Mechanical Processing in Internally Coupled Ears

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TMP Thesis Defence
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Outline for Section 1

Auditory Systems

Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.

Coupled Ears

Wide eustachian tubes open into the mouth cavity.

Eardrums vibrations influence eachother.

Binaural Hearing Cues

Localization using frequency dependent phase and amplitude differences between the ears.

Interaural Time Difference

Equivalent to phase difference between membrane vibrations.

Interaural Level Difference

Equivalent to amplitude difference between membrane vibrations.

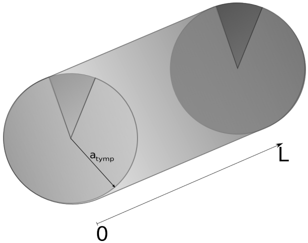
Advantages of Coupled Ears

- Low frequencies result in reduced degradation of hearing cues in dense enviroments.

Outline for Section 2

Mouth Cavity

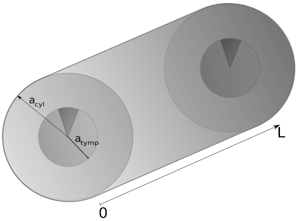
Previous Model



a_{tymp} fixed.

$$V_{\text{cyl}} = \pi a_{\text{tymp}}^2 L$$

Our Model



$a_{\text{tymp}}, \; V_{\text{cyl}}$ fixed.

$$a_{\text{cyl}} = \sqrt{V_{\text{cyl}}/\pi L}$$

Mouth Cavity

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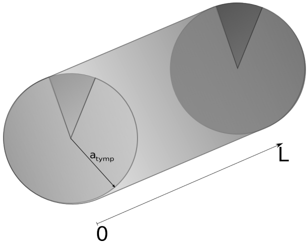

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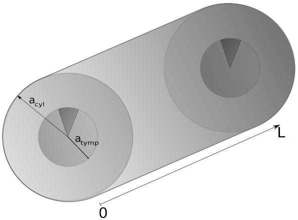
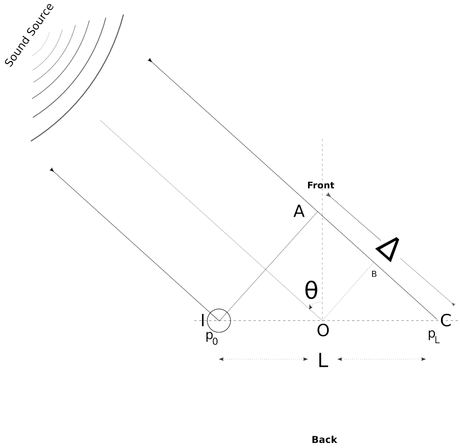


$a_{\mathrm{tymp}},\ V_{\mathrm{cyl}}$ fixed.

$a_{\mathrm{cyl}} = \sqrt{V_{\mathrm{cyl}}/\pi L}$

Acoustic Head Model

- **I** - Ipsilateral ear, **C** - Contralateral ear. p_0, p_L - sound pressure on eardrums, θ - sound source direction.
- Sound source “far away”.
- No appreciable amplitude difference, $|p_0| = |p_L|$.
- Phase difference between sound at both ears - $\Delta = kL \sin\theta$.
- $p_0 = pe^{j\omega t - .5kL\sin\theta}$
  $p_L = pe^{j\omega t + .5kL\sin\theta}$



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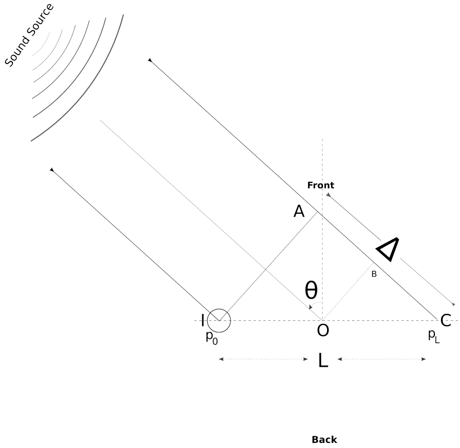

