Solve the following equation using bisection method, correct upto third (3^{rd}) decimal place

i) $e^x - x - 2 = 0$

Given equation,

$e^x - x - 2 = 0$

$\therefore f(x) = 0$

$\therefore f(x) = e^x - x - 2$

Now,

x	0	-1	-2
$f(x)$	-1	-0.6321	0.1353

This shows root lies in between -1 and -2

$\therefore$ Inilial guess, $x_1 = -1$; $x_2 = -2$

computation Table:

Iteration	x_1	x_2	$x = \frac{x_1 + x_2}{2}$	$f(x_1)$	$f(x_2)$	$f(x)$	Error$(x_2 - x)$
1	-1	-2	-1.5	-0.632	0.1353	-0.2768	1
2	-1.5	-2	-1.75	-0.2768	0.1353	-0.07622	0.5
3	1.75	-2	-1.875	-0.07622	0.1353	0.0283	0.25
4	-1.75	-1.875	-1.8125	-0.07622	0.0283	-0.0242	0.125
5	-1.8125	-1.875	-1.8437	-0.0242	0.0283	0.00193	0.0625
6	-1.8125	-1.8437	-1.8281	-0.0243	0.00193	-0.011	0.0312
7	-1.8281	-1.8437	-1.8359	-0.011	0.00193	-0.00463	0.0156
8	-1.8359	-1.8437	-1.8398	-0.00463	0.00193	-0.00135	0.0078
9	-1.8398	-1.8437	-1.84175	-0.00135	0.00193	0.00028	0.0039
10	-1.8398	-1.84175	-1.840775	-0.00135	0.00028	-0.00053	0.00195
11	-1.840775	-1.84175	-1.841126	-0.00053	0.00028	-0.00012	0.000975

$\therefore$ Approximate root is:

$x = -1.841$

(ii) $\sin x - 2x + 1 = 0$

Soln

Given Equation,

$\sin x - 2x + 1 = 0$

$\therefore f(x) = 0$

$\therefore f(x) = \sin x - 2x + 1$

Now,

x	0	1
$f(x)$	1	-0.1585

This shows root lies in between $x_1 = 0$ and $x_2 = 1$

$\therefore$ Initial guess, $x_1 = 0$ and $x_2 = 1$

Computational Table

Iteration	x_1	x_2	$x = \frac{x_1 + x_2}{2}$	$f(x_1)$	$f(x_2)$	$f(x)$	Error $(x_2 - x_1)$
1	0	1	0.5	1	-0.1585	0.4794	1
2	0.5	1	0.75	0.4794	-0.1585	0.1816	0.5
3	0.75	1	0.875	0.1816	-0.1585	0.01754	0.25
4	0.875	1	0.9375	0.01754	-0.1585	-0.0689	0.125
5	0.875	0.9375	0.90625	0.01754	-0.0689	-0.0253	0.0625
6	0.875	0.90625	0.890625	0.01754	-0.0253	-0.000378	0.03125
7	0.875	0.890625	0.8828125	0.01754	-0.000378	0.000690	0.0156
8	0.8828125	0.890625	0.88671875	0.000690	-0.000378	0.000156	0.0078
9	0.88671875	0.890625	0.88867	0.000156	-0.000378	-0.00011	0.0039
10	0.88671875	0.88867	0.88769	0.000156	-0.00011	0.00023	0.0019
11	0.88769	0.88867	0.88818	0.00023	-0.00011	-0.00043	0.00098

$\therefore$ Approximate root is:

