Modelling Internally Coupled Ears

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Abstract

Hier steht eine maximal einseitige Zusammenfassung der Dissertation.

Dies ist ein neuer Absatz.

Chapter 1

Introduction

Chapter 2

The ICE Model

Our goal is to model the middle-ear of the vertebrates in question in the simplest possible way while ensuring an accurately replication of its main properties. The main components of such a system are the mouth-cavity, the two tympani and the two extracollumellar footplates (one on each tympanum). In general, the shape of the mouth-cavity is highly irregular and therefore not conducive to an analytical treatment. Moreover, the system corresponds to a pair of second-order PDE's with moving boundaries. For this reason we will need to make further approximations for the sake of expediency.

2.1 Description

In the earlier treatment of the ICE model, the mouth canal is modelled as a simple cylinder closed at both ends by rigidly clamped (baffled) circular membranes. As shown in [2], The length of the cylinder was chosen to be equal to the interaural distance. The advantage of using a cylindrical cavity model for the mouth cavity is that the pressure distribution inside the cavity is easy to calculate - something that is even more important at higher frequencies as the pressure distribution inside the cavity is highly non-uniform.

The problem with this description is the fact that the volume of the model's cavity is an order of magnitude smaller than that of the mouth-cavity in the corresponding animal with similar tympani and a the same interaural distance. In general, a smaller volume results in a stronger coupling - both in terms of an increased iTD and an increased iLD. For this reason, the earlier model overestimates the iTDs at low frequencies and the iLDs at high frequencies respectively.

2.2 Internal Cavity

We assume that the air inside the cavity obeys linear acoustics (described briefly in A). The pressure distribution inside the cavity is therefore given by the 3D acoustic wave-equation in cylindrical polar coordinates,

$$\frac{1}{c^2}\frac{\partial^2 p(x,r,\phi,t)}{\partial t^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p(x,r,\phi,t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial p(x,r,\phi,t)}{\partial \phi^2} + \frac{\partial p(x,r,\phi,t)}{\partial x^2} \tag{2.1}$$

where c is the sound propagation velocity. The complete solution must take into account the boundary conditions at and within the cavity walls and the ones at the air-membrane interface. We also note that the above equation implies that the animal's mouth is closed, which is typical for a waiting animal. In order to solve (2.1) for a particular frequency f (angular frequency $\omega = 2\pi f$), we use the following separation ansatz

$$p(x,r,\phi,t) = f(x)g(r)h(\phi)e^{j\omega t} \tag{2.2}$$

which after substitution into the acoustic wave-equation leads to,

$$\begin{aligned} k^2 f(x)g(r)h(\phi) + f(x)h(\phi)\left[\frac{\partial^2 g(r)}{\partial r^2} + \frac{1}{r}\frac{\partial g(r)}{\partial r}\right] \\ + f(x)g(r)\frac{1}{r^2}\frac{\partial h(\phi)}{\partial \phi} + \frac{\partial^2 f(x)}{\partial x^2} = 0 \end{aligned} \tag{2.3}$$

with $k := \omega/c$. This yields the following set of equations (ODEs):

$$\frac{d^2 f(x)}{dx^2} + k_x^2 f(x) = 0 \tag{2.4}$$

$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \tag{2.5}$$

$$\frac{\partial^2 g(r)}{\partial r^2} + \frac{1}{r}\frac{\partial g(r)}{\partial r} + \left[\underbrace{(k^2 - k_x^2)}_{=:\nu^2} - \frac{q^2}{r^2}\right] = 0 \tag{2.6}$$

with separation constants q and k_x. The last equation is the Bessel differential equation [1, p. 313] and its general solution is given by,

$$g(r) = C_{qs}J_q(\nu r) + D_{qs}Y_q(\nu r) \tag{2.7}$$

where J_q and Y_q are the order-q Bessel functions of the first and second kind respectively. The Bessel function of the second kind can be ignored as it diverges at $r = 0$. The solutions to the separated equations are therefore given by,

$$f(x) = e^{\pm k_x x},\ \ h(\phi) = e^{\pm j\phi}, \text{and } g(r) = J_q(\nu r) \tag{2.8}$$

with a specific solution to (2.1),

$$p(x,r,\phi;t) = \left[(A^+e^{jq\phi} + A^-e^{-jq\phi})e^{jk_x x} + (B^+e^{jq\phi} + B^-e^{-jq\phi})e^{-jk_x x}\right] J_q(\nu r)e^{j\omega t}. \tag{2.9}$$

The coefficients $A^\pm$, $B^\pm$, q, k_x and ν will be subsequently determined by the boundary conditions.

Pressure Boundary Conditions

There are three sets of boundary conditions -

- Continuity and smoothness in ϕ which is equivalent to $h(0) = h(2\pi)$ and $h'(0) = h'(2\pi)$ where, $h' = dh/d\phi$.
- Vanishing of the normal derivative at the cavity walls - $g'(a_{cyl}) = 0$ (a_{cyl} is the radius of the cylinder).
- Equating the membrane velocity to the air velocity at the membrane boundaries (to be discussed in the next section).

The first set of requirements is obvious. The second and third are a result of the so called "no-penetration" boundary-conditions of fluid-mechanics. They arise from the requirement that at the boundary surface, the normal fluid-particle velocity should vanish. The fluid-particle velocity ($\mathbf{v}$) is related to the pressure by,

$$-\rho\frac{\partial \mathbf{v}}{\partial t} = \nabla p \tag{2.10}$$

The general solution is therefore given by,

$$p(x, r, \phi; t) = \sum_{q=0}^{\infty}\sum_{s=0}^{\infty}\left(A_{qs}e^{j\zeta_{qs}x} + B_{qs}e^{-j\zeta_{qs}x}\right)\cos(m\phi)J_q(\nu_{qs}r)e^{j\omega t} \tag{2.11}$$

2.3 Vibration of the Membrane

As a preliminary exercise, we will first derive expressions for the free and force-driven vibrations of a circular membrane. We will then use our results to move on to the sectoral membrane which corresponds to the tympanum loaded by the extracollumella.

2.3.1 Circular Membrane

The equation of motion for a rigidly clamped circular membrane of radius a is given by,

$$-\frac{\partial^2 u(r,\phi,t)}{\partial t^2} - 2\alpha\frac{\partial u(r,\phi,t)}{\partial t} + c_M^2\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u(r,\phi,t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial u(r,\phi,t)}{\partial \phi^2}\right) = \frac{1}{\rho_m d}\Psi(r,\phi,t) \tag{2.12}$$

subject to the boundary condition $u(r,\phi,t)|_{r=a} = 0$.

2.3.2 Sectoral Membrane

Chapter 3

Analysis of the ICE–Model

Chapter 4

Conclusion

Appendix A

Acoustic Theory

Hier steht der erste Anhang.

Appendix B

Second Appendix Chapter

Hier kommt der zweite Anhang.

Bibliography

[1] E.T. Copson: *Introduction to the Theory of Functions of a Complex Variable*. Clarendon Press, Oxford, 1973.

[2] C. Voßen: *Auditory Information Processing in Systems with Internally Coupled Ears*. Technische Universität München, Dissertation, 2010.

Acknowledgements

Danke.