# Mechanical Processing in Internally Coupled Ears

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The Model

## Introduction

Introduction

The Model Mouth Cavity Eardrum

Coupled Membranes

Evaluation

Conclusio

### **Auditory Systems**



**Independent Ears** Eustachian tubes typically very narrow.

Effectively independent eardrum vibrations.



#### **Coupled Ears**

Eardrums connected through wide eustachian tubes and a large mouth cavity.

Eardrums vibrations influence eachother.

#### Advantages of Coupled Ears

► Low frequencies result in reduced degradation of hearing cues in dense environments.



### Outline for section 2

Introduction

The Model

Mouth Cavity

Eardrum

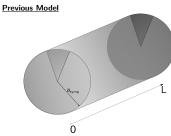
Coupled Membranes

Evaluation



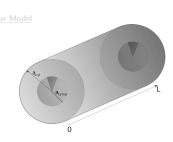
Mouth Cavity

### Mouth Cavity



 $a_{\mathrm{tymp}}$  fixed.

$$V_{
m cyl} = \pi a_{
m tymp}^2 L$$



 $a_{\rm tymp}, V_{\rm cyl}$  fixed

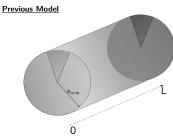
$$a_{\rm cyl} = \sqrt{V_{\rm cyl}/\pi}$$

The Model

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Mouth Cavity

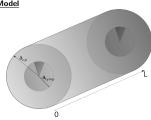
### Mouth Cavity



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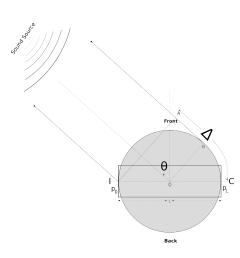


$$a_{
m tymp}, \ V_{
m cyl}$$
 fixed.  $a_{
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m cyl}/\pi L}$ 

Mouth Cavity

#### Acoustic Head Model

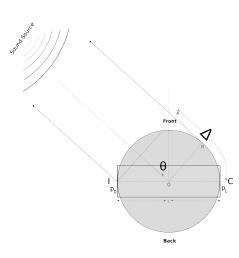
- I Ipsilateral ear, C Contralateral ear.
   p<sub>0</sub>, p<sub>L</sub> sound pressure on
  eardrums, θ sound source
  direction.
- ▶ Sound source "far away".
- No appreciable amplitude difference,  $|p_0| = |p_L|$ .
- Phase difference between sound at both ears  $\Delta = 1.5kL\sin\theta$ .
- $p_0 = pe^{j\omega t .75kL\sin\theta}$   $p_L = pe^{j\omega t + .75kL\sin\theta}$





#### Acoustic Head Model

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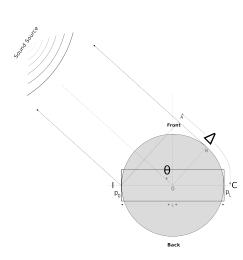




Introduction

#### Acoustic Head Model

- I Ipsilateral ear, C Contralateral ear.
   p<sub>0</sub>, p<sub>L</sub> sound pressure
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Mouth Cavity

### Cavity Pressure

### 3D Wave Equation

$$\frac{1}{c^2} \partial_t^2 p(x, r, \phi, t) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p(x, r, \phi, t)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p(x, r, \phi, t)}{\partial \phi^2} + \frac{\partial p(x, r, \phi, t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

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$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}.$$

### Separated Equations and their Solutions

The Model

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$$x$$
- and  $\phi$ - directions

Introduction

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$

$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$$
(3)

Evaluation

Conclusion

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q^2}{r^2}\right]g(r) = 0 \longrightarrow g(r) = J_q(\nu r)$$
where,  $\nu^2 = k^2 - \zeta^2$ 

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### Separated Equations and their Solutions

### x- and $\phi$ - directions

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$

$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$$
(3)

#### r-direction. Bessel functions

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q^2}{r^2}\right]g(r) = 0 \longrightarrow g(r) = J_q(\nu r)$$
where,  $\nu^2 = k^2 - \zeta^2$  (4)

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Conclusion

### Boundary Conditions - $\phi$

Smoothness and Continuity in 
$$\phi$$
.

