

# Mechanical Processing in Internally Coupled Ears

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**T<sub>M</sub>P**  
The Master Course  
Theoretical and Mathematical Physics

# Outline for Section 1

## Introduction

### Auditory Systems

### Hearing Cues

## The Model

### Mouth Cavity

Pressure Derivation

### Eardrum

Model

Membrane Vibrations

### Acoustic Head Model

## Coupled Membranes

Ansatz

Boundary Conditions

Solution

## Evaluation

### Parameters

### Vibration Amplitude

### Directional Cues

Internal Level Difference

Internal Time Difference

## Conclusion

# Auditory Systems



## Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.



## Coupled Ears

Wide eustachian tubes open into the mouth cavity.

Eardrums vibrations influence each other.

# Binaural Hearing Cues

## Interaural Time Difference

Phase difference between the (pressure) **inputs** to the ears.

- ▶ ITD

## Interaural Level Difference

Amplitude difference between the **inputs**.

- ▶ ILD

## Internal Time Difference

Phase difference between the eardrum **vibrations**.

- ▶ iTD

## Internal Level Difference

Amplitude difference between the **vibrations**.

- ▶ iLD

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Phase difference between the eardrum **vibrations**.

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## Internal Level Difference

Amplitude difference between the **vibrations**.

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## Advantages of Coupled Ears

- ▶ Low frequencies result in reduced degradation of hearing cues in dense environments.

# Outline for Section 2

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Hearing Cues

## The Model

Mouth Cavity

Pressure Derivation

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Acoustic Head Model

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## ICE Model

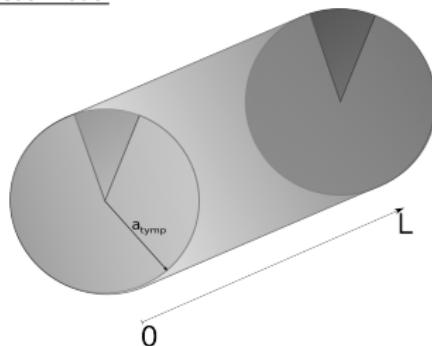
- ▶ An extension of the model first presented by Vossen et al (2010).
  - ▶ Circular eardrums connected by a cylindrical cavity.
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- ▶ The Aim is to accurately reproduce the direction and frequency dependence of the system.

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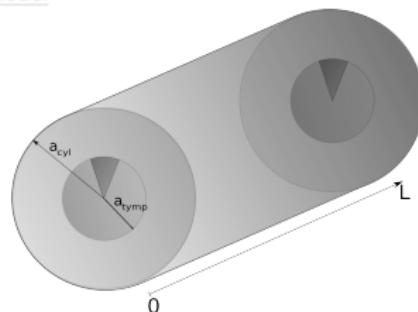
## Mouth Cavity

# Mouth Cavity

Previous Model

$a_{\text{tym}}^*$  fixed.

$$V_{\text{cyl}} = \pi a_{\text{tym}}^*{}^2 L$$

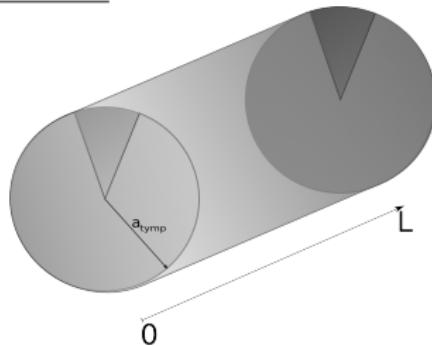
Our Model

$a_{\text{tym}}^*, V_{\text{cyl}}$  fixed.

$$a_{\text{cyl}} = \sqrt{V_{\text{cyl}} / \pi L}$$

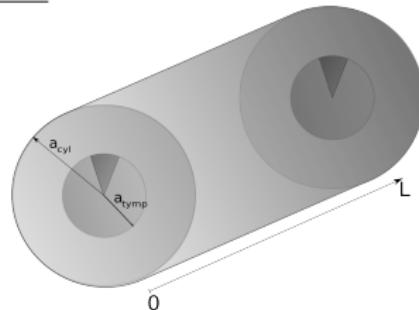
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$a_{\text{tym}}$ ,  $V_{\text{cyl}}$  fixed.

$$a_{\text{cyl}} = \sqrt{V_{\text{cyl}}/\pi L}$$

# Cavity Pressure

## 3D Wave Equation

$$\frac{1}{c^2} \partial_t^2 p(x, r, \phi, t) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p(x, r, \phi, t)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p(x, r, \phi, t)}{\partial \phi^2} + \frac{\partial p(x, r, \phi, t)}{\partial x^2} \quad (1)$$

To be solved using the separation ansatz

$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}.$$

# Separated Equations

$x$ - and  $\phi$ - directions

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x} \quad (2)$$

$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi} \quad (3)$$

$r$ -direction, Bessel functions

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial g(r)}{\partial r} \right) + \left[ \nu^2 - \frac{q^2}{r^2} \right] g(r) = 0 \longrightarrow g(r) = J_q(\nu r) \quad (4)$$

where,  $\nu^2 = k^2 - \zeta^2$

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## Boundary Conditions - $\phi$

