Perceptron

Supervised Learning

$\boxed{\text{AND OR gate}}$

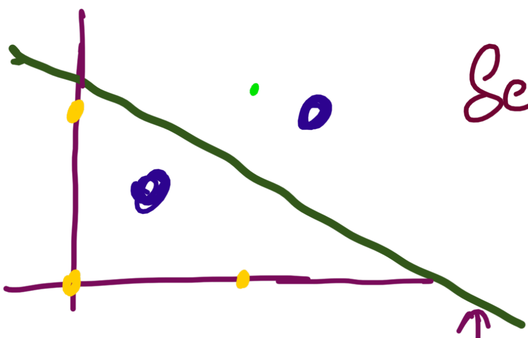
AND $x_1 x_2$

x_1	x_2	Y
0	0	0
0	1	0
1	0	0
1	1	1 ✓

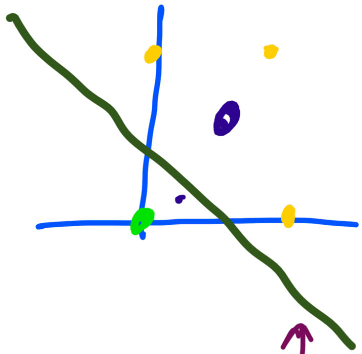
OR $x_1 + x_2$

x_1	x_2	Y
0	0	0 ✓
0	1	1
1	0	1
1	1	1

Linearly Seperable



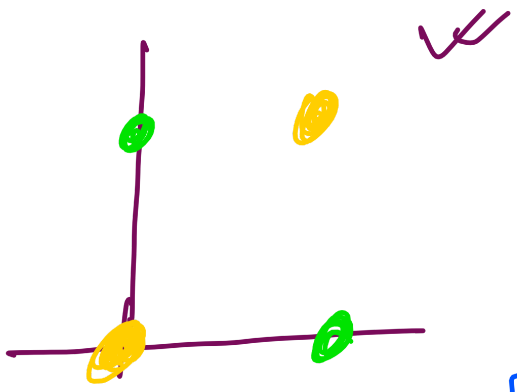
DS



DS

$x_1\bar{x}_2 + \bar{x}_1 x_2 \rightarrow$ XOR

x_1	x_2	Y
0	0	0
0	1	1 ✓
1	0	1 ✓
1	1	0



?

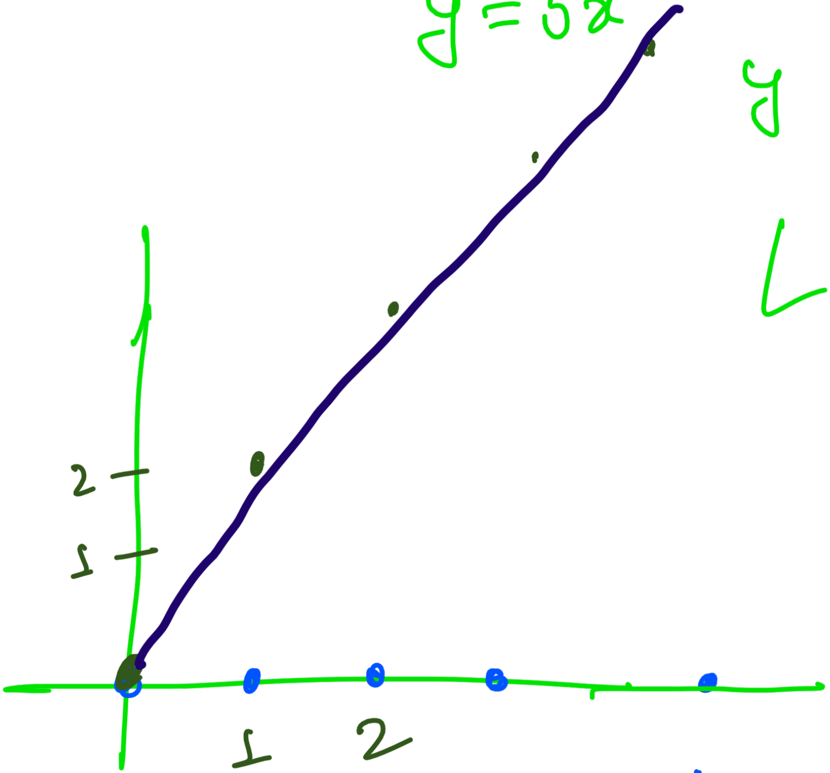
No

Can we have a $\boxed{\text{linear}}$ DS ?

