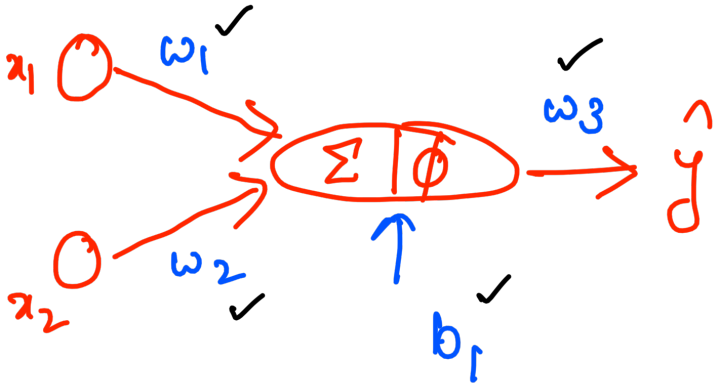
# Perceptron Learning

**Learning** → weights and bias ✓✓

↳ Algorithm can change itself.

**Hyper-parameter** : →

Adjust the hyper-parameter.




$\hat{y}$ = predicted

$$\phi = \begin{cases} 0 & w.x + b \leq 0 \\ 1 & w.x + b > 0 \end{cases}$$

↳ Fixed

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**Learning**

$$w_i = w_i + \Delta w$$

$$\Delta w_i = \eta (y - \hat{y}) x_i$$

$\eta$ (eta)

→ Learning Rate

$w_i$ = weights

$\Delta w$ = amount of change required

$y$ = true class

$\hat{y}$ = predicted class

$x_i$ = input sample.

$\eta$ → very small number $< 1$

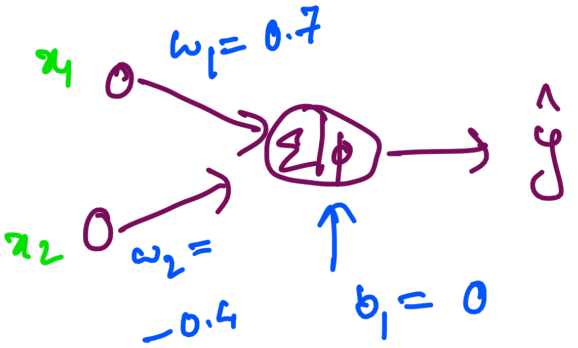
→ not negative

eg: $1 \times 10^{-3}$, $1 \times 10^{-2}$

# Example

## OR gate

| $x_1$ | $x_2$ | $y$ |
|---|---|---|
| 0 | 0 | 0 ✓ |
| 0 | 1 | 1 ✓ |
| 1 | 0 | 1 ✓ |
| 1 | 1 | 1 ✓ |




$w_1 = 0.7 \quad w_2 = -0.4$

$\eta = 0.6$

$$\phi = \begin{cases} 0 & w \cdot x + b \leq 0 \\ 1 & \text{else} \end{cases}$$

$w_i = w_i + \Delta w$

$\Delta w_i = \eta (y - \hat{y}) x_i$

$y - \hat{y} \to 0$

$= 0$

$\boxed{w_i = w_i}$

### 1st case :

$x_1 = 0 \qquad x_2 = 0$

$$\phi(\Sigma) = \phi(w_1 x_1 + w_2 x_2 + b_1)$$
$$= \phi(0.7 \times 0 + (-0.4) \times 0 + 0)$$
$$= \phi(0)$$
$$= 0$$

### 2nd case

$x_1 = 0 \qquad x_2 = 1$

$$\phi(\Sigma) = \phi(w_1 x_1 + w_2 x_2 + b_1)$$
$$= \phi(0.7 \times 0 + (-0.4) \times 1 + 0)$$
$$= \phi(-0.4)$$
$$= 0$$

