ICE Model Continued

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March 9, 2013

We denote the membrane displacements on either side by the functions $u_{0/L}(r, \phi, t)$ where 0 and L correspond to the ipsi- and contra-lateral membranes respectively. They satisfy the boundary condition

$$u_{0/L}(a, \phi, t) = 0 \tag{1}$$

where a is the membrane radius. We choose a convention in which the displacements into the cavity are positive and outward displacements negative. Thus, for a given displacement of the membranes, the cavity volume changes by

$$\Delta V = -\int_S dS(u0 + uL) \tag{2}$$

where $dS = r dr d\phi$ is the area element in 2D polar coordinates and the integral is over the surface of the disk. Assuming the membrane displacements are slow enough to consider the air inside the cavity to be quasi-static, the corresponding change in pressure (linearized in volume-change) is given by

$$\Delta P \approx -\gamma \frac{P0}{V0} \Delta V \tag{3}$$

$P0$ is the atmospheric pressure and V_0. The membrane equations of motion are then given by

$$-\ddot{u}_{0/L} - 2\alpha \dot{u}_{0/L} + c_M^2 \Delta u_{0/L} = \frac{1}{\rho_M d} \left[p_{0/L} - \Delta P \right] \tag{4}$$

Where $p_{0/L}$ is the sound pressure on the ipsi- and contra-lateral membranes respectively. We define a new set of variables, $u_+ = u_L + u_0$ and $u_- = u_L - u_0$ and add and subtract the above equations to get a new system in terms of these newly defined variables.

$$-\ddot{u}_+ - 2\alpha \dot{u}_+ + c_M^2 \Delta u_+ = \frac{1}{\rho_M d} \left[p_+ - 2\Delta P \right] \tag{5}$$

$$-\ddot{u}_- - 2\alpha \dot{u}_- + c_M^2 \Delta u_- = \frac{p_-}{\rho_M d} \tag{6}$$

Where, similar to the above definitions, $p_+ = p_L + p_0$ and $p_- = p_L - p_0$. The second equation can be solved exactly.