

Mechanical Processing in Internally Coupled Ears

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Auditory Systems



Independent Ears

Eustachian tubes typically very narrow.

Effectively independent eardrum vibrations.



Coupled Ears

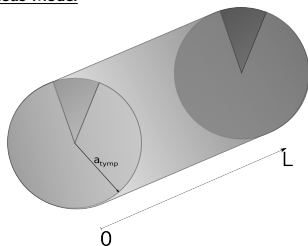
Eardrums connected through wide eustachian tubes and a large mouth cavity.

Eardrums vibrations influence each other.

Advantages of Low Frequency Hearing

Mouth Cavity

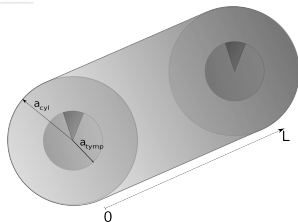
Previous Model



a_{tymp} fixed.

$$V_{\text{cyl}} = \pi a_{\text{tymp}}^2 L$$

Our Model

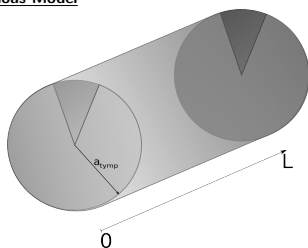


$a_{\text{tymp}}, V_{\text{cyl}}$ fixed.

$$a_{\text{cyl}} = \sqrt{V_{\text{cyl}}/\pi L}$$

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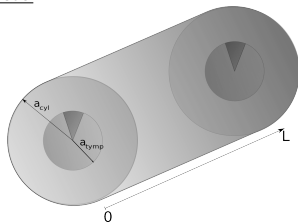
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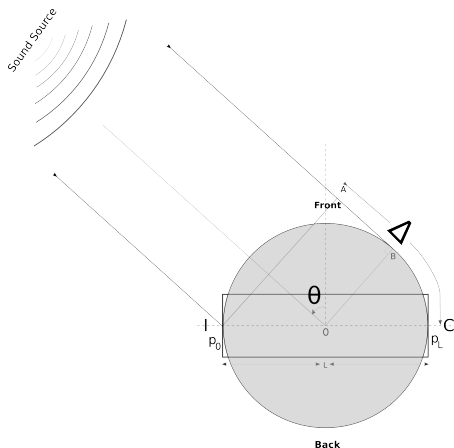


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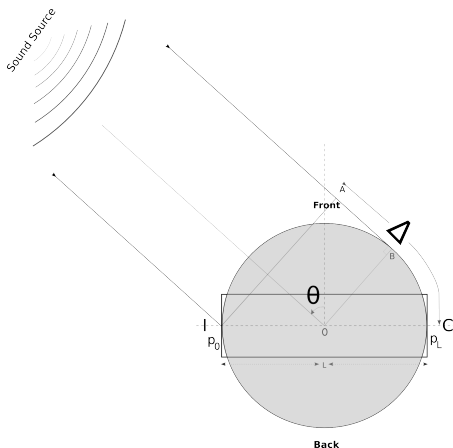
Acoustic Head Model

- ▶ **I** - Ipsilateral ear, **C** - Contralateral ear.
 p_0 , p_L - sound pressure on eardrums, θ - sound source direction.
- ▶ Sound source "far away".
- ▶ No appreciable amplitude difference, $|p_0| = |p_L|$.
- ▶ Phase difference between sound at both ears - $\Delta = 1.5kL \sin \theta$.
- ▶ $p_0 = p e^{j\omega t - .75kL \sin \theta}$
 $p_L = p e^{j\omega t + .75kL \sin \theta}$



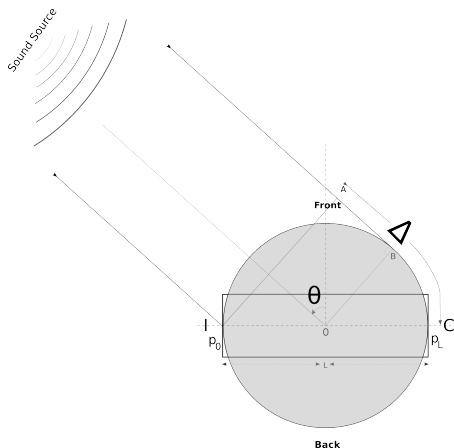
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Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2} \partial_t^2 p(x, r, \phi, t) = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p(x, r, \phi, t)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p(x, r, \phi, t)}{\partial \phi^2} + \frac{\partial^2 p(x, r, \phi, t)}{\partial x^2} \quad (1)$$

