Mechanical Processing in Internally Coupled Ears

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TMP Thesis Defence July 13, 2013





Outline for Section 1

Introduction

Introduction
Auditory Systems
Hearing Cues

The Mode

Mouth Cavity
Acoustic Head Mode
Pressure Derivation

Eardrum

Model

Coupled Membranes

Evaluation

Vibration Amplitude
Internal Level Difference
Internal Amplitude
Difference

Auditory Systems



Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.



Wide eustachian tubes open into the mouth cavity.

Eardrums vibrations influence eachother.

Hearing Cues

Binaural Hearing Cues

Localization using frequency dependent phase and amplitude differences between the ears.

Interaural Time Difference

Equivalent to phase difference between membrane vibrations.

Interaural Level Difference

Equivalent to amplitude difference between membrane vibrations.

Advantages of Coupled Ears

► Low frequencies result in reduced degradation of hearing cues in dense environments.

Outline for Section 2

Introduction

Hearing Cues

The Model

Mouth Cavity

Acoustic Head Mod Pressure Derivation

Eardrum

Model

Coupled Membranes

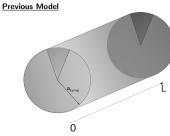
Roundany Condition

Evaluation

Vibration Amplitude Internal Level Difference Internal Amplitude

Introduction

Mouth Cavity



The Model

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 $a_{
m tymp}$ fixed. $V_{
m cyl} = \pi a_{
m tymp}^2 L$

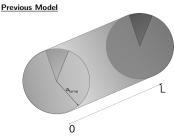


$$a_{\text{tymp}}, v_{\text{cyl}} \cap \lambda \in U$$

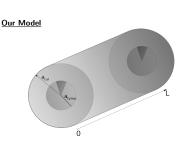


Mouth Cavity

Mouth Cavity



$$a_{
m tymp}$$
 fixed. $V_{
m cyl} = \pi a_{
m tymp}^2 L$

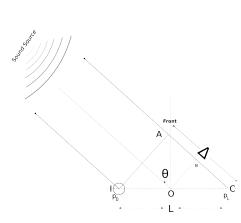


$$a_{
m tymp}, \; V_{
m cyl} \; {
m fixed}.$$
 $a_{
m cyl} = \sqrt{V_{
m cyl}/\pi L}$

Introduction

Acoustic Head Model

- I Ipsilateral ear, C Contralateral ear.
 p₀, p_L sound pressure on
 eardrums, θ sound source
 direction.
- ▶ Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_I|$.
- ▶ Phase difference between sound at both ears $\Delta = kL \sin \theta$.
- $p_0 = pe^{j\omega t .5kL\sin\theta}$ $p_L = pe^{j\omega t + .5kL\sin\theta}$



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Internally Coupled Ears

Introduction

▶ I - Ipsilateral ear, **C** - Contralateral ear.

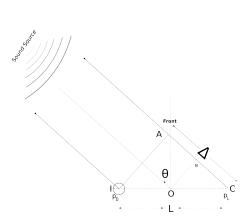
Acoustic Head Model

 p_0 , p_L - sound pressure of eardrums, θ - sound source direction.

- Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_L|$.
- ▶ Phase difference between sound at both ears $\Delta = kL \sin \theta$.

$$p_0 = pe^{j\omega t - .5kL\sin\theta}$$

$$p_L = pe^{j\omega t + .5kL\sin\theta}$$



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Internally Coupled Ears

Mouth Cavity

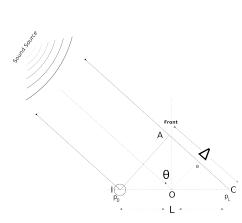
Introduction

Acoustic Head Model

I - Ipsilateral ear, C Contralateral ear.
 p₀, p_L - sound pressure on
eardrums, θ - sound source
direction.