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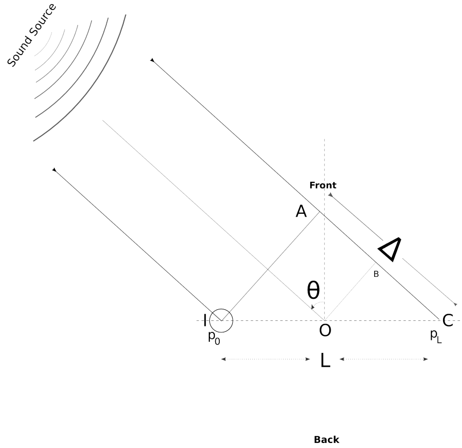
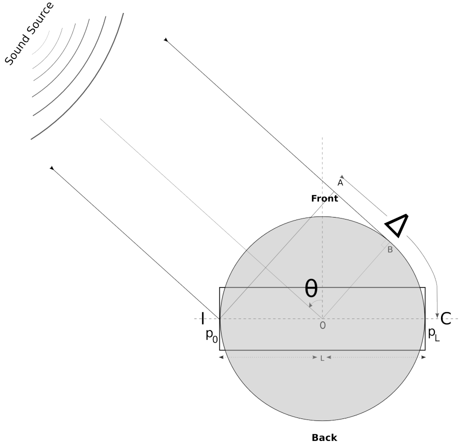


Acoustic Head Model contd.

- $|p_0| = |p_L|$.
- Increased phase difference due to diffraction - $\Delta = 1.5kL\sin\theta$.
- $p_0 = pe^{j\omega t - .75kL\sin\theta}$
  $p_L = pe^{j\omega t + .75kL\sin\theta}$



Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x,r,\phi,t) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p(x,r,\phi,t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p(x,r,\phi,t)}{\partial \phi^2} + \frac{\partial p(x,r,\phi,t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

$$p(x,r,\phi,t) = f(x)g(r)h(\phi)e^{j\omega t}.$$

Separated Equations

x- and ϕ- directions

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x} \tag{2}$$

$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi} \tag{3}$$

r-direction, Bessel functions

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q^2}{r^2}\right] g(r) = 0 \longrightarrow g(r) = J_q(\nu r) \tag{4}$$

where, $\nu^2 = k^2 - \zeta^2$

Separated Equations

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where, $\nu^2 = k^2 - \zeta^2$

Boundary Conditions - ϕ

Smoothness and Continuity in ϕ.

$$\phi \rightarrow h(0) = h(2\pi) \quad \text{and} \quad h'(0) = h'(2\pi)$$

$$\Rightarrow h(\phi) = \cos q\phi, \;\; q = 0, 1, 2, \ldots \tag{5}$$

Boundary Conditions - r

Impenetrable boundary at $r = a_{\text{cyl}}$, i.e. normal derivative vanishes

$$-j\rho\omega\mathbf{v} = \mathbf{n}.\,\nabla p(x, r, \phi; t)\big|_{r=a_{\text{cyl}}} \equiv \left.\frac{\partial g}{\partial r}\right|_{r=a_{\text{cyl}}} = 0 \qquad (6)$$

$$\Rightarrow g(r) = J_q(\nu_{\text{qs}} r / a_{\text{cyl}}) \qquad (7)$$

Bessel Prime Zeros

- ν_{qs} - zeros of J_q', $s = 0, 1, 2, \ldots$
- ν_{00}=0

General Solution

Pressure Modes

$$p(x, r, \phi, t) = \sum_{q=0,s=0}^{\infty} \left[A_{\mathrm{qs}} e^{j\zeta_{\mathrm{qs}} x} + B_{\mathrm{qs}} e^{-j\zeta_{\mathrm{qs}} x} \right] p_{\mathrm{qs}}(r, \phi) e^{j\omega t} \quad (8)$$

$$p_{\mathrm{qs}}(r, \phi) = \cos q\phi J_q(\nu_{\mathrm{qs}} r / a_{\mathrm{cyl}}) \quad (9)$$

$$\text{where, } \zeta_{\mathrm{qs}} = \sqrt{k^2 - \nu_{\mathrm{qs}}^2 / a_{\mathrm{cyl}}^2}$$

Plane Wave Mode

$$p_{\mathrm{pw}}(x, r, \phi; t) = \left[A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t} \quad (10)$$

General Solution

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$$p(x, r, \phi, t) = \sum_{q=0,s=0}^{\infty} \left[A_{\mathrm{qs}} e^{j\zeta_{\mathrm{qs}} x} + B_{\mathrm{qs}} e^{-j\zeta_{\mathrm{qs}} x} \right] p_{\mathrm{qs}}(r, \phi) e^{j\omega t} \quad (8)$$

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Plane Wave Mode

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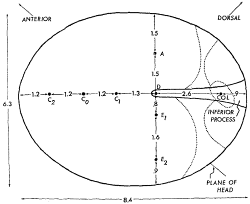
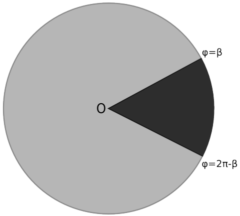
Eardrum

Sketch of a Tokay eardrum as seen from the outside[a].



COL - approximate position opposite the extracolumella insertion.

