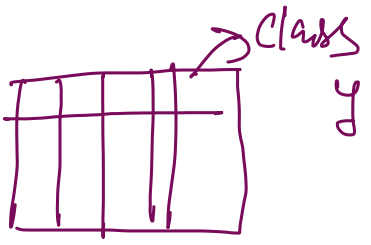
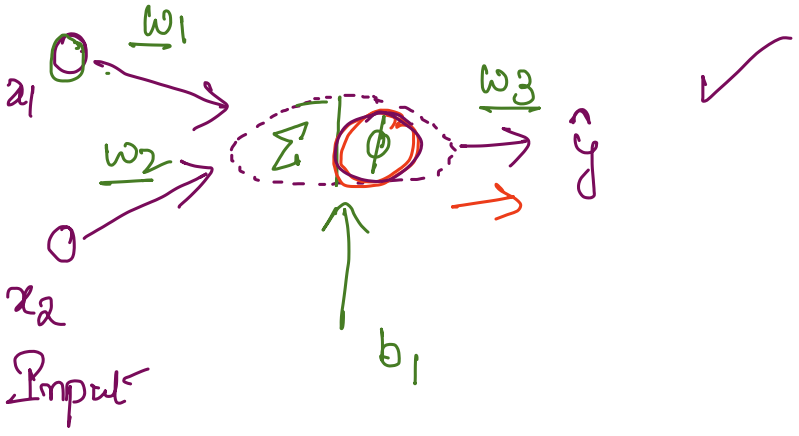
Perceptron