The Model

- Sound source "far away".No appreciable amplitude
- difference, $|p_0| = |p_L|$.
- Phase difference between sound at both ears $\Delta = kL \sin \theta$.
- $p_0 = pe^{j\omega t .5kL\sin\theta}$ $p_L = pe^{j\omega t + .5kL\sin\theta}$



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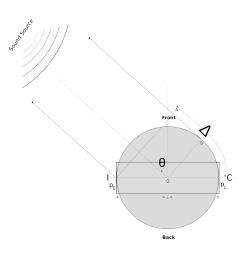
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Introduction

Mouth Cavity

Acoustic Head Model contd.

- $|p_0| = |p_L|.$
- ▶ Increased phase difference due to diffraction $\Delta = 1.5kL \sin \theta$.
- $p_0 = p e^{j\omega t .75kL\sin\theta}$ $p_L = p e^{j\omega t + .75kL\sin\theta}$



Mouth Cavity

Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x, r, \phi, t) = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p(x, r, \phi, t)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p(x, r, \phi, t)}{\partial \phi^2} + \frac{\partial p(x, r, \phi, t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}$$
.

Introduction

x- and
$$\phi$$
- directions
$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$
(2)
$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$$
(3)

The Model

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$$a\phi^2$$

r-direction, Bessel function

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q}{r}\right]$$

where,
$$v^2=k^2-\zeta^2$$

Introduction

$$egin{aligned} rac{d^2f(x)}{dx^2} + \zeta^2f(x) &= 0 \longrightarrow f(x) = e^{\pm j\zeta x} \ rac{d^2h(\phi)}{d\phi^2} + q^2h(\phi) &= 0 \longrightarrow h(\phi) = e^{\pm jq\phi} \end{aligned}$$

x- and ϕ - directions

$$\frac{2h(\phi)}{d\phi^2} + q^2h(\phi)$$

The Model 00000000

$$=e^{\pm jq\phi}$$

Conclusion

 $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q^2}{r^2}\right]g(r) = 0 \longrightarrow g(r) = J_q(\nu r)$ where, $\nu^2 = k^2 - \zeta^2$

$$f(x) = \epsilon$$

Evaluation

$$=e^{\pm j\zeta^{j}}$$

$$a^{\pm i\zeta x}$$

(5)

Conclusion

Evaluation

Introduction

Boundary Conditions -
$$\phi$$

Smoothness and Continuity in
$$\phi$$
.

The Model

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$$\Rightarrow h(\phi) = \cos q\phi, \ q = 0, 1, 2, \dots$$

Impenetrable boundary at $r = a_{cvl}$, i.e. normal derivative

The Model

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$$\Rightarrow g(r) = J_g(\nu_{\rm os} r/a_{\rm cyl}) \tag{7}$$

Evaluation

Bessel Prime Zeros

Boundary Conditions - r

$$\blacktriangleright$$
 $\nu_{\rm qs}$ - zeros of J_a' , $s=0,1,2,\ldots$

$$\nu_{\rm qs}$$
 - 20103 of J_q , 3 = 0, 1, 2,

$$\nu_{00} = 0$$

Introduction

Mouth Cavity

Conclusion

(6)

General Solution

Introduction

Pressure Modes

$$p(x, r, \phi, t) = \sum_{q=0, s=0} \left[A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] p_{qs}(r, \phi) e^{j\omega t}$$
 $p_{qs}(r, \phi) = \cos q\phi J_q(\nu_{qs}r/a_{cvl})$

The Model

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where,
$$\zeta_{\rm qs}=\sqrt{k^2-
u_{\rm qs}^2/a_{\rm cyl}^2}$$

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Conclusion

(8)

(9)

Conclusion

(10)

The Model

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Introduction

Mouth Cavity

Pressure Modes
$$p(x, r, \phi, t) = \sum_{q=0, s=0}^{\infty} \left[A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] p_{qs}(r, \phi) e^{j\omega t} \quad (8)$$

$$p_{qs}(r, \phi) = \cos q\phi J_q(\nu_{qs}r/a_{cyl}) \quad (9)$$
where, $\zeta_{qs} = \sqrt{k^2 - \nu_{rs}^2/a^2}$,

