Mechanical Processing in Internally Coupled Ears

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TMP Thesis Defence
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T35

TMP Elite Master Course Theoretical and Mathematical Physics

Outline

Auditory Systems

Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.

- eg. Mammals.

Coupled Ears

Wide eustachian tubes open into the mouth cavity.

Eardrums vibrations influence eachother.

- eg. Lizards, birds, frogs.

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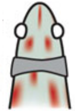

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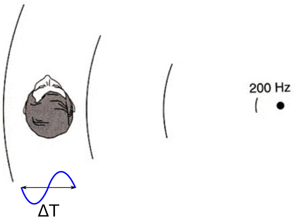
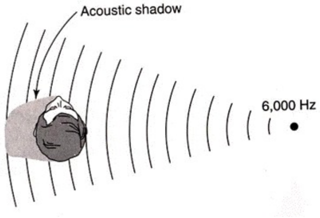
Localization - Independent Ears





Interaural Time Difference

- Phase difference between the (pressure) inputs to the ears.
- ITD
- Used when $\lambda \gg$ head size.

Interaural Level Difference

- Amplitude difference between the inputs.
- ILD
- Used when $\lambda \sim$ head size.

Localization - Coupled Ears

Problems

- Small heads.
- Lower "processing power".
- ITD$\neq$0, "small".
- ILD$\approx$0.

Internal Time Difference

Phase difference between **eardrums**.

- **i**TD
- **i**TD>ITD.

Internal Level Difference

Amplitude difference between the **eardrums**.

- **i**LD
- **i**LD>0.

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ICE Model

Internally Coupled Ears

Aim:

- Accurately reproduce the direction and frequency dependence.
- Analytically tractable.

Solution:

- Circular eardrums.
- Cylindrical mouth cavity.

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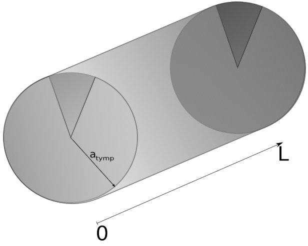
Mouth Cavity

Previous Model[a]



a_{tymp} fixed.

$V_{\text{cyl}} = \pi a_{\text{tymp}}^2 L$

Our Model

$a_{\text{tymp}},\ V_{\text{cyl}}$ fixed.

$a_{\text{cyl}} = \sqrt{V_{\text{cyl}}/\pi L}$

[a] Vossen et al (2010)

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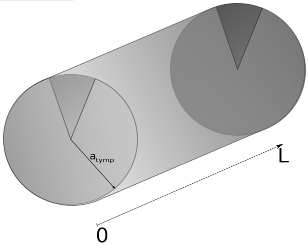

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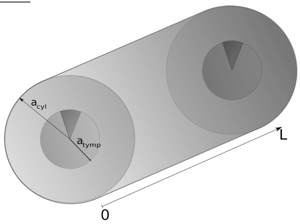

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Cavity Pressure - $p(x, r, \phi; t)$

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \phi^2} + \frac{\partial p}{\partial x^2}$$

Flow Velocity:

$$\dot{\vec{v}}(x, r, \phi; t) = \frac{1}{\rho}\nabla p$$

Separation Ansatz

$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}$$

Separated Equations

x- and ϕ- directions

$$f(x) = Ae^{j\zeta x} + Be^{-j\zeta x}$$
$$h(\phi) = Ce^{jq\phi} + De^{-jq\phi}$$

$$h(\phi) \xrightarrow{cont.,smooth} \cos q\phi$$
$$q = 0, 1, 2, 3, \ldots$$

r-direction, Bessel functions

$$g(r) = J_q(\nu r)$$

$$\nu^2 = k^2 - \zeta^2$$

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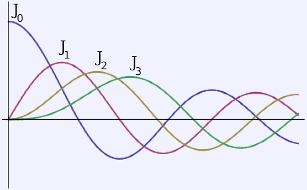
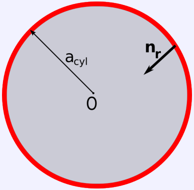


Boundary Conditions - r

Impenetrable boundary at $r = a_{\text{cyl}}$

$$\mathbf{n_r} . \left.\vec{v}\right|_{r=a_{\text{cyl}}} \equiv \left.\frac{\partial g}{\partial r}\right|_{r=a_{\text{cyl}}} = 0$$

$$\Rightarrow g(r) = J_q(\nu_{\text{qs}} r / a_{\text{cyl}})$$



Bessel Prime Zeros

ν_{qs} - extrema of J_q.

