Mechanical Processing in Internally Coupled Ears

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TMP Thesis Defence
July 20, 2013

Outline for Section 1

Auditory Systems

Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.

Coupled Ears

Wide eustachian tubes open into the mouth cavity.

Eardrums vibrations influence eachother.

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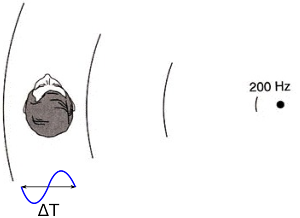
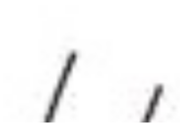
Binaural Hearing Cues





Interaural Time Difference

Phase difference between the (pressure) **inputs** to the ears.

- ITD
- Effective when $\lambda \gg$head size.

Interaural Level Difference

Amplitude difference between the **inputs**.

- ILD
- Effective when $\lambda \sim$head size.

Binaural Hearing Cues contd.

Internal Time Difference

Phase difference between the eardrum **vibrations**.

- **i**TD

Internal Level Difference

Amplitude difference between the **vibrations**.

- **i**LD

Coupled Ears

Input

- ITD$\neq$0, "small".
- ILD$\approx$0.

Response

- **i**TD>ITD.
- **i**LD>0.

Binaural Hearing Cues contd.

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Phase difference between the eardrum **vibrations**.

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- **i**TD$>$ITD.
- **i**LD$>$0.

Outline for Section 2

ICE Model

- An extension of the model first presented by Vossen et al (2010).
- Circular eardrums connected by a cylindrical cavity.

- The Aim is to accurately reproduce the direction and frequency dependence of the system.

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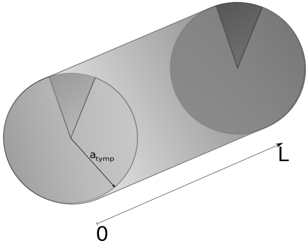
Mouth Cavity

Previous Model



a_{tymp} fixed.

$$V_{\mathrm{cyl}} = \pi a_{\mathrm{tymp}}^2 L$$

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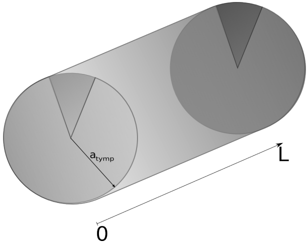

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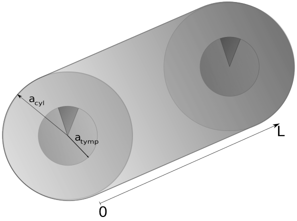
Our Model



$a_{\text{tymp}},\ V_{\text{cyl}}$ fixed.

$a_{\text{cyl}} = \sqrt{V_{\text{cyl}}/\pi L}$

Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x,r,\phi,t) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p(x,r,\phi,t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p(x,r,\phi,t)}{\partial \phi^2} + \frac{\partial p(x,r,\phi,t)}{\partial x^2}$$

Solve using the separation ansatz

$$p(x,r,\phi,t) = f(x)g(r)h(\phi)e^{j\omega t}.$$

Separated Equations

x- and ϕ- directions

$$f(x) = Ae^{j\zeta x} + Be^{-j\zeta x}$$
$$h(\phi) = Ce^{jq\phi} + De^{-jq\phi}$$

$$h(\phi) \xrightarrow{cont.,smooth} \cos q\phi$$
$$q = 0, 1, 2, 3, \ldots$$

r-direction, Bessel functions

$$g(r) = J_q(\nu r)$$
$$\nu^2 = k^2 - \zeta^2$$

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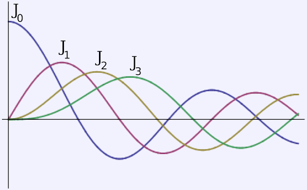