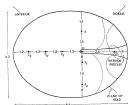
where,
$$\zeta_{\rm qs} = \sqrt{k^2 - \nu_{\rm qs}^2/a_{\rm cyl}^2}$$

Plane Wave Mode
$$p_{\rm pw}(x,r,\phi;t) = \left[A_{00}e^{jkx} + B_{00}e^{-jkx}\right]e^{j\omega t} \qquad (10)$$

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Eardrum

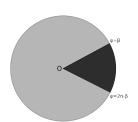
Sketch of a Tokay eardrum as seen from the outside^a.



COL - approximate position opposite the extracolumella insertion.

^aG. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r=a_{\mathrm{tymp}}$ and $\phi=\beta,\ 2\pi-\beta$)

The Model

Membrane Vibrations

Introduction

Membrane EOM

$$-\partial_t^2 u(r,\phi;t) - 2\alpha \partial_t u(r,\phi;t) + c_M^2 \Delta_{(2)} u(r,\phi;t) = \frac{1}{\rho_m d} \Psi(r,\phi;t)$$
(11)

Membrane parameters

$$lpha$$
 - damping coefficient, $\ c_M^2$ - propagation velocity

$$\rho_m$$
 - density, d - thickness.

 $u(r, \phi; t) = f(r)g(\phi)h(t)$

Evaluation

Free-Undamped Membrane, $\alpha \rightarrow$ 0, $\Psi \rightarrow$ 0

The Model

Introduction

$$\frac{d^2g(\phi)}{d\phi^2} + \kappa^2g(\phi) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f(r)}{\partial r}\right) + \left[\mu^2 - \frac{\kappa^2}{r^2}\right]f(r) = 0$$

$$\frac{d^2h(t)}{dt^2} + c_M^2\mu^2h(t) = 0 {(15)}$$

Conclusion

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(13)

(14)

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φ-direction:
$$u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$$

$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$
where, $\kappa = \frac{m\pi}{2(\pi - \beta)}$, $m = 1, 2, 3, ...$ (16)

r-direction:
$$u(a_{\mathrm{tymp}}, \varphi; \tau) = 0$$

$$\Rightarrow f(r) = J_{\kappa}(\mu_{\mathrm{mn}} r / a_{\mathrm{tymp}})$$
where, μ_{mn} is the n^{th} zero of J_{κ} (17)

Introduction

Fardrum

Boundary Conditions

Boundary Conditions

Introduction

φ-direction:
$$u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$$

$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$
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r-direction:
$$u(a_{\text{tymp}}, \phi; t) = 0$$

The Model

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$$\Rightarrow f(r) = J_{\kappa}(\mu_{
m mn} r/a_{
m tymp})$$
 where, $\mu_{
m mn}$ is the $n^{
m th}$ zero of J_{κ}

(17)

$$u_{\mathrm{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t}$$

where, $\omega_{\mathrm{mn}} = c_{M} \mu_{\mathrm{mn}}$

The Model

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Free eigenmodes

$$\widetilde{u}_{\text{free}}(r,\phi;t) = \sum_{m=1}^{\infty} \widetilde{C}_{mn} u_{mn}(r,\phi) e^{j\omega_{mn}t - \alpha t}$$
 (20)

Introduction

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Eardrum

Free eigenmodes

$$u_{\mathrm{mn}}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{\mathrm{mn}} r)$$
 (18)
 $u_{\mathrm{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}} t}$ (19)
where, $\omega_{\mathrm{mn}} = c_{M} \mu_{\mathrm{mn}}$

Damped membrane

$$\widetilde{u}_{\mathrm{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t - \alpha t}$$
 (20)

m = 0, n = 1

Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

Substitute u_{ss} in Membrane EOM.