Smoothness and Continuity in  $\phi$ .

$$\phi \rightarrow h(0) = h(2\pi) \quad \text{and} \quad h'(0) = h'(2\pi)$$

$$\Rightarrow h(\phi) = \cos q\phi, \quad q = 0, 1, 2, \dots \quad (5)$$

## Boundary Conditions - $r$

Impenetrable boundary at  $r = a_{\text{cyl}}$ , i.e. normal derivative vanishes

$$-j\rho\omega \mathbf{v} = \mathbf{n} \cdot \nabla p(x, r, \phi; t) \Big|_{r=a_{\text{cyl}}} \equiv \frac{\partial g}{\partial r} \Bigg|_{r=a_{\text{cyl}}} = 0 \quad (6)$$

$$\Rightarrow g(r) = J_q(\nu_{\text{qs}} r / a_{\text{cyl}}) \quad (7)$$

### Bessel Prime Zeros

- ▶  $\nu_{\text{qs}}$  - zeros of  $J'_q$ ,  $s = 0, 1, 2, \dots$
- ▶  $\nu_{00}=0$

# General Solution

## Pressure Modes

$$p(x, r, \phi, t) = \sum_{q=0, s=0}^{\infty} \left[ A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] p_{qs}(r, \phi) e^{j\omega t} \quad (8)$$

$$p_{qs}(r, \phi) = \cos q\phi J_q(\nu_{qs} r/a_{\text{cyl}}) \quad (9)$$

$$\text{where, } \zeta_{qs} = \sqrt{k^2 - \nu_{qs}^2/a_{\text{cyl}}^2}$$

## Plane Wave Mode

$$p_{\text{pw}}(x, r, \phi; t) = \left[ A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t} \quad (10)$$

# General Solution

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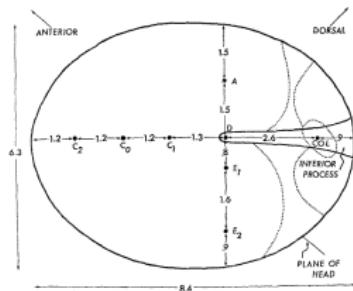
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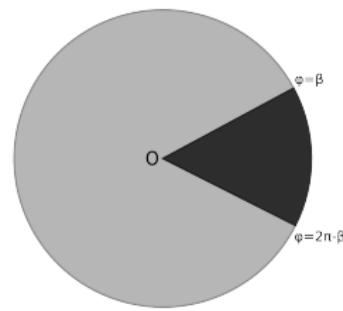
# Eardrum

Sketch of a Tokay eardrum as seen from the outside (G.A. Manley, 1972).



COL - approximate position opposite the extracolumella insertion.

The ICE Eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ( $r = a_{\text{tym}} \text{ and } \phi = \beta, 2\pi - \beta$ )

# Membrane Vibrations

## Membrane EOM

$$-\partial_t^2 u(r, \phi; t) - 2\alpha \partial_t u(r, \phi; t) + c_M^2 \Delta_{(2)} u(r, \phi; t) = \frac{1}{\rho_m d} \Psi(r, \phi; t) \quad (11)$$

## Membrane parameters

$\alpha$  - damping coefficient,  $c_M^2$  - propagation velocity

$\rho_m$  - density,  $d$  - thickness.

# Free-Undamped Membrane, $\alpha \rightarrow 0, \Psi \rightarrow 0$

## Separation Ansatz

$$u(r, \phi; t) = f(r)g(\phi)h(t) \quad (12)$$

## Separated Equations

$$\frac{d^2g(\phi)}{d\phi^2} + \kappa^2 g(\phi) = 0 \quad (13)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f(r)}{\partial r} \right) + \left[ \mu^2 - \frac{\kappa^2}{r^2} \right] f(r) = 0 \quad (14)$$

$$\frac{d^2h(t)}{dt^2} + c_M^2 \mu^2 h(t) = 0 \quad (15)$$

## Boundary Conditions

$\phi$ -direction:  $u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$

$$\Rightarrow g(\phi) = \sin \kappa(\phi - \beta) \quad (16)$$

where,  $\kappa = \frac{m\pi}{2(\pi - \beta)}$ ,  $m = 1, 2, 3, \dots$

$r$ -direction:  $u(a_{\text{tym}} \cos \theta, \phi; t) = 0$

$$\Rightarrow f(r) = J_\kappa(\mu_{mn} r / a_{\text{tym}}) \quad (17)$$

where,  $\mu_{mn}$  is the  $n^{\text{th}}$  zero of  $J_\kappa$

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## Free eigenmodes

$$u_{mn}(r, \phi) = \sin \kappa(\phi - \beta) J_\kappa(\mu_{mn} r) \quad (18)$$

$$u_{\text{free}}(r, \phi; t) = \sum_{m=0, n=1}^{\infty} C_{mn} u_{mn}(r, \phi) e^{j\omega_{mn} t} \quad (19)$$

where,  $\omega_{mn} = c_M \mu_{mn}$

## Damped membrane

$$\tilde{u}_{\text{free}}(r, \phi; t) = \sum_{m=0, n=1}^{\infty} \tilde{C}_{mn} u_{mn}(r, \phi) e^{j\omega_{mn} t - \alpha t} \quad (20)$$