$$\phi 
ightarrow h(0) = h(2\pi)$$
 and  $h'(0) = h'(2\pi)$ 

The Model

$$\Rightarrow h(\phi) = \cos q\phi, \ q = 0, 1, 2, \dots \tag{5}$$

Introduction

Mouth Cavity

Impenetrable boundary at  $r=a_{\rm cyl}$ , i.e. normal derivative vanishes

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$$-j\rho\omega\mathbf{v} = \mathbf{n}. \ \nabla p(x, r, \phi; t)|_{r=\mathbf{a}_{\text{cyl}}} \equiv \left. \frac{\partial g}{\partial r} \right|_{r=\mathbf{a}_{\text{cyl}}} = 0 \tag{6}$$

$$\Rightarrow g(r) = J_q(\nu_{qs}r/a_{cyl}) \tag{7}$$

Evaluation

#### Bessel Prime Zeros

$$\blacktriangleright$$
  $\nu_{\rm qs}$  - zeros of  $J_a'$ ,  $s=0,1,2,\ldots$ 

$$u_{\mathrm{qs}}$$
 - zeros of  $J_q$ ,  $s=0,1,2,\dots$ 

$$u_{00}=0$$

Internally Coupled Ears

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Evaluation

#### Pressure Modes

Introduction

Mouth Cavity

$$p(x, r, \phi, t) = \sum_{q=0, s=0}^{\infty} p_{qs}(x, r, \phi) e^{j\omega t}$$

$$p_{qs}(x, r, \phi) = \left[ A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] \cos q\phi J_q(\nu_{qs}r/a_{cyl})$$
where,  $\zeta_{qs} = \sqrt{k^2 - \nu_{qs}^2/a_{cyl}^2}$  (9)

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The Model

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Introduction

Mouth Cavity

$$p(x, r, \phi, t) = \sum_{q=0, s=0}^{\infty} p_{qs}(x, r, \phi) e^{i\omega t}$$

$$p_{qs}(x, r, \phi) = \left[ A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] \cos q\phi J_q(\nu_{qs}r/a_{cyl})$$
(9)

Plane Wave Mode

$$p_{00}(x, r, \phi; t) = \left[ A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t}$$
 (10)

Evaluation

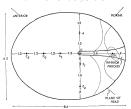
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Eardrum

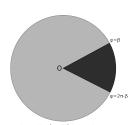
#### Eardrum

#### Sketch of a Tokay eardrum as seen from the outside<sup>a</sup>.



COL - approximate position opposite the extracolumella insertion

#### The ICF eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ( $r = a_{tymp}$ and  $\phi = \beta$ ,  $2\pi - \beta$ )

aG. A. Manley, "The middle ear of the tokay gecko," Journal of Comparative Physiology, vol. 81, no. 3, pp. 239-250, 1972

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Eardrum

### Membrane Vibrations

#### Membrane EOM

$$-\partial_t^2 u(r,\phi;t) - 2\alpha \partial_t u(r,\phi;t) + c_M^2 \Delta_{(2)} u(r,\phi;t) = \frac{1}{\rho_m d} \Psi(r,\phi;t)$$
(11)

#### Membrane parameters

 $\alpha$  - damping coefficient,  $c_M^2$  - propagation velocity

 $ho_m$  - density, d - thickness.

Introduction

### Free-Undamped Membrane, $\alpha \to 0$ , $\Psi \to 0$

The Model

### Separation Ansatz

$$u(r,\phi;t) = f(r)g(\phi)h(t) \tag{12}$$

Evaluation

Conclusion

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### Separated Equations

$$\frac{d^2g(\phi)}{d\phi^2} + \kappa^2 g(\phi) = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f(r)}{\partial r} \right) + \left[ \mu^2 - \frac{\kappa^2}{r^2} \right] f(r) = 0$$
(13)

$$\frac{d^2h(t)}{dt^2} + c_M^2\mu^2h(t) = 0 {15}$$

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$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$

where, 
$$\kappa=rac{m\pi}{2(\pi-eta)}, \quad m=1,2,3,\ldots$$

The Model

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$$p;t)=0$$

$$\rightarrow I(I) - J_{\kappa}(\mu_{\text{mn}} I / a_{\text{tymp}})$$
where  $\mu_{\text{mn}}$  is the  $n^{\text{th}}$  zero of  $J_{\text{mn}}$ 

