

Mechanical Processing in Internally Coupled Ears

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TMP Thesis Defence
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T_{MP}
Theoretical and Mathematical Physics

Auditory Systems



Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.

- eg. Mammals.



Coupled Ears

Wide eustachian tubes open into the mouth cavity.

Eardrums vibrations influence each other.

- eg. Lizards, birds, frogs.

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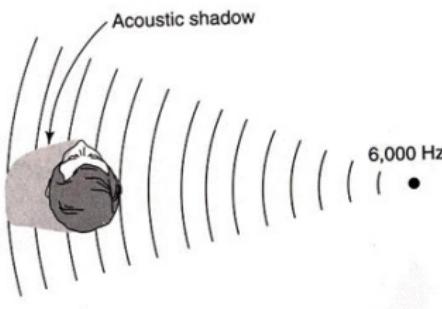
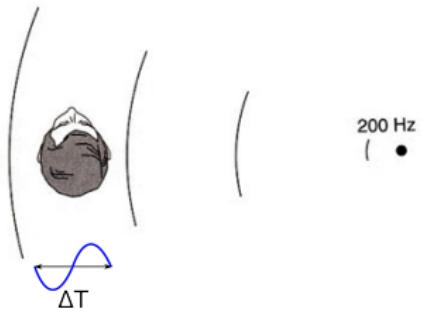
Eardrums vibrations influence each other.

- eg. **Lizards**, birds, frogs.

Outline

- Introduction
 - Auditory Systems
 - Sound Localization
- The Model
 - Mouth Cavity
 - Cavity Pressure
 - Eardrum
 - Model
 - Membrane Vibrations
 - Acoustic Head Model
- Coupled Membranes
 - Ansatz
 - Boundary Conditions
 - Solution
- Evaluation
 - Parameters
 - Vibration Amplitude
 - Directional Cues
 - Internal Level Difference
 - Internal Time Difference
- Conclusion

Localization - Independent Ears



Interaural Time Difference

- ▶ Phase difference between the (pressure) inputs to the ears.
- ▶ **ITD**
- ▶ Used when $\lambda \gg$ head size.

Interaural Level Difference

- ▶ Amplitude difference between the inputs.
- ▶ **ILD**
- ▶ Used when $\lambda \sim$ head size.

Localization - Coupled Ears

Problems

- ▶ Small heads.
- ▶ Lower “processing power”.
- ▶ $iTD \neq 0$, “small”.
- ▶ $iLD \approx 0$.

Internal Time Difference

Phase difference between eardrums.

- ▶ iTD
- ▶ $iTD > ITD$.

Internal Level Difference

Amplitude difference between the eardrums.

- ▶ iLD
- ▶ $iLD > 0$.

Localization - Coupled Ears

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Internal Time Difference

Phase difference between **eardrums**.

- ▶ **iTD**
- ▶ $\text{iTD} > \text{ITD}$.

Internal Level Difference

Amplitude difference between the **eardrums**.

- ▶ **iLD**
- ▶ $\text{iLD} > 0$.

ICE Model

I**nternally Coupled **E**ars**

Aim:

- ▶ Accurately reproduce the direction and frequency dependence.
- ▶ Analytically tractable.

Solution:

- ▶ Circular eardrums.
- ▶ Cylindrical mouth cavity.

ICE Model

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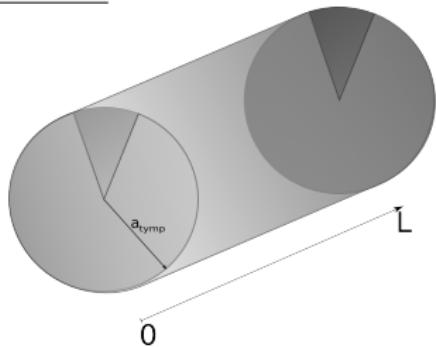
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Mouth Cavity

Mouth Cavity

Previous Model^a

a_{tym} fixed.

$$V_{\text{cyl}} = \pi a_{\text{tym}}^2 L$$

Our Model

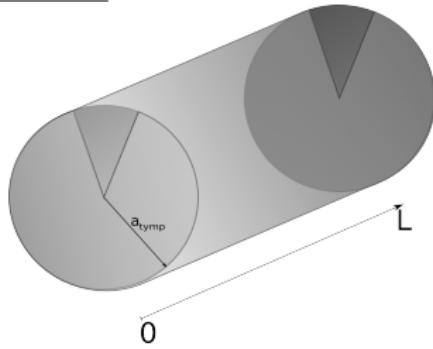
$a_{\text{tym}}, V_{\text{cyl}}$ fixed.

$$a_{\text{cyl}} = \sqrt{V_{\text{cyl}} / \pi L}$$

^aVossen et al (2010)

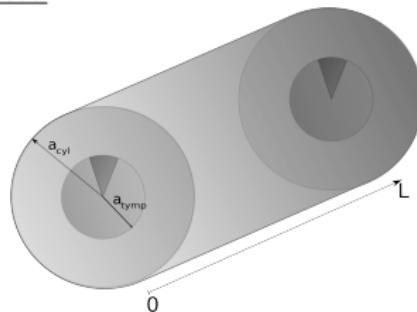
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Cavity Pressure - $p(x, r, \phi; t)$