$y = 2x$

$y = 5x$

$\left.\begin{array}{l} y = 3x \\ y = 0.5x \end{array}\right\}$ Linear



x	1	2	0	3	5
y	2	4	0	6	10

As we increase the value of x, the value of y expands linearly.

Non-linearity $\quad y = 2x + c \quad c = [0, 1]$

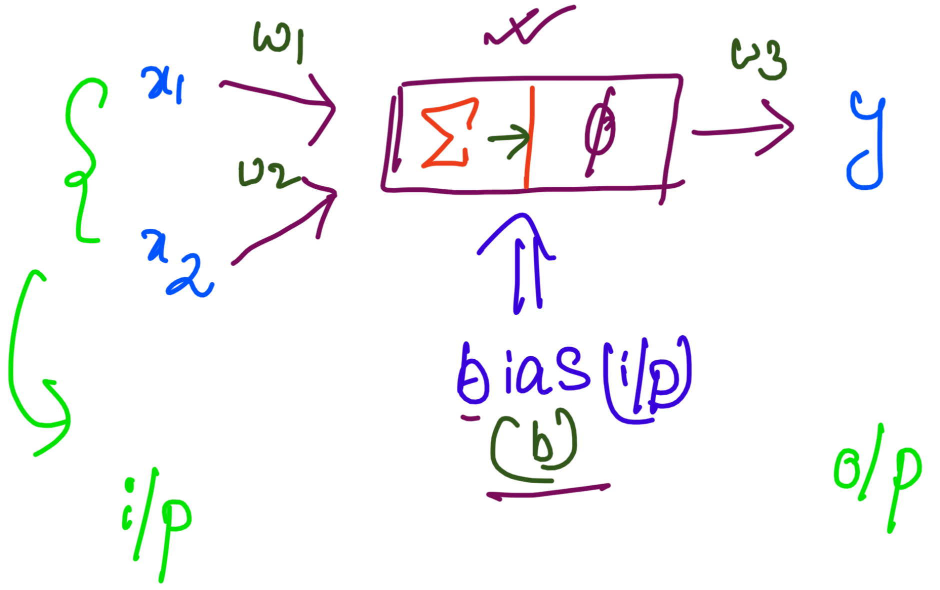
Plot the graph ?

Single Perceptron

Perceptron

Σ = weighted Average

$\boxed{\phi \rightarrow \text{Transfer function (TF)}}$



$$w_3 \leftarrow \phi(x_1 \times w_1 + x_2 \times w_2 + b)$$

If TF was not there and $b = 0$ then

$w_2 = 0$

$$w_3 \leftarrow x_1 w_1 + x_2 w_2$$

$$\boxed{w_3 \leftarrow x_1 w_1}$$

✓

To introduce non linearity = transfer function

has been in place

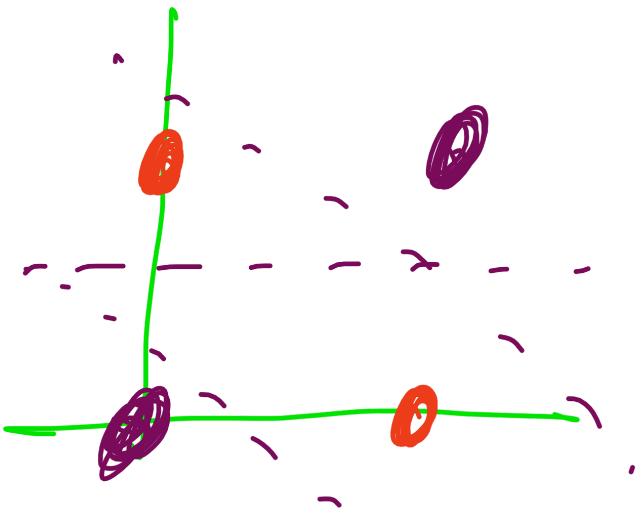
also bias

$$P = x_1 w_1 + x_2 w_2 + b$$

Example

$$\phi(P) = \begin{cases} 0 & \sum xw + b \leq 0 \\ 1 & \sum xw + b > 0 \end{cases}$$

