expression decreases in β, which means that the optimal regulatory regime issues the fewest possible number of false negatives.

A5. Proof of Proposition 4

Case 1: $v\in[0,\frac{1}{3})$. Welfare from a mandatory regime is

$$W^{\mathrm{MR}}=\frac{1}{4}+\frac{3}{4}v\leq\frac{1}{2},$$

where the inequality follows by the restriction on the values of $v\in[0,\frac{1}{3})$. Welfare from the imperfect voluntary regime is greater than $\frac{1}{2}$ (and hence greater than the welfare from the mandatory regime) when the inequality of expression (A6) holds. The proof of proposition 3 demonstrates that error values exist where that is so.

Case 2: $v\in[\frac{1}{3},\frac{1}{2}]$. Welfare from the mandatory regime is $W^{\mathrm{MR}}=\frac{1}{4}+\frac{3}{4}v$. Equation (A10) shows that error values exist that result in welfare greater than W^{MR}.

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