### weight update :

$w_1 = 0.7 \qquad w_2 = 0.2$

$w_i = w_i + \Delta w$

$w_1 = w_1 + \Delta w_1 \quad -(1)$

$\Delta w_1 = \eta (y - \hat{y}) x_1$

$= 0.6 (1 - 0) \times 0$

$= 0$

$\boxed{w_1 = w_1 + 0 = w_1}$

$w_2 = w_2 + \Delta w_2 \quad -(2)$

$\Delta w_2 = \eta (y - \hat{y}) x_2$

$= 0.6 (1 - 0) \times 1$

$= 0.6$

(2) $\Rightarrow \boxed{w_2 = -0.4 + 0.6 = 0.2}$

1st Case

$x_1 = 0 \quad x_2 = 0$

$\phi(\Sigma) = \phi(0) = 0$ ✓

2nd Case

$x_1 = 0 \quad x_2 = 1$

$\phi(\Sigma) = \phi(x_1 w_1 + x_2 w_2 + b_1)$

$= \phi(0 + 0.2 + 0)$

$= \phi(0.2)$

$= 1$

Tuned

$w_1 = 0.7$

$w_2 = 0.2$

↑

New weights

w.r.t dataset

3rd Case

$x_1 = 1 \quad x_2 = 0$

$\phi(\Sigma) = \phi(x_1 w_1 + x_2 w_2 + b_1)$

$= \phi(1 \times 0.7 + 0 + 0)$

$= \phi(0.7)$

$= 1$

4th Case

$x_1 = 1 \quad x_2 = 1$

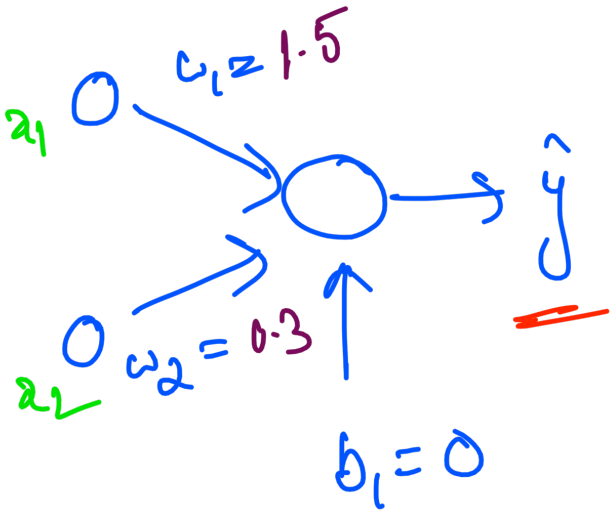
$\phi(\Sigma) = \phi(x_1 w_1 + x_2 w_2 + b_1)$

$= \phi(0.7 + 0.2 + 0)$

$= \phi(0.9)$

$= 1$

# AND gate




| $x_1$ | $x_2$ | $Y$ |
|---|---|---|
| 0 | 0 | 0 ✓ |
| 0 | 1 | 0 ✓ |
| 1 | 0 | 0 ✓ |
| 0 | 0 | 1 ✓ |

$$\phi = \begin{cases} 0 & w.x + b < 1 \\ 1 & w.x + b \geq 1 \end{cases}$$

$w_1 = 1.5$ $\quad \eta = 0.7$

$w_2 = 0.3$

## 1st case

$x_1 = 0 \quad x_2 = 0$

$$\phi(\Sigma) = \phi(x_1 w_1 + x_2 w_2 + b_1)$$
$$= \phi(0)$$
$$= 0$$

$y - \hat{y} = 0$

$0 - 0 = 0$

## 2nd case

$x_1 = 0 \quad x_2 = 1$

$$\phi(\Sigma) = \phi(x_1 w_1 + x_2 w_2 + b_1)$$
$$= \phi(0 + 1 \times 0.3 + 0)$$
$$= \phi(0.3)$$
$$= 0$$

