Mechanical Processing in Internally Coupled Ears

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The Model

Mouth Cavity
Acoustic Head Model
Eardrum
Coupled Membranes

Outline for section 1

Evaluation

Conclusio

Introduction

Introduction



Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.



Coupled Ears

Eardrums connected through wide eustachian tubes and a large mouth cavity.

Eardrums vibrations influence eachother.

Advantages of Coupled Ears

The Model

► Low frequencies result in reduced degradation of hearing cues in dense environments.

Introduction

Outline for section 2

Introduction

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Evaluation



The Model

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Evaluation

Internally Coupled Ears

 $a_{
m tymp}$ fixed. $V_{
m cyl} = \pi a_{
m tymp}^2 L$

Introduction

Mouth Cavity

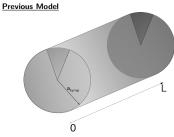
Mouth Cavity
Previous Model

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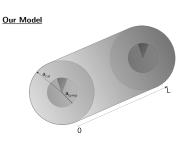
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Mouth Cavity

Mouth Cavity



 $a_{
m tymp}$ fixed. $V_{
m cyl} = \pi a_{
m tymp}^2 L$



 $a_{\mathrm{tymp}},\ V_{\mathrm{cyl}}$ fixed.

$$a_{\rm cyl} = \sqrt{V_{\rm cyl}/\pi L}$$

Conclusion

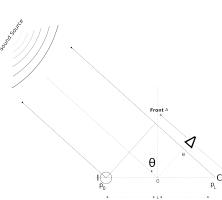
Introduction

Acoustic Head Model

▶ I - Ipsilateral ear, C -Contralateral ear. p_0 , p_L - sound pressure on eardrums, θ - sound source direction.

The Model

- Sound source "far away".
- ► No appreciable amplitude
- Phase difference between sound
- $p_0 = pe^{j\omega t .5kL\sin\theta}$ $p_L = pe^{j\omega t + .5kL\sin\theta}$



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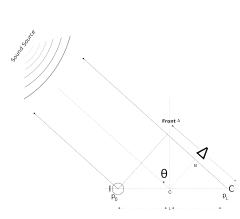
Internally Coupled Ears

Acoustic Head Model

- I Ipsilateral ear, C Contralateral ear.
 p₀, p_L sound pressure on
 eardrums, θ sound source
 direction
- Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_I|$.
- ▶ Phase difference between sound at both ears $\Delta = kL \sin \theta$.

$$p_0 = pe^{j\omega t - .5kL\sin\theta}$$

$$p_L = pe^{j\omega t + .5kL\sin\theta}$$



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Internally Coupled Ears

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Introduction

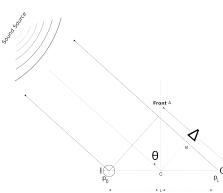
Acoustic Head Model

▶ I - Ipsilateral ear, C -

The Model

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- Sound source "far away". ► No appreciable amplitude
- Phase difference between sound
- $p_0 = p e^{j\omega t .5kL\sin\theta}$ $p_L = p e^{j\omega t + .5kL\sin\theta}$



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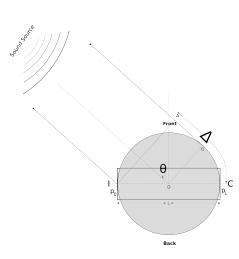
The Model

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Acoustic Head Model

Acoustic Head Model

- $|p_0| = |p_L|.$
- ▶ Increased phase difference due to diffraction $\Delta = 1.5kL \sin \theta$.
- $p_0 = pe^{j\omega t .75kL\sin\theta}$ $p_L = pe^{j\omega t + .75kL\sin\theta}$



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Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x, r, \phi, t) = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p(x, r, \phi, t)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p(x, r, \phi, t)}{\partial \phi^2} + \frac{\partial p(x, r, \phi, t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}$$
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Acoustic Head Model

Introduction

Evaluation

$$e^{\pm j\zeta x}$$

$$\frac{d^2f(x)}{dx^2} + \zeta^2f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$
$$\frac{d^2h(\phi)}{d\phi^2} + q^2h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$$