ν_{00}=0.

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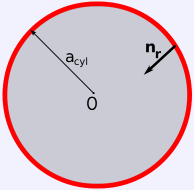

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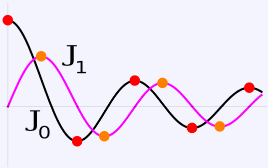
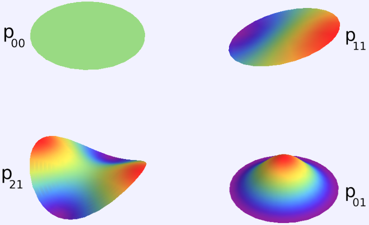


General Solution

Pressure Modes



$$p_{\mathrm{qs}}(r,\phi) = \cos q\phi J_q(\nu_{\mathrm{qs}} r / a_{\mathrm{cyl}})$$

$$f_{\mathrm{qs}}(x) = A_{\mathrm{qs}} e^{j\zeta_{\mathrm{qs}} x} + B_{\mathrm{qs}} e^{-j\zeta_{\mathrm{qs}} x}$$

$$\int dS p_{\mathrm{qs}} \neq 0 \text{ iff } q = 0,\ s = 0$$

$$\int dS p_{\mathrm{qs}} p_{\mathrm{mn}} = \delta_{qm}\delta_{sn}$$

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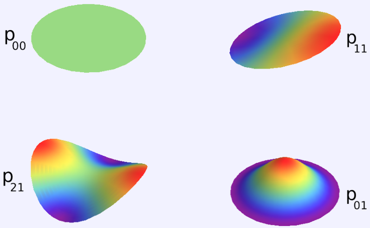

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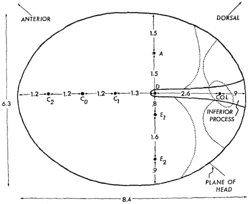
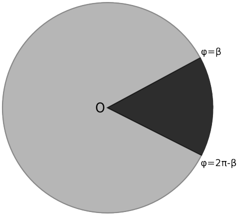
Eardrum

Sketch of a Tokay eardrum as seen from the outside (G.A. Manley, 1972).



COL - approximate position opposite the extracolumella insertion.

The ICE Eardrum.



Extracolumella (dark) - rigid, stationary.

Membrane - linear elastic.

Clamped at boundaries $(r = a_{\text{tymp}},\ \phi = \beta,\ 2\pi - \beta)$

Membrane Vibrations

Membrane Displacement - $u(r, \phi; t)$

$$-\partial_t^2 u - 2\alpha\partial_t u + c_m^2 \Delta_{(2)} u = \frac{1}{\rho_m d}\Psi(r, \phi; t)$$

Membrane Parameters

α - damping coefficient, c_m - propagation velocity

ρ_m - density, d - thickness.

Free-Undamped Membrane, $\alpha \to 0$, $\Psi \to 0$

Separation Ansatz

$$u(r, \phi; t) = f(r)g(\phi)h(t)$$

Boundary Conditions

ϕ-direction: $u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$

$$\Rightarrow g(\phi) = \sin \kappa(\phi - \beta)$$

$$\kappa = \frac{m\pi}{2(\pi - \beta)}, m = 1, 2, \ldots$$

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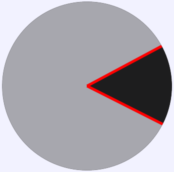