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r = a_{\text{tymp}}$ and $\phi = \beta,\ 2\pi - \beta$)

[a] G. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

Membrane Vibrations

Membrane EOM

$$-\partial_t^2 u(r,\phi;t) - 2\alpha\partial_t u(r,\phi;t) + c_M^2 \Delta_{(2)} u(r,\phi;t) = \frac{1}{\rho_m d}\Psi(r,\phi;t) \tag{11}$$

Membrane parameters

α - damping coefficient, c_M^2 - propagation velocity

ρ_m - density, d - thickness.

Free-Undamped Membrane, $\alpha \to 0$, $\Psi \to 0$

Separation Ansatz

$$u(r,\phi;t) = f(r)g(\phi)h(t) \tag{12}$$

Separated Equations

$$\frac{d^2 g(\phi)}{d\phi^2} + \kappa^2 g(\phi) = 0 \tag{13}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f(r)}{\partial r}\right) + \left[\mu^2 - \frac{\kappa^2}{r^2}\right] f(r) = 0 \tag{14}$$

$$\frac{d^2 h(t)}{dt^2} + c_M^2 \mu^2 h(t) = 0 \tag{15}$$

Boundary Conditions

ϕ-direction: $u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$

$$\Rightarrow g(\phi) = \sin \kappa(\phi - \beta) \tag{16}$$

$$\text{where, } \kappa = \frac{m\pi}{2(\pi - \beta)}, \quad m = 1, 2, 3, \ldots$$

r-direction: $u(a_{\mathrm{tymp}}, \phi; t) = 0$

$$\Rightarrow f(r) = J_\kappa(\mu_{\mathrm{mn}} r / a_{\mathrm{tymp}}) \tag{17}$$

where, μ_{mn} is the n^{th} zero of J_κ

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Free eigenmodes

$$u_{\mathrm{mn}}(r,\phi) = \sin\kappa(\phi-\beta)J_\kappa(\mu_{\mathrm{mn}} r) \tag{18}$$

$$u_{\mathrm{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi)e^{j\omega_{\mathrm{mn}} t} \tag{19}$$

where, $\omega_{\mathrm{mn}} = c_M \mu_{\mathrm{mn}}$

Damped membrane

$$\widetilde{u}_{\mathrm{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi)e^{j\omega_{\mathrm{mn}} t-\alpha t} \tag{20}$$

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Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

$$u_{\rm ss}(r,\phi;t) =: \sum_{m=0,n=1}^{\infty} C_{\rm mn} u_{\rm mn}(r,\phi) e^{j\omega t} \tag{21}$$

Substitute $u_{\rm ss}$ in Membrane EOM.

$$C_{\rm mn} = \frac{p \int dS u_{\rm mn}}{\Omega_{\rm mn} \int dS u_{\rm mn}^2} \tag{22}$$

$$\Omega_{\rm mn} = \rho_M d \left[(\omega^2 - \omega_{\rm mn}^2) - 2j\alpha\omega \right]$$

Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

$$u_{\mathrm{ss}}(r,\phi;t) =: \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} \tag{21}$$

Substitute u_{ss} in Membrane EOM.

$$C_{\mathrm{mn}} = \frac{p \int dS u_{\mathrm{mn}}}{\Omega_{\mathrm{mn}} \int dS u_{\mathrm{mn}}^2} \tag{22}$$

$$\Omega_{\mathrm{mn}} = \rho_M d \left[(\omega^2 - \omega_{\mathrm{mn}}^2) - 2j\alpha\omega \right]$$

Forced Vibrations contd.

Transient Solution

Same as the solution for a free damped membrane

$$u_{\mathrm{t}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}} t - \alpha t} \tag{23}$$

$\widetilde{C}_{\mathrm{mn}}$ determined from the membrane displacement at $t = 0$.
$u_{\mathrm{t}} \to 0$ exponentially as $t \to \infty$.

Steady State Approximation

$u \approx u_{\mathrm{ss}}$ if α is "large" .

Forced Vibrations contd.

Transient Solution

Same as the solution for a free damped membrane

$$u_{\mathrm{t}}(r, \phi; t) = \sum_{m=0, n=1}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r, \phi) e^{j\omega_{\mathrm{mn}} t - \alpha t} \tag{23}$$

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Steady State Approximation

$u \approx u_{\mathrm{ss}}$ if α is "large" .