3D Wave Equation

$$\frac{1}{c^2} \partial_t^2 p = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p}{\partial \phi^2} + \frac{\partial p}{\partial x^2}$$

Flow Velocity:

$$\dot{\vec{v}}(x, r, \phi; t) = \frac{1}{\rho} \nabla p$$

Separation Ansatz

$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}$$

Separated Equations

x - and ϕ - directions

$$f(x) = Ae^{j\zeta x} + Be^{-j\zeta x}$$

$$h(\phi) = Ce^{jq\phi} + De^{-jq\phi}$$

$$h(\phi) \xrightarrow{\text{cont., smooth}} \cos q\phi$$

$$q = 0, 1, 2, 3, \dots$$

r -direction, Bessel functions

$$g(r) = J_q(\nu r)$$

$$\nu^2 = (\omega/c)^2 - \zeta^2$$

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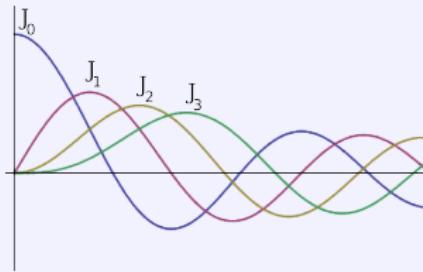
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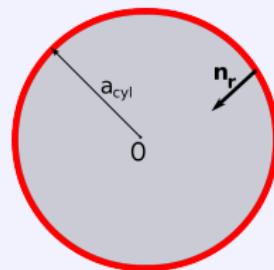


Boundary Conditions - r

Impenetrable boundary at $r = a_{\text{cyl}}$

$$\mathbf{n}_r \cdot \vec{v} \Big|_{r=a_{\text{cyl}}} \equiv \frac{\partial g}{\partial r} \Big|_{r=a_{\text{cyl}}} = 0$$

$$\Rightarrow g(r) = J_q(\nu_{\text{qs}} r / a_{\text{cyl}})$$



Bessel Prime Zeros

ν_{qs} - extrema of J_q .

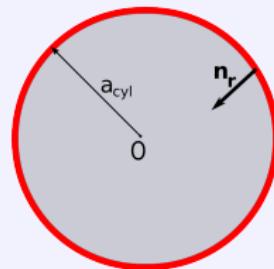
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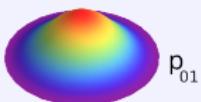


General Solution

Pressure Modes



$$p_{qs}(r, \phi) = \cos q\phi J_q(\nu_{qs} r / a_{cyl})$$



$$f_{qs}(x) = A_{qs} e^{j\zeta_{qs} x} + B_{qs} e^{-j\zeta_{qs} x}$$

$$\int dS p_{qs} \neq 0 \text{ iff } q = 0, s = 0$$

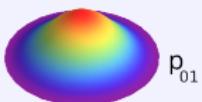
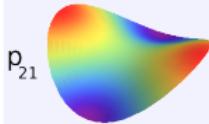
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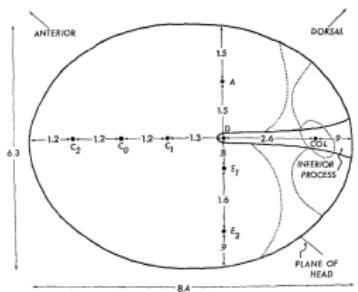
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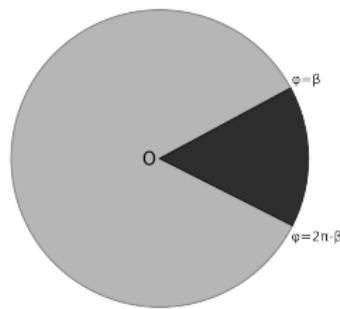
Eardrum

Sketch of a Tokay eardrum as seen from the outside (G.A. Manley, 1972).



COL - approximate position opposite the extracolumella insertion.

The ICE Eardrum.



Extracolumella (dark) - **rigid, stationary.**

Membrane - **linear elastic.**

Clamped at boundaries
 $(r = a_{\text{typ}}, \phi = \beta, 2\pi - \beta)$

Membrane Vibrations

Membrane Displacement - $u(r, \phi; t)$

$$-\partial_t^2 u - 2\alpha \partial_t u + c_m^2 \Delta_{(2)} u = \frac{1}{\rho_m d} \Psi(r, \phi; t)$$

Membrane Parameters

α - damping coefficient, c_m - propagation velocity

ρ_m - density, d - thickness.