To be solved using the separation ansatz

$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}.$$

Separated Equations and their Solutions

x - and ϕ - directions

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x} \quad (2)$$

$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi} \quad (3)$$

r -direction, Bessel functions

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial g(r)}{\partial r} \right) + \left[\nu^2 - \frac{q^2}{r^2} \right] g(r) = 0 \longrightarrow g(r) = J_q(\nu r) \quad (4)$$

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Boundary Conditions - ϕ

Smoothness and Continuity in ϕ .

$$\phi \rightarrow h(0) = h(2\pi) \quad \text{and} \quad h'(0) = h'(2\pi)$$

$$\Rightarrow h(\phi) = \cos q\phi, \quad q = 0, 1, 2, \dots \quad (5)$$

Boundary Conditions - r

Impenetrable boundary at $r = a_{\text{cyl}}$, i.e. normal derivative vanishes

$$-j\rho\omega\mathbf{n} \cdot \nabla p(x, r, \phi; t)|_{r=a_{\text{cyl}}} \equiv \left. \frac{\partial g}{\partial r} \right|_{r=a_{\text{cyl}}} = 0 \quad (6)$$

$$\Rightarrow g(r) = J_q(\nu_{\text{qs}}r/a_{\text{cyl}}) \quad (7)$$

Bessel Prime Zeros

- ▶ ν_{qs} - zeros of J'_q , $s = 0, 1, 2, \dots$
- ▶ $\nu_{00}=0$

General Solution

Pressure Modes

$$p(x, r, \phi, t) = \sum_{q=0, s=0}^{\infty} p_{qs}(x, r, \phi) e^{j\omega t} \quad (8)$$

$$p_{qs}(x, r, \phi) = \left[A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] \cos q\phi J_q(\nu_{qs}r/a_{\text{cyl}}) \quad (9)$$

$$\zeta_{qs} = \sqrt{k^2 - \nu_{qs}^2/a_{\text{cyl}}^2}$$

Plane Wave Mode

$$p_{00}(x, r, \phi; t) = \left[A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t} \quad (10)$$

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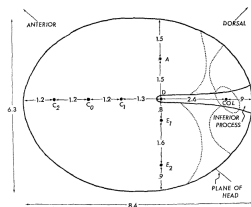
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Eardrum

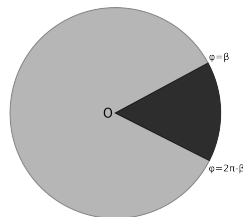
Sketch of a Tokay eardrum as seen from the outside^a.



COL - approximate position opposite the extracolumella insertion.

^aG. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r = a_{\text{tym}}$ and $\phi = \beta, 2\pi - \beta$)

Membrane Vibrations

Membrane EOM

$$-\partial_t^2 u(r, \phi; t) - 2\alpha \partial_t u(r, \phi; t) + c_m^2 \Delta_{(2)} u(r, \phi; t) = \frac{1}{\rho_m d} \Psi(r, \phi; t) \quad (11)$$

Membrane parameters

α - damping coefficient, c_m^2 - propagation velocity

ρ_m - density, d - thickness.

Free-Undamped Membrane

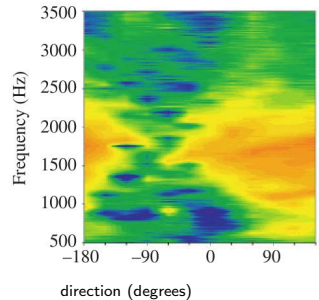
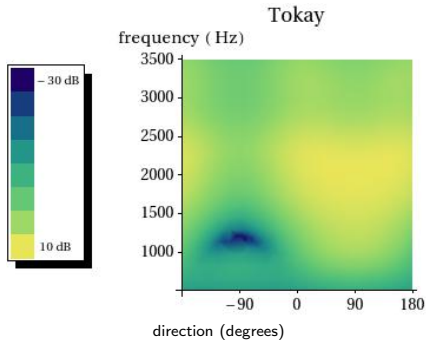
$$\alpha \rightarrow 0, \psi \rightarrow 0$$

Separation Ansatz

$$u(r, \phi; t) = f(r)g(\phi)h(t) \quad (12)$$

- Same set of equations as for pressure case.

Vibration Amplitude



Conclusion

Thank You

