Mechanical Processing in Internally Coupled Ears

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The Model

Mouth Cavity Acoustic Head Model Eardrum

Outline for section 1

Evaluation

Conclusio

Introduction

Auditory Systems



Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.



Wide eustachian tubes open into the mouth cavity.

Eardrums vibrations influence eachother.

Binaural Hearing Cues

Direction and frequency dependent phase and amplitude differences between the ears.

Interaural Time Difference

Equivalent to phase difference between membrane vibrations.

Interaural Level Difference

Equivalent to amplitude difference between membrane vibrations.

Advantages of Coupled Ears

The Model

► Low frequencies result in reduced degradation of hearing cues in dense environments.

Introduction

Outline for section 2

Introduction

The Model

Mouth Cavity

Acoustic Head Model

Eardrum

Coupled Membranes

Evaluation

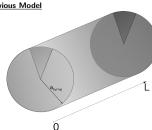


Previous Model

Mouth Cavity

Introduction

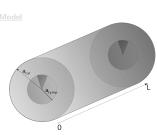
Mouth Cavity



The Model

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$$a_{
m tymp}$$
 fixed. $V_{
m cyl} = \pi a_{
m tymp}^2 L$



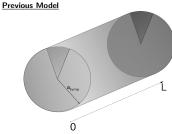
$$2 \cdot 1 = \sqrt{V \cdot 1/\pi I}$$

Evaluation

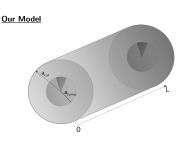
Mouth Cavity

Introduction

Mouth Cavity



 a_{tymp} fixed. $V_{\rm cyl} = \pi a_{
m tymp}^2 L$



 $a_{\rm tymp}, \ V_{\rm cyl}$ fixed.

$$a_{\rm cyl} = \sqrt{V_{\rm cyl}/\pi L}$$

Introduction

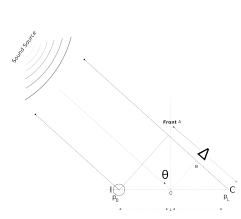
Acoustic Head Model

I - Ipsilateral ear, C Contralateral ear.
 p₀, p_L - sound pressure on
eardrums, θ - sound source
direction.

The Model

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- ► Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_I|$.
- ▶ Phase difference between sound at both ears $\Delta = kL \sin \theta$.
- $p_0 = pe^{j\omega t .5kL\sin\theta}$ $p_L = pe^{j\omega t + .5kL\sin\theta}$



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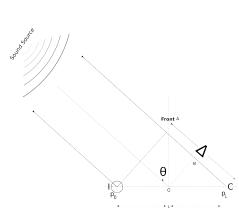
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Acoustic Head Model

- I Ipsilateral ear, C Contralateral ear.
 p₀, p_L sound pressure on
 eardrums, θ sound source
 direction
- Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_I|$.
- ▶ Phase difference between sound at both ears $\Delta = kL \sin \theta$.

$$p_0 = pe^{j\omega t - .5kL\sin\theta}$$

$$p_L = pe^{j\omega t + .5kL\sin\theta}$$





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Internally Coupled Ears

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Introduction

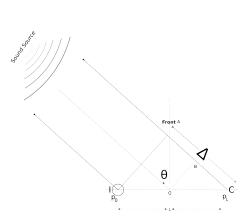
Acoustic Head Model

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The Model

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- Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_L|$.
- ▶ Phase difference between sound at both ears $\Delta = kL \sin \theta$.
- $p_0 = pe^{j\omega t .5kL\sin\theta}$ $p_L = pe^{j\omega t + .5kL\sin\theta}$



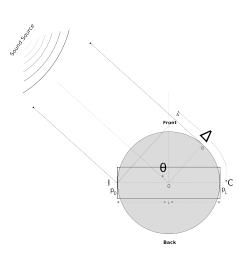
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Acoustic Head Model

Acoustic Head Model

- $|p_0| = |p_L|.$
- ▶ Increased phase difference due to diffraction $\Delta = 1.5kL \sin \theta$.
- $p_0 = p e^{j\omega t .75kL\sin\theta}$ $p_L = p e^{j\omega t + .75kL\sin\theta}$



Introduction

Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x, r, \phi, t) = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p(x, r, \phi, t)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p(x, r, \phi, t)}{\partial \phi^2} + \frac{\partial p(x, r, \phi, t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}$$
.