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Forced Vibrations:  $\Psi = p e^{j\omega t}$

### Steady State Solution

$$u_{ss}(r, \phi; t) =: \sum_{m=0, n=1}^{\infty} C_{mn} u_{mn}(r, \phi) e^{j\omega t} \quad (21)$$

Substitute  $u_{ss}$  in Membrane EOM.

$$C_{mn} = \frac{p \int dS u_{mn}}{\Omega_{mn} \int dS u_{mn}^2} \quad (22)$$

$$\Omega_{mn} = \rho M d [(\omega^2 - \omega_{mn}^2) - 2j\alpha\omega]$$

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## Forced Vibrations contd.

### Transient Solution

Same as the solution for a free damped membrane

$$u_t(r, \phi; t) = \sum_{m=0, n=1}^{\infty} \tilde{C}_{mn} u_{mn}(r, \phi) e^{j\omega_{mn}t - \alpha t} \quad (23)$$

$\tilde{C}_{mn}$  determined from the membrane displacement at  $t = 0$ .  
 $u_t \rightarrow 0$  exponentially as  $t \rightarrow \infty$ .

### Steady State Approximation

$$u \approx u_{ss}.$$

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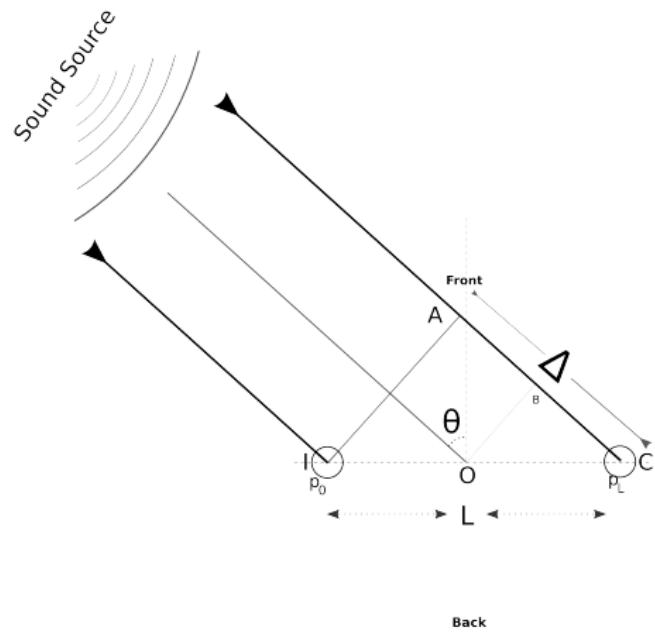
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# Acoustic Head Model

- ▶ **I** - Ipsilateral ear, **C** - Contralateral ear.

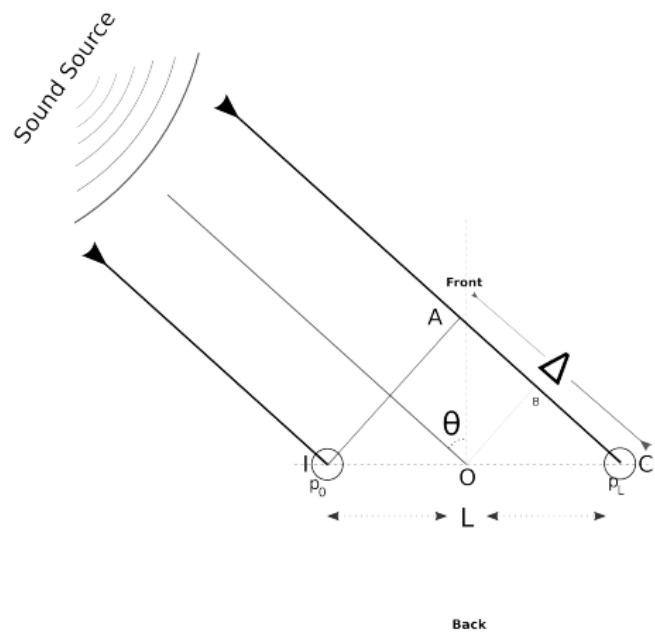
$p_0, p_L$  - sound pressure on eardrums,  $\theta$  - sound source direction.