The Model

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$$C_{\rm mn} = \frac{p \int dS u_{\rm mn}}{Q \int dS u_{\rm mn}^2}$$

$$C_{\rm mn} = \frac{1}{\Omega_{\rm mn} \int dS u_{\rm mn}^2}$$

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Eardrum

Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

$$u_{\mathrm{ss}}(r,\phi;t) =: \sum_{m=0,n=1} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega t}$$

Substitute u_{ss} in Membrane EOM.

$$egin{aligned} C_{ ext{mn}} &= rac{p \int dS u_{ ext{mn}}}{\Omega_{ ext{mn}} \int dS u_{ ext{mn}}^2} \ \Omega_{ ext{mn}} &=
ho_M d \left[(\omega^2 - \omega_{ ext{mn}}^2) - 2jlpha \omega
ight] \end{aligned}$$

(21)

(22)

Eardrum

Forced Vibrations contd.

Transient Solution

Same as the solution for a free damped membrane

$$u_{
m t}(r,\phi;t)=\sum_{m=0,n=1}^{\infty}\widetilde{C}_{
m mn}u_{
m mn}(r,\phi)e^{j\omega_{
m mn}t-\alpha t}$$

 $\widetilde{C}_{
m mn}$ determined from the membrane displacement at t=0. $u_{
m t} o 0$ exponentially as $t o \infty$.

Steady State Approximation

$$u \approx u_{\rm ss}$$
 if α is "large"

(23)

Transient Solution

Same as the solution for a free damped membrane

$$u_{
m t}(r,\phi;t)=\sum_{m=0,n=1}^{\infty}\widetilde{C}_{
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m mn}(r,\phi)e^{j\omega_{
m mn}t-\alpha t}$$

 $\widetilde{C}_{
m mn}$ determined from the membrane displacement at t=0. $u_{
m t} o 0$ exponentially as $t o \infty$.

Steady State Approximation

$$u \approx u_{\rm ss}$$
 if α is "large".

(23)

$u_{0/L} = \sum \Omega_{\rm mn} C_{\rm mn}^{0/L} u_{\rm mn}(r,\phi) e^{j\omega t}$ m = 0, n = 1

Coupled Membranes

The Model

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$$\sum_{n=0}^{\infty} \Omega_{nn} C^{0} \operatorname{Hom}(r, \phi) e^{j\alpha}$$

$$\sum_{m=0,n=1} \Omega_{\rm mn} C_{\rm mn} u_{\rm mn}(r,\phi) e^{jt}$$

$$\sum_{m=0}^{\infty} \Omega_{\rm mn} C_{\rm mn}^L u_{\rm mn}(r,\phi) e^{j\omega t} = p_L e^{j\omega t} - p(L,r,\phi;t)$$

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Coupled Membranes

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Evaluation

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Introduction

Coupled Membranes

$$u_{0/L} = \sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^{0/L} u_{\rm mn}(r,\phi) e^{j\omega t}$$
 (24)

Membrane Equations

m = 0, n = 1

$$\sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^{0} u_{\rm mn}(r,\phi) e^{j\omega t} = p_{0} e^{j\omega t} - p(0,r,\phi;t)$$
 (25)

$$\sum_{mn}^{\infty} \Omega_{mn} C_{mn}^{L} u_{mn}(r, \phi) e^{j\omega t} = p_{L} e^{j\omega t} - p(L, r, \phi; t)$$
 (26)

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Conclusion

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Introduction

"Surface" Velocity

$$U_{0/L} = \begin{cases} u_{0/L}, & 0 < r < a_{\text{tymp}} \text{ and } \beta < \phi < 2\pi - \beta \\ 0, & \text{otherwise} \end{cases}$$
 (27)

Velocity in *x*—direction

$$v_{x} = -\sum_{q=0,s=0}^{\infty} \frac{\zeta_{qs}}{\rho\omega} \left(A_{qs} e^{j\zeta_{qs}x} - B_{qs} e^{-j\zeta_{qs}x} \right) p_{qs}(r,\phi) e^{j\omega t}$$
 (28)