$x = 0.888$

iii) $\log x - \cos x = 0$

Given equation,

$\log x - \cos x = 0$

$f(x) = 0$

$\therefore f(x) = \log x - \cos x$

Now,

x	1	2
$f(x)$	-0.5403	0.717

This shows root lies in between 1 and 2

$\therefore$ Initial guess; $x_1 = 1$ and $x_2 = 2$

Computation Table

Iteration	x_1	x_2	$x = \frac{x_1 + x_2}{2}$	$f(x_1)$	$f(x_2)$	$f(x)$	Error $(x_2 - x_1)$
1	1	2	1.5	-0.5403	0.717	0.1053	1
2	1	1.5	1.25	-0.5403	0.1053	-0.2184	0.5
3	1.25	1.5	1.375	-0.2184	0.1053	-0.0562	0.25
4	1.375	1.5	1.4375	-0.0562	0.1053	0.0247	0.125
5	1.375	1.4375	1.40625	-0.0562	0.0247	-0.0157	0.0625
6	1.40625	1.4375	1.421875	-0.0157	0.0247	0.00448	0.03125
7	1.40625	1.421875	1.4140625	-0.0157	0.00448	-0.0901	0.0156
8	1.4140625	1.421875	1.417968	-0.0901	0.00448	-0.00056	0.0078
9	1.417968	1.421875	1.41992	-0.00056	0.00448	0.00195	0.0039
10	1.417968	1.41992	1.41894	-0.00056	0.00195	0.00069	0.0019
11	1.417968	1.41894	1.41845	-0.00056	0.00069	0.000056	0.00098

$\therefore$ Approximate root is

$x = 1.418$

iv) $x^3 - x - 3 = 0$

Given equation,

$x^3 - x - 3 = 0$

$f(x) = 0$

$\therefore f(x) = x^3 - x - 3$

Now,

x	0	1	2
$f(x)$	-3	-3	3

This shows root lies in between 1 and 2

Initial gauses, $x_1 = 1$ and $x_2 = 2$

Iteration	x_1	x_2	$x = \frac{x_1 + x_2}{2}$	$f(x_1)$	$f(x_2)$	$f(x)$	Error $x_2 - x_1$
1	1	2	1.5	-3	3	-1.125	1
2	1.5	2	1.75	-1.125	3	0.60937	0.5
3	1.5	1.75	1.625	-1.125	0.60937	-0.3339	0.25
4	1.625	1.75	1.6875	-0.339	0.60937	0.1179	0.125
5	1.625	1.6875	1.65625	-0.339	0.1179	-0.1128	0.0625
6	1.65625	1.6875	1.67187	-0.01128	0.1179	0.00125	0.03125
7	1.65625	1.67187	1.66406	-0.001128	0.00125	-0.0061	0.0156
8	1.66406	1.67187	1.66796	-0.0061	0.00125	-0.0075	0.0078
9	1.66796	1.67187	1.669915	-0.0075	0.00125	-0.0031	0.0039
10	1.669915	1.67187	1.67089	-0.0031	0.00125	-0.0055	0.0019
11	1.67089	1.67187	1.67138	-0.0059	0.00125	-0.0023	0.00098

$\therefore$ Approximate root is:

$x = 1.671$

v) $4x^3 - 2x - 6 = 0$

Given equation,

$4x^3 - 2x - 6 = 0$

$f(x) = 0$

$\therefore f(x) = 4x^3 - 2x - 6$

Now,

x	0	1	2
$f(x)$	-6	-4	22

This shows root lies in between 1 and 2

$\therefore$ Initial gauses; $x_1 = 1$ and $x_2 = 2$

computation Table

Iteration	x_1	x_2	$x = \frac{x_1 + x_2}{2}$	$f(x_1)$	$f(x_2)$	$f(x)$	Error $x_2 - x_1$
1	1	2	1.5	-4	22	4.5	1
2	1	1.5	1.25	-4	4.5	-0.6875	0.5
3	1.25	1.5	1.375	-0.6875	4.5	1.6484	0.25
4	1.25	1.375	1.3125	-0.6875	1.6484	0.41894	0.125
5	1.25	1.3125	1.28125	-0.6875	0.41894	-0.14929	0.0625
6	1.28125	1.3125	1.296875	-0.14929	0.41894	0.13102	0.03125
7	1.28125	1.296875	1.2890625	-0.14929	0.13102	-0.01007	0.0156
8	1.2890625	1.296875	1.292968	-0.01007	0.13102	0.06022	0.0078
9	1.2890625	1.292968	1.291015	-0.01007	0.06022	0.02501	0.0039
10	1.2890625	1.291015	1.290038	-0.01007	0.02501	0.00743	0.0019
11	1.2890625	1.290038	1.289550	-0.01007	0.00743	-0.0132	0.00098

$\therefore$ Approximate root is:

$x = 1.289$