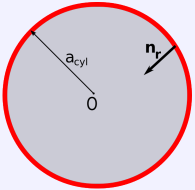
Boundary Conditions - r

Impenetrable boundary at $r = a_{\text{cyl}}$

$$j\omega\rho\mathbf{v} = \nabla p(x, r, \phi; t)$$

$$\mathbf{n_r}.\ \nabla p\big|_{r=a_{\text{cyl}}} \equiv \frac{\partial g}{\partial r}\bigg|_{r=a_{\text{cyl}}} = 0$$

$$\Rightarrow g(r) = J_q(\nu_{\text{qs}} r / a_{\text{cyl}})$$



Bessel Prime Zeros

ν_{qs} - zeros of J_q'.

ν_{00}=0.

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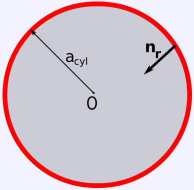

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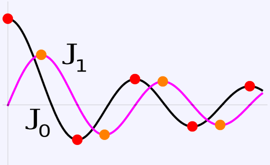


General Solution

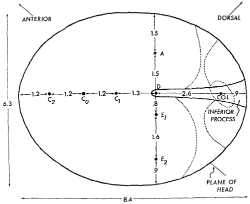
Pressure Modes

$$p(x,r,\phi,t) = \sum_{q=0,s=0}^{\infty} \left[A_{\mathrm{qs}} e^{j\zeta_{\mathrm{qs}} x} + B_{\mathrm{qs}} e^{-j\zeta_{\mathrm{qs}} x}\right] p_{\mathrm{qs}}(r,\phi) e^{j\omega t}$$

$$p_{\mathrm{qs}}(r,\phi) = \cos q\phi J_q(\nu_{\mathrm{qs}} r / a_{\mathrm{cyl}})$$

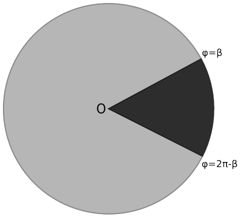
Eardrum

Sketch of a Tokay eardrum as seen from the outside (G.A. Manley, 1972).



COL - approximate position opposite the extracolumella insertion.

The ICE Eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - linear elastic.

Clamped at boundaries $(r = a_{\text{tymp}},\ \phi = \beta,\ 2\pi - \beta)$

Membrane Vibrations

Membrane EOM

$$-\partial_t^2 u(r,\phi;t) - 2\alpha\partial_t u(r,\phi;t) + c_M^2 \Delta_{(2)} u(r,\phi;t) = \frac{1}{\rho_m d}\Psi(r,\phi;t)$$

Membrane parameters

α - damping coefficient, c_M - propagation velocity

ρ_m - density, d - thickness.

Free-Undamped Membrane, $\alpha \to 0$, $\Psi \to 0$

Separation Ansatz

$$u(r, \phi; t) = f(r)g(\phi)h(t)$$

Boundary Conditions

ϕ-direction: $u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$

$$\Rightarrow g(\phi) = \sin \kappa(\phi - \beta)$$
$$\kappa = \frac{m\pi}{2(\pi - \beta)}, m = 1, 2, \ldots$$

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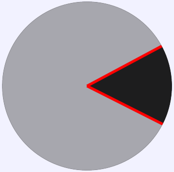

$$\Rightarrow g(\phi) = \sin\kappa(\phi-\beta)$$
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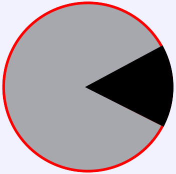
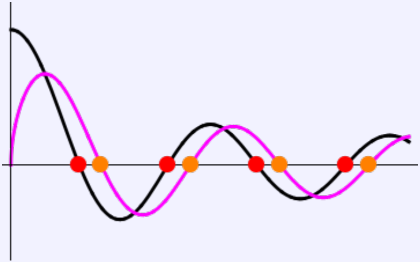
Boundary Conditions contd.

