Mechanical Processing in Internally Coupled Ears

Anupam Prasad Vedurmudi

TMP Thesis Defence July 13, 2013





Outline for Section 1

Introduction
Auditory Systems
Hearing Cues

The Mode

Mouth Cavity
Acoustic Head Mode
Pressure Derivation

Fardrum

Model

Coupled Membranes

Evaluation

Vibration Amplitude Internal Level Difference Internal Amplitude Difference

Introduction

Auditory Systems



Independent Ears

Eustachian tubes generally very narrow.

Effectively independent eardrum vibrations.



Wide eustachian tubes open into the mouth cavity.

Eardrums vibrations influence eachother.

Binaural Hearing Cues

Direction and frequency dependent phase and amplitude differences between the ears.

Interaural Time Difference

Equivalent to phase difference between membrane vibrations.

Interaural Level Difference

Equivalent to amplitude difference between membrane vibrations.

► Low frequencies result in reduced degradation of hearing cues in dense environments.

Outline for Section 2

Introduction

Hearing Cues

The Model

Mouth Cavity

Fardrum

Model

Membrane Vibrations

Coupled Membranes

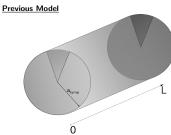
Roundary Condition

Evaluation

Vibration Amplitude Internal Level Difference Internal Amplitude

Introduction

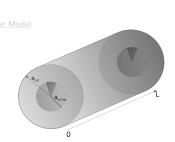
Mouth Cavity



The Model

•0000000

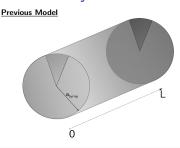
 a_{tymp} fixed. $V_{\rm cyl} = \pi a_{
m tymp}^2 L$



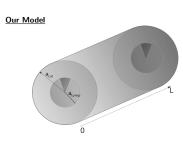
Mouth Cavity

Introduction

Mouth Cavity



 $a_{
m tymp}$ fixed. $V_{
m cyl} = \pi a_{
m tymp}^2 L$



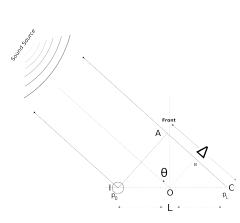
 $a_{
m tymp}, \; V_{
m cyl} \; {
m fixed}.$ $a_{
m cyl} = \sqrt{V_{
m cyl}/\pi L}$

Introduction

Mouth Cavity

Acoustic Head Model

- I Ipsilateral ear, C Contralateral ear.
 p₀, p_L sound pressure on
 eardrums, θ sound source
 direction
- ► Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_I|$.
- ▶ Phase difference between sound at both ears $\Delta = kL \sin \theta$.
- $p_0 = pe^{j\omega t .5kL\sin\theta}$ $p_L = pe^{j\omega t + .5kL\sin\theta}$





Back

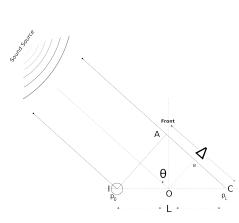
Introduction

Acoustic Head Model

- I Ipsilateral ear, C Contralateral ear.
 p₀, p_L sound pressure on
 eardrums, θ sound source
 direction
- Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_L|$.
- ► Phase difference between sound at both ears $\Delta = kL \sin \theta$.

$$p_0 = pe^{j\omega t - .5kL\sin\theta}$$

$$p_L = pe^{j\omega t + .5kL\sin\theta}$$



Back

Mouth Cavity

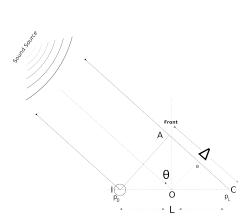
Introduction

Acoustic Head Model

I - Ipsilateral ear, C Contralateral ear.
 p₀, p_L - sound pressure
eardrums, θ - sound sound
discretion

The Model

- Sound source "far away".
- No appreciable amplitude difference, $|p_0| = |p_L|$.
- Phase difference between sound at both ears $\Delta = kL \sin \theta$.
- $p_0 = p e^{j\omega t .5kL\sin\theta}$ $p_L = p e^{j\omega t + .5kL\sin\theta}$



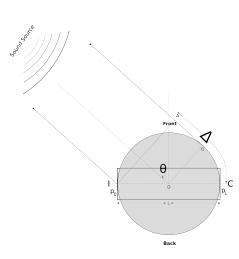


Back

Mouth Cavity

Acoustic Head Model contd.

