Mechanical Processing in Internally Coupled Ears

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TMP Thesis Defence
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Elite Master Course Theoretical and Mathematical Physics

Auditory Systems

Independent Ears

Eustachian tubes typically very narrow.

Effectively independent eardrum vibrations.

Coupled Ears

Eardrums connected through wide eustachian tubes and a large mouth cavity.

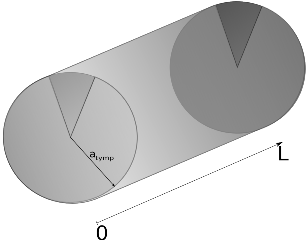
Eardrums vibrations influence eachother.

Advantages of Coupled Ears

- Low frequencies result in reduced degradation of hearing cues in dense enviroments.

Mouth Cavity

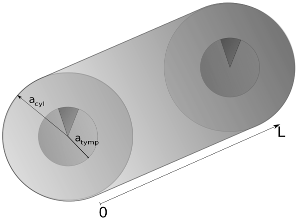
Previous Model



a_{tymp} fixed.

$V_{\mathrm{cyl}} = \pi a_{\mathrm{tymp}}^2 L$

Our Model



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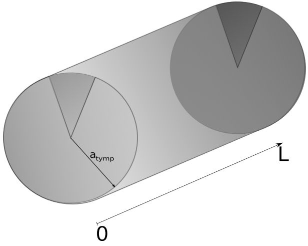

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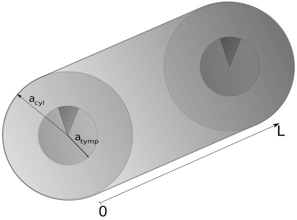

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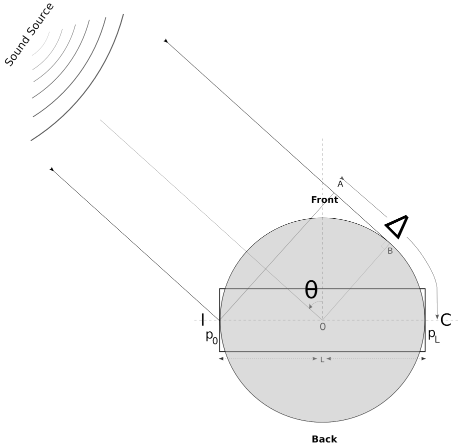
Acoustic Head Model

- **I** - Ipsilateral ear, **C** - Contralateral ear.
  p_0, p_L - sound pressure on eardrums, θ - sound source direction.
- Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_L|$.
- Phase difference between sound at both ears - $\Delta = 1.5kL\sin\theta$.
- $p_0 = pe^{j\omega t - .75kL\sin\theta}$
  $p_L = pe^{j\omega t + .75kL\sin\theta}$



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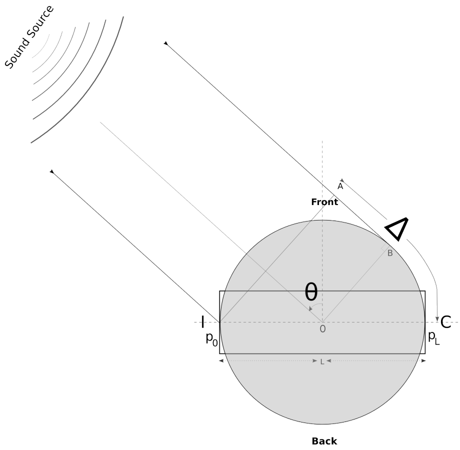

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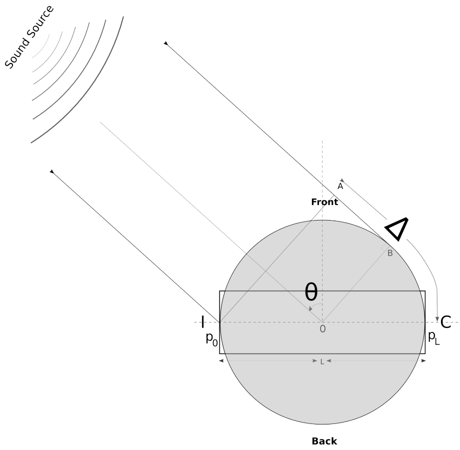

Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x,r,\phi,t) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p(x,r,\phi,t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p(x,r,\phi,t)}{\partial \phi^2} + \frac{\partial p(x,r,\phi,t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

$$p(x,r,\phi,t) = f(x)g(r)h(\phi)e^{j\omega t}.$$

Separated Equations and their Solutions

x- and ϕ- directions

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x} \tag{2}$$

$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi} \tag{3}$$

r-direction, Bessel functions

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q^2}{r^2}\right] g(r) = 0 \longrightarrow g(r) = J_q(\nu r) \tag{4}$$

where, $\nu^2 = k^2 - \zeta^2$

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Boundary Conditions - ϕ

Smoothness and Continuity in ϕ.

