Mechanical Processing in Internally Coupled Ears

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TMP Thesis Defence
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Outline for section 1

Auditory Systems

Independent Ears

Eustachian tubes typically very narrow.

Effectively independent eardrum vibrations.

Coupled Ears

Eardrums connected through wide eustachian tubes and a large mouth cavity.

Eardrums vibrations influence eachother.

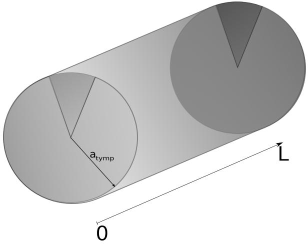
Advantages of Coupled Ears

- Low frequencies result in reduced degradation of hearing cues in dense enviroments.

Outline for section 2

Mouth Cavity

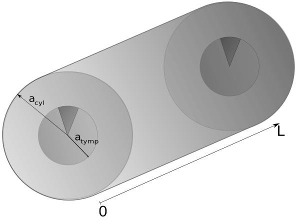
Previous Model



a_{tymp} fixed.

$V_{\mathrm{cyl}} = \pi a_{\mathrm{tymp}}^2 L$

Our Model



a_{tymp}, V_{cyl} fixed.

$a_{\mathrm{cyl}} = \sqrt{V_{\mathrm{cyl}}/\pi L}$

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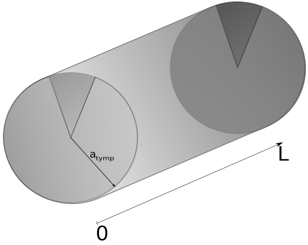

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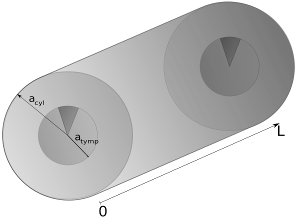

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$a_{\mathrm{tymp}},\ V_{\mathrm{cyl}}$ fixed.

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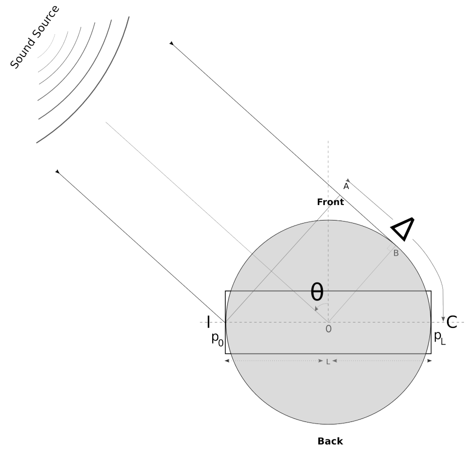
Acoustic Head Model

- **I** - Ipsilateral ear, **C** - Contralateral ear. p_0, p_L - sound pressure on eardrums, θ - sound source direction.
- Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_L|$.
- Phase difference between sound at both ears - $\Delta = 1.5kL\sin\theta$.
- $p_0 = pe^{j\omega t - .75kL\sin\theta}$
  $p_L = pe^{j\omega t + .75kL\sin\theta}$



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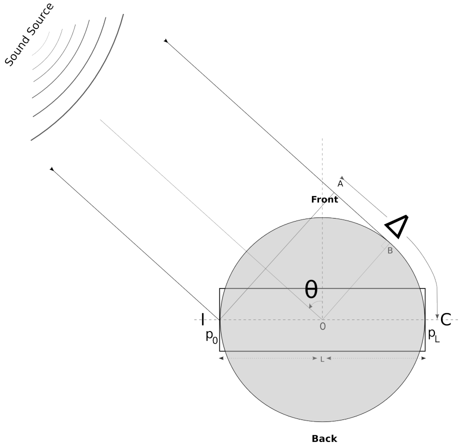