- ▶ Sound source “far away”.
- ▶ No appreciable amplitude difference,  $|p_0| = |p_L|$ .
- ▶ Phase difference between sound at both ears -  $\Delta = kL \sin \theta$ .
- ▶  $p_0 = p e^{j\omega t - .5kL \sin \theta}$
- ▶  $p_L = p e^{j\omega t + .5kL \sin \theta}$



# Acoustic Head Model

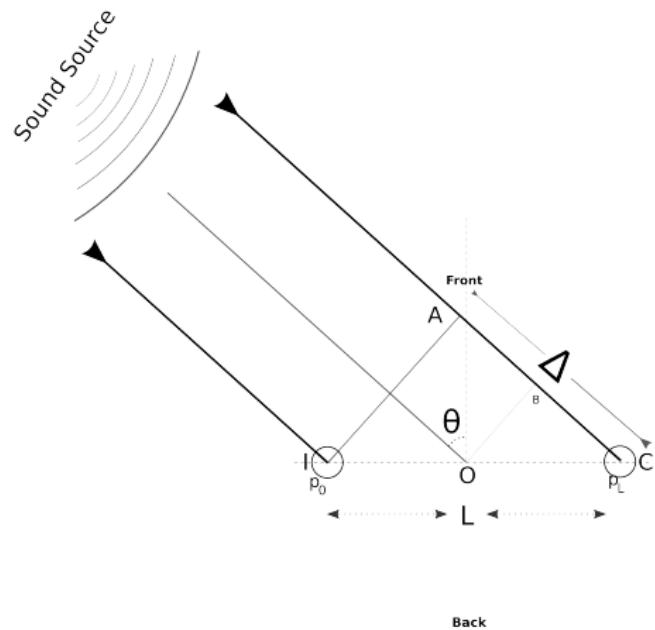
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## Acoustic Head Model

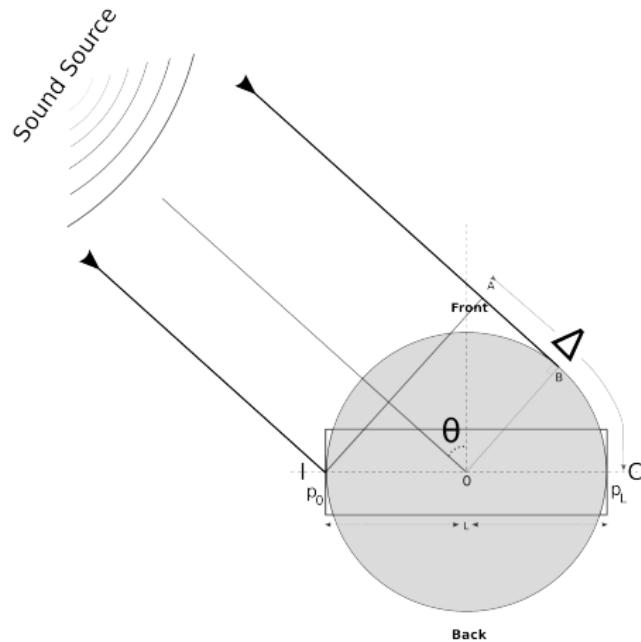
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- ▶  $p_L = p e^{j\omega t + .5kL \sin \theta}$



## Acoustic Head Model contd.

- ▶  $|p_0| = |p_L|$ .
- ▶ Increased phase difference due to diffraction -  $\Delta = 1.5kL \sin \theta$ .
- ▶  $p_0 = p e^{j\omega t - .75kL \sin \theta}$   
 $p_L = p e^{j\omega t + .75kL \sin \theta}$



# Coupled Membranes

$$u_{0/L} = \sum_{m=0,n=1}^{\infty} C_{mn}^{0/L} u_{mn}(r, \phi) e^{j\omega t} \quad (24)$$

## Membrane Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{mn} C_{mn}^0 u_{mn}(r, \phi) e^{j\omega t} = p_0 e^{j\omega t} - p(0, r, \phi; t) \quad (25)$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{mn} C_{mn}^L u_{mn}(r, \phi) e^{j\omega t} = p_L e^{j\omega t} - p(L, r, \phi; t) \quad (26)$$

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## “Surface” Velocity

$$U_{0/L} = \begin{cases} u_{0/L}, & 0 < r < a_{\text{tym}} \text{ and } \beta < \phi < 2\pi - \beta \\ 0, & \text{otherwise} \end{cases} . \quad (27)$$

## Velocity in $x$ -direction

$$v_x = - \sum_{q=0, s=0}^{\infty} \frac{\zeta_{qs}}{\rho\omega} \left( A_{qs} e^{j\zeta_{qs}x} - B_{qs} e^{-j\zeta_{qs}x} \right) p_{qs}(r, \phi) e^{j\omega t} \quad (28)$$

# Boundary Conditions

Exact

$$U_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t) \quad (29)$$

$$U_L = \frac{1}{j\omega} v_x(L, r, \phi; t) \quad (30)$$

Approximate

$$U_{0/L} \approx S^{0/L}(t) =: \int dS U_{0/L} \quad (31)$$

# Boundary Conditions

Exact

$$U_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t) \quad (29)$$

$$U_L = \frac{1}{j\omega} v_x(L, r, \phi; t) \quad (30)$$

Approximate

$$U_{0/L} \approx S^{0/L}(t) =: \int dS U_{0/L} \quad (31)$$

## Boundary Conditions

Higher pressure modes disappear, i.e.