r-direction: $u(a_{\mathrm{tymp}}, \phi; t) = 0$

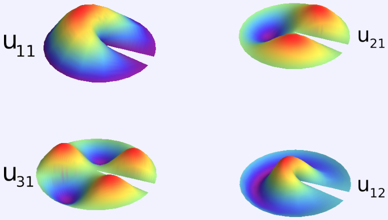


$$\Rightarrow f(r) = J_\kappa(\mu_{\mathrm{mn}} r / a_{\mathrm{tymp}})$$

μ_{mn}: n^{th} zero of J_κ



Free Membrane Eigenmodes - $u_{\mathrm{mn}}(r, \phi)$



Eigenfrequency: $\omega_{\mathrm{mn}} = c_M \mu_{\mathrm{mn}}$

u_{mn}: Orthogonal basis.

Forced Vibrations: $\Psi = \mathrm{p}e^{j\omega t}$, $\alpha \neq 0$

Steady State Solution

$$u_{\mathrm{ss}}(r,\phi;t) =: \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega t}$$

Substitute u_{ss} in Membrane EOM.

$$C_{\mathrm{mn}} = \frac{p \int dS u_{\mathrm{mn}}}{\Omega_{\mathrm{mn}} \int dS u_{\mathrm{mn}}^2}$$

$$\Omega_{\mathrm{mn}} = \rho_M d \left[(\omega^2 - \omega_{\mathrm{mn}}^2) - 2j\alpha\omega \right]$$

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Forced Vibrations contd.

Transient Solution

Same as the solution for a free damped membrane

$$u_{\mathrm{t}}(r,\phi;t)=\sum_{m=0,n=1}^{\infty}\widetilde{C}_{\mathrm{mn}}u_{\mathrm{mn}}(r,\phi)e^{j\widetilde{\omega}_{\mathrm{mn}}t-\alpha t}$$

$\widetilde{\omega}_{\mathrm{mn}}=\sqrt{\omega_{\mathrm{mn}}^2+\alpha^2}$
For $u(t=0)=0$, $\widetilde{C}_{\mathrm{mn}}=-C_{\mathrm{mn}}$.

Steady State Approximation

$u_{\mathrm{t}}\to 0$ exponentially as $t\to\infty$.

$$u\approx u_{\mathrm{ss}}.$$

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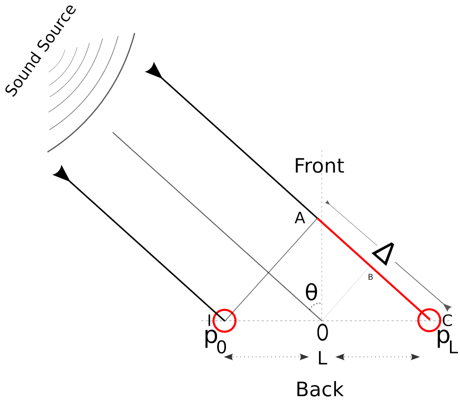
Acoustic Head Model

- **I** - Ipsilateral, **C** - Contralateral.
- $p_0, \; p_L$ - Sound pressure.
- θ - Source direction.
- Sound source “far away”.



- $|p_0| = |p_L|$.
- Phase difference - $\Delta = kL \sin\theta$.

- $p_0 = \text{pexp}\left(-j\Delta/2\right)$
  $p_L = \text{pexp}\left(j\Delta/2\right)$
- ILD=0, ITD=L/c

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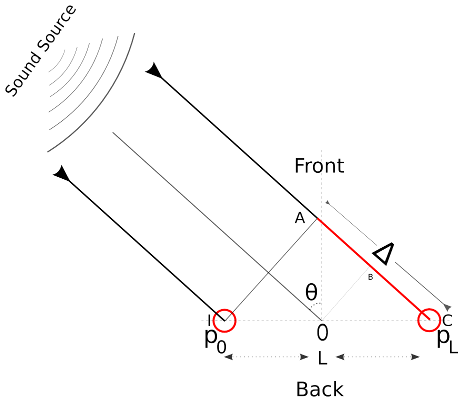

