Exact Steady State Pressure in a Cylindrical Cavity

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0.1 Internal Cavity

Upon solving the wave equation inside the cylindrical, the steady state pressure disturbance p in the internal cavity is given by the following equation,

$$\delta p(x,r,\phi;t) = \sum_{m,n} \left[A_{mn}e^{j\zeta_{mn}x} + B_{mn}e^{-i\zeta_{mn}x}\right]\cos(m\phi)J_m(\nu_{mn}r)e^{j\omega t} \tag{1}$$

$$= \delta p(x,r,\phi)e^{j\omega t} \tag{2}$$

Where $m,n = 0,1,2,\ldots$. We have obtainde the above result after applying the following boundary conditions,

- $\mathbf{n}.\nabla\delta p = 0$ at $r = a$
- $\delta p(x,r,0;t) = \delta p(x,r,2\pi;t)$
- $\frac{\partial\delta p}{\partial\phi}|_{\phi=0} = \frac{\partial\delta p}{\partial\phi}|_{\phi=2\pi}$

The membrane equations of motion are given by,

$$-\ddot{u}_0 - 2\alpha\dot{u}_0 + c_M^2\nabla u_0 = \frac{1}{\rho_M d}\left[p_0e^{j\omega t} - \delta p(0,r,\phi;t)\right] \tag{3}$$

$$-\ddot{u}_L - 2\alpha\dot{u}_L + c_M^2\nabla u_L = \frac{1}{\rho_M d}\left[p_Le^{j\omega t} - \delta p(L,r,\phi;t)\right] \tag{4}$$

We assume the time component of the membrane displacements to be separable in the steady state,

$$u_{0/L}(r,\phi,t) = u_{0/L}(r,\phi)e^{j\omega t} \tag{5}$$

At the membrane surface inside the cavity, we equate the membrane velocity with the velocity of the air fluid particle. This gives us,

$$u_0(r,\phi) = \frac{1}{\rho\omega^2}\frac{\partial\delta p}{\partial x}|_{x=0} \tag{6}$$

$$u_L(r,\phi) = -\frac{1}{\rho\omega^2}\frac{\partial\delta p}{\partial x}|_{x=L} \tag{7}$$

Where we've used the convention that directions into the cavity are postive and those outward from the cavity are negative.

It might be tempting to use the expression for δp in (1) and substitute for $u_{0/L}$ in (3) and (4) to get expressions for the coefficients A_{mn} and B_{mn}. The problem here is that, there is no direct way to impose the membrane boundary conditions - i.e. $u_{0/L}(a, \phi; t) = 0$.

In order to study the pressure inside the membrane, we can consider the following simpler problem - Given a fixed velocity for both the membranes, what is the pressure distribution inside the cavity? This effectively means that we use the boundary conditions (6) and (7) to calculate the pressure coefficients.