Coupled Membranes

$$u_{0/L} = \sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C_{\mathrm{mn}}^{0/L} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} \tag{24}$$

Membrane Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C_{\mathrm{mn}}^{0} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} = p_0 e^{j\omega t} - p(0,r,\phi;t) \tag{25}$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C_{\mathrm{mn}}^{L} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} = p_L e^{j\omega t} - p(L,r,\phi;t) \tag{26}$$

Coupled Membranes

$$u_{0/L} = \sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C_{\mathrm{mn}}^{0/L} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} \tag{24}$$

Membrane Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C_{\mathrm{mn}}^{0} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} = p_0 e^{j\omega t} - p(0,r,\phi;t) \tag{25}$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C_{\mathrm{mn}}^{L} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} = p_L e^{j\omega t} - p(L,r,\phi;t) \tag{26}$$

"Surface" Velocity

$$U_{0/L} = \begin{cases} u_{0/L}, \ 0 < r < a_{\mathrm{tymp}} \text{ and } \beta < \phi < 2\pi - \beta \\ 0, \text{ otherwise} \end{cases} . \tag{27}$$

Velocity in $x-$direction

$$v_x = -\sum_{q=0,s=0}^{\infty} \frac{\zeta_{\mathrm{qs}}}{\rho\omega} \left(A_{\mathrm{qs}} e^{j\zeta_{\mathrm{qs}} x} - B_{\mathrm{qs}} e^{-j\zeta_{\mathrm{qs}} x} \right) p_{\mathrm{qs}}(r, \phi) e^{j\omega t} \tag{28}$$

Boundary Conditions

Exact

$$U_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t) \quad (29)$$

$$U_L = \frac{1}{j\omega} v_x(L, r, \phi; t) \quad (30)$$

Approximate

$$U_{0/L} \approx S^{0/L}(t) =: \int dS U_{0/L} \quad (31)$$

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Exact

$$U_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t) \tag{29}$$

$$U_L = \frac{1}{j\omega} v_x(L, r, \phi; t) \tag{30}$$

Approximate

$$U_{0/L} \approx S^{0/L}(t) =: \int dS U_{0/L} \tag{31}$$

Boundary Conditions

Higher pressure modes disappear, i.e.

$$p = \left[A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t}$$

$$A_{00} = -\frac{\rho\omega^2}{2k \sin kL} \left(S^0 e^{-jkL} + S^L \right) \tag{32}$$

$$B_{00} = -\frac{\rho\omega^2}{2k \sin kL} \left(S^0 e^{jkL} + S^L \right) \tag{33}$$

Coupled Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C^0_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) = p_0 + \frac{\rho\omega^2}{k}\left(\frac{S^0}{\tan kL} + \frac{S^L}{\sin kL}\right) \quad (34)$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C^L_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) = p_L + \frac{\rho\omega^2}{k}\left(\frac{S^0}{\sin kL} + \frac{S^L}{\tan kL}\right) \quad (35)$$

Decoupling

Decouple by taking the sum and difference of the above equations.

Coupled Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C^{0}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) = p_0 + \frac{\rho\omega^2}{k}\left(\frac{S^0}{\tan kL} + \frac{S^L}{\sin kL}\right) \quad (34)$$

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Decouple by taking the sum and difference of the above equations.

Decoupled Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\text{mn}} C_{\text{mn}}^{+} u_{\text{mn}}(r,\phi) = p_{+} + \frac{\rho\omega^2}{k} S^{+} \cot \frac{kL}{2} \tag{36}$$

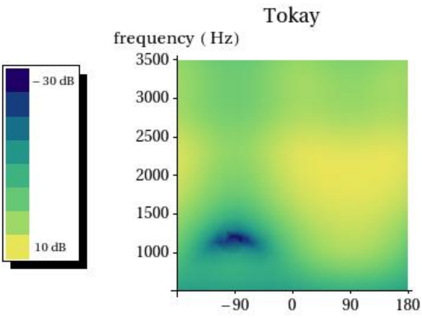
$$\sum_{m=0,n=1}^{\infty} \Omega_{\text{mn}} C_{\text{mn}}^{-} u_{\text{mn}}(r,\phi) = p_{-} - \frac{\rho\omega^2}{k} S^{-} \tan \frac{kL}{2} \tag{37}$$

$$C_{\text{mn}}^{+} = \left[p_{+} + \frac{\rho\omega^2}{k} S^{+} \cot \frac{kL}{2} \right] \frac{\int dS u_{\text{mn}}}{\Omega_{\text{mn}} \int dS u_{\text{mn}}^2} \tag{38}$$

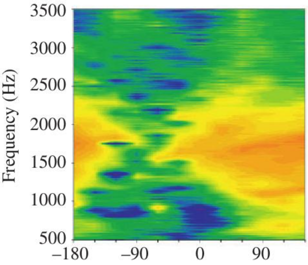
$$C_{\text{mn}}^{-} = \left[p_{-} - \frac{\rho\omega^2}{k} S^{-} \tan \frac{kL}{2} \right] \frac{\int dS u_{\text{mn}}}{\Omega_{\text{mn}} \int dS u_{\text{mn}}^2} \tag{39}$$

Outline for Section 3

Vibration Amplitude



direction (degrees)



direction (degrees)

Outline for Section 4

Conclusion

Thank You