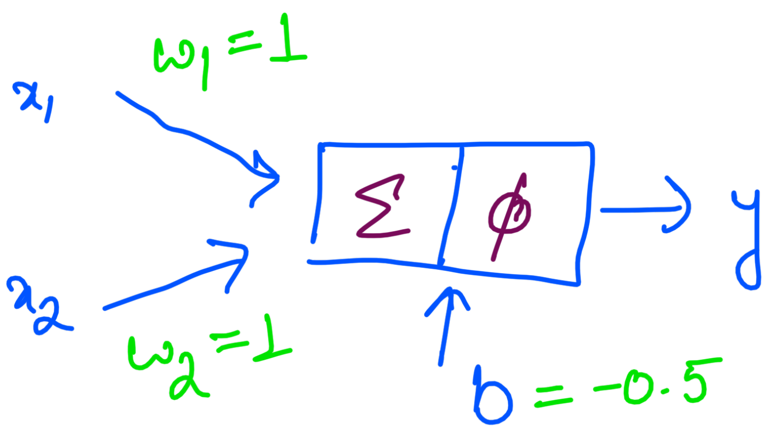
XNOR



Yes

non-linearity

Perceptron for OR gate



x_1	x_2	Y
0	0	0 ✓
0	1	1 ✓
1	0	1
1	1	1

✓

$\phi \leftarrow$ transfer

$$y = \begin{cases} 0 & w.x+b \leq 0 \\ 1 & w.x+b > 0 \end{cases}$$

1st case

$x_1 = 0 \qquad x_2 = 0$

$$P = x_1 w_1 + x_2 w_2 + b$$

$$= 0 \times 1 + 0 \times 1 + (-0.5)$$

$$= -0.5$$

$$\phi(P) = 0$$

2nd case

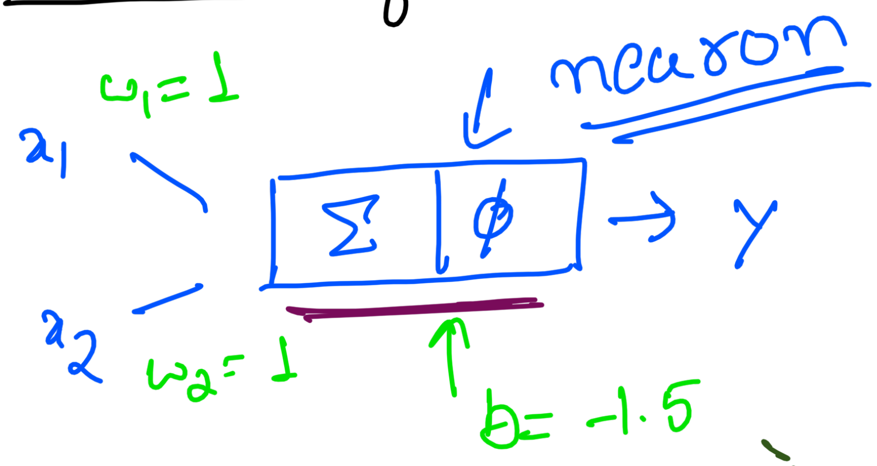
$x_1 = 0 \qquad x_2 = 1$

$$P \Rightarrow x_1 w_1 + x_2 w_2 + b = 0 \times 1 + 1 \times 1 + -0.5$$

$$= 0.5 > 0$$

$$\phi(P) = 1$$

Perceptron for AND gate



x_1	x_2	Y
0	0	0
0	1	0
1	0	0
1	1	1

$\boxed{\text{Fixed}}$

$x_1 x_2$

$$\phi = \begin{cases} 0 & w.x + b \leq 0 \\ 1 & w.x + b > 0 \end{cases}$$

3rd Case :