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The Model

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$$U_{2} = \frac{1}{2} v_{1}(0, r, \phi; t)$$

$$egin{align} U_0 &= -rac{1}{j\omega} v_{\mathsf{x}}(0,r,\phi;t) \ U_L &= rac{1}{j\omega} v_{\mathsf{x}}(L,r,\phi;t) \ \end{pmatrix}$$

Evaluation

Approximate
$$U_{0/I} \approx S^{0/L}(t) =: \int dS U_{0/I}$$
 (31)

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(30)

The Model

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$$U_0 = -\frac{1}{i} v_x(0, r, \phi; t)$$
 (29)

Evaluation

$$U_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t)$$

$$U_L = \frac{1}{j\omega} v_x(L, r, \phi; t)$$
(30)

 $U_{0/L} \approx S^{0/L}(t) =: \int dS U_{0/L}$ (31)

Introduction

Coupled Membranes

Boundary Conditions

Approximate

Boundary Conditions

Higher pressure modes disappear, i.e.

$$p = \left[A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t}$$

$$A_{00} = -\frac{\rho\omega^2}{2k\sin kL} \left(S^0 e^{-jkL} + S^L \right)$$

$$B_{00} = -\frac{\rho\omega^2}{2k\sin kL} \left(S^0 e^{jkL} + S^L \right)$$
(32)

Introduction

Coupled Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^0 u_{\rm mn}(r,\phi) = p_0 + \frac{\rho \omega^2}{k} \left(\frac{S^0}{\tan kL} + \frac{S^L}{\sin kL} \right)$$
(34)
$$\sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^L u_{\rm mn}(r,\phi) = p_L + \frac{\rho \omega^2}{k} \left(\frac{S^0}{\sin kL} + \frac{S^L}{\tan kL} \right)$$
(35)

Decoupling

Decouple by taking the sum and difference of the above

Internally Coupled Ears

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Coupled Equations

$$\sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^0 u_{\rm mn}(r,\phi) = p_0 + \frac{\rho \omega^2}{k} \left(\frac{S^0}{\tan kL} + \frac{S^L}{\sin kL} \right) \quad (34)$$

$$\sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^L u_{\rm mn}(r,\phi) = p_L + \frac{\rho \omega^2}{k} \left(\frac{S^0}{\sin kL} + \frac{S^L}{\tan kL} \right) \quad (35)$$

Decoupling

Decouple by taking the sum and difference of the above equations.

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Decoupled Equations

$$\sum_{n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^+ u_{\rm mn}(r,\phi) = p_+ + \frac{\rho \omega^2}{k} S^+ \cot \frac{kL}{2}$$
 (36)

$$\sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^{-} u_{\rm mn}(r,\phi) = p_{-} - \frac{\rho \omega^{2}}{k} S^{-} \tan \frac{kL}{2}$$
 (37)

(38)

(39)

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$$C_{\rm mn}^{+} = \left[p_{+} + \frac{\rho \omega^{2}}{k} S^{+} \cot \frac{kL}{2} \right] \frac{\int dS u_{\rm mn}}{\Omega_{\rm mn} \int dS u_{\rm mn}^{2}}$$

$$C_{\rm mn}^{-} = \left[p_{-} - \frac{\rho \omega^{2}}{k} S^{-} \tan \frac{kL}{2} \right] \frac{\int dS u_{\rm mn}}{\Omega_{\rm mn} \int dS u_{\rm mn}^{2}}$$

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Outline for Section 3

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Introduction
Auditory Systems

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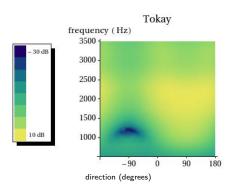
Coupled Membranes

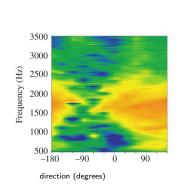
Evaluation

Vibration Amplitude
Internal Level Difference
Internal Amplitude
Difference

Vibration Amplitude

Vibration Amplitude





Outline for Section 4

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