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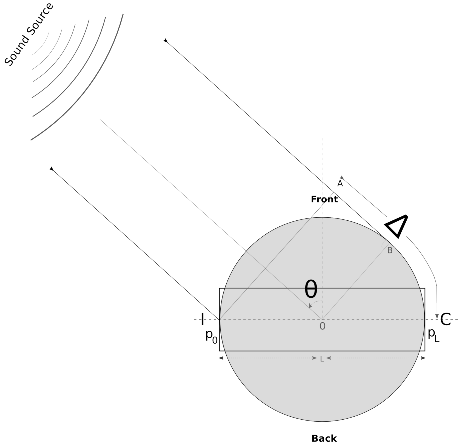

Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x,r,\phi,t) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p(x,r,\phi,t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p(x,r,\phi,t)}{\partial \phi^2} + \frac{\partial p(x,r,\phi,t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

$$p(x,r,\phi,t) = f(x)g(r)h(\phi)e^{j\omega t}.$$

Separated Equations and their Solutions

x- and ϕ- directions

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x} \tag{2}$$

$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi} \tag{3}$$

r-direction, Bessel functions

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q^2}{r^2}\right] g(r) = 0 \longrightarrow g(r) = J_q(\nu r) \tag{4}$$

where, $\nu^2 = k^2 - \zeta^2$

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Boundary Conditions - ϕ

Smoothness and Continuity in ϕ.

$$\phi \to h(0) = h(2\pi) \quad \text{and} \quad h'(0) = h'(2\pi)$$

$$\Rightarrow h(\phi) = \cos q\phi, \;\; q = 0, 1, 2, \ldots \tag{5}$$

Boundary Conditions - r

Impenetrable boundary at $r = a_{\mathrm{cyl}}$, i.e. normal derivative vanishes

$$-j\rho\omega\mathbf{v} = \mathbf{n}.\, \nabla p(x, r, \phi; t)\big|_{r=a_{\mathrm{cyl}}} \equiv \left.\frac{\partial g}{\partial r}\right|_{r=a_{\mathrm{cyl}}} = 0 \qquad (6)$$

$$\Rightarrow g(r) = J_q(\nu_{\mathrm{qs}} r / a_{\mathrm{cyl}}) \qquad (7)$$

Bessel Prime Zeros

- ν_{qs} - zeros of J_q', $s = 0, 1, 2, \ldots$
- ν_{00}=0

General Solution

Pressure Modes

$$p(x,r,\phi,t) = \sum_{q=0,s=0}^{\infty} p_{\mathrm{qs}}(x,r,\phi)e^{j\omega t} \tag{8}$$

$$p_{\mathrm{qs}}(x,r,\phi) = \left[A_{\mathrm{qs}}e^{j\zeta_{\mathrm{qs}}x} + B_{\mathrm{qs}}e^{-j\zeta_{\mathrm{qs}}x}\right]\cos q\phi J_q(\nu_{\mathrm{qs}}r/a_{\mathrm{cyl}}) \tag{9}$$

where, $\zeta_{\mathrm{qs}} = \sqrt{k^2 - \nu_{\mathrm{qs}}^2/a_{\mathrm{cyl}}^2}$

Plane Wave Mode

$$p_{00}(x,r,\phi;t) = \left[A_{00}e^{jkx} + B_{00}e^{-jkx}\right]e^{j\omega t} \tag{10}$$

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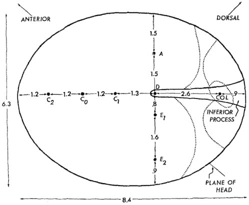
Eardrum

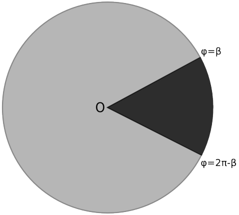
Sketch of a Tokay eardrum as seen from the outside[a].



COL - approximate position opposite the extracolumella insertion.

[a] G. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r = a_{\mathrm{tymp}}$ and $\phi = \beta,\ 2\pi - \beta$)

Membrane Vibrations

Membrane EOM

$$-\partial_t^2 u(r,\phi;t) - 2\alpha\partial_t u(r,\phi;t) + c_M^2 \Delta_{(2)} u(r,\phi;t) = \frac{1}{\rho_m d}\Psi(r,\phi;t) \qquad (11)$$

Membrane parameters

α - damping coefficient, c_M^2 - propagation velocity

ρ_m - density, d - thickness.