$x_1 = 1 \qquad x_2 = 0$

$$P = x_1 w_1 + x_2 w_2 + b$$

$$= 1 \times 1 + 0 \times 1 - 1.5$$

$$= -0.5 < 0 \rightarrow 0$$

4th Case

$x_1 = 1 \qquad x_2 = 1$

$$P = x_1 w_1 + x_2 w_2 + b$$

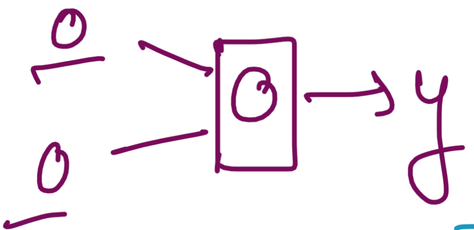
$$= 1 \times 1 + 1 \times 1 - 1.5$$

$$= 2 - 1.5$$

$$= 0.5 \geq 0 \rightarrow 1$$

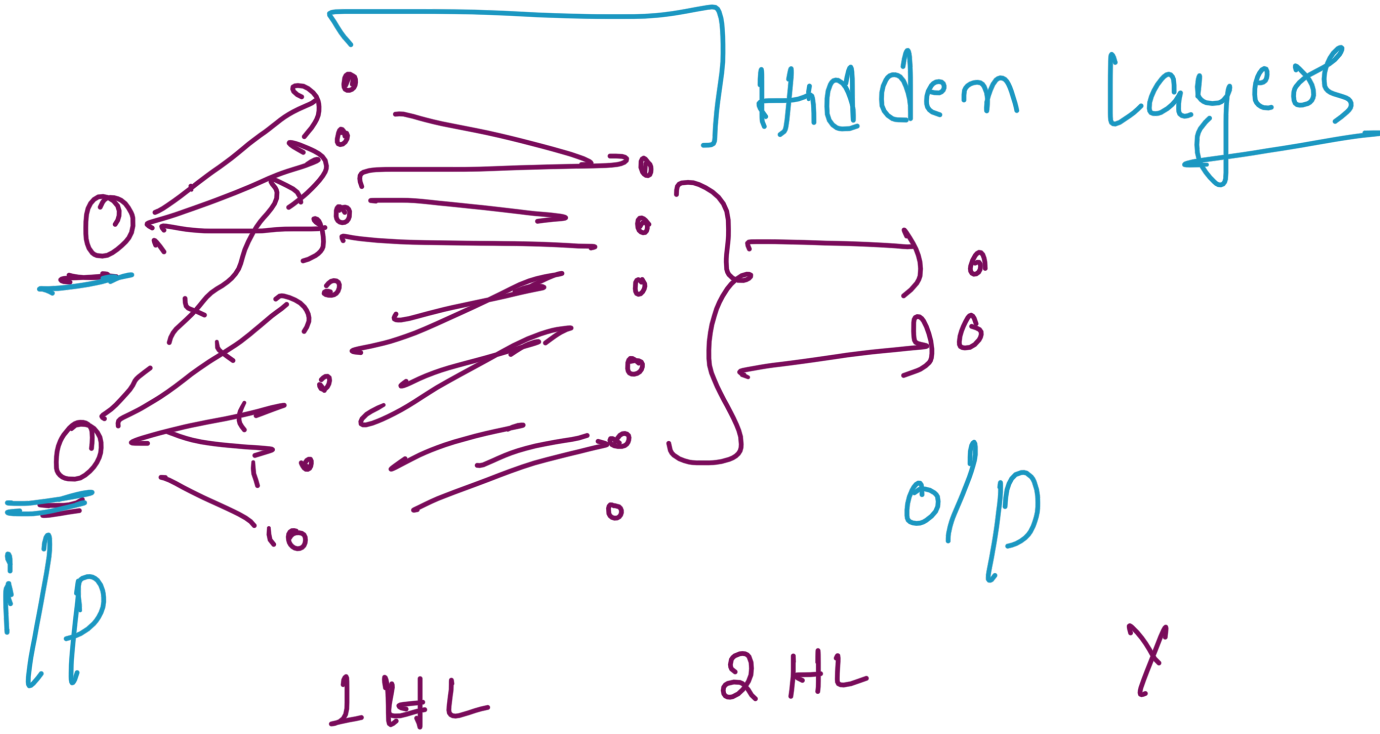
XOR — No

$\boxed{\text{1 single neuron enough ?}}$



O ← Single Neuron

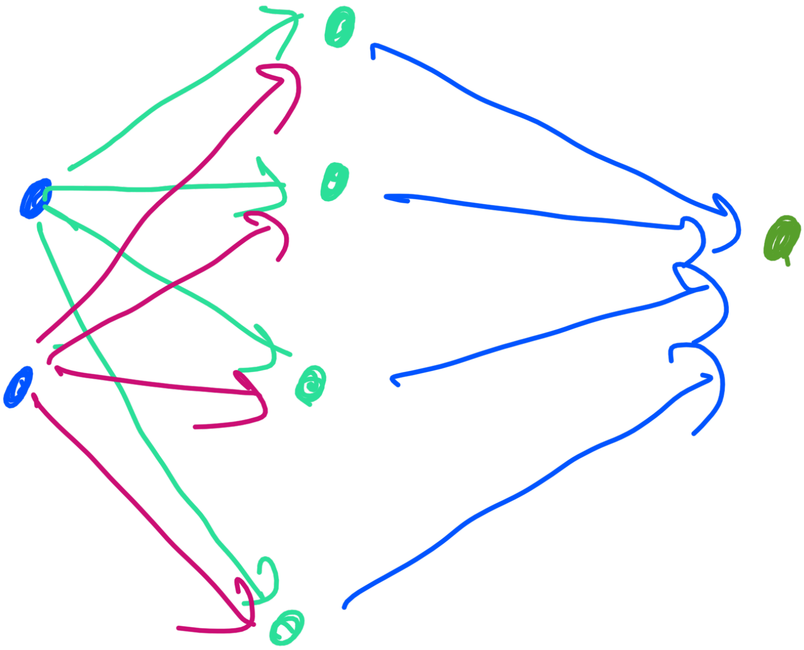
Hidden Layer ← 1



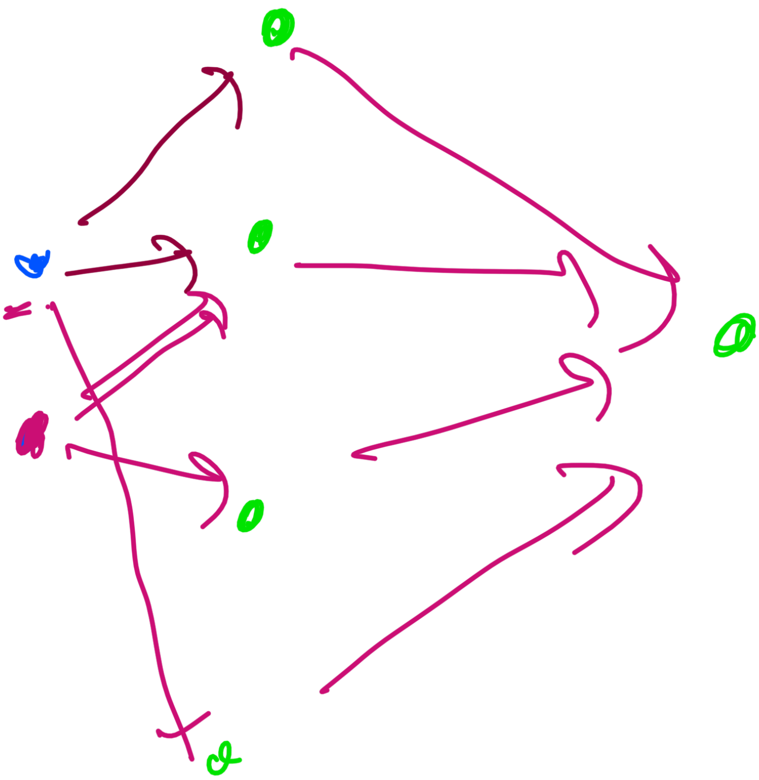
$\boxed{\text{Fully Connected}}$ ← Network