$$\phi \rightarrow h(0) = h(2\pi) \quad \text{and} \quad h'(0) = h'(2\pi)$$

$$\Rightarrow h(\phi) = \cos q\phi, \;\; q = 0, 1, 2, \ldots \tag{5}$$

Boundary Conditions - r

Impenetrable boundary at $r = a_{\mathrm{cyl}}$, i.e. normal derivative vanishes

$$-j\rho\omega\mathbf{v} = \mathbf{n}.\,\nabla p(x, r, \phi; t)\big|_{r=a_{\mathrm{cyl}}} \equiv \left.\frac{\partial g}{\partial r}\right|_{r=a_{\mathrm{cyl}}} = 0 \tag{6}$$

$$\Rightarrow g(r) = J_q(\nu_{\mathrm{qs}} r / a_{\mathrm{cyl}}) \tag{7}$$

Bessel Prime Zeros

- ν_{qs} - zeros of J'_q, $s = 0, 1, 2, \ldots$
- ν_{00}=0

General Solution

Pressure Modes

$$p(x,r,\phi,t) = \sum_{q=0,s=0}^{\infty} p_{\mathrm{qs}}(x,r,\phi)e^{j\omega t} \tag{8}$$

$$p_{\mathrm{qs}}(x,r,\phi) = \left[A_{\mathrm{qs}}e^{j\zeta_{\mathrm{qs}}x} + B_{\mathrm{qs}}e^{-j\zeta_{\mathrm{qs}}x}\right]\cos q\phi J_q(\nu_{\mathrm{qs}}r/a_{\mathrm{cyl}}) \tag{9}$$

where, $\zeta_{\mathrm{qs}} = \sqrt{k^2 - \nu_{\mathrm{qs}}^2/a_{\mathrm{cyl}}^2}$

Plane Wave Mode

$$p_{00}(x,r,\phi;t) = \left[A_{00}e^{jkx} + B_{00}e^{-jkx}\right]e^{j\omega t} \tag{10}$$

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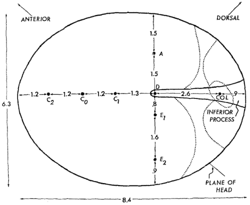
Eardrum

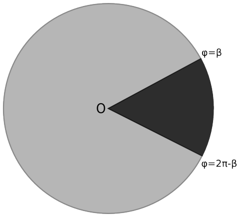
Sketch of a Tokay eardrum as seen from the outside[a].



COL - approximate position opposite the extracolumella insertion.

[a] G. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r = a_{\mathrm{tymp}}$ and $\phi = \beta,\ 2\pi - \beta$)

Membrane Vibrations

Membrane EOM

$$-\partial_t^2 u(r,\phi;t) - 2\alpha\partial_t u(r,\phi;t) + c_m^2 \Delta_{(2)} u(r,\phi;t) = \frac{1}{\rho_m d}\Psi(r,\phi;t) \tag{11}$$

Membrane parameters

α - damping coefficient, c_m^2 - propagation velocity

ρ_m - density, d - thickness.

Free-Undamped Membrane, $\alpha \to 0$, $\Psi \to 0$

Separation Ansatz

$$u(r,\phi;t) = f(r)g(\phi)h(t) \tag{12}$$

Separated Equations

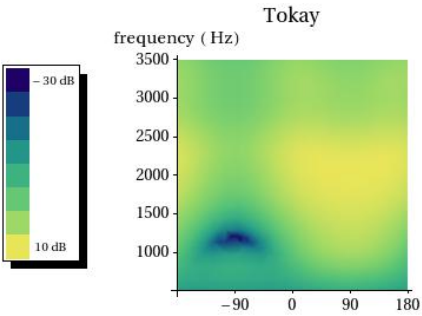
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f(r)}{\partial r}\right) + \left[\mu^2 - \frac{m^2}{r^2}\right] f(r) = 0 \tag{13}$$

$$\frac{d^2 g(\phi)}{d\phi^2} + m^2 g(\phi) = 0 \tag{14}$$

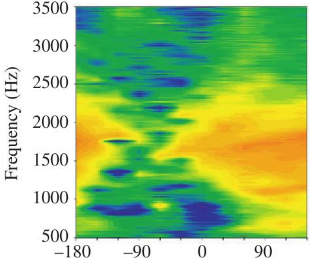
$$\frac{d^2 h(t)}{dt^2} + c_M^2 \mu^2 h(t) = 0 \tag{15}$$

Boundary Conditions

Vibration Amplitude



direction (degrees)



direction (degrees)

Conclusion

Thank You