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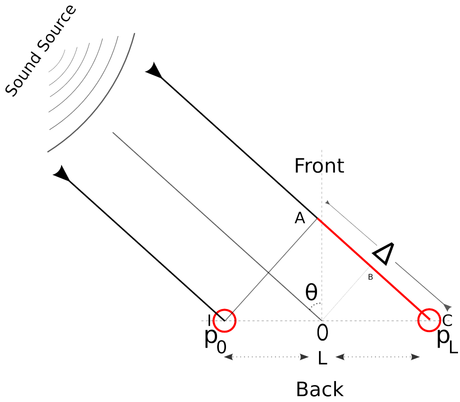

Acoustic Head Model

- $|p_0| = |p_L|$.
  - Increased phase difference due to diffraction - $\Delta = \frac{3}{2}kL\sin\theta$.
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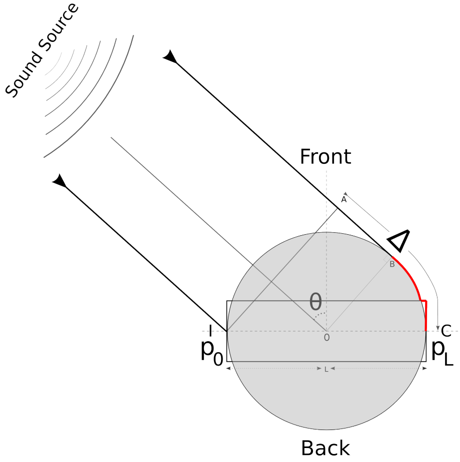
  $p_L = \text{pexp}\left(j\Delta/2\right)$
- ILD=0, ITD=$\frac{3}{2}$L/c



Coupled Membranes

$$u_{0/L} = \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}}^{0/L} u_{\mathrm{mn}}(r,\phi) e^{j\omega t}$$

Membrane Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C_{\mathrm{mn}}^{0} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} = p_0 e^{j\omega t} - p(0,r,\phi;t)$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C_{\mathrm{mn}}^{L} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} = p_L e^{j\omega t} - p(L,r,\phi;t)$$

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Boundary Conditions

Exact

$$u_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t)$$

$$u_L = \frac{1}{j\omega} v_x(L, r, \phi; t)$$

Approximate - $S^{0/L}(t) =: (\pi a_{\text{cyl}}^2)^{-1} \int dS U_{0/L}$

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Boundary Conditions

$\int dS \mathsf{p}_{qs} = 0 \Rightarrow$ higher pressure modes disappear, i.e.

$$p = \left[A_{00}e^{jkx} + B_{00}e^{-jkx}\right] e^{j\omega t}$$

Coupled Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C^0_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) = p_0 + \frac{\rho\omega^2}{k}\left(\frac{S^0}{\tan kL} + \frac{S^L}{\sin kL}\right)$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C^L_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) = p_L + \frac{\rho\omega^2}{k}\left(\frac{S^0}{\sin kL} + \frac{S^L}{\tan kL}\right)$$

Decoupling - $\pm$

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C^+_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) = p_+ + \frac{\rho\omega^2}{k} S^+ \cot\frac{kL}{2}$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{\mathrm{mn}} C^-_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) = p_- - \frac{\rho\omega^2}{k} S^- \tan\frac{kL}{2}$$

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Decoupled Equations contd.

$$S^+ = \frac{p_L + p_0}{\Lambda + \Gamma_+} \qquad S^- = \frac{p_L - p_0}{\Lambda + \Gamma_-}$$

$$\Gamma_+ = -\frac{\rho\omega^2}{k}\cot\frac{kL}{2}, \qquad \Gamma_- = \frac{\rho\omega^2}{k}\tan\frac{kL}{2}$$

$$\frac{1}{\Lambda} = \frac{1}{\pi a_{\mathrm{cyl}}^2}\sum_{m=0,n=1}^{\infty}\frac{\left(\int dS u_{\mathrm{mn}}\right)^2}{\Omega_{\mathrm{mn}}\int dS u_{\mathrm{mn}}^2}$$