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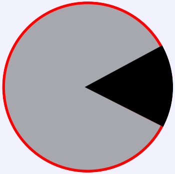
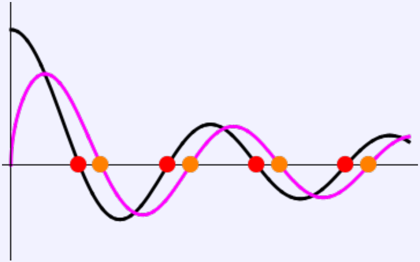
Boundary Conditions contd.

r-direction: $u(a_{\text{tymp}}, \phi; t) = 0$

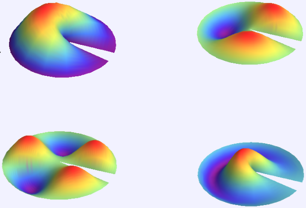


$\Rightarrow f(r) = J_\kappa(\mu_{\text{mn}} r / a_{\text{tymp}})$

μ_{mn}: n^{th} zero of J_κ



Free Membrane Eigenmodes - $u_{\mathrm{mn}}(r, \phi)$



u_{11} u_{21} u_{31} u_{12}

$$\omega_{\mathrm{mn}} = c_m \mu_{\mathrm{mn}}$$

$$h(t) = \exp\left(j\omega_{\mathrm{mn}} t\right)$$

$$\int dS u_{\mathrm{kl}} u_{\mathrm{mn}} = \delta_{km}\delta_{ln}$$

Damped: $\alpha \neq 0$

$$h(t) = \exp\left(-\alpha t + j\widetilde{\omega}_{\mathrm{mn}} t\right)$$

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Forced Vibrations: $\Psi = \mathrm{p}e^{j\omega t}$, $\alpha \neq 0$

Full Solution

$$u(r,\phi;t) = u_{\mathrm{ss}}(r,\phi;t) + u_{\mathrm{t}}(r,\phi;t)$$

Steady State Solution

$$u_{\mathrm{ss}} = \sum C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega t}$$

From Membrane equation.

$$C_{\mathrm{mn}} = \frac{p\int dS u_{\mathrm{mn}}}{\Omega_{\mathrm{mn}} \int dS u_{\mathrm{mn}}^2}$$

$$\Omega_{\mathrm{mn}} = \rho_M d\left[\omega^2 - 2j\alpha\omega - \omega_{\mathrm{mn}}^2\right]$$

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Forced Vibrations contd.

Transient Solution

$\equiv$ free damped membrane

$$u_{\mathrm{t}} = \sum \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\widetilde{\omega}_{\mathrm{mn}} t - \alpha t}$$

$u(t=0) = 0 \Rightarrow \ \widetilde{C}_{\mathrm{mn}} = -C_{\mathrm{mn}}.$

Steady State Approximation

$u_{\mathrm{t}} \to 0$ exponentially as $t \to \infty$.

$$u \approx u_{\mathrm{ss}}$$

Forced Vibrations contd.

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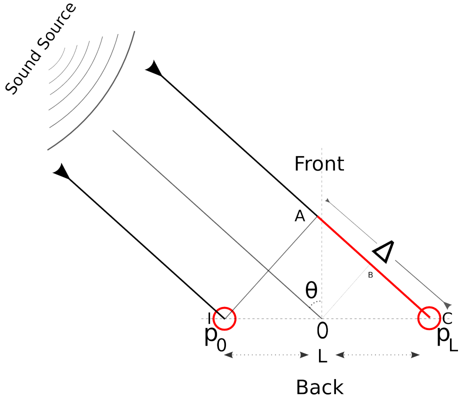
Acoustic Head Model - previous

- **I** - Ipsilateral, **C** - Contralateral.
- $p_0,\ p_L$ - Sound pressure.
- θ - Source direction.
- Sound source "far away".



- $|p_0| = |p_L|$.
- $\Delta = kL\sin\theta$.