$$p = [A_{00}e^{jkx} + B_{00}e^{-jkx}] e^{j\omega t}$$

$$A_{00} = -\frac{\rho\omega^2}{2k \sin kL} (S^0 e^{-jkL} + S^L) \quad (32)$$

$$B_{00} = -\frac{\rho\omega^2}{2k \sin kL} (S^0 e^{jkL} + S^L) \quad (33)$$

## Coupled Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{mn} C_{mn}^0 u_{mn}(r, \phi) = p_0 + \frac{\rho\omega^2}{k} \left( \frac{S^0}{\tan kL} + \frac{S^L}{\sin kL} \right) \quad (34)$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{mn} C_{mn}^L u_{mn}(r, \phi) = p_L + \frac{\rho\omega^2}{k} \left( \frac{S^0}{\sin kL} + \frac{S^L}{\tan kL} \right) \quad (35)$$

## Decoupling

Decouple by taking the sum and difference of the above equations.

## Coupled Equations

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## Decoupled Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{mn} C_{mn}^+ u_{mn}(r, \phi) = p_+ + \frac{\rho\omega^2}{k} S^+ \cot \frac{kL}{2} \quad (36)$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{mn} C_{mn}^- u_{mn}(r, \phi) = p_- - \frac{\rho\omega^2}{k} S^- \tan \frac{kL}{2} \quad (37)$$

$$C_{mn}^+ = \left[ p_+ + \frac{\rho\omega^2}{k} S^+ \cot \frac{kL}{2} \right] \frac{\int dS u_{mn}}{\Omega_{mn} \int dS u_{mn}^2} \quad (38)$$

$$C_{mn}^- = \left[ p_- - \frac{\rho\omega^2}{k} S^- \tan \frac{kL}{2} \right] \frac{\int dS u_{mn}}{\Omega_{mn} \int dS u_{mn}^2} \quad (39)$$

## Decoupled Equations contd.

$$S^+ = \frac{p_L + p_0}{\Lambda + \Gamma_+} \quad S^- = \frac{p_L - p_0}{\Lambda + \Gamma_-} \quad (40)$$

$$\Gamma_+ = -\frac{\rho\omega^2}{k} \cot \frac{kL}{2}, \quad \Gamma_- = \frac{\rho\omega^2}{k} \tan \frac{kL}{2} \quad (41)$$

$$\frac{1}{\Lambda} = \frac{1}{\pi a_{\text{cyl}}^2} \sum_{m=0,n=1}^{\infty} \frac{\left( \int dS u_{mn} \right)^2}{\Omega_{mn} \int dS u_{mn}^2} \quad (42)$$

# Final Expressions

## Membrane Displacement

$$S_0(t) = G_{ipsi}^s p_0 + G_{contra}^s p_L \quad (43)$$

$$S_L(t) = G_{contra}^s p_0 + G_{ipsi}^s p_L \quad (44)$$

$$G_{ipsi}^s = \left( \frac{1}{\Lambda + \Gamma_+} + \frac{1}{\Lambda + \Gamma_-} \right) / 2 \quad (45)$$

$$G_{contra}^s = \left( \frac{1}{\Lambda + \Gamma_+} - \frac{1}{\Lambda + \Gamma_-} \right) / 2 \quad (46)$$