- $|p_0| = |p_L|.$
- ▶ Increased phase difference due to diffraction $\Delta = 1.5kL\sin\theta$.
- $p_0 = p e^{j\omega t .75kL\sin\theta}$ $p_L = p e^{j\omega t + .75kL\sin\theta}$



Mouth Cavity

Cavity Pressure

3D Wave Equation

$$\frac{1}{c^2}\partial_t^2 p(x,r,\phi,t) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p(x,r,\phi,t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p(x,r,\phi,t)}{\partial \phi^2} + \frac{\partial p(x,r,\phi,t)}{\partial x^2} \tag{1}$$

To be solved using the separation ansatz

$$p(x, r, \phi, t) = f(x)g(r)h(\phi)e^{j\omega t}$$
.

Introduction

$$x$$
- and ϕ - directions

$$\frac{d^2f(x)}{dx^2} + \zeta^2f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$
$$\frac{d^2h(\phi)}{d\phi^2} + q^2h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$$

The Model

00000000

$$p(\phi) = 0 \longrightarrow h(\phi)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial g(r)}{\partial r} \right) + \left[r^2 - \frac{q^2}{r^2} \right]$$

(2)

(3)

where,
$$\nu^2 = k^2 - \zeta^2$$

Separated Equations

Introduction

Internally Coupled Ears

$$d^2f(x)$$

x- and ϕ - directions

$$\frac{d^2 f(x)}{dx^2} + \zeta^2 f(x) = 0 \longrightarrow f(x) = e^{\pm j\zeta x}$$
$$\frac{d^2 h(\phi)}{d\phi^2} + q^2 h(\phi) = 0 \longrightarrow h(\phi) = e^{\pm jq\phi}$$

The Model

00000000

Evaluation

$$\pm jq\phi$$
 (3)

Conclusion

(2)

11/30

 $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial g(r)}{\partial r}\right) + \left[\nu^2 - \frac{q^2}{r^2}\right]g(r) = 0 \longrightarrow g(r) = J_q(\nu r)$ where, $\nu^2 = k^2 - \zeta^2$

(5)

Conclusion

Evaluation

Introduction

Boundary Conditions - ϕ

Smoothness and Continuity in
$$\phi$$
.

The Model

00000000

$$\Rightarrow h(\phi) = \cos q\phi, \ q = 0, 1, 2, \dots$$

The Model

00000000

$$\Rightarrow g(r) = J_q(\nu_{qs}r/a_{cyl}) \tag{7}$$

Evaluation

Boundary Conditions - r

$$\blacktriangleright$$
 $\nu_{\rm qs}$ - zeros of J_a' , $s=0,1,2,\ldots$

$$\nu_{00} = 0$$

Introduction

Mouth Cavity

Conclusion

(6)

The Model

0000000

General Solution

Introduction

Pressure Modes

$$p(x,r,\phi,t) = \sum_{q=0,s=0}^{\infty} \left[A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] p_{qs}(r,\phi) e^{j\omega t} \quad (8)$$

$$p_{
m qs}(r,\phi)=\cos q\phi J_q(
u_{
m qs}r/a_{
m cyl})$$
 where, $\zeta_{
m qs}=\sqrt{k^2-
u_{
m qs}^2/a_{
m cyl}^2}$

Plane Wave Mod

Conclusion

(9)

