Mechanical Processing in Internally Coupled Ears

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TMP Thesis Defence
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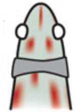
Auditory Systems

Independent Ears

Eustachian tubes typically very narrow.

Effectively independent eardrum vibrations.



Coupled Ears

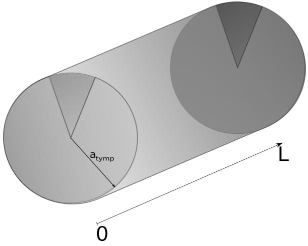
Eardrums connected through wide eustachian tubes and a large mouth cavity.

Eardrums vibrations influence eachother.

Advantages of Low Frequency Hearing

Mouth Cavity

Previous Model

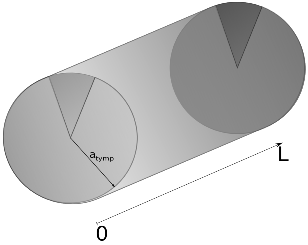


a_{tymp} fixed.

$V_{\mathrm{cyl}} = \pi a_{\mathrm{tymp}}^2 L$

Mouth Cavity

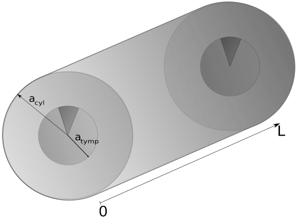
Previous Model



a_{tymp} fixed.

$V_{\text{cyl}} = \pi a_{\text{tymp}}^2 L$

Our Model

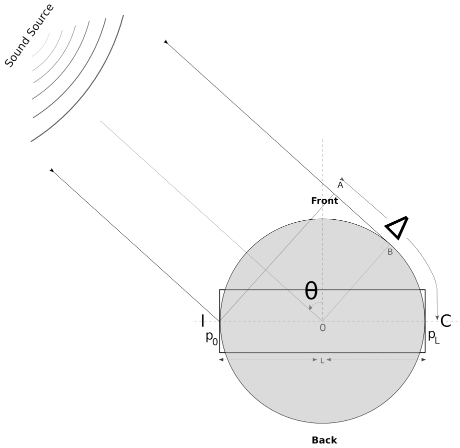


$a_{\text{tymp}},\ V_{\text{cyl}}$ fixed.

$a_{\text{cyl}} = \sqrt{V_{\text{cyl}}/\pi L}$

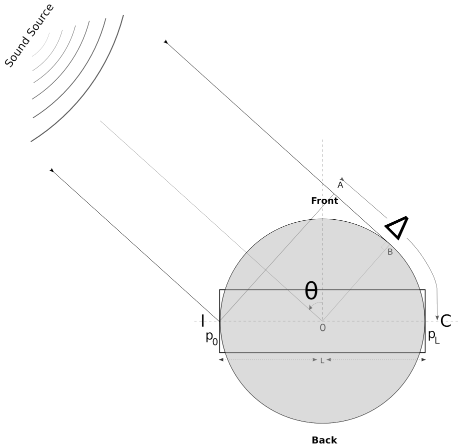
Acoustic Head Model

- **I** - Ipsilateral ear, **C** - Contralateral ear. $p_0, \ p_L$ - sound pressure on eardrums, θ - sound source direction.



Acoustic Head Model

- Sound source "far away".
- Phase difference between sound at both ears - $\Delta = 1.5kL\sin\theta$.
- No appreciable amplitude difference, $|p_0| = |p_L|$.



Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x,r,\phi,t) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p(x,r,\phi,t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p(x,r,\phi,t)}{\partial \phi^2} + \frac{\partial p(x,r,\phi,t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

$$p(x,r,\phi,t) = f(x)g(r)h(\phi)e^{j\omega t}$$

.

No-penetration at the cavity boundary, i.e. normal derivative vanishes

$$-j\rho\omega\mathbf{v} = \mathbf{n}.\,\nabla p(x, r, \phi; t)\big|_{r=a_{\mathrm{cyl}}} = \left.\frac{\partial p}{\partial r}\right|_{r=a_{\mathrm{cyl}}} = 0 \qquad (2)$$

No-penetration at the cavity boundary, i.e. normal derivative vanishes

$$-j\rho\omega\mathbf{v} = \mathbf{n}.\nabla p(x,r,\phi;t)\big|_{r=a_{\mathrm{cyl}}} = \left.\frac{\partial p}{\partial r}\right|_{r=a_{\mathrm{cyl}}} = 0 \qquad (2)$$

Pressure Modes

$$p_{\mathrm{qs}}(x,r,\phi;t) = \left[A_{\mathrm{qs}}e^{j\zeta_{\mathrm{qs}}x} + B_{qs}e^{-j\zeta_{\mathrm{qs}}x}\right]\cos q\phi J_q(\nu_{qs}r)e^{j\omega t} \qquad (3)$$

such that, $\left.\dfrac{\partial J_q(\nu_{qs}r)}{\partial r}\right|_{r=a_{\mathrm{cyl}}} = 0$ and $\zeta_{\mathrm{qs}} = \sqrt{k^2 - \nu_{qs}^2}$

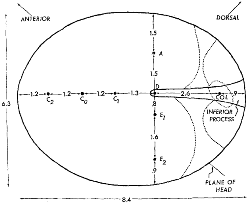
Plane Wave Mode

$$p_{00}(x, r, \phi; t) = \left[Ae^{jkx} + B_{qs}e^{-jkx}\right] e^{j\omega t} \tag{4}$$

Trivially satisfies the no-penetration condition.

Eardrum

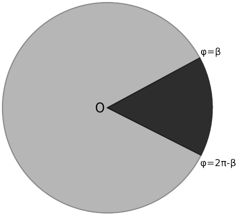
Sketch of a Tokay eardrum as seen from the outside[a].



COL - approximate position opposite the extracolumella insertion.

[a]G. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r = a_{\text{tymp}}$ and $\phi = \beta,\ 2\pi - \beta$)

Membrane Vibrations

Membrane EOM

$$-\partial_t^2 u(r,\phi;t) - 2\alpha\partial_t u(r,\phi;t) + c_M^2 \Delta_{(2)} u(r,\phi;t) = \frac{1}{\rho_m d}\Psi(r,\phi;t) \tag{5}$$

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Membrane EOM

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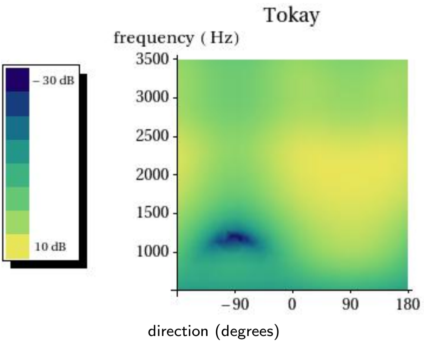
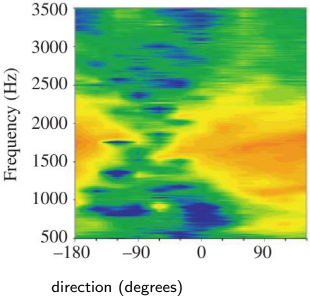
Membrane parameters

α - damping coefficient, c_M^2 - propagation velocity
ρ_m - density, d - thickness.

Vibration Amplitude





Conclusion

Thank You