Free-Undamped Membrane, $\alpha \rightarrow 0, \Psi \rightarrow 0$

Separation Ansatz

$$u(r, \phi; t) = f(r)g(\phi)h(t)$$

Boundary Conditions

ϕ -direction: $u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$

$$\Rightarrow g(\phi) = \sin \kappa(\phi - \beta)$$

$$\kappa = \frac{m\pi}{2(\pi - \beta)}, m = 1, 2, \dots$$

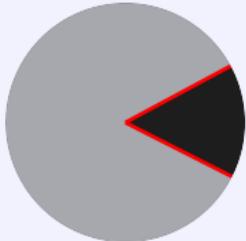
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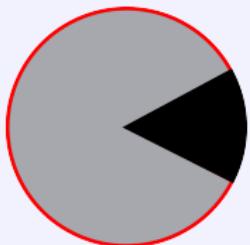


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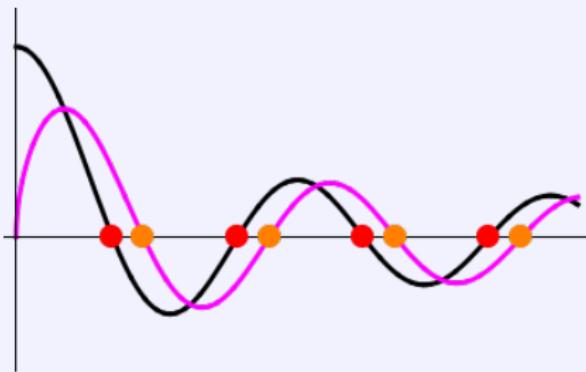
Boundary Conditions contd.

r -direction: $u(a_{\text{tym}} \phi; t) = 0$

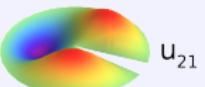
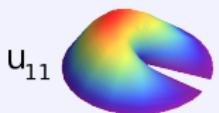


$$\Rightarrow f(r) = J_\kappa(\mu_{mn} r / a_{\text{tym}})$$

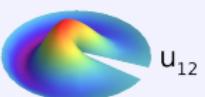
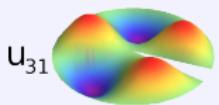
μ_{mn} : n^{th} zero of J_κ



Free Membrane Eigenmodes - $u_{mn}(r, \phi)$



$$\omega_{mn} = c_m \mu_{mn}$$



$$h(t) = \exp(j\omega_{mn}t)$$

$$\int dS u_{kl} u_{mn} = \delta_{km} \delta_{ln}$$

Damped: $\alpha \neq 0$

$$h(t) = \exp(-\alpha t + j\tilde{\omega}_{mn}t)$$

$$\tilde{\omega}_{mn} = \sqrt{\omega_{mn}^2 + \alpha^2}$$

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Forced Vibrations: $\Psi = pe^{j\omega t}$, $\alpha \neq 0$

Full Solution

$$u(r, \phi; t) = u_{ss}(r, \phi; t) + u_t(r, \phi; t)$$

Steady State Solution

From Membrane equation:

$$u_{ss} = \sum C_{mn} u_{mn}(r, \phi) e^{j\omega t}$$

$$C_{mn} = \frac{p \int dS u_{mn}}{\Omega_{mn} \int dS u_{mn}^2}$$

$$\Omega_{mn} = \rho_M d [\omega^2 - 2j\alpha\omega - \omega_{mn}^2]$$

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Forced Vibrations contd.

Transient Solution

≡ free damped membrane

$$u_t = \sum \tilde{C}_{mn} u_{mn}(r, \phi) e^{j\tilde{\omega}_{mn}t - \alpha t}$$

$$u(t=0) = 0 \Rightarrow \tilde{C}_{mn} = -C_{mn}.$$

Steady State Approximation

$u_t \rightarrow 0$ exponentially as $t \rightarrow \infty$.

$$\textcolor{red}{u} \approx u_{ss}$$

Forced Vibrations contd.

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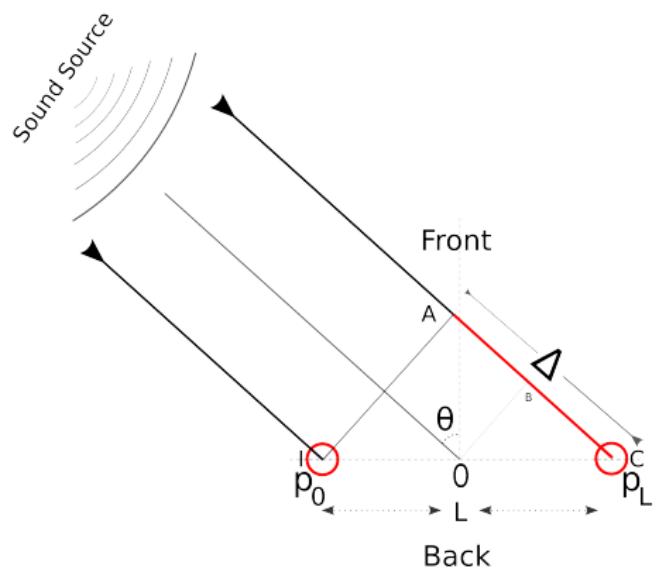
$$u \approx u_{ss}$$

Acoustic Head Model - previous

- **I** - Ipsilateral, **C** - Contralateral.
- p_0, p_L - Sound pressure.
- θ - Source direction.
- Sound source “far away”.