- $p_0 = \text{pexp}\left(-j\Delta/2\right)$
  $p_L = \text{pexp}\left(j\Delta/2\right)$
- ILD=0,
  ITD:= $\frac{1}{\omega}\text{Arg}\left(\frac{p^0}{p^L}\right)$ =L/c

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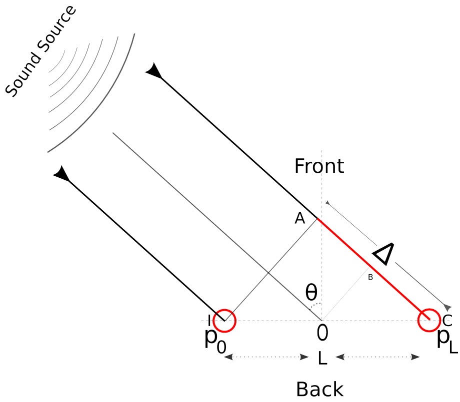

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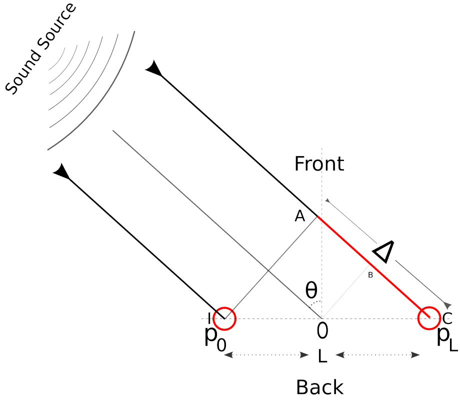
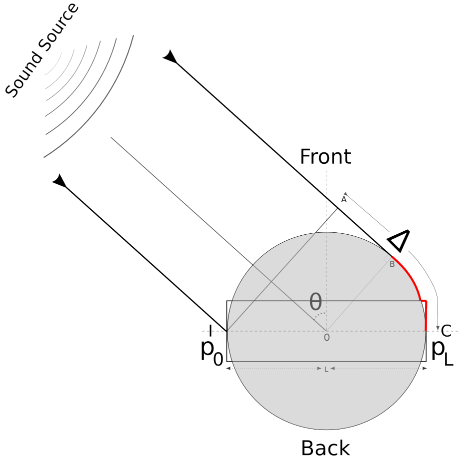


Acoustic Head Model - new

- $|p_0| = |p_L|$.
  - $\Delta = \frac{3}{2}kL\sin\theta$.
- $p_0 = \mathrm{pexp}\left(-j\Delta/2\right)$
  $p_L = \mathrm{pexp}\left(j\Delta/2\right)$
- ILD=0, ITD= $\frac{3}{2}$L/c



Coupled Membranes

$$u_{0/L} = \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}}^{0/L} u_{\mathrm{mn}}(r,\phi) e^{j\omega t}$$

Membrane Equations

$$-\partial_t^2 u_0 - 2\alpha\partial_t u_0 + c_m^2 \Delta_{(2)} u_0 = \frac{1}{\rho_m d}\left[p_0 e^{j\omega t} - p(0,r,\phi;t)\right]$$

$$-\partial_t^2 u_L - 2\alpha\partial_t u_L + c_m^2 \Delta_{(2)} u_L = \frac{1}{\rho_m d}\left[p_L e^{j\omega t} - p(L,r,\phi;t)\right]$$

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Boundary Conditions

Exact

$$u_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t)$$

$$u_L = \frac{1}{j\omega} v_x(L, r, \phi; t)$$

Approximate - $S^{0/L}(t) =: (\pi a_{\text{cyl}}^2)^{-1} \int dS u_{0/L}$

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Average Velocity

Higher pressure modes disappear, i.e.

$$p = \left[A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t}$$

$$\left(\Lambda - \frac{\rho\omega^2}{k \tan kL} \right) S^0 - \frac{\rho\omega^2}{k \sin kL} S^L = p_0$$