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Internal Level Difference

Internal Time Difference

## Conclusion

# Tokay Gecko

$$L=22 \text{ mm} \quad a_{\text{tym}}=2.6 \text{ mm}$$

$$Q = 1.33 \quad \rho_m=1 \text{ mg/mm}^3$$

$$d=10 \mu\text{m} \quad V_{\text{cav}}=3.5 \text{ ml}$$

$$\beta=\pi/25 \quad a_{\text{cyl}} \approx 6.6 \text{ mm}$$

$$f_0 = 1.05 \text{ kHz}$$

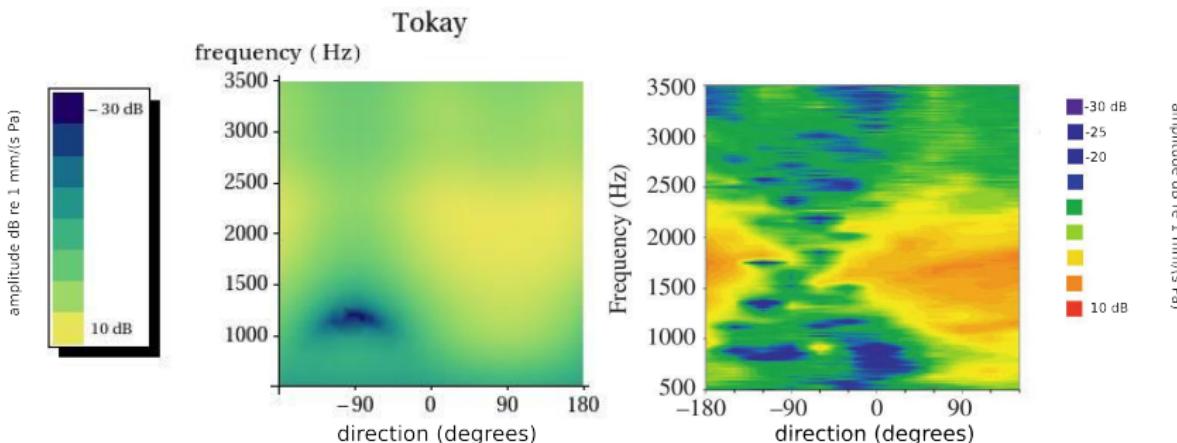
$$f_0 = \omega_{01}/2\pi$$



Tokay gecko. Left: Head illuminated from the opposite side<sup>a</sup>. Right: Mouth casts.

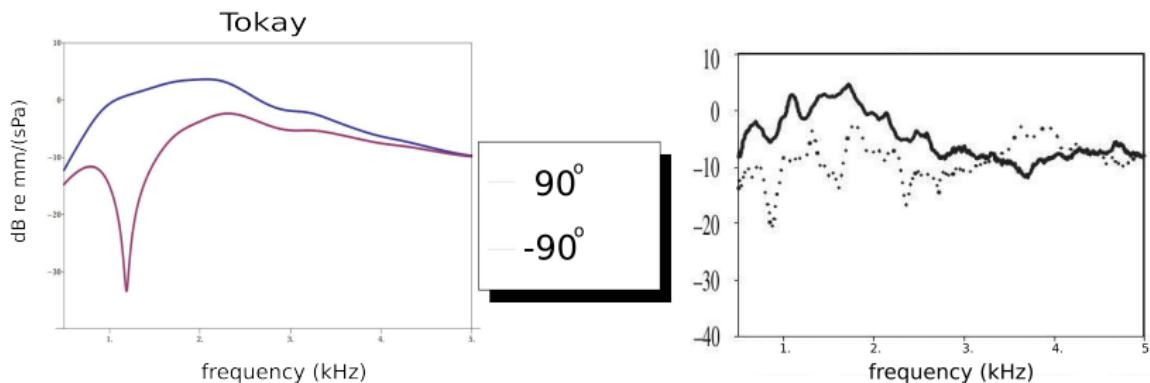
<sup>a</sup>Courtesy J.C. Dalsgaard (Syddansk Universitet)

# Density Plot



- ▶ Plot of vibration amplitude w.r.t direction & frequency (Left: Calculated, Right: Experimental (Christensen-Dalsgaard et al, 2005)).
- ▶  $|p_0| = |p_L| = 1 \text{ Pa}$ .

# Frequency Dependence

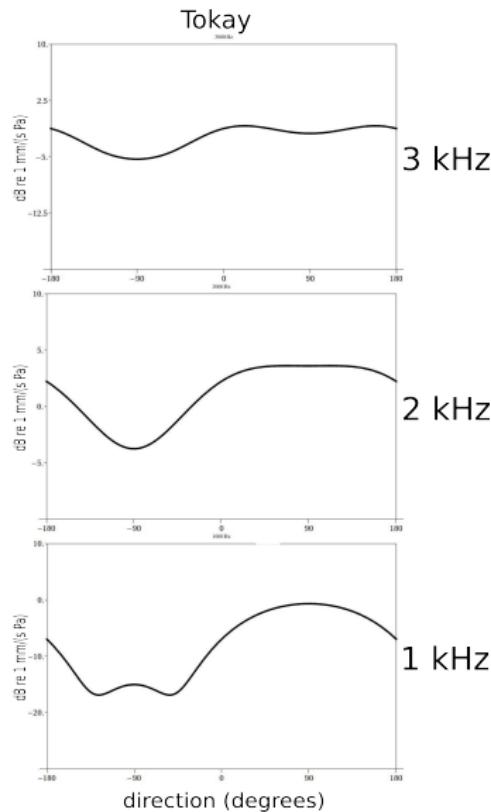


- ▶ Frequency dependence of  $20\log_{10} \left| \dot{S}^0 / (\pi a_{cyl}^2) \right|$  for  $\theta = 90^\circ$ . (Left: Calculated, Right: Experimental)
- ▶  $|p_0| = |p_L| = 1 \text{ Pa}$ .
- ▶ Ipsilateral response > Contralateral response.