Introduction

Acoustic Head Model

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$
$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$$

The Model

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$$\frac{1}{d\phi^2} + q n(q)$$

$$ightarrow$$
 $h(\phi)$ =

Evaluation

$$)=e^{\pm j\zeta x}$$

(3)

Conclusion



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Acoustic Head Model

Introduction

$$x$$
- and ϕ - directions

$$rac{d^2f(x)}{dx^2} + \zeta^2f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$
 $rac{d^2h(\phi)}{d\phi^2} + q^2h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$

The Model

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Evaluation

(2)

(3)

 $\phi \to h(0) = h(2\pi)$ and $h'(0) = h'(2\pi)$

 $\Rightarrow h(\phi) = \cos q\phi, \ q = 0, 1, 2, \dots$

Evaluation

Conclusion

(5)

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The Model

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Introduction

Acoustic Head Model

Internally Coupled Ears

Boundary Conditions - ϕ

Impenetrable boundary at $r = a_{cvl}$, i.e. normal derivative

$$-j\rho\omega\mathbf{v} = \mathbf{n}. \left.\nabla p(\mathbf{x}, r, \phi; t)\right|_{r=\mathbf{a}_{\mathrm{cyl}}} \equiv \left.\frac{\partial \mathbf{g}}{\partial r}\right|_{r=\mathbf{a}_{\mathrm{cyl}}} = 0$$
 (6)

Evaluation

$$\Rightarrow g(r) = J_q(\nu_{qs}r/a_{cyl}) \tag{7}$$

Bessel Prime 7eros

Boundary Conditions - r

$$\blacktriangleright$$
 ν_{qs} - zeros of J_{a}' , $s=0,1,2,\ldots$

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$$\nu_{00}=0$$

Introduction

Acoustic Head Model

Conclusion

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Introduction

Acoustic Head Model

$$p(x, r, \phi, t) = \sum_{q=0, s=0}^{\infty} p_{qs}(x, r, \phi) e^{j\omega t}$$

$$p_{qs}(x, r, \phi) = \left[A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] \cos q\phi J_q(\nu_{qs}r/a_{cyl})$$
where, $\zeta_{qs} = \sqrt{k^2 - \nu_{qs}^2/a_{cyl}^2}$ (9)

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$$p_{00}(x, r, \phi; t) = \left[A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t}$$
 (10)

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The Model

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$$p_{\mathrm{qs}}(x,r,\phi) =$$

$$= \left[A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] \cos q\phi$$
$$= \sqrt{k^2 + k^2 + k^2}$$

Evaluation

$$p_{00}(x, r, \phi; t) = \left[A_{00}e^{jkx} + B_{00}e^{-jkx}\right]e^{j\omega t}$$

Conclusion

Introduction

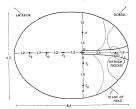
Acoustic Head Model

General Solution

Eardrum

Eardrum

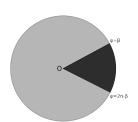
Sketch of a Tokay eardrum as seen from the outside^a.



COL - approximate position opposite the extracolumella insertion.

^aG. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r=a_{\mathrm{tymp}}$ and $\phi=\beta,\ 2\pi-\beta$)

$$lpha$$
 - damping coefficient, $\ c_M^2$ - propagation velocity

d - thickness.

The Model

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 ρ_m - density,

Introduction

Eardrum

Membrane parameters

Conclusion

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The Model

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Introduction

$$u(r, \phi; t) = f(r)g(\phi)h(t)$$

Separation Ansatz

$$d^2g(\phi)$$

$$\frac{d^2g(\phi)}{d\phi^2} + \kappa^2g(\phi) = 0$$

Evaluation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f(r)}{\partial r}\right) + \left[\mu^2 - \frac{\kappa^2}{r^2}\right]f(r) = 0$$

$$\frac{d^2h(t)}{dt} + c_1^2\mu^2h(t) = 0$$

(12)

(13)

(14)

Conclusion

 $\frac{d^2h(t)}{dt^2} + c_M^2\mu^2h(t) = 0$

The Model

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$$\phi\text{-direction: } u(r,\beta;t) = u(r,2\pi - \beta,t) = 0$$

$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$
where, $\kappa = \frac{m\pi}{2(\pi - \beta)}, \quad m = 1,2,3,...$ (16)

r-direction:
$$u(a_{\mathrm{tymp}}, \phi; t) = 0$$

$$\Rightarrow f(r) = J_{\kappa}(\mu_{\mathrm{mn}} r / a_{\mathrm{tymp}})$$
where, μ_{mn} is the n^{th} zero of J_{κ} (17)