Learning → weights & bias
Hyper-parameter → Adjust

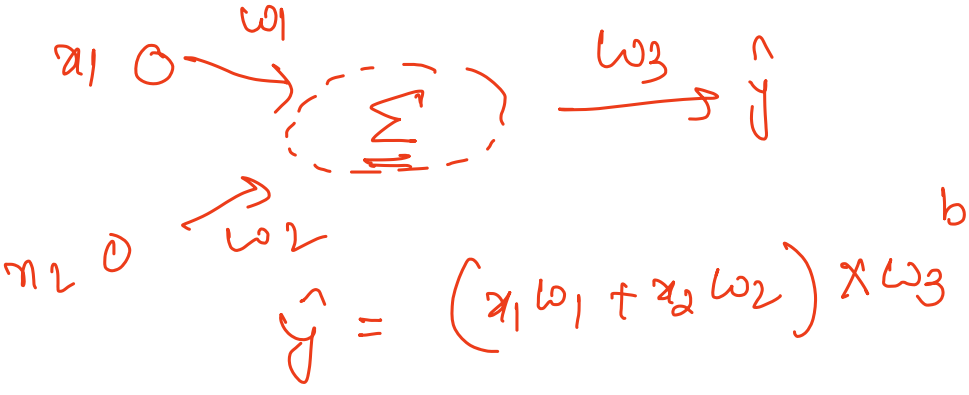




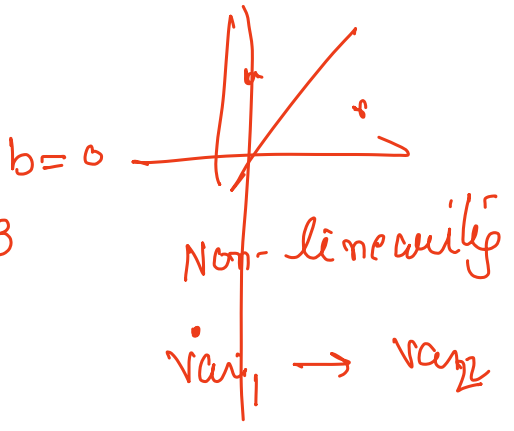
$x_i \rightarrow y_i$

$y \rightarrow$ original
$\hat{y} \rightarrow$ predicted
as close as possible

$\min (y - \hat{y})$



$$\hat{y} = (x_1 w_1 + x_2 w_2) \times w_3$$



$$\phi = \begin{cases} 0 & w \cdot x + b \leq 0 \\ 1 & w x + b > 0 \end{cases}$$

Activation function

↳ Fixed

Learning

$$w_i = w_i + \Delta w$$

$$\Delta w = \eta (y - \hat{y}) x_i$$

$\eta \rightarrow$ Learning rate

$w_i =$ weights

$\Delta w =$ amount of change required

$y =$ true class

$\hat{y} =$ predicted class

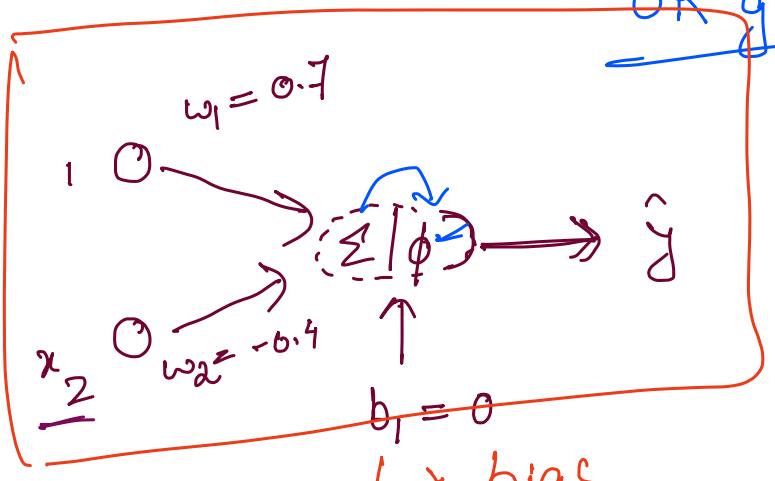
$x_i =$ i/p sample.

$\eta \rightarrow$ very small < 1

1×10^{-3} 0.001

Hyper parameter

OR gate



x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$w_1 = 0.7$

$w_2 = -0.4$

$b_1 = 0$

$\eta = 0.6$

1st Case

$x_1 = 0$ $x_2 = 0$

$$\phi(\Sigma) = \phi((x_1 \cdot w_1 + x_2 w_2) + b_1)$$

$$= \phi(0 \times 0.7 + 0 \times -0.4 + 0)$$

$$= \phi(0)$$

$$= 0$$

$$\phi(x) = \begin{cases} 0 & w \cdot x + b < 0 \\ 1 & \text{else} \end{cases}$$

2nd Case

$x_1 = 0 \quad x_2 = 1$

$$\phi(\Sigma) = \phi(w_1 x_1 + w_2 x_2 + b_1)$$
$$= \phi(0 \times 0.7 + 1 \times (-0.4) + 0)$$
$$= \phi(-0.4)$$
$$= 0$$

✓

$$w_2 = w_2 + \Delta w_2$$
$$\Delta w_2 = \eta(y - \hat{y}) \times x_2$$
$$= 0.6(1-0) \times 1$$
$$= 0.6$$
$$w_2 = -0.4 + 0.6$$
$$= 0.2$$

Update the weights

$$w_i = w_1 + \Delta w$$

$$w_1 = w_1 + \Delta w_1$$ ✓

$$\Delta w_1 = \eta(y - \hat{y}) x_1$$
$$= 0.6(0-1) \times 0$$
$$= 0$$
$$w_1 = w_1 + \Delta w_1$$
$$0$$

$$w_1 = w_1$$

$$w_1 = 0.7 \quad w_2 = 0.2$$

1st Case

$$\phi(\Sigma) = (0 \times w_1 + 0 \times w_2 + b_1)$$
$$= 0$$

2nd Case

$$\phi(\Sigma) = \phi(0 \times w_1 + 1 \times 0.2 + b_1)$$
$$= \phi(0.2)$$
$$= 1$$