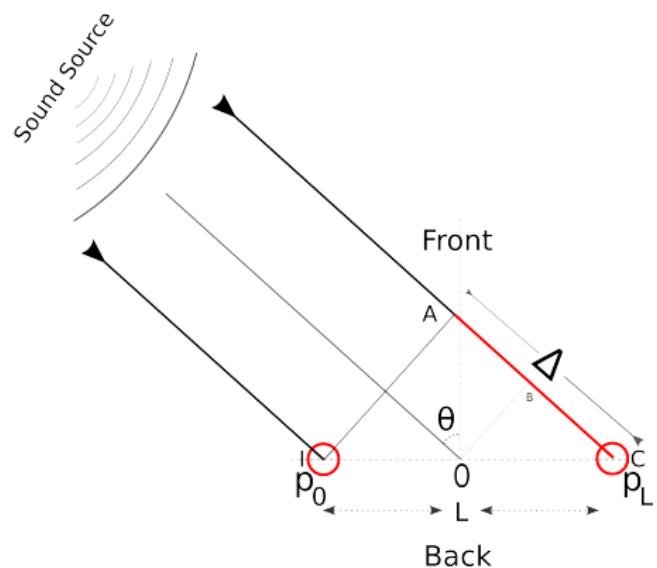
- ▶ $|p_0| = |p_L|$.
- ▶ $\Delta = kL \sin \theta$.

- $p_0 = p \exp(-j\Delta/2)$
- $p_L = p \exp(j\Delta/2)$
- ILD=0,
- $ITD := \frac{1}{\omega} \text{Arg} \left(\frac{p^0}{p^L} \right) = L/c$



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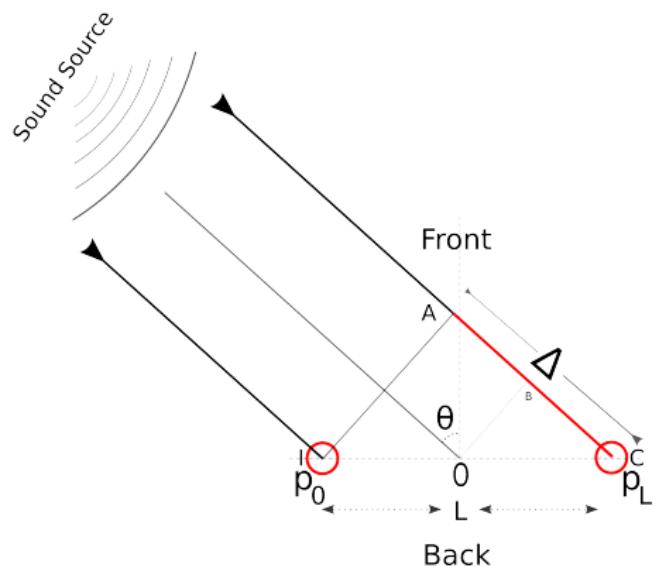
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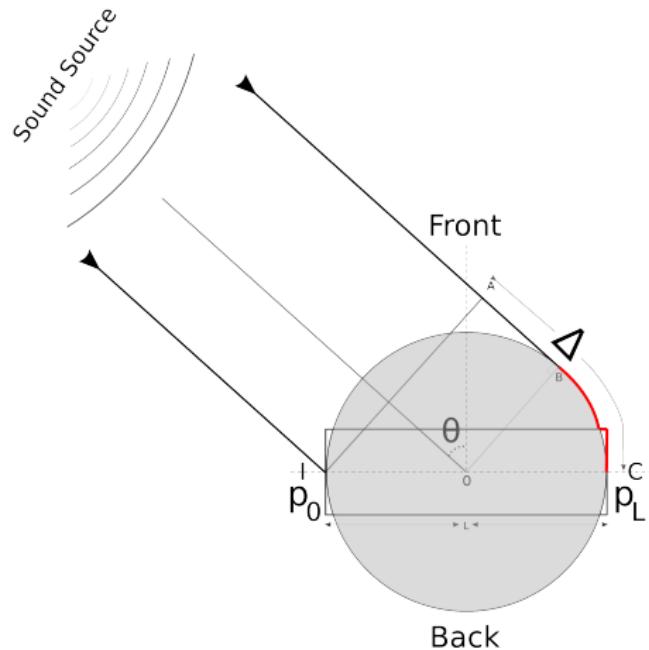
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Acoustic Head Model - new

- $|p_0| = |p_L|.$
- ▶ $\Delta = \frac{3}{2}kL \sin \theta.$
- $p_0 = p \exp(-j\Delta/2)$
 $p_L = p \exp(j\Delta/2)$
- ILD=0, ITD= $\frac{3}{2}L/c$



Coupled Membranes

$$u_{0/L} = \sum_{m=0,n=1}^{\infty} C_{mn}^{0/L} u_{mn}(r, \phi) e^{j\omega t}$$

Membrane Equations

$$-\partial_t^2 u_0 - 2\alpha \partial_t u_0 + c_m^2 \Delta_{(2)} u_0 = \frac{1}{\rho_m d} [p_0 e^{j\omega t} - p(0, r, \phi; t)]$$

$$-\partial_t^2 u_L - 2\alpha \partial_t u_L + c_m^2 \Delta_{(2)} u_L = \frac{1}{\rho_m d} [p_L e^{j\omega t} - p(L, r, \phi; t)]$$