Conclusion

The Model

0000000

Introduction

Mouth Cavity

Pressure Modes
$$p(x, r, \phi, t) = \sum_{q=0, s=0}^{\infty} \left[A_{qs} e^{j\zeta_{qs}x} + B_{qs} e^{-j\zeta_{qs}x} \right] p_{qs}(r, \phi) e^{j\omega t} \quad (8)$$

$$p_{qs}(r, \phi) = \cos q\phi J_q(\nu_{qs}r/a_{cyl}) \quad (9)$$
where $\zeta_{ro} = \sqrt{k^2 - \nu^2/a^2}$.

$$p_{\rm qs}(r,\phi) = \cos q\phi J_q(\nu_{\rm qs}r/a_{\rm cyl})$$

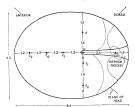
$$\text{where, } \zeta_{\rm qs} = \sqrt{k^2 - \nu_{\rm qs}^2/a_{\rm cyl}^2}$$

$$\text{Plane Wave Mode}$$

$$p_{\rm pw}(x,r,\phi;t) = \left[A_{00}e^{jkx} + B_{00}e^{-jkx}\right]e^{j\omega t}$$
(10)

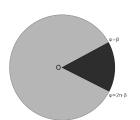
Eardrum

Sketch of a Tokay eardrum as seen from the outside^a.



COL - approximate position opposite the extracolumella insertion.

The ICE eardrum.



Extracolumella (dark) - rigid, stationary.

Tympanum - assumed linear elastic.

Rigidly clamped at the boundaries ($r=a_{\mathrm{tymp}}$ and $\phi=\beta,\ 2\pi-\beta$)

^aG. A. Manley, "The middle ear of the tokay gecko," *Journal of Comparative Physiology*, vol. 81, no. 3, pp. 239–250, 1972

The Model

Membrane Vibrations

Introduction

Membrane EOM

$$-\partial_t^2 u(r,\phi;t) - 2\alpha \partial_t u(r,\phi;t) + c_M^2 \Delta_{(2)} u(r,\phi;t) = \frac{1}{\rho_m d} \Psi(r,\phi;t)$$
(11)

Membrane parameters

$$lpha$$
 - damping coefficient, $\ c_M^2$ - propagation velocity

$$\rho_m$$
 - density, d - thickness.

 $u(r, \phi; t) = f(r)g(\phi)h(t)$

Free-Undamped Membrane, $\alpha \to 0$, $\Psi \to 0$

The Model

$$d^2g(\phi)$$
 , 2.(1)

$$\frac{d^2g(\phi)}{d\phi^2} + \kappa^2 g(\phi) = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f(r)}{\partial r} \right) + \left[\mu^2 - \frac{\kappa^2}{r^2} \right] f(r) = 0$$

 $\frac{d^2h(t)}{dt^2} + c_M^2\mu^2h(t) = 0$

Evaluation

(12)

(13)

(14)

Introduction



The Model

0000000

ction:
$$u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$$

$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$
where, $\kappa = \frac{m\pi}{2(\pi - \beta)}$, $m = 1, 2, 3, ...$ (16)

Evaluation

$$\Rightarrow f(r) = J_{\kappa}(\mu_{\rm mn}r/a_{\rm tymp})$$
 where, $\mu_{\rm mn}$ is the $n^{\rm th}$ zero of J_{κ}

Introduction

Fardrum

Boundary Conditions

Boundary Conditions

Introduction

φ-direction:
$$u(r, \beta; t) = u(r, 2\pi - \beta, t) = 0$$

$$\Rightarrow g(\phi) = \sin \kappa (\phi - \beta)$$
where, $\kappa = \frac{m\pi}{2(\pi - \beta)}, \quad m = 1, 2, 3, ...$
(16)

r-direction:
$$u(a_{\text{tymp}}, \phi; t) = 0$$

The Model

$$\Rightarrow f(r) = J_{\kappa}(\mu_{
m mn} r / a_{
m tymp})$$
 where, $\mu_{
m mn}$ is the $n^{
m th}$ zero of J_{κ}

(17)

Conclusion

4日 → 4 日 → 4 日 → 4 日 → 9 へ ○

 $u_{\rm mn}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{\rm mn} r)$

Evaluation

$$u_{\mathrm{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega_{\mathrm{mn}}t}$$