(3)



x- and ϕ - directions

The Model

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$$rac{d^2f(x)}{dx^2} + \zeta^2f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$
 $rac{d^2h(\phi)}{d\phi^2} + q^2h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$

$$ightarrow$$
 $h(\phi)$

$$\phi$$
) =

Evaluation

$$\mathrm{e}^{\pm jq\phi}$$

Conclusion

 $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q^2}{r^2}\right]g(r) = 0 \longrightarrow g(r) = J_q(\nu r)$ where, $\nu^2 = k^2 - \zeta^2$





Introduction

Acoustic Head Model

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$$\phi o h(0) = h(2\pi) \quad \text{and} \quad h'(0) = h'(2\pi)$$
 $\Rightarrow h(\phi) = \cos q\phi, \ q = 0, 1, 2, \dots$ (5)

Introduction

Acoustic Head Model

Boundary Conditions - ϕ

$$\Rightarrow g(r) = J_a(\nu_{GS}r/a_{CV})$$

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$$lacksquare$$
 $u_{
m qs}$ - zeros of J_q' , $s=0,1,2,\ldots$

Bessel Prime Zeros

Impenetrable boundary at
$$r = a_{cyl}$$
, i.e. normal derivative vanishes

Evaluation

daily at
$$r = a_{\rm cyl}$$
, i.e. normal derivative

$$=\frac{\partial g}{\partial g}$$
 = 0 (6)

$$\left. \frac{\partial g}{\partial r} \right|_{r=a_{\rm cyl}} = 0$$
 (6)

Conclusion

Introduction

Acoustic Head Model

Boundary Conditions - r

Conclusion

General Solution

Introduction

Acoustic Head Model

$$p(x,r,\phi,t) = \sum_{q=0,s=0}^{\infty} p_{qs}(x,r,\phi)e^{j\omega t}$$

$$p_{qs}(x,r,\phi) = \left[A_{qs}e^{j\zeta_{qs}x} + B_{qs}e^{-j\zeta_{qs}x}\right]\cos q\phi J_q(\nu_{qs}r/a_{cyl})$$
where, $\zeta_{qs} = \sqrt{k^2 - \nu_{qs}^2/a_{cyl}^2}$ (9)

$$p_{00}(x, r, \phi; t) = \left[A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t}$$
 (10)

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The Model

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The Model

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$$p(x, r, \varphi, \iota)$$
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Introduction

Acoustic Head Model

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m s}=\sqrt{}$$

Evaluation

$$\cos q\phi J_q (
u_{
m qs} r/a_{
m cy})$$

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$$p_{00}(x, r, \phi; t) = \left[A_{00}e^{jkx} + B_{00}e^{-jkx}\right]e^{j\omega t}$$

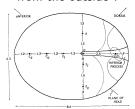
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Eardrum

Introduction

Eardrum

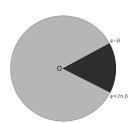
Sketch of a Tokay eardrum as seen from the outside^a.



COL - approximate position opposite the extracolumella insertion.

^aG. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r=a_{\mathrm{tymp}}$ and $\phi=\beta,\ 2\pi-\beta$)

 c_M^2 - propagation velocity α - damping coefficient,

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d - thickness.

Introduction

Eardrum

Membrane Vibrations

Membrane EOM

 ρ_m - density,

Conclusion

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$$d^2g(\phi)$$
 . 2 (1)

 $u(r, \phi; t) = f(r)g(\phi)h(t)$

$$\frac{d^2g(\phi)}{d\phi^2} + \kappa^2 g(\phi) = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f(r)}{\partial r} \right) + \left[\mu^2 - \frac{\kappa^2}{r^2} \right] f(r) = 0$$

 $\frac{d^2h(t)}{dt^2} + c_M^2\mu^2h(t) = 0$

Evaluation

Internally Coupled Ears

Conclusion

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(15)

Conclusion

The Model

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Introduction

Eardrum

$$\phi$$
-direction: $u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$

$$\Rightarrow \sigma(\phi) = \sin v(\phi - \beta)$$
(18)