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Boundary Conditions

Exact

$$u_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t)$$

$$u_L = \frac{1}{j\omega} v_x(L, r, \phi; t)$$



Approximate - $S^{0/L}(t) =: (\pi a_{\text{cyl}}^2)^{-1} \int dS u_{0/L}$

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Average Velocity

Higher pressure modes disappear, i.e.

$$p = [A_{00}e^{j kx} + B_{00}e^{-j kx}] e^{j \omega t}$$

$$\left(\Lambda - \frac{\rho \omega^2}{k \tan kL} \right) S^0 - \frac{\rho \omega^2}{k \sin kL} S^L = p_0$$

$$\left(\Lambda - \frac{\rho \omega^2}{k \tan kL} \right) S^L - \frac{\rho \omega^2}{k \sin kL} S^0 = p_L$$

$$\frac{1}{\Lambda} = \frac{1}{\pi a_{\text{cyl}}^2} \sum_{m=0, n=1}^{\infty} \frac{\left(\int dS u_{mn} \right)^2}{\Omega_{mn} \int dS u_{mn}^2}$$

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Final Expressions

Membrane Displacement

$$S_0(t) = G_{ippsi}^s p_0 + G_{cont}^s p_L$$

$$S_L(t) = G_{cont}^s p_0 + G_{ippsi}^s p_L$$

$$G_{ippsi}^s = \frac{1}{2} \left(\frac{1}{\Lambda + \Gamma_+} + \frac{1}{\Lambda + \Gamma_-} \right)$$

$$\Gamma_+ = -\frac{\rho\omega^2}{k} \cot \frac{kL}{2}$$

$$G_{cont}^s = \frac{1}{2} \left(\frac{1}{\Lambda + \Gamma_+} - \frac{1}{\Lambda + \Gamma_-} \right)$$

$$\Gamma_- = \frac{\rho\omega^2}{k} \tan \frac{kL}{2}$$

Tokay Gecko

$$L = 22 \text{ mm}$$

$$a_{\text{tym}} = 2.6 \text{ mm}$$

$$c_m = 5.41 \text{ ms}^{-1}$$

$$\rho_m = 1 \text{ mg/mm}^3$$

$$d = 10 \mu\text{m}$$

$$V_{\text{cav}} = 3.5 \text{ ml}$$

$$\beta = \pi/25$$

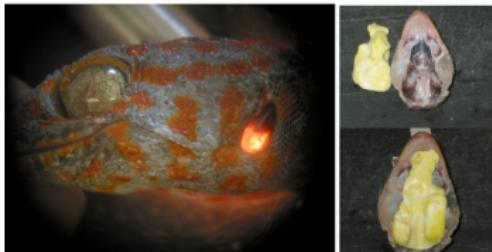
$$a_{\text{cyl}} \approx 6.6 \text{ mm}$$

$$\alpha \approx 2474 \text{ s}^{-1}$$

$$f_0 = c_m \mu_{01} / 2\pi.$$

$$\text{ITD} = 96.21 \mu\text{s.}$$

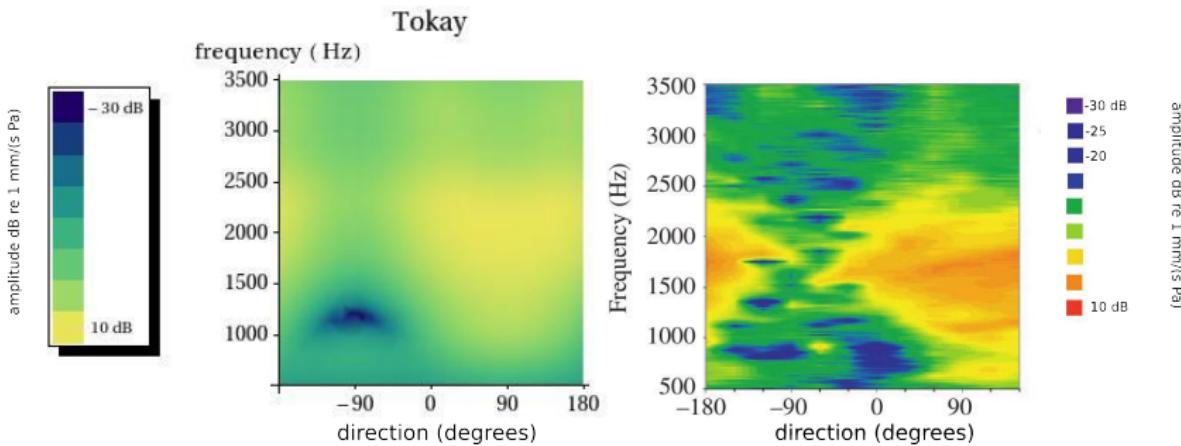
$\alpha \rightarrow$ steady state.



Tokay gecko. Left: Head illuminated from the opposite side^a. Right: Mouth casts.