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$$\frac{1}{\Lambda} = \frac{1}{\pi a_{\mathrm{cyl}}^2} \sum_{m=0,n=1}^{\infty} \frac{\left(\int dS u_{\mathrm{mn}} \right)^2}{\Omega_{\mathrm{mn}} \int dS u_{\mathrm{mn}}^2}$$

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Final Expressions

Membrane Displacement

$$S_0(t) = G^s_{ipsi} p_0 + G^s_{cont} p_L$$
$$S_L(t) = G^s_{cont} p_0 + G^s_{ipsi} p_L$$

$$G^s_{ipsi} = \frac{1}{2}\left(\frac{1}{\Lambda + \Gamma_+} + \frac{1}{\Lambda + \Gamma_-}\right) \qquad \Gamma_+ = -\frac{\rho\omega^2}{k}\cot\frac{kL}{2}$$
$$G^s_{cont} = \frac{1}{2}\left(\frac{1}{\Lambda + \Gamma_+} - \frac{1}{\Lambda + \Gamma_-}\right) \qquad \Gamma_- = \frac{\rho\omega^2}{k}\tan\frac{kL}{2}$$

Tokay Gecko

L$=22\,$mm $\quad a_{\text{tymp}}=2.6\,$mm

$c_m = 5.41\,\text{ms}^{-1}$ $\quad \rho_m=1\,\text{mg}/\text{mm}^3$

$d=10\,\mu$m $\quad V_{\text{cav}}=3.5\,$ml

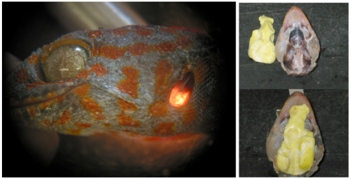
$\beta=\pi/25$ $\quad a_{cyl} \approx 6.6\,$mm

$\alpha \approx 2474\ \text{s}^{-1}$

$f_0 = c_m \mu_{01}/2\pi.$

ITD $= 96.21\,\mu$s.

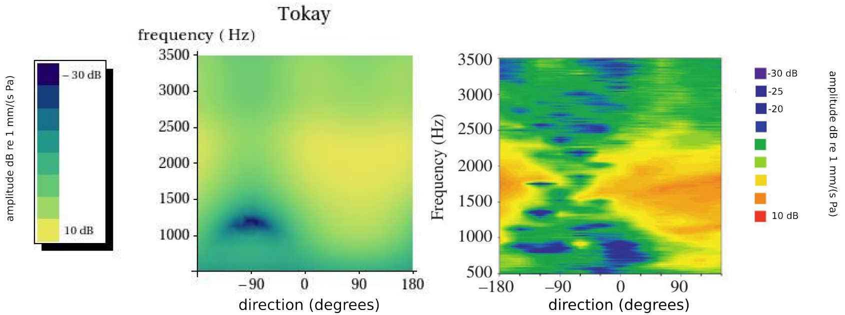
$\alpha \to$ steady state.



Tokay gecko. Left: Head illuminated from the opposite side[a]. Right: Mouth casts.

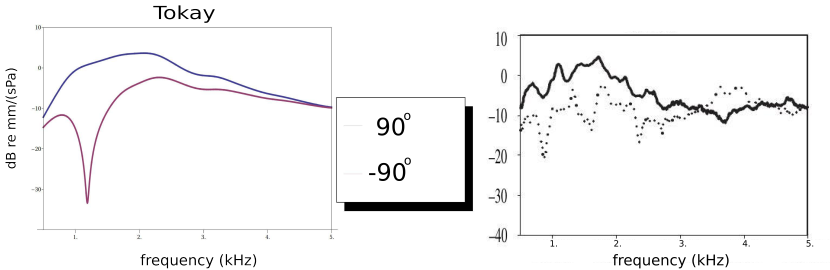
[a] Courtesy J.C. Dalsgaard (Syddansk Universitet)

Density Plot



- Plot of vibration amplitude w.r.t direction & frequency (Left: Calculated, Right: Experimental (Christensen-Dalsgaard et al, 2005).
- $|p_0| = |p_L| = 1$ Pa.