all neurons are connected to all other neuron

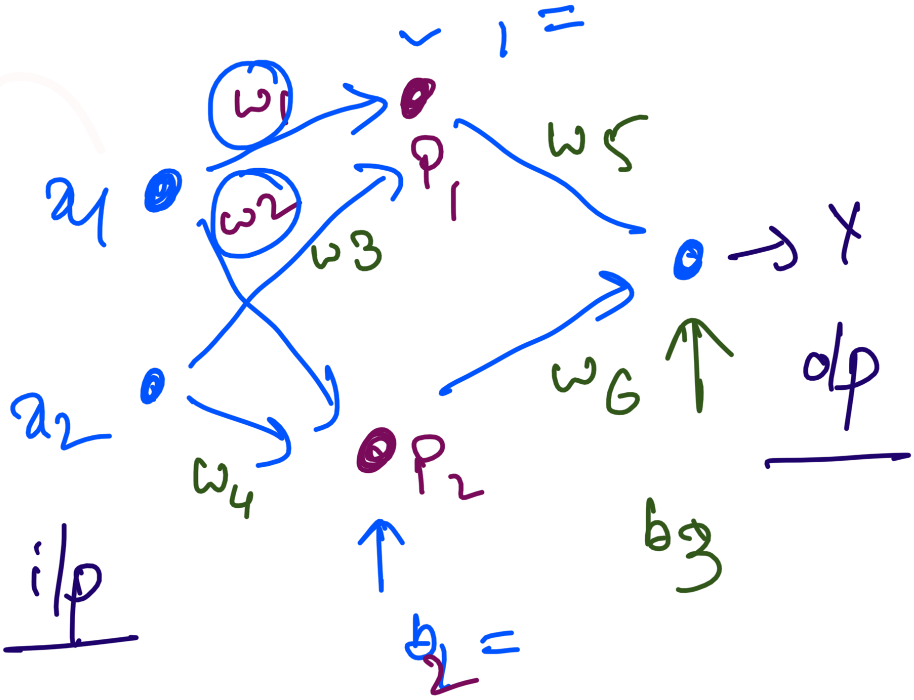
✓ **MLP**: Multi Layer Perceptron



FC



Not FC



XOR

x_1	x_2	Y
0	0	0
0	1	1
1	0	1
1	1	0

1st Case

$x_1 = 0 \quad x_2 = 0$

$$P_1 = w_1 x_1 + w_2 x_2 + b_1$$
$$= 1 \times 0 + 1 \times 0 + -0.5$$
$$= 0 - 0.5$$
$$= -0.5 \leq 0$$
$$\phi(P_1) = 0$$

$$P_2 = w_3 x_1 + w_4 x_2 + b_2$$
$$= 1 \times 0 + 1 \times 0 - 1.5$$
$$= -1.5 \leq 0$$
$$\phi(P_2) = 0$$

$w_1 = 1 \quad w_3 = 1$

$w_2 = 1 \quad w_4 = 1$

$b_1 = -0.5$

$b_2 = -1.5$

$b_3 = -0.5$

$w_5 = 1$

$w_6 = -1$

Every neuron $\boxed{\Sigma \mid \phi}$

$$\phi \begin{cases} 0 & w \cdot x + b \leq 0 \\ 1 & w \cdot x + b > 0 \end{cases}$$

$$Y = \phi(P_1) \times \omega_5 + \phi(P_2)\,\omega_6 + b_3$$

$$= 0 \times 1 + 0 \times 1 - 0.5$$

$$= -0.5 < 0$$

$$y = 0$$

2nd Case

$x_1 = 0 \quad x_2 = 1$

$$P_1 = x_1 \omega_1 + x_2 \omega_2 + b_1$$

$$= 0 + 1 \times 1 + -0.5$$

$$= 0.5 \geq 0 \qquad \underline{\phi(P_1)} = 1$$

$$P_2 = x_1 \omega_3 + x_2 \omega_4 + b_2$$

$$= 0 + 1 \times 1 - 1.5$$

$$= -0.5 < 0 \qquad \underline{\phi(P_2) = 0}$$

$$Y = \phi\left(\phi(P_1) \times \omega_5 + \phi(P_2)\omega_6 + b_3\right)$$

$$= \phi\left(1 \times 1 + 0 \times 1 + -0.5\right)$$

$$= \phi\left(1 - 0.5\right)$$

$$= \phi(0.5) > 0$$

$$= 1$$

1. If we start from any arbitary weights then is it possible that the n/w is able to tune the weights?

Yes → Learning

2. Weights that are fixed today is this the only representation or there could be other representation which results the same?

Representation: the structure is same the weights are different.

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