The Model

0000000

where, $\omega_{\mathrm{mn}} = c_{M} \mu_{\mathrm{mn}}$

Damned membrane

$$\widetilde{u}_{\text{free}}(r,\phi;t) = \sum_{n=0}^{\infty} \widetilde{C}_{\text{mn}} u_{\text{mn}}(r,\phi) e^{j\omega_{\text{mn}}t - \alpha t}$$
 (20)

↓□▶ ↓□▶ ↓□▶ ↓□▶ □ ♥♀○

Introduction

Eardrum

Conclusion

(18)

(19)

m=0, n=1

Eardrum

Free eigenmodes

$$u_{\rm mn}(r,\phi) = \sin \kappa (\phi - \beta) J_{\kappa}(\mu_{\rm mn} r)$$

$$u_{\rm free}(r,\phi;t) = \sum_{m}^{\infty} C_{\rm mn} u_{\rm mn}(r,\phi) e^{j\omega_{\rm mn} t}$$
(18)

where, $\omega_{\mathrm{mn}} = c_{M} \mu_{\mathrm{mn}}$

Damped membrane

$$\widetilde{u}_{\text{free}}(r,\phi;t) = \sum_{m=0,n=1}^{\infty} \widetilde{C}_{\text{mn}} u_{\text{mn}}(r,\phi) e^{j\omega_{\text{mn}}t - \alpha t}$$
 (20)

m=0, n=1

Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

Substitute $u_{\rm ss}$ in Membrane EOM.

The Model

0000000

$$\Omega_{\rm mn} \int dS u_{\rm mn}^2$$

$$\Omega_{\rm mn} = \rho_M d \left[(\omega^2 - \omega_{\rm mn}^2) - 2j\alpha\omega \right]$$

Evaluation

Introduction

Eardrum

Eardrum

Forced Vibrations: $\Psi = pe^{j\omega t}$

Steady State Solution

$$u_{\mathrm{ss}}(r,\phi;t) =: \sum_{m=0,n=1} C_{\mathrm{mn}} u_{\mathrm{mn}}(r,\phi) e^{j\omega t}$$

Substitute u_{ss} in Membrane EOM.

$$egin{aligned} C_{
m mn} &= rac{p \int dS u_{
m mn}}{\Omega_{
m mn} \int dS u_{
m mn}^2} \ \Omega_{
m mn} &=
ho_M d \left[\left(\omega^2 - \omega_{
m mn}^2
ight) - 2 j lpha \omega
ight] \end{aligned}$$

4日 → 4周 → 4 差 → 4 差 → 2 ● 99℃

(21)

(22)

Forced Vibrations contd.

Transient Solution

Same as the solution for a free damped membrane

$$u_{
m t}(r,\phi;t)=\sum_{m=0,n=1}^{\infty}\widetilde{C}_{
m mn}u_{
m mn}(r,\phi)e^{j\omega_{
m mn}t-\alpha t}$$

 $\widetilde{C}_{
m mn}$ determined from the membrane displacement at t=0. $u_{
m t} o 0$ exponentially as $t o \infty$.

Steady State Approximation

$$u \approx u_{\rm ss}$$
 if α is "large"

(23)

Transient Solution

Same as the solution for a free damped membrane

$$u_{
m t}(r,\phi;t)=\sum_{m=0,n=1}^{\infty}\widetilde{C}_{
m mn}u_{
m mn}(r,\phi)e^{j\omega_{
m mn}t-\alpha t}$$
 where the membrane displacement at $t=0$

 $\widetilde{C}_{\mathrm{mn}}$ determined from the membrane displacement at t=0. $u_{\mathrm{t}} \to 0$ exponentially as $t \to \infty$.

Steady State Approximation

$$u \approx u_{\rm ss}$$
 if α is "large".