Frequency Dependence

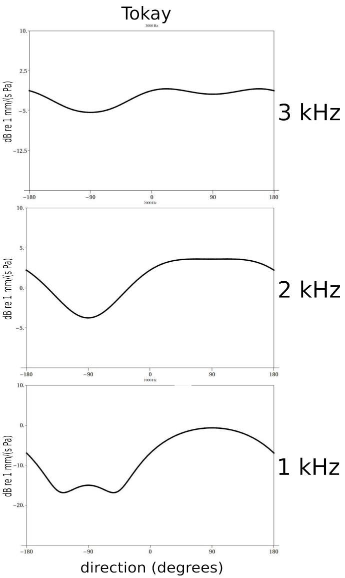


- Frequency dependence of $20\mathrm{Log}_{10}\left|\dot{S}^0/\pi a_{\mathrm{cyl}}^2\right|$ for $\theta = 90^\circ$. (Left:Calculated, Right:Experimental)
- $|p_0| = |p_L| = 1$ Pa.
- Ipsilateral response $>$ Contralateral response.

Direction Dependence.

- Inputs to ears
  - Negligible level (amplitude) difference
  - Small time (phase) difference
- Response is highly directional.

- Independent vibration amplitudes not enough.
- Localization requires using information from both ears.



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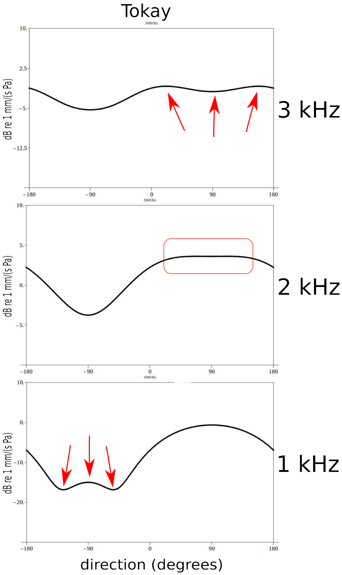

Hearing Cues

Internal Level Difference (iLD) -

$$\text{iLD} := 20\text{Log}_{10}\left|\frac{\dot{S}^0}{\dot{S}^L}\right|$$

Internal Time Difference (iTD) -

$$\text{iTD} := \text{Arg}\left(\frac{\dot{S}^0}{\dot{S}^L}\right)/\omega$$

Requirements

Both:

1. increase with the nearness of the sound source.
2. vanish at $\theta = 0°,\ \pm 180°$.

iTD:

$\sim$constant in a certain frequency range.

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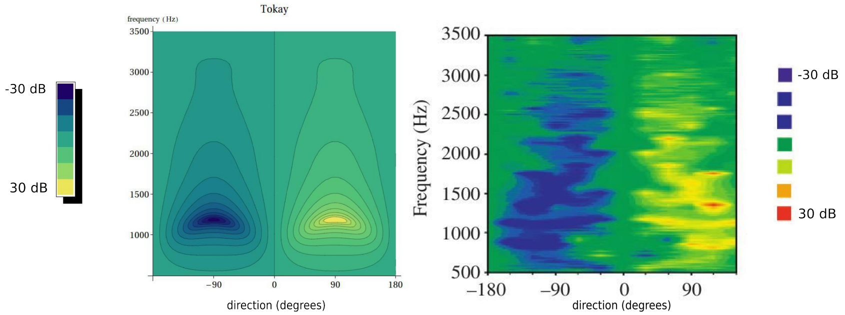
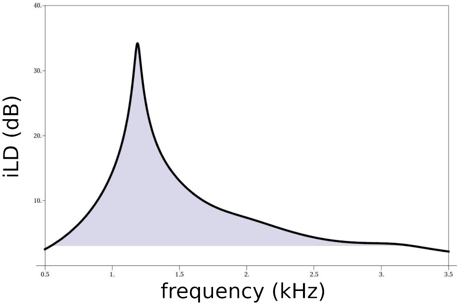
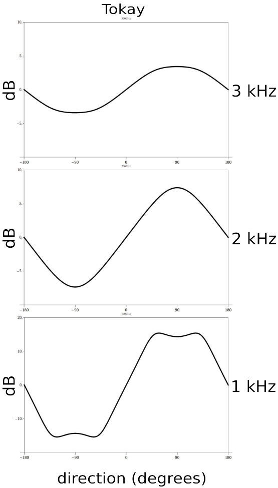
iLD Density Plot