Introduction

Eardrum

Introduction

Eardrum

$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$

where,
$$\kappa=\dfrac{m\pi}{2(\pi-eta)}, \quad \emph{m}=1,2,3,\ldots$$

The Model

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$$\phi \cdot t = 0$$

r-direction:
$$u(a_{\text{tymp}}, \phi; t) = 0$$

(16)

$$\Rightarrow$$
 $f(r) = J_{\kappa}(\mu_{
m mn} r/a_{
m tymp})$ where, $\mu_{
m mn}$ is the $n^{
m th}$ zero of J_{κ}

 $u_{\rm mn}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{\rm mn} r)$ (18) $u_{
m free}(r,\phi;t) = \sum_{}^{\infty} C_{
m mn} u_{
m mn}(r,\phi) e^{j\omega_{
m mn}t}$ (19)

> m = 0, n = 1where, $\omega_{\rm mn} = c_M \mu_{\rm mn}$

The Model

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Free eigenmodes

$$\widetilde{u}_{ ext{free}}(r,\phi;t) = \sum_{m=0}^{\infty} \widetilde{C}_{ ext{mn}} u_{ ext{mn}}(r,\phi) e^{j\omega_{ ext{mn}}t - \alpha t}$$
 (20)

Introduction

Eardrum

Free eigenmodes

Introduction

$$u_{\rm mn}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{\rm mn} r)$$

$$u_{\rm free}(r,\phi;t) = \sum_{m}^{\infty} C_{\rm mn} u_{\rm mn}(r,\phi) e^{j\omega_{\rm mn} t}$$
(18)

Evaluation

$$_{m=0,n=1}^{m=0,n=1}$$
 where, $\omega_{
m mn}=c_{M}\mu_{
m mn}$

The Model

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Damped membrane

$$\widetilde{u}_{\mathrm{free}}(r,\phi;t) = \sum_{n=0}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t - \alpha t}$$
 (20)

m = 0, n = 1

 $u_{\rm ss}(r, \varphi, \iota) = \sum_{m=0, n=1}^{\infty}$

Forced Vibrations: $\Psi = pe^{j\omega t}$

Introduction

Eardrum

Substitute $u_{\rm ss}$ in Membrane EOM.

in Membrane EOM.

The Model

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$$C_{\rm mn} = \frac{p \int dS u_{\rm mn}}{Q_{\rm mn} \int dS u_{\rm mn}^2}$$

$$\Omega_{\rm mn} = \Omega_{\rm mn} \int dS u_{\rm mn}^2$$

Conclusion

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Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

$$u_{\mathrm{ss}}(r,\phi;t) =: \sum_{m=0,n=1} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega t}$$

The Model

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Substitute u_{ss} in Membrane EOM.

$$egin{aligned} C_{ ext{mn}} &= rac{p \int dS u_{ ext{mn}}}{\Omega_{ ext{mn}} \int dS u_{ ext{mn}}^2} \ \Omega_{ ext{mn}} &=
ho_{ ext{M}} d \left[(\omega^2 - \omega_{ ext{mn}}^2) - 2jlpha \omega
ight] \end{aligned}$$

Introduction

Conclusion

(21)

(22)

Eardrum

Forced Vibrations contd.

Transient Solution

Same as the solution for a free damped membrane

$$u_{\mathrm{t}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t - \alpha t}$$

 \widetilde{C}_{mn} determined from the membrane displacement at t=0.

$$u_{\rm t} \to 0$$
 as $t \to \infty$.

(23)

The Model

Mouth Cavity Acoustic Head Model Eardrum

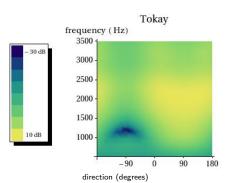
Outline for section 3

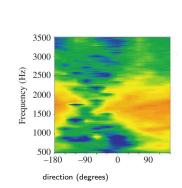
Evaluation

Conclusio

Introduction

Vibration Amplitude





The Model

Mouth Cavity
Acoustic Head Model
Eardrum
Coupled Membranes

Outline for section 4

Evaluation

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Introduction

The Model

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Introduction

Conclusion

The Model

Thank You

Introduction