(23)

$u_{0/L} = \sum \Omega_{\rm mn} C_{\rm mn}^{0/L} u_{\rm mn}(r,\phi) e^{j\omega t}$

The Model

0000

$$m=0, n=1$$

Evaluation

$$\sum_{m}^{\infty} \Omega_{mn} C_{mn}^{L} u_{mn}(r, \phi) e^{j\omega t} = p_{L} e^{j\omega t} - p(L, r, \phi; t)$$

(24)

Conclusion

Coupled Membranes

$$\infty$$

$$m=0, n=1$$

$$m=0, n=1$$

Introduction

Coupled Membranes

Coupled Membranes

Coupled Membranes

$$u_{0/L} = \sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^{0/L} u_{\rm mn}(r,\phi) e^{j\omega t}$$
 (24)

Membrane Equations

m = 0, n = 1

$$\sum_{m=0,n=1}^{\infty} \Omega_{\rm mn} C_{\rm mn}^{0} u_{\rm mn}(r,\phi) e^{j\omega t} = p_{0} e^{j\omega t} - p(0,r,\phi;t)$$
 (25)

$$\sum_{m}^{\infty} \Omega_{mn} C_{mn}^{L} u_{mn}(r,\phi) e^{j\omega t} = p_{L} e^{j\omega t} - p(L,r,\phi;t)$$
 (26)

4 D > 4 D > 4 E > 4 E > E 990

22/30

Coupled Membranes

"Surface" Velocity

$$U_{0/L} = \begin{cases} u_{0/L}, \ 0 < r < a_{\text{tymp}} \ \text{and} \ \beta < \phi < 2\pi - \beta \\ 0, \ \text{otherwise} \end{cases}$$
 (27)

Velocity in *x*—direction

$$v_{x} = -\sum_{q=0,s=0}^{\infty} \frac{\zeta_{qs}}{\rho\omega} \left(A_{qs} e^{j\zeta_{qs}x} - B_{qs} e^{-j\zeta_{qs}x} \right) p_{qs}(r,\phi) e^{j\omega t}$$
 (28)

The Model

0000

$$U_0 = \frac{1}{2} v_1(0, r, \phi; t) \tag{20}$$

Evaluation

$$U_0 = -\frac{1}{j\omega} v_x(0, r, \phi; t)$$

$$U_L = \frac{1}{j\omega} v_x(L, r, \phi; t)$$
(29)

Introduction

Coupled Membranes

Boundary Conditions

Introduction

Coupled Membranes

$$U_0 = -rac{1}{j\omega}v_x(0,r,\phi;t)$$
 $U_L = rac{1}{j\omega}v_x(L,r,\phi;t)$

$$U_{0/L} = S^{0/L}(t) =: \int dS U_{0/L}$$

The Model

0000

Conclusion

◆□▶ ◆□▶ ◆豆▶ ◆豆▶ ・豆・ 夕♀@

Coupled Membranes

Boundary Conditions

Higher pressure modes disappear, i.e.

$$p = \left[A_{00} e^{jkx} + B_{00} e^{-jkx} \right] e^{j\omega t}$$

Outline for Section 3

Introduction
Auditory Systems

Hearing Cues

The Model

Mouth Cavity
Acoustic Head Mode

Fordrum

Model

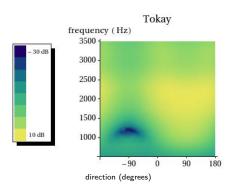
Coupled Membranes

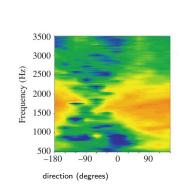
Evaluation

Vibration Amplitude Internal Level Difference Internal Amplitude Difference

Vibration Amplitude

Vibration Amplitude





The Model

Outline for Section 4

Introduction

Introduction

Hearing Cues

The Mode

Mouth Cavity
Acoustic Head Mode

Eardrun

Model

Evoluation

Evaluation

valuation

Vibration Amplitude
Internal Level Difference
Internal Amplitude

Difference

Conclusion

The Model

Introduction

Conclusion

《□》《圖》《意》《意》 [] []

Conclusion

Introduction