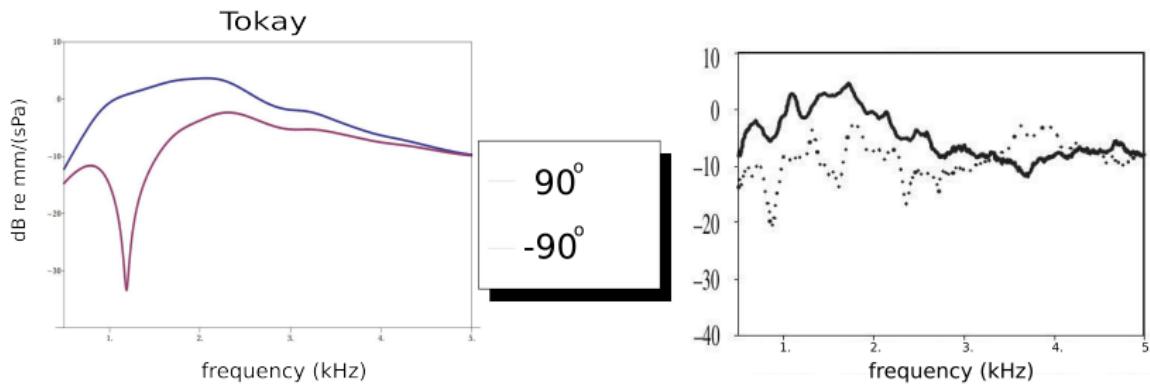
^aCourtesy J.C. Dalsgaard (Syddansk Universitet)

Density Plot



- ▶ Plot of vibration amplitude w.r.t direction & frequency (Left: Calculated, Right: Experimental (Christensen-Dalsgaard et al, 2005)).
- ▶ $|p_0| = |p_L| = 1 \text{ Pa}$.

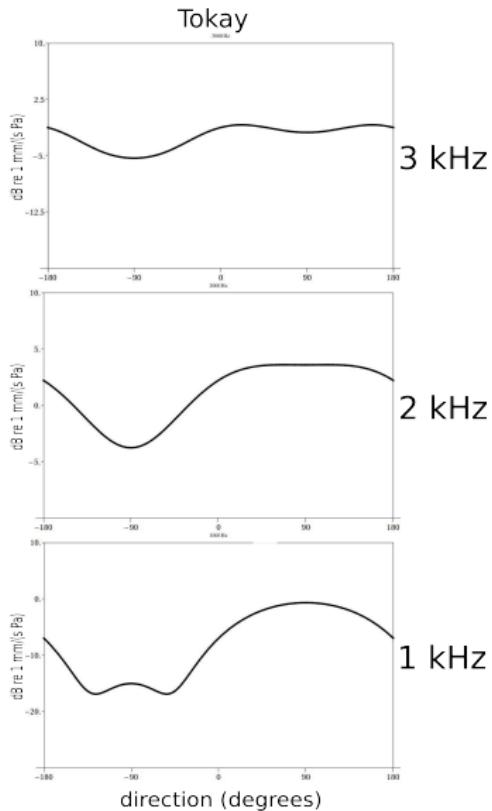
Frequency Dependence



- ▶ Frequency dependence of $20\log_{10} \left| \dot{S}^0 / \pi a_{cy1}^2 \right|$ for $\theta = 90^\circ$. (Left:Calculated, Right:Experimental)
- ▶ $|p_0| = |p_L| = 1 \text{ Pa}$.
- ▶ Ipsilateral response > Contralateral response.

Direction Dependence.

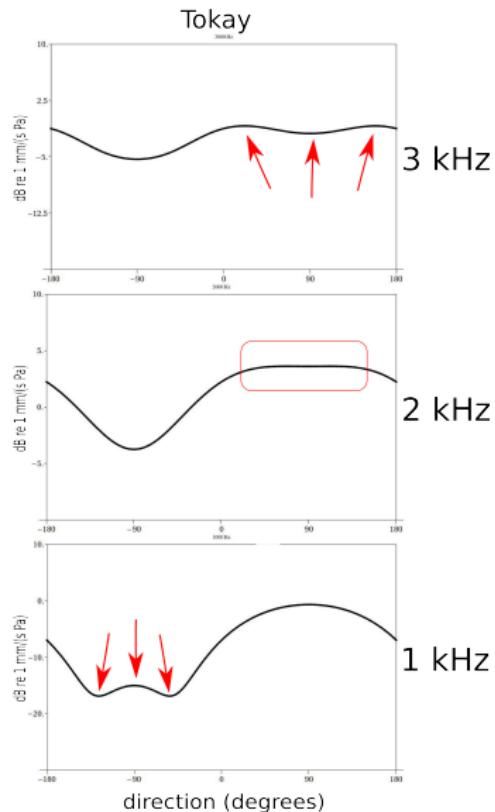
- ▶ Inputs to ears
 - ▶ Negligible level (amplitude) difference
 - ▶ Small time (phase) difference
 - ▶ Response is highly directional.
-
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Vibration Amplitude

Direction Dependence.