## Vibration Amplitude

## Direction Dependence.

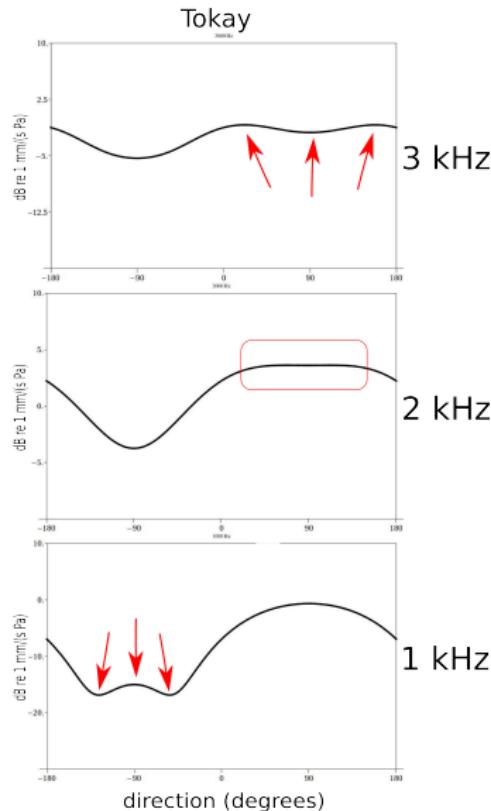
- ▶ Inputs to ears
    - ▶ Negligible level (amplitude) difference
    - ▶ Small time (phase) difference
  - ▶ Response is highly directional.
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- ▶ Independent vibration amplitudes not enough.
  - ▶ Localization requires using information from both ears.



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## Hearing Cues

Internal Level Difference (iLD) -

$$iLD := 20 \log_{10} \left| \frac{\dot{S}^0}{\dot{S}^L} \right| \quad (47)$$

Internal Time Difference (iTD) -

$$iTD := \text{Arg} \left( \frac{\dot{S}^0}{\dot{S}^L} \right) / \omega \quad (48)$$

### Requirements

1. Both increase with the adjacency of the sound source. Max at  $\theta = 90^\circ$  and min at  $\theta = -90^\circ$ .
2. Both vanish at  $\theta = 0^\circ, \pm 180^\circ$ .
3. iTD  $\approx$  constant for a given frequency range. Advantageous for neuronal processing.

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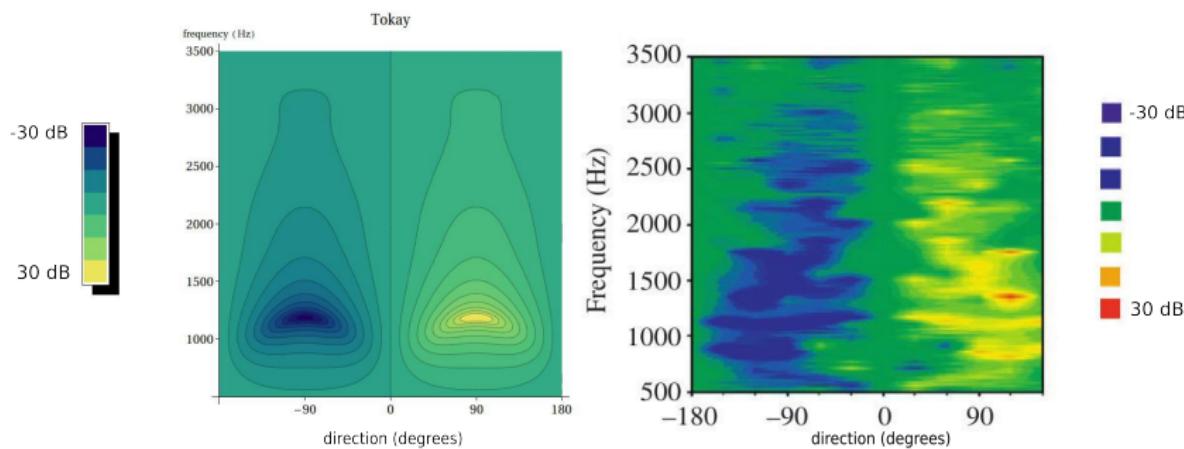
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$$iTD := \text{Arg} \left( \frac{\dot{S}^0}{\dot{S}^L} \right) / \omega \quad (48)$$

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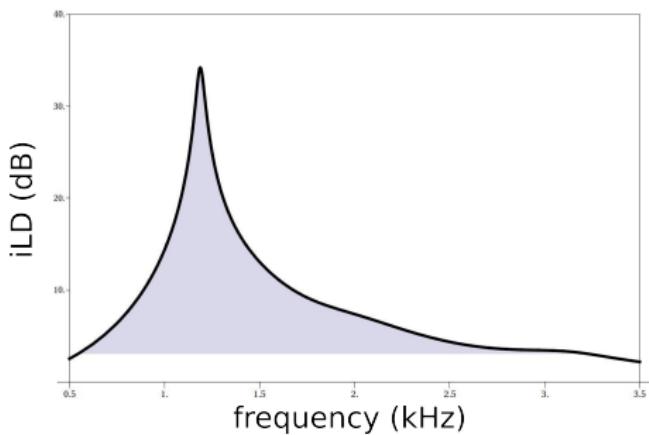
# iLD Density Plot