$y - \hat{y} = 0$

3rd Case

$x_1 = 1 \quad x_2 = 0$

$$\phi(\Sigma) = \phi(x_1 w_1 + x_2 w_2 + b_1)$$
$$= \phi(1 \times 1.5 + 0 + 0)$$
$$= \phi(1.5)$$
$$= 1$$

Apply weight update

$$w_i = w_i + \Delta w_i \; ; \quad \Delta w_i = \eta (y - \hat{y}) x_i$$

$$w_1 = w_1 + \eta (y - \hat{y}) x_1$$
$$= 1.5 + 0.7 (0 - 1) 1$$
$$= 1.5 - 0.7 \; *$$
$$= 0.8$$

↳ Adjusted

Adjusted weights

$w_1 = 0.8$

$w_2 = 0.3$

$$w_2 = w_2 + \eta (y - \hat{y}) x_2$$
$$= w_2 + \eta (y - \hat{y}) 0$$
$$= w_2$$
$$= 0.3$$

$\eta = 0.7$ X

$\eta = 0.2$ ✓

* $1.5 - 0.2$

$= 1.3$ It will

Repeat 3rd case

$x_1 = 1 \quad x_2 = 0$

$w_1 = 1.3$

$w_2 = 0.3$

fal c
more
x
time

$$\phi(\Sigma) = \phi(x_1 w_1 + w_2 x_2 + b_1)$$

$$= \phi(1 \times 0.8 + 0 + 0)$$

$$= \phi(0.5)$$

$$= 0$$

4th case

$x_1 = 1 \quad x_2 = 1$

$$\phi(\Sigma) = \phi(x_1 w_1 + x_2 w_2 + b_1)$$

$$= \phi(0.5 + 0.3)$$

$$= \phi(1.1)$$

$$= 1$$

✓

$(y - \hat{y})$ ×

↳ Error

$\Delta w_i = \eta (y - \hat{y}) x_i$

↳ Error

↳ Adjust

Learning Rate

$w_i = w_i + \Delta w_i$

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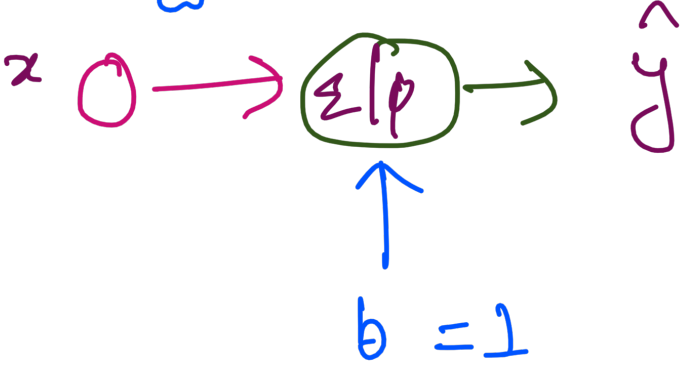
NOT gate

| x | Y |
|---|---|
| 0 | 1 ✓ |
| 1 | 0 ✓ |




$$\phi = \begin{cases} 0 & w \cdot x + b \leq 0 \\ 1 & w \cdot x + b > 0 \end{cases}$$