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Hearing Cues

Internal Level Difference (iLD) -

$$iLD := 20 \log_{10} \left| \frac{\dot{S}^0}{\dot{S}^L} \right|$$

Internal Time Difference (iTД) -

$$iTД := \text{Arg} \left(\frac{\dot{S}^0}{\dot{S}^L} \right) / \omega$$

Requirements

Both:

1. increase with the nearness of the sound source.
2. vanish at $\theta = 0^\circ, \pm 180^\circ$.

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~constant in a certain frequency range.

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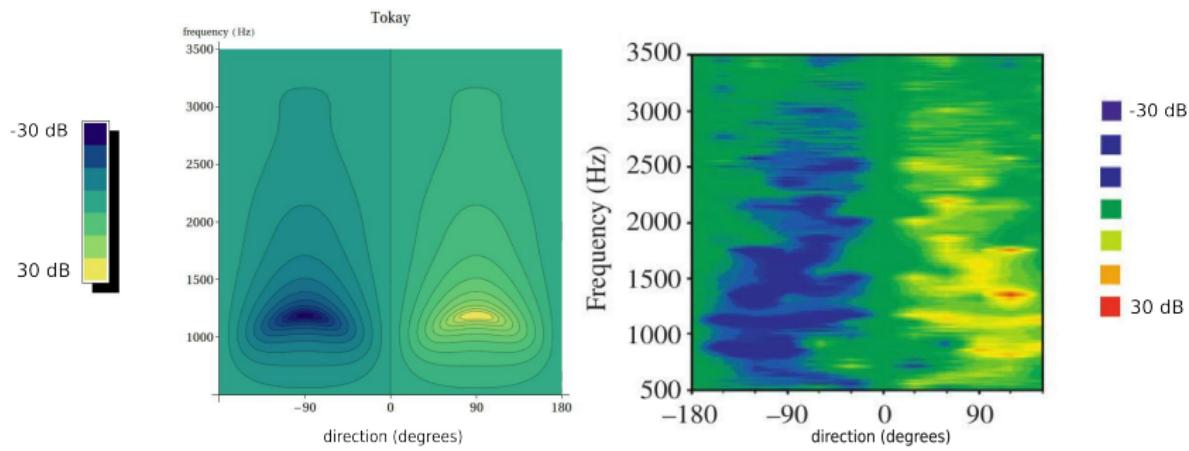
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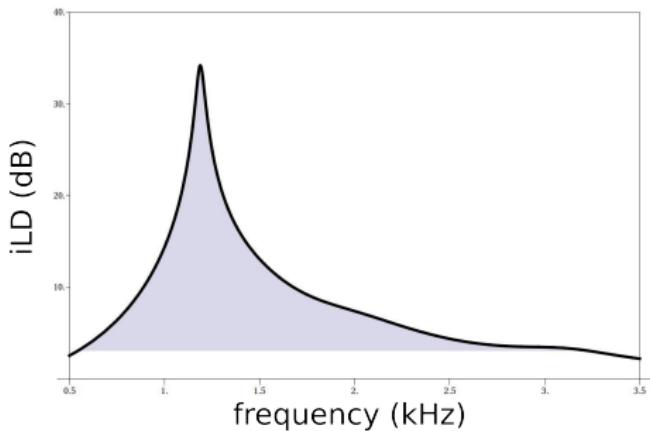
iLD Density Plot



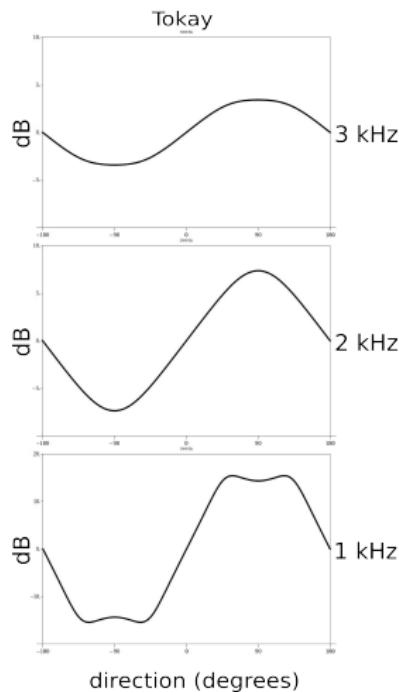
- ▶ Plot of iLD, against frequency and direction.
- ▶ Left: Calculated, Right: Experimental