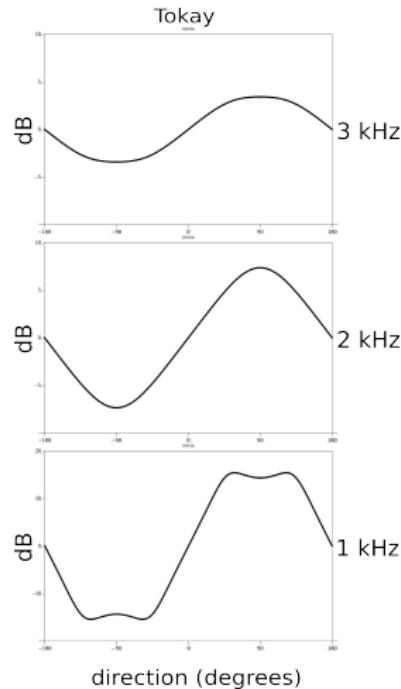
- ▶ Plot of iLD, against frequency and direction.
- ▶ Left: Calculated, Right: Experimental

## Directional Cues

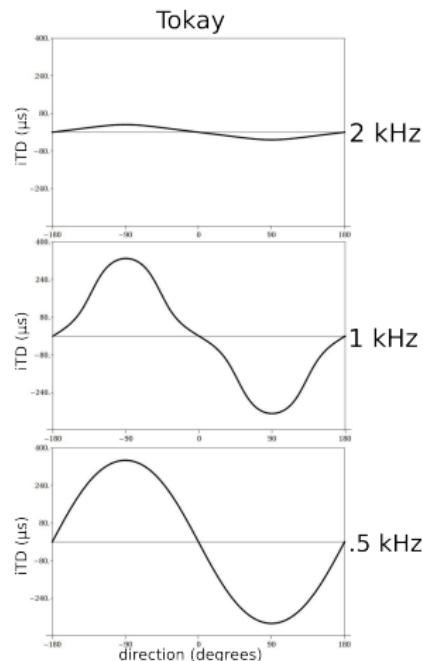
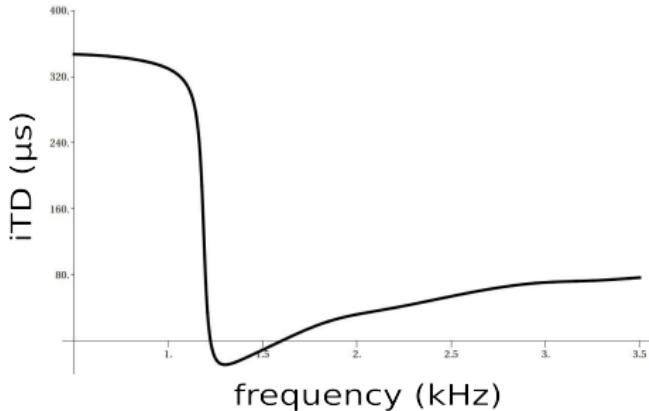
# iLD Frequency/Direction Dependence



- ▶ iLD is a better cue at higher frequencies.
- ▶ Peak response at  $\sim f_0$ .



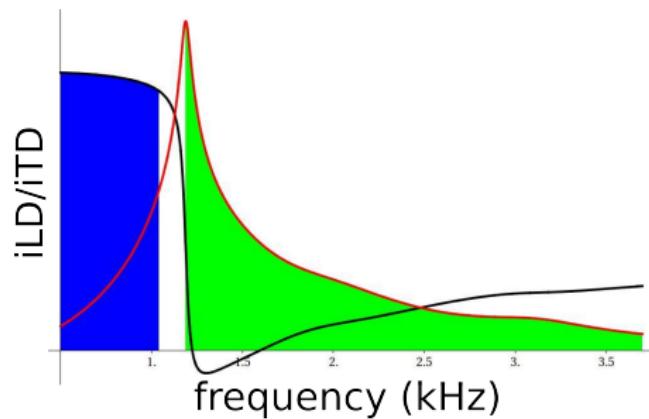
# iTD Frequency/Direction Dependence



- ▶ iTD is a better cue at lower frequencies.
- ▶ constant upto  $\sim f_0$ .
- ▶  $iTD \approx 3 \times ITD$

# iTD/iLD Frequency Regimes

- ▶ The frequency for transition from iTD to iLD based localization is determined by  $f_0$ .
- ▶ Possibility of a frequency regime where both cues can simultaneously be used.



# Outline for Section 4

## Introduction

Auditory Systems

Hearing Cues

## The Model

Mouth Cavity

Pressure Derivation

Eardrum

Model

Membrane Vibrations

Acoustic Head Model

## Coupled Membranes

Ansatz

Boundary Conditions

Solution

## Evaluation

Parameters

Vibration Amplitude

Directional Cues

Internal Level Difference

Internal Time Difference

## Conclusion

# Conclusion

- ▶ Single model describes both low and high frequency behaviour.

# Thank You