3rd

$x_1 = 1 \quad x_2 = 0$

$\phi(\Sigma) = \phi(x_1 \times 0.7 + 0 \times \omega_2 + 0)$

$= \phi(0.7)$

$= 1$ ✓

4th

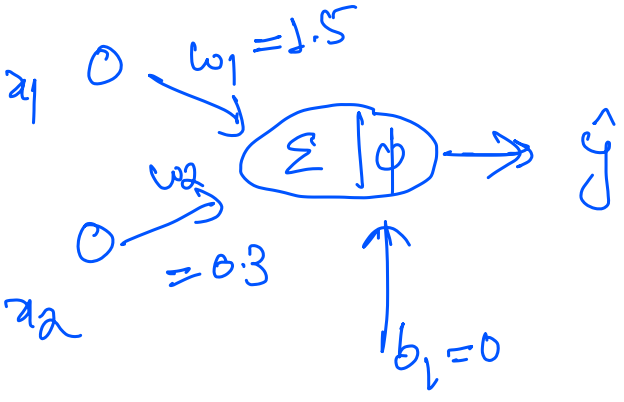
$x_1 = 1 \quad x_2 = 1$

$\phi(\Sigma) = \phi(1 \times 0.7 + 1 \times 0.2 + b_1)$

$= \phi(0.9)$

$= 1$ ✓

AND



x_1	x_2	y
0	0	0 ✓
0	1	0 ✓
1	0	0 ✓
1	1	1 ✓

1st Case

$x_1 = 0 \quad x_2 = 0$

$\phi(\Sigma) = \phi(0 \times \omega_1 + 0 \times \omega_2 + 0)$

$= \phi(0)$

$= 0$ ✓

$$\phi = \begin{cases} 0, & \omega \cdot a + b < 1 \\ 1, & \omega \cdot a + b \geq 1 \end{cases}$$ ✓

$\eta = 0.7$

2nd case

$x_1 = 0 \quad x_2 = 1$

$\phi(\Sigma) = \phi(0 \times w_1 + 1 \times 0.3 + 0)$

$= \phi(0.3)$

$= 0$ ✓

3rd case

$x_1 = 1 \quad x_2 = 0$

$\phi(\Sigma) = \phi(1 \times 1.5 + 0 \times w_2 + b_1)$

$= \phi(1.5)$

$= 1 \leftarrow \hat{y}$

Weight update

$w_i = w_i + \Delta w_i$

$w_1 = w_1 + \Delta w_1$

$\Delta w_1 = \eta (y - \hat{y}) x_1$

$= 0.7 (0-1) \times 1$

$= -0.7$

$w_1 = 1.5 + (-0.7)$

$= 0.8$ ✓

$w_2 = w_2 + \Delta w_2$

$\Delta w_2 = \eta (y - \hat{y}) x_2$

$= 0.7 (0-1) \times 0$

$= 0$

$w_2 = w_2$ ✓ 0.3

$w_1 = 0.8$
$w_2 = 0.3$ ← updated weights

Check weather for 1st 100 case?

3rd case

$$\phi(\Sigma) = \phi(1 \times 0.8 + 0 \times 0.3 + 0)$$
$$= \phi(0.8)$$
$$= 0 \checkmark$$

4th case

$$\phi(\Sigma) = \phi(1 \times 0.8 + 1 \times 0.3 + 0)$$
$$= \phi(0.8 + 0.3)$$
$$= \phi(1.1)$$
$$= 1$$

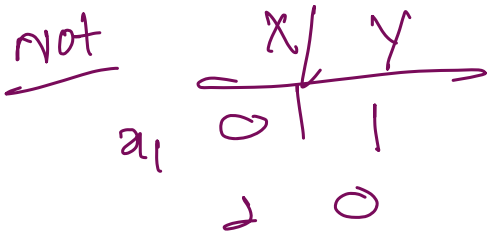
OR, AND ✓ | NOT, XOR, NOR ✓

$(y - \hat{y})$ → Error

$$\Delta w_1 = \eta (y - \hat{y}) x_1$$

↑ adjust

not



$\phi(\xi) =$

$$\phi = \begin{cases} 0 & \omega_1 . x + b \le 0 \\ 1 & 0 \text{ therwise} \end{cases}$$