- Plot of iLD, against frequency and direction.
- Left: Calculated, Right: Experimental

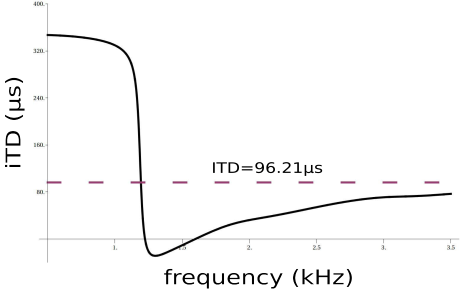
iLD Frequency/Direction Dependence

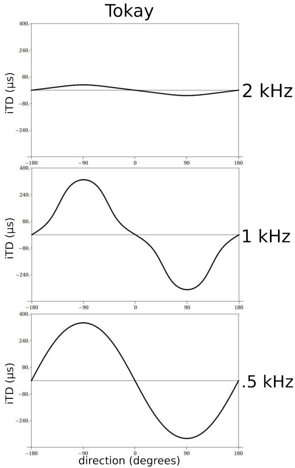




- iLD is a better cue at higher frequencies.
- Peak response at $\sim f_0\sqrt{1+1/4Q^2}$.

iTD Frequency/Direction Dependence

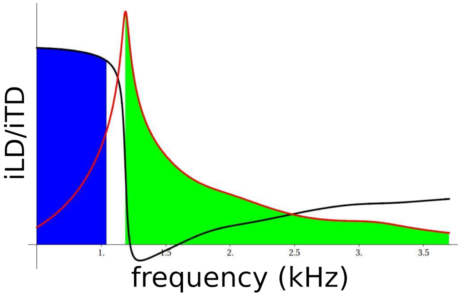




- iTD is a better cue at lower frequencies.
- constant up to $\sim f_0$.
- iTD $\approx 3\times$ITD

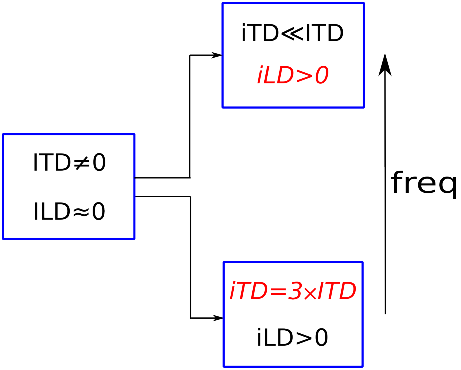
iTD/iLD Frequency Regimes

- Presence of phase and amplitude sensitive neurons.
- Transition-frequency determined by $f_0,\ \alpha$. NOT by head size.
- Possibile regime where both cues can simultaneously be used.



Conclusion

- Single description for both regimes.
- Transition determined by f_0, α.



Future Work

- Extracolumella motion.
- Crocodiles
  - Independent above water.
  - ICE underwater.

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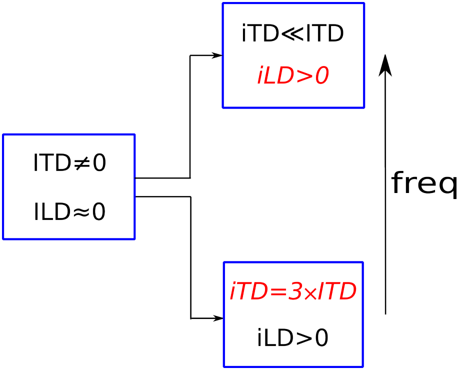

Thank You