Useful Coefficients

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Final expression for volume displacements of the membranes.

$$S_0 = G_{ipsi}p_0 + G_{contra}p_L \tag{1}$$

$$S_L = G_{ipsi}p_L + G_{contra}p_0 (2)$$

Defining the other useful objects in terms of the membrane/cylinder properties

$$\frac{1}{\Lambda} = \sum_{m,n} \frac{\left(\int dS f_{mn}(r,\phi)\right)^2}{\Omega_{mn} \int dS f_{mn}^2(r,\phi)} \tag{3}$$

$$\Omega_{mn} = \rho_m d \left(\omega^2 - 2j\alpha\omega - \omega_{mn}^2 \right) \tag{4}$$

(5)

As we will see, the above definition of Λ makes subsequent calculations simpler. Easier to calculate the sum and difference of the membrane displacements.

$$S^{+} = (S^{L} + S^{0}) = \frac{p_{L} + p_{0}}{\Lambda + \Gamma^{+}}$$
 (6)

$$S^{-} = (S^{L} - S^{0}) = \frac{p_{L} - p_{0}}{\Lambda + \Gamma^{-}}$$
 (7)

Where,

$$\Gamma^{+} = -\frac{\rho c \omega \cot \frac{kL}{2}}{\pi a_{cyl}^{2}}$$

$$\Gamma^{-} = \frac{\rho c \omega \tan \frac{kL}{2}}{\pi a_{cyl}^{2}}$$

$$(8)$$

$$\Gamma^{-} = \frac{\rho c \omega \tan \frac{kL}{2}}{\pi a_{cyl}^2} \tag{9}$$

We therefore have,

$$G_{ipsi} = \left(\frac{1}{\Lambda + \Gamma^{+}} + \frac{1}{\Lambda + \Gamma^{-}}\right)/2 \tag{10}$$

$$G_{contra} = \left(\frac{1}{\Lambda + \Gamma^{+}} - \frac{1}{\Lambda + \Gamma^{-}}\right)/2 \tag{11}$$

It is also convenient to define,

$$\frac{G_{contra}}{G_{ipsi}} = \frac{\frac{\rho c \omega \csc kL}{\pi a_{cyl}^2}}{\Lambda - \frac{\rho c \omega \cot kL}{\pi a_{cyl}^2}}$$
(12)

$$= \frac{1}{\eta \sin kL - \cos kL} \tag{13}$$

Where we've also defined,

$$\eta = \frac{\pi a_{cyl}^2 \Lambda}{\rho c \omega} \tag{14}$$

The pressure coefficients,

$$A = -\frac{\Lambda}{\eta \sin kL} \left(S^0 e^{-jkL} + S^L \right)$$

$$B = -\frac{\Lambda}{\eta \sin kL} \left(S^0 e^{jkL} + S^L \right)$$
(15)

$$B = -\frac{\Lambda}{n\sin kL} \left(S^0 e^{jkL} + S^L \right) \tag{16}$$