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Introduction

**Boundary Conditions** 

Eardrum

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Introduction

### **Boundary Conditions**

$$\phi$$
-direction:  $u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$ 

The Model

$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$
 (16) where,  $\kappa = \frac{m\pi}{2(\pi - \beta)}, \quad m = 1, 2, 3, \dots$ 

Evaluation

r-direction: 
$$u(a_{\text{tymp}}, \phi; t) = 0$$

$$\Rightarrow f(r) = J_{\kappa}(\mu_{\rm mn} r / a_{\rm tymp})$$
 where,  $\mu_{\rm mn}$  is the  $n^{\rm th}$  zero of  $J_{\kappa}$ 

### Free eigenmodes

Introduction

Eardrum

$$u_{ ext{free}}(r,\phi;t) = \sum_{-\infty}^{\infty} C_{ ext{mn}} u_{ ext{mn}}(r,\phi) e^{j\omega_{ ext{mn}}t}$$

The Model

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$$m=0, n=1$$

 $u_{\rm mn}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{\rm mn} r)$ 

where, 
$$\omega_{\rm mn} = c_{\rm M} \mu_{\rm mn}$$

$$\widetilde{u}_{\text{free}}(r,\phi;t) = \sum_{n=0}^{\infty} \widetilde{C}_{\text{mn}} u_{\text{mn}}(r,\phi) e^{j\omega_{\text{mn}}t - \alpha t}$$
 (20)

Evaluation

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Conclusion

(18)

(19)

Eardrum

#### Free eigenmodes

$$u_{
m free}(r,\phi;t) = \sum_{}^{\infty} C_{
m mn} u_{
m mn}(r,\phi) e^{j\omega_{
m mn}t}$$

 $u_{\rm mn}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{\rm mn} r)$ 

m = 0, n = 1

where, 
$$\omega_{\mathrm{mn}} = c_{M} \mu_{\mathrm{mn}}$$

### Damped membrane

$$\widetilde{u}_{\text{free}}(r,\phi;t) = \sum_{m=0}^{\infty} \widetilde{C}_{\text{mn}} u_{\text{mn}}(r,\phi) e^{j\omega_{\text{mn}}t - \alpha t}$$
 (20)

(18)

(19)

m=0,n=1

The Model

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Forced Vibrations:  $\Psi = pe^{j\omega t}$ 

Steady State Solution

Substitute  $u_{ss}$  in Membrane EOM.

$$C_{\rm mn} = \frac{p \int dS u_{\rm mn}}{\Omega_{\rm mn} \int dS u_{\rm mn}^2}$$

$$\Omega_{\rm mn} = a_{\rm mn} d \left[ (\omega^2 - \omega^2) \right] = 2i\alpha \omega$$

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Eardrum

### Forced Vibrations: $\Psi = pe^{j\omega t}$

### Steady State Solution

$$u_{\rm ss}(r,\phi;t) =: \sum_{m=0,n=1}^{\infty} C_{\rm mn} u_{\rm mn}(r,\phi) e^{j\omega t}$$
 (21)

Substitute  $u_{ss}$  in Membrane EOM.

$$C_{\rm mn} = \frac{p \int dS u_{\rm mn}}{\Omega_{\rm mn} \int dS u_{\rm mn}^2}$$

$$\Omega_{\rm mn} = \rho_{M} d \left[ (\omega^2 - \omega_{\rm mn}^2) - 2j\alpha\omega \right]$$
(22)

Eardrum

### Forced Vibrations contd.

#### Transient Solution

Same as the solution for a free damped membrane

$$u_{\rm t}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\rm mn} u_{\rm mn}(r,\phi) e^{j\omega_{\rm mn}t - \alpha t}$$
 (23)

 $\widetilde{\textit{C}}_{
m mn}$  determined from the membrane displacement at t=0.

$$u_{\rm t} \to 0$$
 as  $t \to \infty$ .

The Model

Mouth Cavity
Eardrum
Coupled Membranes

Outline for section 3

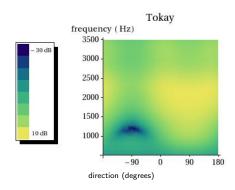
**Evaluation** 

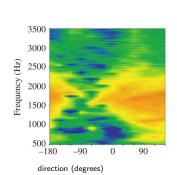
Introduction

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### Vibration Amplitude

Introduction





Outline for section 4

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Conclusion

### Thank You

