Useful Coefficients

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Final expression for volume displacements of the membranes.

$$S_0 = G_{ipsi} p_0 + G_{contra} p_L \tag{1}$$
$$S_L = G_{ipsi} p_L + G_{contra} p_0 \tag{2}$$

Defining the other useful objects in terms of the membrane/cylinder properties

$$\frac{1}{\Lambda} = \sum_{m,n} \frac{\left(\int dS f_{mn}(r,\phi)\right)^2}{\Omega_{mn} \int dS f_{mn}^2(r,\phi)} \tag{3}$$
$$\Omega_{mn} = \rho_m d \left(\omega^2 - 2j\alpha\omega - \omega_{mn}^2\right) \tag{4}$$
$$\tag{5}$$

As we will see, the above definition of Λ makes subsequent calculations simpler.

Easier to calculate the sum and difference of the membrane displacements.

$$S^+ = (S^L + S^0) = \frac{p_L + p_0}{\Lambda + \Gamma^+} \tag{6}$$
$$S^- = (S^L - S^0) = \frac{p_L - p_0}{\Lambda + \Gamma^-} \tag{7}$$

Where,

$$\Gamma^+ = -\frac{\rho c \omega \cot \frac{kL}{2}}{\pi a_{cyl}^2} \tag{8}$$
$$\Gamma^- = \frac{\rho c \omega \tan \frac{kL}{2}}{\pi a_{cyl}^2} \tag{9}$$

We therefore have,

$$G_{ipsi} = \left(\frac{1}{\Lambda + \Gamma^+} + \frac{1}{\Lambda + \Gamma^-}\right)/2 \tag{10}$$
$$G_{contra} = \left(\frac{1}{\Lambda + \Gamma^+} - \frac{1}{\Lambda + \Gamma^-}\right)/2 \tag{11}$$

It is also convenient to define,

$$\frac{G_{contra}}{G_{ipsi}} = \frac{\frac{\rho c \omega \csc kL}{\pi a_{cyl}^2}}{\Lambda - \frac{\rho c \omega \cot kL}{\pi a_{cyl}^2}} \tag{12}$$
$$= \frac{1}{\eta \sin kL - \cos kL} \tag{13}$$

Where we've also defined,

$$\eta = \frac{\pi a_{cyl}^2 \Lambda}{\rho c \omega} \tag{14}$$

The pressure coefficients,

$$A = -\frac{1}{\eta \sin kL} \left(S^0 e^{-jkL} + S^L \right) \tag{15}$$

$$B = -\frac{1}{\eta \sin kL} \left(S^0 e^{jkL} + S^L \right) \tag{16}$$