Final Expressions

Membrane Displacement

$$S_0(t) = G^s_{ipsi} p_0 + G^s_{contra} p_L$$
$$S_L(t) = G^s_{contra} p_0 + G^s_{ipsi} p_L$$

$$G^s_{ipsi} = \left(\frac{1}{\Lambda + \Gamma_+} + \frac{1}{\Lambda + \Gamma_-} \right) /2$$
$$G^s_{contra} = \left(\frac{1}{\Lambda + \Gamma_+} - \frac{1}{\Lambda + \Gamma_-} \right) /2$$

Outline for Section 3

Tokay Gecko

L$=22$ mm $a_{\text{tymp}}=2.6$ mm

$f_0 = 1.05$ kHz $\rho_m=1$ mg/mm^3

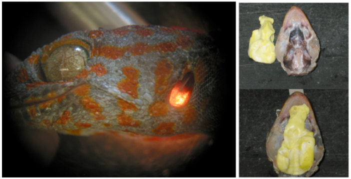
$d=10\,\mu$m $V_{\text{cav}}=3.5$ ml

$\beta=\pi/25$ $a_{cyl} \approx 6.6$ mm

$\alpha \approx 2474$ s^{-1}

$f_0 = \omega_{01}/2\pi$, ITD $= 96.21\,\mu$s.

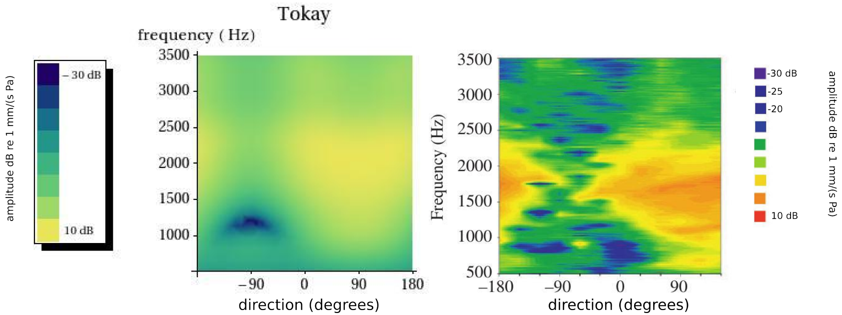
$\alpha \rightarrow$ steady state.



Tokay gecko. Left: Head illuminated from the opposite side[a]. Right: Mouth casts.

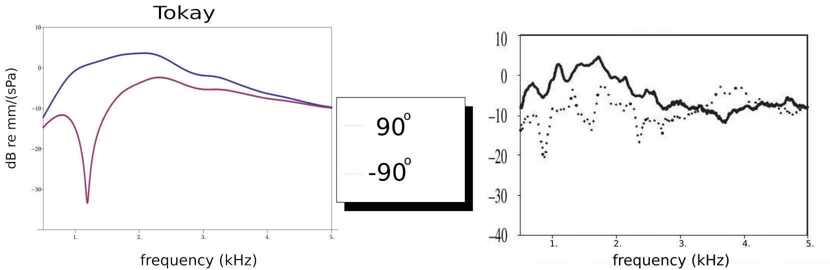
[a] Courtesy J.C. Dalsgaard (Syddansk Universitet)

Density Plot



- Plot of vibration amplitude w.r.t direction & frequency (Left: Calculated, Right: Experimental (Christensen-Dalsgaard et al, 2005).
- $|p_0| = |p_L| = 1$ Pa.