Directional Cues

iLD Frequency/Direction Dependence

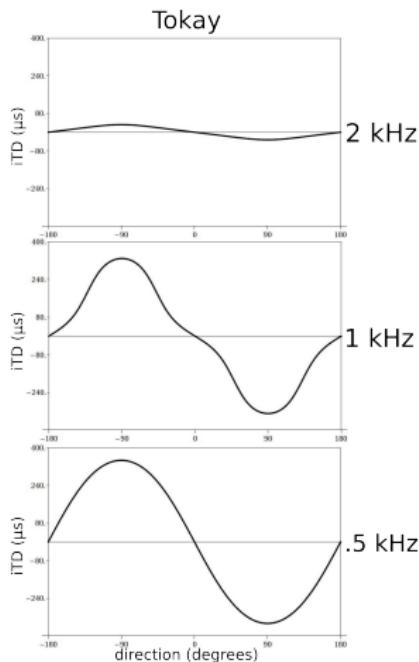
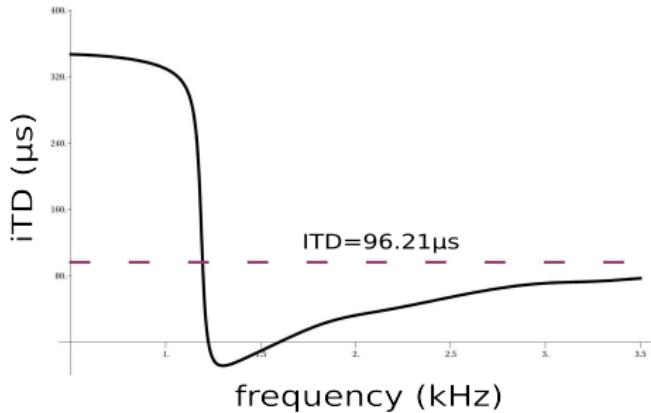


- ▶ iLD is a better cue at higher frequencies.
- ▶ Peak response at $\sim f_0 \sqrt{1 + 1/4Q^2}$.



Directional Cues

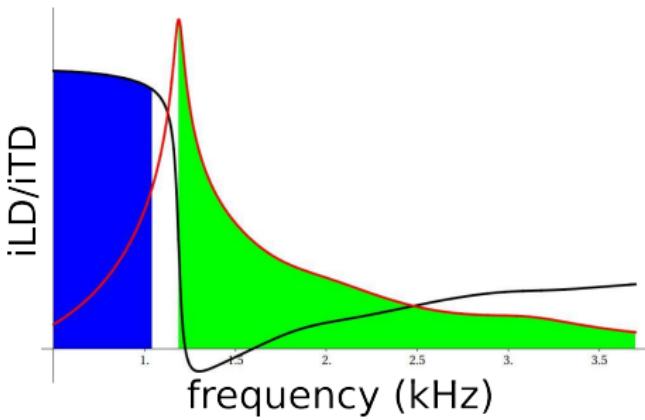
iTD Frequency/Direction Dependence



- ▶ iTD is a better cue at lower frequencies.
- ▶ constant up to $\sim f_0$.
- ▶ $iTD \approx 3 \times ITD$

iTD/iLD Frequency Regimes

- ▶ Presence of phase and amplitude sensitive neurons.
- ▶ Transition-frequency determined by f_0 , α . NOT by head size.
- ▶ Possible regime where both cues can simultaneously be used.

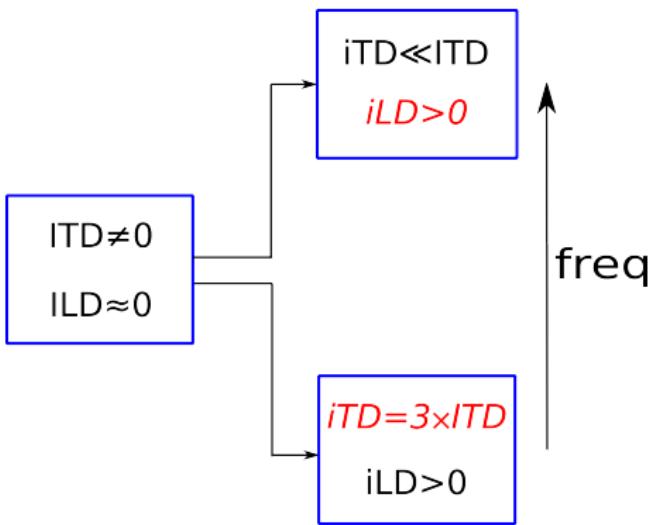


Conclusion

- ▶ Single description for both regimes.
- ▶ Transition determined by f_0 , α .

Future Work

- ▶ Extracolumella motion.
- ▶ Crocodiles
 - ▶ Independent above water.
 - ▶ ICE underwater.

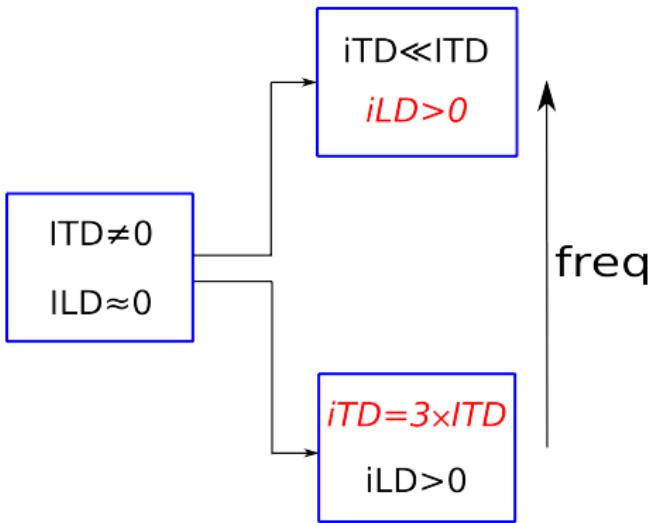


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Thank You