## 1st Case

$x = 0 \rightarrow 1$

$\phi(\Sigma) = \phi(xw + b)$

$= \phi(0 \times 1 + 1)$

$= \phi(1)$

$= 1$

## 2nd Case

$x = 1 \rightarrow$

$\phi(\Sigma) = \phi(xw + b)$

$= \phi(1(-1) + 1)$

$= \phi(0)$

$\rightarrow 0$

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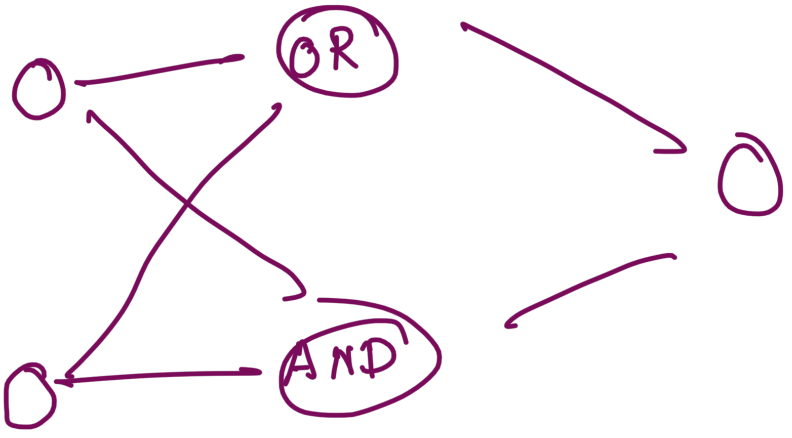
## XOR




Is it the only way to solve it?

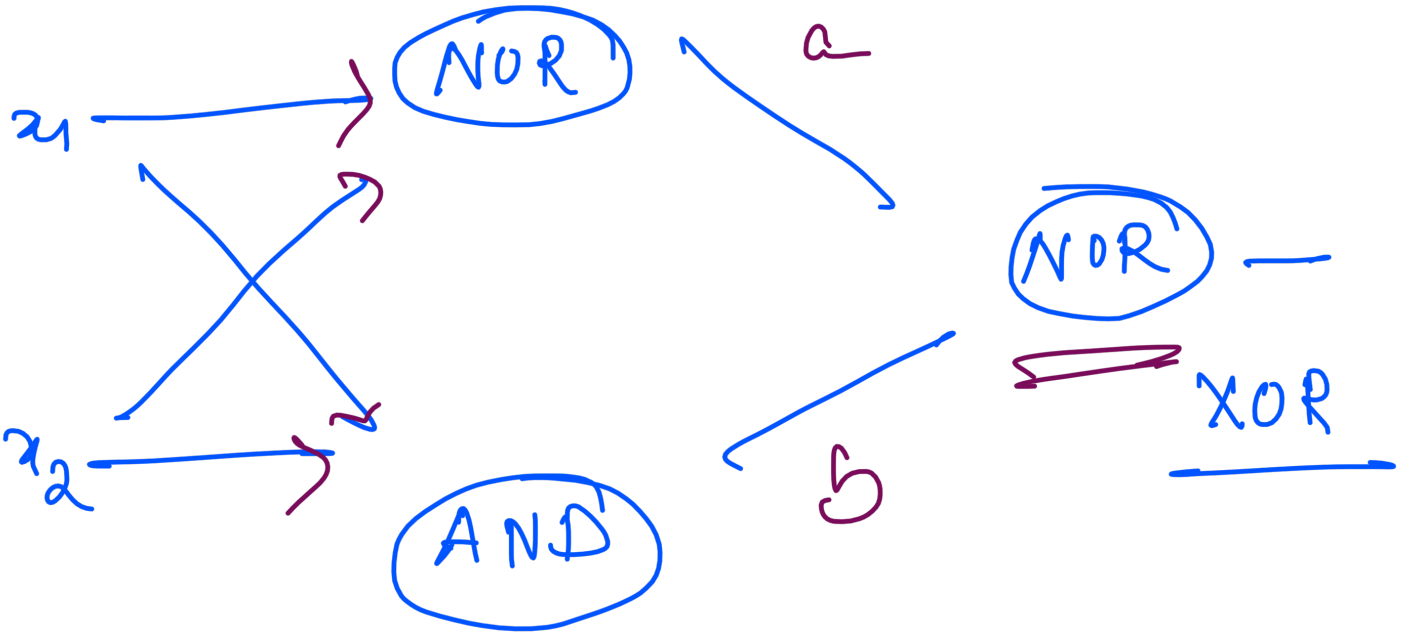
NOR

OR + NOT

OR NOR

| $x_1$ | $x_2$ | Y | |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

NOR




OR AND NOT

| $x_1$ | $x_2$ | NOR | AND | NOR(a,b) |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |

XOR