Frequency Dependence

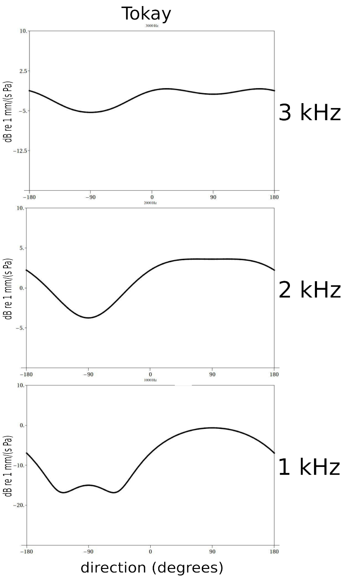


- Frequency dependence of $20\mathrm{Log}_{10}\left|\dot{S}^0/\pi a_{\mathrm{cyl}}^2\right|$ for $\theta = 90^\circ$. (Left:Calculated, Right:Experimental)
- $|p_0| = |p_L| = 1$ Pa.
- Ipsilateral response $>$ Contralateral response.

Direction Dependence.

- Inputs to ears
  - Negligible level (amplitude) difference
  - Small time (phase) difference
- Response is highly directional.

- Independent vibration amplitudes not enough.
- Localization requires using information from both ears.



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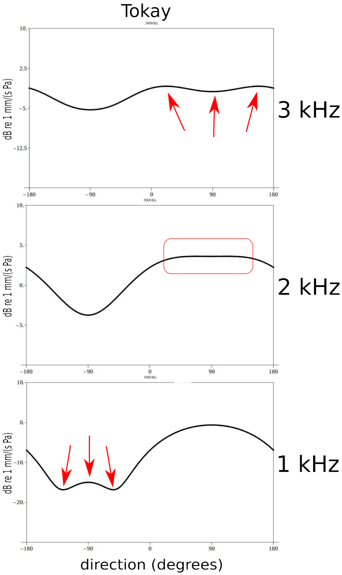

Hearing Cues

Internal Level Difference (iLD) -

$$\text{iLD} := 20\text{Log}_{10}\left|\frac{\dot{S}^0}{\dot{S}^L}\right|$$

Internal Time Difference (iTD) -

$$\text{iTD} := \text{Arg}\left(\frac{\dot{S}^0}{\dot{S}^L}\right)/\omega$$

Requirements

Both

1. increase with the nearness of the sound source.
2. vanish at $\theta = 0°, \pm 180°$.

iTD

$\sim$constant in a certain frequency range.

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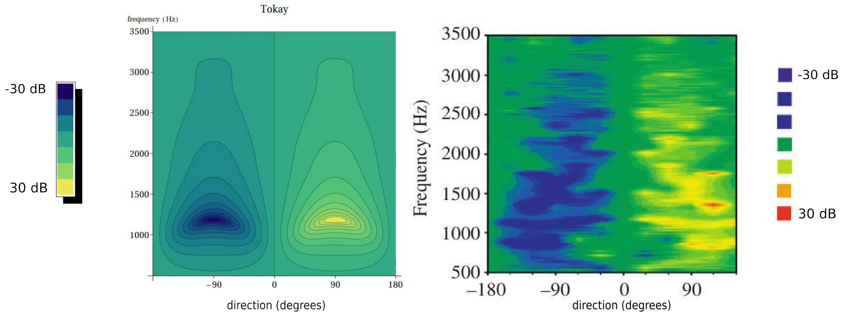
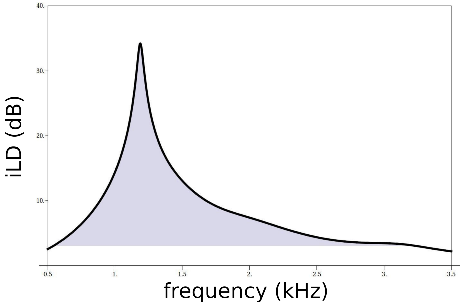
iLD Density Plot