Free-Undamped Membrane, $\alpha \to 0$, $\Psi \to 0$

Separation Ansatz

$$u(r,\phi;t) = f(r)g(\phi)h(t) \tag{12}$$

Separated Equations

$$\frac{d^2 g(\phi)}{d\phi^2} + \kappa^2 g(\phi) = 0 \tag{13}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f(r)}{\partial r}\right) + \left[\mu^2 - \frac{\kappa^2}{r^2}\right] f(r) = 0 \tag{14}$$

$$\frac{d^2 h(t)}{dt^2} + c_M^2 \mu^2 h(t) = 0 \tag{15}$$

Boundary Conditions

ϕ-direction: $u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$

$$\Rightarrow g(\phi) = \sin \kappa(\phi - \beta) \tag{16}$$

$$\text{where, } \kappa = \frac{m\pi}{2(\pi - \beta)}, \quad m = 1, 2, 3, \ldots$$

r-direction: $u(a_{\mathrm{tymp}}, \phi; t) = 0$

$$\Rightarrow f(r) = J_\kappa(\mu_{\mathrm{mn}} r / a_{\mathrm{tymp}}) \tag{17}$$

where, μ_{mn} is the n^{th} zero of J_κ

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Free eigenmodes

$$u_{\mathrm{mn}}(r,\phi) = \sin\kappa(\phi-\beta)J_\kappa(\mu_{\mathrm{mn}}r) \tag{18}$$

$$u_{\mathrm{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}}u_{\mathrm{mn}}(r,\phi)e^{j\omega_{\mathrm{mn}}t} \tag{19}$$

where, $\omega_{\mathrm{mn}} = c_M\mu_{\mathrm{mn}}$

Damped membrane

$$\widetilde{u}_{\mathrm{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\mathrm{mn}}u_{\mathrm{mn}}(r,\phi)e^{j\omega_{\mathrm{mn}}t-\alpha t} \tag{20}$$

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Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

$$u_{\mathrm{ss}}(r,\phi;t) =: \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega t} \tag{21}$$

Substitute u_{ss} in Membrane EOM.

$$C_{\mathrm{mn}} = \frac{p \int dS u_{\mathrm{mn}}}{\Omega_{\mathrm{mn}} \int dS u_{\mathrm{mn}}^2} \tag{22}$$

$$\Omega_{\mathrm{mn}} = \rho_M d \left[(\omega^2 - \omega_{\mathrm{mn}}^2) - 2j\alpha\omega \right]$$

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Forced Vibrations contd.

Transient Solution

Same as the solution for a free damped membrane

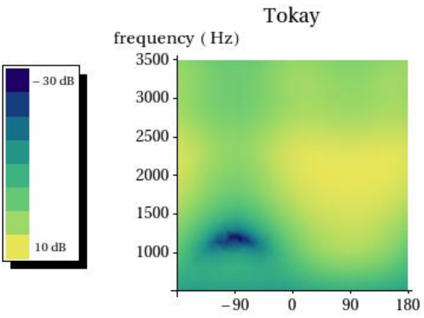
$$u_{\mathrm{t}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t-\alpha t} \tag{23}$$

$\widetilde{C}_{\mathrm{mn}}$ determined from the membrane displacement at $t = 0$.

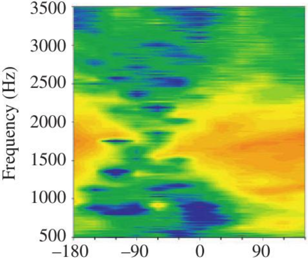
$u_{\mathrm{t}} \to 0$ as $t \to \infty$.

Outline for section 3

Vibration Amplitude



direction (degrees)



direction (degrees)

Outline for section 4

Conclusion

Thank You