$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$
where, $\kappa = \frac{m\pi}{2(\pi - \beta)}$, $m = 1, 2, 3, ...$

r-direction:
$$u(a_{\rm tymp}, \phi; t) = 0$$

$$\Rightarrow f(r) = J_{\kappa}(\mu_{\rm mn} r / a_{\rm tymp})$$
where, $\mu_{\rm mn}$ is the $n^{\rm th}$ zero of J_{κ} (17)

$$\Rightarrow$$
 $g(\phi)=\sin\kappa(\phi-eta)$ where, $\kappa=rac{m\pi}{2(\pi-eta)}, \quad m=1,2,3,\ldots$

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r-direction:
$$u(a_{\text{tymp}}, \phi; t) = 0$$

Boundary Conditions

$$\Rightarrow$$
 $f(r) = J_{\kappa}(\mu_{
m mn} r/a_{
m tymp})$ where, $\mu_{
m mn}$ is the $n^{
m th}$ zero of J_{κ}

Evaluation

(16)

Conclusion

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Introduction

Eardrum

$$u_{
m mn}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{
m mn} r)$$
 (18)
 $u_{
m free}(r,\phi;t) = \sum_{
m mn} C_{
m mn} u_{
m mn}(r,\phi) e^{j\omega_{
m mn} t}$ (19)

m = 0, n = 1where, $\omega_{\rm mn} = c_M \mu_{\rm mn}$

The Model

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Free eigenmodes

Damped membrane
$$\widetilde{u}_{\mathrm{free}}(r,\phi;t) = \sum_{-\infty}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t - \alpha t}$$
 (20)

Introduction

Eardrum

Free eigenmodes

Introduction

Eardrum

$$u_{\rm mn}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{\rm mn} r)$$

$$u_{\rm free}(r,\phi;t) = \sum_{m}^{\infty} C_{\rm mn} u_{\rm mn}(r,\phi) e^{j\omega_{\rm mn} t}$$
(18)

Evaluation

$$_{m=0,n=1}^{m=0,n=1}$$
 where, $\omega_{
m mn}=c_{M}\mu_{
m mn}$

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Damped membrane

$$\widetilde{u}_{\mathrm{free}}(r,\phi;t) = \sum_{n=0}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t - \alpha t}$$
 (20)

m = 0, n = 1

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 $u_{\rm ss}(r, \psi, t) = \sum_{m=0, n=1}^{\infty}$

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Substitute u_{ss} in Membrane EOM.

Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

$$\Omega_{\rm mn} = \rho_M d \left[(\omega^2 - \omega_{\rm mn}^2) - 2j\alpha\omega \right]$$

Evaluation

Introduction

Eardrum

Steady State Solution

$$u_{\rm ss}(r,\phi;t) =: \sum_{m=0,n=1} C_{\rm mn} u_{\rm mn}(r,\phi) e^{j\omega t}$$

The Model

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Substitute u_{ss} in Membrane EOM.

$$C_{
m mn} = rac{p \int dS u_{
m mn}}{\Omega_{
m mn} \int dS u_{
m mn}^2}$$

$$\Omega_{\rm mn} = \rho_{M} d \left[(\omega^2 - \omega_{\rm mn}^2) - 2j\alpha\omega \right]$$

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Eardrum

Forced Vibrations contd.

Transient Solution

Same as the solution for a free damped membrane

$$u_{\mathrm{t}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t - \alpha t}$$

 $\widetilde{C}_{\mathrm{mn}}$ determined from the membrane displacement at t=0.

$$u_{\rm t} \to 0$$
 as $t \to \infty$.

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The Model

Mouth Cavity Acoustic Head Model Eardrum

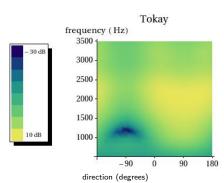
Outline for section 3

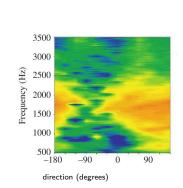
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Vibration Amplitude





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Outline for section 4

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Conclusion

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Introduction