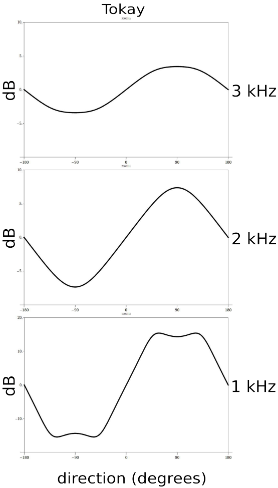


- Plot of iLD, against frequency and direction.
- Left: Calculated, Right: Experimental

iLD Frequency/Direction Dependence



- iLD is a better cue at higher frequencies.
- Peak response at $\sim f_0\sqrt{1+1/4Q^2}$.



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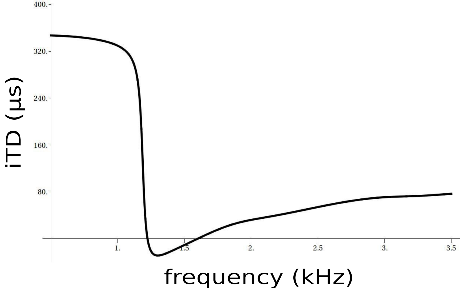
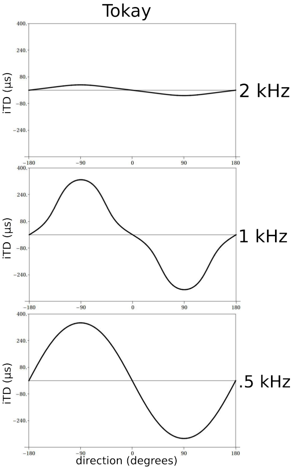
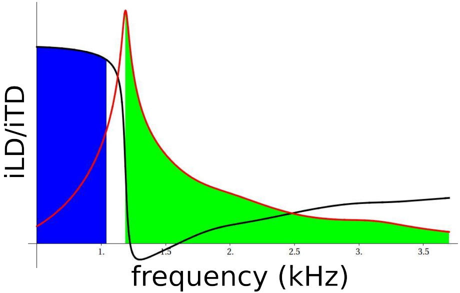




- iTD is a better cue at lower frequencies.
- constant up to $\sim f_0$.
- iTD $\approx 3\times$ITD

iTD/iLD Frequency Regimes

- The iTD to iLD tFransition-frequency is determined by $f_0,\ \alpha$.
- Possibile regime where both cues can simultaneously be used.



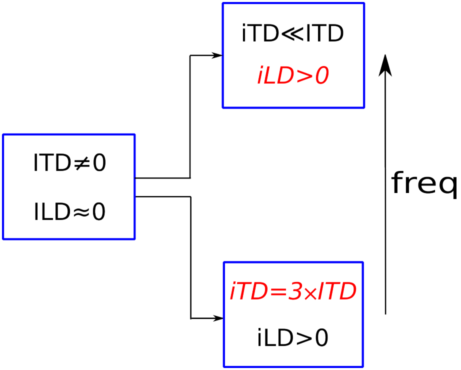
Outline for Section 4

Conclusion

- Single description for all frequencies.
- Transition determined by f_0, α.

Future Work

- Extracolumella motion.
- Crocodiles
  - Independent above water.
  - ICE underwater.



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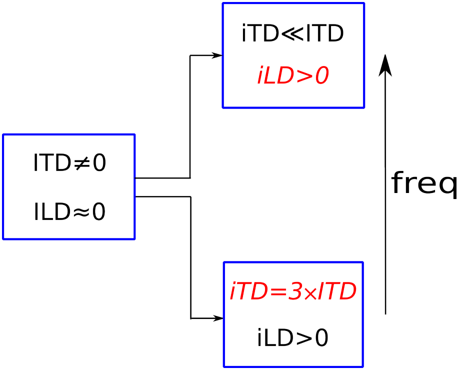

Thank You